

Assignment 6: Decomposition and Normal Forms

Hien Tu - tun1

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Part 1: The analysis of a quick-event wizard for a local community

1. Minimal cover of all realistic non-trivial functional dependencies:

- Since each event is organized by a user, we know that if the event id (*id*) is the same, then the organizer (*user_id*) must be the same. So, we can have $id \rightarrow user_id$. The reverse would not hold since an organizer could organize many events.
- Furthermore, each event happens in a day, so, we know that if the event id (*id*) is the same, the date that the event happens in (*date*) must be the same. So, we can have $id \rightarrow date$.
- We also have $id, inv_id \rightarrow inv_confirmed$ since each guest would determine what event to go to. We need both the event id (*id*) and the guest (*inv_id*) to determine if the invitation is confirmed (*inv_confirmed*).
- Furthermore, we have $id, product \rightarrow p_amount$ since we would only know how much to bring if we know what to bring and where we need to bring it to. Only the product wouldn't be able to determine the amount since different event could need different amount of the same product. For example, in the given table, event 1 needs 4 chips while event 2 only needs 2 chips.
- We have $product \rightarrow p_price$ since each product has its own price. The price of the product wouldn't depend on the event that the product is brought into. An example is that, in the given table,

the chips cost \$2 no matter if it is brought into event with id 1 or 2 and the cola would cost \$4 no matter if it is brought into event with id 1 or 2.

2. An example of a non-trivial dependency is $id \rightarrow inv_id, inv_confirmed$. This would hold since, for example, in the given table, from the first row and the fourth row, we know that there would exist a row where inv_id and $inv_confirmed$ are the same as the first row and the rest of the attributes ($user_id$, $date$, $product$, p_price , p_amount) are the same as the fourth row. This row is the third row. We also know that there would exist a row where inv_id and $inv_confirmed$ are the same as the fourth row and the rest of the attributes ($user_id$, $date$, $product$, p_price , p_amount) are the same as the first row. This row is the second row.

Part 2: Refinement of an order-table for a cinema chain

We will use the short hand notation for each attribute for brevity.

3. The relational schema is not in 3NF since $I \rightarrow St, Si, Ss, Sd$ would violate the 3NF property since $\{St, Si, Ss, Sd\} \not\subseteq I$, I is not a (super)key and each attribute in $\{St, Si, Ss, Sd\} \setminus I = \{St, Si, Ss, Sd\}$ is not part of a key.

To decompose, first, we need to compute the minimal cover of the functional dependencies

- We can use Decomposition on the first three functional dependencies to get
 $\{I \rightarrow St; I \rightarrow Si; I \rightarrow Ss; I \rightarrow Sd; I \rightarrow Fi; I \rightarrow Fl; I \rightarrow Fs; I \rightarrow Ri; I \rightarrow Rs; Si \rightarrow Si; Si \rightarrow Ss; Si \rightarrow Sd; Ss \rightarrow Sd; Sd \rightarrow Ss; St, Ri \rightarrow Fi; Fi \rightarrow Fl; Fi, Si \rightarrow Fs; Ri \rightarrow Rs\}$
- From Reflexivity, since $Si \subseteq Si$, we can get $Si \rightarrow Si$. So, we don't need to include $Si \rightarrow Si$ in the minimal cover.
 By Transitivity on $I \rightarrow Si$ and $Si \rightarrow Ss$, we can get $I \rightarrow Ss$. So, we don't need $I \rightarrow Ss$.
 By Transitivity on $I \rightarrow Si$ and $Si \rightarrow Sd$, we can get $I \rightarrow Sd$.

So, we don't need $I \rightarrow Sd$.

By Union on $I \rightarrow St$ and $I \rightarrow Ri$; we can get $I \rightarrow St, Ri$. By Transitivity on $I \rightarrow St, Ri$ and $St, Ri \rightarrow Fi$; we can get $I \rightarrow Fi$. So we don't need $I \rightarrow Fi$.

By Transitivity on $I \rightarrow Fi$ (that we just proved) and $Fi \rightarrow Fl$, we can get $I \rightarrow Fl$. So, we don't need $I \rightarrow Fl$.

By Union on $I \rightarrow Fi$ and $I \rightarrow Si$; we can get $I \rightarrow Fi, Si$. By Transitivity on $I \rightarrow Fi, Si$ and $Fi, Si \rightarrow Fs$, we can get $I \rightarrow Fs$. So, we don't need $I \rightarrow Fs$.

By Transitivity on $Si \rightarrow Ss$ and $Ss \rightarrow Sd$, we can get $Si \rightarrow Sd$. So, we don't need $Si \rightarrow Sd$.

Thus, we would get

$\{I \rightarrow St; I \rightarrow Si; I \rightarrow Ri; Si \rightarrow Ss; Ss \rightarrow Sd; Sd \rightarrow Ss; St, Ri \rightarrow Fi; Fi \rightarrow Fl; Fi, Si \rightarrow Fs; Ri \rightarrow Rs\}$

- Notice that the set above is already minimal. So it is the minimal cover.

Starting the algorithm, we get know

- $result = \{\}$
- $cover = \{I \rightarrow St; I \rightarrow Si; I \rightarrow Ri; Si \rightarrow Ss; Ss \rightarrow Sd; Sd \rightarrow Ss; St, Ri \rightarrow Fi; Fi \rightarrow Fl; Fi, Si \rightarrow Fs; Ri \rightarrow Rs\}$

First, we have $I \rightarrow St$

- Since we have $I \rightarrow St, I \rightarrow Si, I \rightarrow Ri \in cover$, we get $B = \{St, Si, Ri\}$
- So, $result = \{(I, St, Si, Ri)\}$

Next, we have $Si \rightarrow Ss$

- Since only $Si \rightarrow Ss \in cover$ that starts with Si , $B = \{Ss\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss)\}$

Then, we have $Ss \rightarrow Sd$

- Since only $Ss \rightarrow Sd \in cover$ that starts with Ss , $B = \{Sd\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd)\}$

Then, we have $Sd \longrightarrow Ss$

- Since only $Sd \longrightarrow Ss \in cover$ that starts with Sd , $B = \{Ss\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (Sd, Ss)\}$

Next, we have $St, Ri \longrightarrow Fi$

- Since only $St, Ri \longrightarrow Fi \in cover$ that starts with St, Ri ; $B = \{Fi\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (Sd, Ss), (St, Ri, Fi)\}$

Next, we have $Fi \longrightarrow Fl$

- Since only $Fi \longrightarrow Fl \in cover$ that starts with Fi , $B = \{Fl\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (Sd, Ss), (St, Ri, Fi), (Fi, Fl)\}$

Next, we have $Fi, Si \longrightarrow Fs$

- Since only $Fi, Si \longrightarrow Fs \in cover$ that starts with Fi, Si ; $B = \{Fs\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (Sd, Ss), (St, Ri, Fi), (Fi, Fl), (Fi, Si, Fs)\}$

Finally, we have $Ri \longrightarrow Rs$

- Since only $Ri \longrightarrow Rs \in cover$ that starts with Ri , $B = \{Rs\}$
- So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (Sd, Ss), (St, Ri, Fi), (Fi, Fl), (Fi, Si, Fs), (Ri, Rs)\}$

Since $result$ doesn't contain the key, which is "I, P, Rp", we need to add it to $result$. So, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (Sd, Ss), (St, Ri, Fi), (Fi, Fl), (Fi, Si, Fs), (Ri, Rs), (I, P, Rp)\}$

Since $(Sd, Ss) \subseteq (Ss, Sd)$, we will remove (Sd, Ss) from $result$. Thus, $result = \{(I, St, Si, Ri), (Si, Ss), (Ss, Sd), (St, Ri, Fi), (Fi, Fl), (Fi, Si, Fs), (Ri, Rs), (I, P, Rp)\}$

Minimal cover for each relational schema of the resulting decomposition

- (I, St, Si, Ri)
 $\implies \{I \longrightarrow St; I \longrightarrow Si, I \longrightarrow Ri\}$

- (Si, Ss)
 $\implies \{\text{Si} \longrightarrow \text{Ss}\}$
- (Ss, Sd)
 $\implies \{\text{Ss} \longrightarrow \text{Sd}\}$
- (St, Ri, Fi)
 $\implies \{\text{St}, \text{Ri} \longrightarrow \text{Fi}\}$
- (Fi, Fl)
 $\implies \{\text{Fi} \longrightarrow \text{Fl}\}$
- (Fi, Si, Fs)
 $\implies \{\text{Fi}, \text{Si} \longrightarrow \text{Fs}\}$
- (Ri, Rs)
 $\implies \{\text{Ri} \longrightarrow \text{Rs}\}$

From the minimal covers of the relational schemas of the resulting decomposition, we can see that the decomposition is dependency-preserving. It is also proved in the lecture that Decompose-3NF is dependency-preserving.

Furthermore, it is proved in the lecture that Decompose-3NF is also lossless-join. Thus, this decomposition should be lossless-join as well.

Decompose the example dataset

- (I, St, Si, Ri)

I	St	Si	Ri
1	Nov. 1, 1pm	1	7
2	Nov. 1, 1pm	2	7
3	Nov. 7, 2pm	2	3

- (Si, Ss)

Si	Ss
1	Oct. 1
2	Oct. 3

- (Ss, Sd)

Ss	Sd
Oct. 1	31
Oct. 3	29

- (St, Ri, Fi)

St	Ri	Fi
Nov. 1, 1pm	7	5
Nov. 7, 2pm	3	9

- (Fi, Fl)

Fi	Fl
5	120
9	99

- (Fi, Si, Fs)

Fi	Si	Fs
5	1	great
5	2	awful
9	2	not-scored

- (Ri, Rs)

Ri	Rs
7	medium
3	large

- The relational schema is not in BCNF since $I \longrightarrow St, Si, Ss, Sd \in \mathfrak{S}^+$ but I is not a (super)key.

DECOMPOSE-BCNF:

Since $I \longrightarrow St, Si, Ss, Sd \in \mathfrak{S}^+$ violates the BCNF constraint since I is not the (super)key, then

- $\mathbf{R}_1 = I^+ = \{I, St, Si, Ss, Sd, Fi, Fl, Fs, Ri, Rs\}$
- $\mathbf{R}_2 = I \cup Z = \{I, P, Rp\}$
- Note that $\{I, P, Rp\}$ satisfies the BCNF constraint, so they would be part of the resulting decomposition.

For \mathbf{R}_1 , since $Si \longrightarrow Ss, Sd \in \mathfrak{S}_1^+$ violates the BCNF constraint since Si is not the (super)key, then

- $\mathbf{R}_{1,1} = (\text{Si})^+ = \{\text{Si}, \text{Ss}, \text{Sd}\}$
- $\mathbf{R}_{1,2} = \text{Si} \cup \text{Z} = \{\text{Si}, \text{I}, \text{St}, \text{Fi}, \text{Fl}, \text{Fs}, \text{Ri}, \text{Rs}\}$

For $\mathbf{R}_{1,1}$, since $\text{Ss} \longrightarrow \text{Sd} \in \mathfrak{S}_{1,1}^+$ violates the BCNF constraint, then

- $\mathbf{R}_{1,1,1} = (\text{Ss})^+ = \{\text{Ss}, \text{Sd}\}$
- $\mathbf{R}_{1,1,2} = \text{Ss} \cup \text{Z} = \{\text{Ss}, \text{Si}\}$
- Note that $\{\text{Ss}, \text{Sd}\}$ and $\{\text{Ss}, \text{Si}\}$ satisfies BCNF, so they would be part of the resulting decomposition.

For $\mathbf{R}_{1,2}$, since $\text{St}, \text{Ri} \longrightarrow \text{Fi} \in \mathfrak{S}_{1,2}^+$ violates the BCNF constraint, then

- $\mathbf{R}_{1,2,1} = \{\text{St}, \text{Ri}\}^+ = \{\text{St}, \text{Ri}, \text{Fi}, \text{Fl}, \text{Rs}\}$
- $\mathbf{R}_{1,2,2} = \{\text{St}, \text{Ri}\} \cup \text{Z} = \{\text{St}, \text{Ri}, \text{I}, \text{Fs}, \text{Si}\}$

For $\mathbf{R}_{1,2,1}$, since $\text{Fi} \longrightarrow \text{Fl} \in \mathfrak{S}_{1,2,1}^+$ violates the BCNF constraint, then

- $\mathbf{R}_{1,2,1,1} = (\text{Fi})^+ = \{\text{Fi}, \text{Fl}\}$
- $\mathbf{R}_{1,2,1,2} = \text{Fi} \cup \text{Z} = \{\text{Fi}, \text{St}, \text{Ri}, \text{Rs}\}$
- Note that $\{\text{Fi}, \text{Fl}\}$ satisfies the BCNF constraint, so they would be part of the resulting decomposition.

For $\mathbf{R}_{1,2,1,2}$, since $\text{Ri} \longrightarrow \text{Rs} \in \mathfrak{S}_{1,2,1,2}^+$ violates the BCNF constraint, then

- $\mathbf{R}_{1,2,1,2,1} = (\text{Ri})^+ = \{\text{Ri}, \text{Rs}\}$
- $\mathbf{R}_{1,2,1,2,2} = \text{Ri} \cup \text{Z} = \{\text{Ri}, \text{Fi}, \text{St}\}$
- Note that $\{\text{Ri}, \text{Rs}\}$ and $\{\text{Ri}, \text{Fi}, \text{St}\}$ satisfies the BCNF constraint, so they would be part of the resulting decomposition.

For $\mathbf{R}_{1,2,2} = \{\text{I}, \text{St}, \text{Si}, \text{Fs}, \text{Ri}\}$: We have $\text{St}, \text{Ri} \longrightarrow \text{Fi}$. By Augmentation on that functional dependency with Si ; we get $\text{St}, \text{Ri}, \text{Si} \longrightarrow \text{Fi}$. From the one we just got and $\text{Fi}, \text{Si} \longrightarrow \text{Fs}$; using Transitivity; we get $\text{St}, \text{Ri}, \text{Si} \longrightarrow \text{Fs}$. However, this dependency violates the BCNF constraint since $(\text{St}, \text{Ri}, \text{Si})$ is not (super)key. Thus, we will decompose $\mathbf{R}_{1,2,2}$ into

- $\mathbf{R}_{1,2,2,1} = \{\text{St}, \text{Ri}, \text{Si}\}^+ = \{\text{St}, \text{Ri}, \text{Si}, \text{Fs}\}$

- $\mathbf{R}_{1,2,2,2} = \{\text{St, Ri, Si}\} \cup Z = \{\text{St, Ri, Si, I}\}$
- Note that $\{\text{St, Ri, Si, Fs}\}$ and $\{\text{St, Ri, Si, I}\}$ satisfy the BCNF constraint, so they would be part of the resulting decomposition.

All relational schemas in the resulting decomposition are $\{\text{I, P, Rp}\}, \{\text{Ss, Sd}\}, \{\text{Si, Ss}\}, \{\text{Fi, Fl}\}, \{\text{Ri, Rs}\}, \{\text{Ri, Fi, St}\}, \{\text{St, Ri, Si, Fs}\}, \{\text{St, Ri, Si, I}\}$.

Minimal cover for each relational schema

- (I, P, Rp)
 $\implies \{\}$
- (Ss, Sd)
 $\implies \{\text{Ss} \longrightarrow \text{Sd}, \text{Sd} \longrightarrow \text{Ss}\}$
- (Si, Ss)
 $\implies \{\text{Si} \longrightarrow \text{Ss}\}$
- (Fi, Fl)
 $\implies \{\text{Fi} \longrightarrow \text{Fl}\}$
- (Ri, Rs)
 $\implies \{\text{Ri} \longrightarrow \text{Rs}\}$
- (Ri, Fi, St)
 $\implies \{\text{St, Ri} \longrightarrow \text{Fi}\}$
- (St, Ri, Si, Fs)
 $\implies \{\text{St, Ri, Si} \longrightarrow \text{Fs}\}$
- (St, Ri, Si, I)
 $\implies \{\text{I} \longrightarrow \text{St}, \text{I} \longrightarrow \text{Ri}, \text{I} \longrightarrow \text{Si}\}$

This decomposition is not dependency-preserving since we lost the dependency $\text{Fi, Si} \longrightarrow \text{Fs}$.

It is proved in the lecture that DECOMPOSE-BCNF is lossless-join. Thus, the decomposition is lossless-join.

Decompose the example data set

- (I, P, Rp)

I	P	Rp
1	ticket	3D
1	ticket	Dolby
1	3D	3D
1	3D	Dolby
2	ticket	3D
2	ticket	Dolby
2	3D	3D
2	3D	Dolby
3	ticket	IMAX
3	IMAX	IMAX
3	ticket	4D
3	IMAX	4D

- (Ss, Sd)

Ss	Sd
Oct. 1	31
Oct. 3	29

- (Si, Ss)

Si	Ss
1	Oct. 1
2	Oct. 3

- (Fi, Fl)

Fi	Fl
5	120
9	99

- (Ri, Rs)

Ri	Rs
7	medium
3	large

- (St, Fi, Ri)

St	Fi	Ri
Nov. 1, 1pm	5	7
Nov. 7, 2pm	9	3

- (St, Si, Fs, Ri)

St	Si	Fs	Ri
Nov. 1, 1pm	1	great	7
Nov. 1, 1pm	2	awful	7
Nov. 7, 2pm	2	not-scored	3

- (I, St, Si, Ri)

I	St	Si	Ri
1	Nov. 1, 1pm	1	7
2	Nov. 1, 1pm	2	7
3	Nov. 7, 2pm	2	3

5. The relational schema is not in 4NF since, although the multi-valued functional dependencies $ID \rightarrow P$ and $ID \rightarrow Rp$ satisfy the 4NF constraint (as ID is the super key), we have other multi-valued dependencies in \mathfrak{S}^+ that does not satisfy the 4NF constraint. More specifically, since $I \rightarrow St, Si, Ss, Sd$; by Replication, we have $I \twoheadrightarrow St, Si, Ss, Sd$. Thus, $I \twoheadrightarrow St, Si, Ss, Sd \in \mathfrak{S}^+$. However, I is not a (super)key. Therefore, it violates the 4NF constraint and the relational schema is not in 4NF.

We will decompose the relational schema using the DECOMPOSE-4NF algorithm

- As mentioned before, by using Decomposition rule on $I \rightarrow St, Si, Ss, Sd$ and $I \rightarrow Fi, Fl, Fs, Ri, Rs$; we get
 $I \rightarrow St, I \rightarrow Si, I \rightarrow Ss, I \rightarrow Sd, I \rightarrow Fi, I \rightarrow Fl, I \rightarrow Fs, I \rightarrow Ri, I \rightarrow Rs$
- By Replication on $I \rightarrow St$, we get $I \twoheadrightarrow St$, which violates the 4NF constraint since I is not a (super)key. We will decompose into
 - $\mathbf{R}_1 = \{I, St\}$
 - $\mathbf{R}_2 = \{I, Si, Ss, Sd, Fi, Fl, Fs, Ri, Rs, P, Rp\}$
 - Note that $\{I, St\}$ is in 4NF since I is the key of \mathbf{R}_1

- For \mathbf{R}_2 , since $I \longrightarrow Si$, by Replication, we get $I \twoheadrightarrow Si$, which violates the 4NF constraint since I is not the (super)key. Thus, we will decompose \mathbf{R}_2 into
 - $\mathbf{R}_{2,1} = \{I, Si\}$
 - $\mathbf{R}_{2,2} = \{I, Ss, Sd, Fi, Fl, Fs, Ri, Rs, P, Rp\}$
 - Note that $\{I, Si\}$ is in 4NF since I is the key of $\mathbf{R}_{2,1}$
- By repeatedly applying the same argument to $I \longrightarrow Ss, I \longrightarrow Sd, I \longrightarrow Fi, I \longrightarrow Fl, I \longrightarrow Fs, I \longrightarrow Ri, I \longrightarrow Rs$; we get $\{I, Ss\}, \{I, Sd\}, \{I, Fi\}, \{I, Fl\}, \{I, Fs\}, \{I, Ri\}, \{I, Rs\}, \{I, P, Rp\}$
 Note that from the above sets, all except $\{I, P, Rp\}$ are in 4NF since I is the (super)key.
- For $\{I, P, Rp\}$, by Reflexivity, we know $I \longrightarrow I$, by Union on $I \longrightarrow St, Si, Ss, Sd$ and $I \longrightarrow Fi, Fl, Fs, Ri, Rs$; we get $I \longrightarrow St, Si, Ss, Sd, Fi, Fl, Fs, Ri, Rs$. Applying this with $I \longrightarrow I$ using Union rule, we get $I \longrightarrow I, St, Si, Ss, Sd, Fi, Fl, Fs, Ri, Rs$. Note that $ID = \{I, St, Si, Ss, Sd, Fi, Fl, Fs, Ri, Rs\}$, so $I \longrightarrow ID$. Then, using Replication, we get $I \twoheadrightarrow ID$. From the question, we also have $ID \twoheadrightarrow P$. By using Transitivity, we get $I \twoheadrightarrow P \setminus ID$. Since ID does not include P , $P \setminus ID = P$. Thus, we get $I \twoheadrightarrow P$. Since $I \twoheadrightarrow P$ violates the 4NF constraint, we will decompose $\{I, P, Rp\}$ into $\{I, P\}$ and $\{I, Rp\}$, which are in 4NF.
- Therefore, all of the relational schemas in the resulting decomposition are $\{I, St\}, \{I, Si\}, \{I, Ss\}, \{I, Sd\}, \{I, Fi\}, \{I, Fl\}, \{I, Fs\}, \{I, Ri\}, \{I, Rs\}, \{I, P\}, \{I, Rp\}$.

Minimal cover of each relational schema in the resulting decomposition:

- $\{I, St\} \implies \{I \longrightarrow St\}$
- $\{I, Si\} \implies \{I \longrightarrow Si\}$
- $\{I, Ss\} \implies \{I \longrightarrow Ss\}$
- $\{I, Sd\} \implies \{I \longrightarrow Sd\}$
- $\{I, Fi\} \implies \{I \longrightarrow Fi\}$
- $\{I, Fl\} \implies \{I \longrightarrow Fl\}$
- $\{I, Fs\} \implies \{I \longrightarrow Fs\}$

- $\{I, Ri\} \implies \{I \longrightarrow Ri\}$
- $\{I, Rs\} \implies \{I \longrightarrow Rs\}$
- $\{I, P\} \implies \{I \twoheadrightarrow P\}$
- $\{I, Rp\} \implies \{I \twoheadrightarrow Rp\}$

The decomposition is lossless-join since we can do natural join on all the decomposed relational schemas on I (the event id id).

The decomposition is not dependency-preserving since we lost $Si \longrightarrow Si, Ss, Sd$ and more.

Decompose the example data set

- (I, St)

I	St
1	Nov. 1, 1pm
2	Nov. 1, 1pm
3	Nov. 7, 2pm

- (I, Si)

I	Si
1	1
2	2
3	2

- (I, Ss)

I	Ss
1	Oct. 1
2	Oct. 3
3	Oct. 3

- (I, Sd)

I	Sd
1	31
2	29
3	29

- (I, Fi)

- | I | Fi |
|---|----|
| 1 | 5 |
| 2 | 5 |
| 3 | 9 |

- (I, Fl)

I	Fl
1	120
2	120
3	99

- (I, Fs)

I	Fs
1	great
2	awful
3	not-scored

- (I, Ri)

I	Ri
1	7
2	7
3	3

- (I, Rs)

I	Rs
1	medium
2	medium
3	large

- (I, P)

I	P
1	ticket
1	3D
2	ticket
2	3D
3	ticket
3	IMAX

- (I, Rp)

I	Rp
1	3D
1	Dolby
2	3D
2	Dolby
3	IMAX
3	4D