CS3SD3 - Assignment 3

Hien Tu - tun1

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Question 8

Let lpr_i be local processing of reader i where $i = 1, 2$
lpw_i be local processing of writer i where $i = 1, 2$
tr_i be request reading of reader i where $i = 1, 2$
tw_i be request writing of writer i where $i = 1, 2$
r_i be reading of reader i where $i = 1, 2$
w_i be writing of writer i where $i = 1, 2$

We also introduce additional boolean variables (or atomic predicates): turn=w1 (writer 1 will write), turn=w2 (writer 2 will write) and turn=r (one or both readers will read).
Now states can be defined as by atomic predicates of the form:

(str1, str2, stw1, stw2, turn)

where str1 $\in \{\text{lpr}_1, \text{tr}_1, \text{r}_1\}$ stands for status of reader 1
str2 $\in \{\text{lpr}_2, \text{tr}_2, \text{r}_2\}$ stands for status of reader 2
stw1 $\in \{\text{lpw}_1, \text{tw}_1, \text{w}_1\}$ stands for status of writer 1
stw2 $\in \{\text{lpw}_2, \text{tw}_2, \text{w}_2\}$ stands for status of writer 2
turn $\in \{\text{turn=w1}, \text{turn=w2}, \text{turn=r}\}$ stands for status of turns

A cycle of reader 1 is

$$(\text{lpr}_1, \_, \_, \_, \_) \to (\text{tr}_1, \_, \_, \_, \_) \to (\text{r}_1, \_, \_, \_, \_) \to \text{back to the beginning}$$

A cycle of reader 2 is

$$(\text{lpr}_2, \_, \_, \_, \_) \to (\text{tr}_2, \_, \_, \_, \_) \to (\text{r}_2, \_, \_, \_, \_) \to \text{back to the beginning}$$

A cycle of writer 1 is

$$(\_, \_, \text{lpw}_1, \_, \_) \to (\_, \_, \text{tw}_1, \_, \_) \to (\_, \_, \text{w}_1, \_, \_) \to \text{back to the beginning}$$

A cycle of writer 2 is

$$(\_, \_, \text{lpw}_2, \_, \_) \rightarrow (\_, \_, \text{tw}_2, \_, \_) \rightarrow (\_, \_, \text{w}_2, \_, \_) \rightarrow \text{back to the beginning}$$

This would result in $3 \times 3 \times 3 \times 3 \times 3 = 243$ states in the diagram.
However, not all combinations of atomic predicates are allowed. For example, if the readers are reading, then no writers can write (the other reader can still read). That is

$$\text{str1} = \text{r}_1 \Rightarrow \text{stw1} \neq \text{w}_1 \wedge \text{stw2} \neq \text{w}_2$$

Similarly,

$$\text{str2} = \text{r}_2 \Rightarrow \text{stw1} \neq \text{w}_1 \wedge \text{stw2} \neq \text{w}_2$$

If one of the writer is writing, then the other writer cannot write and the two readers cannot read. That is

$$\text{stw1} = \text{w}_1 \Rightarrow \text{str1} \neq \text{r}_1 \wedge \text{str2} \neq \text{r}_2 \wedge \text{stw2} \neq \text{w}_2$$

and

$$\text{stw2} = \text{w}_2 \Rightarrow \text{str1} \neq \text{r}_1 \wedge \text{str2} \neq \text{r}_2 \wedge \text{stw1} \neq \text{w}_1$$

In total, there would be 203 states in the diagram, which is impossible to draw.
Safety properties:

- LTL: $\text{G}(\text{r}_1 \Rightarrow \neg(\text{w}_1 \vee \text{w}_2))$
  $\text{G}(\text{r}_2 \Rightarrow \neg(\text{w}_1 \vee \text{w}_2))$
  $\text{G}(\text{w}_1 \Rightarrow \neg(\text{r}_1 \vee \text{r}_2 \vee \text{w}_2))$
  $\text{G}(\text{w}_2 \Rightarrow \neg(\text{r}_1 \vee \text{r}_2 \vee \text{w}_1))$

- LTL: $\text{AG}(\text{r}_1 \Rightarrow \neg(\text{w}_1 \vee \text{w}_2))$
  $\text{AG}(\text{r}_2 \Rightarrow \neg(\text{w}_1 \vee \text{w}_2))$
  $\text{AG}(\text{w}_1 \Rightarrow \neg(\text{r}_1 \vee \text{r}_2 \vee \text{w}_2))$
  $\text{AG}(\text{w}_2 \Rightarrow \neg(\text{r}_1 \vee \text{r}_2 \vee \text{w}_1))$

Liveness properties:

- LTL: $\text{G}(\text{tr}_1 \Rightarrow \text{F } \text{r}_1)$
  $\text{G}(\text{tr}_2 \Rightarrow \text{F } \text{r}_2)$
  $\text{G}(\text{tw}_1 \Rightarrow \text{F } \text{w}_1)$
  $\text{G}(\text{tw}_2 \Rightarrow \text{F } \text{w}_2)$

- LTL: $\mathrm{AG}(\mathrm{tr}_1 \Rightarrow \mathrm{AF}\ \mathrm{r}_1)$
  $\mathrm{AG}(\mathrm{tr}_2 \Rightarrow \mathrm{AF}\ \mathrm{r}_2)$
  $\mathrm{AG}(\mathrm{tw}_1 \Rightarrow \mathrm{AF}\ \mathrm{w}_1)$
  $\mathrm{AG}(\mathrm{tw}_2 \Rightarrow \mathrm{AF}\ \mathrm{w}_2)$