0.1 Overview

Frequency-shift keying (FSK) is a frequency modulation scheme in which digital information is encoded on a carrier signal by periodically shifting the frequency of the carrier between several discrete frequencies. The technology is used for communication systems such as telemetry, weather balloon radiosondes, caller ID, garage door openers, and low frequency radio transmission in the VLF and ELF bands. The simplest FSK is $binary\ FSK$ ($BFSK$), in which the carrier is shifted between two discrete frequencies to transmit binary (0s and 1s) information.

0.2 Perform the FSK modulation

- Generating and Plotting the Carrier Signals: The first step in FSK modulation is to generate the carrier signals. The carrier signals are 2 cosine waves with same amplitude and phase but different frequencies. $0s$ are represented by $\cos(2\pi f_1 t)$ and $1s$ are represented by $\cos(2\pi f_2 t)$ and $f_2 = 2\ f_1$

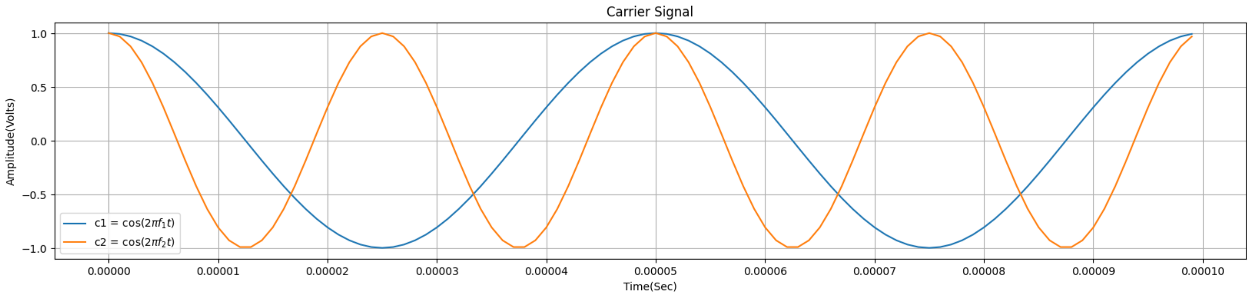


Figure 0.1: Carrier signal

- Generating and Plotting the Binary Data Sequence: The next step is to generate a random binary data sequence, which consist of $1s$ and $0s$. In this assignment, I use a stream bit with the length of 50. The binary data sequence is then plotted to visualize the pattern of data.

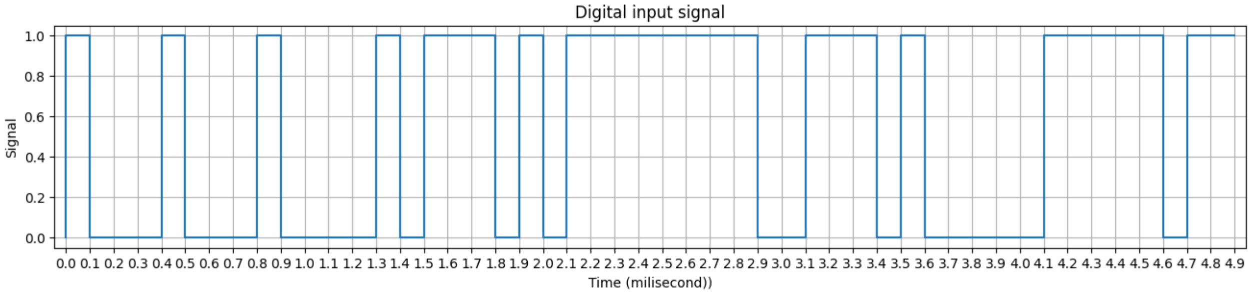


Figure 0.2: Binary data sequence

- Performing FSK Modulation and Plotting the FSK Modulated Signal: The final step is to perform the FSK modulation of the binary data sequence using the carrier signals. The FSK modulated signal is generated by multiplying the binary data sequence with the carrier signals. The FSK modulated signal is then plotted to visualize the waveform with the help of $matplotlib$ library.

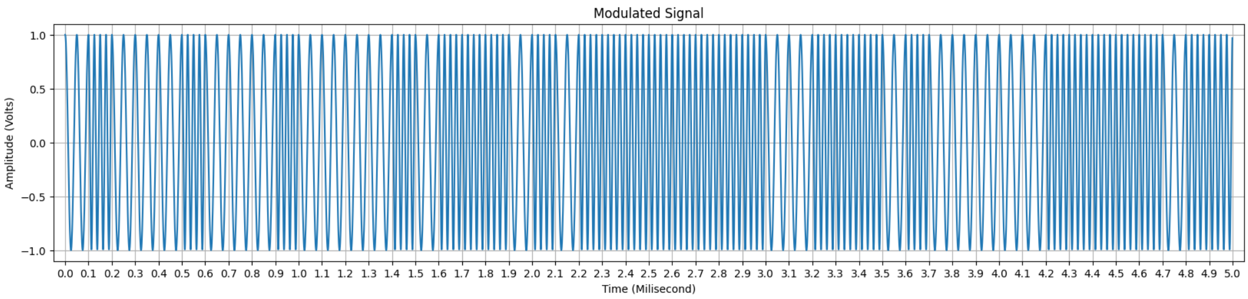


Figure 0.3: Modulated signal

0.3 Perform the FSK demodulation

The method in FSK demodulation is to correlate the FSK modulated signal with the carrier signals. The purpose of correlation is to generate decision variables that will be used to determine the demodulated binary data. The correlation is performed using the Maximum Likelihood Criterion, which is a mathematical method that determines the most likely value of the demodulated binary data based on the FSK modulated signal and the carrier signals. This will define whether the demodulated binary data is 0 or 1.

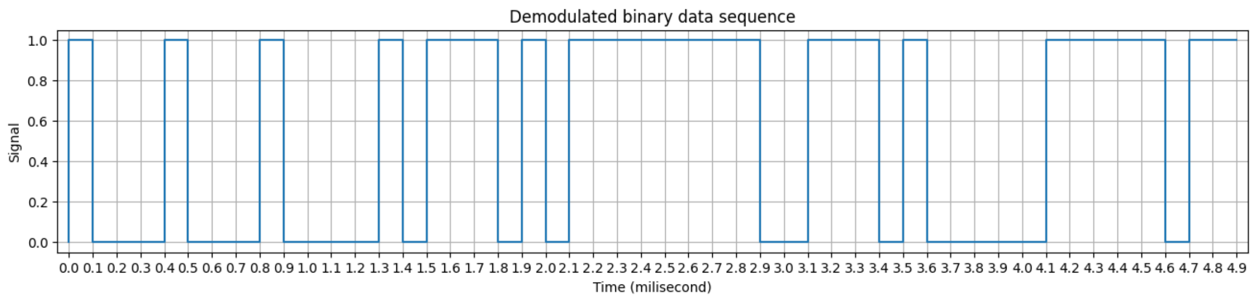


Figure 0.4: Demodulated signal

0.4 FSK modulation/demodulation under the effects of Gaussian noise

Gaussian noise is a type of random noise that is commonly present in communication systems. It is characterized by a zero mean and a variance of $N_0/2$. A low value of N_0 indicates a low level of noise, while a high value of N_0 indicates a high level of noise. The value of N_0 can be used in conjunction with the SNR to determine the error rate in the system, as well as to design and evaluate the performance of communication systems. The noise is added to the transmitted waveform as $r(t) = s(t) + n(t)$, where $s(t)$ is the transmitted waveform and $n(t)$ is the Gaussian noise. In the project, I choose $N_0 = 18$, which makes the standard variation to be 3.

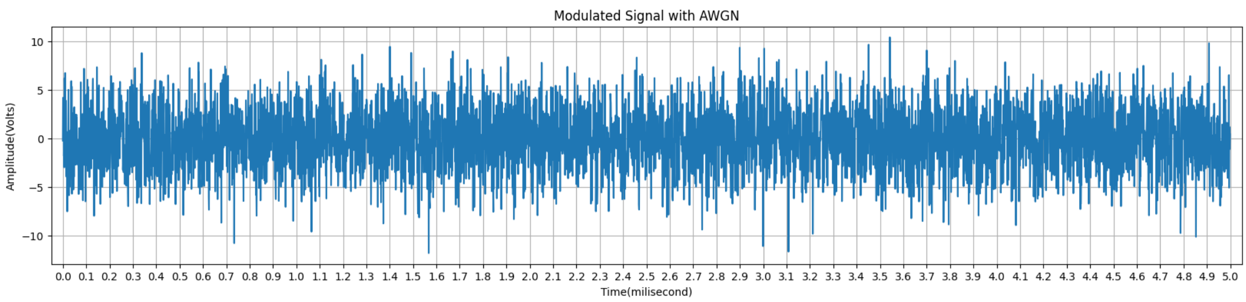


Figure 0.5: Modulated signal under the effect of $AWGN$

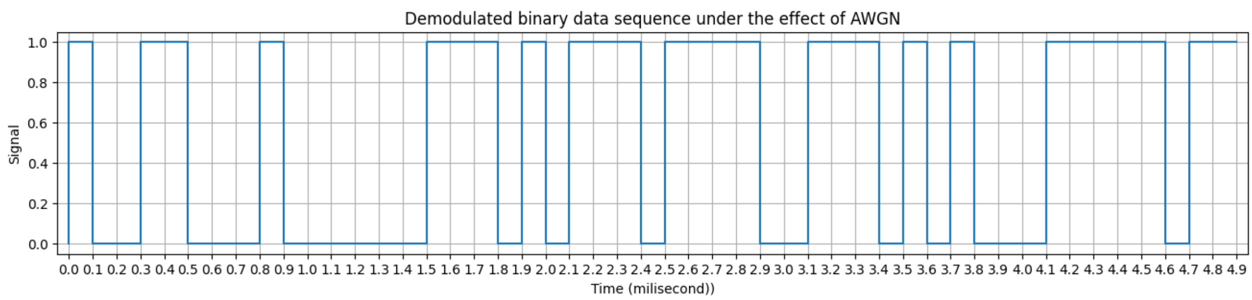


Figure 0.6: Demodulated signal under the effect of $AWGN$

0.5 Compute the Error rate

The error probability is calculated by comparing the original binary data sequence with the demodulated binary data under the effect of noise. The number of difference bits between the input and output bits divided by the total number of input bits represent the bit error rate.

Bit error rate: $P(e) = P(u_R[i] \neq u_T[i])$

From the above experiment, it can be seen that there are 4 bits difference between the input binary sequence and the demodulated signal after adding the noise, therefore the $P(e) = \frac{4}{50} = 8\%$

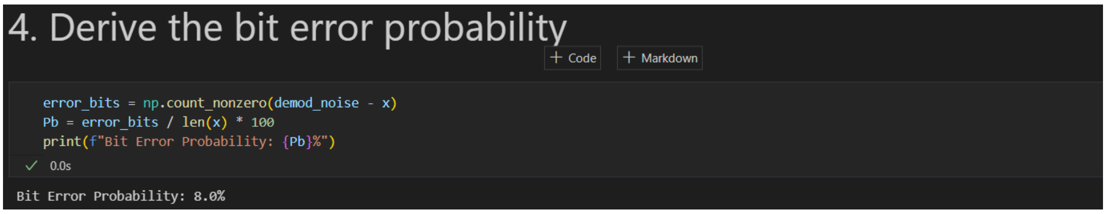


Figure 0.7: Bit Error probability under the effect of $AWGN$

0.6 Derive the BER of a Gaussian channel using the FSK modulation and demodulation

We have:

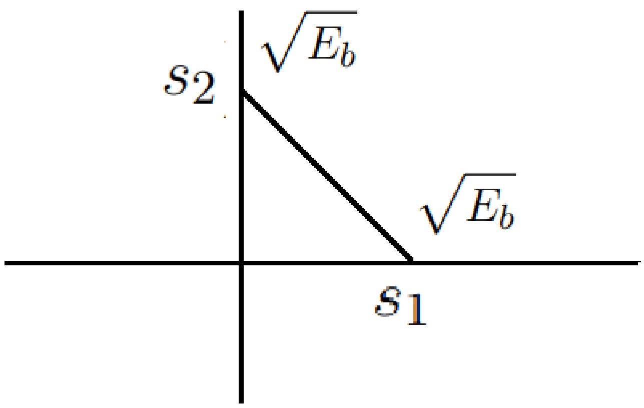
$$
\begin{aligned}
M &= \Big\{ s_1(t) = \cos(2\pi f_1 t),\ s_2(t) = \cos(2\pi f_2 t) \Big\} \\
E_1 &= \int_{-\infty}^{\infty} s_1^2(t)\,dt = \int_0^{T_b} \cos^2(2\pi f_1 t)\,dt = \frac{1}{2}\int_0^{T_b} \big[1 + cos(4\pi f_1 t)\big]\,dt \\
&= \frac{1}{2}T_b + \frac{1}{2}\int_0^{T_b} \cos(4\pi f_1 t)\,\frac{d(4\pi f_1 t)}{4\pi f_1} = \frac{T_b}{2} + \frac{1}{8\pi f_1}\sin(4\pi f_1 t)\Big|_0^{T_b} \\
&= \frac{T_b}{2} + \frac{\sin(4\pi f_1 T_b)}{8\pi f_1}
\end{aligned}
$$

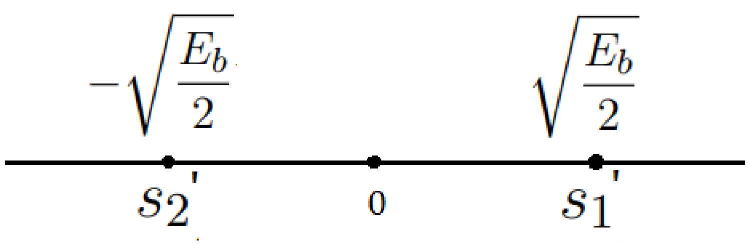
In this project, I choose $f_1 = 2\times$ bit rate $= \frac{2}{T_b}$ and $f_2 = 4\times$ bit rate $= \frac{4}{T_b}$, therefore $E_1 = \frac{T_b}{2}$.

$$E_2 = \int_{-\infty}^{\infty} s_2^2(t)\,dt = \int_0^{T_b} \cos^2(2\pi f_2 t)\,dt = \frac{T_b}{2}$$
$$E_s = \frac{E_1 + E_2}{2} = \frac{T_b}{2} \Rightarrow E_b = \frac{E_b}{k} = \frac{T_b}{2}\ (k=1)$$
$$\varphi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \sqrt{\frac{2}{T_b}}\cos\left(\frac{4\pi t}{T_b}\right)$$
$$s_{21}(t) = \int_0^{T_b} s_2(t)\varphi_1(t)\,dt$$
$$= \sqrt{\frac{2}{T_b}}\int_0^{T_b} \cos\left(\frac{8\pi t}{T_b}\right)\cos\left(\frac{4\pi t}{T_b}\right)dt$$
$$= \sqrt{\frac{1}{2T_b}}\int_0^{T_b} \left(\cos\left(\frac{12\pi t}{T_b}\right) + \cos\left(\frac{4\pi t}{T_b}\right)\right)dt = 0$$
$$g_2(t) = s_2(t) - s_{21}(t).\varphi_1(t) = \cos\left(\frac{8\pi t}{T_b}\right) \Rightarrow \varphi_2(t) = \frac{g_2}{\sqrt{E_2}} = \sqrt{\frac{2}{T_b}}\cos\left(\frac{8\pi t}{T_b}\right)$$

Then we have $B = \left\{\varphi_1(t) = \sqrt{\frac{2}{T_b}}\cos\left(\frac{4\pi t}{T_b}\right), \varphi_2(t) = \sqrt{\frac{2}{T_b}}\cos\left(\frac{8\pi t}{T_b}\right)\right\}$ is the orthonormal basis.

$$\Rightarrow s_1\Big(\sqrt{E_b}, 0\Big),\ s_2\Big(0, \sqrt{E_b}\Big)$$





According to the BER computation:

$$P\big(e|s_T = s_1\big) = P\big(s_1' + n < 0\big) = P\left(n < -\sqrt{\frac{E_b}{2}}\right) = P\left(n > \sqrt{\frac{E_b}{2}}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$P\big(e|s_T = s_2\big) = P\left(s_2' + n > \sqrt{\frac{E_b}{2}}\right) = P\left(-\sqrt{\frac{E_b}{2}} + n > 0\right) = P\left(n > \sqrt{\frac{E_b}{2}}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$P(e) = \frac{P\big(e|s_T = s_1\big) + P\big(e|s_T = s_2\big)}{2} = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$