# Assignment 5: Dependency Theory

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**D1 Reasoning with dependencies**

1.
   - Assume $AB \longrightarrow C, A \longrightarrow D$ and $CD \longrightarrow EF$
   - Apply Augmentation on $AB \longrightarrow C$ with $A$ to derive $AB \longrightarrow CA$
   - Apply Augmentation on $A \longrightarrow D$ with $C$ to dervie $CA \longrightarrow CD$
   - Apply Transitivity on $AB \longrightarrow CA$ and $CA \longrightarrow CD$ to derive $AB \longrightarrow CD$
   - Apply Transitivity on $AB \longrightarrow CD$ and $CD \longrightarrow EF$ to derive $AB \longrightarrow EF$
   - Apply Reflection on $F \subseteq EF$ to derive $EF \longrightarrow F$
   - Apply Transitivity on $AB \longrightarrow EF$ and $EF \longrightarrow F$ to derive $AB \longrightarrow F$
   - Hence, $AB \longrightarrow F$

2.
   - Assume we have rows $r_1, r_2 \in I$ of instance $I$ such that $r_1[XW] = r_2[XW]$
   - By $r_1[XW] = r_2[XW]$, we have $r_1[X] = r_2[X]$ and $r_1[W] = r_2[W]$
   - Using $X \longrightarrow Y$ and $r_1[X] = r_2[X]$, we conclude that $r_1[Y] = r_2[Y]$
   - By $r_1[Y] = r_2[Y]$ and $r_1[W] = r_2[W]$, we have $r_1[YW] = r_2[YW]$
   - Using $YW \longrightarrow Z$ and $r_1[YW] = r_2[YW]$, we conclude that $r_1[Z] = r_2[Z]$

- Hence, $r_1[Z] = r_2[Z]$ holds.

3.
- Assume we have $r_1[X]$ for every instance $I_1$ of $R$ and every row $r_1 \in I_1$
- Using $R[X] \subseteq S[Y]$ and $r_1[X]$, there exists a row in instance $I_2$ of $S$ with $r_1[X] = r_2[X]$
- Using $S[Y] \subseteq T[Z]$ and $r_2[Y]$, there exists a row in instance $I_3$ of $T$ with $r_2[Y] = r_3[Z]$
- Thus, for every instance $I_1$ of $R$ and every row $r_1 \in R$, there exists a row in instance $I_3$ of $T$ such that $r_1[X] = r_3[Z]$
- Hence, $R[X] \subseteq T[Z]$

4.
- Assume $X \twoheadrightarrow Y$ and $XY \longrightarrow Z$
- Apply Complementation on $X \twoheadrightarrow Y$ to derive $X \twoheadrightarrow Z$ (with $Z$ all attributes of $\mathbf{R}$ not in $X$ and $Y$)
- Apply Reflexivity on $Z \setminus (X \cup Y) \subseteq Z$ to derive $Z \longrightarrow Z \setminus (X \cup Y)$
- Apply Transitivity on $XY \longrightarrow Z$ and $Z \longrightarrow Z \setminus (X \cup Y)$ to derive $XY \longrightarrow Z \setminus (X \cup Y)$
- Since $Z$ is all attributes of $\mathbf{R}$ not in $X$ and $Y$, $Z \cap XY = \emptyset$
- Apply Coalensence on $X \twoheadrightarrow Z$, $XY \longrightarrow Z \setminus (X \cup Y)$, $Z \cap XY = \emptyset$ and $Z \setminus (X \cup Y) \subseteq Z$, we conclude that $X \longrightarrow Z \setminus (X \cup Y)$
- Hence, $X \longrightarrow Z \setminus (X \cup Y)$

5. Consider the following table of the schema

**person**(name, number, birthdate, age)

| **name** | **number** | **birthdate** | **age** |
|---|---|---|---|
| Alice | 1 | 2001-01-01 | 20 |
| Alice | 2 | 2005-09-05 | 16 |

Since name, number are the primary keys, they determine all attributes. Thus, "name, number $\longrightarrow$ birthdate".
We know from the lecture that birthdate determines age since people who have the same birthdate would have the same age, and so "birthdate $\longrightarrow$ age".
Since {birthdate} $\subseteq$ {name, birthdate}, by applying Reflexivity, we get "name, birthdate $\longrightarrow$ birthdate".
Applying Transitivity on "name, birthdate $\longrightarrow$ birthdate" and "birthdate $\longrightarrow$ age", we derive "name, birthdate $\longrightarrow$ age"
Hence, we have "name, number $\longrightarrow$ birthdate" and "name, birthdate $\longrightarrow$ age".
However, from the table, we can see that "name $\longrightarrow$ age" does not hold.
Let $X$ = name, $W$ = number, $Y$ = birthdate, $Z$ = age, we have shown that the inference rule from the question is not sound.

6. TODO: Do Question 6

7. The attribute closure of set of attributes $C$:

- Initially, $closure = \{C\}$
- From $C \longrightarrow A$, since $C \subseteq closure$ and $A \not\subseteq closure$, $closure = \{C, A\}$
- From $AC \longrightarrow E$, since $AC \subseteq closure$ and $E \not\subseteq closure$, $closure = \{C, A, E\}$
- From $E \longrightarrow B$, since $E \subseteq closure$ and $B \not\subseteq closure$, $closure = \{C, A, E, B\}$
- From $AB \longrightarrow D$, since $AB \subseteq closure$ and $D \not\subseteq closure$, $closure = \{C, A, E, B, D\}$
- From $BC \longrightarrow D$, since $BC \subseteq closure$ and $D \subseteq closure$, we don't need to add $D$ into $closure$ more (as $D$ is already in $closure$)
- From $D \longrightarrow A$, since $D \subseteq closure$ and $A \subseteq closure$, we don't need to add $A$ into $closure$ more (as $A$ is already in $closure$)
- Therefore, $C^+ = \{A, B, C, D, E\}$

The attribute closure of set of attributes $(EA)$:

- Initially, $closure = \{E, A\}$
- From $E \longrightarrow B$, since $E \subseteq closure$ and $B \not\subseteq closure$, $closure = \{E, A, B\}$
- From $AB \longrightarrow D$, since $AB \subseteq closure$ and $D \not\subseteq closure$, $closure = \{E, A, B, D\}$
- From $AC \longrightarrow E$, since $AC \not\subseteq closure$, we don't need to add $E$ into $closure$
- From $BC \longrightarrow D$, since $BC \not\subseteq closure$, we don't need to add $D$ into $closure$
- From $C \longrightarrow A$, since $C \not\subseteq closure$, we don't need to add $A$ into $closure$
- From $D \longrightarrow A$, since $D \subseteq closure$ and $A \subseteq closure$, we don't need to add $A$ into $closure$ more (as $A$ is already in $closure$)
- Therefore, $(EA)^+ = \{A, B, D, E\}$

8. Compute $X^+$ for every $X \subseteq \{A, B, C, D, E\}$

- $A^+ = \{A\}$ since there is no $A \longrightarrow \ldots$ in $\mathfrak{S}$
- $B^+ = \{B\}$ since there is no $B \longrightarrow \ldots$ in $\mathfrak{S}$
- $C^+ = \{A, B, C, D, E\}$ from question 7
- $D^+ = \{A, D\}$ since there is only $D \longrightarrow A$ that would satisfy the closure algorithm
- $E^+ = \{B, E\}$ since there is only $E \longrightarrow B$ that would satisfy the closure algorithm
- $(AB)^+ = \{A, B, D\}$ since there is only $AB \longrightarrow D$ that would satisfy the closure algorithm
- $(AC)^+ = \{A, B, C, D, E\}$ since $AC$ includes $C$ and we can reach others from only $C$
- $(AD)^+ = \{A, D\}$ since there is no dependency that would satisfy the closure algorithm

- $(AE)^+ = \{A, B, E\}$ since there is only $E \longrightarrow B$ that would satisfy the closure algorithm
- $(BC)^+ = \{A, B, C, D, E\}$ since $BC$ includes $C$ and we can reach others from only $C$
- $(BD)^+ = \{A, B, D\}$ since there is only $D \longrightarrow A$ that would satisfy the closure algorithm
- $(BE)^+ = \{B, E\}$ since there is no dependency that would satisfy the closure algorithm
- Any combination that contains $C$ is the superkey
- $C$ is the (candidate) key

9.

- Starting with $\{AB \longrightarrow D, AC \longrightarrow E, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B\}$
- From $C \longrightarrow A$, we can use Augmentation with $C$ to derive $C \longrightarrow AC$. We also have $AC \longrightarrow E$. Then, by Transitivity, we conclude $C \longrightarrow E$. Thus, we can add $C \longrightarrow E$ to our set.
  $\{AB \longrightarrow D, AC \longrightarrow E, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E\}$
- From $C \longrightarrow E$, we can use Augmentation with $A$ to derive $AC \longrightarrow AE$. Then, we can use Decomposition to get $AC \longrightarrow A$ and $AC \longrightarrow E$. $AC \longrightarrow A$ is trivial since $A \subseteq AC$. This means that, from $C \longrightarrow E$, we can get $AC \longrightarrow E$. So, we can get rid of $AC \longrightarrow E$.
  $\{AB \longrightarrow D, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E\}$
- From $C \longrightarrow E$ and $E \longrightarrow B$, by Transitivity, we get $C \longrightarrow B$. Then, we get apply Augmentation on $C \longrightarrow B$ with $C$ to derive $C \longrightarrow BC$. We also have $BC \longrightarrow D$. So, by Transitivity, we derive $C \longrightarrow D$. Thus, we can add $C \longrightarrow D$ to our set.
  $\{AB \longrightarrow D, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E, C \longrightarrow D\}$
- From $C \longrightarrow D$, we can apply Augmentation with $B$ to get $BC \longrightarrow BD$. Then, we can use Decomposition to get $BC \longrightarrow B$ and

$BC \longrightarrow D$. $BC \longrightarrow B$ is trivial since $B \subseteq BC$. This means that from $C \longrightarrow D$, we can get $BC \longrightarrow D$. So, we can get rid of $BC \longrightarrow D$.
$\{AB \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E, C \longrightarrow D\}$

- By Transitivity on $C \longrightarrow D$ and $D \longrightarrow A$, we can derive $C \longrightarrow A$. Thus, we can get rid of $C \longrightarrow A$.
  $\{AB \longrightarrow D, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E, C \longrightarrow D\}$