Assignment 5: Dependency Theory

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D1 Reasoning with dependencies

1.
   - Assume $AB \longrightarrow C, A \longrightarrow D$ and $CD \longrightarrow EF$
   - Apply Augmentation on $AB \longrightarrow C$ with A to derive $AB \longrightarrow CA$
   - Apply Augmentation on $A \longrightarrow D$ with C to dervie $CA \longrightarrow CD$
   - Apply Transitivity on $AB \longrightarrow CA$ and $CA \longrightarrow CD$ to derive $AB \longrightarrow CD$
   - Apply Transitivity on $AB \longrightarrow CD$ and $CD \longrightarrow EF$ to derive $AB \longrightarrow EF$
   - Apply Reflection on $F \subseteq EF$ to derive $EF \longrightarrow F$
   - Apply Transitivity on $AB \longrightarrow EF$ and $EF \longrightarrow F$ to derive $AB \longrightarrow F$
   - Hence, $AB \longrightarrow F$

2.
   - Assume we have rows $r_1, r_2 \in I$ of instance I such that $r_1[XW] = r_2[XW]$
   - By $r_1[XW] = r_2[XW]$, we have $r_1[X] = r_2[X]$ and $r_1[W] = r_2[W]$
   - Using $X \longrightarrow Y$ and $r_1[X] = r_2[X]$, we conclude that $r_1[Y] = r_2[Y]$
   - By $r_1[Y] = r_2[Y]$ and $r_1[W] = r_2[W]$, we have $r_1[YW] = r_2[YW]$
   - Using $YW \longrightarrow Z$ and $r_1[YW] = r_2[YW]$, we conclude that $r_1[Z] = r_2[Z]$
   - Hence, $r_1[Z] = r_2[Z]$ holds.

3.
   - Assume we have $r_1[X]$ for every instance I_1 of R and every row $r_1 \in I_1$
   - Using $R[X] \subseteq S[Y]$ and $r_1[X]$, there exists a row in instance I_2 of S with $r_1[X] = r_2[X]$
   - Using $S[Y] \subseteq T[Z]$ and $r_2[Y]$, there exists a row in instance I_3 of T with $r_2[Y] = r_3[Z]$

- Thus, for every instance I_1 of R and every row $r_1 \in R$, there exists a row in instance I_3 of T such that $r_1[X] = r_3[Z]$
- Hence, $R[X] \subseteq T[Z]$