CS3SD3 - Assignment 3

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Question 1

The colour petri net is at the end of the file.
The invariants of the petri net are:

(i1) $M(p_1) + M(p_2) + M(p_3) + M(p_4) + M(p_5) = ph1 + ph2 + ph3 + ph4 + ph5$

(i2) $\text{LEFT}(M(p_3)) + \text{LEFT}(M(p_4)) + \text{RIGHT}(M(p_4)) + \text{RIGHT}(M(p_5)) + M(p_6) = f_1 + f_2 + f_3 + f_4 + f_5$
where $\text{LEFT}(X) = \sum_{x\in X} \text{LEFT}(x)$ and $\text{RIGHT}(X) = \sum_{x \in X} \text{RIGHT}(x)$

If $M(p_4) \neq \emptyset$, i.e., ph$j \in M(p_4)$, then (return_left_fork, x = ph$j$) can be fired.
If $M(p_5) \neq \emptyset$, i.e., ph$j \in M(p_5)$, then (return_right_fork_and_exit_dining_room, x = ph$j$) can be fired.
If $M(p_4) = \emptyset$ and $M(p_5) = \emptyset$

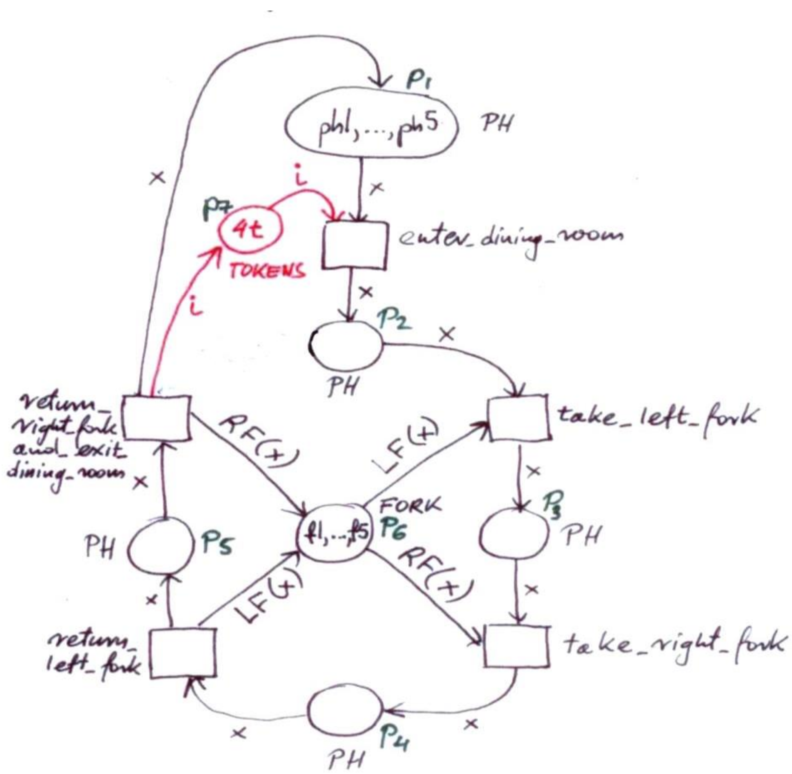
- If $M(p_3) \neq \emptyset$
  - Since we only have 4 tokens t, we know that only at most 4 philosophers (ph) can reach p_2 and p_3
  - Thus, only at most 4 forks (f) are not in p_6
  - This means there is at least 1 fork in p_6
  - So $M(p_6) \neq \emptyset$
  - Since $M(p_3) \neq \emptyset$ and $M(p_6) \neq \emptyset$, (take_right_fork, x = ph$i$) where ph$i \in$ PH, can be fired.
- If $M(p_3) = \emptyset$
  - If $M(p_2) \neq \emptyset$
    * Since $M(p_3) = \emptyset$, $M(p_4) = \emptyset$ and $M(p_5) = \emptyset$, from (i2), we know that $M(p_6) = f_1 + f_2 + f_3 + f_4 + f_5$
    * This means $M(p_6) \neq \emptyset$
    * Since $M(p_2) \neq \emptyset$ and $M(p_6) \neq \emptyset$, (take_left_fork, x = ph$i$) where ph$i \in$ PH, can be fired

- If $M(p_2) = \emptyset$
  * Then we know $M(p_2) = \emptyset$, $M(p_3) = \emptyset$, $M(p_4) = \emptyset$, $M(p_5) = \emptyset$
  * Thus, from (i1), we have $M(p_1) = ph1 + ph2 + ph3 + ph4 + ph5$ and $M(p_7) \neq \emptyset$
  * This case happens right at the start of the process (at the initial marking) and so, (enter_dining_room, x = ph$i$) where ph$i \in PH$, can be fired.

Therefore, in any case, at least one state will be able to fire.

Colour Petri Nets Solution to Dining Philosophers with Butler.

colour PH = with ph1 | ph2 | ph3 | ph4 | ph
colour FORK = with f1 | f2 | f3 | f4 | f5
colour TOKENS = with t
var x : PH
var i: TOKENS
fun LF x = case of ph1 ⇒ f2 | ph2 ⇒ f3 | ph3 ⇒ f4 | ph4 ⇒ f5 | ph5 ⇒ f1
fun RF x = case of ph1 ⇒ f1 | ph2 ⇒ f2 | ph3 ⇒ f3 | ph4 ⇒ f4 | ph5 ⇒ f5



Interpretation of places:
p_1 - thinking room

p_2 - philosophers without forks in the dining room
p_3 - philosophers with left forks in the dining room
p_4 - philosophers that are eating
p_5 - philosophers that finished eating and still with right forks in the dining room
p_6 - unused forks
p_7 - butler or counter