ML

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1 Problem 1

To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution:
P(A|B)=0.05
P(B)=0.95
P(AB)=P(A|B)P(B)=(0.05)(0.98)
P(A$\bar{\text{B}}$)=P(A$\bar{\text{B}}$)P($\bar{\text{B}}$)=(0.95)(0.98)

$$P(B|A) = \frac{P(A|B)P(B)}{P(AB) + P(A\bar{B})} = \frac{(0.05)(0.98)}{(0.05)(0.98) + (0.95)(0.05)} = 0.632$$

2 Problem 2

Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution: Univariate normal distribution

a)Mean

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x-\mu^2}{2\sigma^2}}dx$$

Let:

$$t = \frac{x-\mu}{\sqrt{2}\sigma} => x = \sqrt{2}\sigma t + \mu => dx = \sqrt{2}\sigma dt$$

$$=> E(X) = \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu)\frac{1}{\sigma\sqrt{2\pi}}e^{-t^2}\sqrt{2}\sigma dt = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}(\sqrt{2}\sigma t + \mu)e^{-t^2}dt$$

$$= \frac{1}{\sqrt{\pi}}\left(\sqrt{2}\sigma\int_{-\infty}^{\infty} te^{-t^2}dt + \mu\int_{-\infty}^{\infty} e^{-t^2}dt\right) = \frac{1}{\sqrt{\pi}}\left(\sqrt{2}\sigma\left[-\frac{1}{2}e^{-t^2}\right]_{-\infty}^{\infty} + \mu\sqrt{\pi}\right) = \frac{\mu\sqrt{\pi}}{\sqrt{\pi}}$$

=μ

b)Variance:

$$V(X) = \int_{-\infty}^{\infty} x^2 f(x)dx - (E(X))^2 = \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x-\mu^2}{2\sigma^2}} dx - \mu^2$$

Let:

$$t = \frac{x-\mu}{\sqrt{2}\sigma} => x = \sqrt{2}\sigma t + \mu => dx = \sqrt{2}\sigma dt$$

$$V(X) = \int_{-\infty}^{\infty} (\sqrt{2}\sigma t+\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2} \sqrt{2}\sigma dt - \mu^2 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\sqrt{2}\sigma t+\mu)^2 e^{-t^2} dt - \mu^2$$

$$= \frac{1}{\sqrt{\pi}} \left(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \int_{-\infty}^{\infty} t e^{-t^2} dt + \mu^2 \int_{-\infty}^{\infty} e^{-t^2} dt \right) - \mu^2$$

$$= \frac{1}{\sqrt{\pi}} \left(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \left[-\frac{1}{2} e^{-t^2} \right]_{-\infty}^{\infty} + \mu^2\sqrt{\pi} \right) - \mu^2$$

$$= \frac{1}{\sqrt{\pi}} \left(2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \cdot 0 \right) + \mu^2 - \mu^2 = \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left(\left[-\frac{t}{2} e^{-t^2} \right]_{-\infty}^{\infty} + \frac{1}{2} \int_{-\infty}^{\infty} e^{-t^2} dt \right) = \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \frac{2\sigma^2\sqrt{\pi}}{2\sqrt{\pi}} = \sigma^2$$