

# Assignment 5: Dependency Theory

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November 14, 2021

## D1 Reasoning with dependencies

1.
  - Assume  $AB \rightarrow C, A \rightarrow D$  and  $CD \rightarrow EF$
  - Apply Augmentation on  $AB \rightarrow C$  with  $A$  to derive  $AB \rightarrow CA$
  - Apply Augmentation on  $A \rightarrow D$  with  $C$  to derive  $CA \rightarrow CD$
  - Apply Transitivity on  $AB \rightarrow CA$  and  $CA \rightarrow CD$  to derive  $AB \rightarrow CD$
  - Apply Transitivity on  $AB \rightarrow CD$  and  $CD \rightarrow EF$  to derive  $AB \rightarrow EF$
  - Apply Reflection on  $F \subseteq EF$  to derive  $EF \rightarrow F$
  - Apply Transitivity on  $AB \rightarrow EF$  and  $EF \rightarrow F$  to derive  $AB \rightarrow F$
  - Hence,  $AB \rightarrow F$
  
2.
  - Assume we have rows  $r_1, r_2 \in I$  of instance  $I$  such that  $r_1[XW] = r_2[XW]$
  - By  $r_1[XW] = r_2[XW]$ , we have  $r_1[X] = r_2[X]$  and  $r_1[W] = r_2[W]$
  - Using  $X \rightarrow Y$  and  $r_1[X] = r_2[X]$ , we conclude that  $r_1[Y] = r_2[Y]$
  - By  $r_1[Y] = r_2[Y]$  and  $r_1[W] = r_2[W]$ , we have  $r_1[YW] = r_2[YW]$
  - Using  $YW \rightarrow Z$  and  $r_1[YW] = r_2[YW]$ , we conclude that  $r_1[Z] = r_2[Z]$

- Hence,  $r_1[Z] = r_2[Z]$  holds.
- 3.
- Assume we have  $r_1[X]$  for every instance  $I_1$  of  $R$  and every row  $r_1 \in I_1$
  - Using  $R[X] \subseteq S[Y]$  and  $r_1[X]$ , there exists a row in instance  $I_2$  of  $S$  with  $r_1[X] = r_2[X]$
  - Using  $S[Y] \subseteq T[Z]$  and  $r_2[Y]$ , there exists a row in instance  $I_3$  of  $T$  with  $r_2[Y] = r_3[Z]$
  - Thus, for every instance  $I_1$  of  $R$  and every row  $r_1 \in R$ , there exists a row in instance  $I_3$  of  $T$  such that  $r_1[X] = r_3[Z]$
  - Hence,  $R[X] \subseteq T[Z]$
- 4.
- Assume  $X \twoheadrightarrow Y$  and  $XY \longrightarrow Z$
  - Apply Complementation on  $X \twoheadrightarrow Y$  to derive  $X \twoheadrightarrow Z$  (with  $Z$  all attributes of  $\mathbf{R}$  not in  $X$  and  $Y$ )
  - Apply Reflexivity on  $Z \setminus (X \cup Y) \subseteq Z$  to derive  $Z \longrightarrow Z \setminus (X \cup Y)$
  - Apply Transitivity on  $XY \longrightarrow Z$  and  $Z \longrightarrow Z \setminus (X \cup Y)$  to derive  $XY \longrightarrow Z \setminus (X \cup Y)$
  - Since  $Z$  is all attributes of  $\mathbf{R}$  not in  $X$  and  $Y$ ,  $Z \cap XY = \emptyset$
  - Apply Coalensence on  $X \twoheadrightarrow Z$ ,  $XY \longrightarrow Z \setminus (X \cup Y)$ ,  $Z \cap XY = \emptyset$  and  $Z \setminus (X \cup Y) \subseteq Z$ , we conclude that  $X \longrightarrow Z \setminus (X \cup Y)$
  - Hence,  $X \longrightarrow Z \setminus (X \cup Y)$

5. Consider the following table of the schema

**person**(name, number, birthdate, age)

name	number	birthdate	age
Alice	1	2001-01-01	20
Alice	2	2005-09-05	16

Since name, number are the primary keys, they determine all attributes. Thus, “name, number  $\rightarrow$  birthdate”.

We know from the lecture that birthdate determines age since people who have the same birthdate would have the same age, and so “birthdate  $\rightarrow$  age”.

Since  $\{\text{birthdate}\} \subseteq \{\text{name, birthdate}\}$ , by applying Reflexivity, we get “name, birthdate  $\rightarrow$  birthdate”.

Applying Transitivity on “name, birthdate  $\rightarrow$  birthdate” and “birthdate  $\rightarrow$  age”, we derive “name, birthdate  $\rightarrow$  age”

Hence, we have “name, number  $\rightarrow$  birthdate” and “name, birthdate  $\rightarrow$  age”.

However, from the table, we can see that “name  $\rightarrow$  age” does not hold.

Let  $X = \text{name}$ ,  $W = \text{number}$ ,  $Y = \text{birthdate}$ ,  $Z = \text{age}$ , we have shown that the inference rule from the question is not sound.

6. TODO: Do Question 6

7. The attribute closure of set of attributes  $C$ :

- Initially,  $\text{closure} = \{C\}$
- From  $C \rightarrow A$ , since  $C \subseteq \text{closure}$  and  $A \not\subseteq \text{closure}$ ,  $\text{closure} = \{C, A\}$
- From  $AC \rightarrow E$ , since  $AC \subseteq \text{closure}$  and  $E \not\subseteq \text{closure}$ ,  $\text{closure} = \{C, A, E\}$
- From  $E \rightarrow B$ , since  $E \subseteq \text{closure}$  and  $B \not\subseteq \text{closure}$ ,  $\text{closure} = \{C, A, E, B\}$
- From  $AB \rightarrow D$ , since  $AB \subseteq \text{closure}$  and  $D \not\subseteq \text{closure}$ ,  $\text{closure} = \{C, A, E, B, D\}$
- From  $BC \rightarrow D$ , since  $BC \subseteq \text{closure}$  and  $D \subseteq \text{closure}$ , we don't need to add  $D$  into  $\text{closure}$  more (as  $D$  is already in  $\text{closure}$ )
- From  $D \rightarrow A$ , since  $D \subseteq \text{closure}$  and  $A \subseteq \text{closure}$ , we don't need to add  $A$  into  $\text{closure}$  more (as  $A$  is already in  $\text{closure}$ )
- Therefore,  $C^+ = \{A, B, C, D, E\}$

The attribute closure of set of attributes  $(EA)$ :

- Initially,  $closure = \{E, A\}$
- From  $E \longrightarrow B$ , since  $E \subseteq closure$  and  $B \not\subseteq closure$ ,  $closure = \{E, A, B\}$
- From  $AB \longrightarrow D$ , since  $AB \subseteq closure$  and  $D \not\subseteq closure$ ,  $closure = \{E, A, B, D\}$
- From  $AC \longrightarrow E$ , since  $AC \not\subseteq closure$ , we don't need to add  $E$  into  $closure$
- From  $BC \longrightarrow D$ , since  $BC \not\subseteq closure$ , we don't need to add  $D$  into  $closure$
- From  $C \longrightarrow A$ , since  $C \not\subseteq closure$ , we don't need to add  $A$  into  $closure$
- From  $D \longrightarrow A$ , since  $D \subseteq closure$  and  $A \subseteq closure$ , we don't need to add  $A$  into  $closure$  more (as  $A$  is already in  $closure$ )
- Therefore,  $(EA)^+ = \{A, B, D, E\}$

8. Compute  $X^+$  for every  $X \subseteq \{A, B, C, D, E\}$

- $A^+ = \{A\}$  since there is no  $A \longrightarrow \dots$  in  $\mathfrak{S}$
- $B^+ = \{B\}$  since there is no  $B \longrightarrow \dots$  in  $\mathfrak{S}$
- $C^+ = \{A, B, C, D, E\}$  from question 7
- $D^+ = \{A, D\}$  since there is only  $D \longrightarrow A$  that would satisfy the closure algorithm
- $E^+ = \{B, E\}$  since there is only  $E \longrightarrow B$  that would satisfy the closure algorithm

9.