logistic

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1 Ex1

biến đổi lại thuật toán logistic regression:

solution:

the posterior probability for class C1(the case of 2 classes):

$$p(C_1|x) = \frac{p(x|C_1) * p(C_1}{p(x|C_1) * p(C_1) + p(x|C_2) * p(C_2)} = \frac{1}{1 + \frac{p(x|C_2)*p(C_2)}{p(x|C_1)*p(C_1)}}$$

we have:

$$a = log\frac{p(x|C_1) * p(C_1)}{p(x|C_2) * p(C_2)}$$

$\Rightarrow \quad \sigma(a) = \frac{1}{1+e^{-a}}$

$\Rightarrow \quad \sigma(a) \quad is \quad the \quad logistic \quad sigmoid \quad function$

đạo hàm $\sigma(x)$:

$$\frac{d}{dx}\sigma(x) = \frac{d}{dx}\left[\frac{1}{1+e^{-x}}\right] = \frac{d}{dx}(1+e^{-x})^{-1} = -(1+e^{-x})^{-2}(-e^{-x})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} * \frac{e^{x}}{1+e^{-x}} = \frac{1}{1+e^{-x}} * \frac{1+e^{x}-1}{1+e^{-x}} = \frac{1}{1+e^{-x}}\left(1 - \frac{1}{1+e^{-x}}\right)$$

=$\sigma(x)(1-\sigma(x))$

The model logistic regression

p(C$_1|\phi) = y(\phi) = \sigma(w^T\phi)$

p(C$_2|\phi) = 1 - p(C_1|\phi)$

For the data set $\phi_n, t_n where t_n \in (0;1) and \phi_n = \phi(x_n) with n = 1, 2, .., N$

=> Likelihood function

p(t|w)=$\prod_{n=1}^{N} y_n^{t_n}(1-y_n)^{1-t_n}$

$\Rightarrow max \quad p(t|w)$

L=-log(p(t|w)=-$\sum_{n=1}^{N}(t_n log y_n + 1 - t_n)$

với y$_n = \sigma(\phi_n^T w) = w_0 + w_1\phi_1$

=-[tlog(y)+(1-t)log(1-y)]

y=$\sigma(w_0 + w_1\phi_1 + ... + w_D\phi_D)$
Let: z=w$_0 + w_1\phi_1 + ... + w_D\phi_D \Rightarrow y = \sigma(z)$
we have:

$$\frac{dL}{dy} = -\left(\frac{t}{y} - \frac{1-t}{1-y}\right) = -\frac{t(1-y) - y(1-t)}{y(1-y)} = -\frac{t - ty - y + ty}{y(1-y)} = \frac{-t+y}{y(1-y)}$$

$$\frac{dy}{dz} = \frac{d\sigma(z)}{dz} = \sigma(z)(1-\sigma(z)) = y(1-y)$$

$$\frac{dz}{dw_1} = \phi_1$$

$\Rightarrow \frac{dL}{dw_1} = \frac{dL}{dy} * \frac{dy}{dz} * \frac{dz}{dw_1} = \frac{-t+y}{y(1-y)} * y(1-y) * \phi_1 = t(y-t)\phi_1$
$\Rightarrow \frac{dL}{w_0} = y - t$
$\Rightarrow \frac{dL}{dw_2} = (y-t)\phi_2$
We have:

$$\frac{dL}{dw} = \begin{bmatrix} \frac{dL}{dw_0} \\ \frac{dL}{dw_1} \\ .... \\ \frac{dL}{dw_n} \end{bmatrix} = \begin{bmatrix} (y-t)*1 \\ (y-t)*\phi_1 \\ .... \\ (y-t)*\phi_N \end{bmatrix}$$

$\Rightarrow \nabla L = \sum_{n=1}^{N}(y_n - t_n)\phi_n$

2 Ex2

Tìm hàm f(x), biết f'(x) = f(x)(1-f(x))
Solution: f'(x) = f(x)(1-f(x))=f(x)-f$^2(x)$

$$\Rightarrow \frac{df}{dx} = f(x) - f^2(x)$$

$$\Rightarrow \frac{df}{f(x) - f^2(x)} = dx$$

$$\int \frac{df}{f(x) - f^2(x)} = \int dx$$

$$\int \frac{1}{f(x)(1-f(x))} df = -\int dx$$

$$\Rightarrow \int \frac{1}{1-f(x)} - \frac{1}{f(x)})df = -\int dx$$

$$\int \frac{1}{1-f(x)} df - \int \frac{1}{f(x)})df = -\int dx$$

$$\Rightarrow ln|1-f(x)| - ln|f(x)| + a = -x + b$$

$$ln\left|\frac{1-f(x)}{f(x)}\right| = -x + b - a$$

$$\frac{f(x)}{f(x)-1} = e^{x+b-a} = e^{-x} * (b-a)$$

$$\Rightarrow 1 - f(x) = f(x)e^{-x}(b-a)$$

$$\Rightarrow f(x)(1 + e^{-x}(b-a)) = 1$$

$$\Rightarrow f(x) = \frac{1}{1 + e^{-x}(b-a)}$$