CS3SD3 - Assignment 3

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Question 7

(a) (i) $\neg p \Rightarrow r \equiv \neg(\neg p) \vee r) \equiv p \vee r$. Since we have $L(s_0) = \{r\}$, $M, s_0 \vDash \varphi$. We have $L(s_2) = \{p, q\}$, so $M, s_2 \vDash \varphi$

(ii) Since $r \in L(s_0)$, $r \in L(s_1)$, and we can have path $s_0 \rightarrow s_1 \rightarrow s_1 \rightarrow s_1 \rightarrow \ldots$, we know that $M, s_0 \vDash \text{EG } r$. Therefore, $M, s_0 \vDash \neg\text{EG } r$ is false. Since $r \notin L(s_2)$, we know that $M, s_0 \vDash \neg\text{EG } r$ is true as future also includes present.

(iii) Since $t \notin L(s_0)$, we know that $M, s_0 \vDash \text{E}(t \text{ U } q)$ is false. Since $q \in L(s_2)$, we know that q already holds in s_2, thus, we don't need t to hold anymore. Therefore, $M, s_2 \vDash \text{E}(t \text{ U } q)$ is true.

(iv) Since $q \in L(s_2)$, and we have a path $s_0 \rightarrow s_2 \rightarrow \ldots$, we know $M, s_0 \vDash \text{F } q$ is true. Since $q \in L(s_2)$, we also know $M, s_2 \vDash \text{F } q$ is true since future also includes present.

(b) LTL: $\text{G}(\text{F } p \wedge (p \Rightarrow \text{F } s) \wedge (p \Rightarrow \text{F } t))$
CTL: $\text{AG}(\text{F } p \wedge \text{AG}(p \Rightarrow \text{AF } s) \wedge \text{AG}(p \Rightarrow \text{AF } t))$

(c) LTL: $\text{G}(\text{F } q \wedge \text{F } r \wedge (q \Rightarrow (\neg p \text{ U } r)) \wedge (q \Rightarrow (t \text{ U } r)))$
CTL: $\text{AG}(\text{F } q \wedge \text{F } r) \wedge \text{AG}(q \Rightarrow \text{A}(\neg p \text{ U } r)) \wedge \text{AG}(q \Rightarrow \text{A}(t \text{ U } r))$

(d) LTL: $s \vDash \text{G}(\text{F } \Phi)$
CTL: $s \vDash \text{AG}(\text{AF } \Phi)$

(e) LTL: $\text{G}((p \Rightarrow \text{F } q) \Rightarrow (\neg r \text{ U } t))$
CTL: $\text{AG}((p \Rightarrow \text{F } q) \Rightarrow \text{A}(\neg r \text{ U } t))$

(f) LTL: $\text{G}(\text{F } q \wedge \text{F } r \wedge (q \Rightarrow (\neg p \text{ U } r)))$
CTL: $\text{AG}(\text{F } q \wedge \text{F } r) \wedge \text{AG}(q \Rightarrow \text{A}(\neg p \text{ U } r))$