Assignment 5: Dependency Theory

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D1 Reasoning with dependencies

1.
- Assume $AB \longrightarrow C, A \longrightarrow D$ and $CD \longrightarrow EF$
- Apply Augmentation on $AB \longrightarrow C$ with A to derive $AB \longrightarrow CA$
- Apply Augmentation on $A \longrightarrow D$ with C to dervie $CA \longrightarrow CD$
- Apply Transitivity on $AB \longrightarrow CA$ and $CA \longrightarrow CD$ to derive $AB \longrightarrow CD$
- Apply Transitivity on $AB \longrightarrow CD$ and $CD \longrightarrow EF$ to derive $AB \longrightarrow EF$
- Apply Reflection on $F \subseteq EF$ to derive $EF \longrightarrow F$
- Apply Transitivity on $AB \longrightarrow EF$ and $EF \longrightarrow F$ to derive $AB \longrightarrow F$
- Hence, $AB \longrightarrow F$

2.
- Assume we have rows $r_1, r_2 \in I$ of instance I such that $r_1[XW] = r_2[XW]$
- By $r_1[XW] = r_2[XW]$, we have $r_1[X] = r_2[X]$ and $r_1[W] = r_2[W]$
- Using $X \longrightarrow Y$ and $r_1[X] = r_2[X]$, we conclude that $r_1[Y] = r_2[Y]$
- By $r_1[Y] = r_2[Y]$ and $r_1[W] = r_2[W]$, we have $r_1[YW] = r_2[YW]$
- Using $YW \longrightarrow Z$ and $r_1[YW] = r_2[YW]$, we conclude that $r_1[Z] = r_2[Z]$

- Hence, $r_1[Z] = r_2[Z]$ holds.

3. - Assume we have $r_1[X]$ for every instance I_1 of R and every row $r_1 \in I_1$
   - Using $R[X] \subseteq S[Y]$ and $r_1[X]$, there exists a row in instance I_2 of S with $r_1[X] = r_2[X]$
   - Using $S[Y] \subseteq T[Z]$ and $r_2[Y]$, there exists a row in instance I_3 of T with $r_2[Y] = r_3[Z]$
   - Thus, for every instance I_1 of R and every row $r_1 \in R$, there exists a row in instance I_3 of T such that $r_1[X] = r_3[Z]$
   - Hence, $R[X] \subseteq T[Z]$

4. - Assume $X \twoheadrightarrow Y$ and $XY \longrightarrow Z$
   - Apply Complementation on $X \twoheadrightarrow Y$ to derive $X \twoheadrightarrow Z$ (with Z all attributes of **R** not in X and Y)
   - Apply Reflexivity on $Z \setminus (X \cup Y) \subseteq Z$ to derive $Z \longrightarrow Z \setminus (X \cup Y)$
   - Apply Transitivity on $XY \longrightarrow Z$ and $Z \longrightarrow Z \setminus (X \cup Y)$ to derive $XY \longrightarrow Z \setminus (X \cup Y)$
   - Since Z is all attributes of **R** not in X and Y, $Z \cap XY = \emptyset$
   - Apply Coalensence on $X \twoheadrightarrow Z$, $XY \longrightarrow Z \setminus (X \cup Y)$, $Z \cap XY = \emptyset$ and $Z \setminus (X \cup Y) \subseteq Z$, we conclude that $X \longrightarrow Z \setminus (X \cup Y)$
   - Hence, $X \longrightarrow Z \setminus (X \cup Y)$

5. Consider the following table of the schema

person(<u>name</u>, <u>number</u>, birthdate, age)

name	**number**	**birthdate**	**age**
Alice	1	2001-01-01	20
Alice	2	2005-09-05	16

Since <u>name</u>, <u>number</u> are the primary keys, they determine all attributes. Thus, "name, number $\longrightarrow$ birthdate".
We know from the lecture that birthdate determines age since people who have the same birthdate would have the same age, and so "birthdate $\longrightarrow$ age".
Since {birthdate} $\subseteq$ {name, birthdate}, by applying Reflexivity, we get "name, birthdate $\longrightarrow$ birthdate".
Applying Transitivity on "name, birthdate $\longrightarrow$ birthdate" and "birthdate $\longrightarrow$ age", we derive "name, birthdate $\longrightarrow$ age"
Hence, we have "name, number $\longrightarrow$ birthdate" and "name, birthdate $\longrightarrow$ age".
However, from the table, we can see that "name $\longrightarrow$ age" does not hold.
Let X = name, W = number, Y = birthdate, Z = age, we have shown that the inference rule from the question is not sound.

6. TODO: Do Question 6

7. The attribute closure of set of attributes C:

- Initially, $closure = \{C\}$
- From $C \longrightarrow A$, since $C \subseteq closure$ and $A \not\subseteq closure$, $closure = \{C, A\}$
- From $AC \longrightarrow E$, since $AC \subseteq closure$ and $E \not\subseteq closure$, $closure = \{C, A, E\}$
- From $E \longrightarrow B$, since $E \subseteq closure$ and $B \not\subseteq closure$, $closure = \{C, A, E, B\}$
- From $AB \longrightarrow D$, since $AB \subseteq closure$ and $D \not\subseteq closure$, $closure = \{C, A, E, B, D\}$
- From $BC \longrightarrow D$, since $BC \subseteq closure$ and $D \subseteq closure$, we don't need to add D into $closure$ more (as D is already in $closure$)
- From $D \longrightarrow A$, since $D \subseteq closure$ and $A \subseteq closure$, we don't need to add A into $closure$ more (as A is already in $closure$)
- Therefore, $C^+ = \{A, B, C, D, E\}$

The attribute closure of set of attributes (EA):

- Initially, $closure = \{E, A\}$
- From $E \longrightarrow B$, since $E \subseteq closure$ and $B \not\subseteq closure$, $closure = \{E, A, B\}$
- From $AB \longrightarrow D$, since $AB \subseteq closure$ and $D \not\subseteq closure$, $closure = \{E, A, B, D\}$
- From $AC \longrightarrow E$, since $AC \not\subseteq closure$, we don't need to add E into $closure$
- From $BC \longrightarrow D$, since $BC \not\subseteq closure$, we don't need to add D into $closure$
- From $C \longrightarrow A$, since $C \not\subseteq closure$, we don't need to add A into $closure$
- From $D \longrightarrow A$, since $D \subseteq closure$ and $A \subseteq closure$, we don't need to add A into $closure$ more (as A is already in $closure$)
- Therefore, $(EA)^+ = \{A, B, D, E\}$

8. Compute X^+ for every $X \subseteq \{A, B, C, D, E\}$

   - $A^+ = \{A\}$ since there is no $A \longrightarrow \ldots$ in $\mathfrak{S}$
   - $B^+ = \{B\}$ since there is no $B \longrightarrow \ldots$ in $\mathfrak{S}$
   - $C^+ = \{A, B, C, D, E\}$ from question 7
   - $D^+ = \{A, D\}$ since there is only $D \longrightarrow A$ that would satisfy the closure algorithm
   - $E^+ = \{B, E\}$ since there is only $E \longrightarrow B$ that would satisfy the closure algorithm

9. - starting with $\{AB \longrightarrow D, AC \longrightarrow E, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B\}$

- From $C \longrightarrow A$, we can use Augmentation with C to derive $C \longrightarrow AC$. We also have $AC \longrightarrow E$. Then, by Transitivity, we conclude $C \longrightarrow E$. Thus, we can add $C \longrightarrow E$ to our set.
  $\{AB \longrightarrow D, AC \longrightarrow E, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E\}$
- From $C \longrightarrow E$, we can use Augmentation with A to derive $AC \longrightarrow AE$. Then, we can use Decomposition to get $AC \longrightarrow A$ and $AC \longrightarrow E$. $AC \longrightarrow A$ is trivial since $A \subseteq AC$. This means that, from $C \longrightarrow E$, we can get $AC \longrightarrow E$. So, we can get rid of $AC \longrightarrow E$.
  $\{AB \longrightarrow D, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E\}$
- From $C \longrightarrow E$ and $E \longrightarrow B$, by Transitivity, we get $C \longrightarrow B$. Then, we get apply Augmentation on $C \longrightarrow B$ with C to derive $C \longrightarrow BC$. We also have $BC \longrightarrow D$. So, by Transitivity, we derive $C \longrightarrow D$. Thus, we can add $C \longrightarrow D$ to our set.
  $\{AB \longrightarrow D, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E, C \longrightarrow D\}$
- From $C \longrightarrow D$, we can apply Augmentation with B to get $BC \longrightarrow BD$. Then, we can use Decomposition to get $BC \longrightarrow B$ and $BC \longrightarrow D$. $BC \longrightarrow B$ is trivial since $B \subseteq BC$. This means that from $C \longrightarrow D$, we can get $BC \longrightarrow D$. So, we can get rid of $BC \longrightarrow D$.
  $\{AB \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B, C \longrightarrow E, C \longrightarrow D\}$