week 1

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1 Problem 1

To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution:
A: Know to have the disease =¿ P(A)= 0.05
B: Know not to have the disease =¿ P(B)=0.95
D: testing positive for the disease
P(D—A)=0.98
P(D—B)=0.03

$$P(A|D) = \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|B)P(B)} = \frac{(0.98)(0.05)}{(0.98)(0.05) + (0.03)(0.95)} = 0.632$$

2 Problem 2

Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution: Univariate normal distribution

a)Mean

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x-\mu^2}{2\sigma^2}}dx$$

Let:

$$t = \frac{x-\mu}{\sqrt{2}\sigma} => x = \sqrt{2}\sigma t + \mu => dx = \sqrt{2}\sigma dt$$

$$=> E(X) = \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu)\frac{1}{\sigma\sqrt{2\pi}}e^{-t^2}\sqrt{2}\sigma dt = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}(\sqrt{2}\sigma t + \mu)e^{-t^2}dt$$

$$= \frac{1}{\sqrt{\pi}}\left(\sqrt{2}\sigma\int_{-\infty}^{\infty} te^{-t^2}dt + \mu\int_{-\infty}^{\infty} e^{-t^2}dt\right) = \frac{1}{\sqrt{\pi}}\left(\sqrt{2}\sigma\left[-\frac{1}{2}e^{-t^2}\right]_{-\infty}^{\infty} + \mu\sqrt{\pi}\right) = \frac{\mu\sqrt{\pi}}{\sqrt{\pi}}$$

$=\mu$

b)Variance:

$$V(X) = \int_{-\infty}^{\infty} x^2 f(x)dx - (E(X))^2 = \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x-\mu^2}{2\sigma^2}} dx - \mu^2$$

Let:

$$t = \frac{x-\mu}{\sqrt{2}\sigma} => x = \sqrt{2}\sigma t + \mu => dx = \sqrt{2}\sigma dt$$

$$V(X) = \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2} \sqrt{2}\sigma dt - \mu^2 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu)^2 e^{-t^2} dt - \mu^2$$

$$= \frac{1}{\sqrt{\pi}} \left(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \int_{-\infty}^{\infty} t e^{-t^2} dt + \mu^2 \int_{-\infty}^{\infty} e^{-t^2} dt \right) - \mu^2$$

$$= \frac{1}{\sqrt{\pi}} \left(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \left[-\frac{1}{2} e^{-t^2} \right]_{-\infty}^{\infty} + \mu^2 \sqrt{\pi} \right) - \mu^2$$

$$= \frac{1}{\sqrt{\pi}} \left(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \cdot 0 \right) + \mu^2 - \mu^2 = \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left(\left[-\frac{t}{2} e^{-t^2} \right]_{-\infty}^{\infty} + \frac{1}{2} \int_{-\infty}^{\infty} e^{-t^2} dt \right) = \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \frac{2\sigma^2 \sqrt{\pi}}{2\sqrt{\pi}} = \sigma^2$$