week 1

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1 Problem 1

To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution:

A: Know to have the disease

$\Rightarrow P(A) = 0.05$

B: Know not to have the disease

$\Rightarrow P(B) = 0.95$

D: testing positive for the disease

P(D|A)=0.98 and P(D|B)=0.03

$$P(A|D) = \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|B)P(B)} = \frac{(0.98)(0.05)}{(0.98)(0.05) + (0.03)(0.95)} = 0.632$$

2 Problem 2

Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution: Univariate normal distribution

a)Mean

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} x\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x-\mu^2}{2\sigma^2}}dx$$

Let:

$$t = \frac{x-\mu}{\sqrt{2}\sigma} => x = \sqrt{2}\sigma t + \mu => dx = \sqrt{2}\sigma dt$$

$$=> E(X) = \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu)\frac{1}{\sigma\sqrt{2\pi}}e^{-t^2}\sqrt{2}\sigma dt = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}(\sqrt{2}\sigma t + \mu)e^{-t^2}dt$$

$$= \frac{1}{\sqrt{\pi}}\left(\sqrt{2}\sigma\int_{-\infty}^{\infty} te^{-t^2}dt + \mu\int_{-\infty}^{\infty} e^{-t^2}dt\right) = \frac{1}{\sqrt{\pi}}\left(\sqrt{2}\sigma\left[-\frac{1}{2}e^{-t^2}\right]_{-\infty}^{\infty} + \mu\sqrt{\pi}\right) = \frac{\mu\sqrt{\pi}}{\sqrt{\pi}}$$

$=\mu$
b)Variance:

$$V(X)=\int_{-\infty}^{\infty}x^2f(x)dx-(E(X))^2=\int_{-\infty}^{\infty}x^2\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x-\mu^2}{2\sigma^2}}dx-\mu^2$$

Let:

$$t=\frac{x-\mu}{\sqrt{2}\sigma}=>x=\sqrt{2}\sigma t+\mu=>dx=\sqrt{2}\sigma dt$$

$$V(X)=\int_{-\infty}^{\infty}(\sqrt{2}\sigma t+\mu)^2\frac{1}{\sigma\sqrt{2\pi}}e^{-t^2}\sqrt{2}\sigma dt-\mu^2=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}(\sqrt{2}\sigma t+\mu)^2e^{-t^2}dt-\mu^2$$

$$=\frac{1}{\sqrt{\pi}}\left(2\sigma^2\int_{-\infty}^{\infty}t^2e^{-t^2}dt+2\sqrt{2}\sigma\mu\int_{-\infty}^{\infty}te^{-t^2}dt+\mu^2\int_{-\infty}^{\infty}e^{-t^2}dt\right)-\mu^2$$

$$=\frac{1}{\sqrt{\pi}}\left(2\sigma^2\int_{-\infty}^{\infty}t^2e^{-t^2}dt+2\sqrt{2}\sigma\mu\left[-\frac{1}{2}e^{-t^2}\right]_{-\infty}^{\infty}+\mu^2\sqrt{\pi}\right)-\mu^2$$

$$=\frac{1}{\sqrt{\pi}}\left(2\sigma^2\int_{-\infty}^{\infty}t^2e^{-t^2}dt+2\sqrt{2}\sigma\mu\cdot 0\right)+\mu^2-\mu^2=\frac{2\sigma^2}{\sqrt{\pi}}\int_{-\infty}^{\infty}t^2e^{-t^2}dt$$

$$=\frac{2\sigma^2}{\sqrt{\pi}}\left(\left[-\frac{t}{2}e^{-t^2}\right]_{-\infty}^{\infty}+\frac{1}{2}\int_{-\infty}^{\infty}e^{-t^2}dt\right)=\frac{2\sigma^2}{\sqrt{\pi}}\cdot\frac{1}{2}\int_{-\infty}^{\infty}e^{-t^2}dt$$

$$=\frac{2\sigma^2\sqrt{\pi}}{2\sqrt{\pi}}=\sigma^2$$