gAGE

gAGE research group of Astronomy and Geomatics
Barcelona, Spain

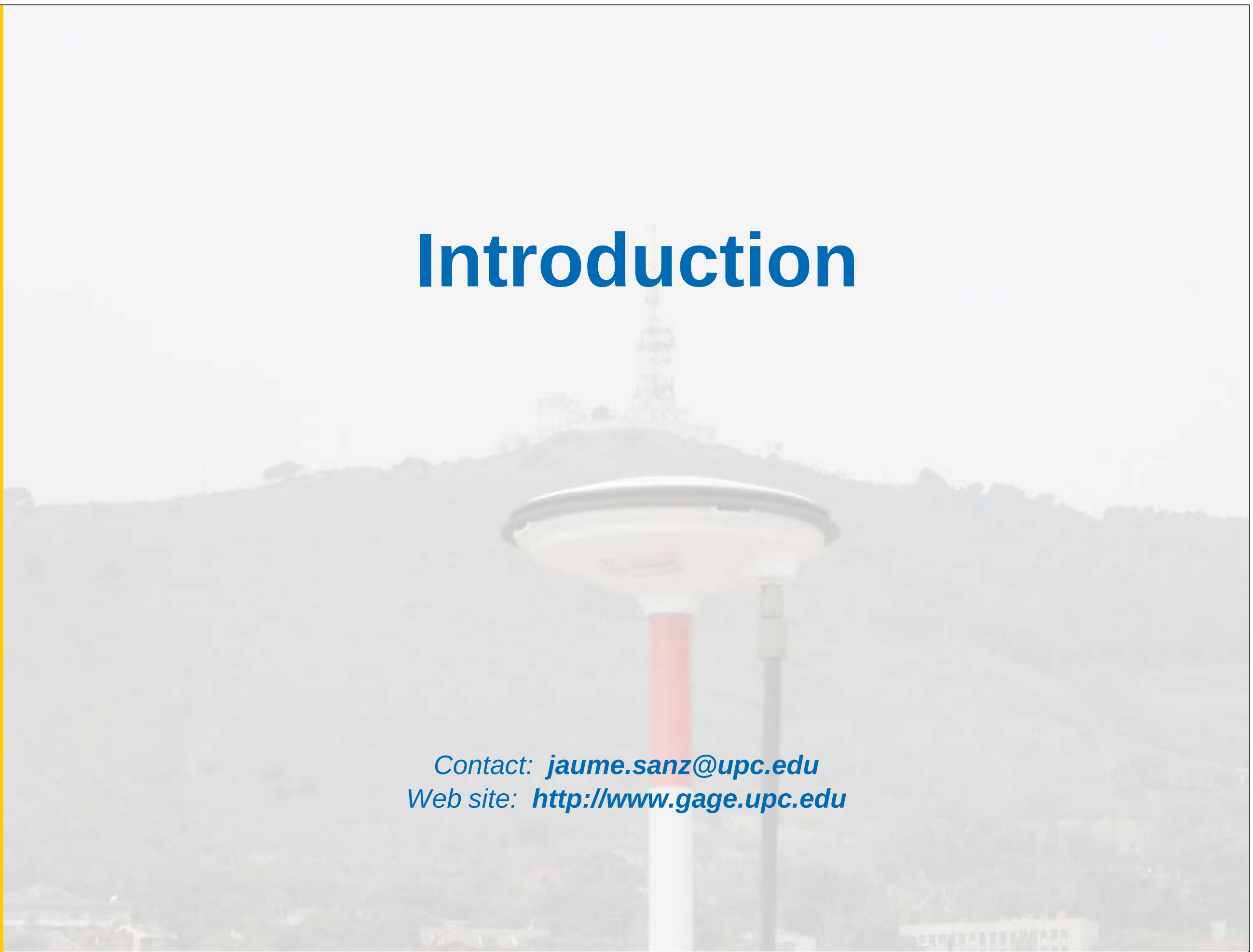
GNSS Data Processing

Theory

Slides

http://www.gage.upc.edu

Contents



Introduction

Contact: ***jaume.sanz@upc.edu***
Web site: ***http://www.gage.upc.edu***

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Specific Objectives:

- To learn about GNSS observables (code and phase), their characteristics, properties, combinations and applications.
- To learn how to calculate satellites orbits and clocks from navigation message. To know the achievable precision.
- To learn how to model pseudodistance for code and phase measurements. This includes calculation of: 1) Coordinates at emission epoch, 2) Ionospheric delay (Klobuchar model), 3) Tropospheric delay, 4) relativistic correction, 5) clocks offsets and satellite instrumental delays, 6) phase wind-up, etc.
- To learn how to set and solve the navigation equation system using least-squares or Kalman filter (algorithm level).
- To know how to use phase differential positioning: Floating and fixing ambiguities.
- To learn Carrier Phase Ambiguity Fixing techniques.

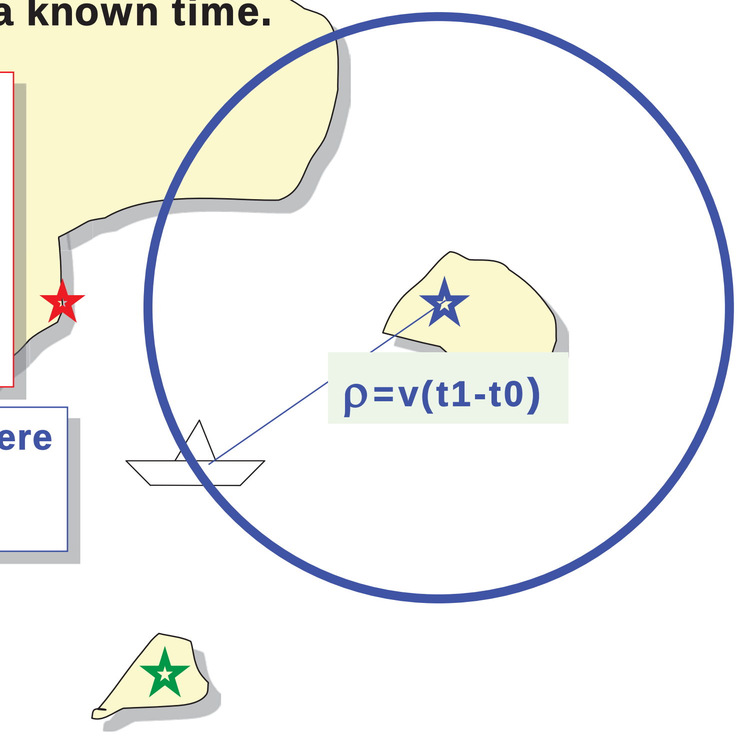
To get tools and skills to process and analyze GNSS data. To implement algorithms for satellite navigation.

An intuitive approach to GNSS positioning

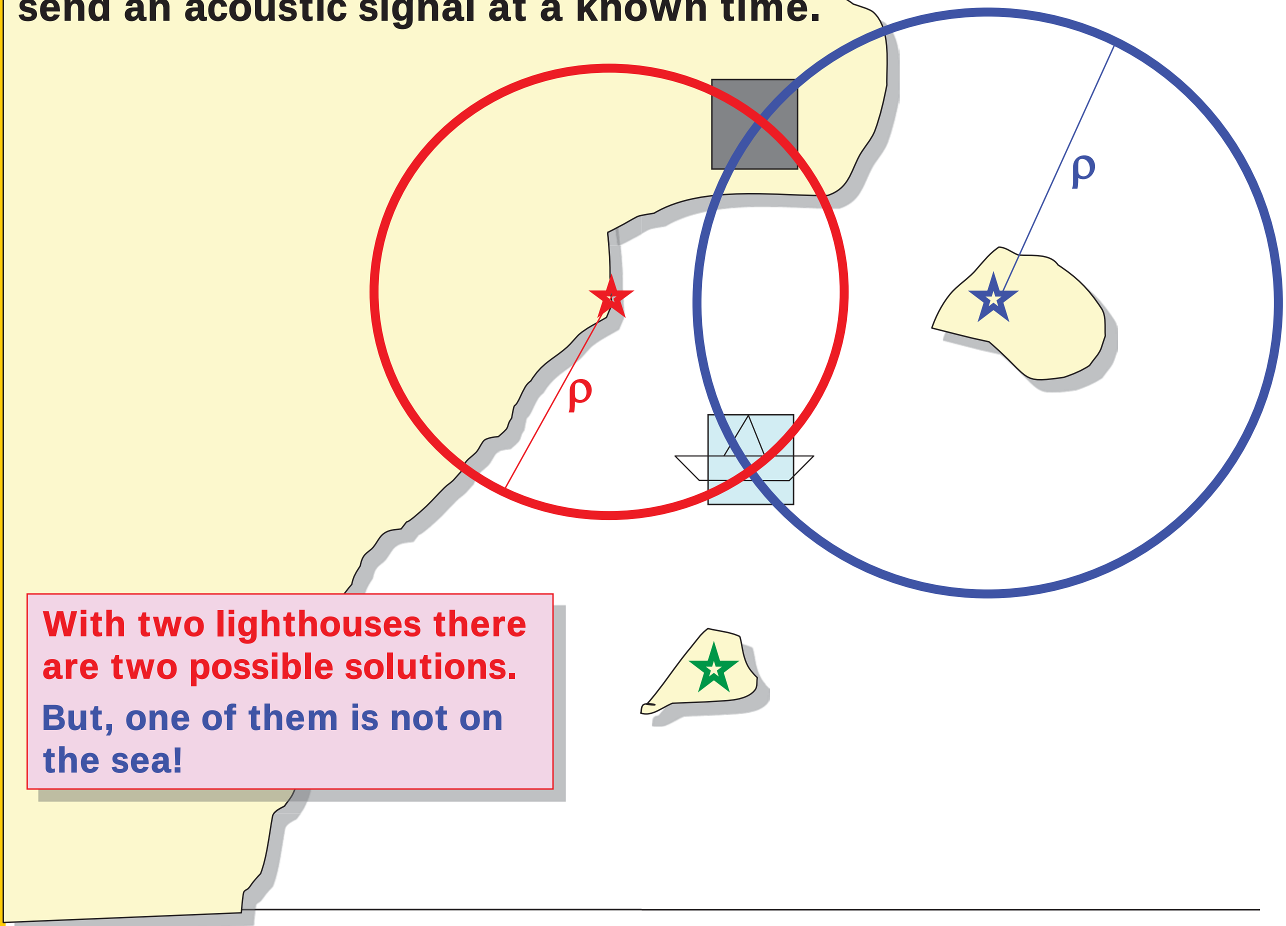
A ship determines its location from a set on lighthouses that send an acoustic signal at a known time.

Knowing the emission time "t0" in the lighthouse and the reception time "t1" in the ship, the traveling time "t1-t0", and the geometric range "ρ=v(t1-t0)" may be computed.

With only one lighthouse there is a whole circumference of possible locations



A ship determines its location from a set on lighthouses that send an acoustic signal at a known time.



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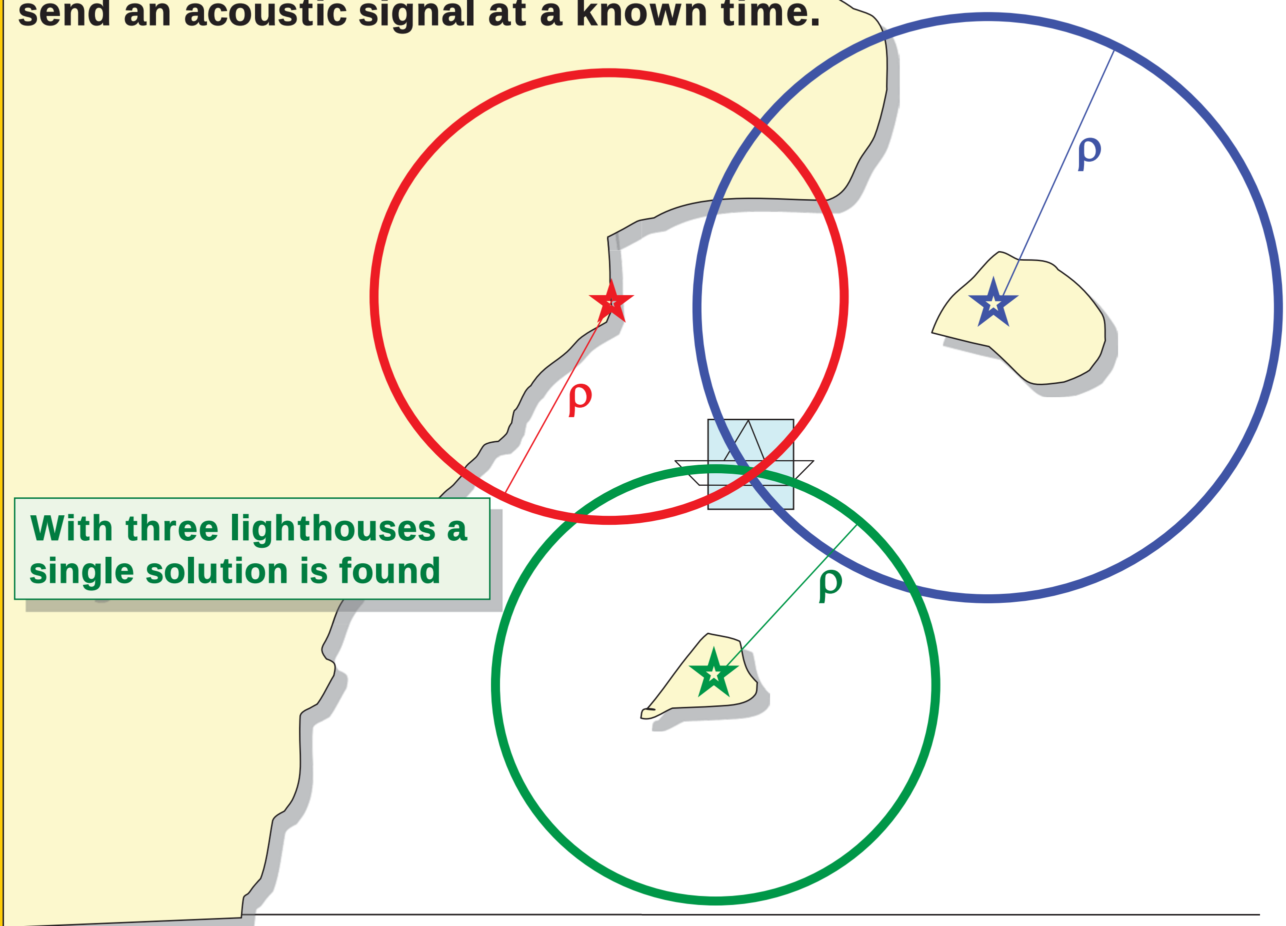
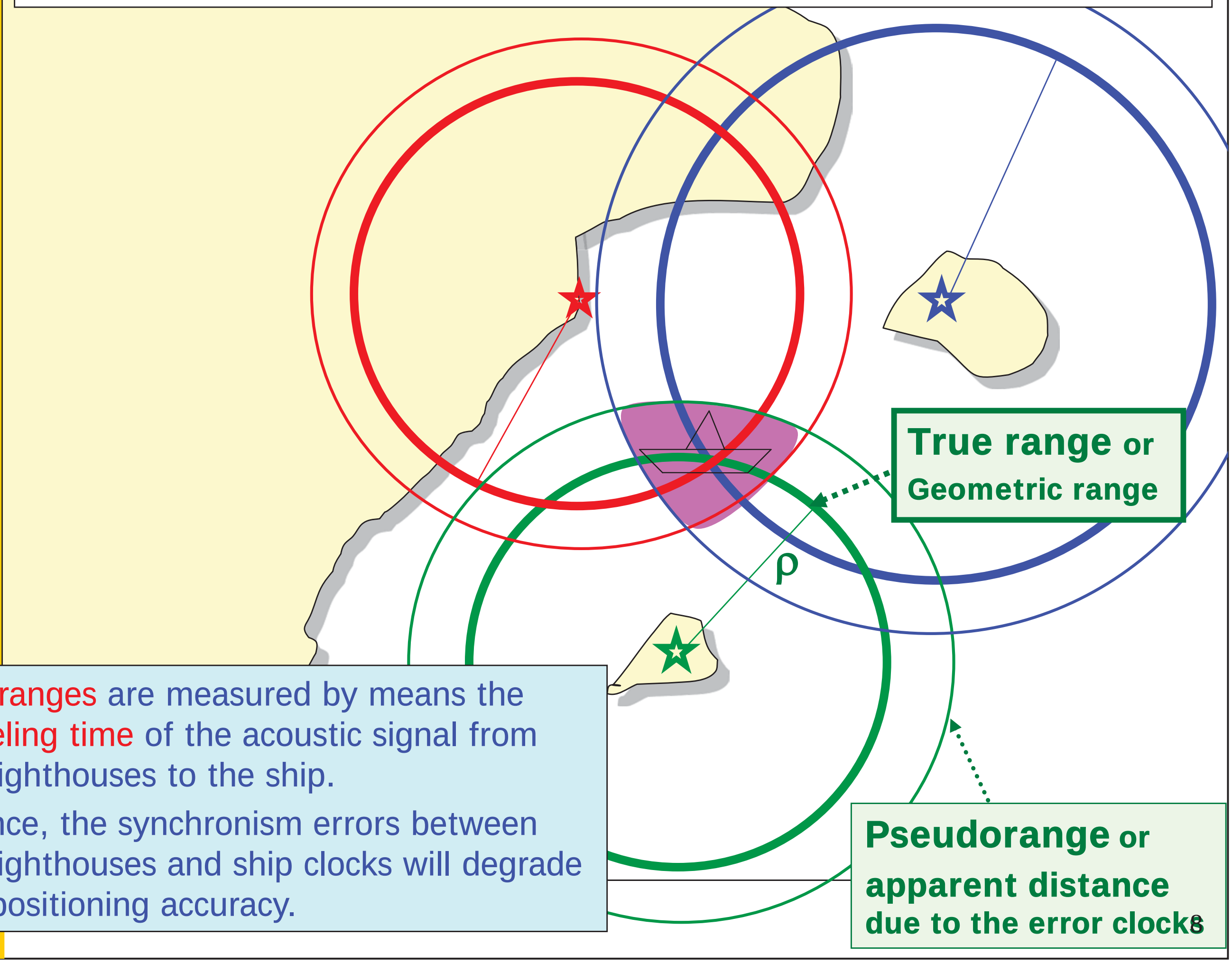
A ship determines its location from a set on lighthouses that send an acoustic signal at a known time.



Errors in the clocks (lighthouses and ship) synchronism affects the accuracy

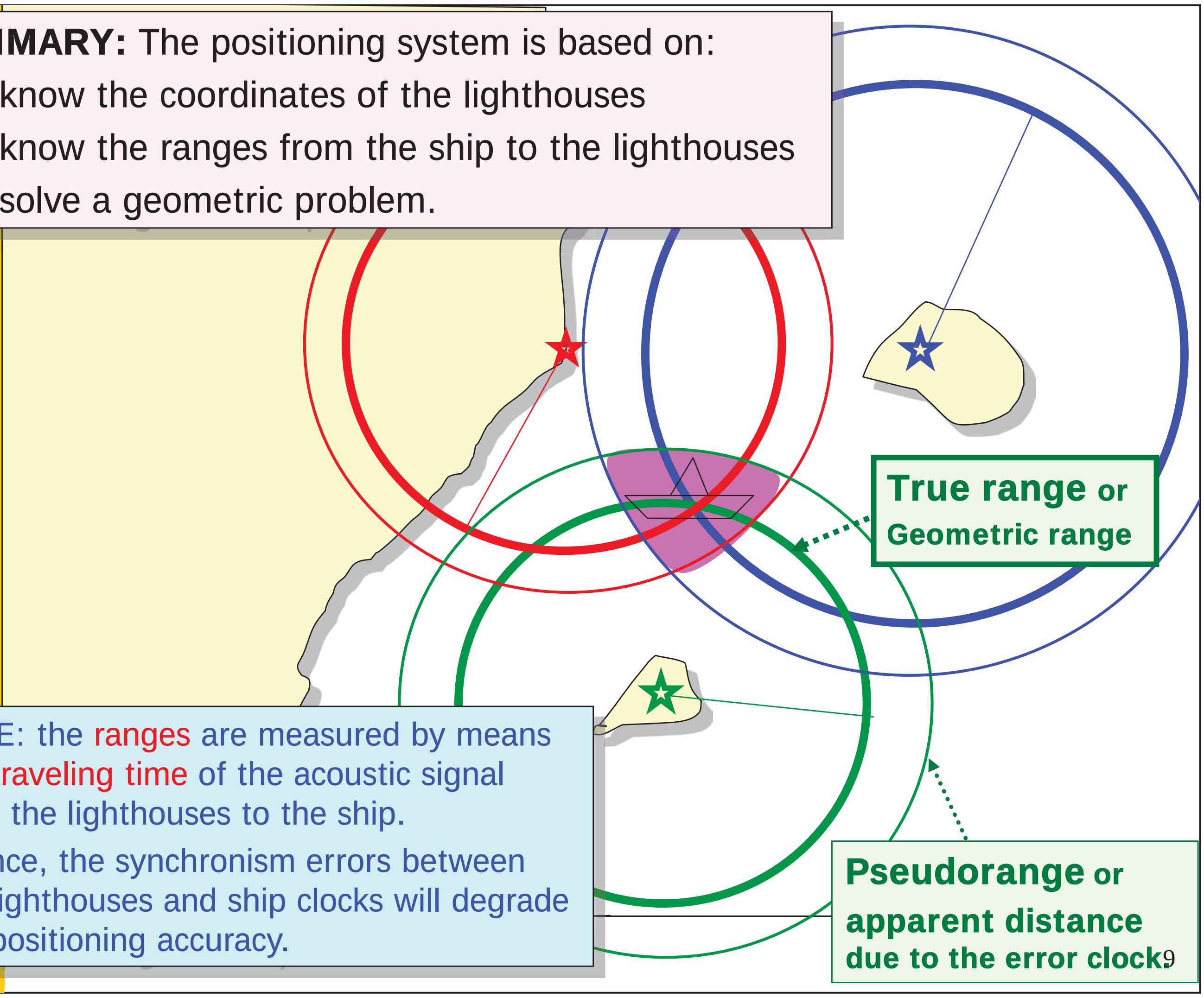


The ranges are measured by means the traveling time of the acoustic signal from the lighthouses to the ship.

Thence, the synchronism errors between the lighthouses and ship clocks will degrade the positioning accuracy.

SUMMARY: The positioning system is based on:

- To know the coordinates of the lighthouses
- To know the ranges from the ship to the lighthouses
- To solve a geometric problem.



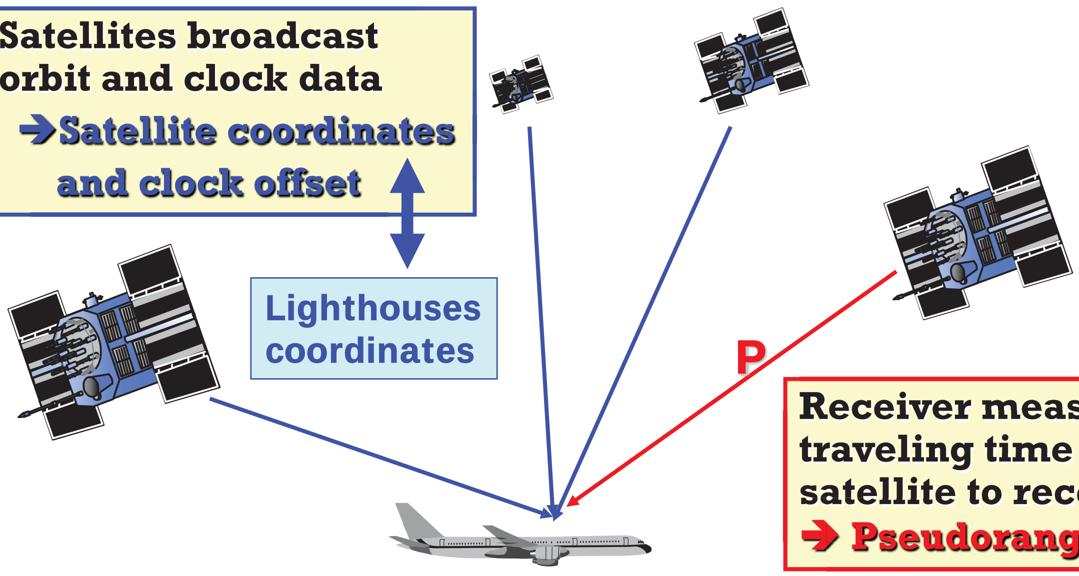
NOTE: the ranges are measured by means the traveling time of the acoustic signal from the lighthouses to the ship.

Thence, the synchronism errors between the lighthouses and ship clocks will degrade the positioning accuracy.

How GNSS Works

Satellites broadcast orbit and clock data
➔Satellite coordinates and clock offset

Lighthouses coordinates

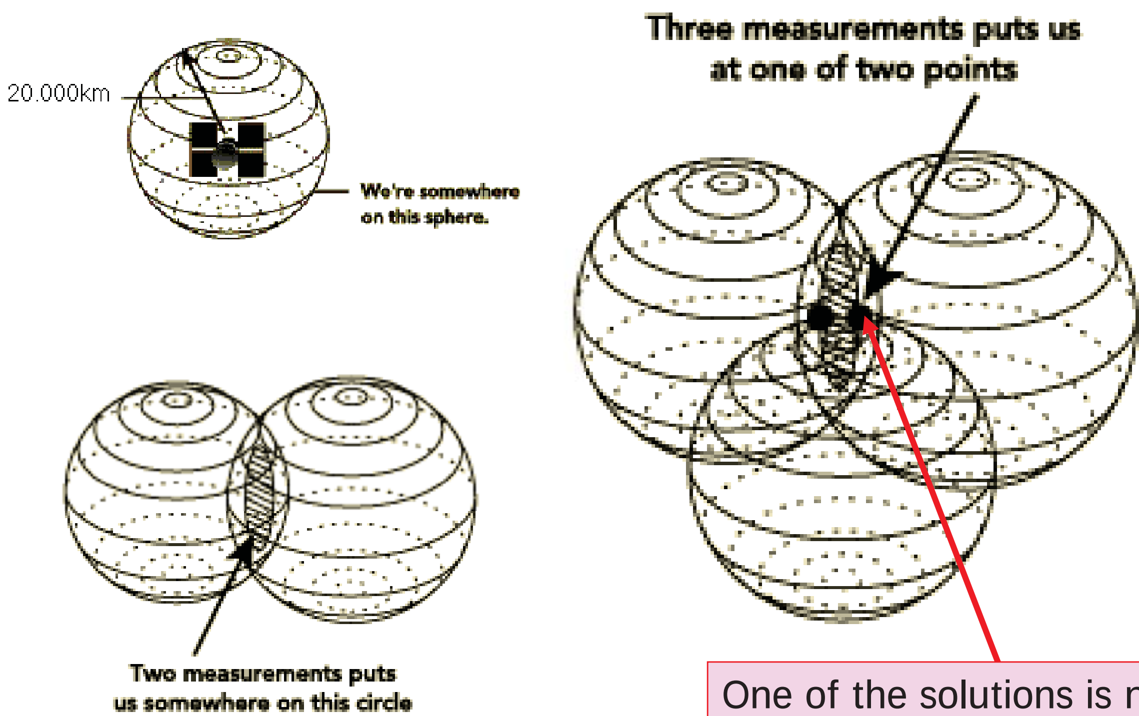


Receiver measures traveling time from satellite to receiver
➔ Pseudorange (P)

Lighthouses-ship ranges.

Thence, the receiver coordinates are found **solving a geometrical problem**: from sat. coordinates and ranges

How GNSS Works



One of the solutions is not on the Earth surface.

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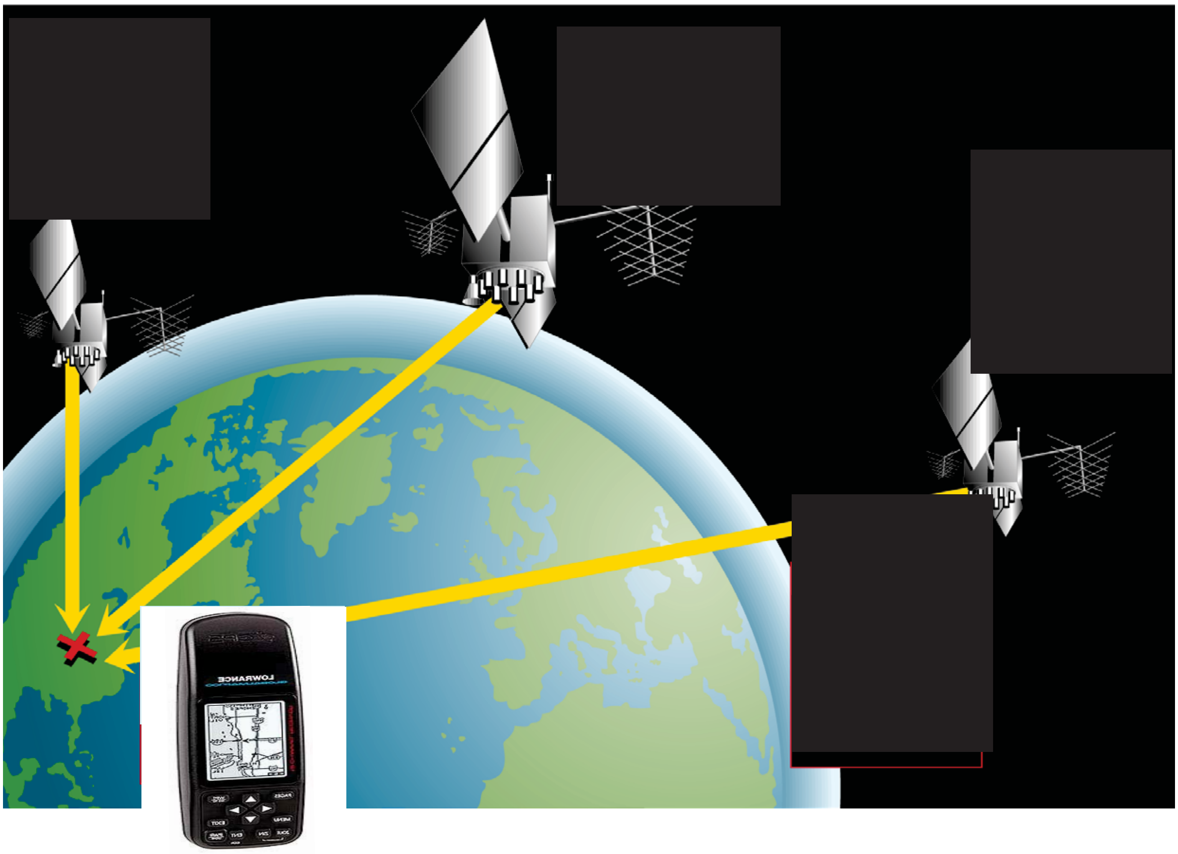
How GNSS Works



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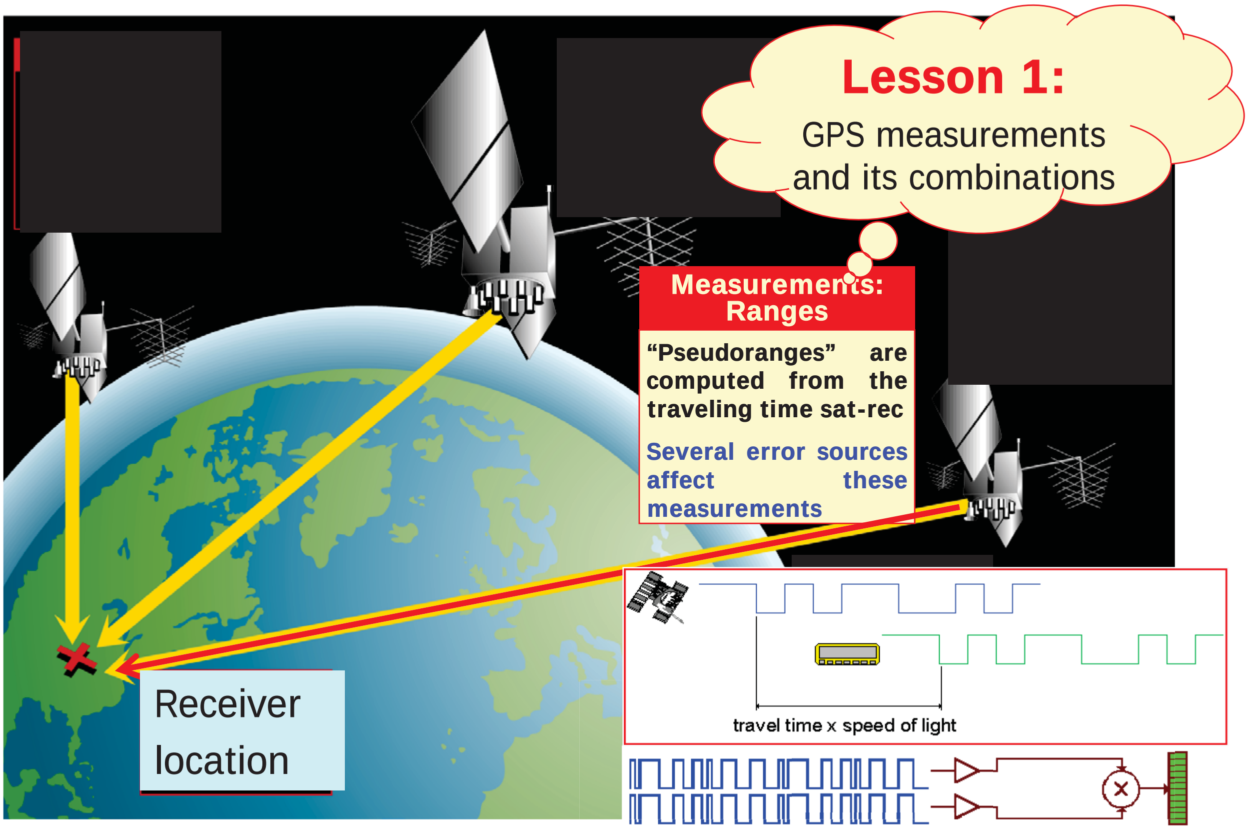
How GNSS Works



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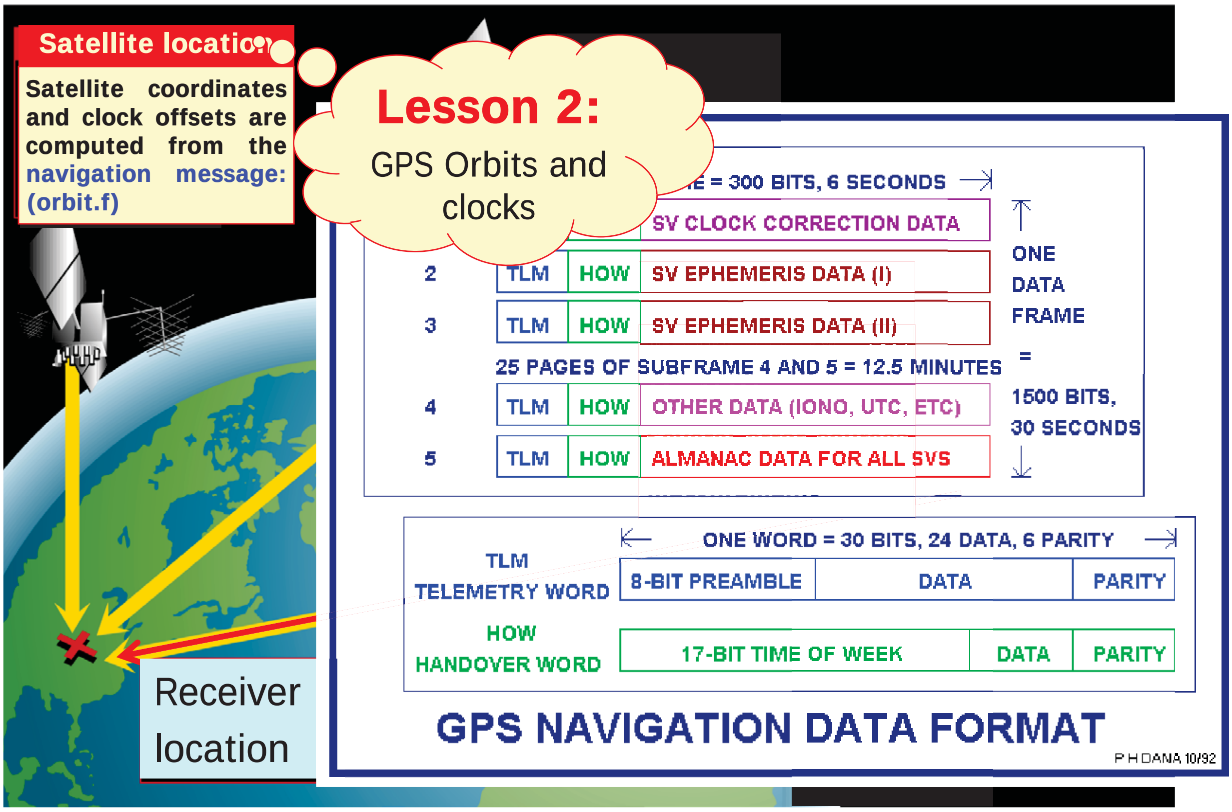
How GNSS Works



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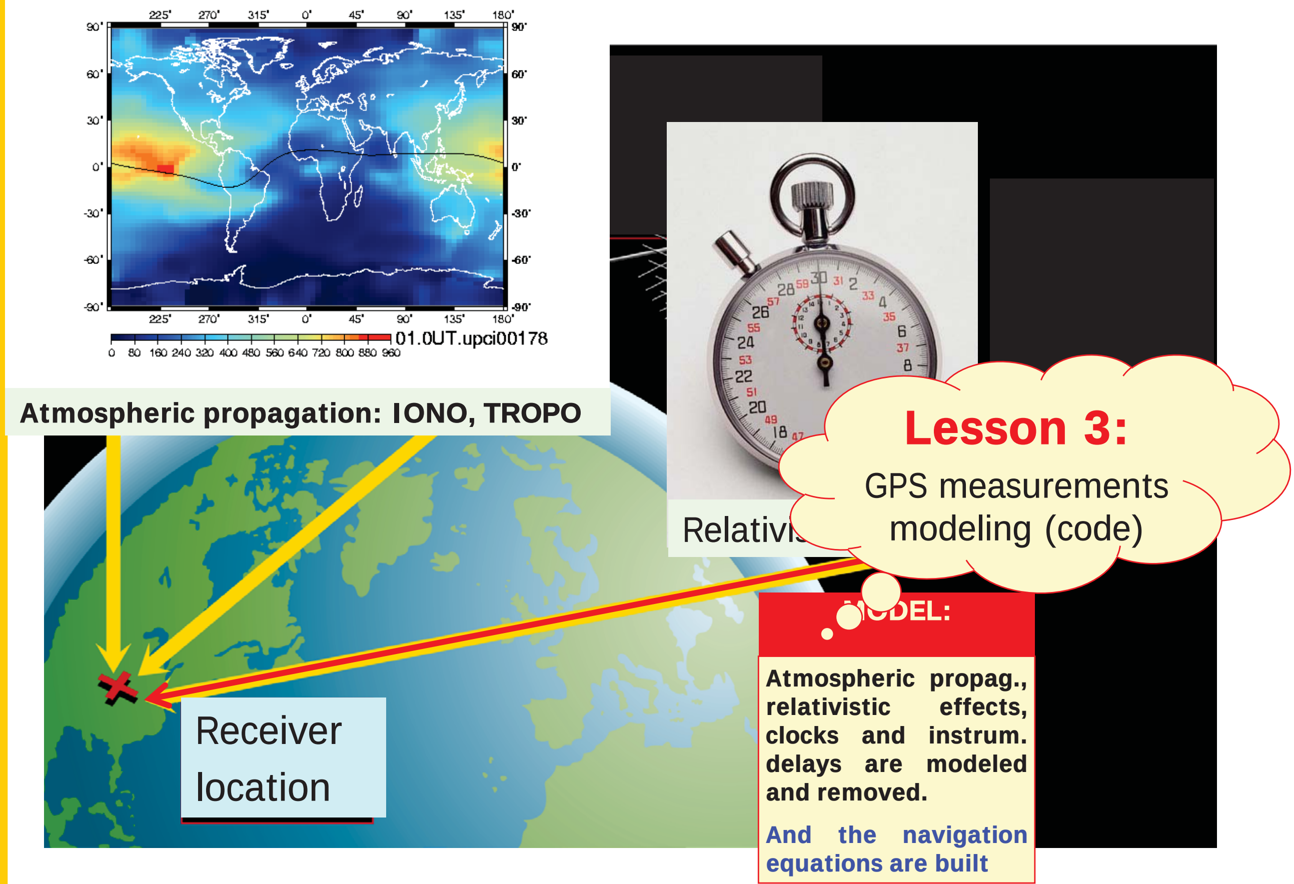
How GNSS Works



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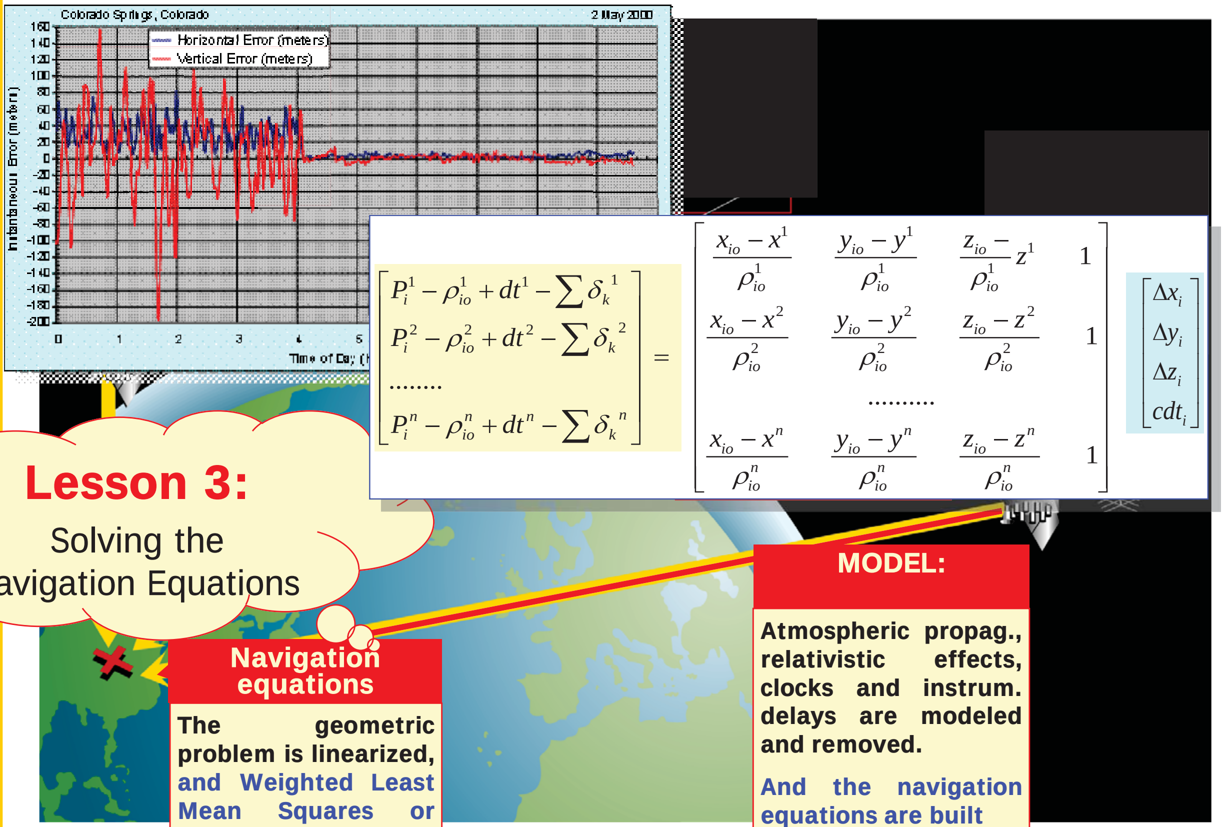
How GNSS Works



$$\begin{bmatrix} P_i^1-\rho_{io}^1+dt^1-\sum\delta_k{}^1 \\ P_i^2-\rho_{io}^2+dt^2-\sum\delta_k{}^2 \\ \cdots\cdots \\ P_i^n-\rho_{io}^n+dt^n-\sum\delta_k{}^n \end{bmatrix} = \begin{bmatrix} \frac{x_{io}-x^1}{\rho_{io}^1} & \frac{y_{io}-y^1}{\rho_{io}^1} & \frac{z_{io}-z^1}{\rho_{io}^1} & 1 \\ \frac{x_{io}-x^2}{\rho_{io}^2} & \frac{y_{io}-y^2}{\rho_{io}^2} & \frac{z_{io}-z^2}{\rho_{io}^2} & 1 \\ & \cdots\cdots\cdots & & \\ \frac{x_{io}-x^n}{\rho_{io}^n} & \frac{y_{io}-y^n}{\rho_{io}^n} & \frac{z_{io}-z^n}{\rho_{io}^n} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ cdt_i \end{bmatrix}$$

Navigation equations

The geometric problem is linearized, and Weighted Least Mean Squares or Kalman filter are used to compute the solution.

MODEL:

Atmospheric propag., relativistic effects, clocks and instrum. delays are modeled and removed.

And the navigation equations are built

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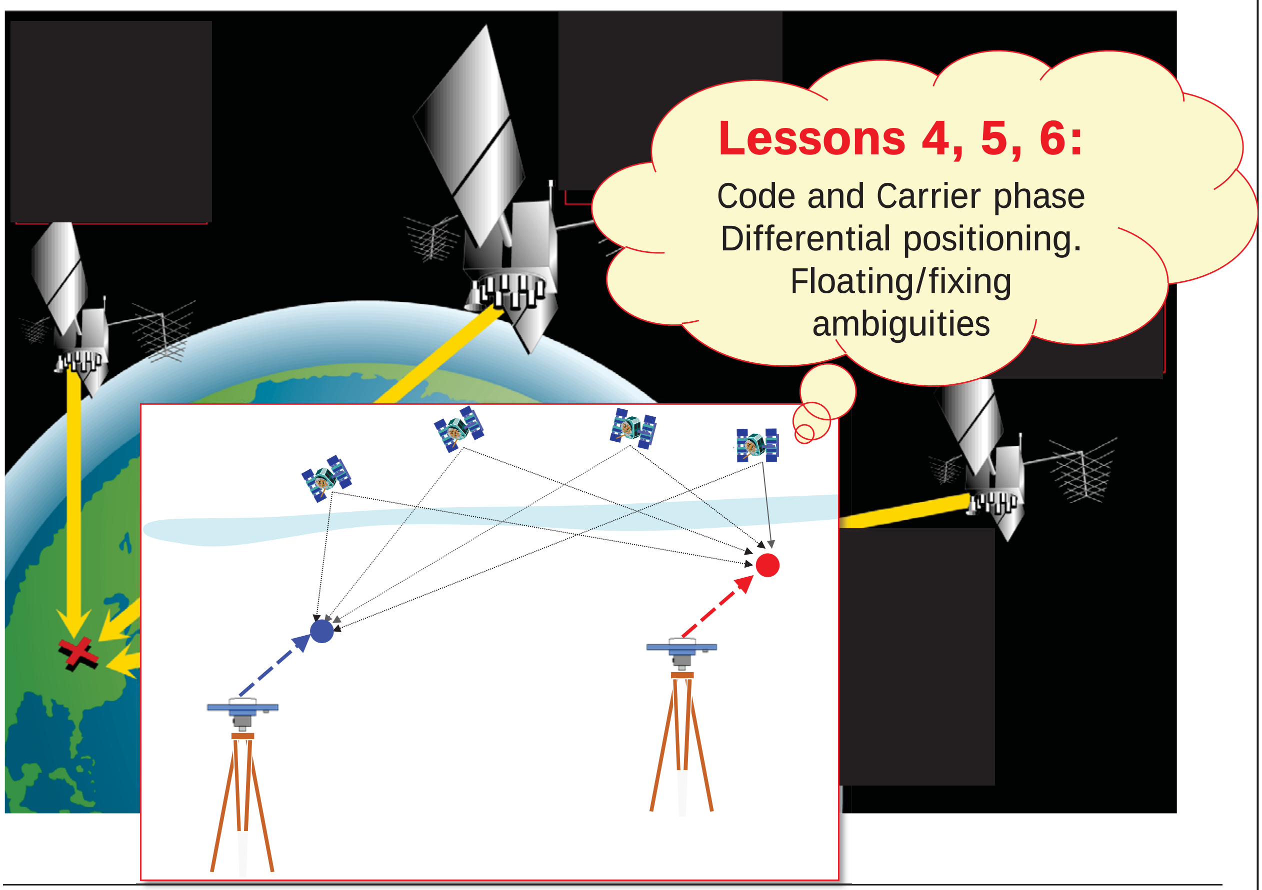
How GNSS Works



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References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

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Thank you!

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Lecture 1

GNSS measurements and their combinations

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Authorship statement

22 Jan 2015

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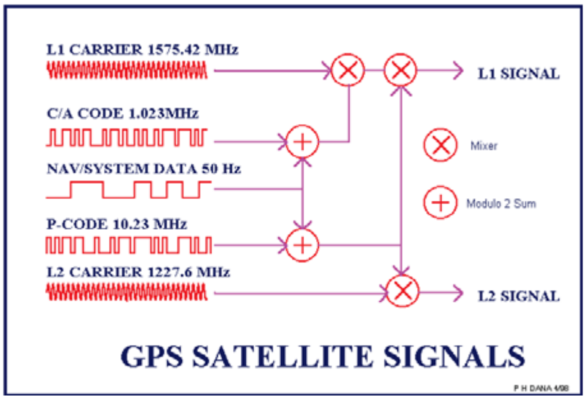
GPS SIGNAL STRUCTURE

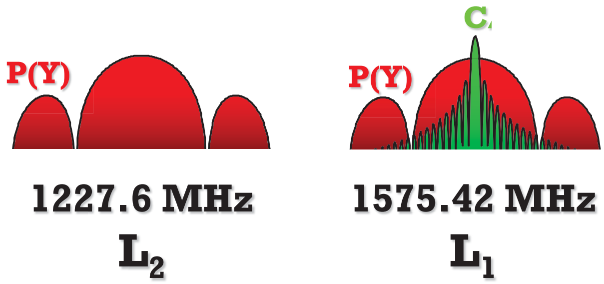
Two carriers in L-band:

- L_1=154 fo=1575.42 MHz
- L_2=120 fo=1227.60 MHz

where fo=10.23 MHz

- C/A-code for civilian users *[$X_C(t)$]*
- P-code only for military and authorized users *[$X_P(t)$]*
- Navigation message with satellite ephemeris and clock corrections *[$D(t)$]*





$$S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_1 t + \phi_{L_1}) + a_C X_C^{(k)}(t)\, D^{(k)}(t) \cos(\omega_1 t + \phi_{L_1})$$

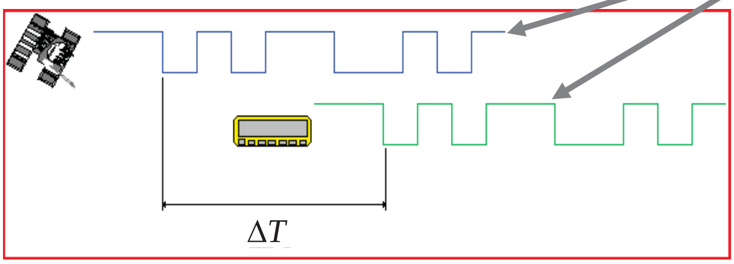
$$S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_2 t + \phi_{L_2})$$

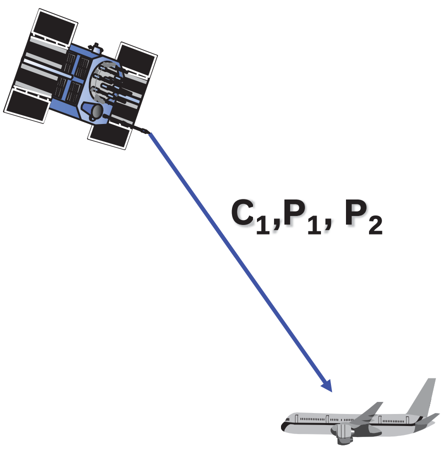
GPS Code Pseudorange Measurements

$$S_{L_1}^{(k)}(t) = a_P \boxed{X_P^{(k)}(t)}\, D^{(k)}(t) \sin(\omega_1 t + \varphi_{L_1}) + a_C X_C^{(k)}(t)\, D^{(k)}(t) \cos(\omega_1 t + \varphi_{L_1})$$

$$S_{L_2}^{(k)}(t) = b_P \boxed{X_P^{(k)}(t)}\, D^{(k)}(t) \sin(\omega_2 t + \varphi_{L_2})$$

binary code $X_P(t)$





$$\mathrm{P}(T) = c\,\Delta T = c\left[t_{rec}(T) - t^{sat}(T - \Delta T)\right]$$

From hereafter we will call:

- C_1 pseudorange computed from $X_C(t)$ binary code (on frequency 1)
- P_1 pseudorange computed from $X_P(t)$ binary code (on frequency 1)
- P_2 pseudorange computed from $X_P(t)$ binary code (on frequency 2)

GPS Carrier Phase Measurements

$$S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_1 t + \varphi_{L_1}) + a_C X_C^{(k)}(t)\, D^{(k)}(t) \cos(\omega_1 t + \varphi_{L_1})$$

$$S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t)\, D^{(k)}(t) \sin(\omega_2 t + \varphi_{L_2})$$

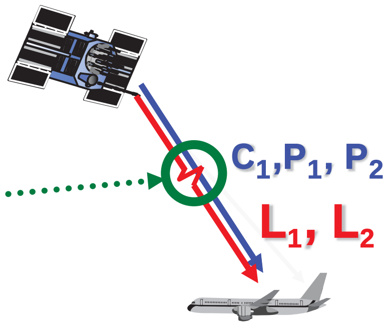
Carrier phase

Carrier beat phase:

$$\phi_L(T) = \phi_{L\,rec}(T) - \phi_L^{sat}(T - \Delta\tilde{T})$$

$$= \frac{c}{\lambda}\Delta\tilde{T} + N$$

Unknown ambiguity

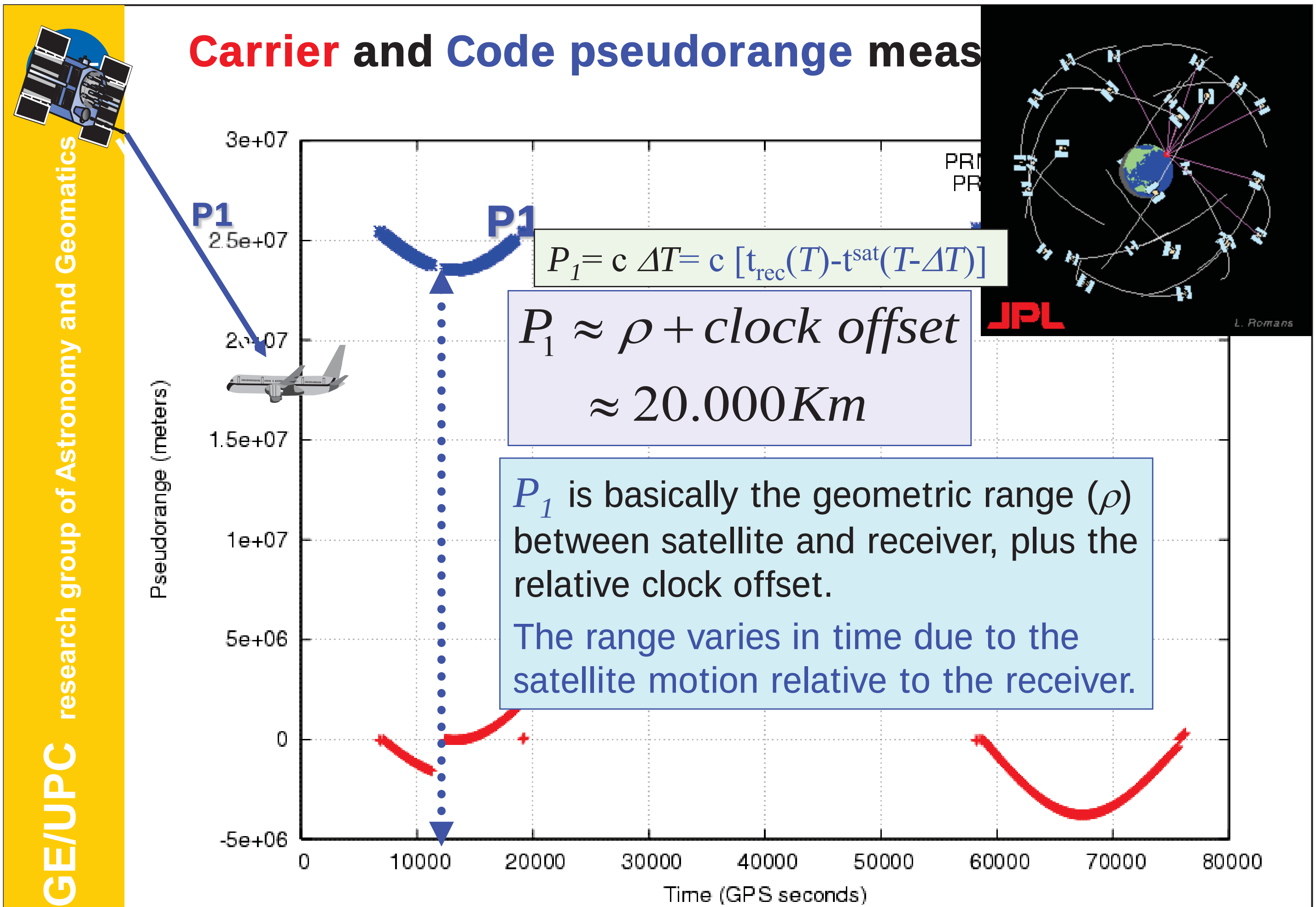


From hereafter we will call:

- $L_1 = \lambda_1 \phi_{L1}$ measur. computed from the carrier phase on frequency 1
- $L_2 = \lambda_2 \phi_{L2}$ measur. computed from the carrier phase on frequency 2

- C_1 pseudorange computed from $X_C(t)$ binary code (on frequency 1)
- P_1 pseudorange computed from $X_P(t)$ binary code (on frequency 1)
- P_2 pseudorange computed from $X_P(t)$ binary code (on frequency 2)

Carrier and **Code pseudorange meas**



$P_1 = c\,\Delta T = c\,[t_{rec}(T) - t^{sat}(T-\Delta T)]$

$$P_1 \approx \rho + clock\ offset$$

$$\approx 20.000\,Km$$

P_1 is basically the geometric range (ρ) between satellite and receiver, plus the relative clock offset.

The range varies in time due to the satellite motion relative to the receiver.

P_1 is an **absolute** measurement **(unambiguous)**

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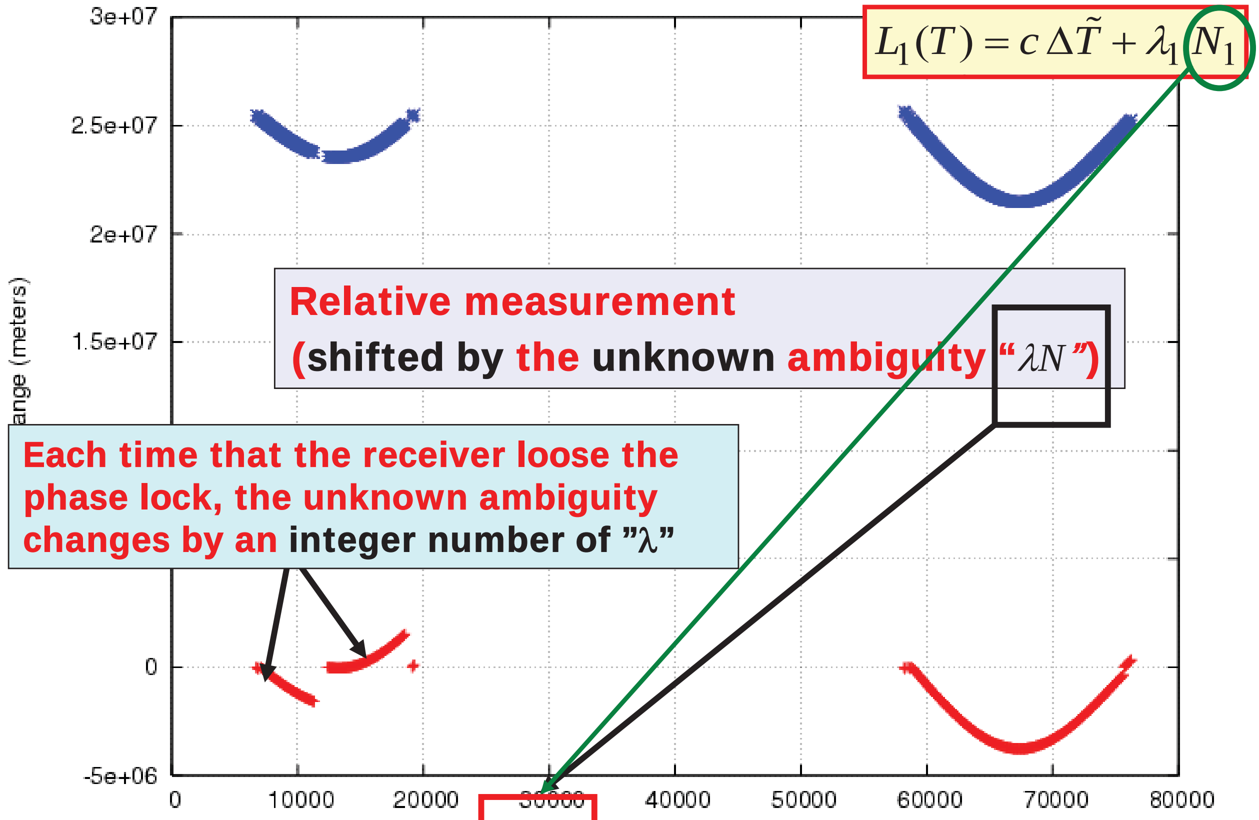
Phase and **Code pseudorange** measurements



$$L_1 \approx \rho + clock\ offset + \boxed{\lambda_1 N_1}$$

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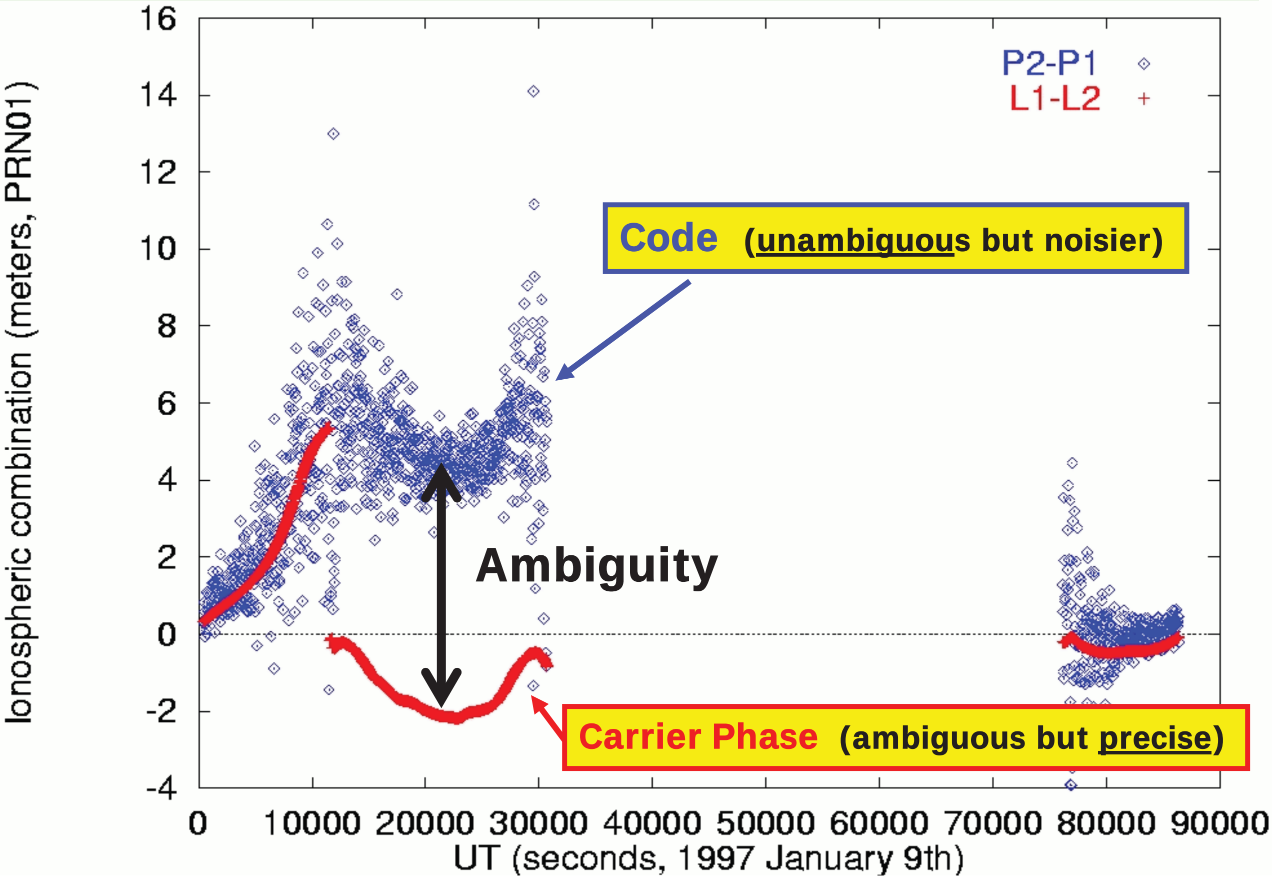
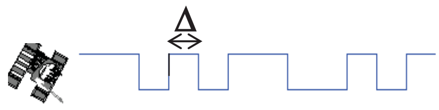
Code and Carrier Phase measurements



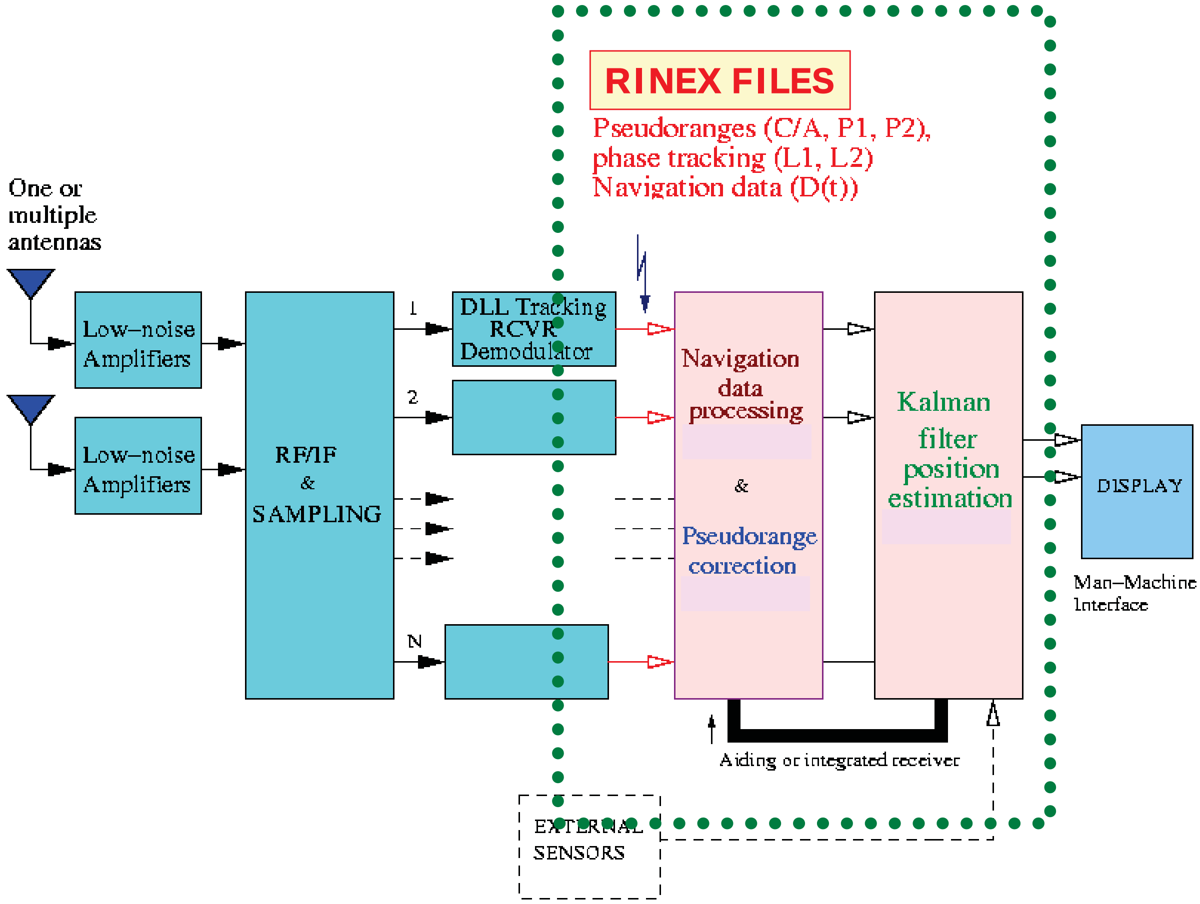
GPS measurements: Code and Carrier Phase



Antispoofing (A/S):
The code P is encrypted to Y.
➔ Only the code C at frequency L1 is available.

	Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
	Code measurements		
C_1	300 m	3 m	**Unambiguous** but noisier
P_1 (Y1): encrypted	30 m	30 cm	
P_2 (Y2): encrypted	30 m	30 cm	
	Phase measurements		
L_1	19.05 cm	2 mm	**Precise** but ambiguous
L_2	24.45 cm	2 mm	

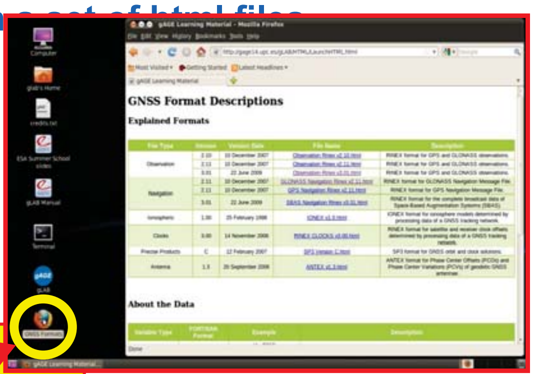
[*] the codes can be smoothed with the phases in order to reduce noise (i.e, C_1 smoothed with L_1 ➔ 50 cm noise)

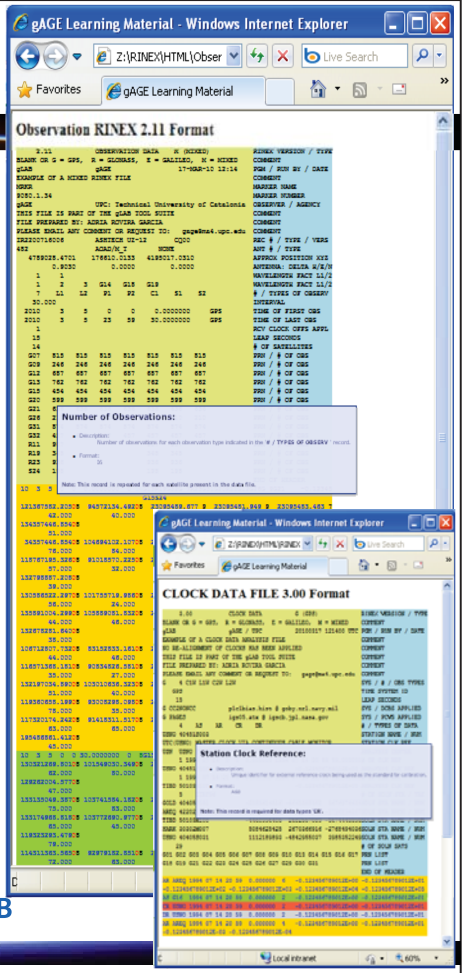


GNSS Format Descriptions

- GNSS data files follow a well defined set of standards formats: RINEX, ANTEX, SINEX…
- Understanding a format description is a tough task.
- These standards are explained in a very easy and friendly way through a set of html files.
- Described formats:
  - Observation RINEX
  - Navigation RINEX
  - RINEX CLOCKS
  - SP3 Version C
  - ANTEX

Open GNSS Formats with **Firefox** internet browser





More details at: http://www.gage.es/gLAB

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RINEX measurement file

HEADER

```
        2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX     AUSLIG              10-JAN-97 10:19     PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN)     - COCOS ISLAND      COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION    COMMENT
        -0.000000000103              HARDWARE CALIBRATION (S)  COMMENT
        -0.000000054663              CLOCK OFFSET (S)          COMMENT
COCO                                                           MARKER NAME
AU18                                                           MARKER NUMBER
mrh                    uslig                                   OBSERVER / AGENCY
126                    ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                    DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                     APPROX POSITION XYZ
        0.0040        0.0000        0.0000                     ANTENNA: DELTA H/E/N
     1     1                                                   WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                           # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                     COMMENT
 SNR:    >500  >100   >50   >10    >5    >0   bad   n/a        COMMENT
 sig:       9     8     7     6     5     4     1     0        COMMENT
    30                                                         INTERVAL
  1997     1     9     0     7   30.0000000                    TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                    TIME OF LAST OBS
```

MEASUREMENTS

```
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127                                 1118481.28445  22127685.4014  <====  1
  22672                                 8969469.30045  22672158.5184  <==== 25
  22594902.367    -12949753.825 7   -10090708.53945  22594903.7394  <====  9
  22731128.796    -11621184.951 7    -9055464.16945  22731130.0094  <====  5
  24610920.702      -924108.174 6     -720085.67045  24610920.0404  <==== 23
  20718775.074    -18605935.474 9   -14498133.97346  20718775.6074  <==== 17
  20842713.610    -19083282.892 9   -14870090.55546  20842713.4814  <====  6
```

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     -0.000000054663         CLOCK OFFSET (S)               COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  741950.3241  6190961.9624 -1337769.9813                   APPROX POSITION XYZ
                                                            ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:    >500  >100   >50   >10    >5    >0   bad   n/a     COMMENT
 sig:       9     8     7     6     5     4     1     0     COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014   <===  1
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184   <=== 25
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394   <===  9
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094   <===  5
  24610920.702     -924108.174 6   -720085.67045  24610920.0404   <=== 23
  20718775.074   -18605935.474 9 -14498133.97346  20718775.6074   <=== 17
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COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
                        SNR-8100        93.05.25 / 2.8.33.2 REC # / TYPE / VERS
                      MARGOLIN T                            ANT # / TYPE
                    9624 -1337769.9813                      APPROX POSITION XYZ
                    0000        0.0000                      ANTENNA: DELTA H/E/N
                                                            WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:    >500  >100                                         COMMENT
 sig:       9     8                                         COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014   <===  1
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184   <=== 25
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394   <===  9
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094   <===  5
  24610920.702     -924108.174 6   -720085.                       <=== 23
  20718                          9 -14498133.                          17
  20842                          9 -14870090.                           6
```

Measurement time (receive time tags)

Number of tracked satellites

Epoch flag 0: OK

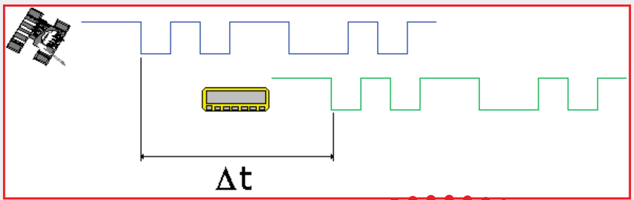
One satellite per row

RINEX measurement file

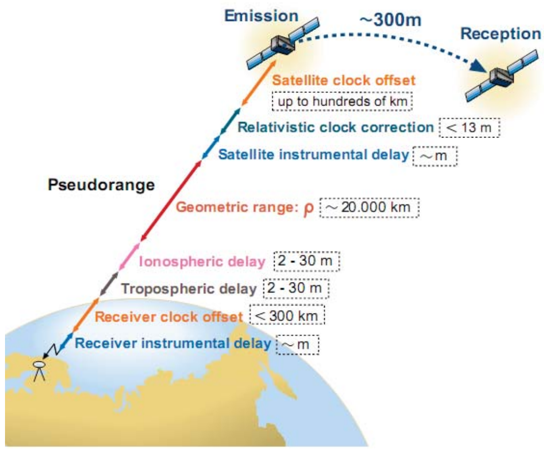
```
     2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX  AUSLIG              10-JAN-97 10:19     PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN) - COCOS ISLAND       COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
     -0.000000000103           HARDWARE CALIBRATION (S)     COMMENT
     -0.000000054663           CLOCK OFFSET (S)             COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:    >500  >100   >50   >10    >5    >0   bad   n/a     COMMENT
 sig:      9     8     7     6     5     4     1     0      COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105  -14268715.899 8 -11118481.28445  22127685.4014   <====== 1
  22672158.746  -11510817.892 7  -8969469.30045  22672158.5184   <====== 25
  22594902.367  -12949753.825 7 -10090708.53945  22594903.7394   <====== 9
  22731128.796  -11621184.951 7  -9055464.16945  22731130.0094   <====== 5
  24610920.702    -924108.174 6   -720085.67045  24610920.0404   <====== 23
  20718775.074  -18605935.474 9 -14498133.97345  20718775.6074   <====== 17
  20842713.610  -19083282.892 9 -14870090.55545  20842713.4814   <====== 6
```

Synthetic P2 (A/S=on)

S/N indicator **Loss of lock indicator**

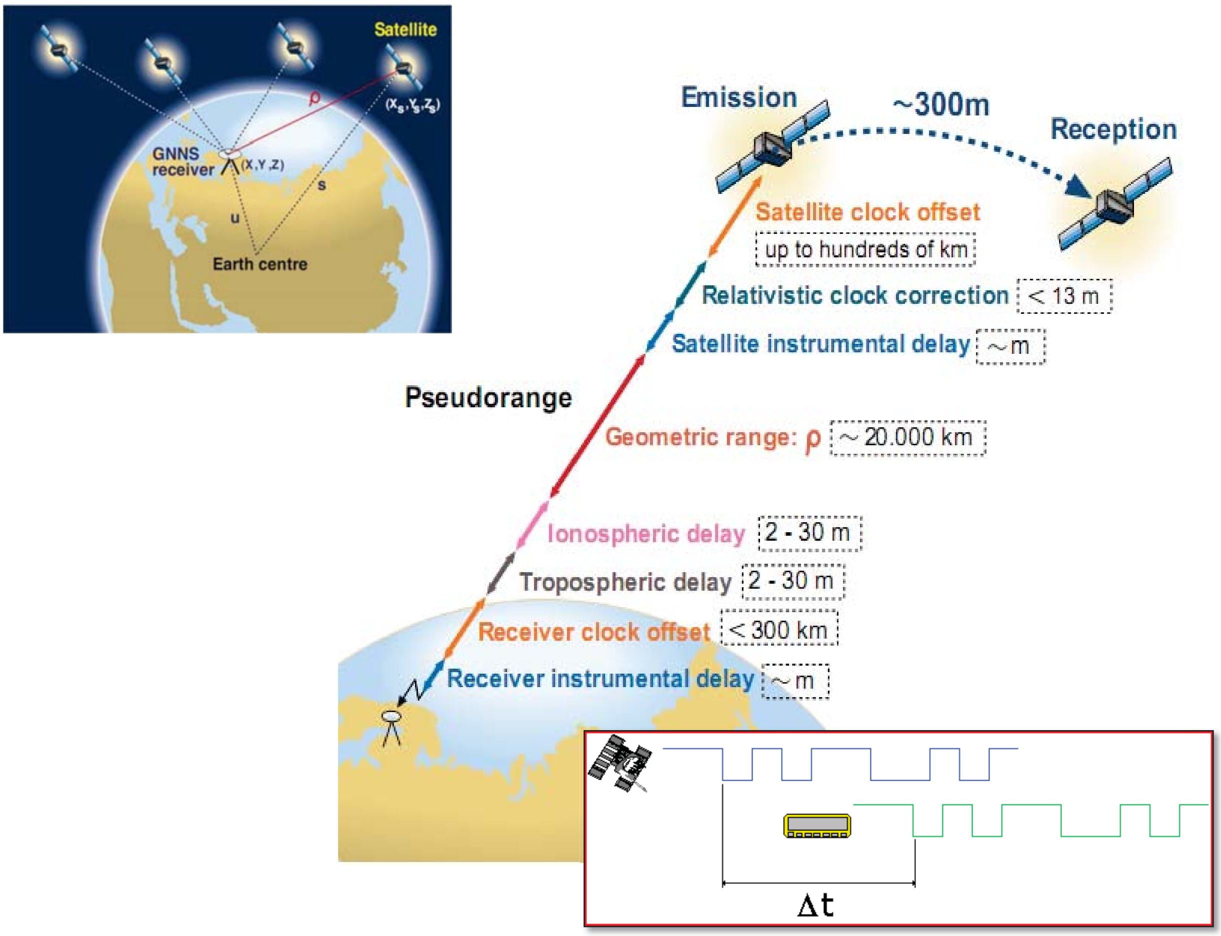


$$P = c\,\Delta T = c\,[t_{rec}(T) - t^{sat}(T-\Delta T)]$$



$$P_{rec}^{sat} = \underbrace{\rho_{rec}^{sat}}_{\text{Geometric range}} + c\cdot(\underbrace{dt_{rec} - dt^{sat}}_{\text{Clock offsets}}) + \sum \delta$$

$$\sum \delta = \underbrace{Trop_{rec}^{sat}}_{\text{Tropospheric delay}} + \underbrace{Ion_{rec}^{sat}}_{\text{Ionospheric delay}} + \underbrace{K_{rec} + K^{sat}}_{\text{Instrumental delays}} + \underbrace{\varepsilon}_{\text{noise}}$$



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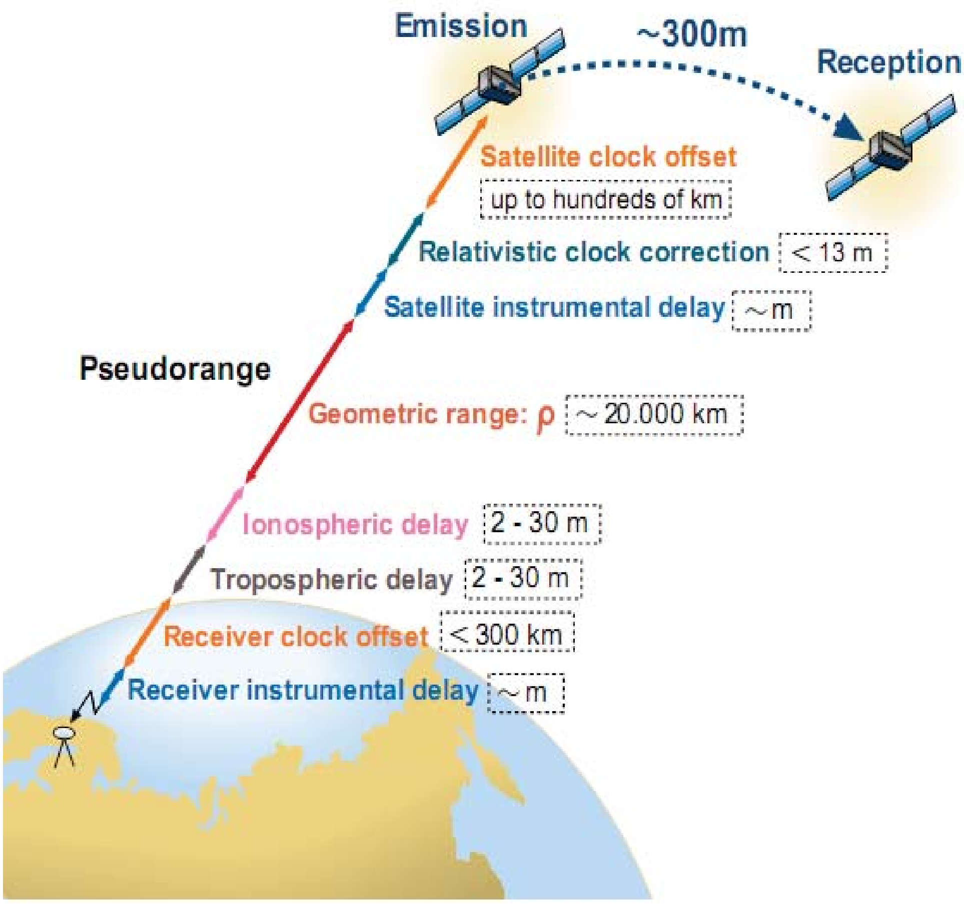

Exercise:

a) Using the file 95oct18casa___r0.rnx, generate the "txt" file 95oct18casa.a (with data ordered in columns).

b) Plot code and phase measurements for satellite PRN28 and discuss the results.

Resolution:

a) **gLAB_linux** -input:cfg **meas.cfg** -input:obs **coco0090.97o**

b) See next plots:

An example of program to read the RINEX: gLAB

RINEX file → gLAB → txt file

```
     2              OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
RGRINEXO V2.4.1 UX  AUSLIG              10-JAN-97 10:19     PGM / RUN BY / DATE
Australian Regional GPS Network (ARGN)  - COCOS ISLAND      COMMENT
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION COMMENT
     -0.000000000103           HARDWARE CALIBRATION (S)     COMMENT
     -0.000000054663           CLOCK OFFSET (S)             COMMENT
COCO                                                        MARKER NAME
AU18                                                        MARKER NUMBER
mrh                 auslig                                  OBSERVER / AGENCY
126                 ROGUE SNR-8100      93.05.25 / 2.8.33.2 REC # / TYPE / VERS
327                 DORNE MARGOLIN T                        ANT # / TYPE
  -741950.3241  6190961.9624 -1337769.9813                  APPROX POSITION XYZ
        0.0040        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    C1    L1    L2    P2    P1                        # / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]                  COMMENT
 SNR:    >500  >100   >50   >10    >5    >0   bad   n/a     COMMENT
 sig:       9     8     7     6     5     4     1     0     COMMENT
    30                                                      INTERVAL
  1997     1     9     0     7   30.0000000                 TIME OF FIRST OBS
  1997     1     9    23    59   30.0000000                 TIME OF LAST OBS
                                                            END OF HEADER
 97  1  9  0  7 30.0000000  0  7  1 25  9  5 23 17  6
  22127685.105   -14268715.899 8 -11118481.28445  22127685.4014   <=====  1
  22672158.746   -11510817.892 7  -8969469.30045  22672158.5184   <===== 25
  22594902.367   -12949753.825 7 -10090708.53945  22594903.7394   <=====  9
  22731128.796   -11621184.951 7  -9055464.16945  22731130.0094   <=====  5
  24610920.702     -924108.174 6   -720085.67045  24610920.0404   <===== 23
  20718775.074   -18605935.474 9 -14498133.97346  20718775.6074   <===== 17
  20842713.610   -19083282.892 9 -14870090.55546  20842713.4814   <=====  6
```

sta	Doy	sec	PRN	L1	L2	C1/	P2	
casa	291	0.30	14	3832855.061	3832852.989	20764791.163	20764791.889	0
casa	291	0.30	15	-1932753.473	-1932752.152	23025605.133	23025608.420	0
casa	291	0.30	18	-191430.529	-191430.252	24656587.151	24656589.163	0
casa	291	0.30	22	-2444624.551	-2444621.901	22508513.354	22508514.937	0
casa	291	0.30	25	-2949812.363	-2949808.662	22258999.265	22258999.532	0
casa	291	0.30	29	-1670473.859	-1670472.308	22409115.477	22409113.635	0
casa	291	30.30	14	-3840286.848	-3840284.776	20757359.266	20757360.162	0
casa	291	30.30	15	-1914283.549	-1914282.239	23044075.112	23044078.373	0
casa	291	30.30	18	-207329.153	-207328.868	24640688.394	24640690.635	0
casa	291	30.30	22	-2458225.787	-2458223.122	22494912.015	22494913.548	0
casa	291	30.30	25	2935690.698	2935686.992	22273121.015	22273121.208	0
casa	291	30.30	29	1681115.594	1681114.037	22398473.854	22398474.354	0
casa	291	60.30	14	-3847635.893	-3847533.821	20750010.062	20750016.988	0
casa	291	60.30	15	-1895770.976	-1895769.678	23062588.653	23062591.023	0
casa	291	60.30	18	-223219.005	-223218.710	24624798.015	24624800.800	0
casa	291	60.30	22	-2471750.391	-2471747.715	22481387.436	22481388.934	0
casa	291	60.30	25	-2921510.699	-2921507.003	22287301.001	22287301.347	0
casa	291	60.30	29	-1691735.199	-1691733.640	22387854.082	22387854.429	0
casa	291	90.30	14	-3854900.513	-3854898.440	20742745.403	20742746.301	0
casa	291	90.30	15	-1877216.337	-1877215.051	23081143.251	23081145.556	0
casa	291	90.30	18	-239097.495	-239097.188	24608919.738	24608922.372	0
casa	291	90.30	22	2485199.565	2485196.877	22467938.317	22467939.902	0
casa	291	90.30	25	2907272.799	2907269.105	22301538.849	22301539.273	0
casa	291	90.30	29	-1702332.295	-1702330.732	22377257.010	22377257.225	0
casa	291	120.30	14	-3862079.748	-3862077.674	20735566.158	20735566.939	0
casa	291	120.30	15	-1858620.266	-1858618.992	23099739.479	23099741.640	0
casa	291	120.30	18	-254966.373	-254966.052	24593053.725	24593053.168	0

The RINEX file is convert to a "columnar format" to easily plot its contents and to analyze the measurements (the public domain free tool **"gnuplot"** is used in the book to make the plots).

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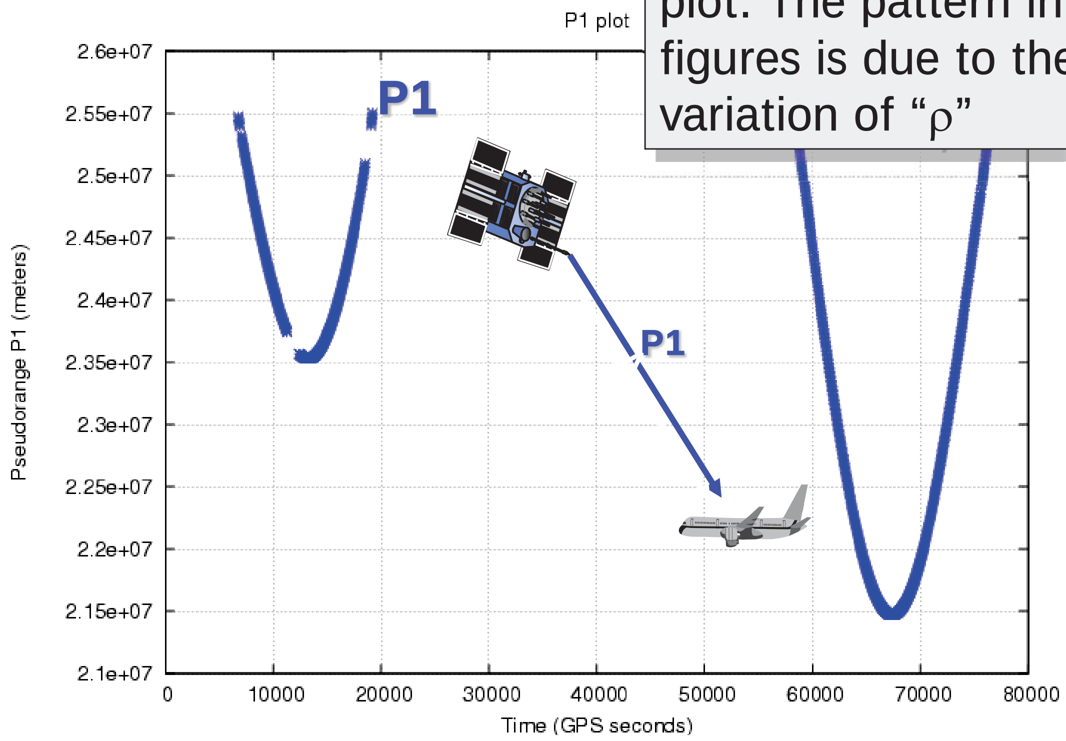
Code measurements

The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ"



$$P_{1sta}^{sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

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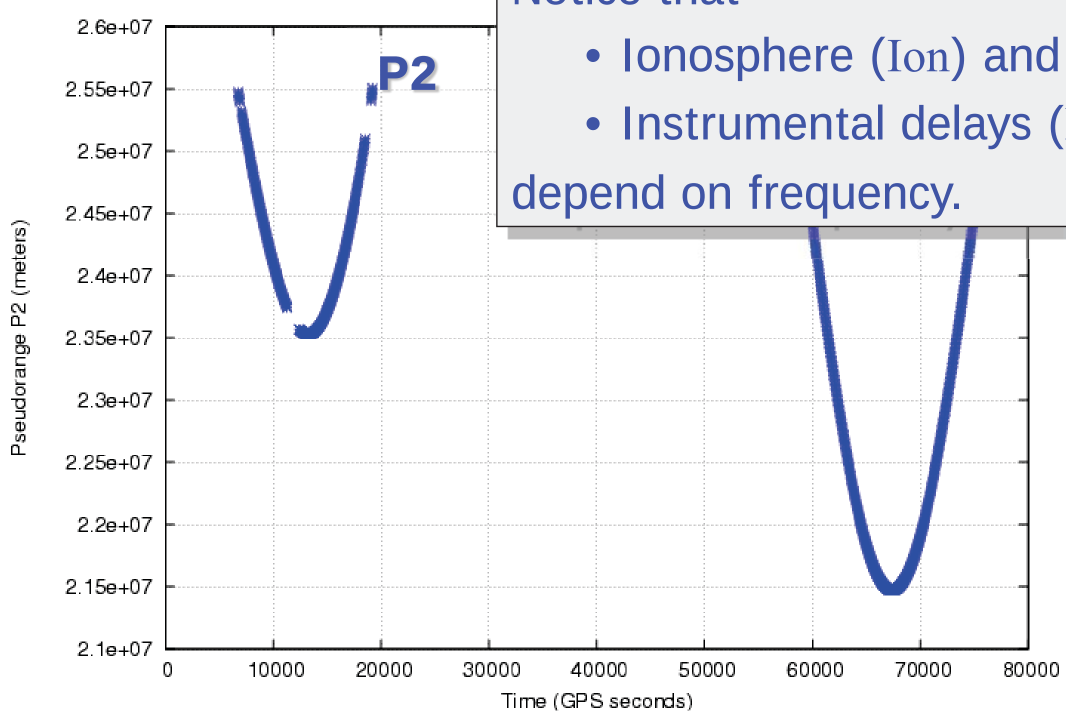
Code measurements

Similar plot for code measurements at f_2.

Notice that

- Ionosphere (Ion) and
- Instrumental delays (K)

depend on frequency.



$$P_{2sta}^{sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_2^{sat} + \varepsilon_2$$

...ase measurements

Ionosphere **delays code** and **advances phase measurements**

C_{1sta}^{sat};

Code measurements: C1,P1,P2

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

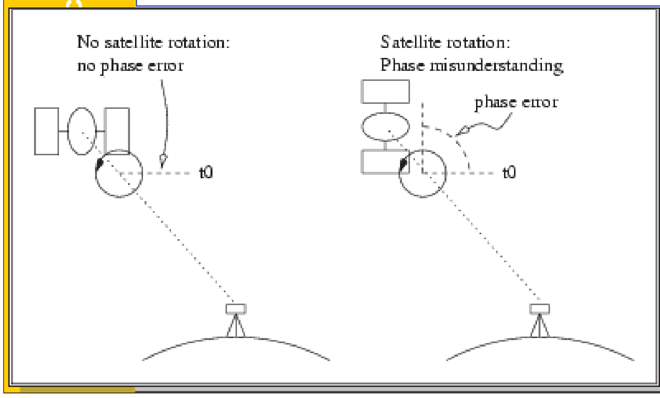
$$P_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_2^{sat} + \varepsilon_2$$

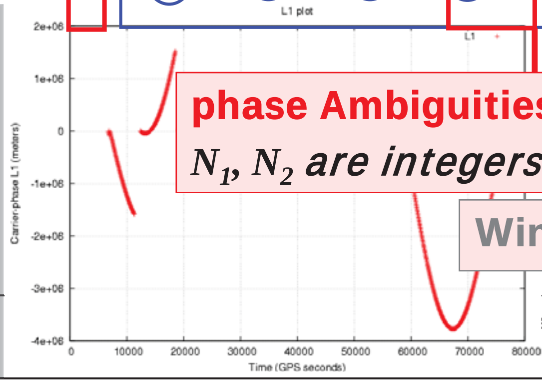
Frequency dependent

Phase measurements: L1,L2

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

$$L_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + b_{2sta} + b_2^{sat} + \lambda_2 N_2 + \lambda_2 w + \nu_2$$



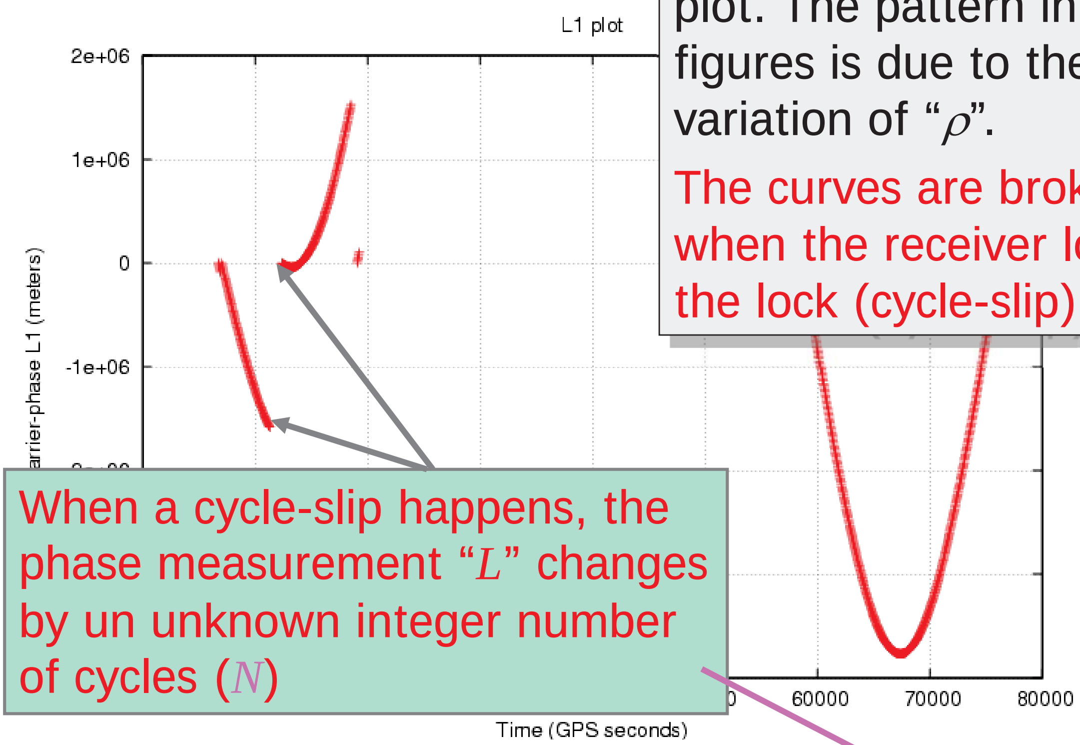


phase Ambiguities

N_1, N_2 are integers

Wind Up

Carrier Phase measurements



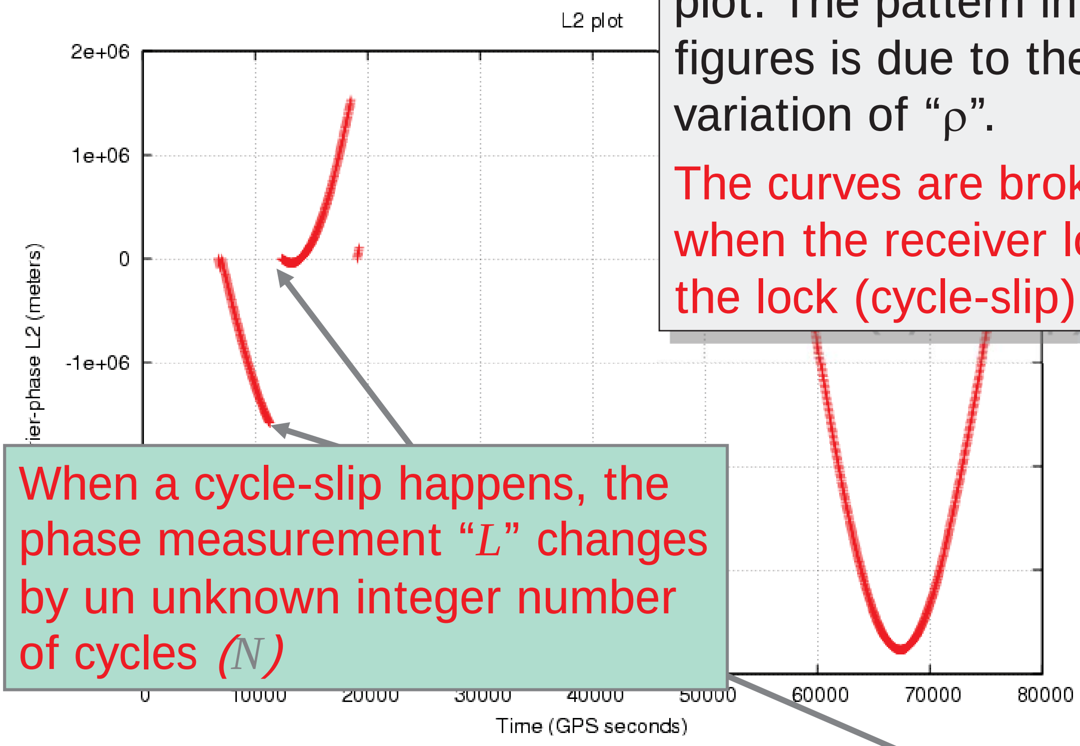
The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ".

The curves are broken when the receiver loss the lock (cycle-slip).

When a cycle-slip happens, the phase measurement "L" changes by un unknown integer number of cycles (N)

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

Carrier Phase measurements



The geometry "ρ" is the dominant term in the plot. The pattern in the figures is due to the variation of "ρ".

The curves are broken when the receiver loss the lock (cycle-slip).

When a cycle-slip happens, the phase measurement "L" changes by un unknown integer number of cycles (N)

$$L_{2\,sta}^{\;sat} = \boxed{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat})} + Trop_{sta}^{sat} - Ion_{2\,sta}^{\;sat} + b_{2\,sta} + b_2^{\;sat} + \boxed{\lambda_2 N_2} + \lambda_2 w + \nu_2$$

Contents

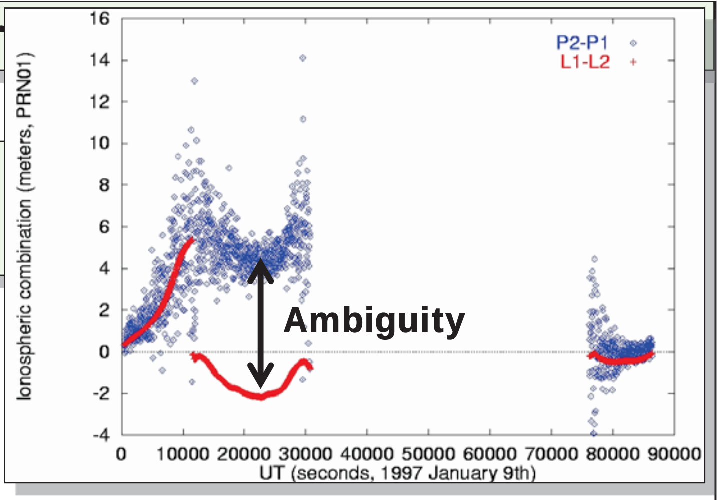
Linear Combinations of measurements:

- Geometry-free (or Ionospheric) combination.
- Ionosphere-Free combination.
- Wide-lane and Narrow-lane combinations.

1. Geometry-free (or

$P_I = P_2 - P_1 = Iono + ctt$

$L_I = L_1 - L_2 = Iono + ctt + Ambig$



Code measurements: C_1, P_1, P_2

$P_2 - P_1$

$$P_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

$$P_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{2sta}^{sat} + K_{2sta} + K_2^{sat} + \varepsilon_2$$

Carrier measurements: L1,L2

$L_1 - L_2$

$$L_{1sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

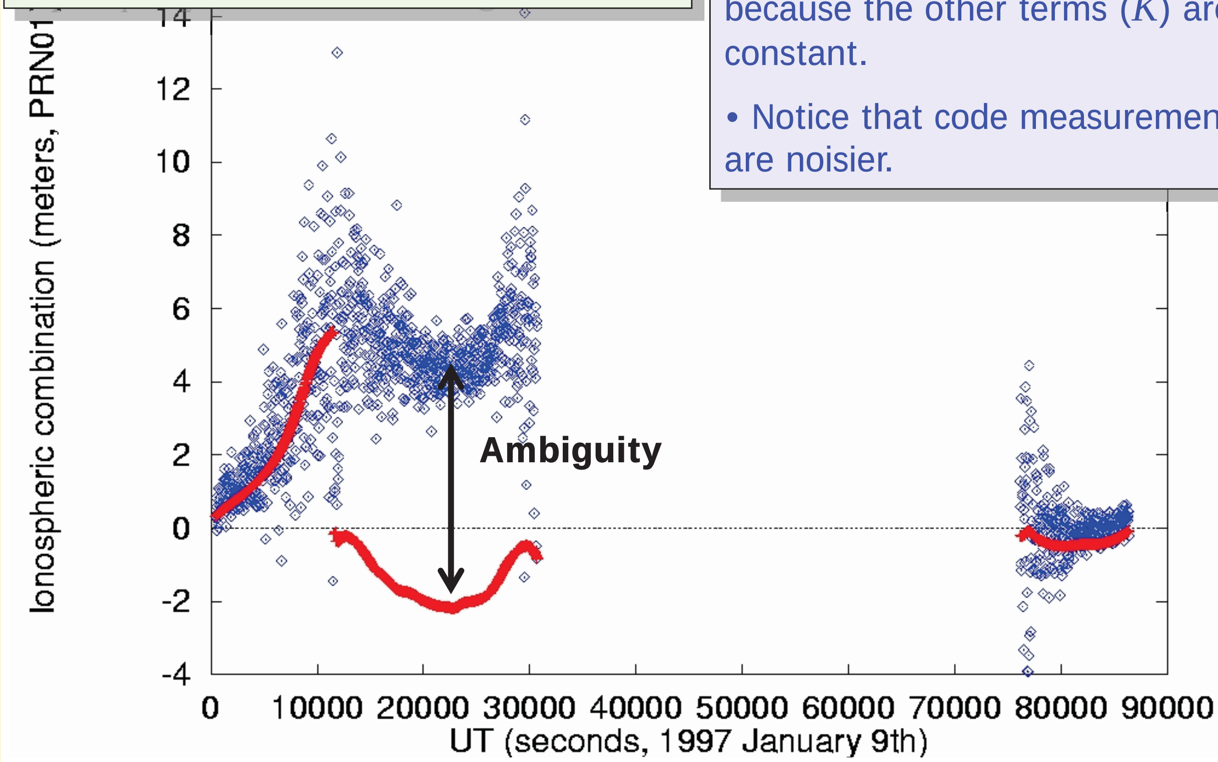
$$L_{2sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{2sta}^{sat} + b_{2sta} + b_2^{sat} + \lambda_2 N_2 + \lambda_2 w + \nu_2$$

Carrier Ambiguities

1. Geometry-free (or ionospheric) combination

$P_I = P_2 - P_1 = Iono + ctt$

$L_I = L_1 - L_2 = Iono + ctt + Ambig$



- The pattern corresponds to the ionospheric refraction (Ion), because the other terms (K) are constant.
- Notice that code measurements are noisier.

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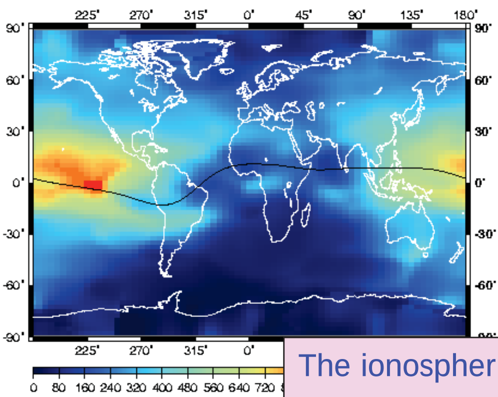
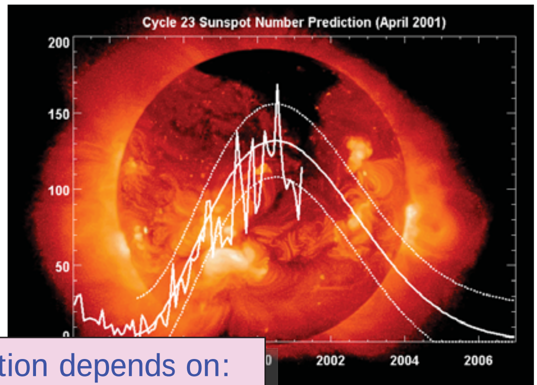
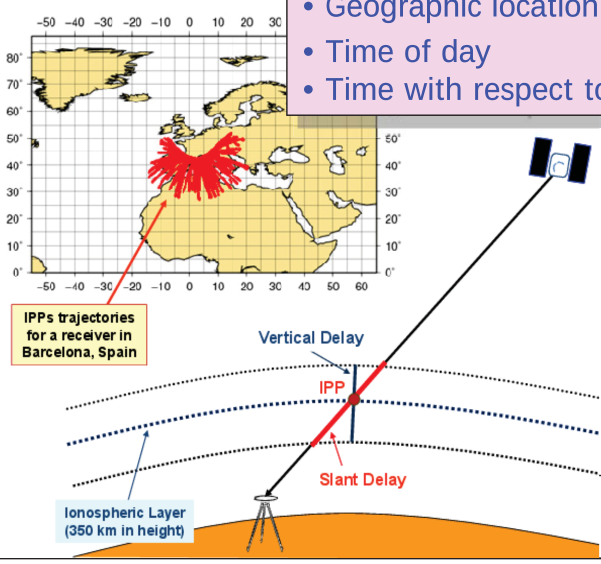
Ionospheric effects

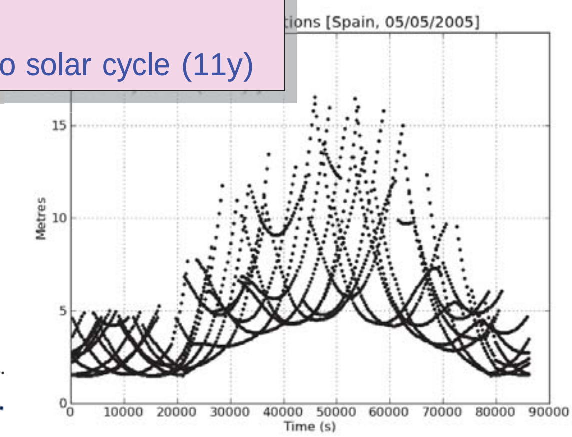




The ionospheric refraction depends on:

- Geographic location
- Time of day
- Time with respect to solar cycle (11y)





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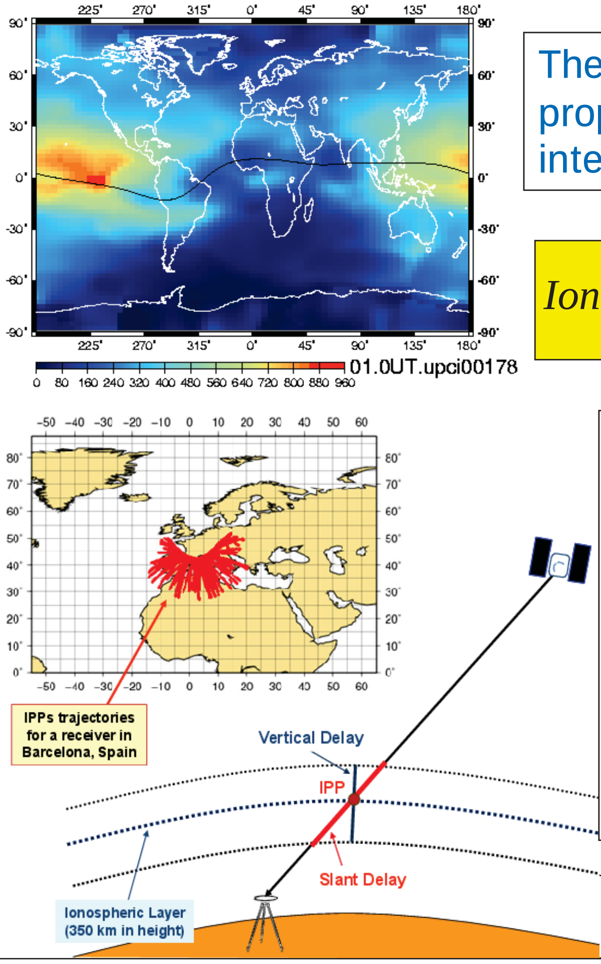
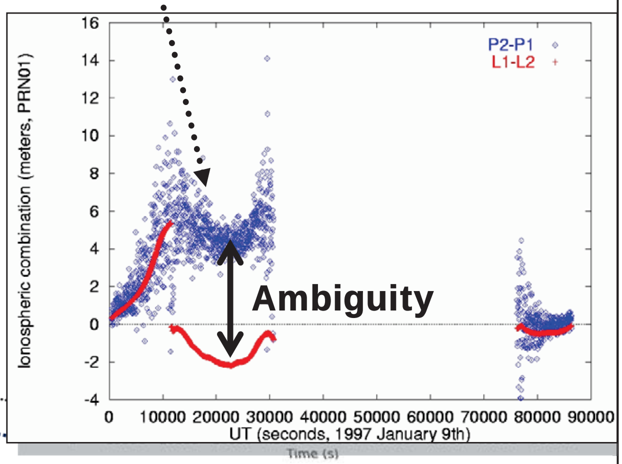
Ionospheric effects



The ionospheric delay (Ion) is proportional to the electron density integrated along the ray path ($STEC$)

$$Ion = \frac{40.3}{f^2} STEC$$

$$STEC = \int_{\vec{r}[GPStransmitter]}^{\vec{r}[GPSreceiver]} N_e(\vec{r},t)dr$$



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2. Ionosphere-free Combination *(Pc,Lc)*

The ionospheric refraction depends on the inverse of the squared frequency and can be removed up to 99.9% combining *f1* and *f2* signals:

$$Ion = \frac{40.3}{f^2} STEC$$

$$Pc = \frac{f_1^{\,2} P_1 - f_2^{\,2} P_2}{f_1^{\,2} - f_2^{\,2}} \qquad Lc = \frac{f_1^{\,2} L_1 - f_2^{\,2} L_2}{f_1^{\,2} - f_2^{\,2}}$$

$$Pc_{sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + \varepsilon_c$$

Note: K^{sat} cancels in Pc and K_{sta} included in dt_{sta}

$$Lc_{sta}^{sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + b_{c,sta} + b_c^{\;sat} + \lambda_N Rc + \lambda_N w + \nu_c$$

- **The ionospheric refraction has been removed in Lc and Pc**

  $\lambda_N = 10.7$ cm, $\lambda_W = 86.2$cm

  The Rc ambiguities are NOT integers!!

$$Rc = N_1 - \frac{\lambda_W}{\lambda_2}(N_1 - N_2)$$

Comments:

Two-frequency receivers are needed to apply the ionosphere-free combination.

If a one-frequency receiver is used, a ionospheric model must be applied to remove the ionospheric refraction. The GPS navigation message provides the parameters of the Klobuchar model which accounts for more than 50% (RMS) of the ionospheric delay.

3.- Narrow-lane (P_N) and Wide-lane Combination (L_W)

The wide-lane combination L_W provides a signal with a large wave-length (λ_W=86.2cm $\sim 4*\lambda_1$). This makes it very useful for detecting cycle-slips through the Melbourne-Wübbena combination: $L_W - P_N$

$$P_N = \frac{f_1\, P_1 + f_2\, P_2}{f_1 + f_2} \qquad L_W = \frac{f_1\, L_1 - f_2\, L_2}{f_1 - f_2}$$

The same sign

$$P_{N\,sta}^{\;sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{W\,sta}^{\;sat} + K_{W\,sta} + K_W^{\;sat} + \varepsilon_N$$

$$L_{W\,sta}^{\;sat} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} + Ion_{W\,sta}^{\;sat} + b_{W\,sta} + b_W^{\;sat} + \lambda_W \boxed{N_W} + \nu_W$$

The ambiguities N_W are INTEGERS!

No wind-up

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Exercises:

1) Consider the wide-lane combination of carrier phase measurements

$$L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$$, where L_W is given in length units (i.e. $L_i = \lambda_i \, \phi_i$).

Show that the corresponding wavelength is: $\lambda_W = \dfrac{c}{f_1 - f_2}$

Hint:

$L_W = \lambda_W \, \phi_W$; $\phi_W = \phi_1 - \phi_2$

2) Assuming L_1, L_2 uncorrelated measurements with equal noise σ_L, show that:

$$\sigma_{L_W} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}-1}}\sigma_L \quad ; \quad \gamma_{12} = \left(\frac{f_1}{f_2}\right)^2$$

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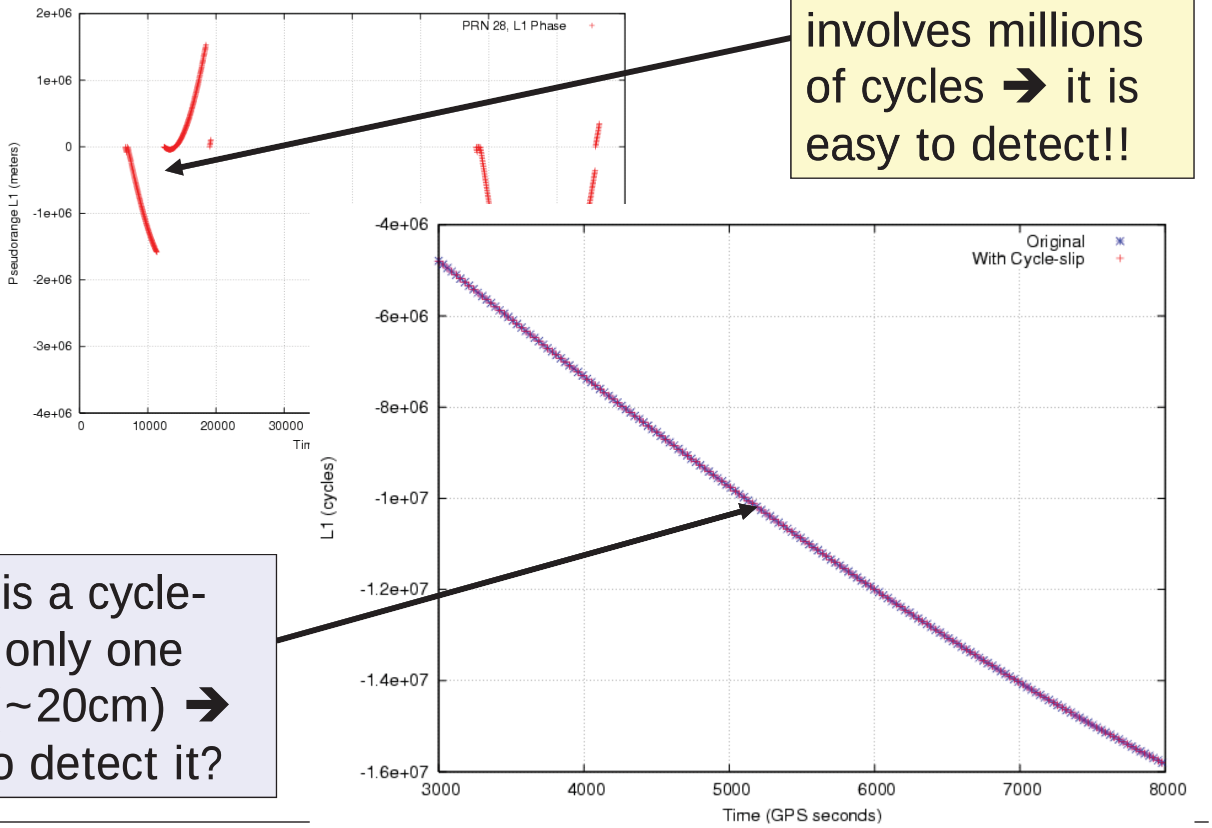
Contents

Detecting cycle-slips



This cycle-slip involves millions of cycles ➔ it is easy to detect!!

There is a cycle-slip of only one cycle (~20cm) ➔ How to detect it?

Exercise:

a) Using the file 95oct18casa___r0.rnx, generate the "txt" file 95oct18casa.a (with data ordered in columns).

b) Insert a cycle-slip of "one wavelength" (19cm) in L1 measurement at t=5000 s (and no cycle-slip in L2).

c) Plot the measurements "L1, L1-P1, LC-PC, Lw-PN and L1-L2" and discuss which combination/s should be used to detect the cycle-slip.

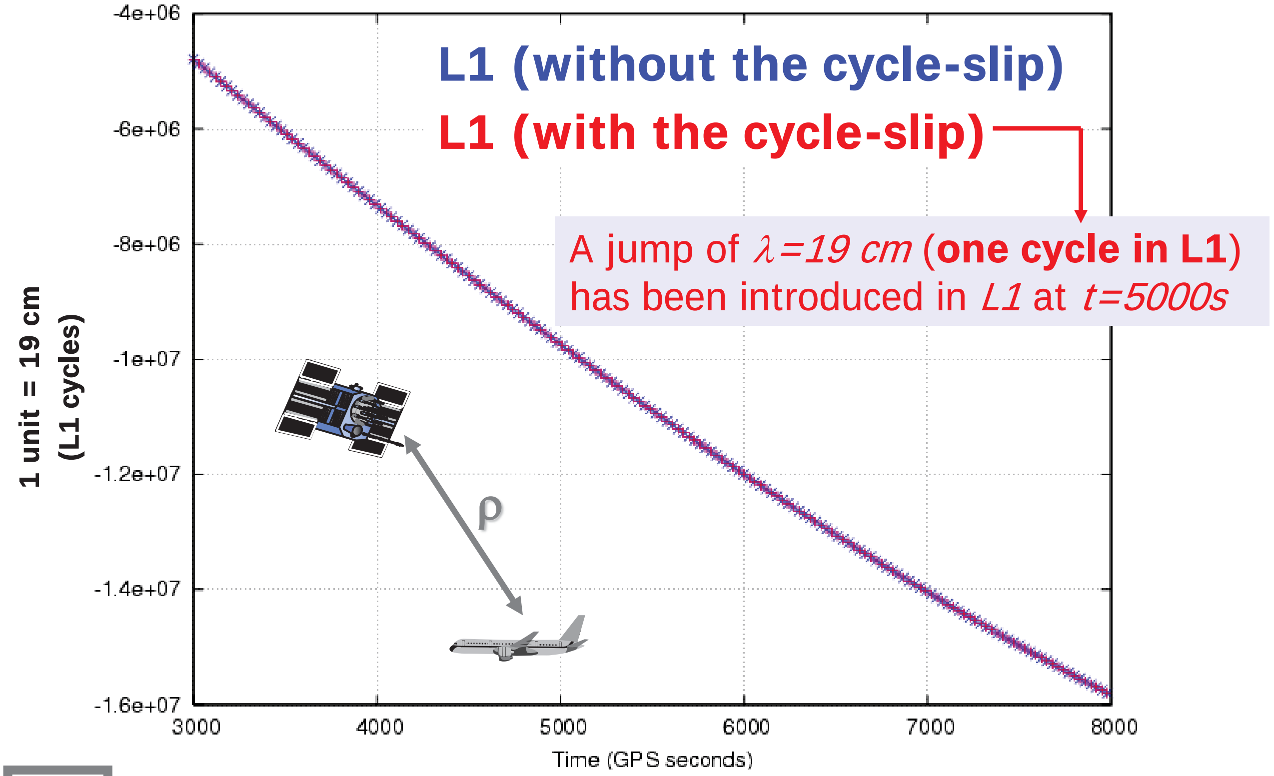
Resolution:

a)
```
gLAB_linux -input:cfg meas.cfg -input:obs 95oct18casa_r0.rnx
```

b)
```
cat 95oct18casa.a | gawk '{if ($4==18)
                         print $3,$5,$6,$7,$8}' > s18.org
cat s18.org | gawk '{if ($1>=5000) $2=$2+0.19;
  printf "%s %f %f %f %f \n", $1,$2,$3,$4,$5}' > s18.cl
```

c) See next plots:

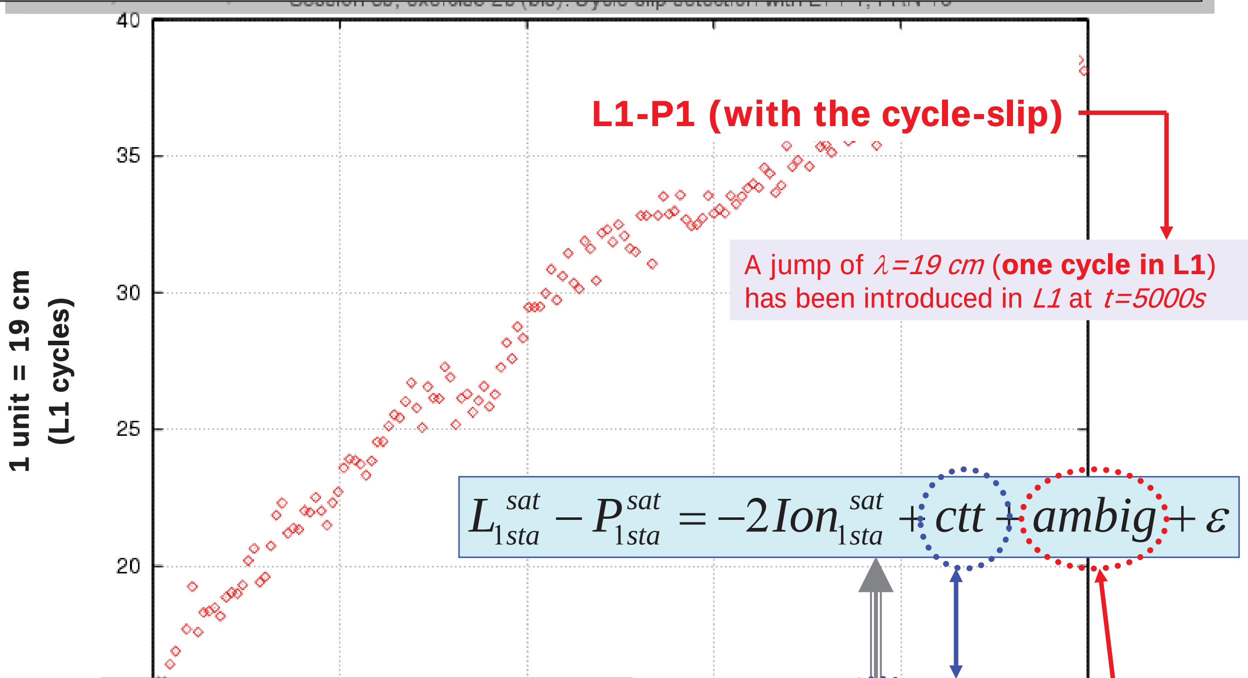
The geometry "ρ" is the dominant term in the plot. The variation of "ρ" in 1 sec may be hundreds of meters, many times greater than the cycle-slip *(19 cm)* ➔ **the variation of ρ shadows the cycle-slip!**



$$\boxed{L_{1sta}^{sat}} = \rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

The geometry and clock offsets have been removed.

The trend is due to the Ionosphere. The *P1* code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



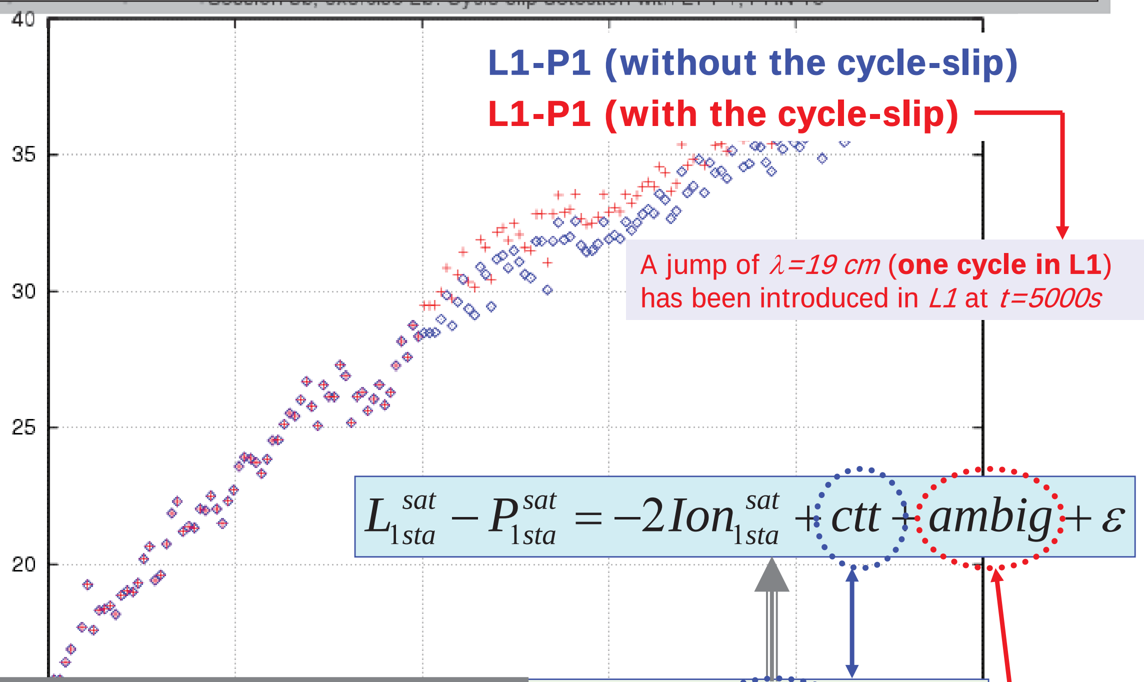
$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + \varepsilon$$

$$P_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

The geometry and clock offsets have been removed.

The trend is due to the Ionosphere. The *P1* code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



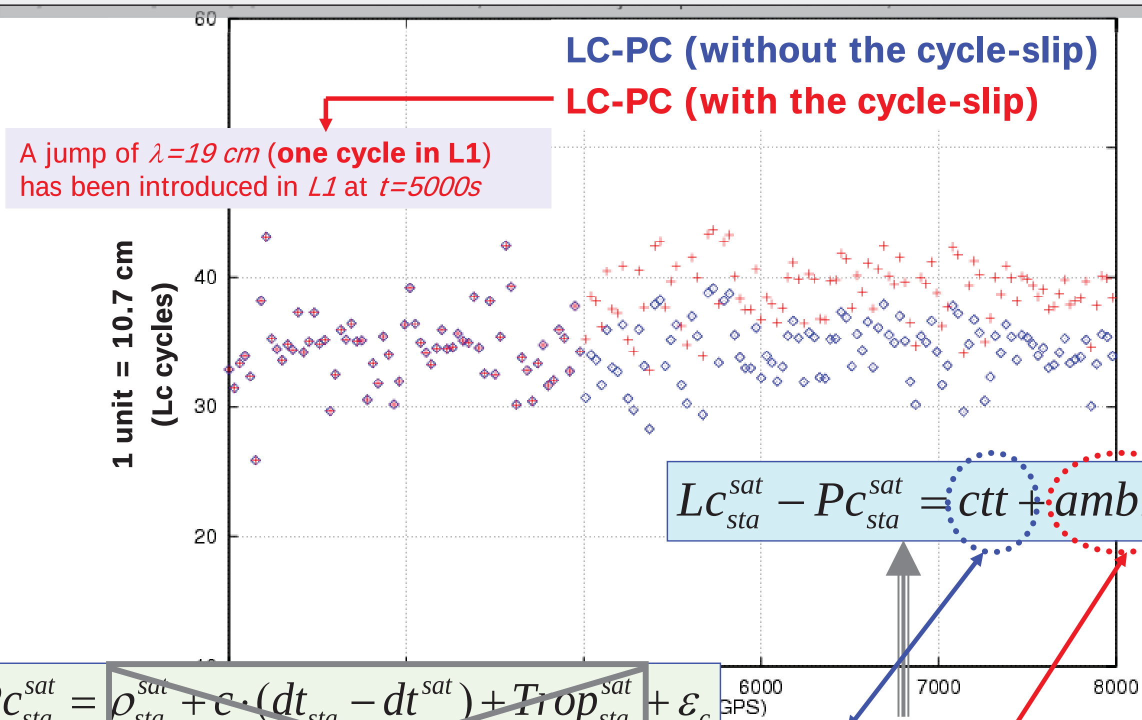
$$L_{1sta}^{sat} - P_{1sta}^{sat} = -2Ion_{1sta}^{sat} + ctt + ambig + \varepsilon$$

$$P_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} + Ion_{1sta}^{sat} + K_{1sta} + K_1^{sat} + \varepsilon_1$$

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. The *P1* code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



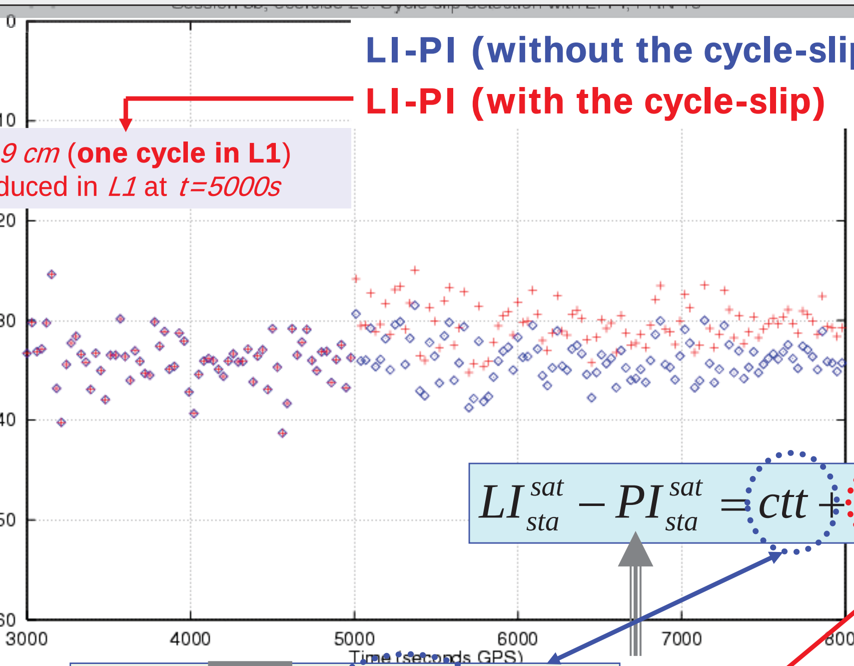
$$Lc_{sta}^{sat} - Pc_{sta}^{sat} = ctt + ambig + \varepsilon$$

$$Pc_{sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} + \varepsilon_c$$

$$Lc_{sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} + b_{C,sta} + b_C^{sat} + \lambda_N Rc + \lambda_N w + \nu_c$$

The geometry, clock offsets and iono have been removed.

There is a constant pattern plus noise. The *P1* code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.



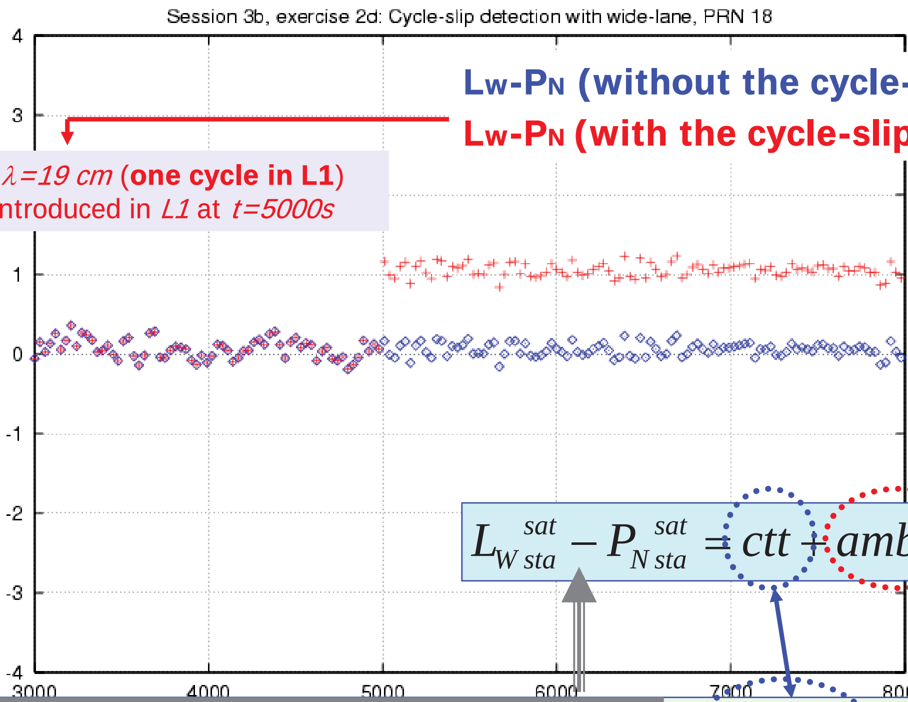
$$LI^{sat}_{sta} - PI^{sat}_{sta} = ctt + ambig + \varepsilon$$

$$PI^{sat}_{sta} = \cancel{Ion_I} + K_{I\,sta} + K_I^{sat} + \varepsilon_I$$

$$LI^{sat}_{sta} = \cancel{Ion_I} + b_{I_{sta}} + b_I^{sat} + \lambda_1 N_1 - \lambda_2 N_2 + (\lambda_1 - \lambda_2) w + \nu_I$$

The geometry , clock offsets and iono have been removed.

There is a constant pattern plus noise. The P_N code noise is under one cycle of L_W. Thence, the cycle-slip is clearly detected



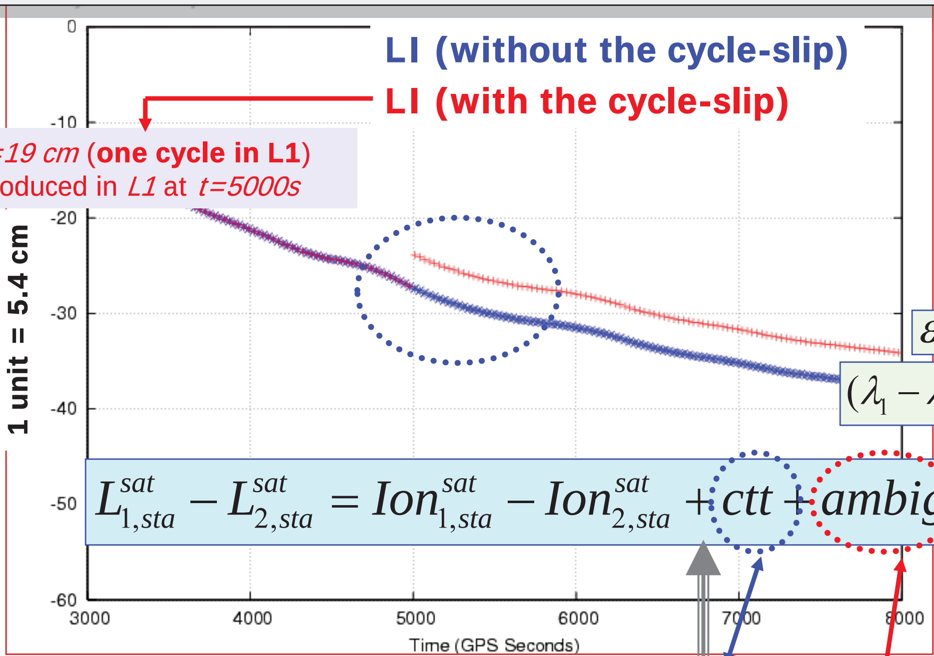
$$L_{W\,sta}^{sat} - P_{N\,sta}^{sat} = ctt + ambig + \varepsilon$$

$$P_{N\,sta}^{sat} = \cancel{\rho^{sat}_{sta} + c\cdot(dt_{sta} - dt^{sat}) + Trop^{sat}_{sta} + Ion_{W\,sta}^{sat}} + K_{W\,sta} + K_W^{sat} + \varepsilon_N$$

$$L_{W\,sta}^{sat} = \cancel{\rho^{sat}_{sta} + c\cdot(dt_{sta} - dt^{sat}) + Trop^{sat}_{sta} - Ion_{W\,sta}^{sat}} + b_{W\,sta} + b_W^{sat} + \lambda_W N_W + \nu_W$$

The geometry and clock offsets have been removed.

The trend is due to the Iono. The *L1* code noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19\ cm$ Thence, the cycle-slip is detected.



ε □ mm

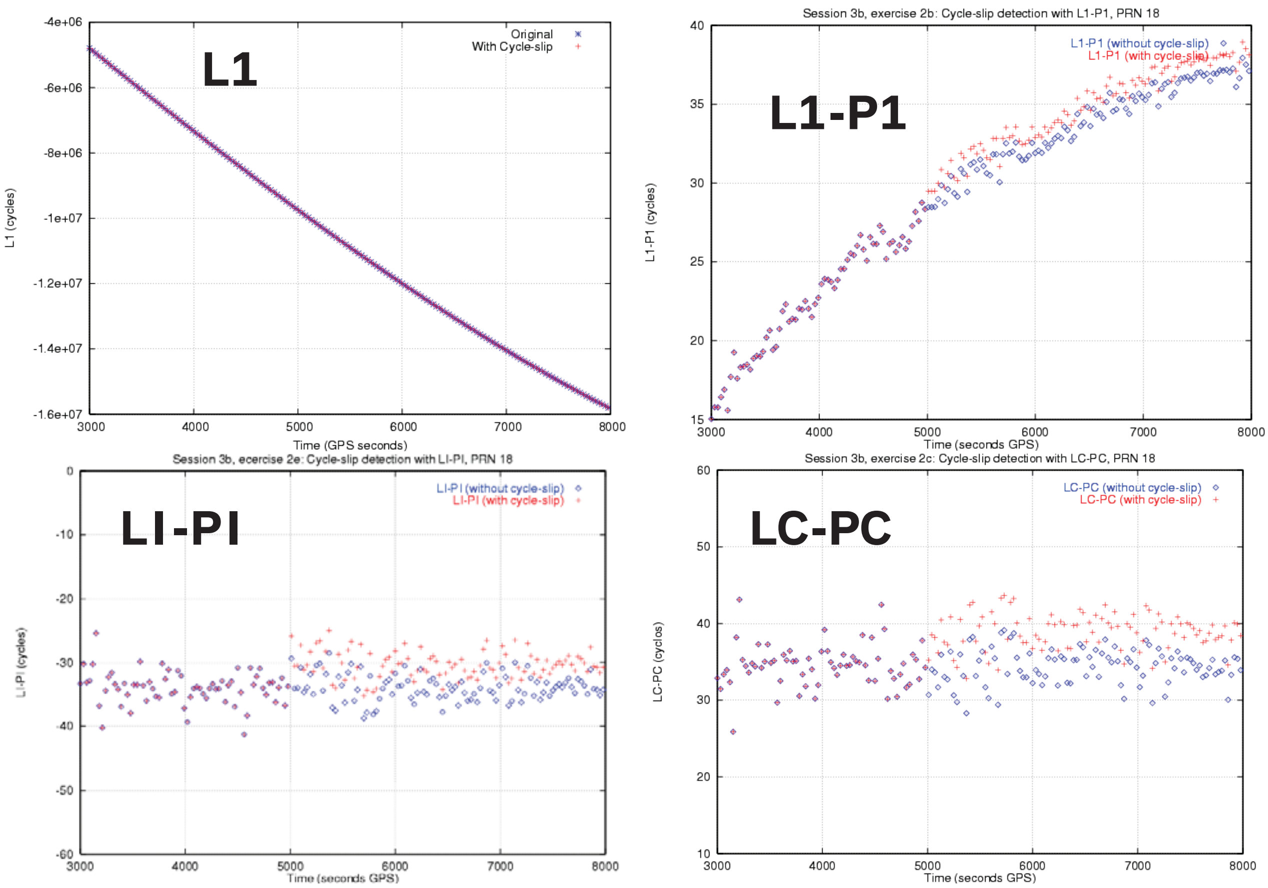
$(\lambda_1 - \lambda_2) w <<$

$$L_{1,sta}^{sat} - L_{2,sta}^{sat} = Ion_{1,sta}^{sat} - Ion_{2,sta}^{sat} + ctt + ambig + \varepsilon$$

$$L_{1sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} - Ion_{1sta}^{sat} + b_{1sta} + b_1^{sat} + \lambda_1 N_1 + \lambda_1 w + \nu_1$$

$$L_{2sta}^{sat} = \cancel{\rho_{sta}^{sat} + c\cdot(dt_{sta} - dt^{sat}) + Trop_{sta}^{sat}} - Ion_{2sta}^{sat} + b_{2sta} + b_2^{sat} + \lambda_2 N_2 + \lambda_2 w + \nu_2$$

Summary



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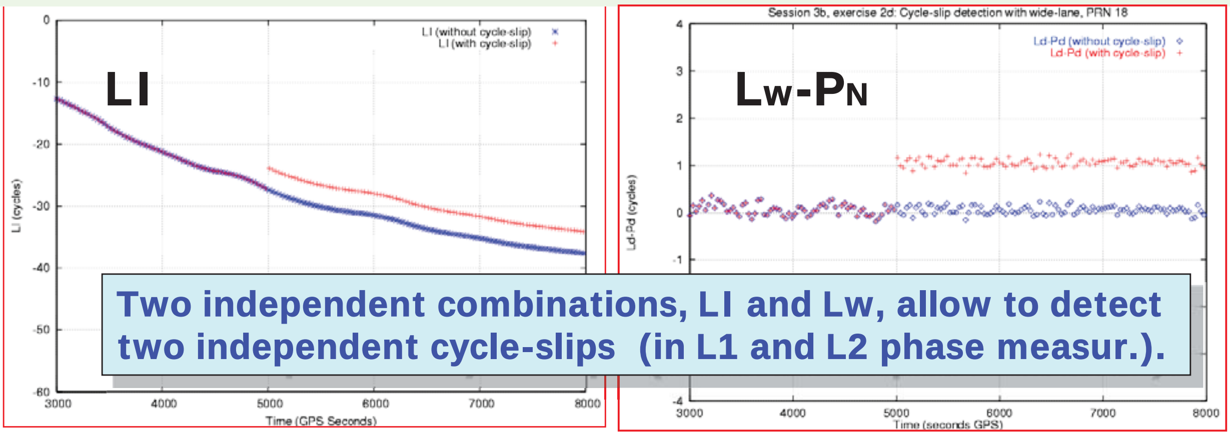
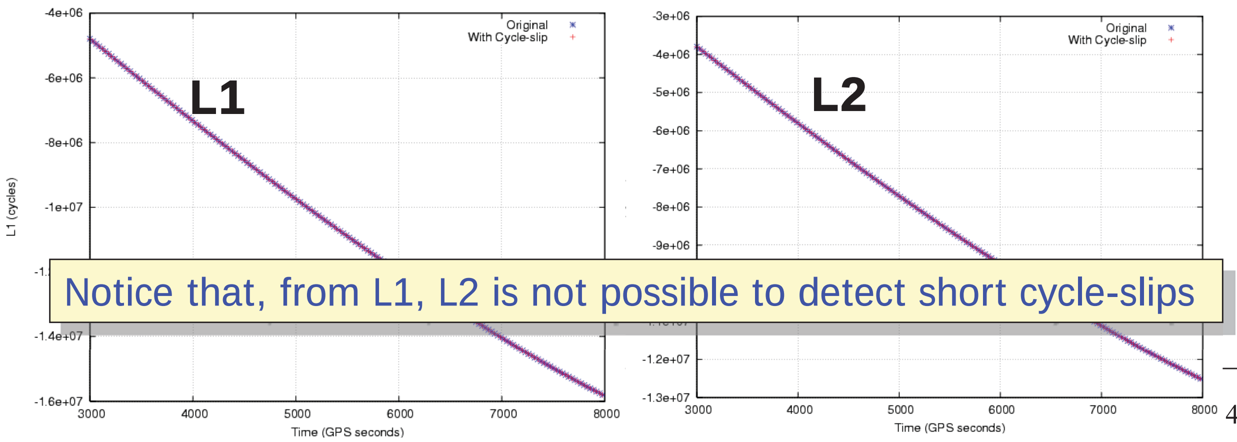
The cycle-slips are detected by the Ionospheric combination (LI=L1-L2) and the Melbourne Wübbena (W=Lw-PN)



Two independent combinations, LI and Lw, allow to detect two independent cycle-slips (in L1 and L2 phase measur.).



Notice that, from L1, L2 is not possible to detect short cycle-slips

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Contents

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Cycle-slip detector based on carrier phase data: The Geometry-free combination

Input data: Geometry-free combination of carrier phase measurements

$$\boxed{L_I = L_1(s;k) - L_2(s;k)}$$

Output: [satellite, time, cycle-slip flag].

The detection is based on fitting a second order polynomial over a sliding window of N_I samples.

The predicted value is compared with the observed one to detect cycle-slip.

For each epoch (k)

For each tracked satellite (s)

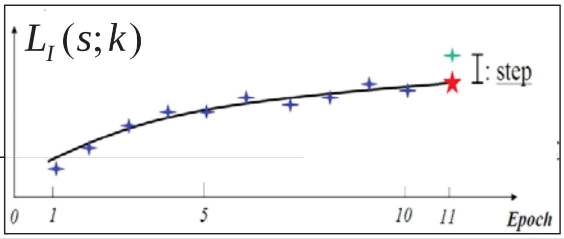
- Declare cycle slip when data gap greater than $tol_{\Delta t}$.[16]
- Fit a second-degree polynomial $p(s;k)$ to the previous values (after the last cycle-slip) $[L_I(s;k-N_I),\ldots,L_I(s;k-1)]$.
- Compare the measured $L_I(s;k)$ and the predicted value $p(s;k)$ at epoch k. If the discrepancy exceeds a given *threshold*, then declare cycle slip. That is,

$$\boxed{\text{if } |L_I(s;k) - p(s;k)| > threshold \text{ then cycle slip.}}$$

- Reset algorithm after cycle slip.

End

End



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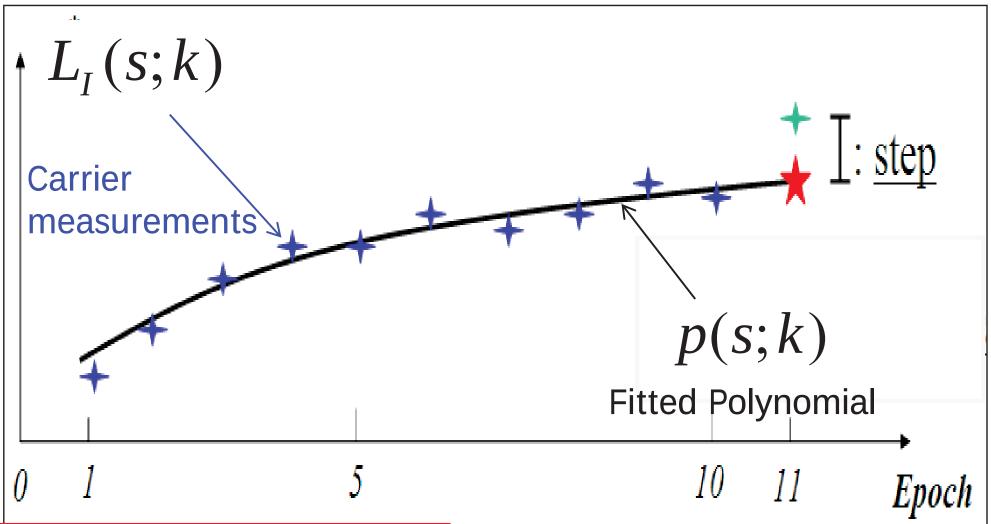

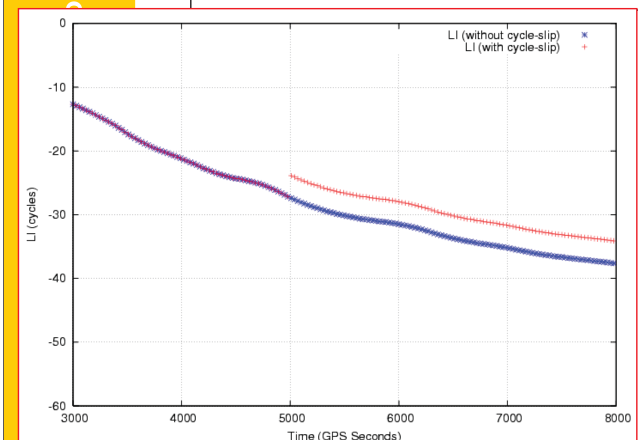

Under not disturbed ionospheric conditions, the geometry-free combination performs as a **very precise and smooth test signal, driven by the ionospheric refraction.**

Although, for instance, the jump produced by a simultaneous one-cycle slip in both signals is smaller in this combination than in the original signals (λ_2-λ_1=5.4cm), it can provide reliable detection even for small jumps

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Cycle-slip detector based on code and carrier phase data: The Melbourne-Wübbena combination

Input data: Melbourne-Wübbena combination $B_W = L_W - P_N = \lambda_W N_W + \varepsilon$

Output: [satellite (PRN), time, cycle-slip flag]

The detection is based on real-time computation of mean (m_{BW}) and sigma (S_{BW}) values of the measurement test data ***Bw***.

For each epoch (k)

For each tracked satellite (s)

- Declare cycle-slip when data hole greater than $tol_{\Delta t}$ (e.g., $60\,s$).
- If no data hole larger than $tol_{\Delta t}$, thence:
- Compare the measurement $B_W(s;k)$ at the epoch k with the mean bias $m_{B_W}(s;k-1)$ computed from the previous values. If the discrepancy is over a *threshold* $= K_{factor} * S_{B_W}$ (e.g., $K_{factor}=4$), declare cycle-slip. That is:

  If $\boxed{|B_W(s;k) - m_{B_W}(s;k-1)| > K_{factor}\, S_{B_W}(s;k-1),}$

  Thence, cycle-slip.
- Update the mean and sigma values according to the equations:

$$m_{B_W}(s;k) = \frac{k-1}{k} m_{B_W}(s;k-1) + \frac{1}{k} B_W(s;k)$$

$$S^2_{B_W}(s;k) = \frac{k-1}{k} S^2_{B_W}(s;k-1) + \frac{1}{k}\left(B_W(s;k) - m_{B_W}(s;k-1)\right)^2 \quad (4.24)$$

Note the S_{B_W} is initialised with an a priori $S_0 = \lambda_w/2$.

End

End

A cycle-slip is declared when the measurement differs form the mean value by a predefined number of standard deviations (S_{BW})

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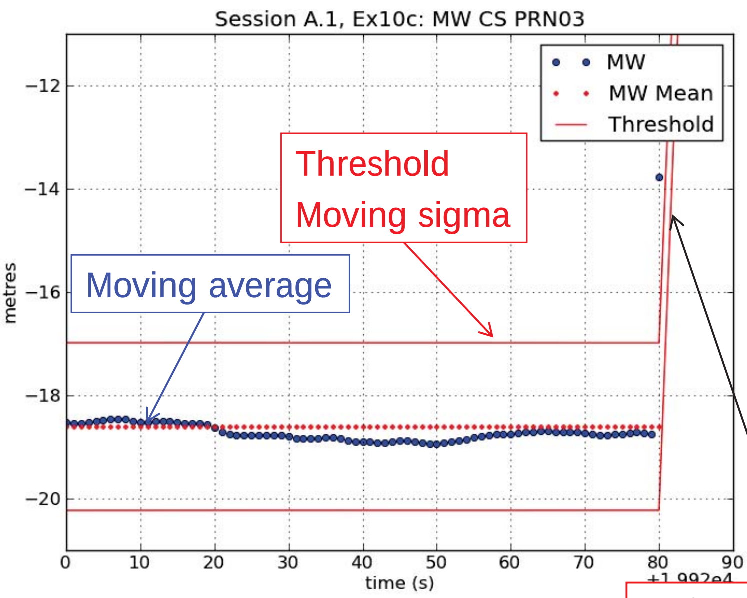
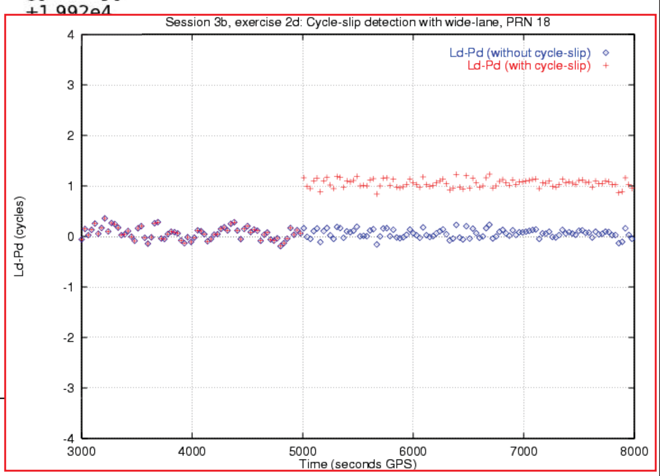


The Melbourne-Wübbena combination has a double benefit:

- The enlargement of the ambiguity spacing, thanks to the larger wavelength λ_W=80.4cm.
- The noise is reduced by the narrow-lane combination of code measurement

Cycle-slip detection

Nevertheless, in spite of these benefits, the performance is worse than in the previous carrier-phase-only based detector and it is used as a secondary test.



Exercises:

1) Show that $\Delta N_1 = 9$ and $\Delta N_2 = 7$
   produces jumps of few millimetres in the geometry-free combination.

2) Show that no jump happens in the geometry-free combination when $\Delta N_1 / \Delta N_2 = 77/60$. In particular when $\Delta N_1 = 77$ and $\Delta N_2 = 60$ the jump in the wide-lane combination is: $17\,\lambda_w \simeq 15\,m$

Hint: Consider the following relationships (from [RD-1]):

The effect of a jump in the integer ambiguities in terms of ΔN_1, ΔN_2 and N_W is given next:

$\Delta\Phi_W$, $\Delta\Phi_I$, $\Delta\Phi_C$ **variations**
$\Delta\Phi_W = \lambda_W \Delta N_W = \lambda_W\,(\Delta N_1 - \Delta N_2)$
$\Delta\Phi_I = \lambda_1 \Delta N_1 - \lambda_2 \Delta N_2 = (\lambda_2 - \lambda_1)\,\Delta N_1 + \lambda_2\,\Delta N_W$
$\Delta\Phi_C = \lambda_N \left(\frac{\lambda_W}{\lambda_1}\Delta N_1 - \frac{\lambda_W}{\lambda_2}\Delta N_2\right) = \lambda_N \left(\Delta N_1 + \frac{\lambda_W}{\lambda_2}\Delta N_W\right)$

(4.20)

Example of Single frequency Cycle-slip detector

Input data: Code pseudorange (P_1) and carrier phase (L_1) measurements.

Output: [satellite (PRN), time, cycle-slip flag]

For each epoch (k)

For each tracked satellite (s)

- Declare cycle-slip when data hole greater than $tol_{\Delta t}$[24].
- If no data hole larger than $tol_{\Delta t}$, thence:
- Update an array with the last N differences of

$$d(s;k) = L_1(s;k) - P_1(s;k)$$

That is: $[d(s;k-N),\ldots,d(s;k-1)]$

- Compute the mean and sigma discrepancy over the previous N samples $[k-N,\ldots,k-1]$:

$$m_d(s;k-1) = \frac{1}{N}\sum_{i=1}^{N} d(s;k-i)$$
$$m_{d^2}(s;k-1) = \frac{1}{N}\sum_{i=1}^{N} d^2(s;k-i) \qquad (4.27)$$
$$S_d(s;k-1) = \sqrt{m_{d^2}(s;k-1) - m_d^2(s;k-1)}$$

- Compare the difference at the epoch k with the mean value of differences computed over the previous N samples window. If the value is over a $threshod = n_T * S_d$ (e.g., $n_T = 5$), declare cycle-slip[25].

That is:

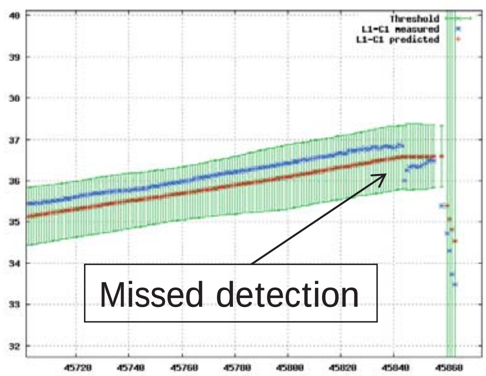
If $|d(s;k) - m_d(s;k-1)| > n_T\,S_d(s;k-1)$,
Thence, cycle-slip.

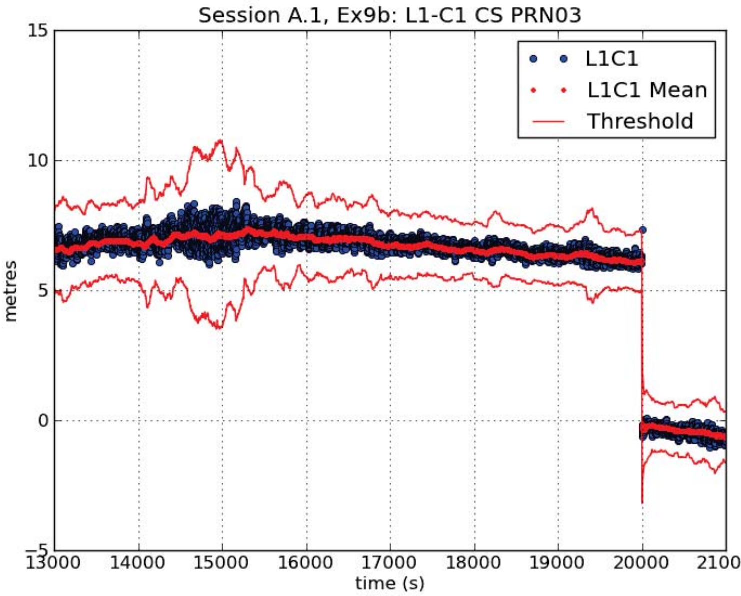
End

End

The detection is based on real-time computation of mean and sigma values of the differences (***d=L₁-P₁***) of the code pseudorange and carrier over a sliding window of ***N*** samples (e.g. ***N=100***).

A cycle-slip is declared when a measurement differs from the mean bias value over a predefined threshold.

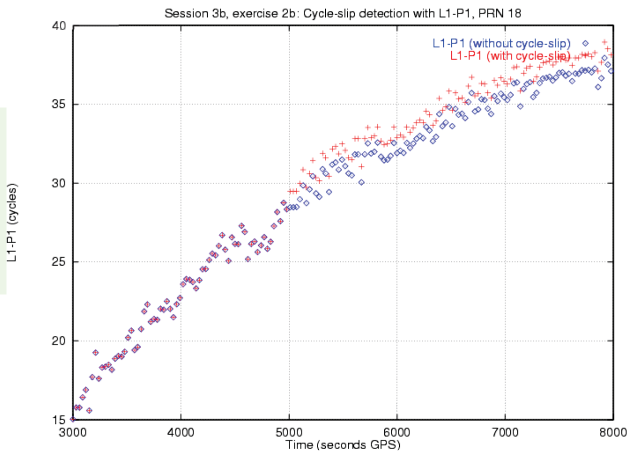




This detector is affected by the code pseudorange noise and multipath as well as the divergence of the ionosphere.

Higher sampling rate improves detection performance, but shortest jumps can still escape from this detector.

On the other hand, a minimum number of samples is needed for filter initialization in order to ensure a reliable value of sigma for the detection threshold



More details, exercises and examples of software code implementation of these detectors can be found in [RD-1] and [RD-2].

Contents

Carrier smoothing of code pseudorange

The noisy (but unambiguous) code pseudorange can be smoothed with the precise (but ambiguous) carrier. A simple algorithm is given next:

Hatch filter:

$$\hat{P}(k) = \frac{1}{n}P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right)$$

where $\hat{P}(1) = P(1)$ and

$n = k; \quad k < N$

$n = N; \quad k \geq N$

This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

$$\hat{P}(k) = \frac{1}{n}P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right) = L(k) + \langle P - L \rangle_{(k)}$$

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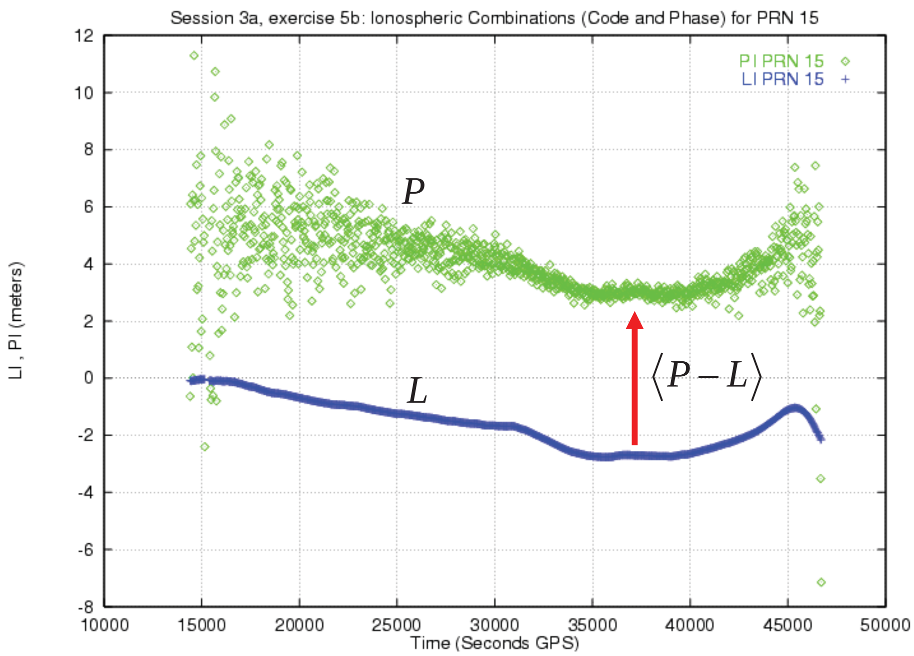

This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

$$\hat{P}(k) = \frac{1}{n}P(k) + \frac{n-1}{n}\left(\hat{P}(k-1) + L(k) - L(k-1)\right) = L(k) + \boxed{\langle P - L \rangle_{(k)}}$$

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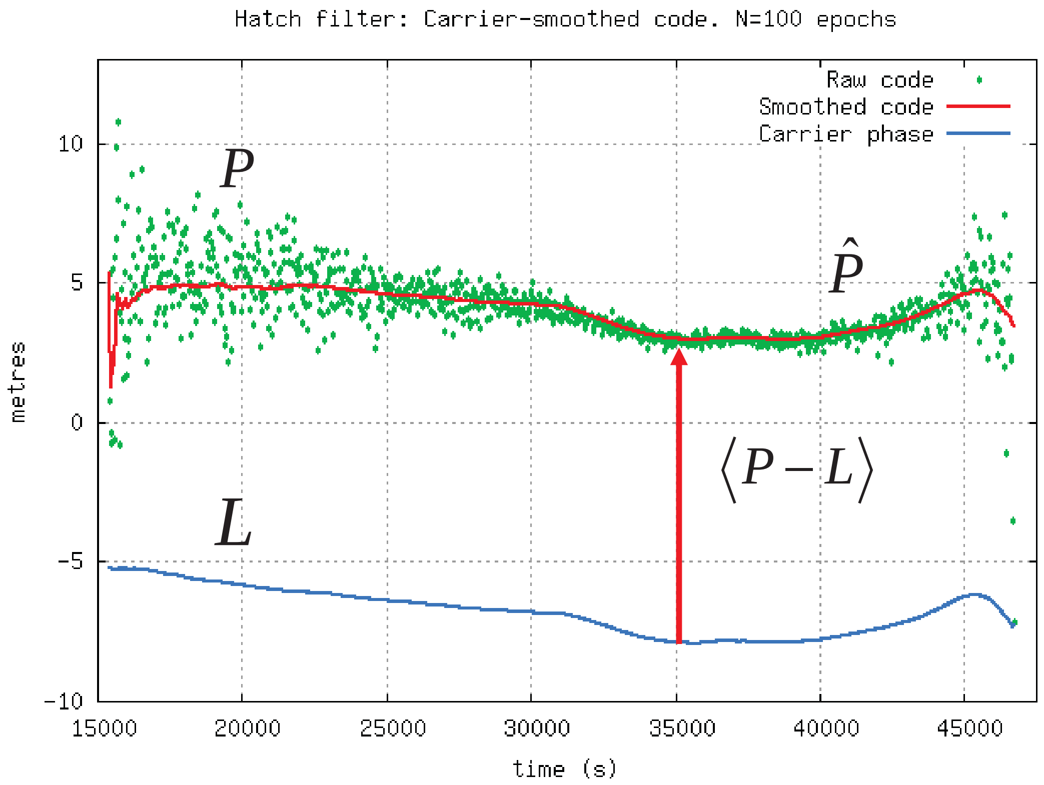

$$\hat{P}(k)=\frac{1}{n}P(k)+\frac{n-1}{n}\left(\hat{P}(k-1)+L(k)-L(k-1)\right)=L(k)+\boxed{\langle P-L\rangle_{(k)}}$$

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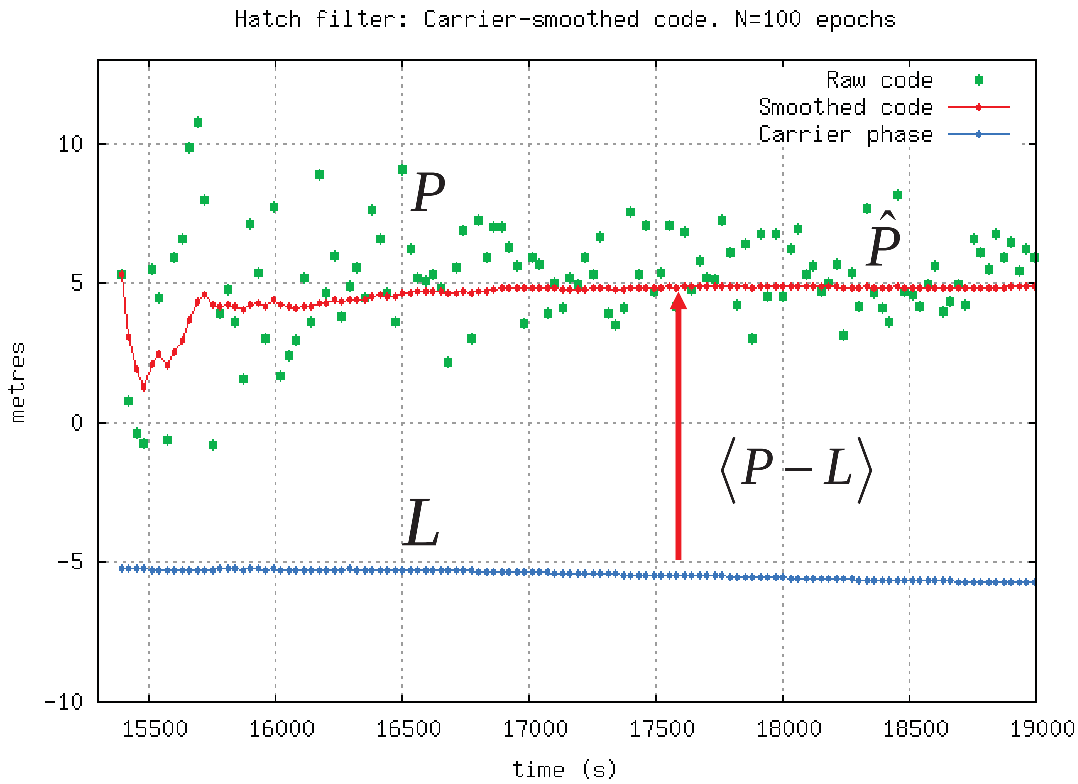

$$\hat{P}(k)=\frac{1}{n}P(k)+\frac{n-1}{n}\left(\hat{P}(k-1)+L(k)-L(k-1)\right)=L(k)+\boxed{\langle P-L\rangle_{(k)}}$$

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Code-carrier divergence: SF smoother

Time varying ionosphere induces a bias in the single frequency (SF) smoothed code when it is averaged in the smoothing filter (Hatch filter).

Let:

$$P_1 = \rho + I_1 + \varepsilon_1$$
$$L_1 = \rho - I_1 + B_1 + \varsigma_1$$

Where ρ includes all non dispersive terms (geometric range, clock offsets, troposphere) and I_1 represents the frequency dependent terms (ionosphere and DCBs). B_1 Is the carrier ambiguity, which is constant along continuous carrier phase arcs and $\varepsilon_1, \varsigma_1$ account for code and carrier multipath and thermal noise.

thence,

$$P_1 - L_1 = 2I_1 - B + \varepsilon_1 \qquad \Rightarrow 2I_1 : \text{ Code-carrier divergence}$$

Substituting $P_1 - L_1$ in Hatch filter equation

$$\hat{P}(k) = L(k) + \langle P - L \rangle_{(k)} = \rho(k) - I_1(k) + B_1 + \langle 2I_1 - B_1 \rangle_{(k)} =$$

$$= \rho(k) + I_1(k) + \underbrace{2\left(\langle I_1 \rangle_{(k)} - I_1(k)\right)}_{bias_I} \qquad \Rightarrow \hat{P}_1 = \rho + I_1 + bias_I + \upsilon_1$$

where, being the ambiguity term B_1 a constant bias, thence $\langle B_1 \rangle_{(k)} \simeq B_1$, and cancels in the previous expression.

where υ_1 is the noise term after smoothing.

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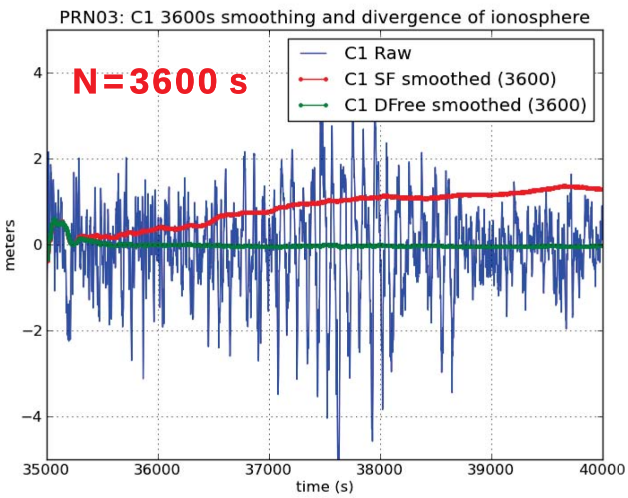
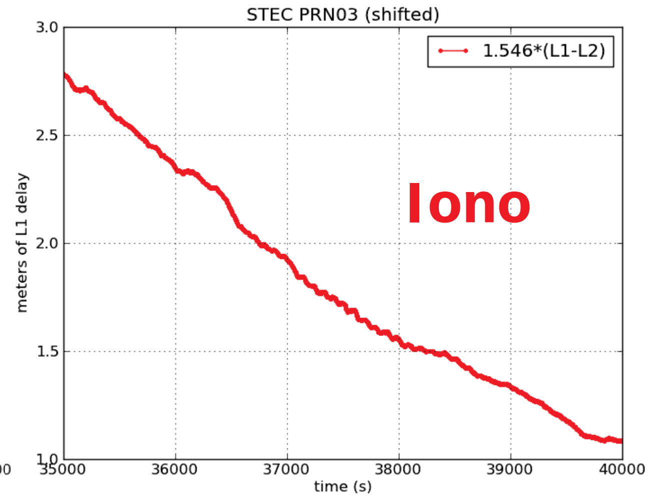
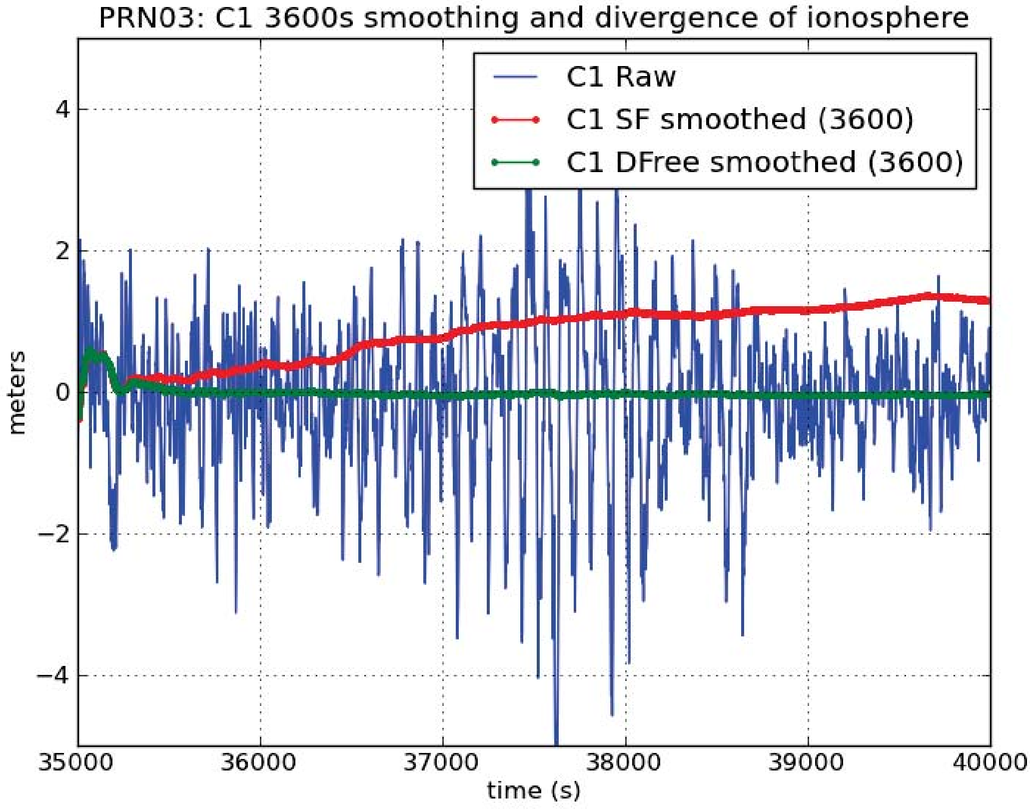
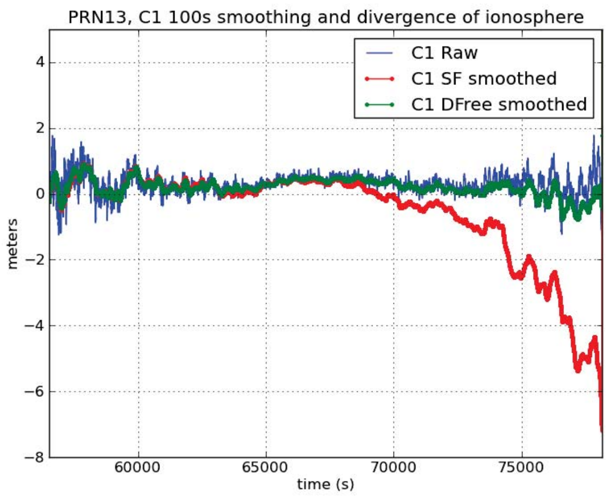


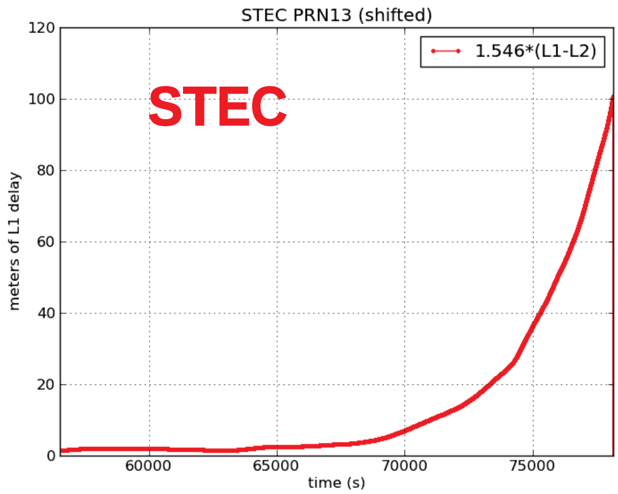




Halloween storm

Data File: amc23030.03o_1Hz





N=100 (i.e. filter smoothing time constant ***τ=100 sec***).

Carrier-smoothed pseudorange: DFree

Divergence-Free (Dfree) smoother:

With two frequency **carrier** measurements **a combination of carriers** with the same ionospheric delay (the same sign) as the code can be generated:

$$\boxed{L_{1,DF} = L_1 + 2\tilde{\alpha}_1 \left(L_1 - L_2\right) = \rho + I_1 + B_{1,DF} + \varsigma_{1,DF}}$$

$$\tilde{\alpha}_1 = \frac{f_2^2}{f_1^2 - f_2^2} = \frac{1}{\gamma - 1} = 1.545$$

$$\gamma = \left(\frac{77}{60}\right)^2$$

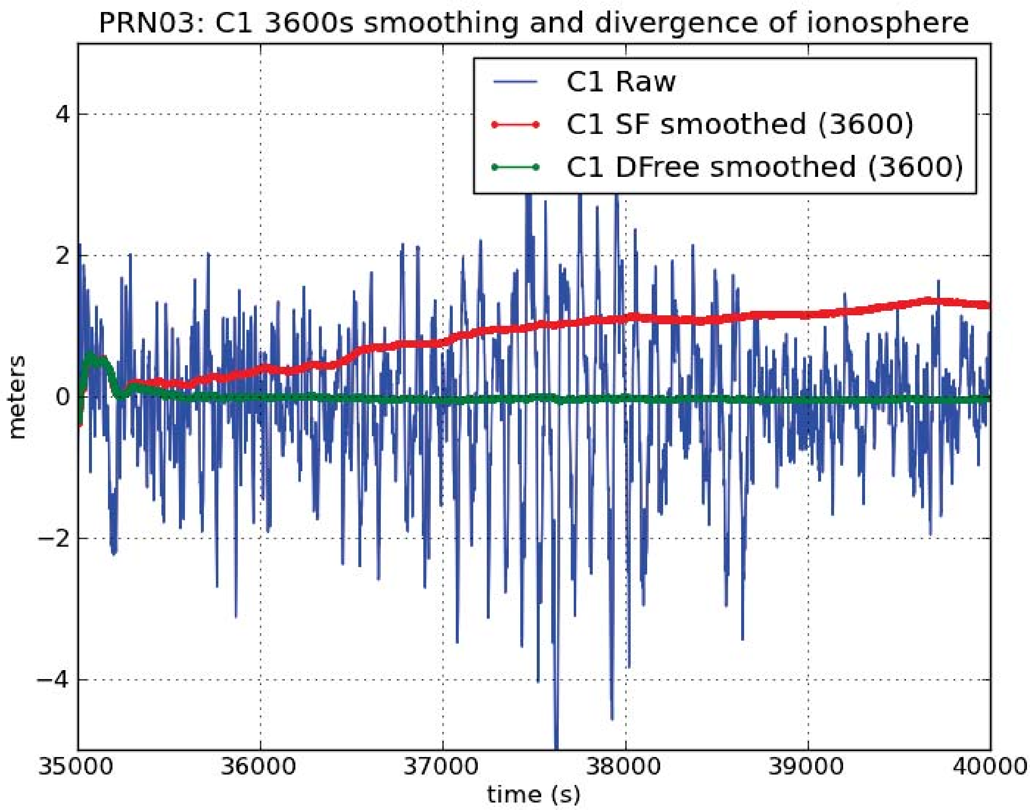
With this new combination we have:

$$P_1 = \rho + I_1 + \varepsilon_1$$
$$L_{1,DF} = \rho + I_1 + B_{1,DF} + \varsigma_{1,DF}$$

Thence,

$$P_1 - L_{1,DF} = B_{1,DF} + \varepsilon_1$$
$$\Rightarrow \textit{No} \text{ Code-carrier divergence!} \Rightarrow \hat{P}_{1,DF} = \rho + I_1 + \upsilon_{12}$$

This smoothed code is immune to temporal gradients (unlike the SF smoother), being the same ionospheric delay as in the original raw code (i.e. I_1). Nevertheless, as it is still affected by the ionosphere, its **spatial decorrelation** must be taken into account in differential positioning.



Carrier-smoothed pseudorange: IFree

Ionosphere-Free (Ifree) smoother:

Using both code and carrier dual-frequency measurements, it is possible to remove the frequency dependent effects using the ionosphere-free combination of code and carriers (PC and LC). Thence:

$$P_C = \rho + \varepsilon_{P_C}$$
$$L_C = \rho + B_{L_C} + \upsilon_{L_C}$$

$$P_{IFree} \equiv P_C = \frac{\gamma P_1 - P_2}{\gamma - 1} \quad ; \quad L_{IFree} \equiv L_C = \frac{\gamma L_1 - L_2}{\gamma - 1} \qquad \gamma = \left(\frac{77}{60}\right)^2$$

Thence,

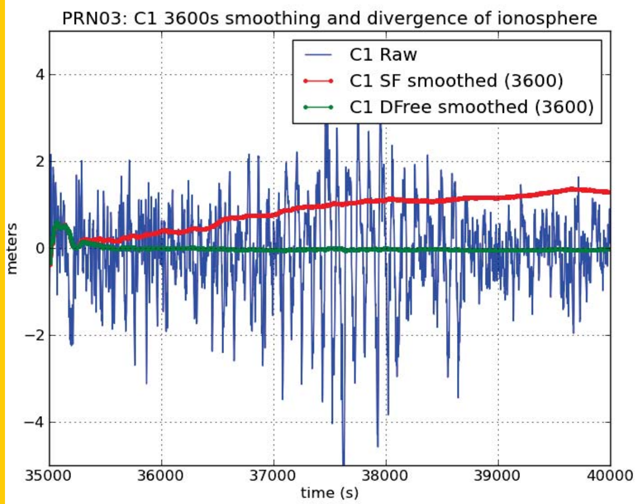
$$P_C - L_C = B_C + \varepsilon_{P_C} \quad \Rightarrow \quad \boxed{\hat{P}_{IFree} \equiv \hat{P}_C = \rho + \upsilon_{IFree}}$$

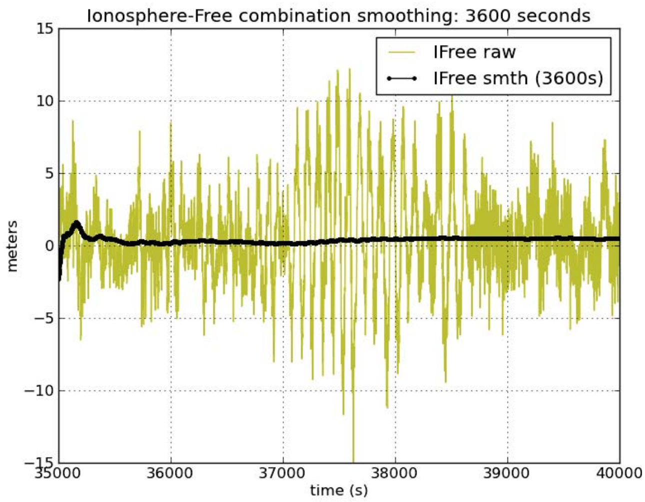
$$\sigma_{Pc} = \frac{\sqrt{\gamma^2 + 1}}{\gamma - 1}\sigma_{P_1} \simeq 3\sigma_{P_1}$$

This smoothed is based on the ionosphere-free combination of measurements, and therefore it is unaffected by either the spatial and temporal inospheric gradients, but has the disadvantage that **the noise in amplified by a factor 3** (using the legacy GPS signals).

Vertical range: [-5 : 5]

Vertical range: [-15:15]





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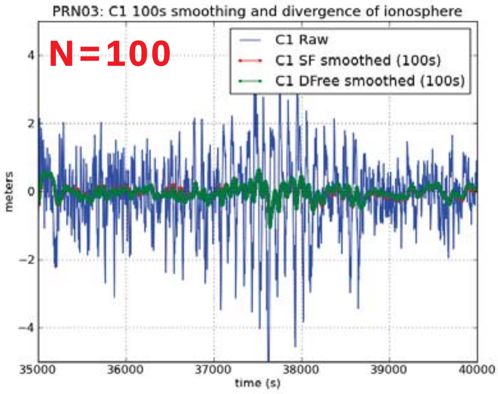
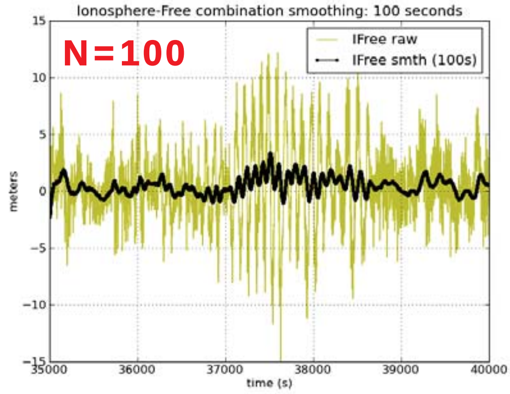
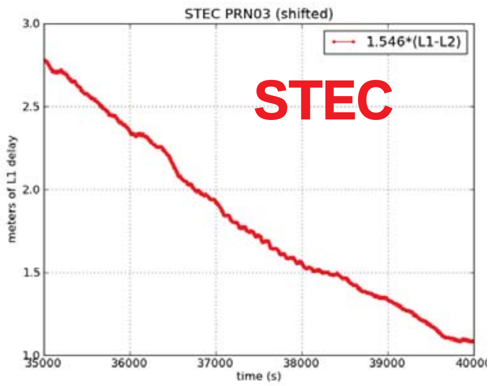
C1, L1

PC, LC







Exercise:

Justify that the ionosphere-free combination (**PC**) is (obviously) not affected by the code-carrier divergence, but it is 3 times noisier.

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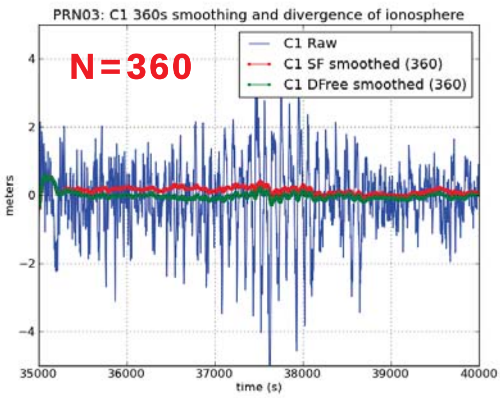
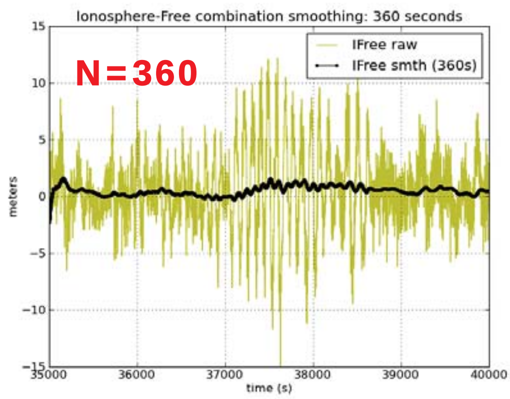
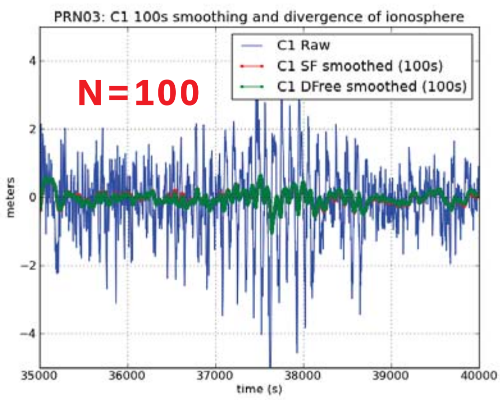
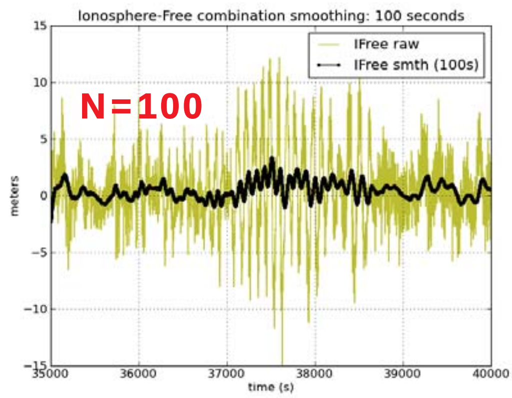
C1, L1

PC, LC

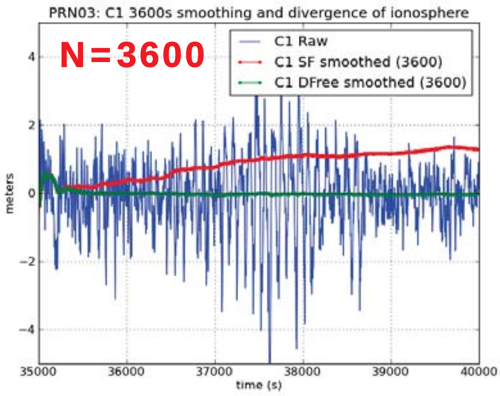


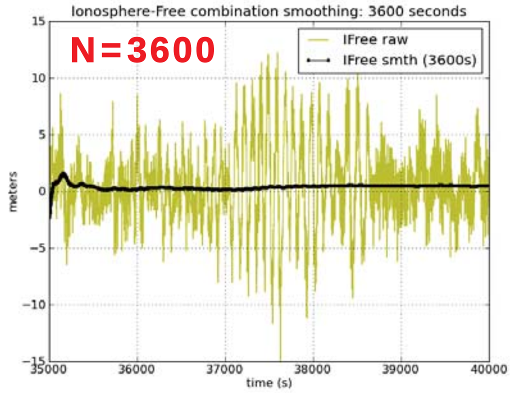


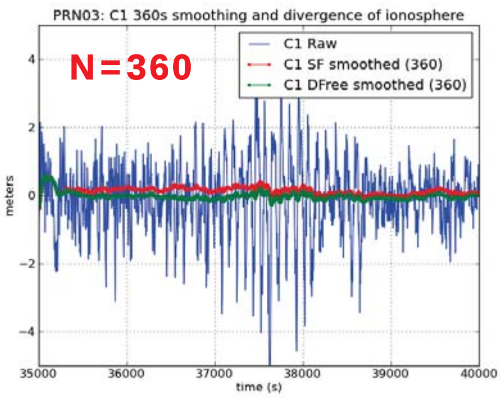


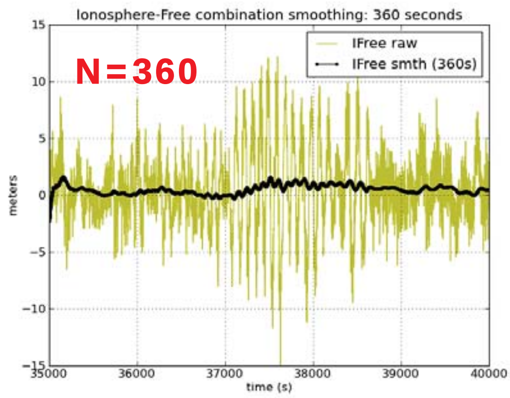


C1, L1 | **PC, LC**



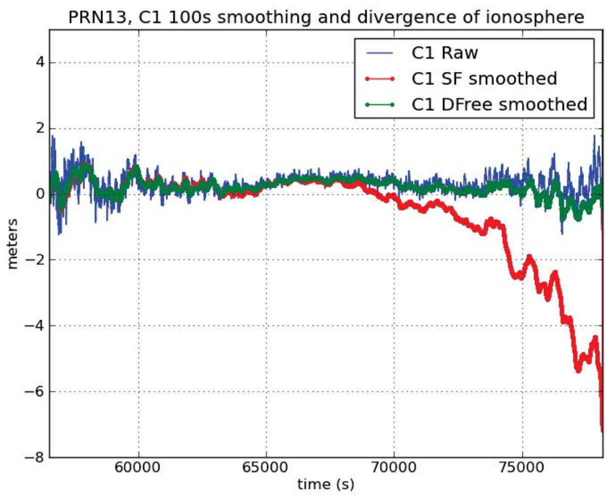


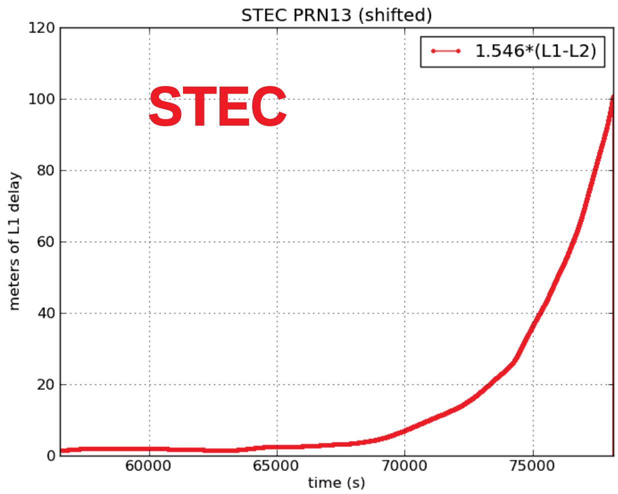




Halloween storm

Data File: amc23030.03o_1Hz





N=100 (i.e. filter smoothing time constant τ*=100 sec*).

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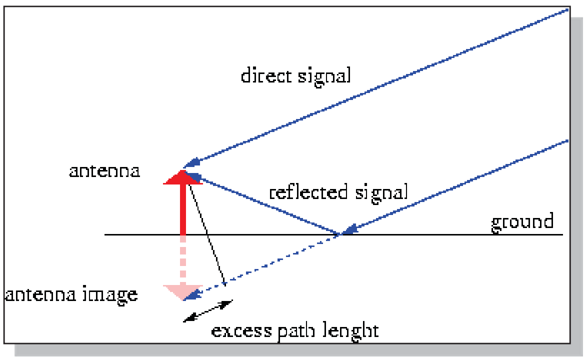
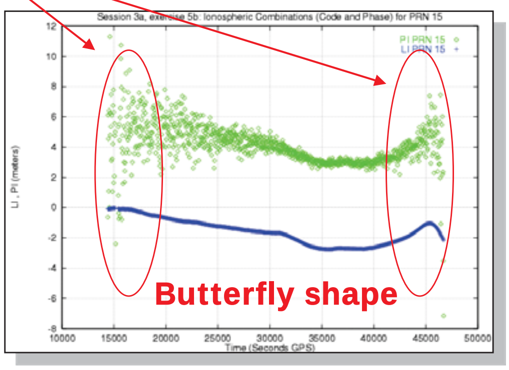
Multipath

One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.

It affects both code and carrier phase measurements, and it is more important at low elevation angles.





Code: up to 1.5 chip-length ➔ up to 450m for C1 [theoretically]
Typically: less than 2-3 m.

Phase: up to $\lambda/4$ ➔ up to 5 cm for L1 and L2 [theoretically]
Typically: less than 1 cm

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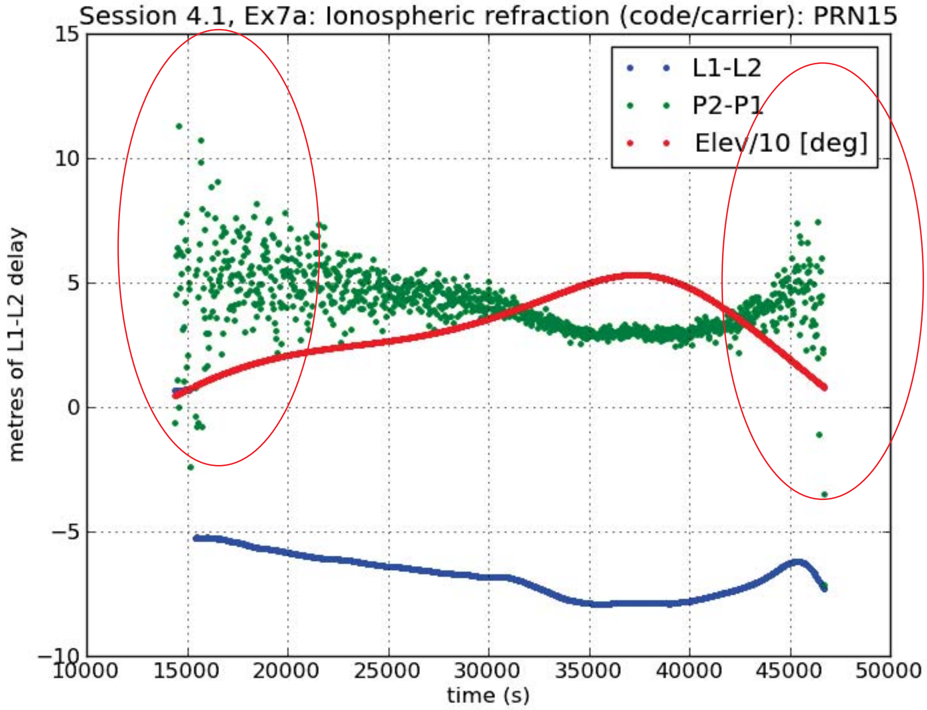
Exercise

Plot code and phase geometry-free combination for satellite PRN 15 of file 97jan09coco_r0.rnx and discuss the results.



Butterfly shape:

High multipath for low elevation rays (when satellite rises and sets)

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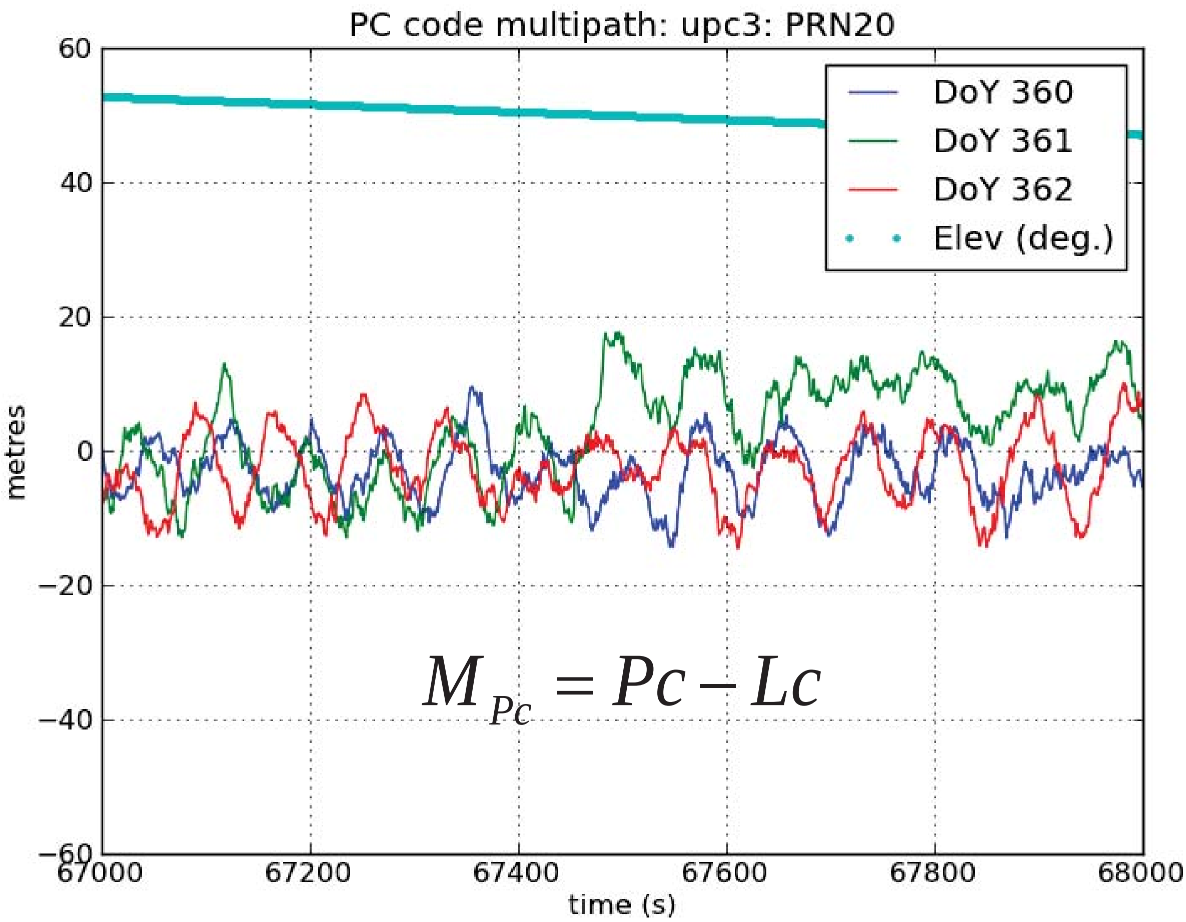
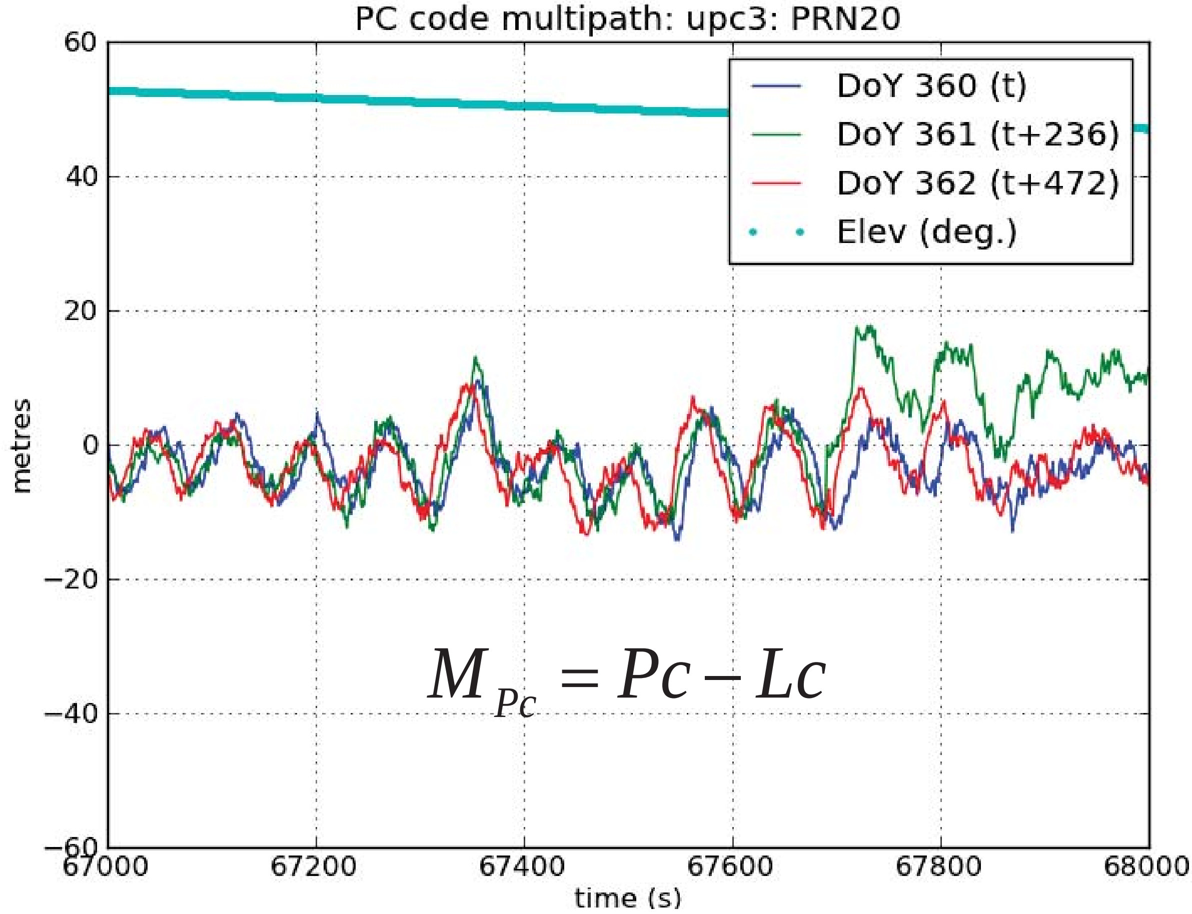


$$M_{Pc} = Pc - Lc$$



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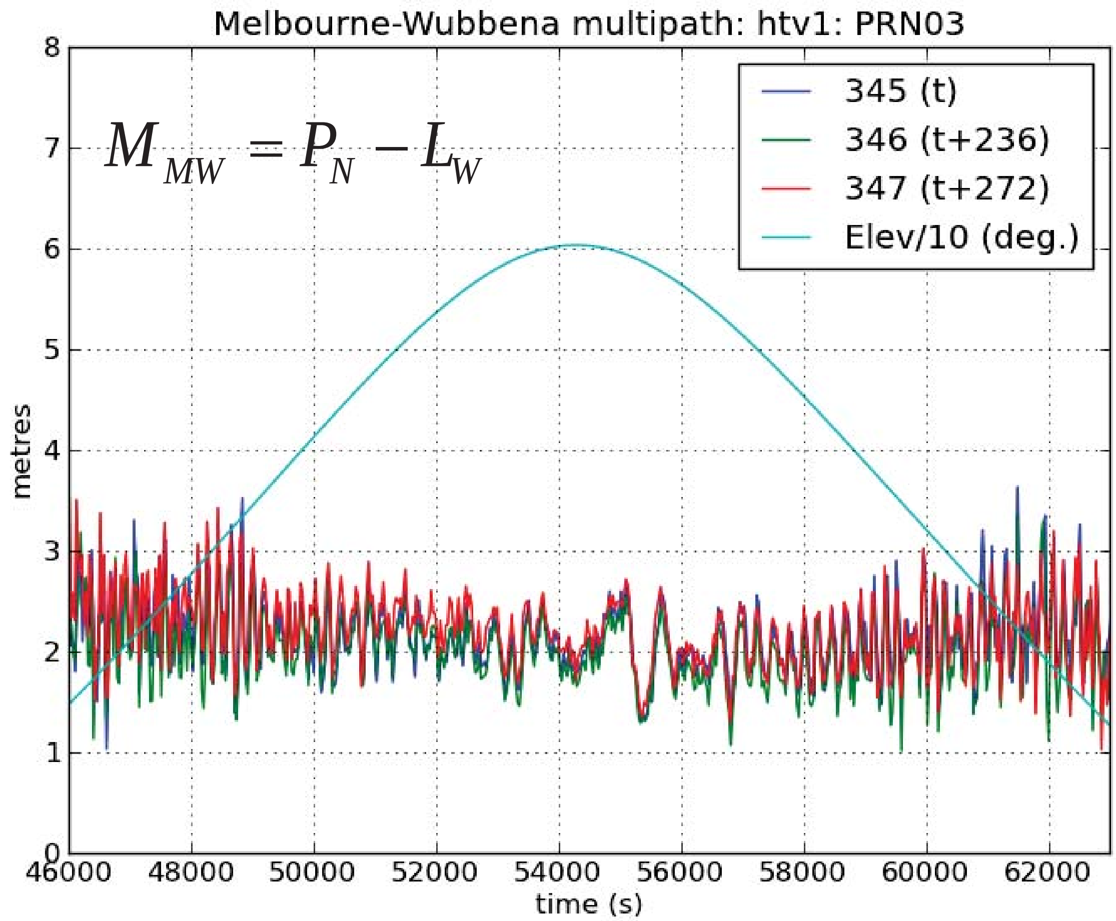
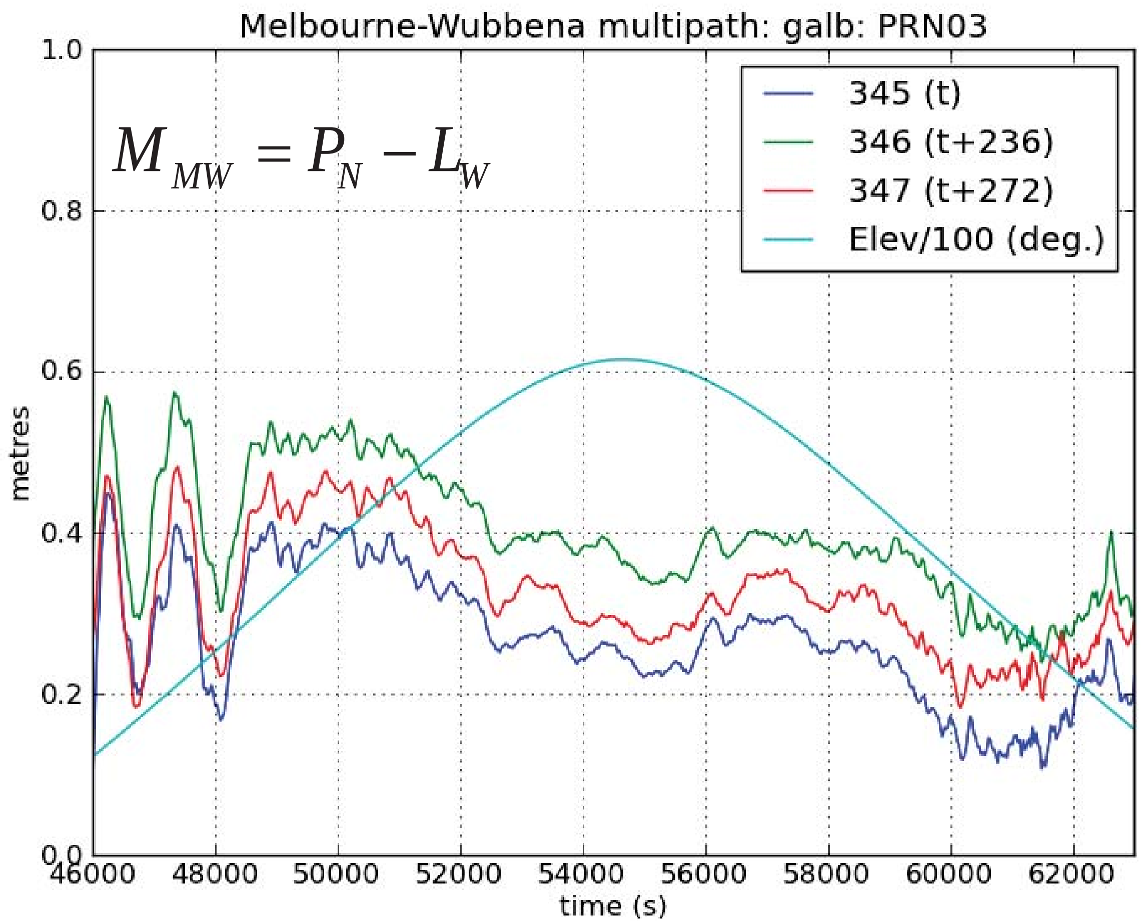
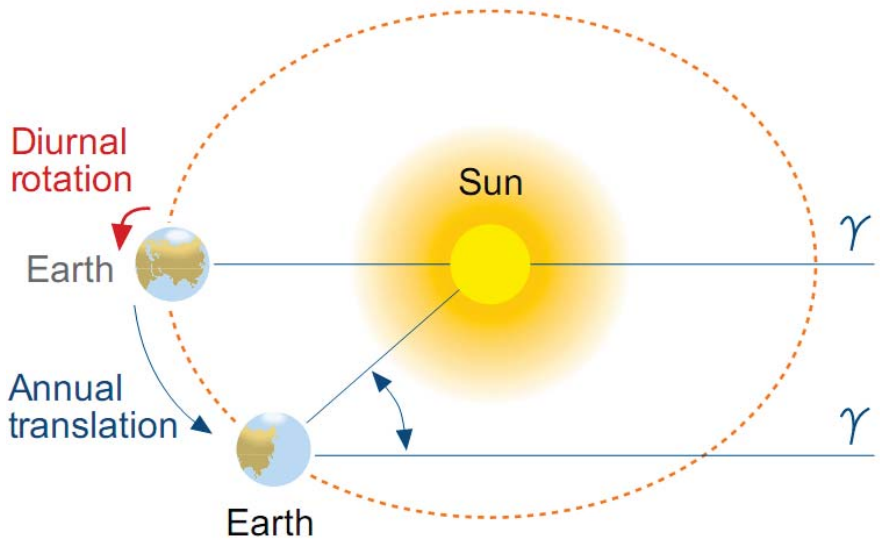




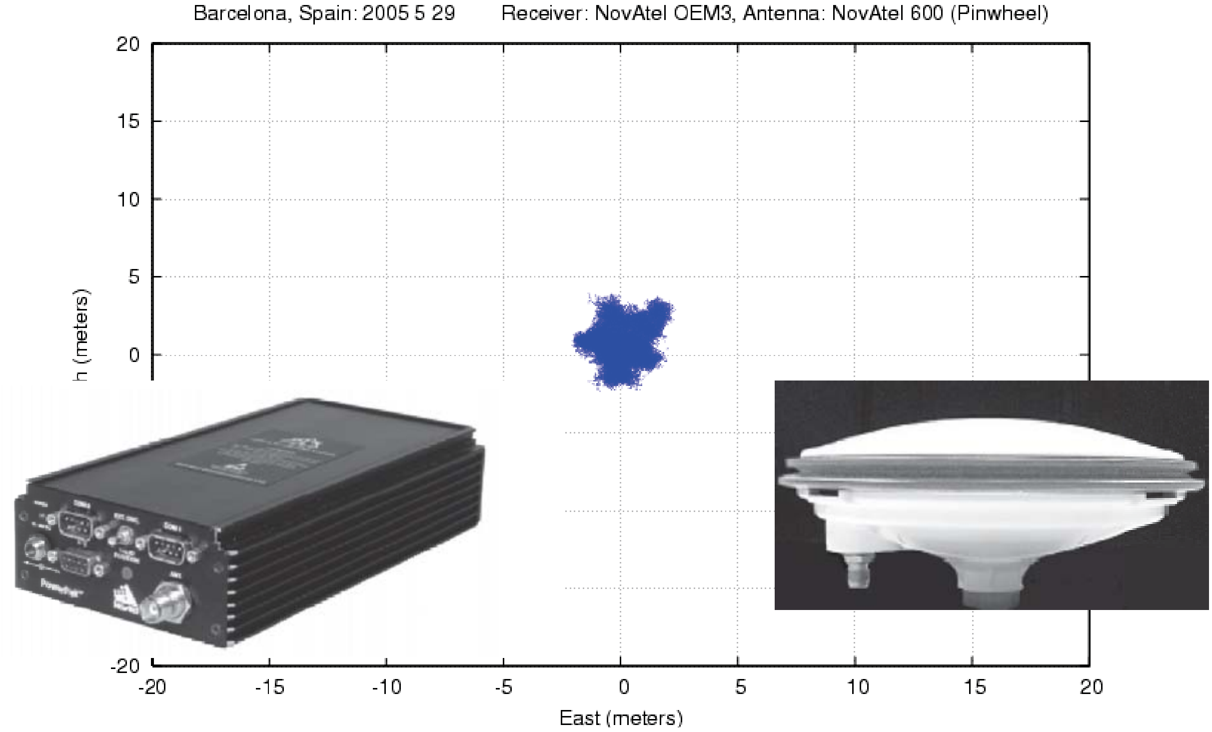


After one year, the directions of the Sun and Aries coincide again, but the number of laps relative to the Sun (solar days) is one less than those relative to Aries (sidereal days).

$$\frac{24\text{h}}{365.2422} \approx 3^{\text{m}}56^{\text{s}}$$

Thus, a **sidereal day is shorter** than a solar day for about $3^{\text{m}}\,56^{\text{s}}$

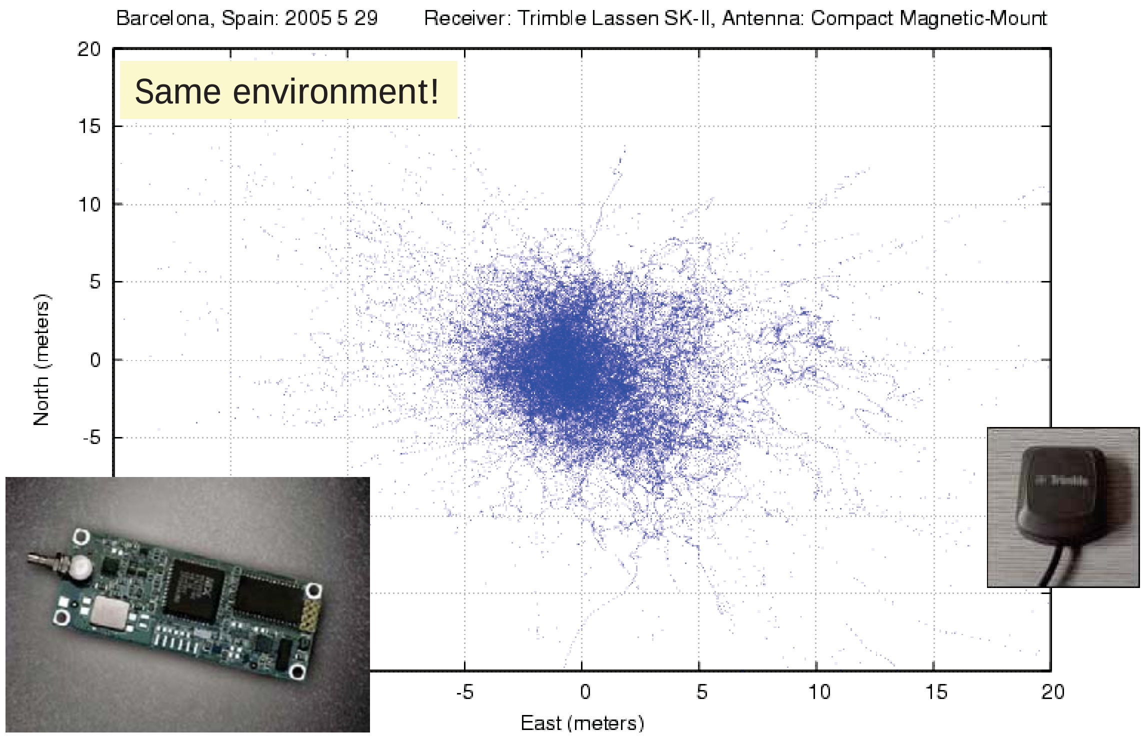
Receiver and multipath noise



GPS standalone (C1 code)

10,000 €

Receiver and multipath noise



GPS standalone (C1 code)

100 €

References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

Thank you!

Lecture 2

Satellite orbits and clocks computation and accuracy

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Authorship statement

24 April 2014

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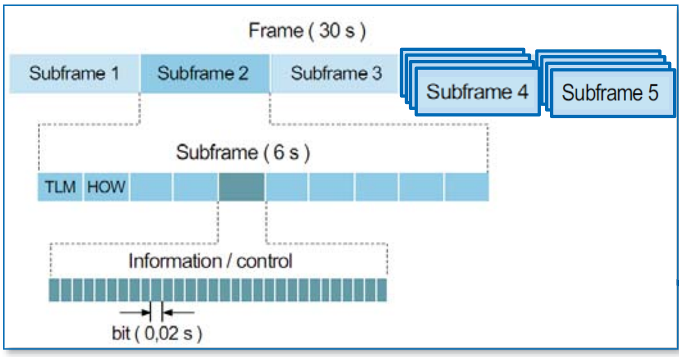
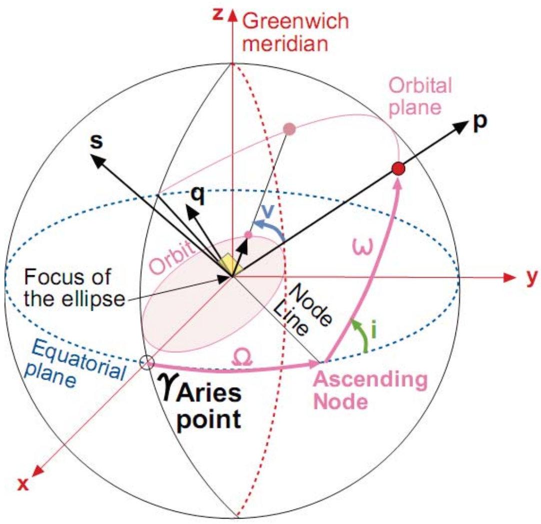
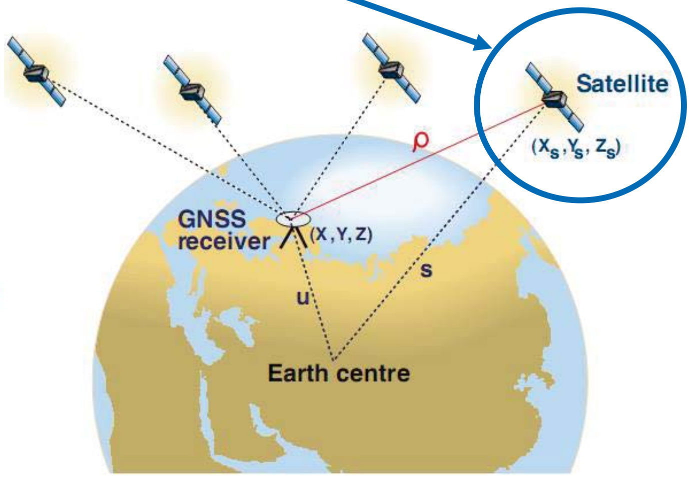
The GPS navigation message provides pseudo-Keplerian elements to compute satellite coordinates







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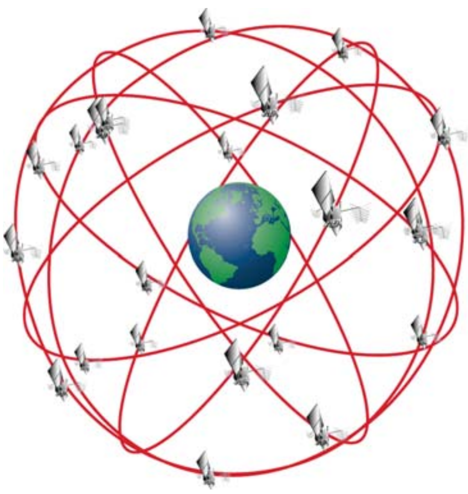
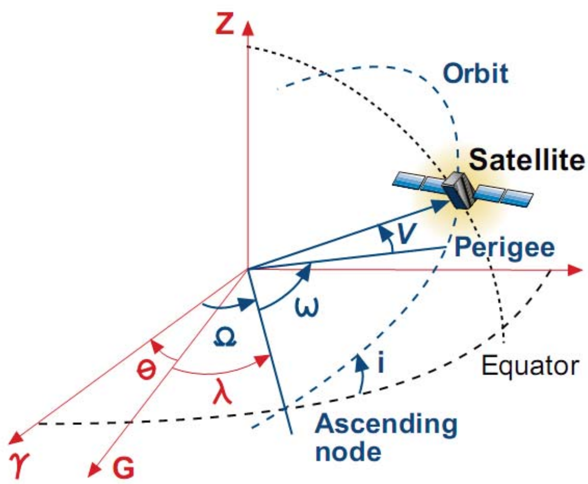
(X, Y, Z, Vx, Vy, Vz) ➔ (*a, e, i, Ω, ω, V*)

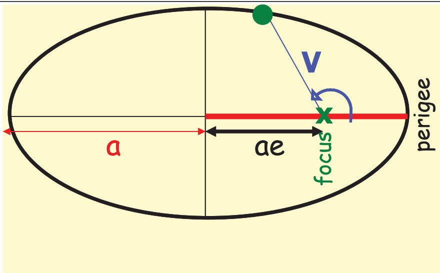
6 values are needed (x,y,z,vx,vy,vz) to provide the position and velocity of a body. They can be map into the **six Keplerian elements** (*a, e, i, Ω, ω, V*), which provides the "natural" representation of the orbit!

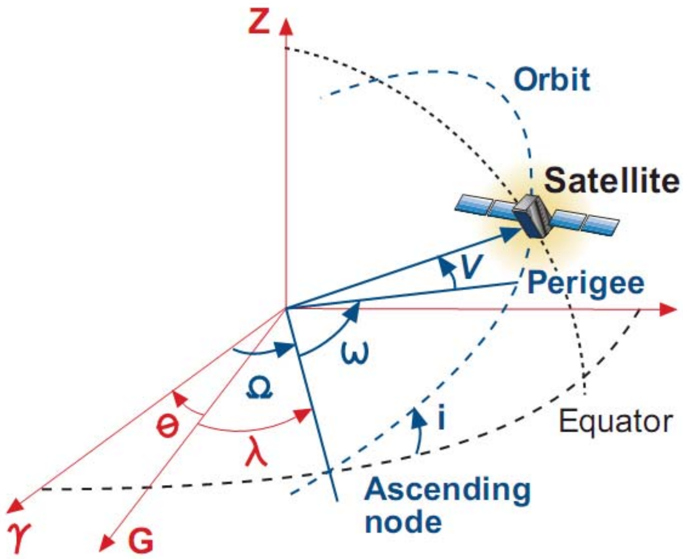




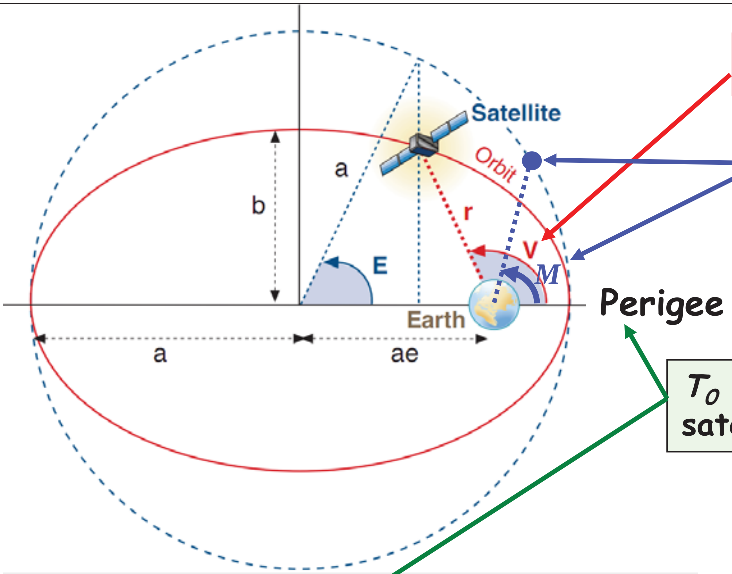
$$(\boxed{a,\ e},\ \boxed{i,\ \Omega,\ \omega},\ \boxed{V})$$

orbit shape | orbit orientation | position in the orbit





- **i** inclination
- **ω** argument of perigee
- **Ω** arg. ascending node (Aries)
- **λ** arg. ascending node (Greenwich)
- **V** true anomaly
- **Θ** sidereal time
- **ϒ** vernal equinox
- **G** Greenwich meridian



True anomaly $V(t)$

Fictitious body moving at velocity $n=2\pi/P=constant$

➔ Mean anomaly $M(t)$

T_0 : time of passage by satellite's perigee

$$t \rightarrow \boxed{\begin{matrix} T_0 \\ n=\dfrac{2\pi}{P} \\ \mu,\ a,\ e \end{matrix}} \rightarrow V(t) \longleftrightarrow$$

$$M(t)=n(t-T_0) \quad ; \quad n=\frac{2\pi}{P}=\sqrt{\frac{\mu}{a^3}}$$

$$E(t)=M(t)+e\sin E(t)$$

$$V(t)=2\arctan\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{E(t)}{2}\right]$$

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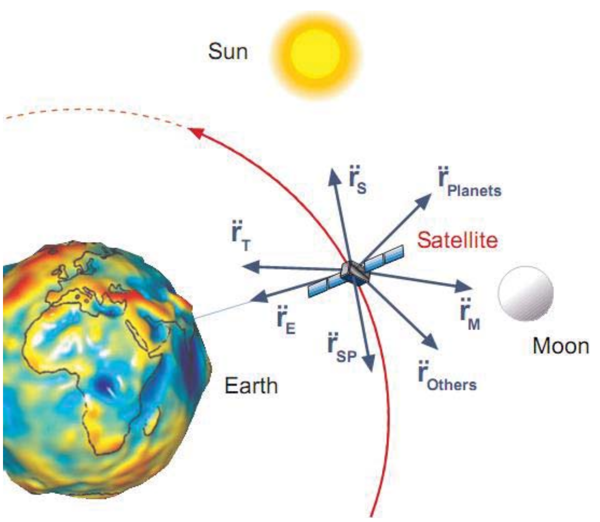
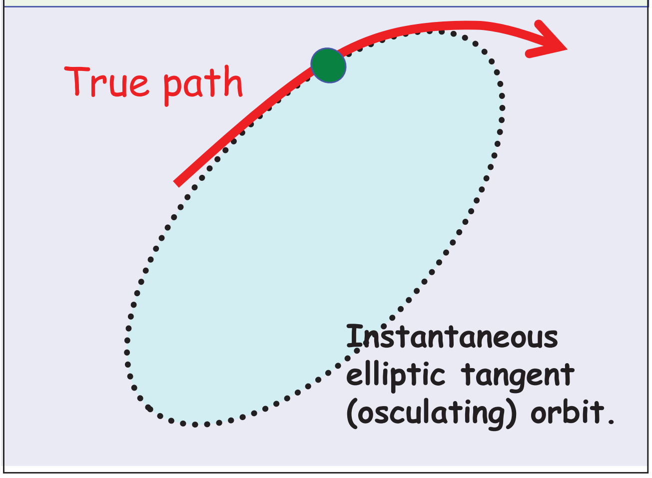
Due to the non-spherical nature of gravitational potential, the attraction of the Sun and Moon, the solar radiation pressure, etc., **the true satellite path deviates from the elliptic orbit**.



At any time an elliptical orbit tangent to the true path can be defined. This is the "osculating orbit", whose Keplerian elements vary with time "t":

$$a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$$



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Different magnitudes of perturbation and their effects on GPS orbits

Perturbation	Acceleration (m/s^2)	Orbital effect in 3 hours	Orbital effect in 3 days
Central force (as a reference)	0.56		
J_2	$5 \cdot 10^{-5}$	2 km	14 km
Rest of the harmonics	$3 \cdot 10^{-7}$	50–80 m	100–1500 m
Solar + Moon grav.	$5 \cdot 10^{-6}$	5–150 m	1000–3000 m
Tidal effects	$1 \cdot 10^{-9}$	–	0.5–1.0 m
Solar rad. pressure	$1 \cdot 10^{-7}$	5–10 m	100–800 m

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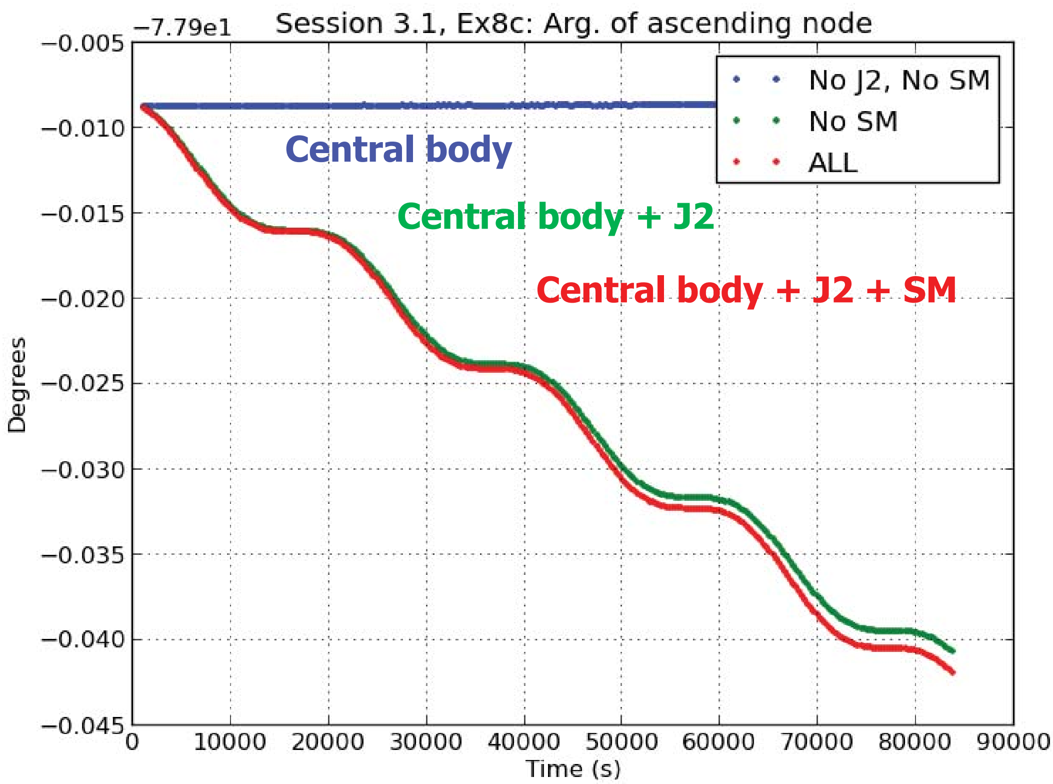
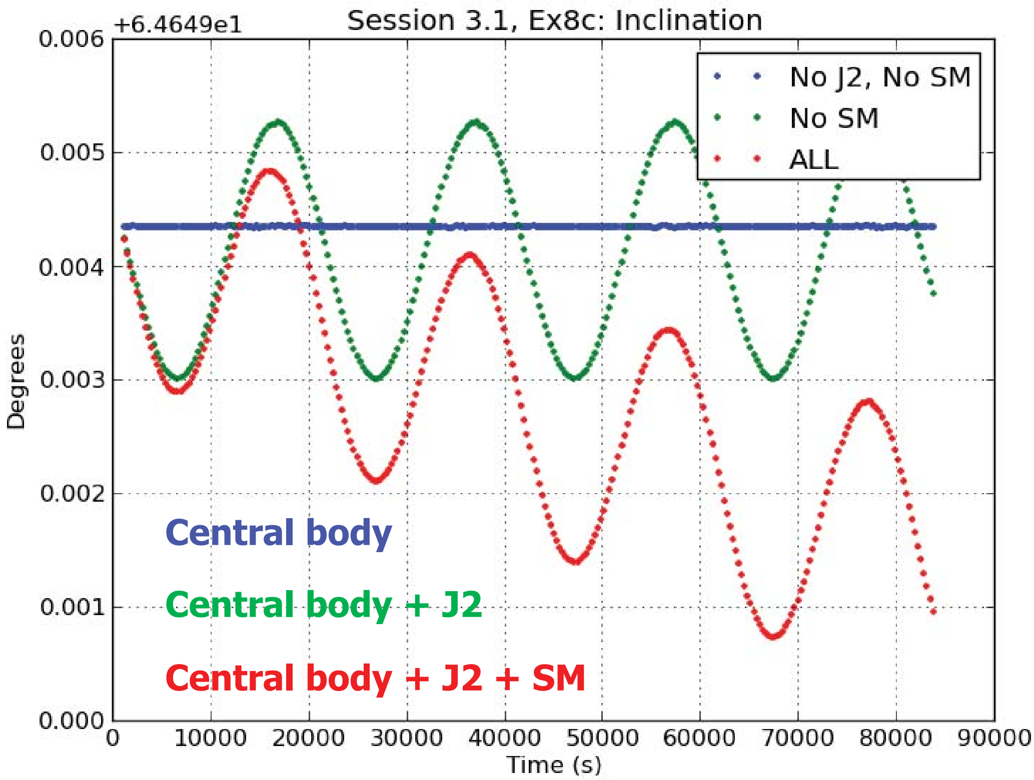
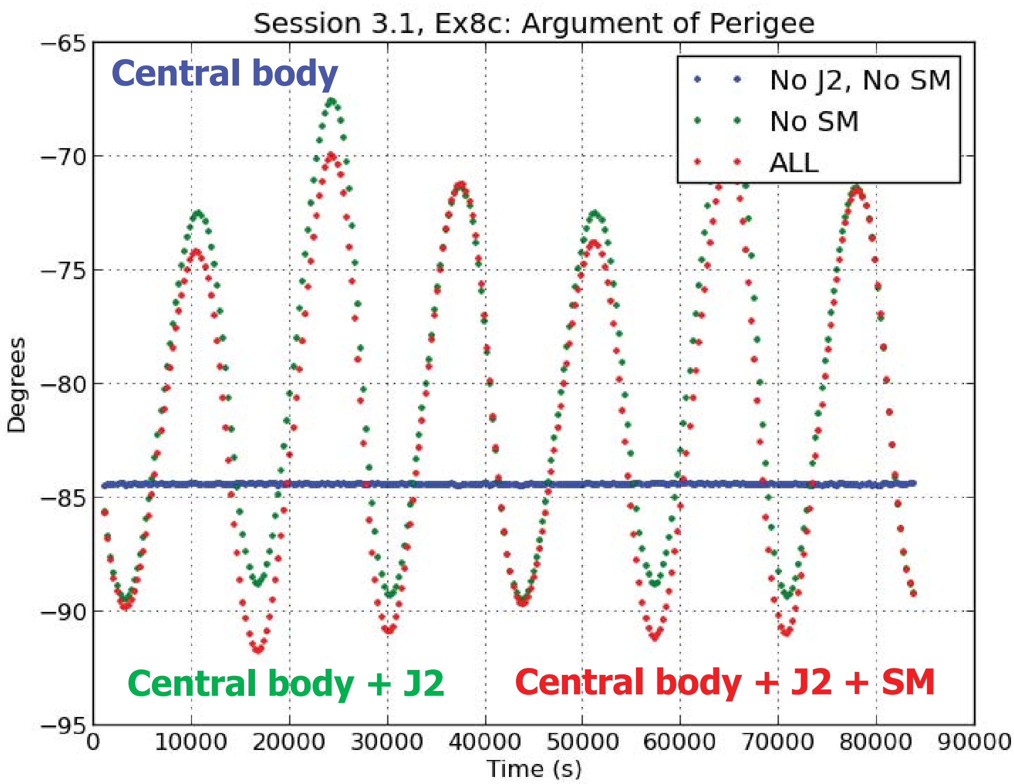



Session 3.1, Ex8c: Inclination
+6.4649e1
0.006
0.005
0.004
0.003
0.002
0.001
0.000
Degrees
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
Time (s)
No J2, No SM
No SM
ALL
Central body
Central body + J2
Central body + J2 + SM


Session 3.1, Ex8c: Argument of Perigee
−65
−70
−75
−80
−85
−90
−95
Degrees
0
10000
20000
30000
40000
50000
60000
70000
80000
90000
Time (s)
Central body
No J2, No SM
No SM
ALL
Central body + J2
Central body + J2 + SM

Calculation of osculating orbital elements from position and velocity (rv2osc.f)

$$\boxed{(x, y, z, v_x, v_z, v_z) \Rightarrow (a, e, i, \Omega, \omega, M)}$$

$$\vec{c} = \vec{r} \times \vec{v} \Longrightarrow p = \frac{c^2}{\mu} \Longrightarrow p$$
$$\mathrm{v}^2 = \mu(2/r - 1/a) \Longrightarrow a$$
$$\mathrm{p} = \mathrm{a}\ (1 - \mathrm{e}^2) \Longrightarrow e$$

$$\vec{c} = c\vec{S} \Longrightarrow \Omega = \arctan(-c_x/c_y); \quad i = \arccos(c_z/c) \Longrightarrow \Omega, i$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cos(\omega+V) - \sin\Omega\sin(\omega+V)\cos i \\ \sin\Omega\cos(\omega+V) + \cos\Omega\sin(\omega+V)\cos i \\ \sin(\omega+V)\sin i \end{pmatrix} \Longrightarrow \omega + V$$

$$r = \frac{p}{1 + e\cos(V)} \Longrightarrow \omega, V$$

$$\tan(E/2) = (\frac{1-e}{1+e})^{1/2}\tan(V/2) \quad ; \quad M = E - e\sin E \Longrightarrow M$$

Calculation of position and velocity from osculating orbital elements (osc2rv.f)

$$\boxed{(a, e, i, \Omega, \omega, \underbrace{T; t}_{V}) \Rightarrow (x, y, z, v_x, v_z, v_z)}$$

$$\boxed{\begin{array}{ccccccc} t & \Longrightarrow & M & \Longrightarrow & E & \Longrightarrow & (r, V) \\ & M = n(t-T) & & M = E - e\sin E & & r = a(1 - e\cos E) & \\ & & & & & \tan(V/2) = (\frac{1+e}{1-e})^{1/2}\tan(E/2) & \end{array}}$$

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \begin{pmatrix} r\cos(V) \\ r\sin(V) \\ 0 \end{pmatrix} \; ; \; \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{na^2}{r}\{\vec{Q}(1-e^2)^{1/2}\cos E - \vec{P}\sin E\}}$$

Where:

$$\begin{aligned} \mathrm{R} &= \mathrm{R}_3(-\Omega)R_1(-i)R_3(-\omega) = \\ &= \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \mathrm{P}_x & \mathrm{Q}_x & \mathrm{S}_x \\ \mathrm{P}_y & \mathrm{Q}_y & \mathrm{S}_y \\ \mathrm{P}_z & \mathrm{Q}_z & \mathrm{S}_z \end{pmatrix} = [\vec{P}\ \vec{Q}\ \vec{S}] \end{aligned}$$

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Exercise: Orbital elements variation:

File 1995-10-18.eci contains the precise position and velocities of GPS satellites every 5 minutes for October 18th, 1995 in a Earth-Centred Inertial system (ECI) **[from JPL/NASA server: ftp://sideshow.jpl.nasa.gov/pub/gipsy_products]**

a) Use program "**rv2osc**" to compute the instantaneous orbital eleme $(X, Y, Z, Vx, Vy, Vz) \rightarrow (a, e, i, \Omega, \omega, V)$

b) Plot the orbital elements in function of time to show their variation: $a(t), e(t), i(t), \Omega(t), \omega(t), V(t)$

c) Compare with the broadcast orbital elements

Solution:

a) cat 1995-10-18.eci|rv2osc> orb.dat

b) See the following plots

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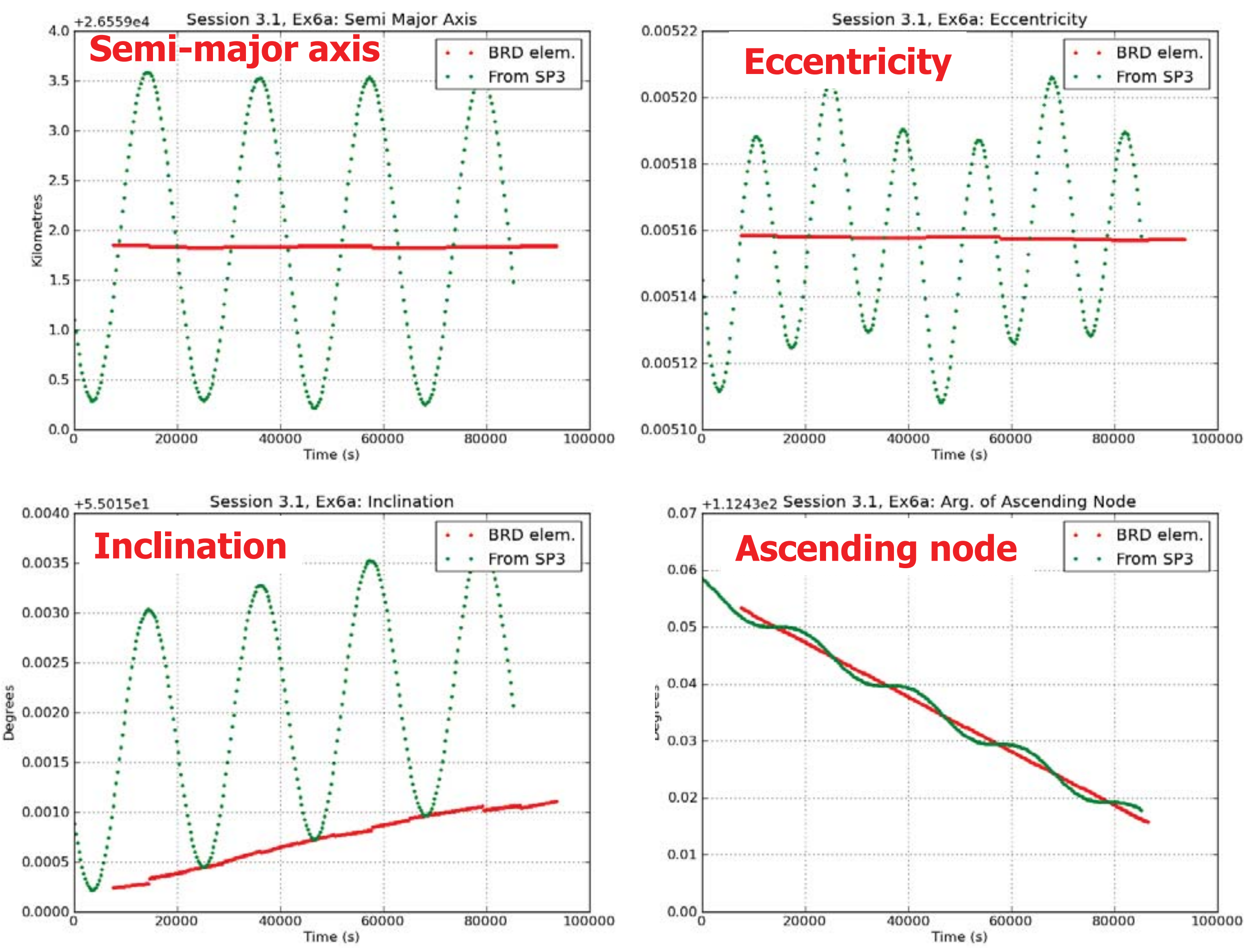
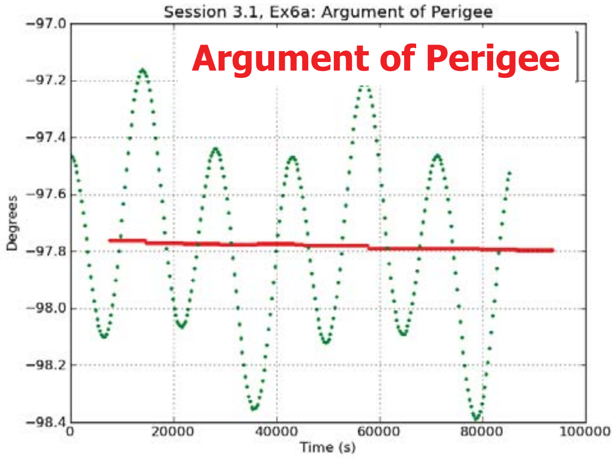
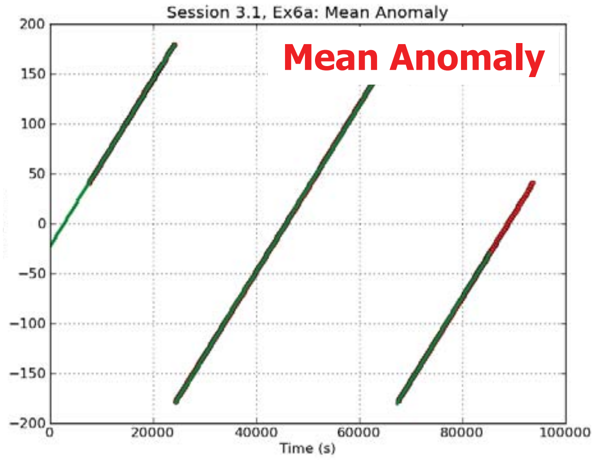



Session 3.1, Ex6a: Argument of Perigee
Argument of Perigee
Degrees
Time (s)


Session 3.1, Ex6a: Mean Anomaly
Mean Anomaly
Time (s)

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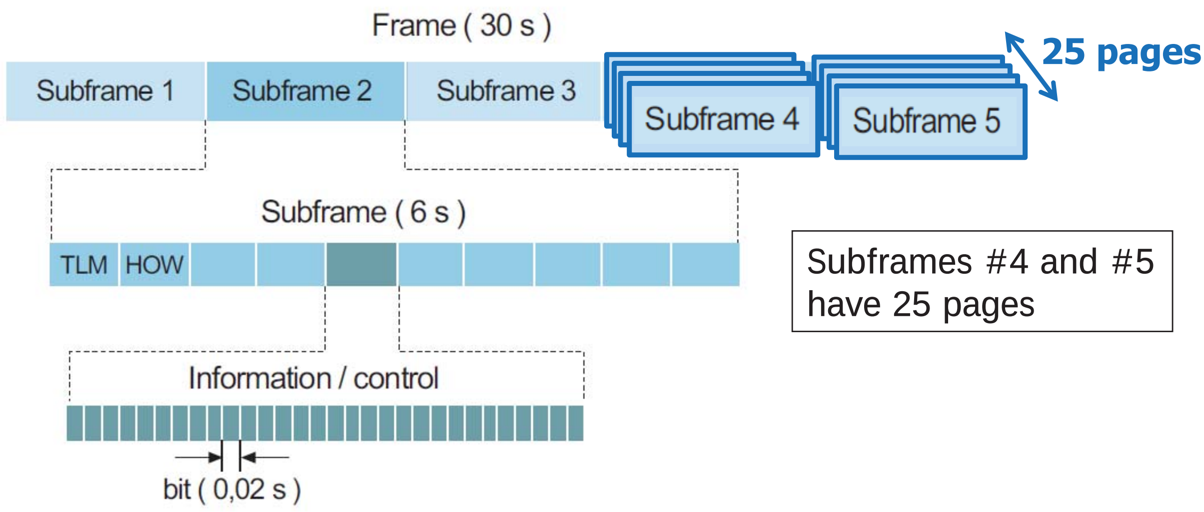
GPS navigation message



Subframes #4 and #5 have 25 pages

One Master Frame includes All 25 pages of Subframes #4 and #5 ➔ 25 x 30s = **12.5 min**

Subframe 1 contains information about the parameters to be applied to **satellite clock** status for its correction. These values are polynomial coefficients that allow time onboard to be converted to GPS time. The subframe also contains information on satellite health condition.

Subframes 2 and 3 contain **satellite ephemerides**.

Subframe 4 provides **ionospheric model** parameters (in order to adjust for ionospheric refraction), UTC information, part of the **almanac**, and indications whether the A/S is activated or not (which transforms the P code into encrypted Y code).

Subframe 5 contains data from the **almanac** and on constellation status. It allows rapid identification of the satellite from which the signal comes. A total of 25 frames are needed to complete the almanac.

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Ephemeris in navigation message

Parameter	Explanation
$IODE$	Series number of ephemerides data
t_{oe}	Ephemerides reference epoch
$\sqrt{a}$	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω	Ascending node's right ascension
Δn	Mean motion difference
$\dot{i}$	rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
c_{uc}, c_{us}	Latitude argument correction
c_{rc}, c_{rs}	Orbital radius correction
c_{ic}, c_{is}	Inclination correction

In order to calculate WGS84 satellite coordinates, you should apply de following algorithm [GPS/SPS-SS, table 2-15] (see in the book FORTRAN subroutine orbit.f)

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RINEX ephemeris file

```
     2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
XPRINT v1.1         gAGE                00/08/17 09:31:37   PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE                               COMMENT
    +1.7695E-08 +2.2352E-08 -1.1921E-07 -1.1921E-07         ION ALPHA
    +1.1878E+05 +1.4746E+05 -1.3107E+05 -3.2768E+05         ION BETA
    +1.955777406693E-08+1.598721155460E-14   405504     1064 DELTA_UTC: A0,A1,T,W
    13                                                      LEAP SECONDS
                                                            END OF HEADER

03 00  5 30 10  0 40.0+7.855705916882E-06+3.524291969370E-12+0.000000000000E+00
    +1.010000000000E+02+6.500000000000E+01+5.456298524109E-09+5.530285585107E-01  Mo
    +3.475695848465E-06+1.308503560722E-03+2.641230821609E-06+5.153678266525E+03  e, √a
    +2.088000000000E+05+1.117587089539E-08+7.472176136643E-01-1.862645149231E-09  TOE, Ω
    +9.412719852649E-01+3.163750000000E+02+1.125448382894E+00-8.826796182859E-09  io, ω
    +1.239337382719E-10+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00
    +4.000000000000E+00+0.000000000000E+00-4.190951585770E-09+6.130000000000E+02  TGD
    +2.044980000000E+05+0.000000000000E+00+0.000000000000E+00+0.000000000000E+00

06 00  5 30 10  0  0.0+1.636799424887E-06+0.000000000000E+00+0.000000000000E+00
    +6.000000000000E+01+5.100000000000E+01+5.198073527168E-09-5.601816471398E-01
    +2.635642886162E-06+6.763593177311E-03+2.468004822731E-06+5.153726325989E+03
    +2.088000000000E+05+1.862645149231E-08+7.894129138508E-01+8.195638656616E-08
    +9.487675576456E-01+3.229687500000E+02-2.409256713064E+00-8.734292400447E-09
    +4.714481929846E-11+1.000000000000E+00+1.064000000000E+03+0.000000000000E+00
```

3.1. Computation of satellite coordinates from navigation message (orbit.f)

- **Computation of t_k time since ephemerids reference epoch t_{oe} (t and t_{oe} are given in GPS seconds of week):**

$$\boxed{t_k = t - t_{oe}}$$

- **Computation of mean anomaly M_k for t_k,**

$$\boxed{M_k = M_0 + \left(\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n \right) t_k}$$

- **Iterative resolution of Kepler's equation in order to compute eccentric anomaly E_k :**

$$\boxed{M_k = E_k - e \sin E_k}$$

- **Calculation of true anomaly v_k :**

$$\boxed{v_k = \arctan\left(\frac{\sqrt{1-e^2}\,\sin E_k}{\cos E_k - e} \right)}$$

- **Computation of latitude argument u_k from perigee argument W, true anomaly v_k and corrections c_{uc} and c_{us}:**

$$\boxed{u_k = \omega + v_k + c_{uc}\cos 2(\omega + v_k) + c_{us}\sin 2(\omega + v_k)}$$

- **Computation of radial distance r_k, taking into consideration corrections c_{rc} and c_{rs}:**

$$\boxed{r_k = a(1 - 2\cos E_k) + c_{rc}\cos 2(\omega + v_k) + c_{rs}\sin 2(\omega + v_k)}$$

- **Calculation of orbital plane inclination i_k from inclination i_o at reference epoch t_{oe} and corrections c_{ic} and c_{is} :**

$$\boxed{i_k = i_0 + i t_k + c_{ic}\cos 2(\omega + v_k) + c_{is}\sin 2(\omega + v_k)}$$

- **Computation of ascending node longitude Ω_k (Greenwich), from longitude Ω_0 at start of GPS week, corrected from apparent variation of sidereal time at Greenwich between start of week and and reference time $t_k = t - t_{oe}$, and also corrected from change of ascending node longitude since reference epoch t_{oe}.**

$$\boxed{\Omega_k = \Omega_0 + (\Omega - \omega_E)t_k - \omega_E t_{oe}}$$

- **Calculation of coordinates in CTS system, applying three rotations (around u_k, i_k, Ω_k) :**

$$\begin{bmatrix} X_k \\ Y_k \\ Z_k \end{bmatrix} = \mathbf{R}_3(-\Omega_k)\mathbf{R}_1(-i_k)\mathbf{R}_3(-u_k)\begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$

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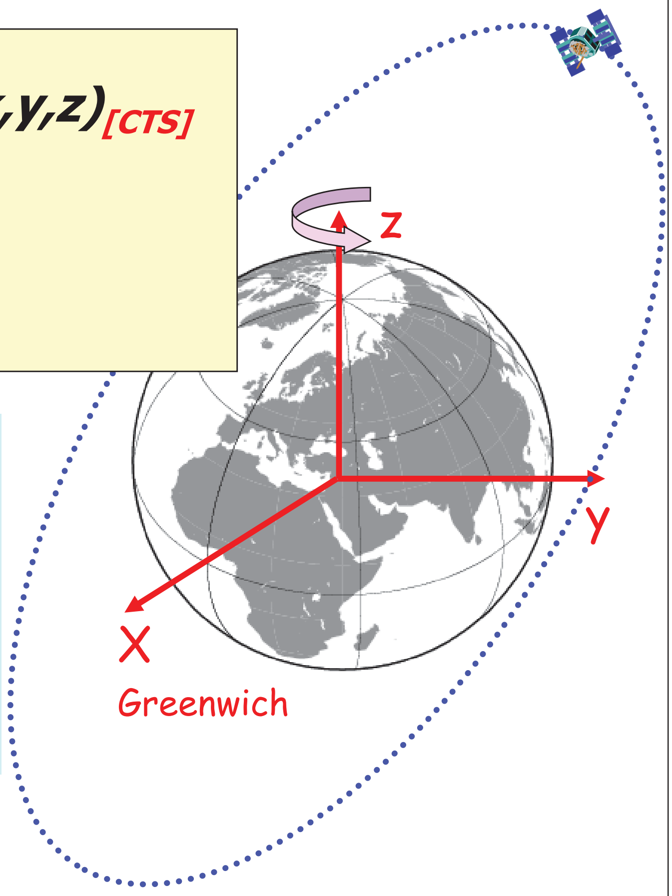
$t \longrightarrow$ **Orbit.f** $\longrightarrow (x,y,z)_{[CTS]}$

Nav. message
(ephemeris)



Conventional Terrestrial System **(CTS):**

Earth Centred, Earth-Fixed (ECEF) System ➔

the reference system rotates with Earth.

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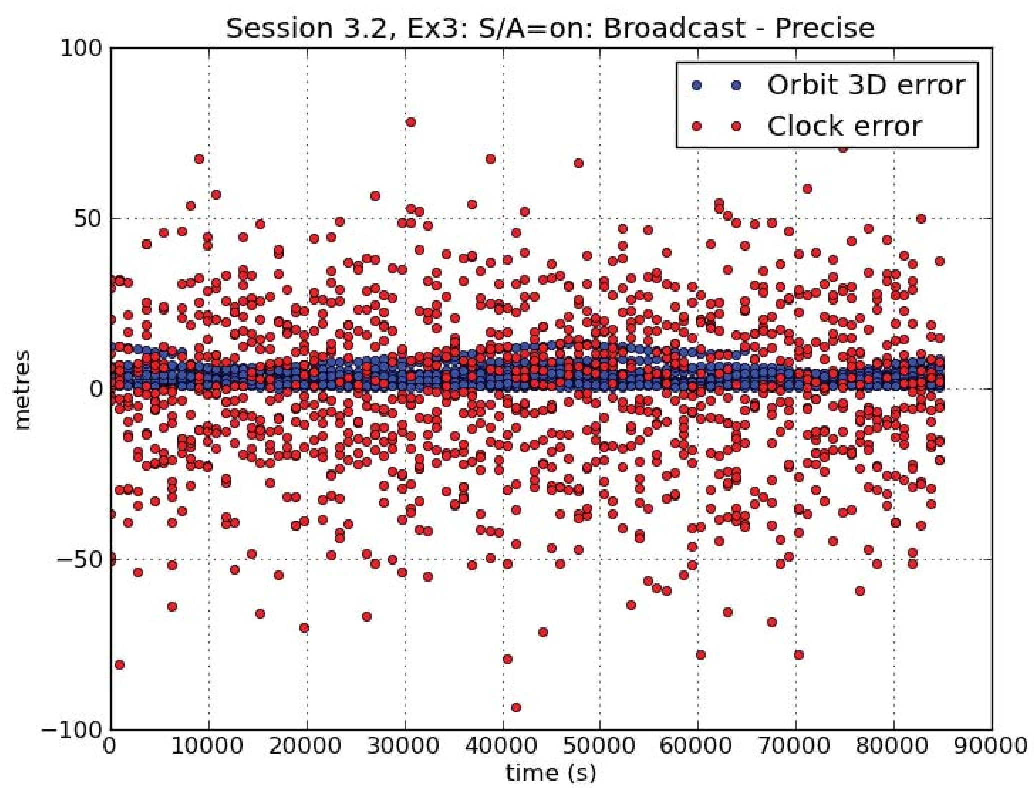
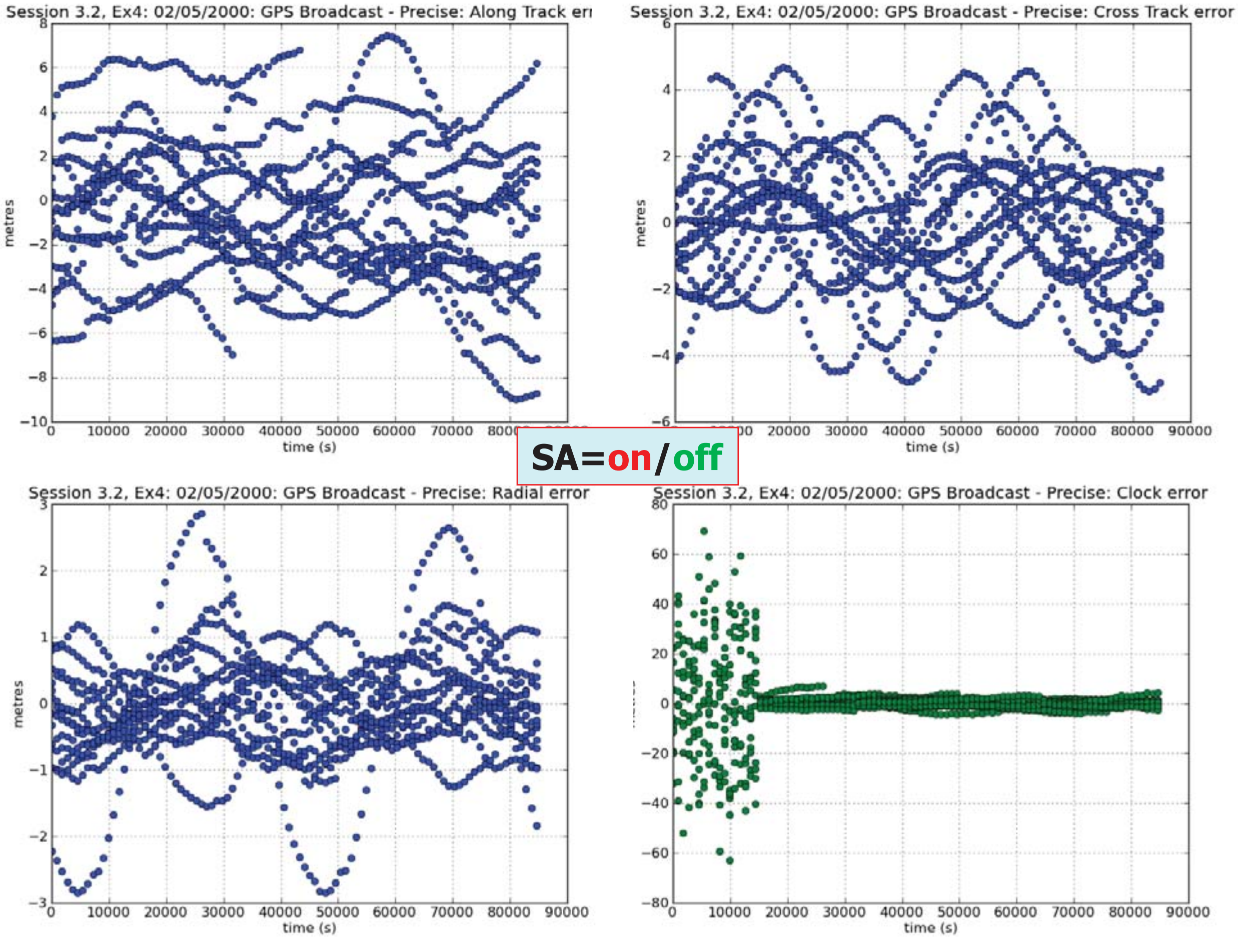
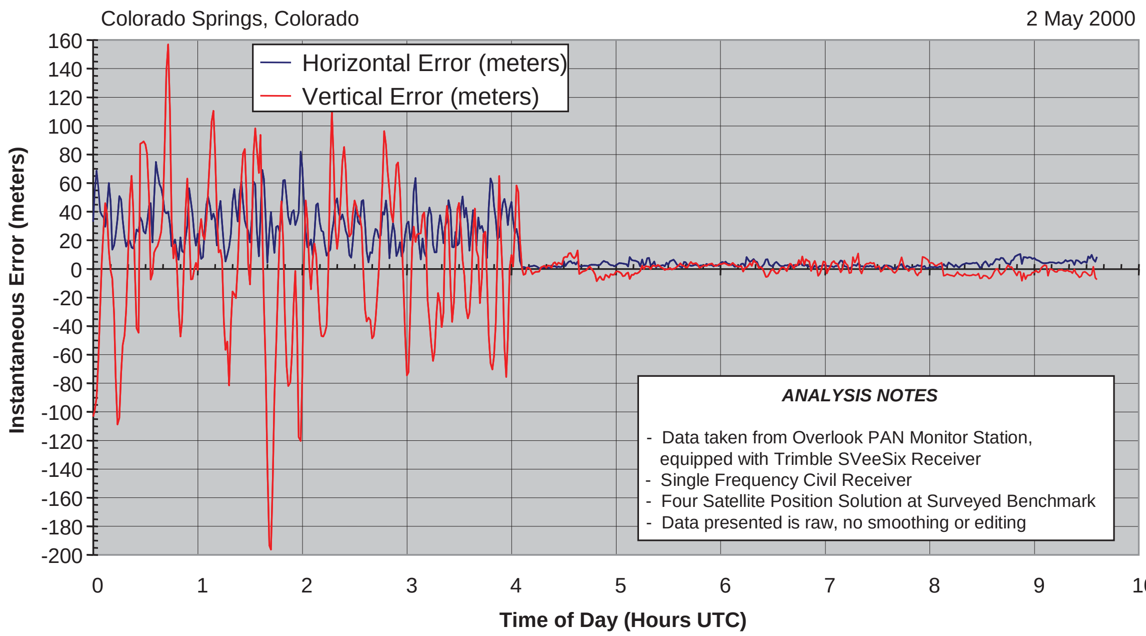




Selective Availability (S/A): Intentional degradation of satellite clocks and broadcast ephemeris. (from 25 March, 1990)

GPS Before and After S/A was switched off



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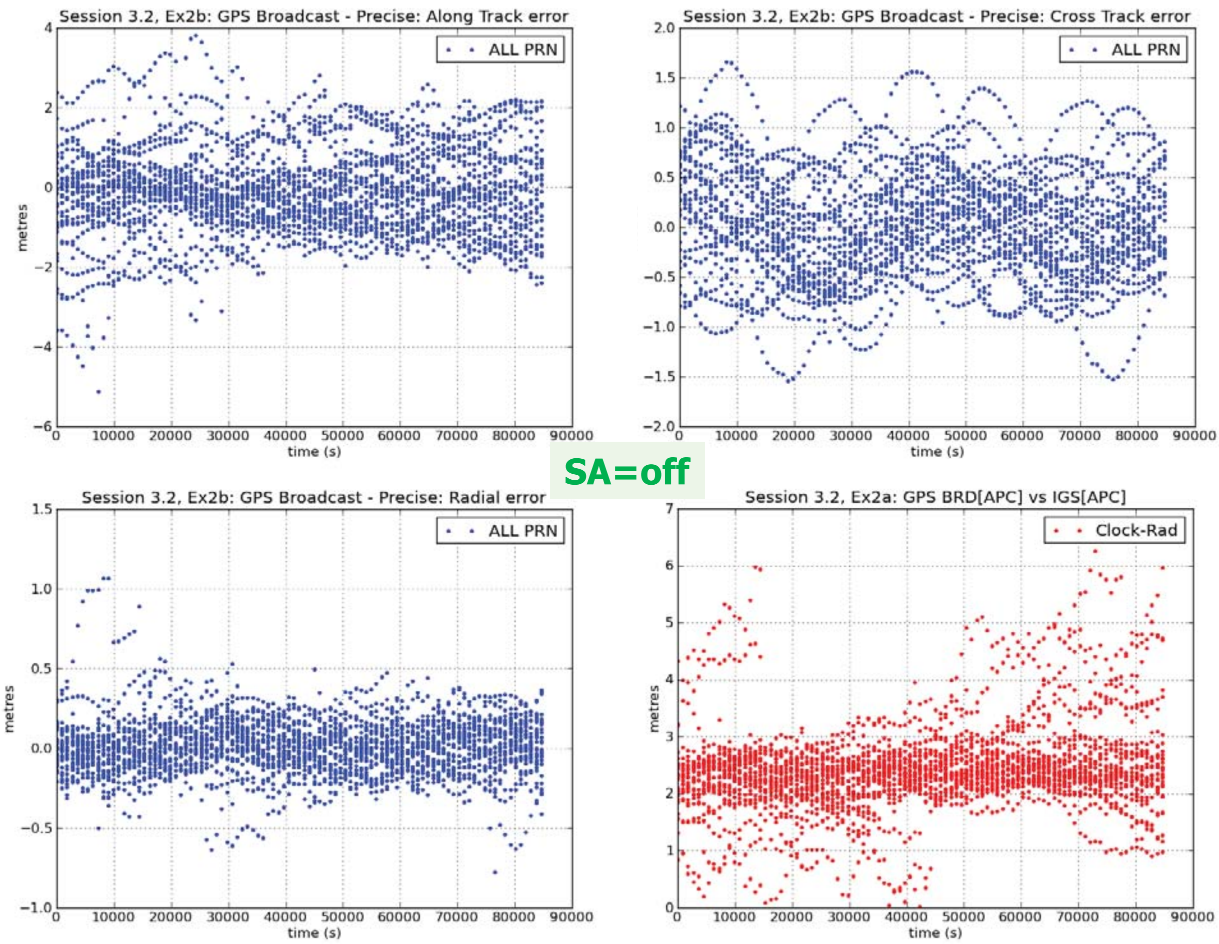

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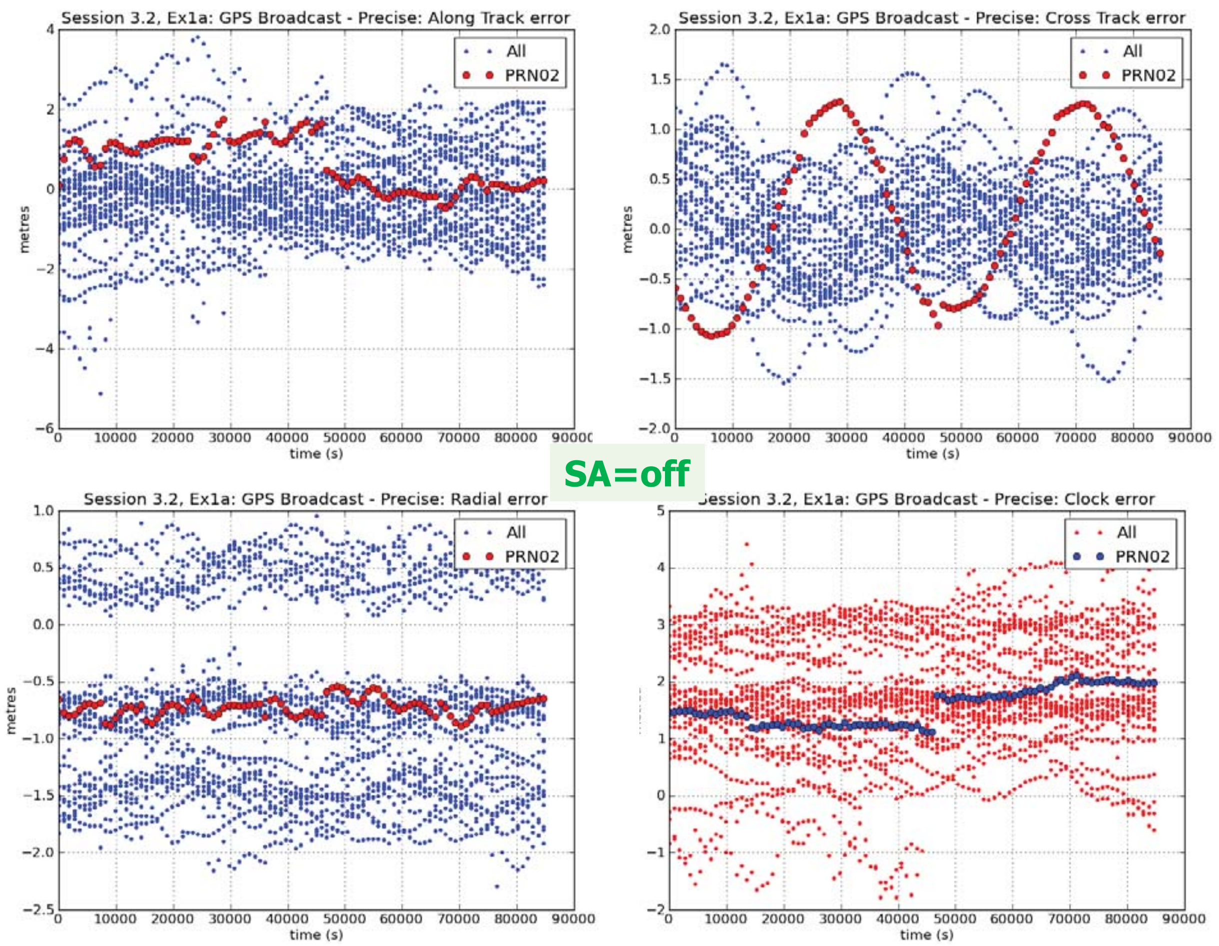

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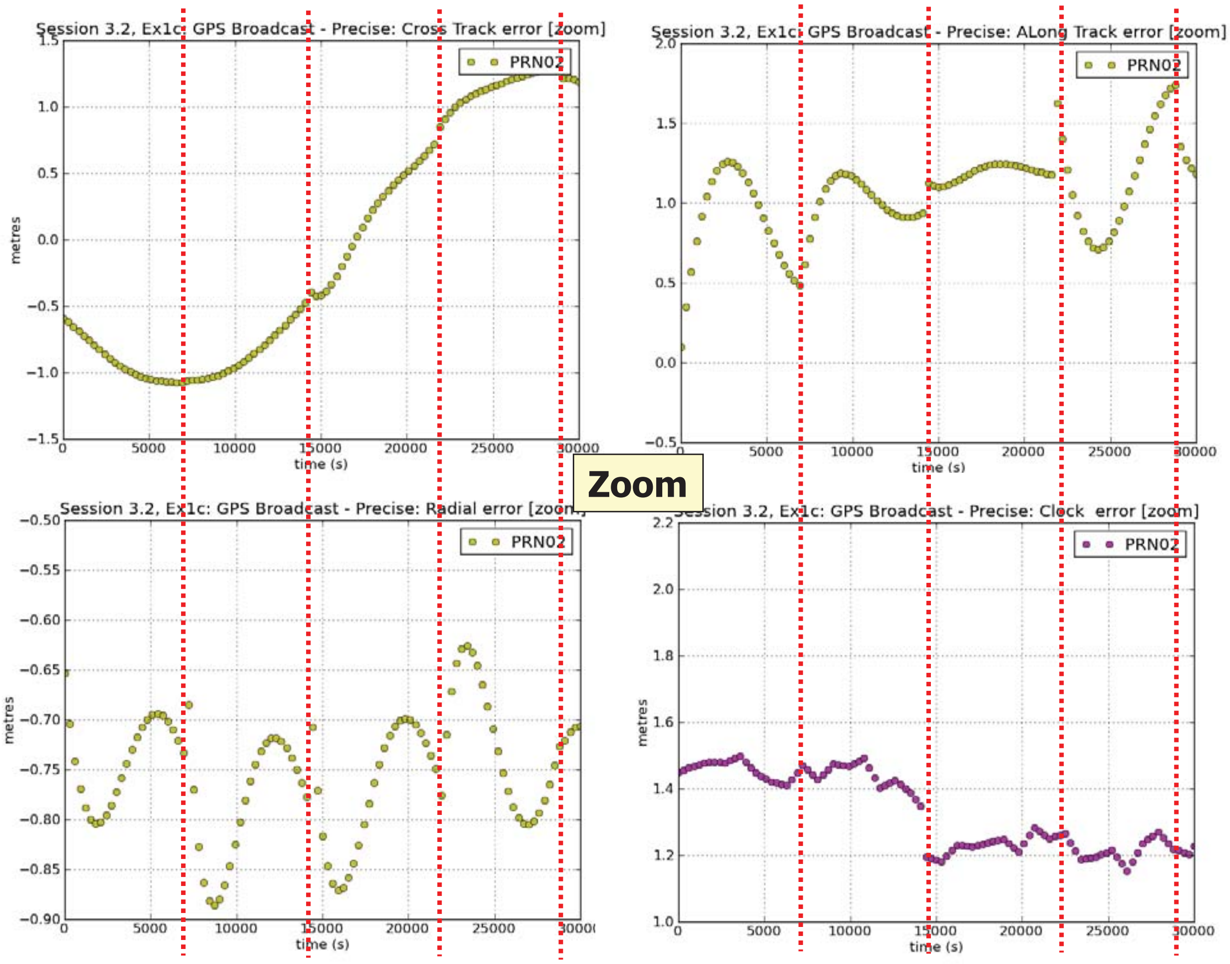

Ephemeris Updates

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Contents

1. Elliptic orbit: Keplerian elements.
2. Perturbed Keplerian orbits: Osculating orbit.
3. GPS satellite coordinates computation and accuracy
   - 3.1. From Broadcast Navigation Message.
   - 3.2. From precise products.
4. GPS Satellite clock computation and accuracy
   - 4.1. From Broadcast Navigation Message.
   - 4.2. From precise products.
5. Geographic decorrelation of ephemeris errors.

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3.2 Computation of satellite coordinates from precise products.

Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**

Orbits are given by (x,y,z) coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

$$
\begin{aligned}
P_n(x) &= \sum_{i=1}^{n} y_i \frac{\prod_{j\neq i}(x-x_j)}{\prod_{j\neq i}(x_i-x_j)} \\
&= y_1 \frac{x-x_2}{x_1-x_2} \cdots \frac{x-x_n}{x_1-x_n} + \cdots \\
&\quad + y_i \frac{x-x_1}{x_i-x_1} \cdots \frac{x-x_{i-1}}{x_i-x_{i-1}} \frac{x-x_{i+1}}{x_i-x_{i+1}} \cdots \frac{x-x_n}{x_i-x_n} + \cdots \\
&\quad + y_n \frac{x-x_1}{x_n-x_1} \cdots \frac{x-x_{n-1}}{x_n-x_{n-1}}
\end{aligned}
$$

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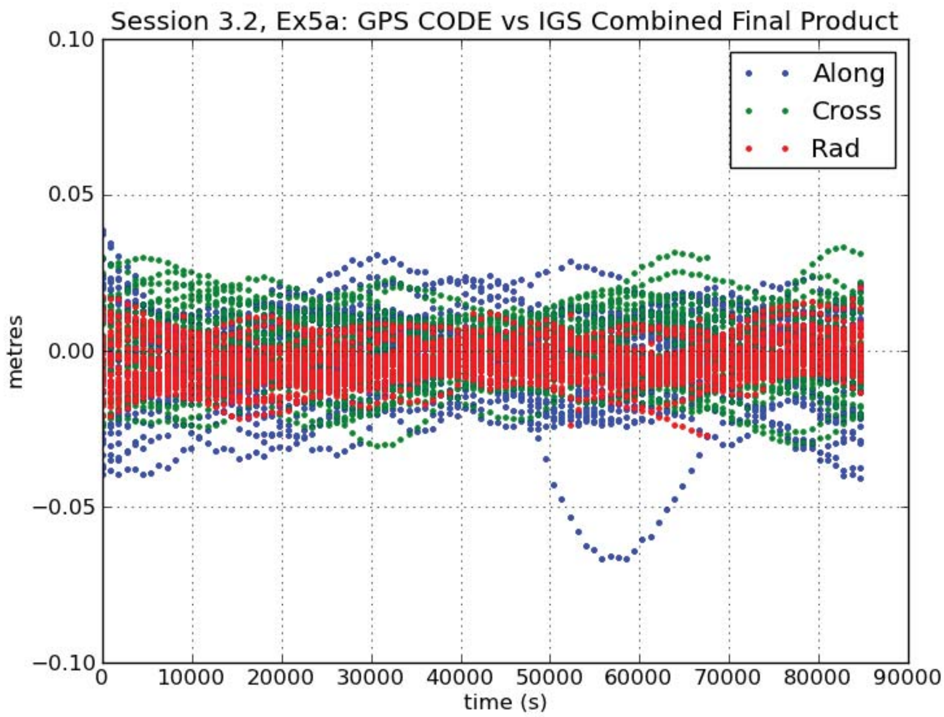
IGS orbit and clock products (for PPP):

Discrepancy between the different centres



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Contents

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GPS Satellite Clock computation: Broadcast message

$$dt^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2$$

t0 a0 a1 a2

YY MM DD H M S

PRN

```
 2              NAVIGATION DATA     GPS                 RINEX VERSION / TYPE
srx/v1.8.1.4         BAI                   95/10/19 03:18:35   PGM / RUN BY / DATE
CASA                                                           COMMENT
 -2444431.2031 -4428688.6270  3875750.1442                     COMMENT
                                                               END OF HEADER
14 95 10 18 00 51 44.0 1.129414886236D-05 1.136868377216D-13 0.000000000000D+00
    1.730000000000D+02-5.175000000000D+01 4.375182243902D-09-5.836427291652D-01
   -2.712011337280D-06 2.427505562082D-03 8.568167686462D-06 5.153718931198D+03
    2.623040000000D+05 4.470348358154D-08 1.698435481558D+00 1.676380634308D-08
    9.636381916043D-01 2.153437500000D+02 3.056960010495D+00-8.030691653399D-09
   -5.178787145843D-11 1.000000000000D+00 8.230000000000D+02 0.000000000000D+00
    3.200000000000D+01 0.000000000000D+00 1.396983861923D-09 1.730000000000D+02
    2.592180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

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Computation of satellite clocks from precise products

Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**

They are providing precise orbits and clock files with a sampling rate of 15 min, as well as precise clock files with a sample rate of 5 min and 30 s in SP3 format.

Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.

Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.

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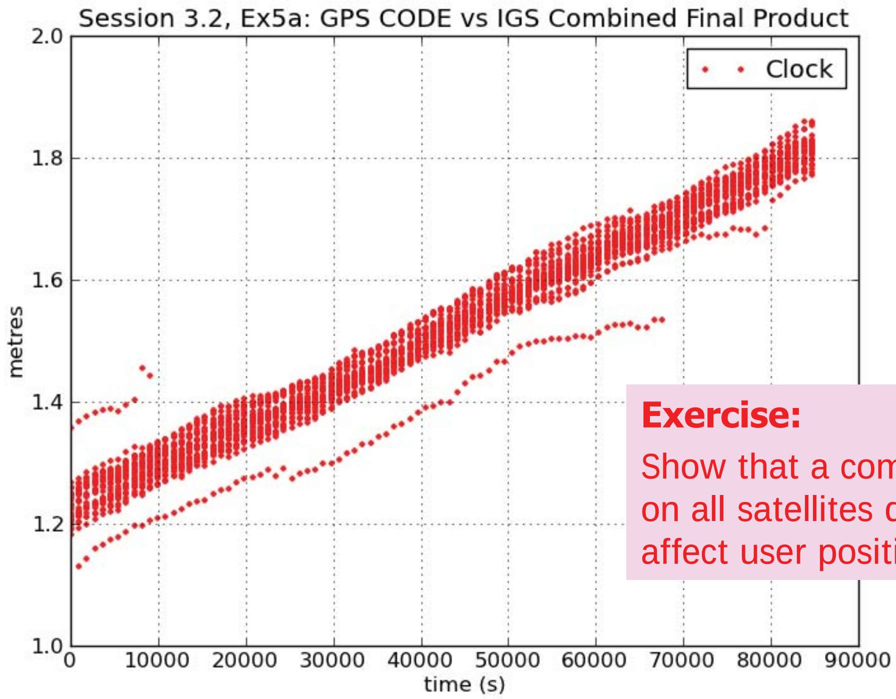
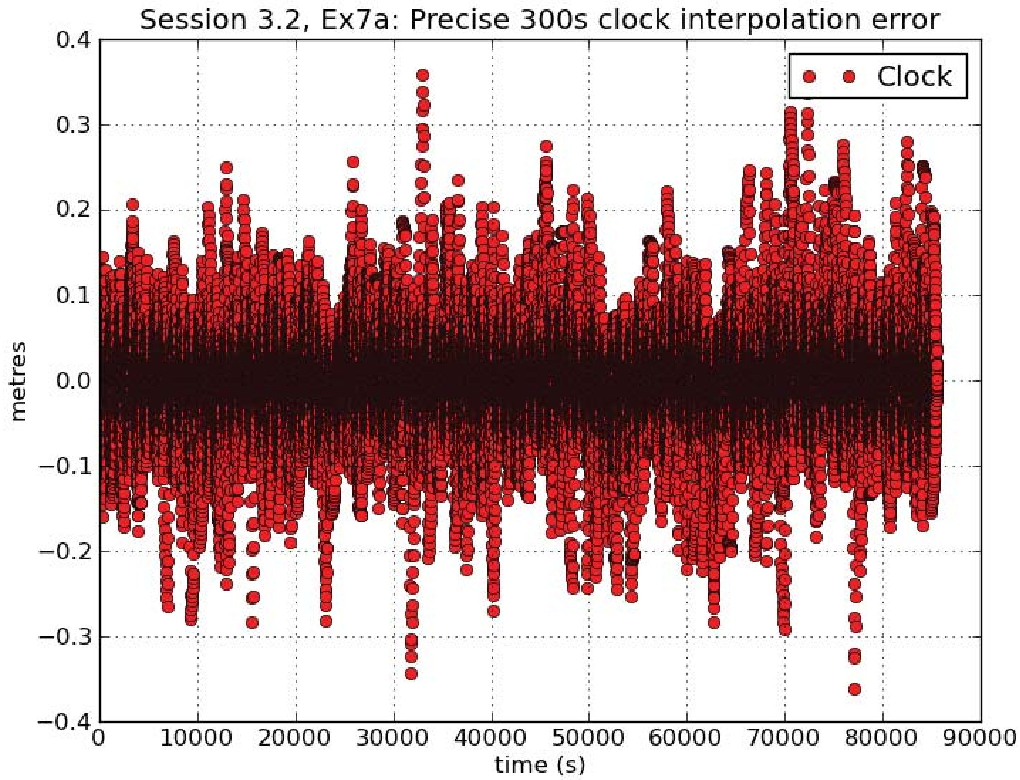
IGS orbit and clock products (for PPP):

Discrepancy between the different centres



Exercise:
Show that a common error on all satellites does not affect user positioning.



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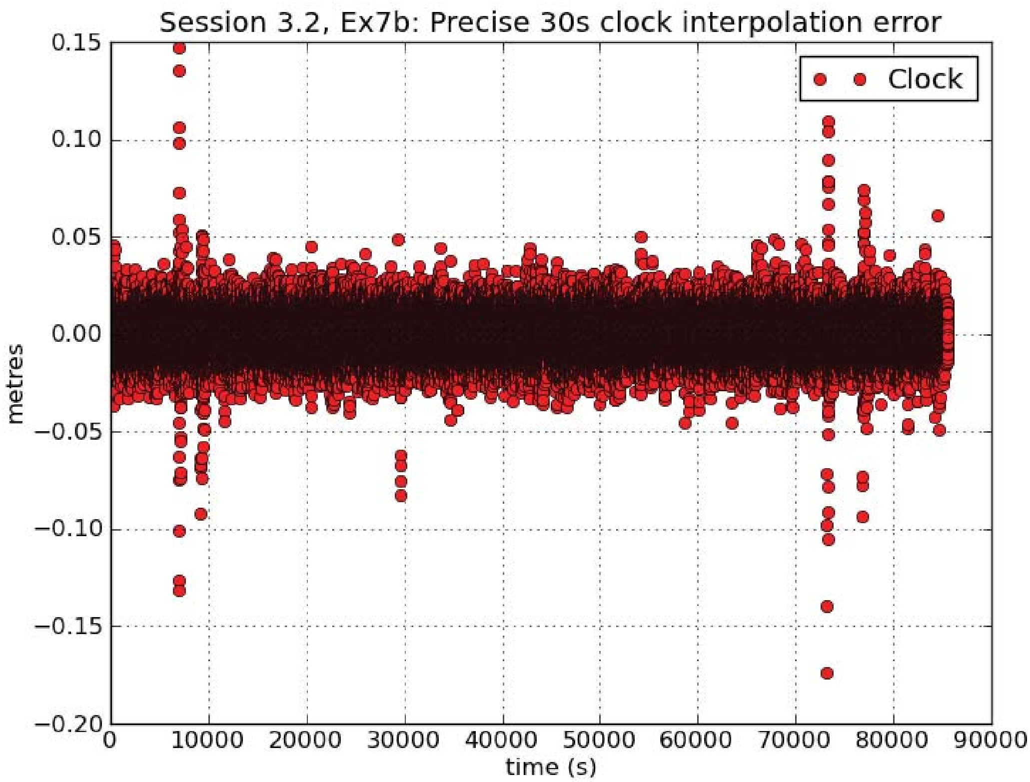
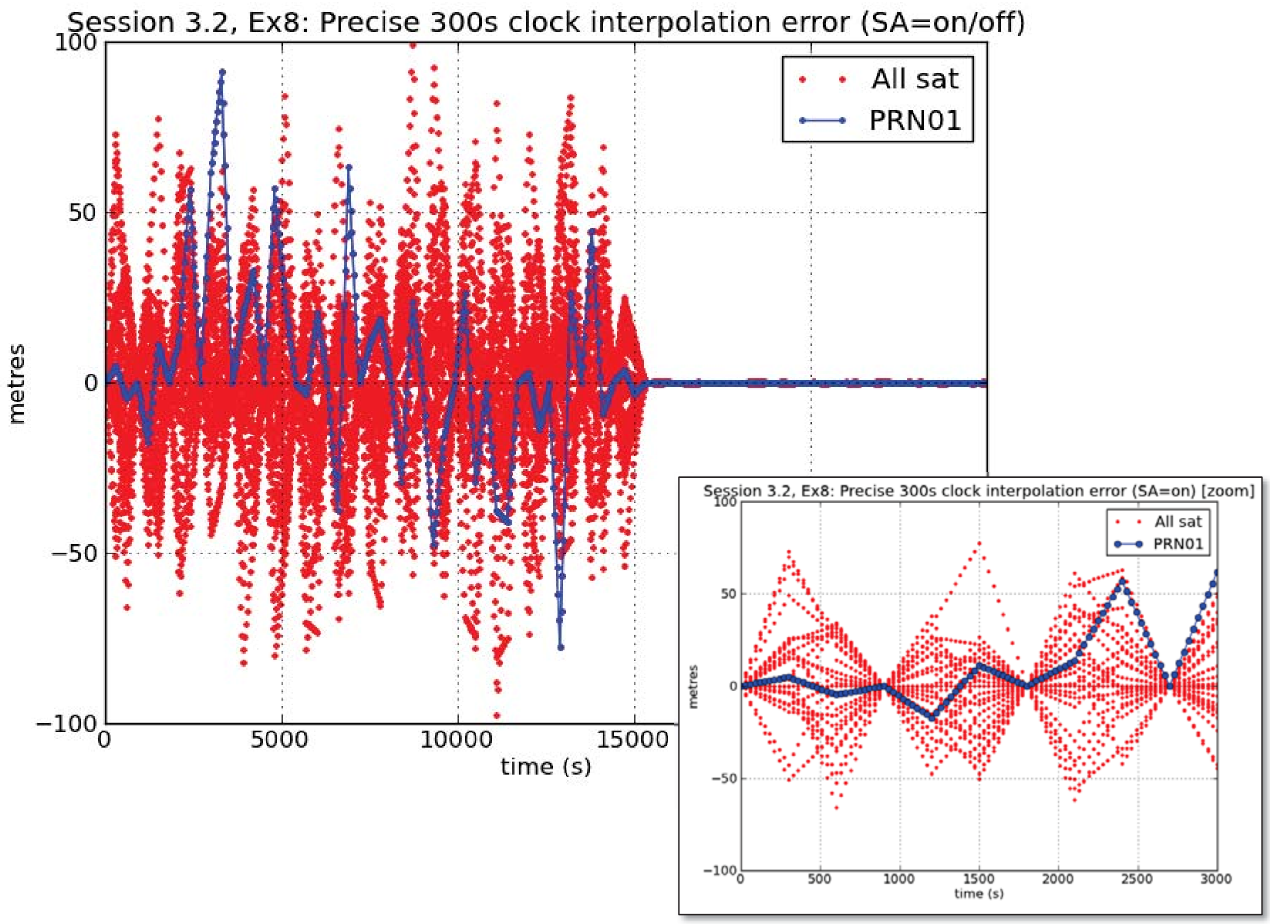




IGS Precise orbit and clock products:

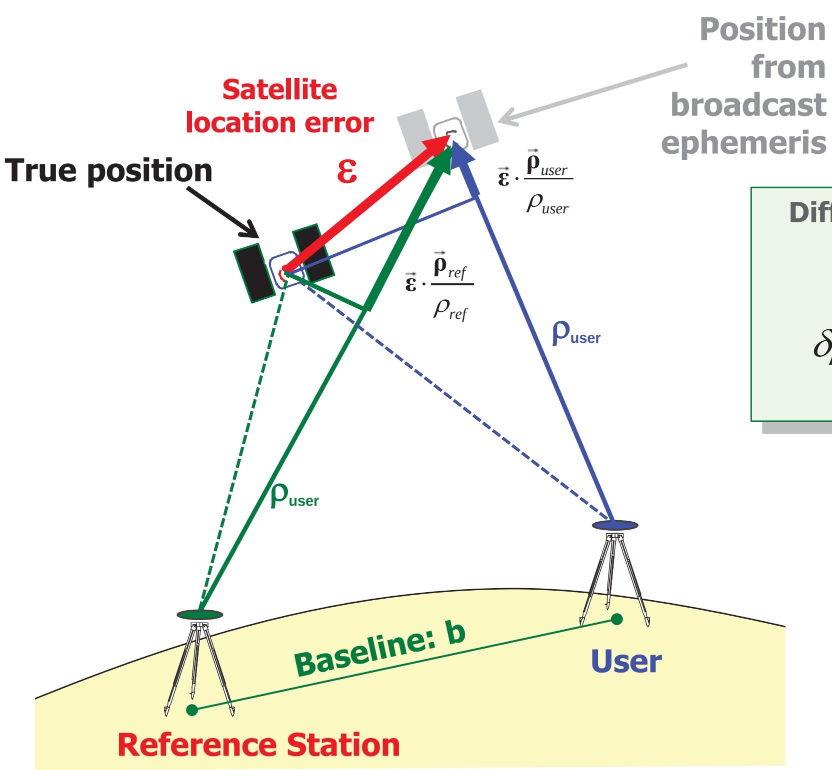
RMS accuracy, latency and sampling

Products (delay)	Broadcast (real time)	Ultra-rapid Predicted (real time)	Ultra-rapid Observed (3–9 h)	Rapid (17–41 h)	Final (12–18 d)
Orbit GPS (sampling)	~100 cm (~2 h)	~5 cm (15 min)	~3 cm (15 min)	~2.5cm (15 min)	~ 2.5 cm (15 min)
Glonass (sampling)					~5 cm (15 min)
Clock GPS (sampling)	~5 ns (daily)	~3 ns (15 min)	~150 ps (15 min)	~75 ps (5 min)	~75 ps (30 s)
Glonass (sampling)					~ TBD (15 min)

http://igscb.jpl.nasa.gov/components/prods.html

Contents

Ephemeris Errors and Geographic decorrelation



Differential range error due to satellite obit error

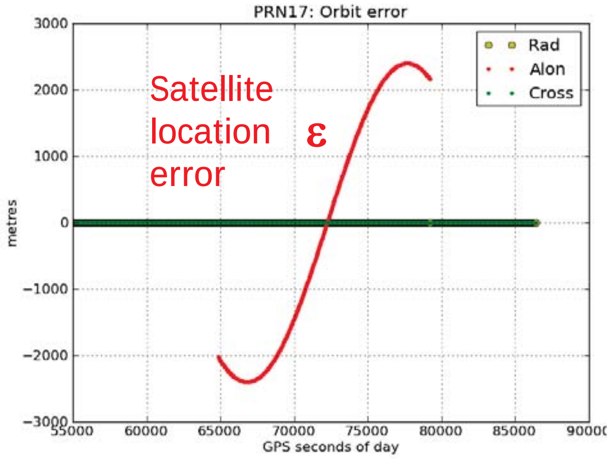
$$\delta\rho = \vec{\varepsilon}\cdot\frac{\vec{\rho}_{user}}{\rho_{user}} - \vec{\varepsilon}\cdot\frac{\vec{\rho}_{ref}}{\rho_{ref}}$$

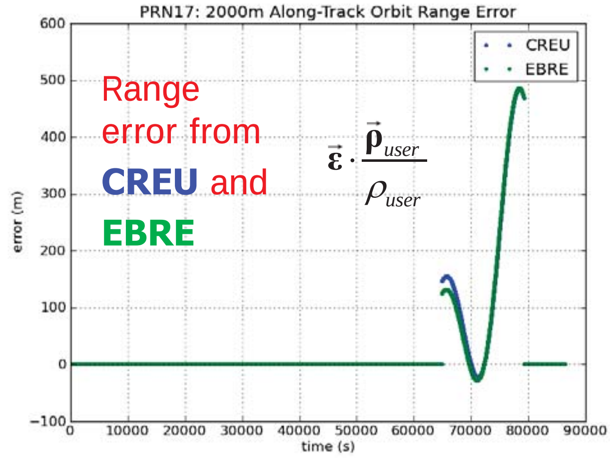
A conservative bound:

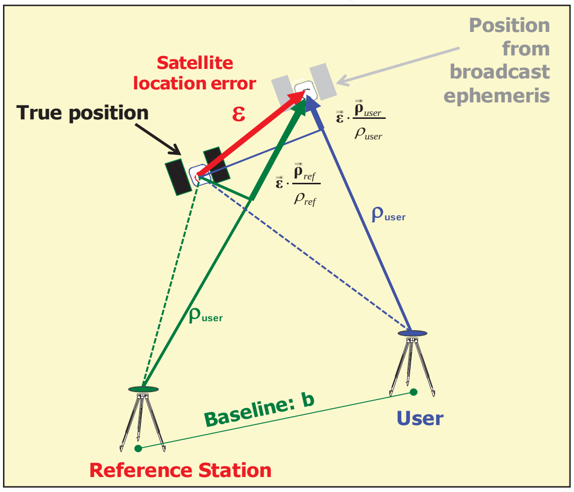
$$\boxed{\delta\rho < \frac{b}{\rho}\varepsilon}$$

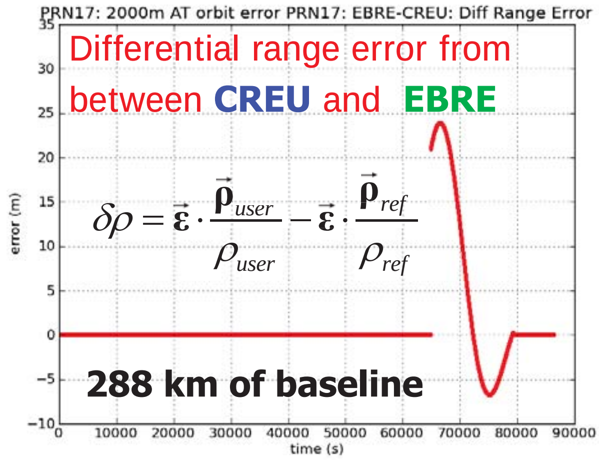
with a baseline $b = 20km$

$$\delta\rho < \frac{20}{20000}\varepsilon = \frac{1}{1000}\varepsilon$$









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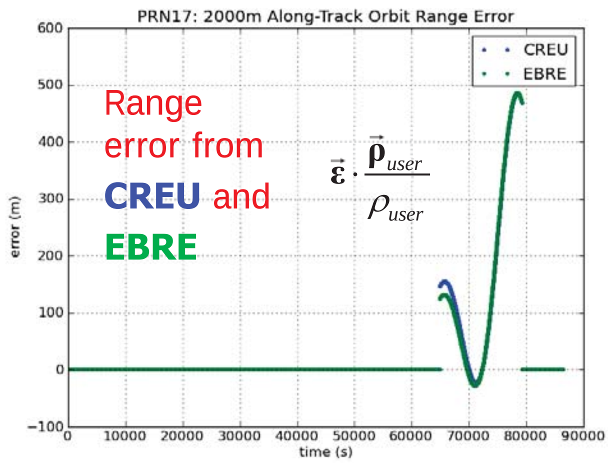

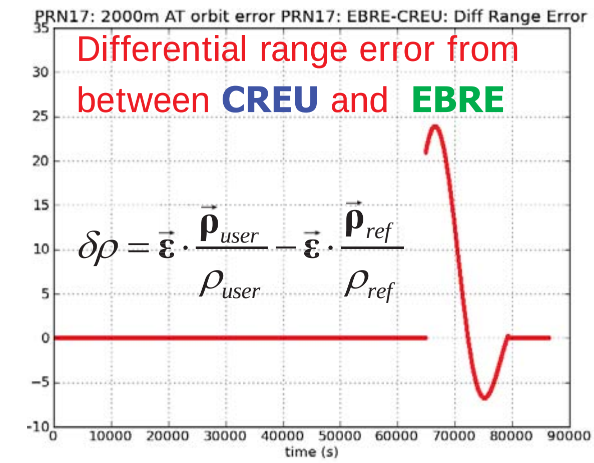

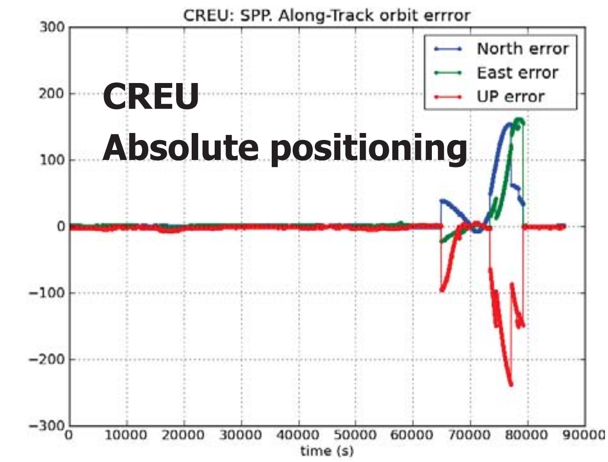

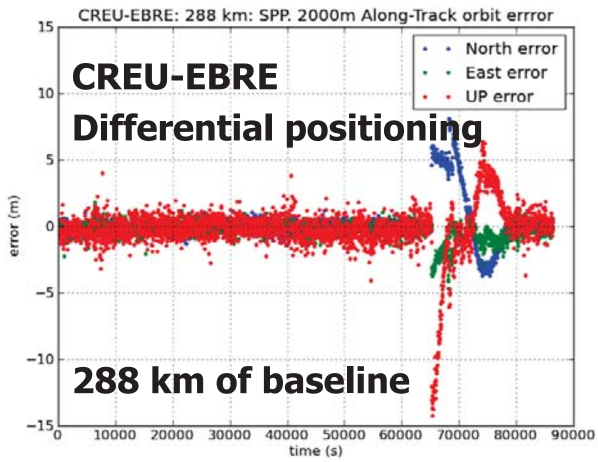

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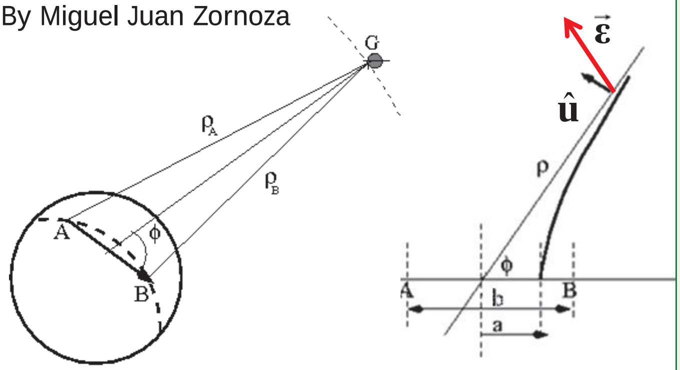
By Miguel Juan Zornoza



$a=(\rho_B-\rho_A)/2$: hyperboloid semiaxis

$b/2$: focal length

where $a=b/2\cdot\cos(\phi)$

Note: in this 3D problem ϕ is NOT the elevation of ray.

- Errors over the hyperboloid (i.e. $\rho_B-\rho_A=ctt$) will not produce differential range errors.
- The highest error is given by the vector $\hat{\mathbf{u}}$, orthogonal to the hyperboloid and over the plain containing the baseline vector $\hat{\mathbf{b}}$ and the LoS vector $\hat{\boldsymbol{\rho}}$.

Note:
Being the baseline **b** much smaller than the distance to the satellite, we can assume that the LoS vectors from A and B receives are essentially identical to ρ.
That is, $\rho_B\cong\rho_A\cong\rho$

$$\mathbf{u}=\hat{\boldsymbol{\rho}}\times\left(\hat{\mathbf{b}}\times\hat{\boldsymbol{\rho}}\right)=\hat{\mathbf{b}}\left(\hat{\boldsymbol{\rho}}^T\cdot\hat{\boldsymbol{\rho}}\right)-\hat{\boldsymbol{\rho}}\left(\hat{\boldsymbol{\rho}}^T\cdot\hat{\mathbf{b}}\right)$$

$$=\mathbf{I}\,\hat{\mathbf{b}}-\left(\hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{\rho}}^T\right)\hat{\mathbf{b}}=\left(\mathbf{I}-\hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{\rho}}^T\right)\hat{\mathbf{b}}$$

Note: $\mathbf{u}=\sin\phi\,\hat{\mathbf{u}}$

Differential range error $\delta\rho$ produced by an orbit error $\varepsilon_{\parallel}$ parallel to vector $\hat{\mathbf{u}}$

Let $\varepsilon\equiv\varepsilon_{\parallel}$

$$\delta\rho\equiv\delta(\rho_B-\rho_A)=2\delta a=$$

$$=2\frac{\partial a}{\partial\varepsilon}\varepsilon=2\frac{\partial a}{\partial\phi}\frac{\partial\phi}{\partial\varepsilon}\varepsilon=-b\sin\phi\frac{\partial\phi}{\partial\varepsilon}\varepsilon$$

$$\simeq-b\sin\phi\frac{1}{\rho}\varepsilon$$

Note: $\vec{\boldsymbol{\varepsilon}}_{\parallel}\perp\vec{\boldsymbol{\rho}}\Rightarrow\varepsilon\simeq\rho\,\delta\phi$

Note: being $\hat{\mathbf{u}}$ a vector orthogonal to the LoS $\hat{\boldsymbol{\rho}}$, thence, $\varepsilon_{\parallel}=\vec{\boldsymbol{\varepsilon}}^T\hat{\mathbf{u}}$

Thence:

$$\delta\rho=-\frac{b\sin\phi}{\rho}\vec{\boldsymbol{\varepsilon}}^T\cdot\hat{\mathbf{u}}=-\vec{\boldsymbol{\varepsilon}}^T\cdot\left(\sin\phi\,\hat{\mathbf{u}}\right)\frac{b}{\rho}$$

$$=-\vec{\boldsymbol{\varepsilon}}^T\left(\mathbf{I}-\hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{\rho}}^T\right)\frac{\mathbf{b}}{\rho}$$

Where: $\mathbf{b}=b\,\hat{\mathbf{b}}$ is the baseline vector

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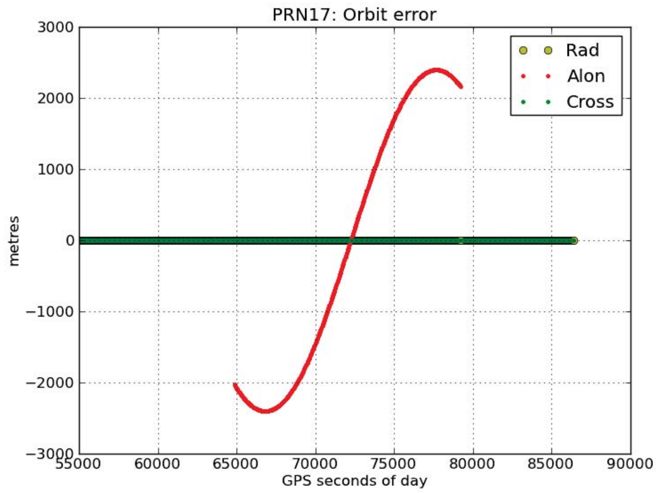
ORBIT TEST :

Broadcast orbits

Along-track Error (PRN17)

PRN17:
Doy=077, Transm. time: 64818 sec




```
17 10  3 18 20  0  0.0 1.379540190101E-04 2.842170943040E-12 0.000000000000E+00
    7.800000000000E+01-5.059375000000E+01 4.506973447820E-09-2.983492318682E+00
   -9.257976353169E-05 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
    4.176000000000E+05-5.401670932770E-08-4.040348681654E-01-7.636845111847E-08
    9.603630515702E-01 2.215312500000E+02-2.547856603060E+00-7.964974630307E-09
   -3.771585673111E-10 1.000000000000E+00 1.575000000000E+03 0.000000000000E+00
    2.000000000000E+00 0.000000000000E+00-1.024454832077E-08 7.800000000000E+01
    4.104180000000E+05 4.000000000000E+00


diff EPH.dat.org EPHcuc_x0.dat --------------------------------------------------
<     -2.579763531685E-06 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
>     -9.257976353169E-05 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
----------------------------------------------------------------------------------
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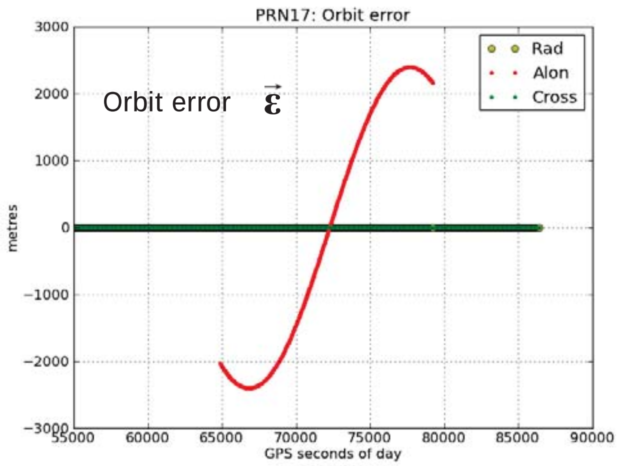

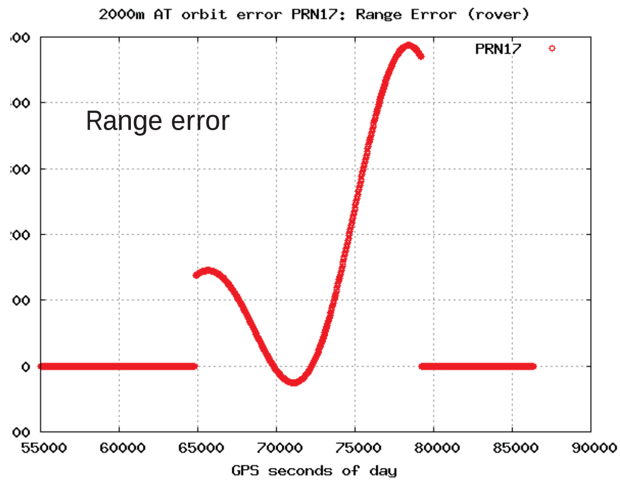

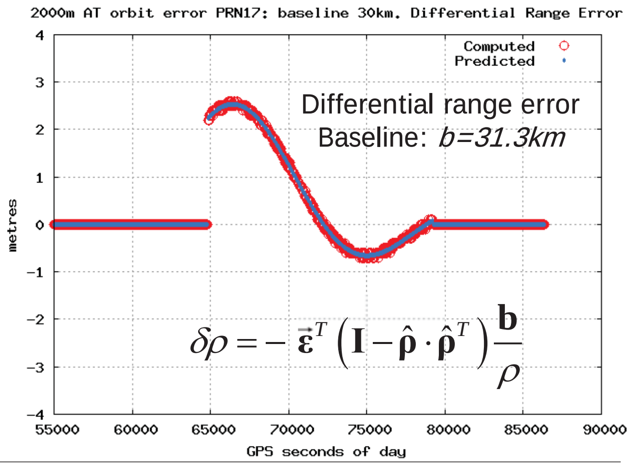

Exercise:

Justify that clock errors completely cancel in differential positioning.

ERRORS on the Signal

- Space Segment Errors:
  - Clock errors — Common
  - Ephemeris errors
- Propagation Errors — Strong spatial correlation
  - Ionospheric delay
  - Tropospheric delay — Weak spatial correlation
- Local Errors — No spatial correlation
  - Multipath
  - Receiver noise

References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

Thank you

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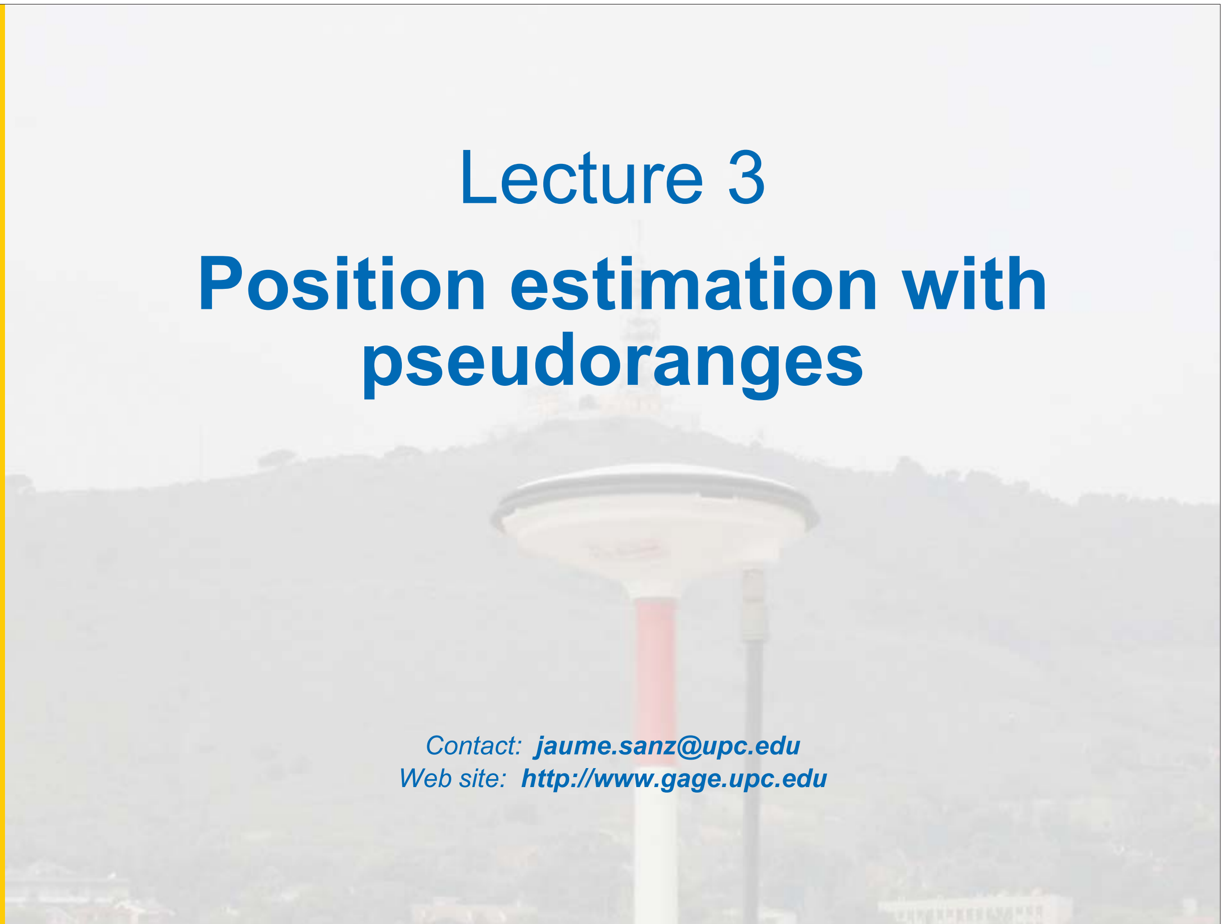

Lecture 3

Position estimation with pseudoranges

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Authorship statement

24 April 2014

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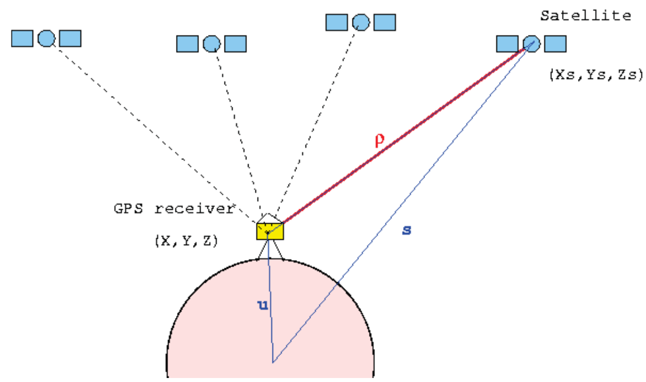
Introduction: Linear model and Prefit-residual



Input:

- ***Pseudoranges*** **(receiver-satellite j):** P^j
- **Navigation message. In particular:**
  - ***satellite position*** **when transmitting signal:** $r^j = (x^j, y^j, z^j)$
  - ***offsets*** **of satellite clocks:** dt^j

  (j = 1,2,...n) (n>=4)

Unknowns:

- **receiver** ***position*****:** $r = (x, y, z)$
- **receiver** ***clock*** **offset:** DT

For each satellite in view

Iono+Tropo+TGD...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + \boxed{\sum \delta_k} + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \boxed{\rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}}\Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}}\Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}}\Delta z_{rec}} + c\left(dt_{rec} - dt^{sat}\right) + \sum \delta_k$$

where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)

$$\boxed{C1_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt^{sat} - \sum \delta_k} = \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}}\Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}}\Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}}\Delta z_{rec} + c\,dt_{rec}$$

measurement computed unknown

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For all satellites in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \cdots\cdots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec}-x^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{y_{0,rec}-y^{sat1}}{\rho_{0,rec}^{sat1}} & \frac{z_{0,rec}-z^{sat1}}{\rho_{0,rec}^{sat1}} & 1 \\ \frac{x_{0,rec}-x^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{y_{0,rec}-y^{sat2}}{\rho_{0,rec}^{sat2}} & \frac{z_{0,rec}-z^{sat2}}{\rho_{0,rec}^{sat2}} & 1 \\ & \cdots\cdots\cdots & & \\ \frac{x_{0,rec}-x^{sat\,n}}{\rho_{0,rec}^{sat\,n}} & \frac{y_{0,rec}-y^{sat\,n}}{\rho_{0,rec}^{sat\,n}} & \frac{z_{0,rec}-z^{sat\,n}}{\rho_{0,rec}^{sat\,n}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

Observations (measured/computed)

Unknowns

Measurements modelling:

Prefit residual is the difference between measured and modeled pseudorange:

$$\boxed{\mathrm{Prefit}_{rec}^{sat} = C1_{rec}^{sat}[\mathrm{measured}] - C1_{rec}^{sat}[\mathrm{modelled}]}$$

where:

$$\boxed{C1_{rec}^{sat}[\mathrm{modelled}] = \rho_{rec,0}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}}$$

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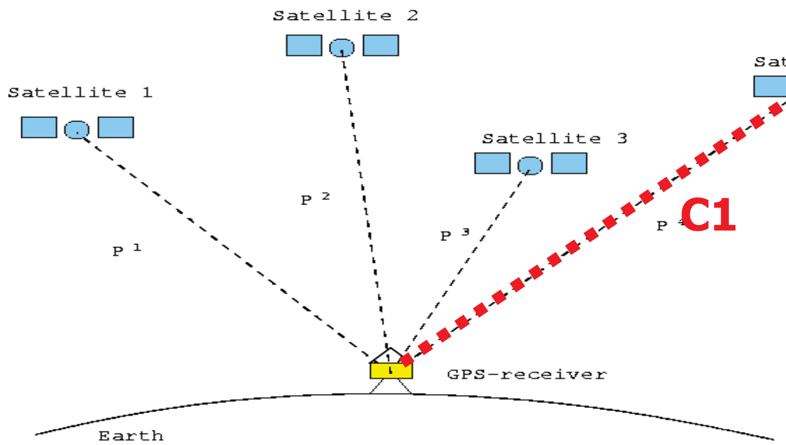
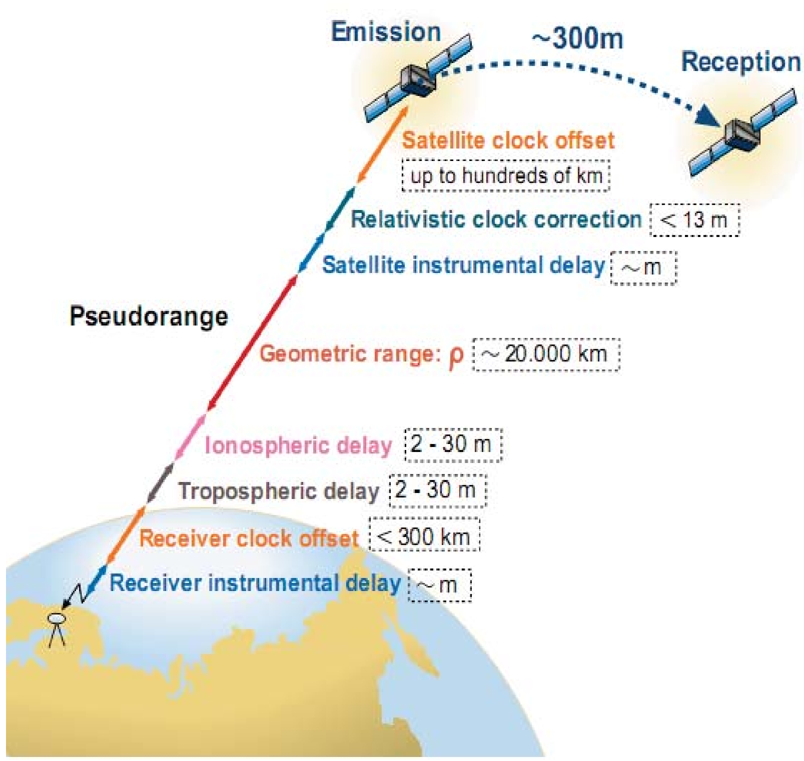
Code Pseudorange modeling



The pseudorange modeling is based in the GPS Standard Positioning Service Signal Specification (GPS/SPS-SS).

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$



$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

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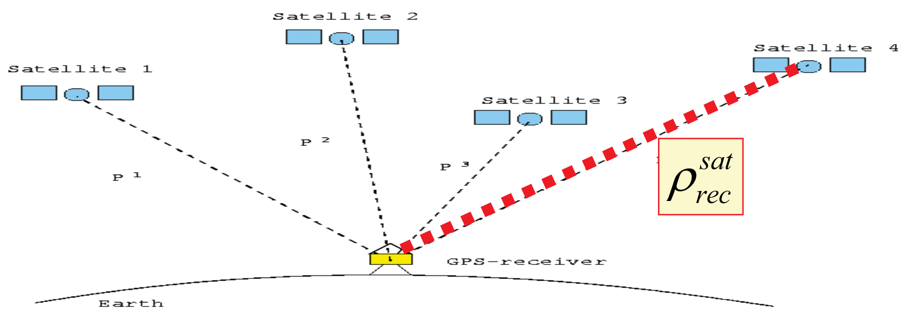
Geometric range



Euclidean distance between satellite coordinates at emission time and receiver coordinates at reception time.

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2}$$

Of course, receiver coordinates are not known (is the target of this problem). But ....

$$C1_{rec}^{sat}[\text{modelled}] = \boxed{\rho_{rec,0}^{sat}} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

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$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2}$$

Of course, receiver coordinates $(x_{rec}, y_{rec}, z_{rec})$ are not known (they are the target of this problem). But, we can always assume that an "approximate position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ is known".

Thence, as it will be shown in next lesson, the navigation problem will consist on:

1.- To start from an approximate value for receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ e.g. the Earth's centre) to linearise the equations.

2.- With the pseudorange measurements and the navigation equations, compute the correction $(\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$ to have improved estimates: $(x_{rec}, y_{rec}, z_{rec}) = (x_{0,rec}, y_{0,rec}, z_{0,rec}) + (\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$

3.- Linearise the equations again, about the new improved estimates, and iterate until the change in the solution estimates is sufficiently small.

The estimates converges quickly. Generally in two to four iterations, even if starting from the Earth's Centre.

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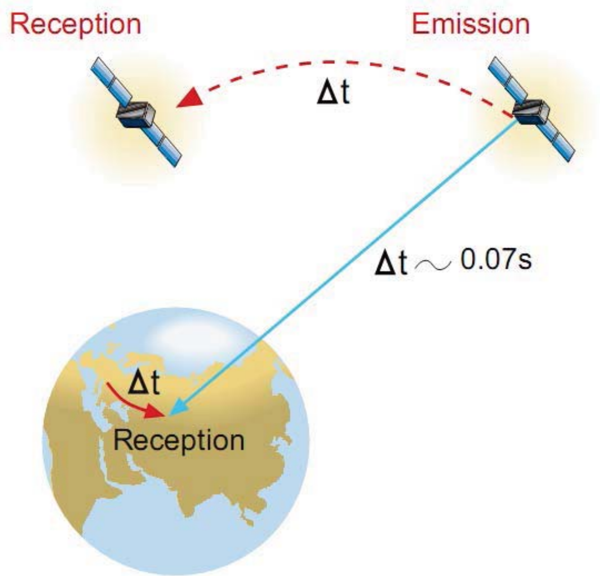
Satellite coordinates **at emission time** (rec2ems.f)

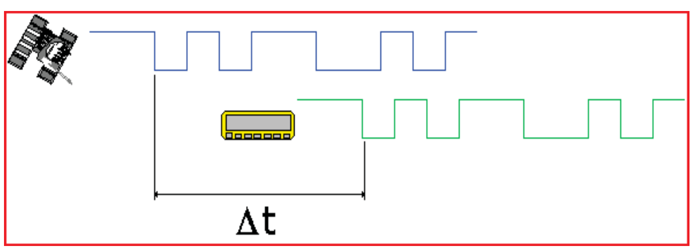


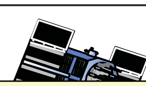
- The GPS signal travels from satellite coordinates at emission time (t^{ems}) to receiver coordinates at reception time (t_{rec}).
- The satellite can move several hundreds of meters from t^{ems} to t_{rec}.

The receiver time-tags are given at reception time and in the receiver clock time.

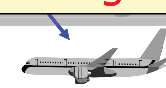
An algorithm is needed to compute the satellite coordinates at **emission time** "in the GPS system time" from reception time in the receiver time tags.

Emission time in the GPS system time *T[ems]*





The satellite offset clock dt^S can be computed from the navigation message



$$C1 = c\,\Delta t = c\,[t_{rec}(T_R) - t^{ems}(T^S)]$$

As it is known, the pseudorange measurements link the "emission time (t^{ems})" in satellite clock (T^S) with reception time (t_{rec}) in receiver clock (T_R) (receiver time tags).

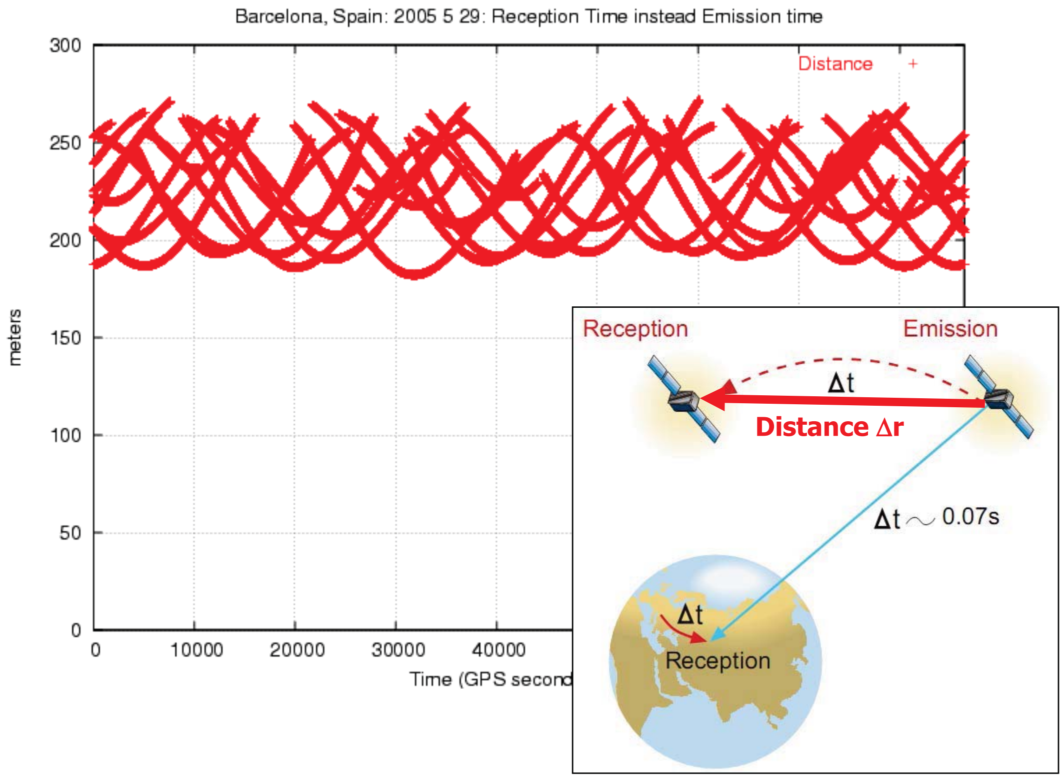
Thence, the emission time in the satellite clock is:

$$t^{ems}(T^S) = t_{rec}(T_R) - C1/c$$

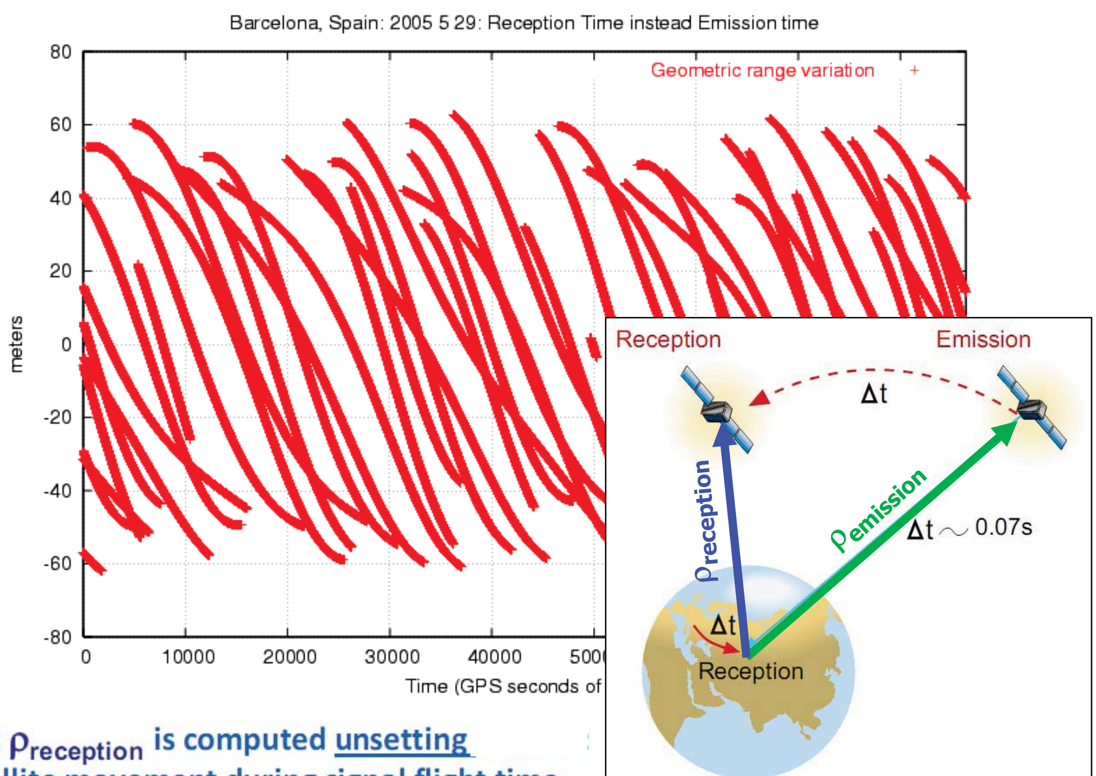
Finally, since $dt^S = t^S - T$ is the time offset between satellite clock (t^S) and **GPS system time** (T), thence:

$$T[ems] = t^{ems}(T^S) - dt^S = t_{rec}(T_R) - (C1/c + dt^S)$$

Distance: Δr



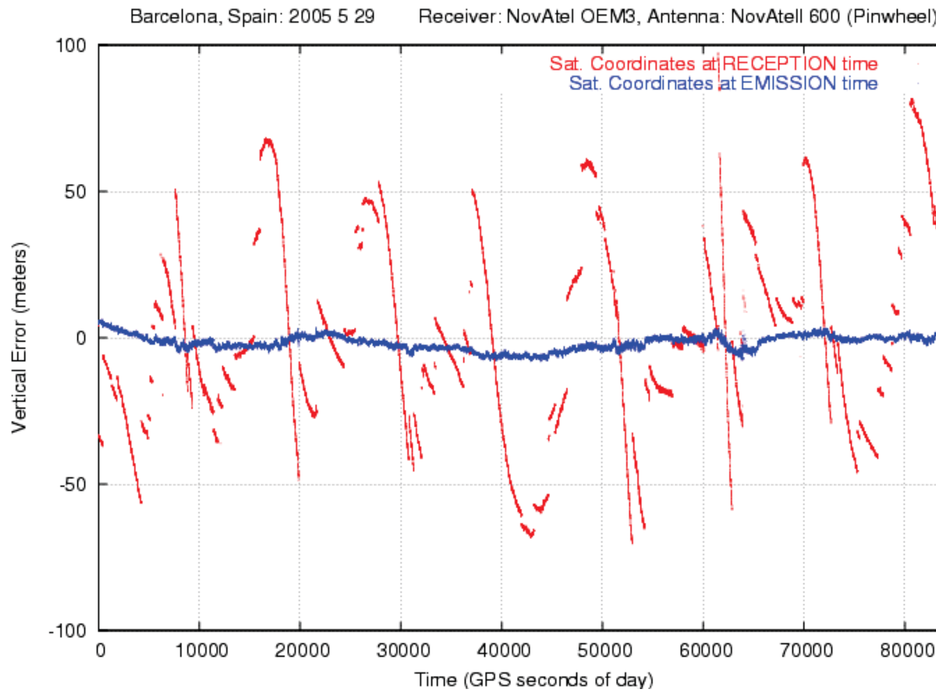
Variation in range: $\Delta\rho = \rho_{emission} - \rho_{reception}$



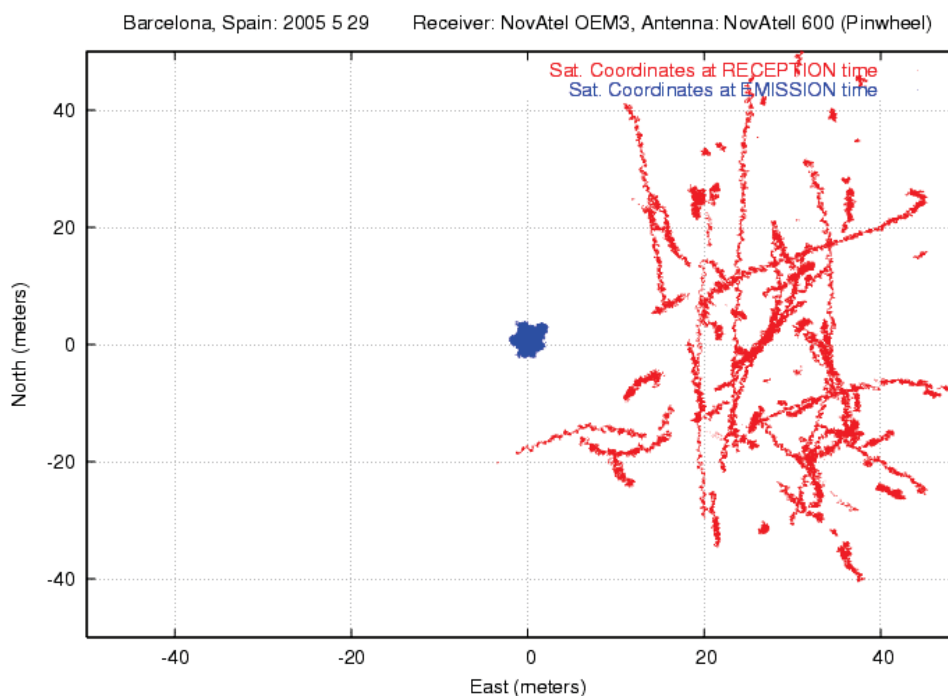
Note: $\rho_{reception}$ is computed unsetting

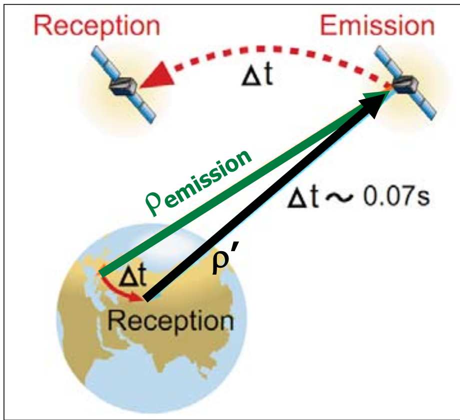
- Satellite movement during signal flight time.
- Earth rotation during signal flight time.

Vertical error comparison



Horizontal error comparison





...rdinates computation **at emission time**

...vided by the GPS/SPS-SS (**orbit.f**) supplies satellite ... an Earth-Fixed reference frame. To compute the ...nates

See **rec2ems.f**

...ne, the following algorithm can be applied:

...time-tags, compute emission time in GPS system time:

$$T[ems] = t_{rec}(T_R) - (C1/c + dt^S)$$

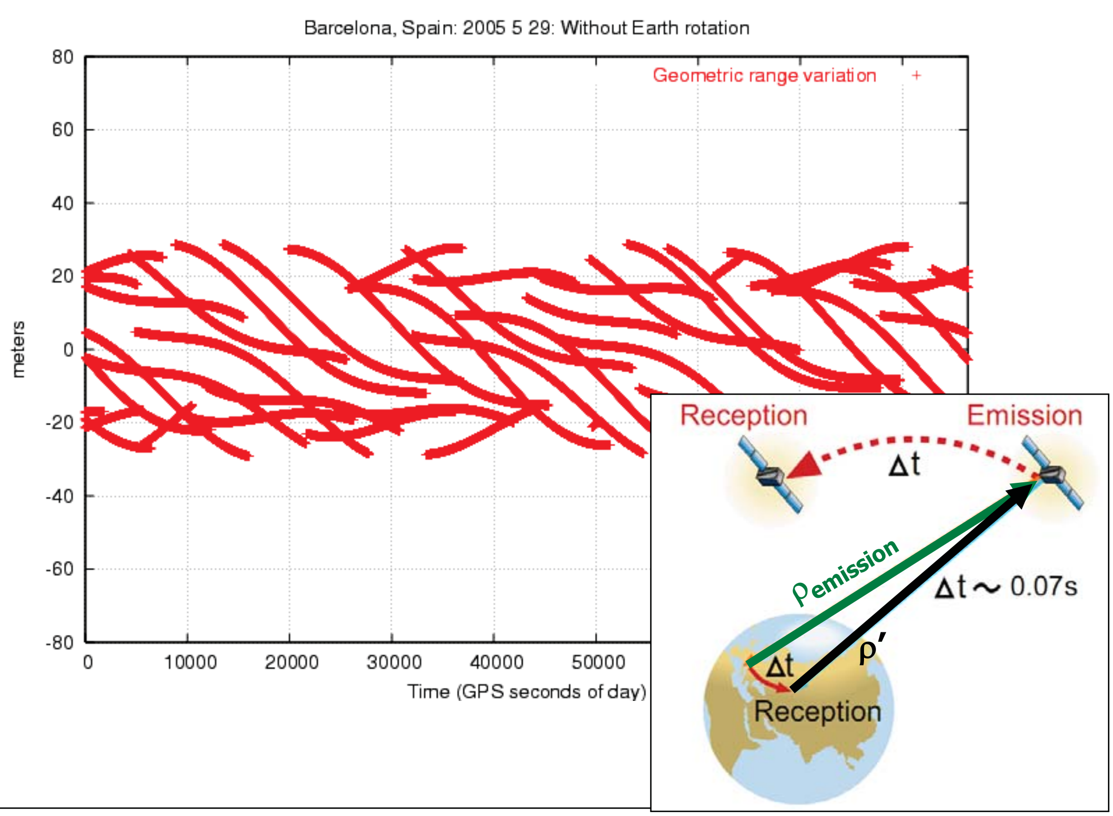
2. Compute satellite coordinates at emission time *T[ems]*

$$T[ems] \rightarrow [\mathbf{orbit}] \rightarrow (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$$

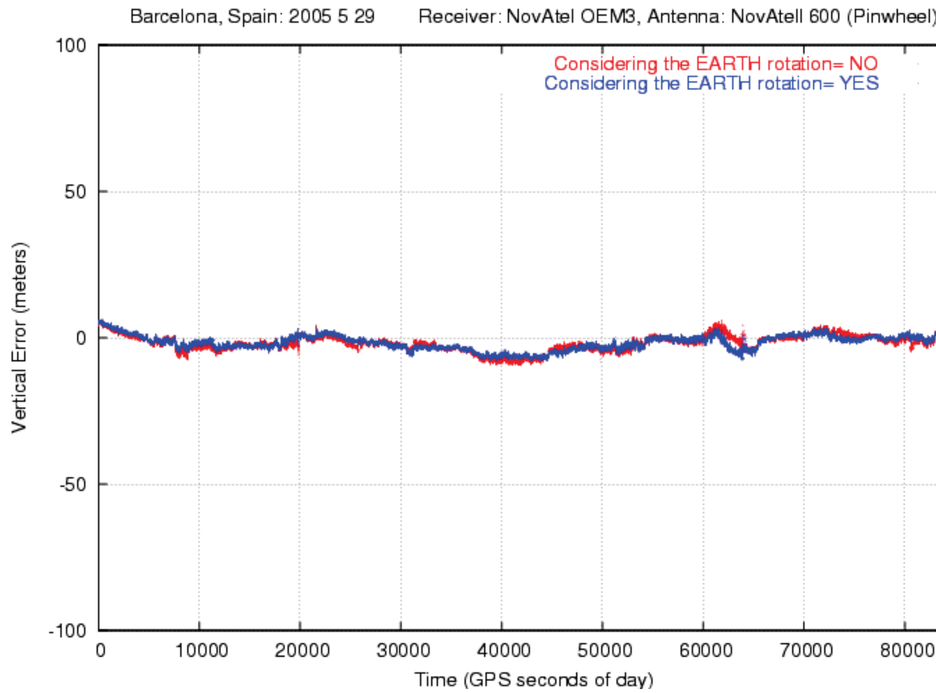
3. Account for Earth rotation during traveling time from emission to reception "Δt" *(CTS reference system at reception time is used to build the navigation equations).*

$$(X^{sat}, Y^{sat}, Z^{sat})_{CTS[reception]} = R_3(\omega_E \, \Delta t) \cdot (X^{sat}, Y^{sat}, Z^{sat})_{CTS[emission]}$$

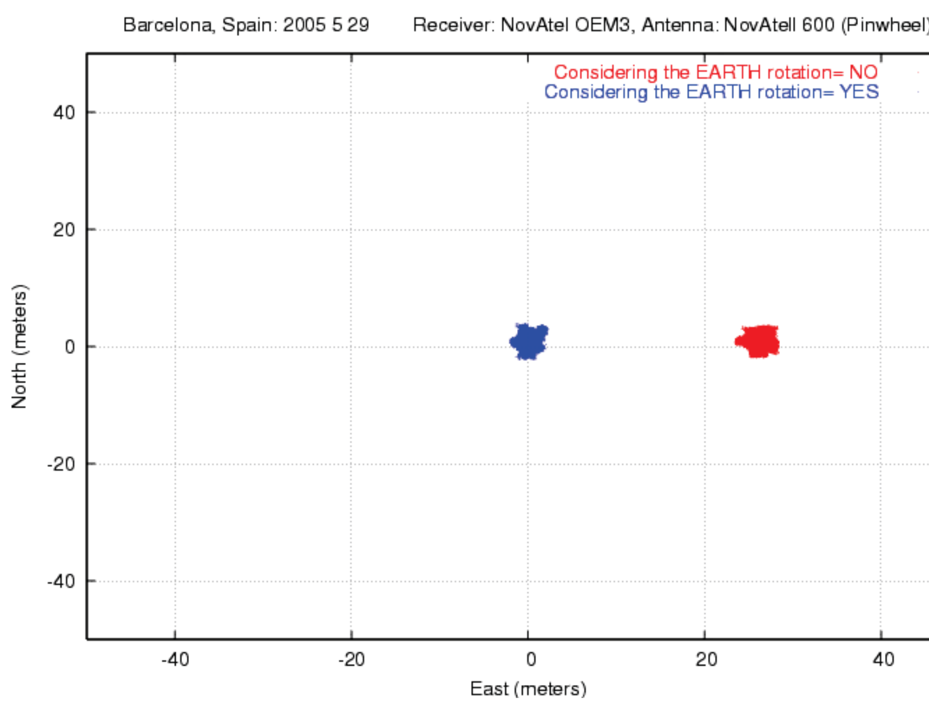
Variation in range: $\Delta\rho = \rho' - \rho_{emission}$



Vertical error comparison



Horizontal error comparison



Satellite and receiver clock offsets

- They are time-offsets between satellite/receiver time and GPS system time (provided by the ground control segment):

  - The receiver clock offset (dt_{rec}) is estimated together with receiver coordinates.

  - Satellite clock offset (dt^{sat}) may be computed from navigation message plus a Relativistic clock correction

$$dt^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 + \Delta_{rel}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(\boxed{d\tilde{t}^{\,sat} + \Delta rel^{sat}}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

$$a_0 + a_1(t-t_0) + a_2(t-t_0)^2$$

t0
YY MM DD H M S

a0 a1 a2

PRN

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    2.592180000000D+05 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
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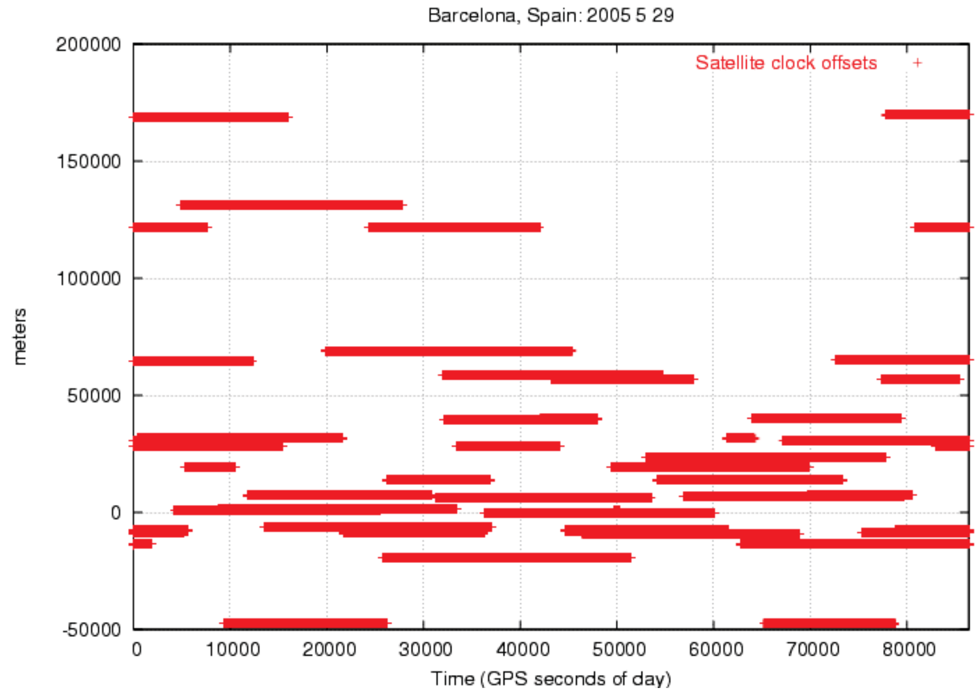
Range variation: satellite clocks



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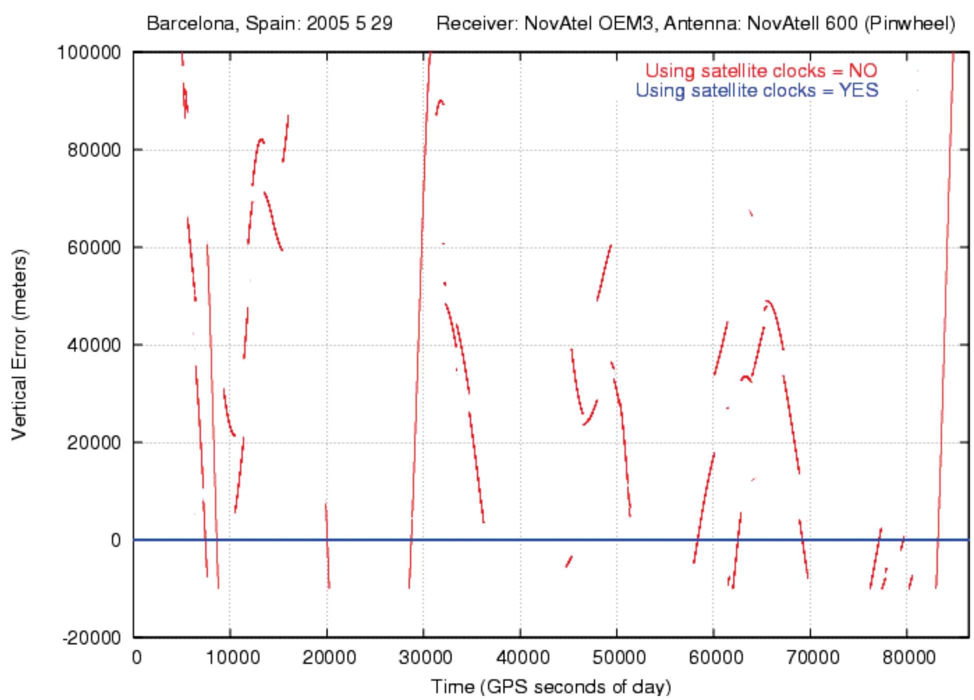
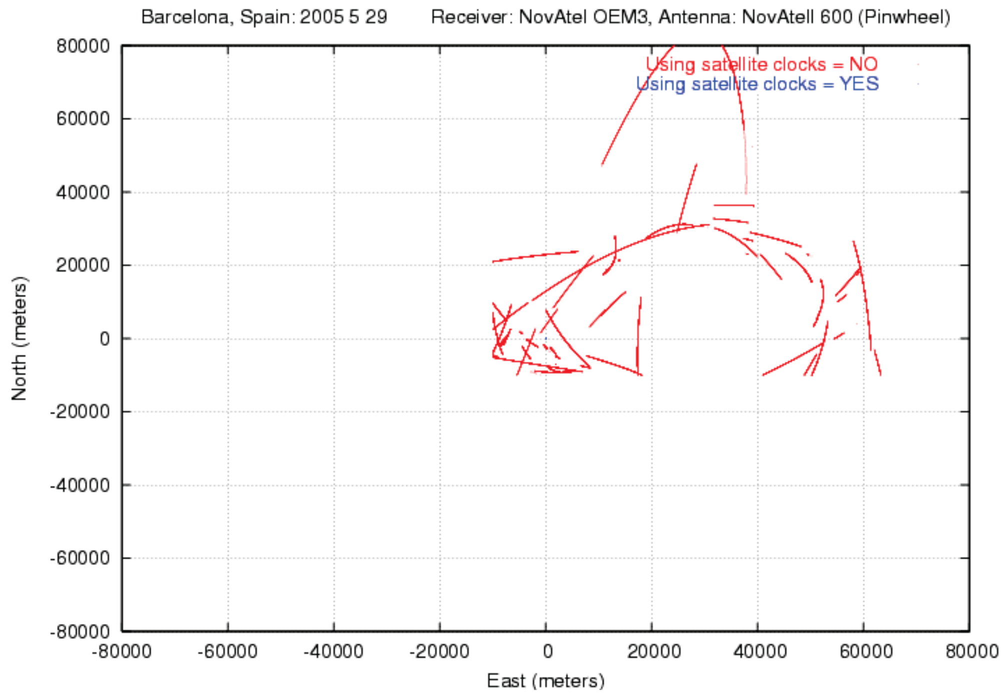
Vertical error comparison



Horizontal error comparison



Relativistic clock correction (Δ_{rel})

- A constant component depending only on nominal value of satellite's orbit major semi-axis, being corrected modifying satellite's clock oscillator frequency*:

$$\frac{f_0' - f_0}{f_0} = \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{\Delta U}{c^2} \simeq -4.464 \cdot 10^{-10}$$

- A periodic component due to orbit eccentricity (to be corrected by user receiver):

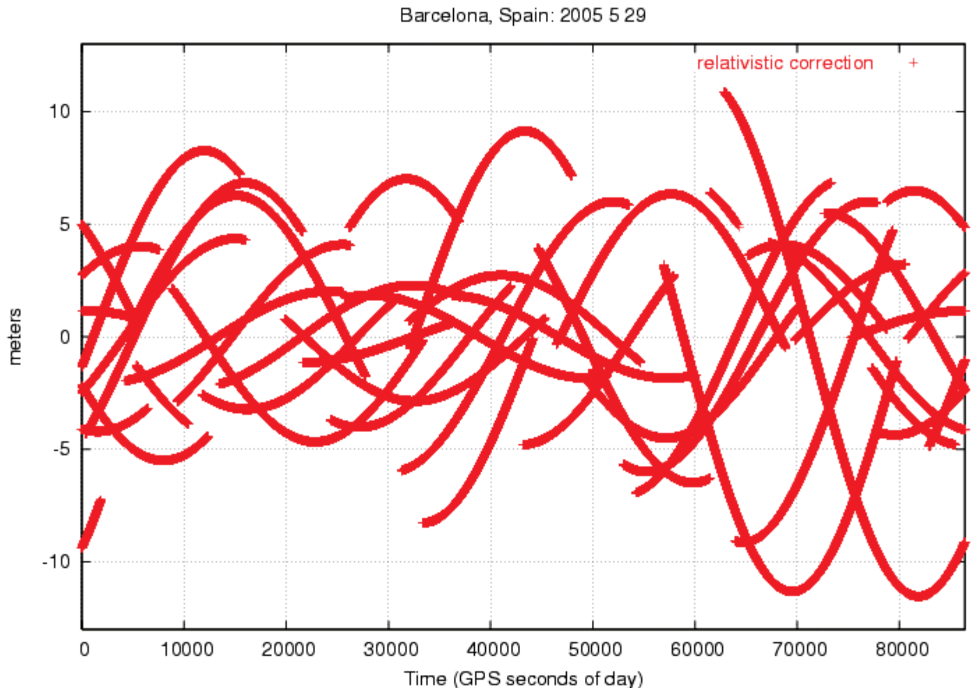
$$\Delta_{rel} = -2\frac{\sqrt{\mu a}}{c^2} e \sin(E) = -2\frac{\mathbf{r}\cdot\mathbf{v}}{c^2}\,(seconds)$$

Being $\mu=3.986005\ 10^{14}\ (m^3/s^2)$ universal gravity constant, $c=299792458\ (m/s)$ light speed in vacuum, a is orbit's major semi-axis, e is its eccentricity, E is satellite's eccentric anomaly, and r and v are satellite's geocentric position and speed in an inertial system.

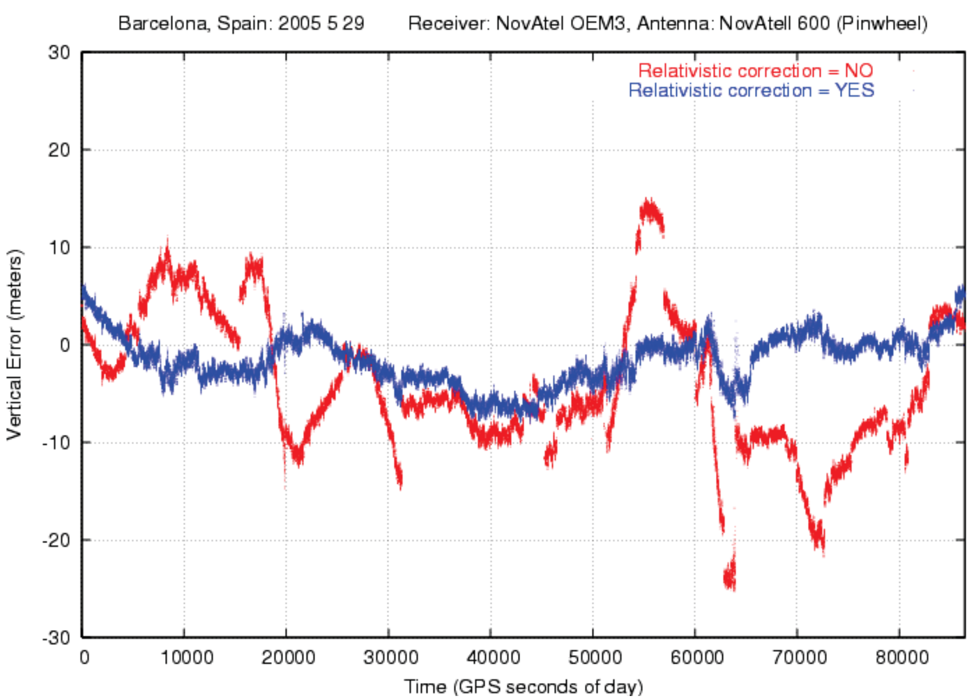
*being f_0 = 10.23 MHz, we have $\Delta f=4.464\ 10^{-10}\ f_0= 4.57\ 10^{-3}$ Hz

so satellite should use $f'o=10.22999999543$ MHz.

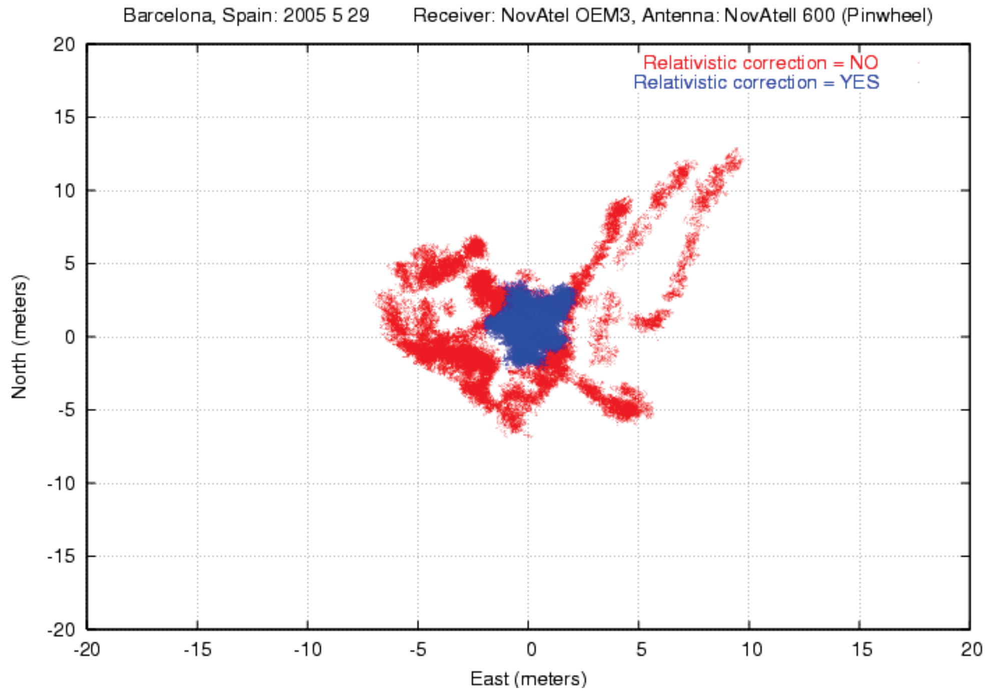
Range variation: relativistic correction



Vertical error comparison



Horizontal error comparison



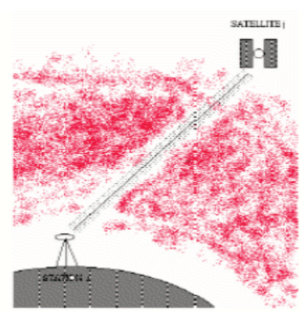
Ionospheric Delay $Ion_{f\ rec}^{\ sat}$

The ionosphere extends from about 60 km in height until more than 2000 km, with a sharp electron density maximum at around 350 km.

The ionosphere delays code and advances carrier by the same amount

The ionospheric delay depends on signal frequency as given by:

$$Ion_{1\ rec}^{\ sat} = \frac{40.3}{f_1^2} I$$



Where I is number of electrons per area unit in the direction of observation, or STEC (*Slant Total Electron Content*)

$$I = \int_{rec}^{sat} N_e\, ds$$

- For two-frequency receivers, it may be cancelled (99.9%) using ionosphere-free combination

$$LC = \frac{f_1^2 L1 - f_2^2 L2}{f_1^2 - f_2^2}$$

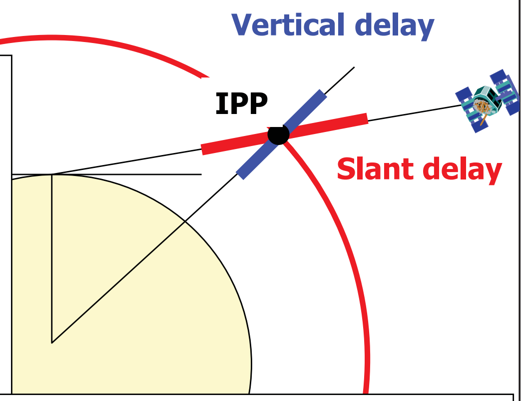
- For one-frequency receivers, it may be corrected (about 60%) using Klobuchar model (defined in GPS/SPS-SS), whose parameters are sent in navigation message. (See program klob.f)

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + \boxed{Ion_{1rec}^{sat}} + TGD^{sat}$$

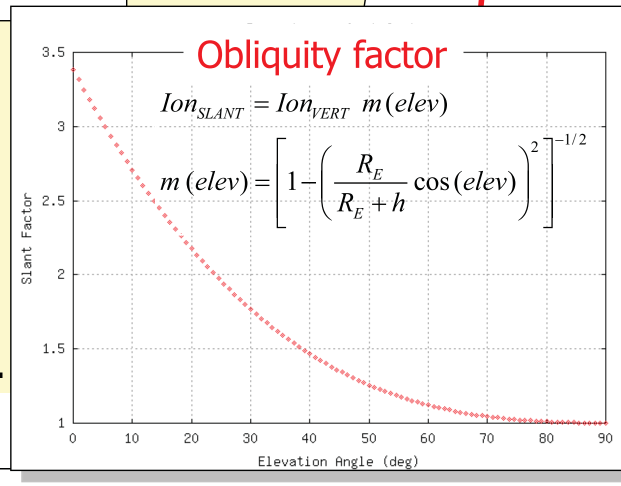
Klobuchar model (klob.f)

It was designed to minimize user computational complexity.

- Minimum user computer storage
- Minimum number of coefficients transmitted on satellite-user link
- At least 50% overall RMS ionospheric error reduction worldwide.



- It is assumed that the electron content is concentrated in a thin layer at 350 Km in height.
- The slant delay is computed from the vertical delay at the ionospheric Pierce Point (IPP), multiplying by the obliquity factor.



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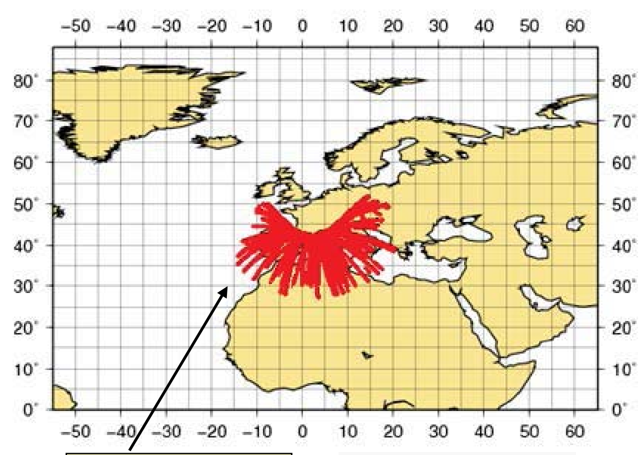
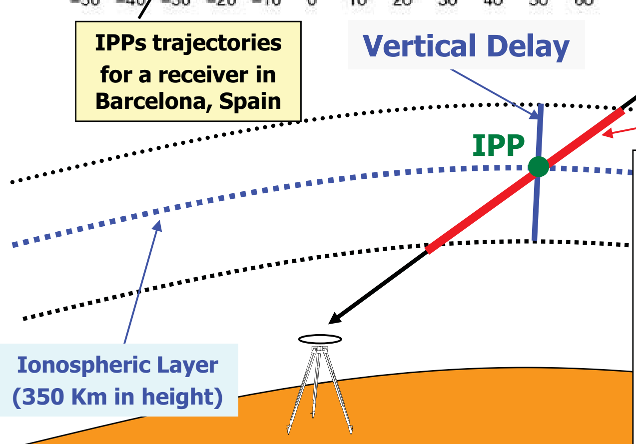
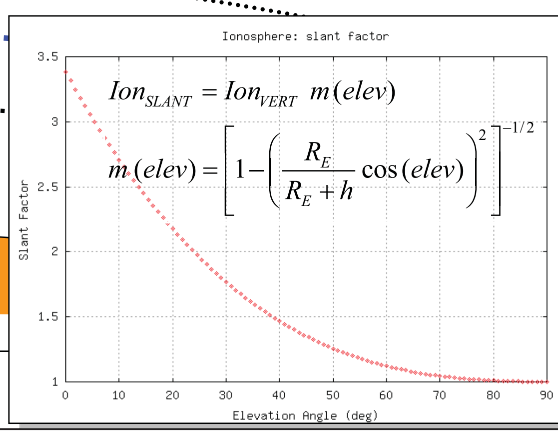
IONOSPHERIC PIERCE POINTS (IPP)



IPPs trajectories for a receiver in Barcelona, Spain





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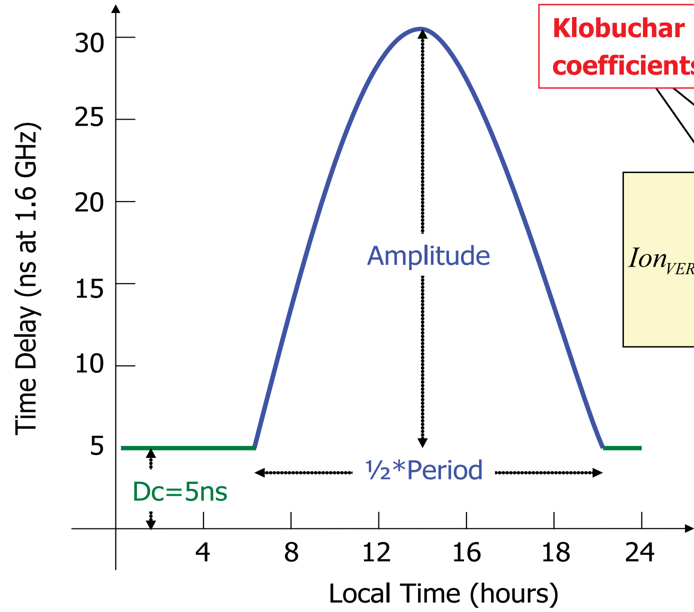
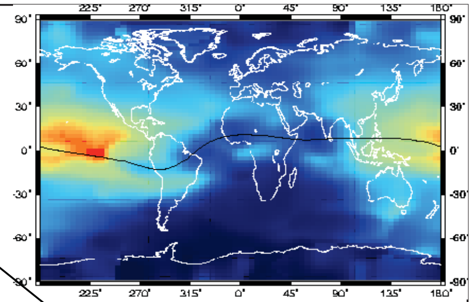
Klobuchar model





Klobuchar coefficients

$$Ion_{VERT} = \begin{cases} DC + A\cos\left[\dfrac{2\pi(t-\Phi)}{P}\right] & (day) \\ DC \quad ; \; if \left[\dfrac{2\pi(t-\Phi)}{P}\right] > \dfrac{\pi}{2} & (night) \end{cases}$$

Being:

$$A = \sum_{n=0}^{3} \alpha_n \varphi^n \quad ; \quad P = \sum_{n=0}^{3} \beta_n \varphi^n$$

φ = **Geomagnetic Latitude**

$$Ion_{SLANT} = Ion_{VERT} \; m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h}\cos(elev)\right)^2\right]^{-1/2}$$

Where:

DC= 5ns

Φ= 14 **(ctt. phase offset)**

t = Local Time

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(time, r_{sta}, r^{sat}, $\alpha0,\alpha1,\alpha2,\alpha3,\beta0,\beta1,\beta2,\beta3$) ➔[Klob] ➔ Iono

elev, ϕ

```
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   -0.540167093277D-07 0.476544268895D-02 0.713579356670D-05 0.515433833885D+04
    0.172800000000D+06-0.260770320892D-07-0.850753478531D+00 0.763684511185D-07
    0.957259887797D+00 0.241437500000D+03-0.167990552187D+01-0.823998608564D-08
    0.174650132022D-09 0.100000000000D+01 0.100200000000D+04 0.000000000000D+00
    0.320000000000D+02 0.000000000000D+00 0.465661287308D-09 0.191000000000D+03
    0.172800000000D+06 0.000000000000D+00 0.000000000000D+00 0.000000000000D+00
```

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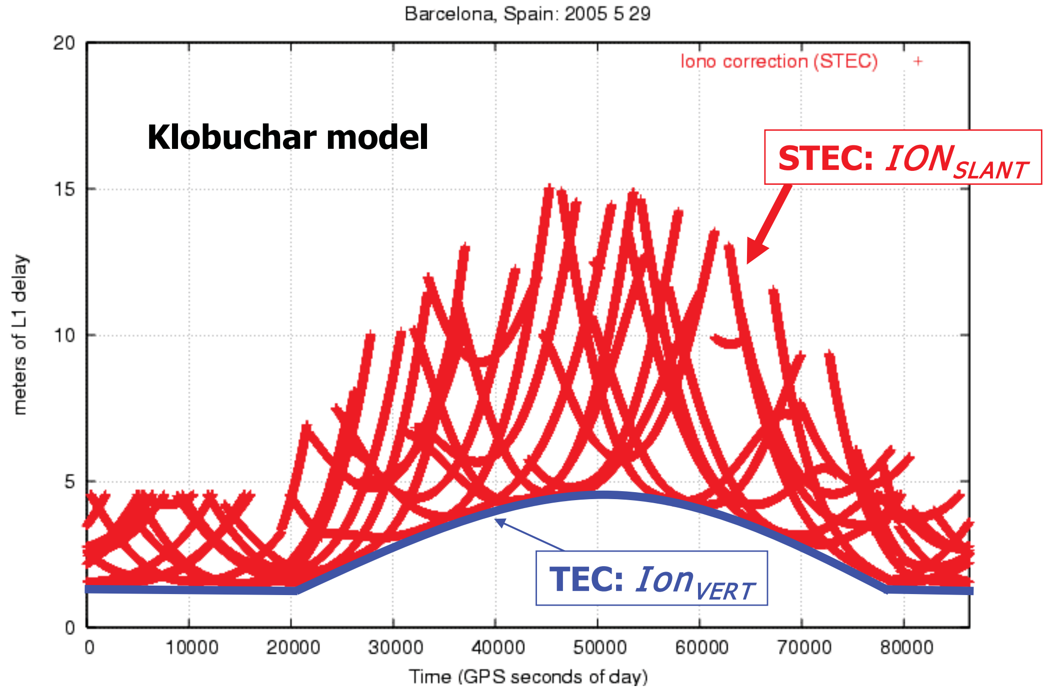
Range variation: Ionospheric correction



$$Ion_{SLANT} = Ion_{VERT} \; m(elev)$$

$$m(elev) = \left[1 - \left(\frac{R_E}{R_E + h} \cos(elev) \right)^2 \right]^{-1/2}$$

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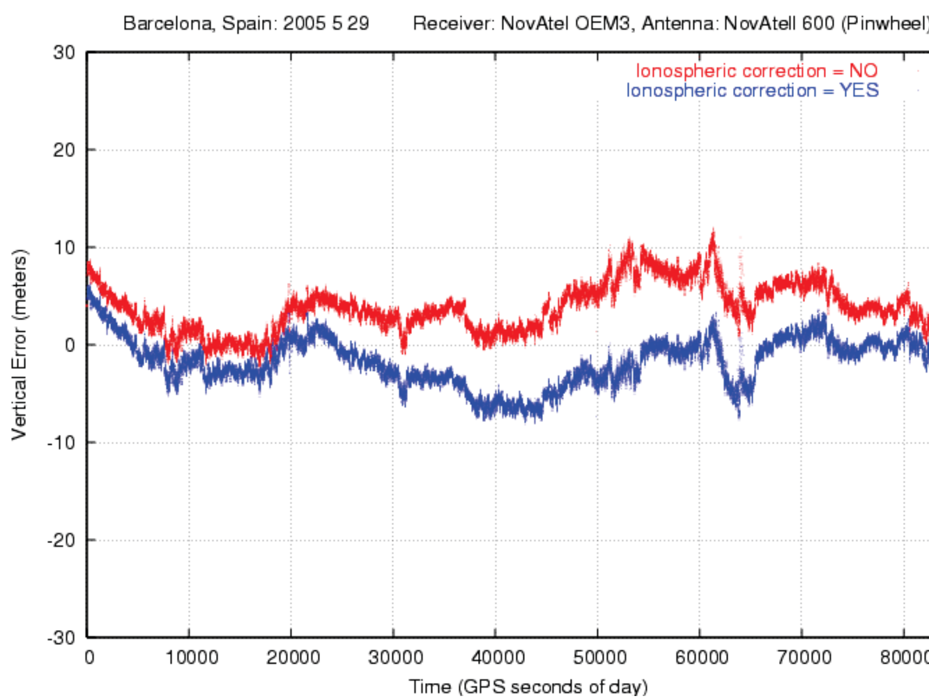
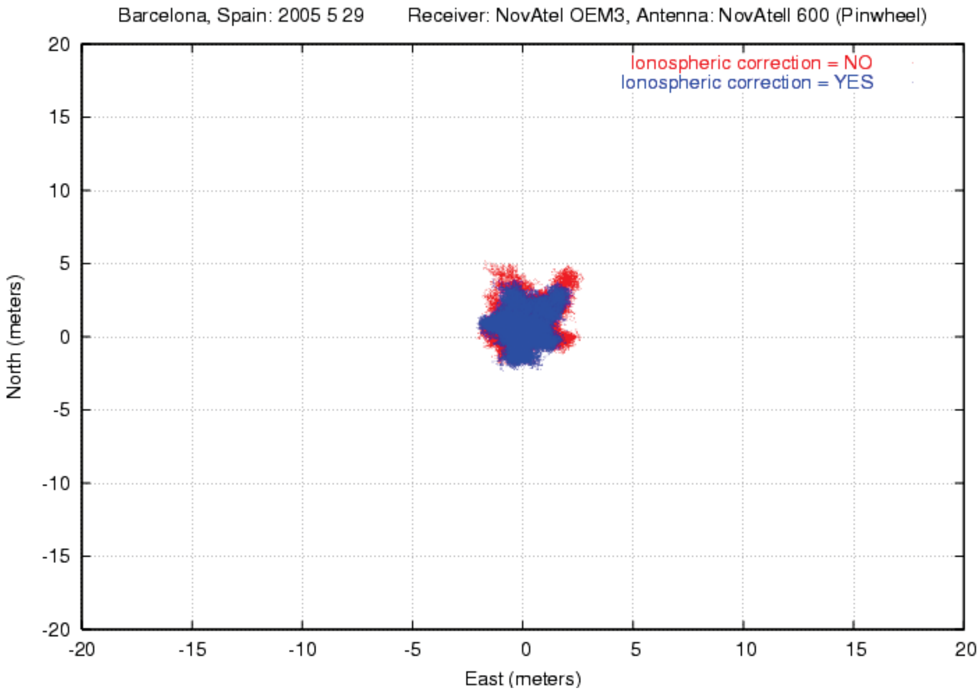
Vertical error comparison



gAGE

Horizontal error comparison



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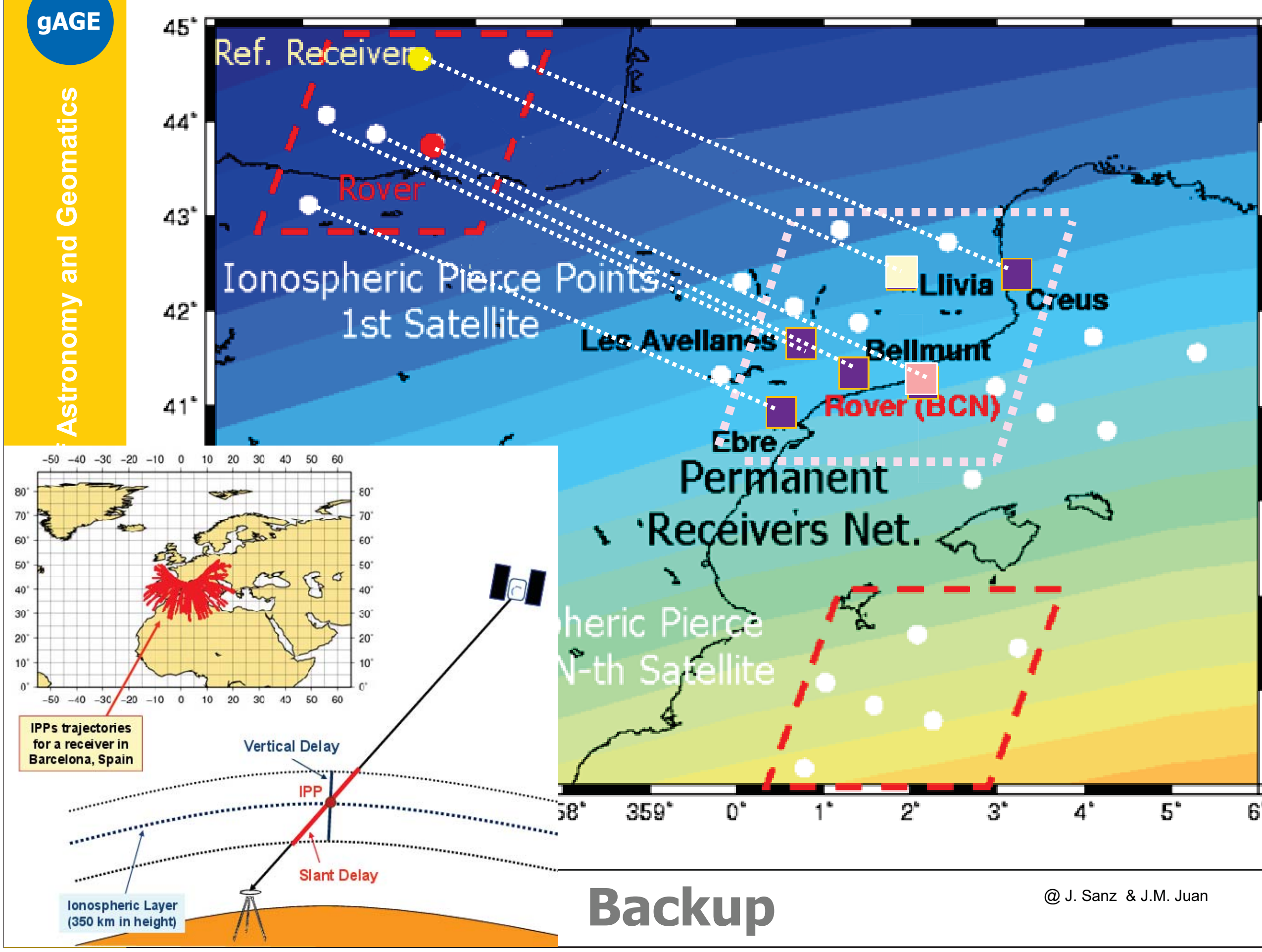

Backup

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Tropospheric Delay

Troposphere is the atmospheric layer placed between Earth's surface and an altitude of about 60km.

The tropospheric delay does not depend on frequency and affects both the code and carrier phases in the same way. It can be modeled (about 90%) as:

- d_{dry} corresponds to the vertical delay of the dry atmosphere (basically oxygen and nitrogen in hydrostatical equilibrium)
  ➔ It can be modeled as an ideal gas.
- d_{wet} corresponds to the vertical delay of the wet component (water vapor) ➔ difficult to model.

A simple model is:

$$Trop_{rec}^{sat} = (d_{dry} + d_{wet}) \cdot m(elev)$$

$$d_{dry} = 2.3 \exp(-0.116 \cdot 10^{-3} H) \quad meters$$
$$d_{wet} = 0.1m \quad ; [H: heigh] \text{ over the sea level}$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + \boxed{Trop_{rec}^{sat}} + Ion_{1rec}^{sat} + TGD^{sat}$$

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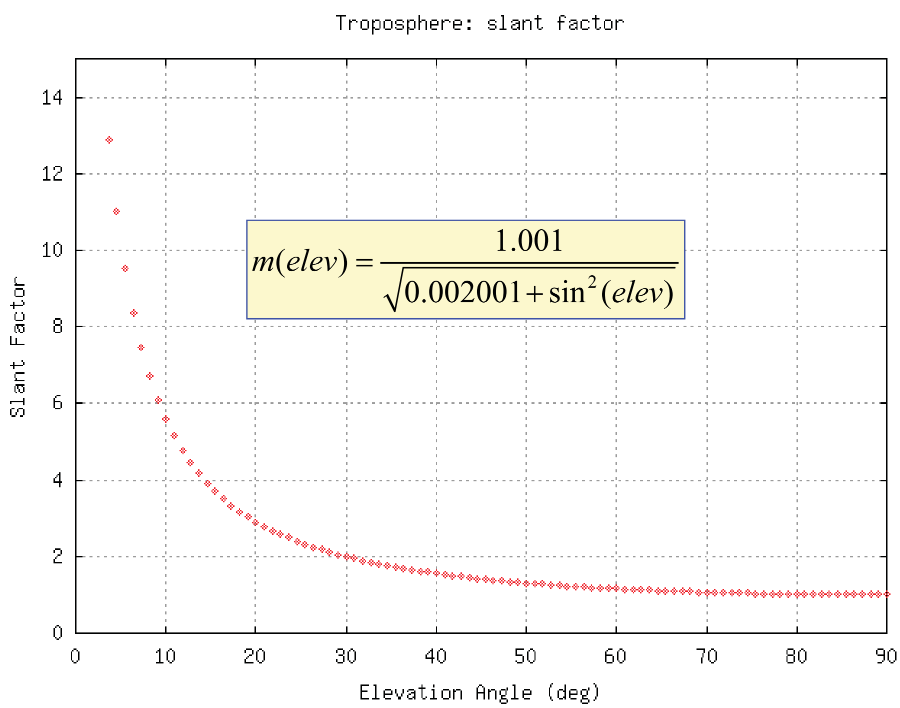

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

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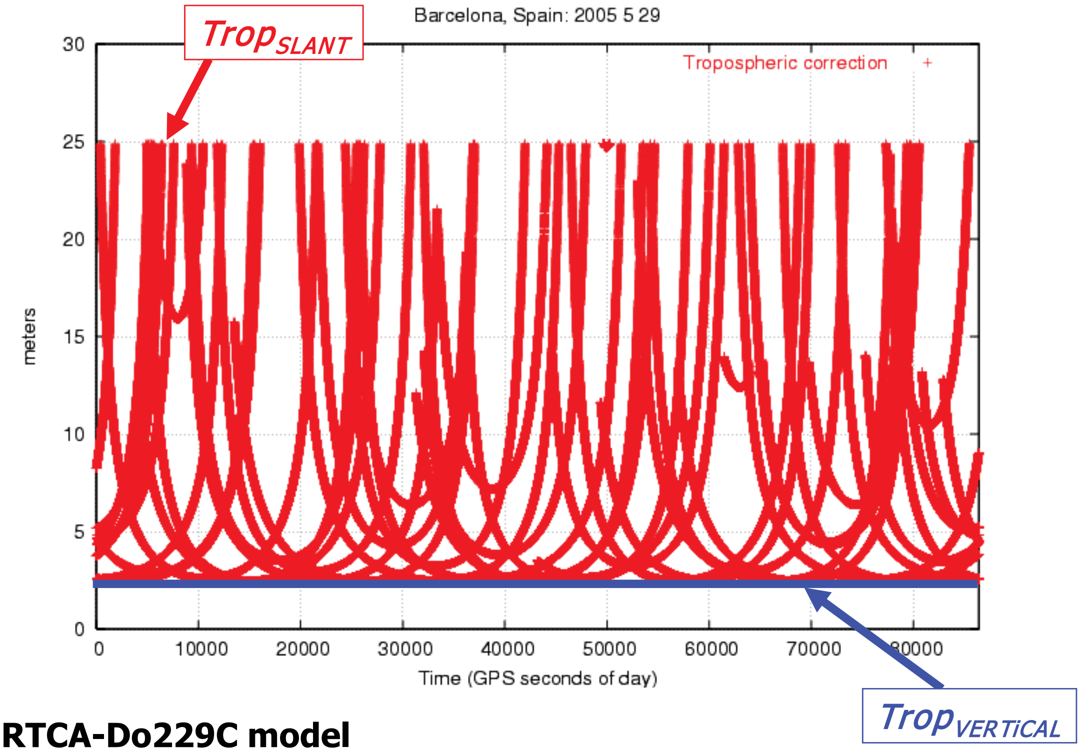
Range variation: Tropospheric correction



RTCA-Do229C model

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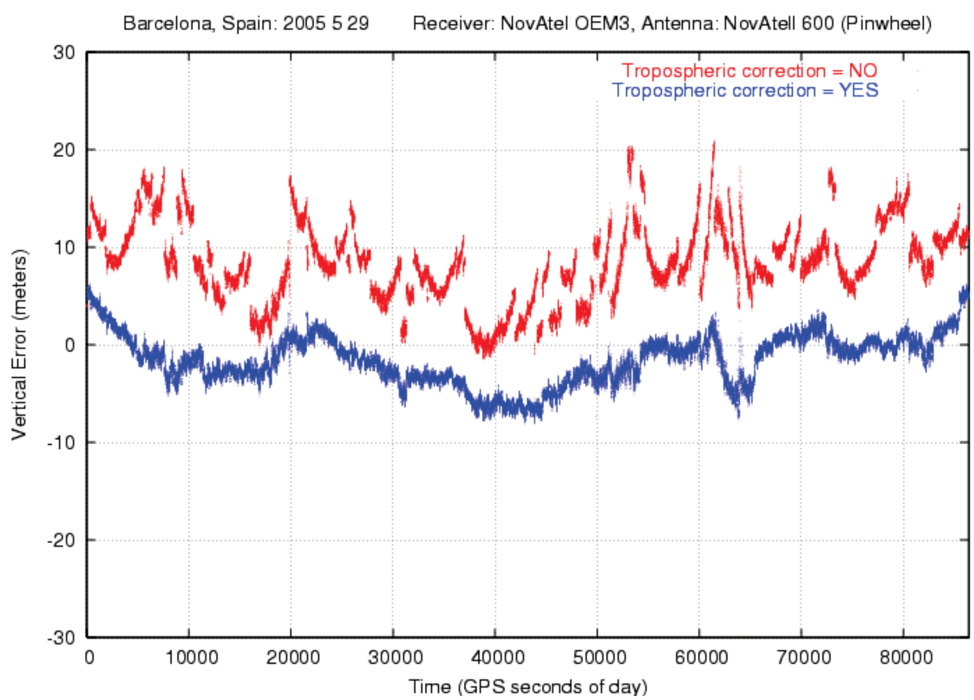
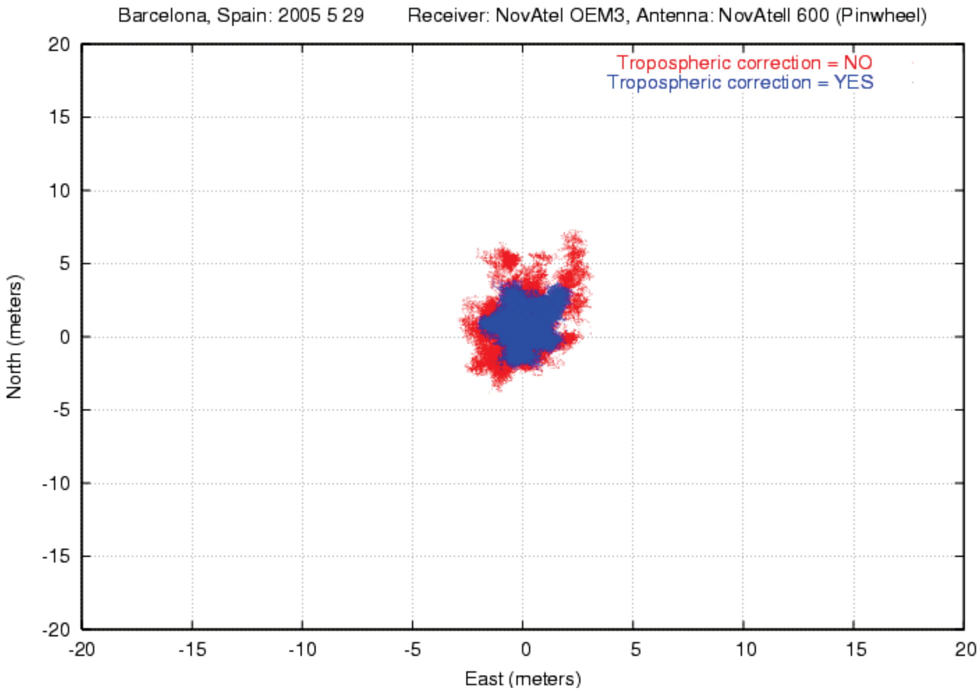
Vertical error comparison



Horizontal error comparison



Instrumental Delays

Some sources for these delays are antennas, cables, as well as several filters used in both satellites and receivers.

They are composed by a delay corresponding to satellite and other to receiver, depending on frequency:

$$K_{1,rec}^{sat} = K_{1,rec} + TGD^{sat}$$

$$K_{2,rec}^{sat} = K_{2,rec} + \frac{f_1^2}{f_2^2} TGD^{sat}$$

- $K1_{rec}$ may be assumed as zero (including it in receiver clock offset).
- TGD^{sat} is transmitted in satellite's navigation message (*Total Group Delay*).

According to ICD GPS-2000, control segment monitors satellite timing, so TGD cancels out when using free-ionosphere combination. That is why we have that particular equation for K_2.

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + \boxed{TGD^{sat}}$$

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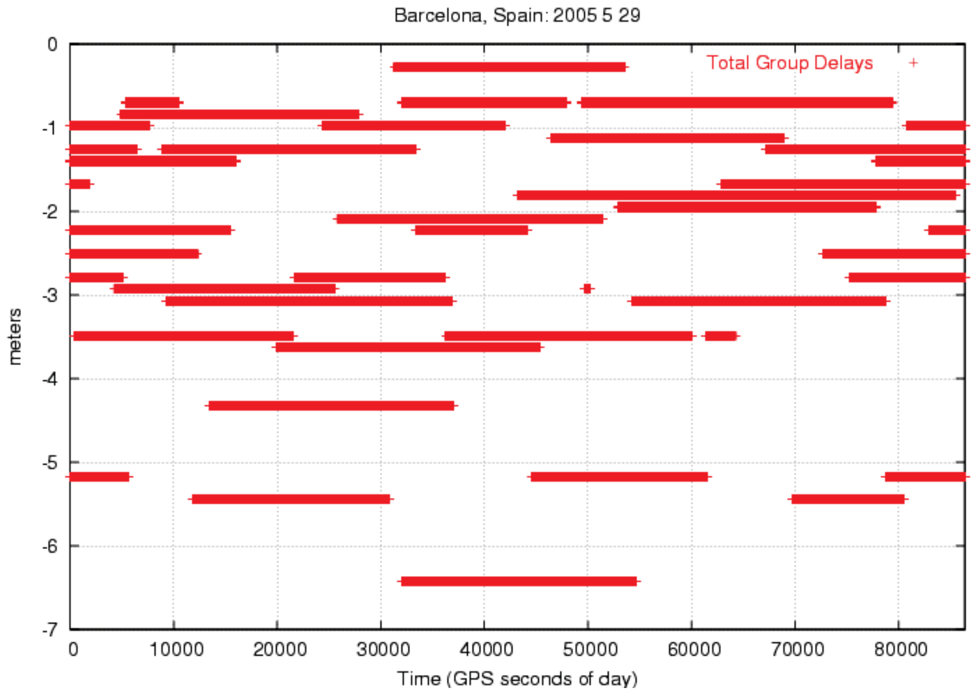
Range variation: Instrumental delays (TGD)



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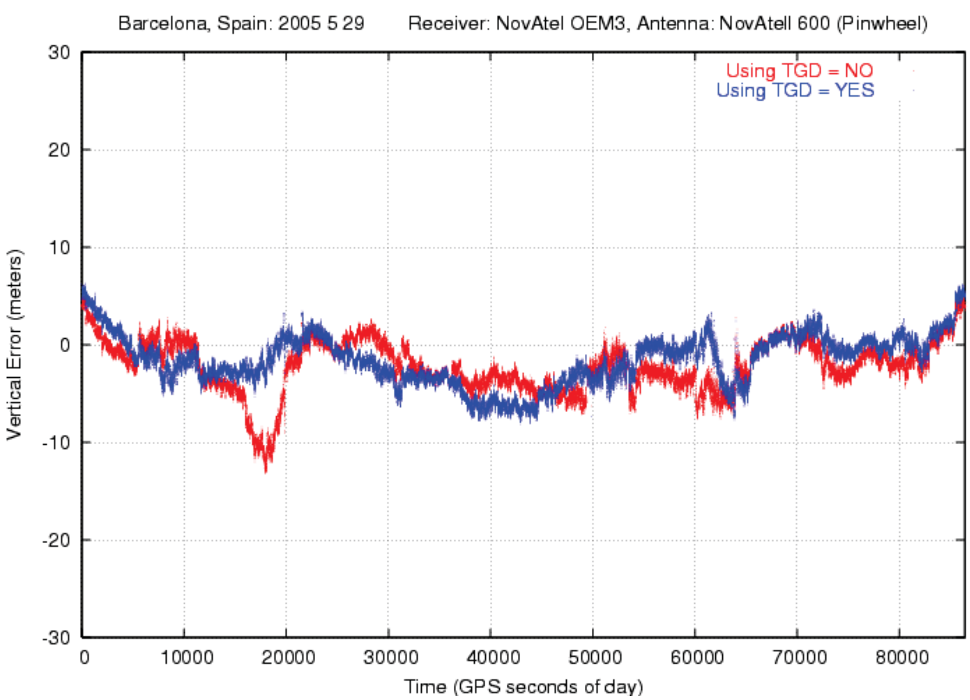
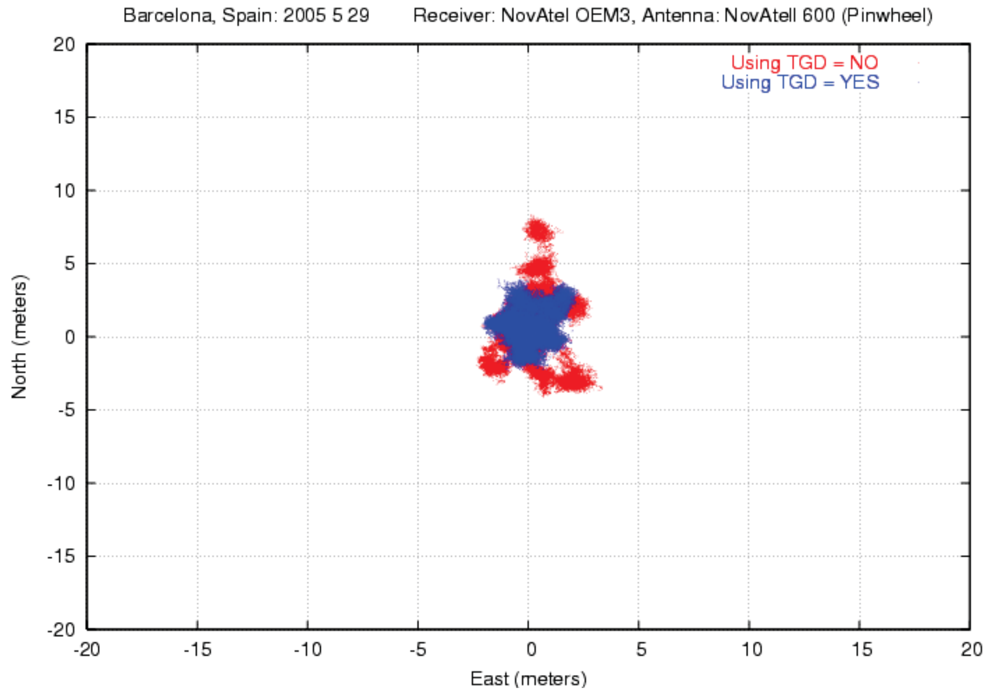
Vertical error comparison



Horizontal error comparison



Measurement noise (thermal noise)

Antispoofing (A/S):
The code **P** is encrypted to **Y**.
➔ Only the code **C** at frequency **L1** is available.

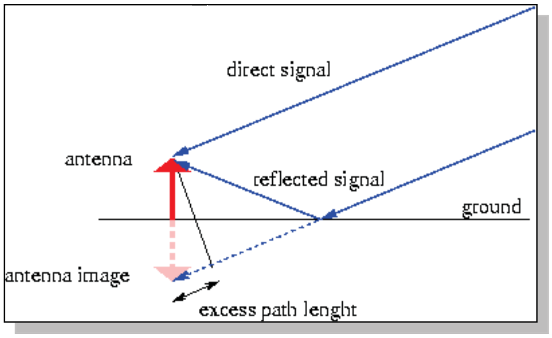
	Wavelength (chip-length)	σ noise (1% of λ) [*]	Main characteristics
Code measurements			
C1	300 m	3 m	Unambiguous but noisier
P1 (Y1): encrypted	30 m	30 cm	
P2 (Y2): encrypted	30 m	30 cm	
Phase measurements			
L1	19.05 cm	2 mm	Precise but ambiguous
L2	24.45 cm	2 mm	

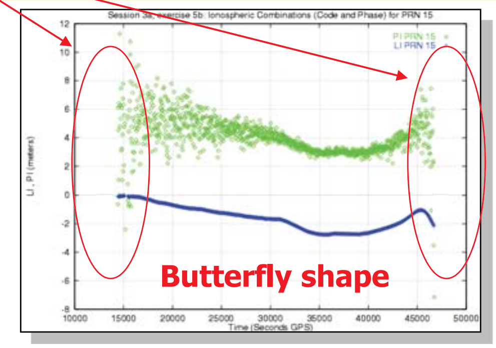
[*] codes may be smoothed with the phases in order to reduce noise (i.e., **C1** smoothed with L1 ➔ **50 cm noise**)

Multipath

- One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.

- It affects both code and carrier phase measurements, and it is more important at low elevation angles.

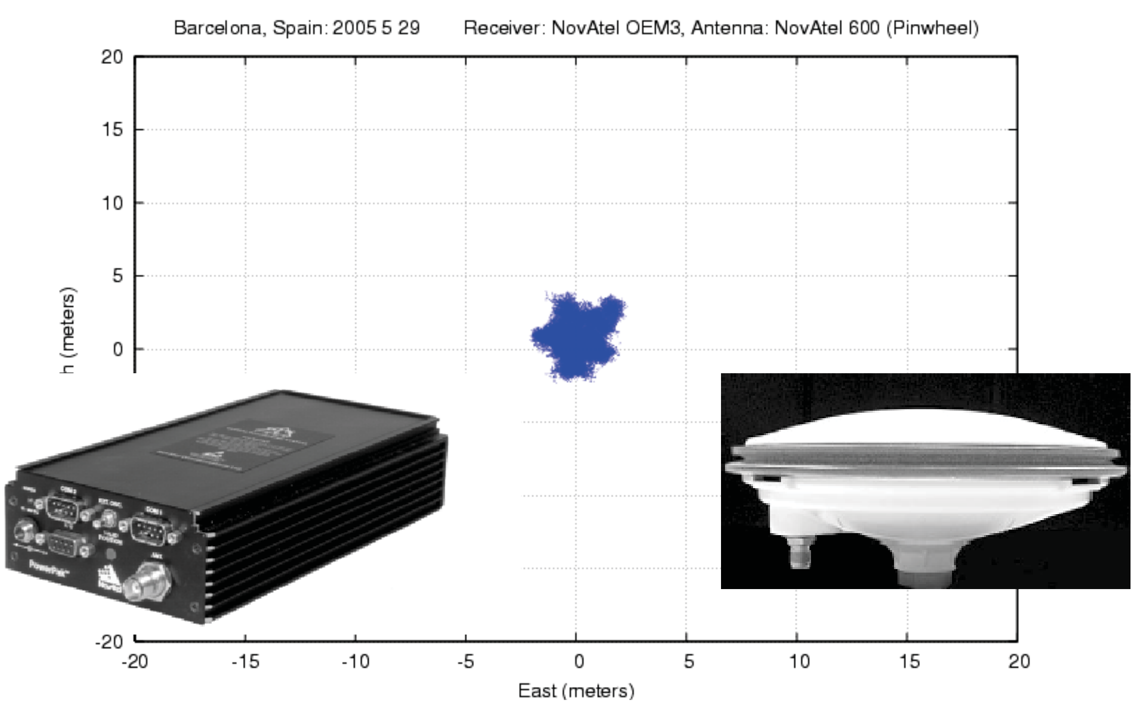




- Code: up to 1.5 chip-length ➔ up to 450m for C1 [theoretically]
  Typically: less than 2-3 m.

- Phase: up to $\lambda/4$ ➔ up to 5 cm for L1 and L2 [theoretically]
  Typically: less than 1 cm

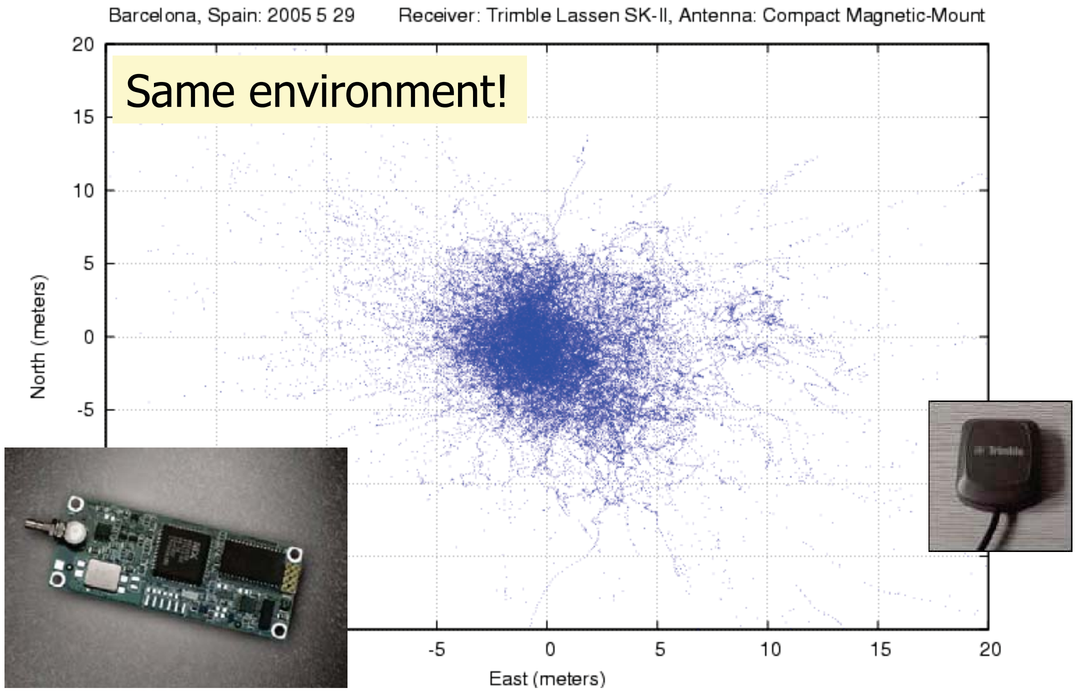
Receiver and multipath noise



GPS standalone (C1 code)

10,000 €

Receiver and multipath noise



GPS standalone (C1 code)

100 €

Contents

Example of Computation of modeled pseudorange

Using data of files **gage2860.98o** and **brdc2860.98n**, compute "by hand" the modeled pseudorange for satellite PRN 14 at t=38230 sec (10h37m10s).

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Follow these steps:

See also exercise 5, Session 5.2 in [RD-2]

1. Select orbital elements closer to 38230
2. Compute satellite clock offset
3. Compute satellite-receiver aprox. geometric range

   3.1 Compute emission time from receiver (reception) time-tags and code pseudorange.

   3.2 Compute satellite coordinates at emission time

   3.3 Compute approximate geometric range.
4. Compute satellite Instrumental delay (TGD):
5. Compute relativistic satllite clock correction
6. Compute tropospheric delay
7. Compute ionospheric delay
8. Compute modeled pseudorange from previous values:

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

1. Selection of orbital elements: From file brdc2860.98n, select the last transmitted navigation message block before instant t=38230 s (10h37m10s).

Transmission time:
979 208818 → 10h 0m 18s

PRN

GPS week

GPS sec of week

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
+2.16000000000E+05 -6.33299350738E-08 +1.004(                .725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.661                  .081050495434E-09
                    .00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
                    .00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```


2. Satellite clock offset computation: From file brdc2860.98n, compute satellite clock offset at time t=3830 s for PRN14:

PRN t_0 a_0 a_1 a_2

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

t = 38230 sec
t_0= 12h 0m 0s= 43200 s

$$d\tilde{t}^{sat} = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 = 5.65 \cdot 10^{-6}\,\mathrm{s}$$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(\boxed{d\tilde{t}^{sat}} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

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3. Satellite-receiver geometric range computation:

Use the following values (4789031, 176612, 4195008) as approximate coordinates.

3.1: Emission time computation from receiver time-tag and code pseudorange: $T[ems]= t_{rec}(T_R)-(C1/c + dt^{sat})$

Measurement file gage2860.98o	→	Pseudorange *C1* at receiver time-tag t=38230: *C1*= 23585247.70 m
Ephemeris file brdc2860.98n	→	Satellite clock offset at t=38230 sec dt^{sat}= 5.65 10^{-6} sec (see previous results)

Thence, the emission time in GPS satellite clock is:

$$T[ems]= 38230 - (23585247.70/c + 5.65\ 10^{-6}) =$$

$$= \mathbf{38229.9213224} \qquad \textbf{(where } c=299792458\textbf{)}$$

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Note:

From RINEX measurement file **gage2860.98o**, select the *C1* pseudorange measurement at receiver time-tag for PRN14:

PRN 14

t = 38230 sec= 10h 37m 10s

```
4     L1     L2     C1     P2                              # / TYPES OF OBSERV

98 10 13 10 37 10.0000000  0  5G18G14G16G 4G19
   5007753.999           0.000     20143892.105          0.000
   -220595.001           0.000     23585247.703          0.000
   1305085.999           0.000     23146887.826          0.000
   6246118.999           0.000     20798091.711          0.000
 -19853878.999           0.000     22235319.057          0.000
```

Thence:

Measurement file gage2860.98o	→	Pseudorange *C1* at receiver time-tag t=38230: *C1*= 23585247.70 m

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3.2: Satellite coordinates at emission time pseudorange:

T[emission]=
38229.921 sec → **Orbit.f** → X= 11453479.346
Y= 122468524.004
Z= 8245076.145
CTS [emission]

↑

Use the selected ephemeris for PRN14 (from file brdc2860.98n)

The previous coordinates are given in an Earth-fixed reference frame (CTS) at *t=T[emission]=* 38229.921 s.

This reference frame rotates by un amount "$\omega_E\,\Delta t$" during traveling time Δt*=T[reception]-T[emission].*

$$(X^{sat},Y^{sat},Z^{sat})_{CTS[reception]} = R_3(\omega_E\,\Delta t).(X^{sat},Y^{sat},Z^{sat})_{CTS[emission]}$$

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$$(X^{sat},Y^{sat},Z^{sat})_{CTS[reception]} = R_3(\omega_E\,\Delta t).(X^{sat},Y^{sat},Z^{sat})_{CTS[emission]}$$

$$\underset{CTS[reception]}{\boxed{\begin{pmatrix} 11453350.377 \\ 122468589.797 \\ 8245076.145 \end{pmatrix}}} = \begin{pmatrix} \cos(\omega_E \Delta t) & \sin(\omega_E \Delta t) & 0 \\ -\sin(\omega_E \Delta t) & \cos(\omega_E \Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \underset{CTS[emission]}{\begin{pmatrix} 11453479.346 \\ 122468524.004 \\ 8245076.145 \end{pmatrix}}$$

$$\omega_E \Delta t = -5.74 \cdot 10^{-6}\, rad. \quad (where \quad \omega_E = 7.2921151467 \cdot 10^{-5}\, rad/\sec)$$

$$\Delta t = -\frac{\rho_{0,rec}^{sat}}{c} = -0.079 \sec$$

$$\rho_{0,rec}^{sat} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2} \simeq 23616673.3m$$

$$(x,y,z)^{satellite} \approx (11453479,\ 22468524,\ 8245076)$$

$$(x_0,y_0,z_0)_{receiver} \approx (4789031,\ \ 176612,\ 4195008)$$

An approximate value is enough to compute Δt.

Note: Both satellite and receiver coordinates must be given in the same reference system!

→the CTS[reception] will be used to build navigation equations.

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3.2: Geometric range computation

The geometric range between satellite coordinates at emission time and the "approximate position of the receiver" at reception time *(both coordinates given in the same reference system [for instance the CTS system at reception time])* is computed by:

$$\rho_{0,receiver}^{satellite} = \sqrt{\left(x^{sat} - x_{0,rec}\right)^2 + \left(y^{sat} - y_{0,rec}\right)^2 + \left(z^{sat} - z_{0,rec}\right)^2} = 23616699.124m$$

$$(x,y,z)^{satellite} = (11453350.2771,\ 22468589.7975,\ 8245076.1448)_{CTS[reception]}$$

$$\boxed{(x_0, y_0, z_0)_{receiver}} = (4789031,\ \ 176612,\ 4195008)_{CTS[reception]}$$

"Approximate" receiver coordinates at reception time.

$$C1_{rec}^{sat}[\text{modelled}] = \boxed{\rho_{0,rec}^{sat}} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

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4. Satellite Instrumental delay (TGD): From file brdc2860.98n, compute the Total Group Delay for PRN14:

PRN

TGD (in sec)

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.000000000000E+00
 +1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.198042513605E-01
 -3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.153795101166E+03
 +2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.725290298462E-09
 +9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.081050495434E-09
 -1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.000000000000E+00
 +3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.280000000000E+02
 +2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.000000000000E+00
```

TGD= -2.32830643654E-09 * c= -0.69801 m

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + \boxed{TGD^{sat}}$$

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5. Relativistic clock correction:

PRN e sqrt(a)

```
14 98 10 13 12 0 0 +5.65452501178E-06 +9.09494701773E-13 +0.00000000000E+00
+1.28000000000E+02 -6.10000000000E+01 +4.38125402624E-09 +8.19804251360 5E-01
-3.31364572048E-06 +1.09227513894E-03 +5.67547976971E-06 +5.15379510116 6E+03
+2.16000000000E+05 -6.33299350738E-08 +1.00409621952E+00 -3.72529029846 2E-09
+9.73658001335E-01 +2.74031250000E+02 +2.66122811383E+00 -8.08105049543 4E-09
-1.45720352451E-10 +1.00000000000E+00 +9.79000000000E+02 +0.00000000000 0E+00
+3.20000000000E+01 +0.00000000000E+00 -2.32830643654E-09 +1.28000000000 0E+02
+2.08818000000E+05 +0.00000000000E+00 +0.00000000000E+00 +0.00000000000 0E+00
```

T[emission] = **38229.921 s** ➔ *Orbit.f* ➔ *E =0.095 rad. (eccentric anomaly)*

$$\Delta rel^{sat} = -2\frac{\sqrt{\mu a}}{c^2}\, e\sin(E) = -2.3\cdot 10^{-10}\,\mathrm{s}$$

$\mu = 3.986005\cdot 10^{14} \quad m^3 s^{-2}$

$c = 299792458 \quad m\, s^{-1}$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{sat} + \boxed{\Delta rel^{sat}}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

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6. Tropospheric correction

$$Trop_{rec}^{sat} = \left(d_{dry} + d_{wet}\right) m(elev) = 6.76m$$

$$d_{dry} = 2.3\,\mathrm{e}^{-0.116\cdot 10^{-3}H} = 2.3m$$

$$d_{wet} = 0.1m$$

$$m(elev) = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}}$$

See klob.f

$elev = 20.57\frac{\pi}{180} = 0.359rad$

$H = 160m \quad (heigh \;\; over\ the\ ellipsoid)$

$(x,y,z)_{rec}$ ➔[car2geo] ➔ $(Lon, Lat, H)_{rec}$

$$C1_{rec}^{sat}[\text{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{sat} + \Delta rel^{sat}\right) + \boxed{Trop_{rec}^{sat}} + Ion_{1rec}^{sat} + TGD^{sat}$$

7. Ionospheric correction

(time, r_{sta}, r^{sat}, $\alpha 0, \alpha 1, \alpha 2, \alpha 3, \beta 0, \beta 1, \beta 2, \beta 3$) ➔[Klob] ➔ Iono=10.26m

```
     2              NAVIGATION DATA     GPS                 RINEX VERSION/ TYPE
XPRINT v1.1             gAGE                00/06/04 17:36:23    PGM / RUN BY / DATE
gAGE BROADCAST EPHEMERIS FILE                                    COMMENT
    +1.9558E-08 +0.0000E+00 -1.1921E-07 +0.0000E+00              ION ALPHA
    +1.2288E+05 -1.6384E+04 -2.6214E+05 +1.9661E+05              ION BETA
    -8.381903171539E-09-1.421085471520E-14   405504        979 DELTA_UTC: A0,A1,T,W
     12                                                          LEAP SECONDS
                                                                 END OF HEADER
```

$$t = 38230\,\mathrm{sec}$$
$$(x,y,z)^{satellite} = (11453350.2771,\ 22468589.7975,\ 8245076.1448)_{CTS[reception]}$$
$$(x_0,y_0,z_0)_{receiver} = (4789031,\ \ 176612,\ 4195008)_{CTS[reception]}$$

Approximate values for receiver or satellite coordinates are enough

$$C1_{rec}^{sat}[\mathrm{modelled}] = \rho_{0,rec}^{sat} - c\left(d\tilde{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + \boxed{Ion_{1rec}^{\,sat}} + TGD^{sat}$$

7. Compute the modeled pseudorange.

$$C1_{rec}^{sat}[\mathrm{modelled}] = \rho_{rec,0}^{sat} - c\left(d\tilde{t}^{\,sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{\,sat} + TGD^{sat}$$

$$\rho_{0,rec}^{sat} = 23616699.124\,\mathrm{m}$$
$$c\,dt^{sat} = 5.65\cdot 10^{-6}\,c = 1693.828\,\mathrm{m}$$
$$c\,\Delta rel^{sat} = -2.33\cdot 10^{-10}\,c = -0.071\,\mathrm{m}$$
$$Trop_{rec}^{sat} = 6.760\,\mathrm{m}$$
$$Ion_{1rec}^{\,sat} = 10.260\,\mathrm{m}$$
$$TGD^{sat} = -0.698\,\mathrm{m}$$

➔ $C1_{rec}^{sat}[\mathrm{modelled}] = 23615021.689\mathrm{m}$

Prefit residual:

Is the difference between measured and modeled pseudorange

$$\mathrm{Pref}_{rec}^{sat} = \boxed{C1_{rec}^{sat} - C1[\mathrm{mod}]_{rec}^{sat}} = \boxed{\rho_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt_{rec} + K_{1rec} + \varepsilon}$$

In the previous example (PRN14 at t = 38230 s):

Pref= 23585247.703 - 23615021.689 = -29773.986 m

Previously calculated

From measurement file

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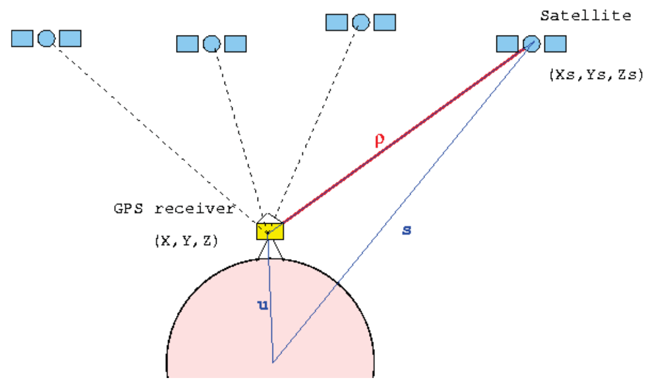
Solving navigation equations



Input:

- ***Pseudoranges* (receiver-satellite j): P^j**
- **Navigation message. In particular:**
  - ***satellite position* when transmitting signal: $r^j = (x^j, y^j, z^j)$**
  - ***offsets* of satellite clocks: dt^j**

  (j = 1,2,...n) (n>=4)

Unknowns:

- **receiver *position*: $r = (x, y, z)$**
- **receiver *clock* offset: DT**

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For each satellite in view

Iono+Tropo+TGD...

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + \sum \delta_k$$

where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)

$$\boxed{C1_{rec}^{sat} - \left(\rho_{0,rec}^{sat} + c\,dt^{sat} - \sum \delta_k\right)} = \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c\,dt_{rec}$$

measurement computed unknown

For all sat. in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec}-x^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{y_{0,rec}-y^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{z_{0,rec}-z^{sat1}}{\rho^{sat1}_{0,rec}} & 1 \\ \frac{x_{0,rec}-x^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{y_{0,rec}-y^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{z_{0,rec}-z^{sat2}}{\rho^{sat2}_{0,rec}} & 1 \\ & \dots\dots\dots & & \\ \frac{x_{0,rec}-x^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{y_{0,rec}-y^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{z_{0,rec}-z^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

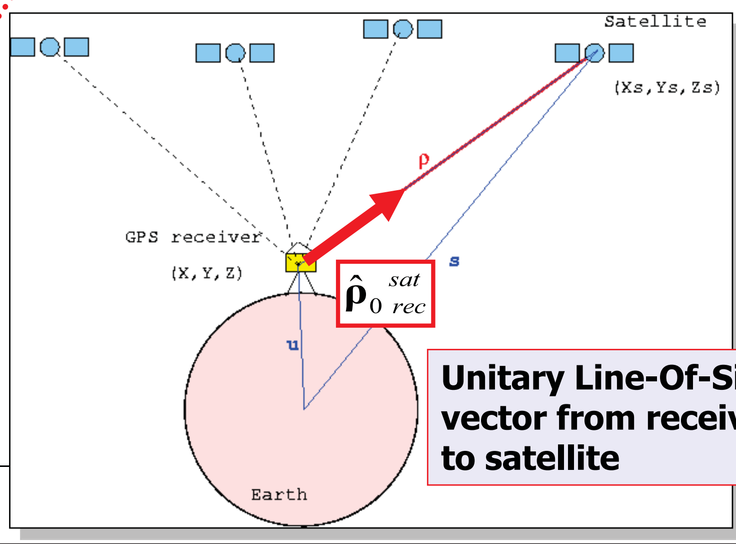
Observations

(measured/computed)

Geometry of rays

$$-\frac{\vec{\boldsymbol{\rho}}^{T\ sat\,n}_{0\ rec}}{\rho^{sat\,n}_{0\ rec}}$$

$$\hat{\boldsymbol{\rho}}^{T\ sat\,n}_{0\ rec} \equiv \frac{\vec{\boldsymbol{\rho}}^{T\ sat\,n}_{0\ rec}}{\rho^{sat1}_{0\ rec}}$$



$\hat{\boldsymbol{\rho}}^{sat}_{0\ rec}$

Unitary Line-Of-Sight vector from receiver to satellite

(x,y,z) coordinates

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec}-x^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{y_{0,rec}-y^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{z_{0,rec}-z^{sat1}}{\rho^{sat1}_{0,rec}} & 1 \\ \frac{x_{0,rec}-x^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{y_{0,rec}-y^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{z_{0,rec}-z^{sat2}}{\rho^{sat2}_{0,rec}} & 1 \\ & \dots\dots\dots & & \\ \frac{x_{0,rec}-x^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{y_{0,rec}-y^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{z_{0,rec}-z^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

Observations

(measured/computed)

Geometry of rays

$$-\hat{\boldsymbol{\rho}}^{T\ sat\,n}_{0\ rec}$$

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} -\hat{\boldsymbol{\rho}}^{T\ sat1}_{0\ rec} & 1 \\ -\hat{\boldsymbol{\rho}}^{T\ sat2}_{0\ rec} & 1 \\ \dots & \dots \\ -\hat{\boldsymbol{\rho}}^{T\ sat\,n}_{0\ rec} & 1 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{r}_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

(e,n,u) coordinates

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \dots\dots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} -\cos el^1 \sin az^1 & -\cos el^1 \cos az^1 & -\sin el^1 & 1 \\ -\cos el^2 \sin az^2 & -\cos el^2 \cos az^2 & -\sin el^2 & 1 \\ & \dots\dots\dots & & \\ -\cos el^n \sin az^n & -\cos el^n \cos az^n & -\sin el^n & 1 \end{bmatrix} \begin{bmatrix} \Delta e_{rec} \\ \Delta n_{rec} \\ \Delta u_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

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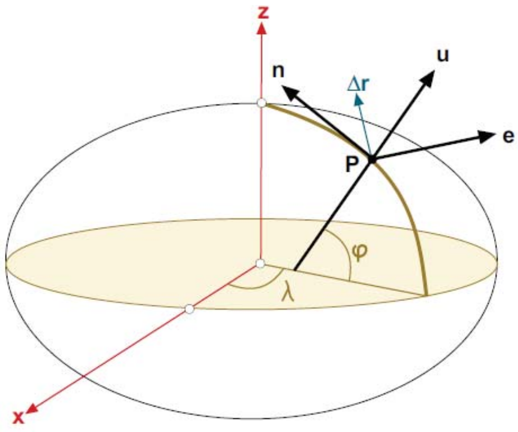
From ECEF (x,y,z) to Local (e,n,u) coordinates



$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \mathbf{R}_1[\pi/2 - \varphi]\,\mathbf{R}_3[\pi/2 + \lambda] \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

$$\hat{\mathbf{e}} = (-\sin\lambda\,,\ \cos\lambda\,,\ 0)$$
$$\hat{\mathbf{n}} = (-\cos\lambda\sin\varphi\,,\ -\sin\lambda\sin\varphi\,,\ \cos\varphi)$$
$$\hat{\mathbf{u}} = (\cos\lambda\cos\varphi\,,\ \sin\lambda\ \cos\varphi\,,\ \sin\varphi)$$

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\varphi & -\sin\lambda\sin\varphi & \cos\varphi \\ \cos\lambda\cos\varphi & \sin\lambda\cos\varphi & \sin\varphi \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

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COMMENTS:

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \cdots\cdots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec}-x^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{y_{0,rec}-y^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{z_{0,rec}-z^{sat1}}{\rho^{sat1}_{0,rec}} & 1 \\ \frac{x_{0,rec}-x^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{y_{0,rec}-y^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{z_{0,rec}-z^{sat2}}{\rho^{sat2}_{0,rec}} & 1 \\ & \cdots\cdots\cdots & & \\ \frac{x_{0,rec}-x^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{y_{0,rec}-y^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{z_{0,rec}-z^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

Of course, receiver coordinates $(x_{rec}, y_{rec}, z_{rec})$ are not known (they are the target of this problem). But, we can always assume that an "approximate position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ is known".

Thence, as it will be shown in next lesson, the navigation problem will consist on:

1.- To start from an approximate value for receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$ e.g. the Earth's centre) to linearise the equations

2.- With the pseudorange measurements and the navigation equations, compute the correction $(\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$ to have improved estimates: $(x_{rec}, y_{rec}, z_{rec}) = (x_{0,rec}, y_{0,rec}, z_{0,rec}) + (\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})$

3.- Linearise the equations again, about the new improved estimates, and iterate until the change in the solution estimates is sufficiently small.

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For all sa[...] in view

$$\begin{bmatrix} Prefit^1 \\ Prefit^2 \\ \ldots\ldots \\ Prefit^n \end{bmatrix} = \begin{bmatrix} \frac{x_{0,rec} - x^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{y_{0,rec} - y^{sat1}}{\rho^{sat1}_{0,rec}} & \frac{z_{0,rec} - z^{sat1}}{\rho^{sat1}_{0,rec}} & 1 \\ \frac{x_{0,rec} - x^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{y_{0,rec} - y^{sat2}}{\rho^{sat2}_{0,rec}} & \frac{z_{0,rec} - z^{sat2}}{\rho^{sat2}_{0,rec}} & 1 \\ & \ldots\ldots\ldots & & \\ \frac{x_{0,rec} - x^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{y_{0,rec} - y^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & \frac{z_{0,rec} - z^{sat\,n}}{\rho^{sat\,n}_{0,rec}} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ c\,dt_{rec} \end{bmatrix}$$

Observations
(measured/computed)

Geometry of rays

Thence, the basic linearized GPS measurement equation can be written as:

$$\mathbf{y} = \mathbf{G}\,\mathbf{x}$$

This is a linear system with, in general, $\boxed{n \geq 4}$ equations which we can solve using LS, WLS, Kalman filter,…

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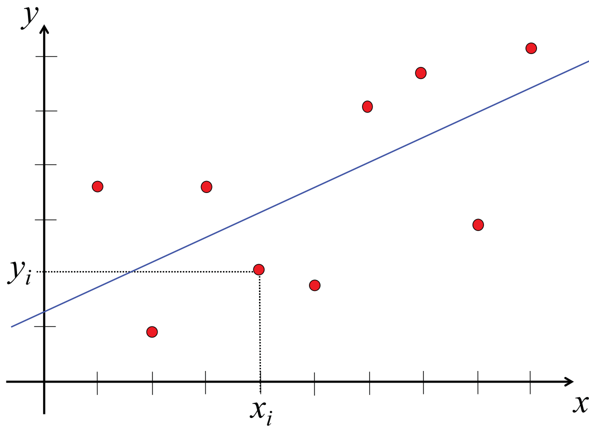
Least Squares solution (conceptual review)

As a driving problem, let us consider the problem of fitting a set of points (noisy measurements) to a straight line $y=mx+n$.



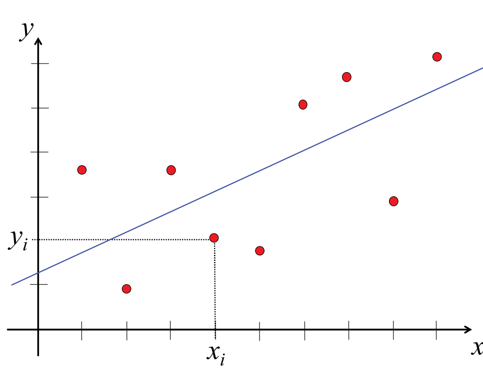
x	y
x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_N	y_N

$$\left\{\begin{array}{l} y_1 \simeq m\,x_1 + n \\ y_2 \simeq m\,x_2 + n \\ \quad\vdots \\ y_N \simeq m\,x_N + n \end{array}\right. \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_N & 1 \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} + \boldsymbol{\varepsilon} \Rightarrow \underset{N\times 1}{\mathbf{y}} = \underset{N\times 2}{\mathbf{G}}\,\underset{2\times 1}{\mathbf{p}} + \boldsymbol{\varepsilon}$$

$$\left\{\begin{array}{l} y_1 \simeq m\,x_1 + n \\ y_2 \simeq m\,x_2 + n \\ \quad\vdots \\ y_N \simeq m\,x_N + n \end{array}\right. \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_N & 1 \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} + \boldsymbol{\varepsilon} \Rightarrow \underset{N\times 1}{\mathbf{y}} = \underset{N\times 2}{\mathbf{G}}\,\underset{2\times 1}{\mathbf{p}} + \boldsymbol{\varepsilon}$$

This is an over-determined (**incompatible**) system of equations (due to the measurement noise $\boldsymbol{\varepsilon}$).

It is evident that there is no straight line passing over all the data points (red points). Thence, **we have to look for a solution that fits the measurements best in some sense.**



Note that, as $\mathbf{G}$ is not an squared matrix ($N>2$), we cannot try:

$$\mathbf{y} = \mathbf{G}\,\mathbf{p} \Rightarrow \mathbf{p} = \cancel{\mathbf{G}^{-1}\mathbf{y}}$$

But, $\mathbf{G}^T\mathbf{G}$ is a squared $(N \times N)$ matrix, thence, we can try:

$$\boxed{\mathbf{G}^T\mathbf{y} = \mathbf{G}^T\mathbf{G}\,\mathbf{p} \Rightarrow \hat{\mathbf{p}} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{y}}$$

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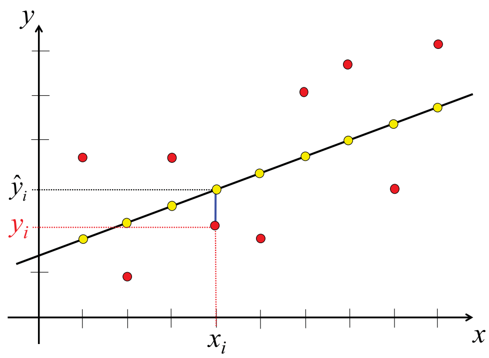
Results from Linear Algebra:

1) $\exists\,(\mathbf{G}^T\mathbf{G})^{-1} \Leftrightarrow$ The columns of matrix G are linearly independents.

2) $\hat{\mathbf{p}} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{y} \Leftrightarrow \min\|\mathbf{y}-\hat{\mathbf{y}}\| = \min\left(\sum_{i=1}^{N}(y_i-\hat{y}_i)^2\right)$ Least Squares solution

where $\hat{\mathbf{y}} = \mathbf{G}\,\hat{\mathbf{p}}$



But, what is the physical meaning of the least square solution?

What does it mean the condition

$$\min\|\mathbf{y}-\hat{\mathbf{y}}\| = \min\left(\sum_{i=1}^{N}(y_i-\hat{y}_i)^2\right) \quad ?$$

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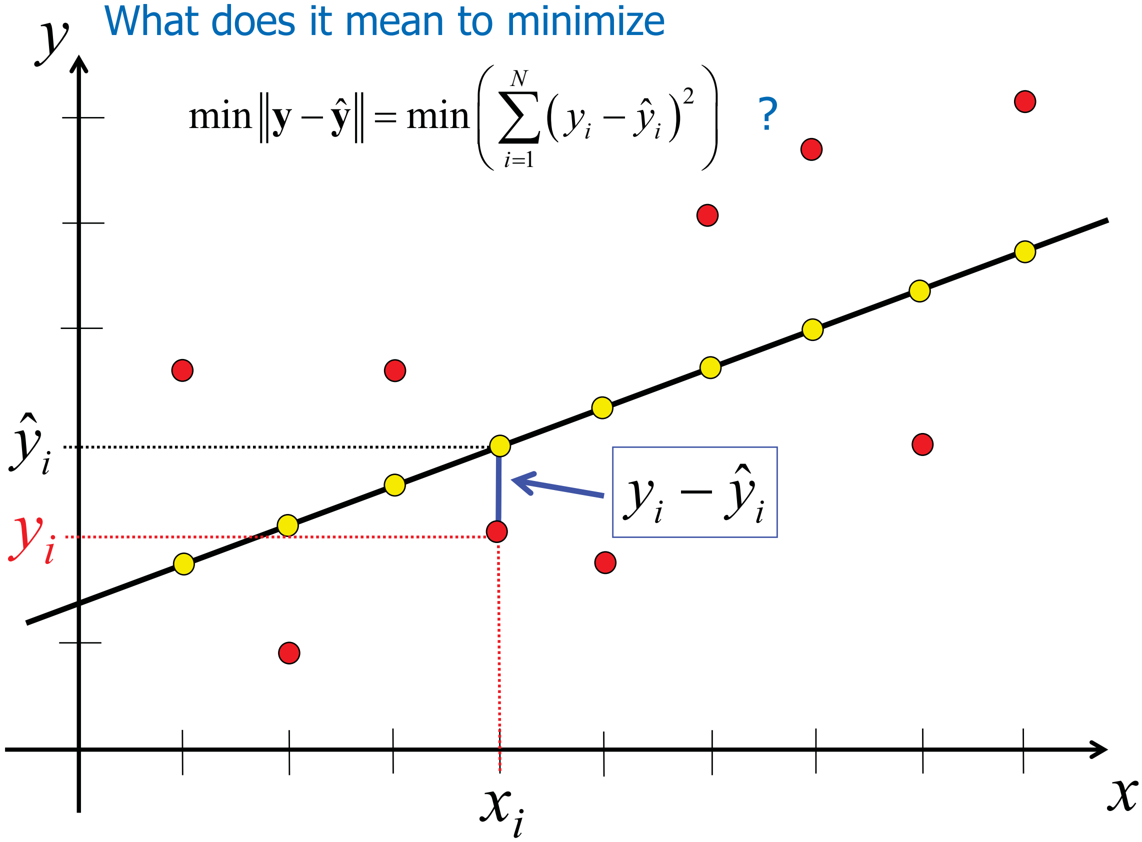
What is the physical meaning of the least square solution?
What does it mean to minimize

$$\min\|\mathbf{y}-\hat{\mathbf{y}}\| = \min\left(\sum_{i=1}^{N}(y_i-\hat{y}_i)^2\right) \quad ?$$



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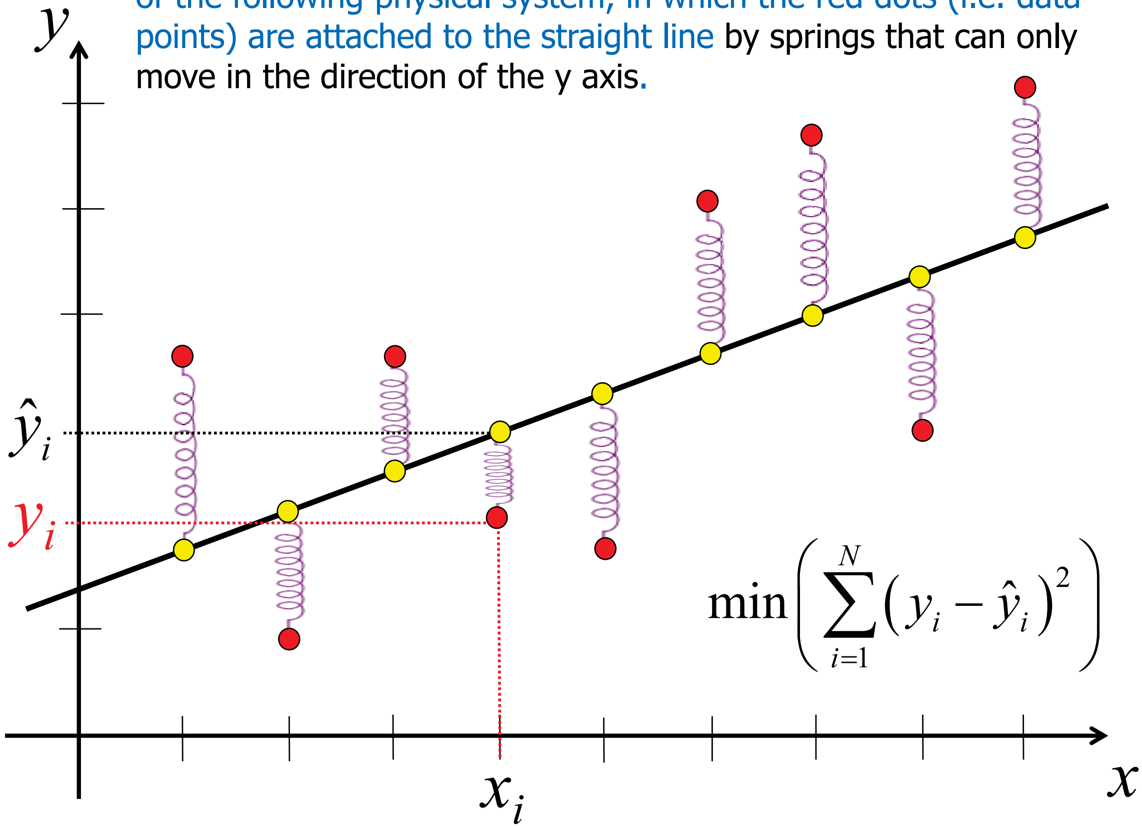
The Least Squares solution gives the solution of equilibrium of the following physical system, in which the red dots (i.e. data points) are attached to the straight line by springs that can only move in the direction of the y axis.



$$\min\left(\sum_{i=1}^{N}\left(y_i-\hat{y}_i\right)^2\right)$$

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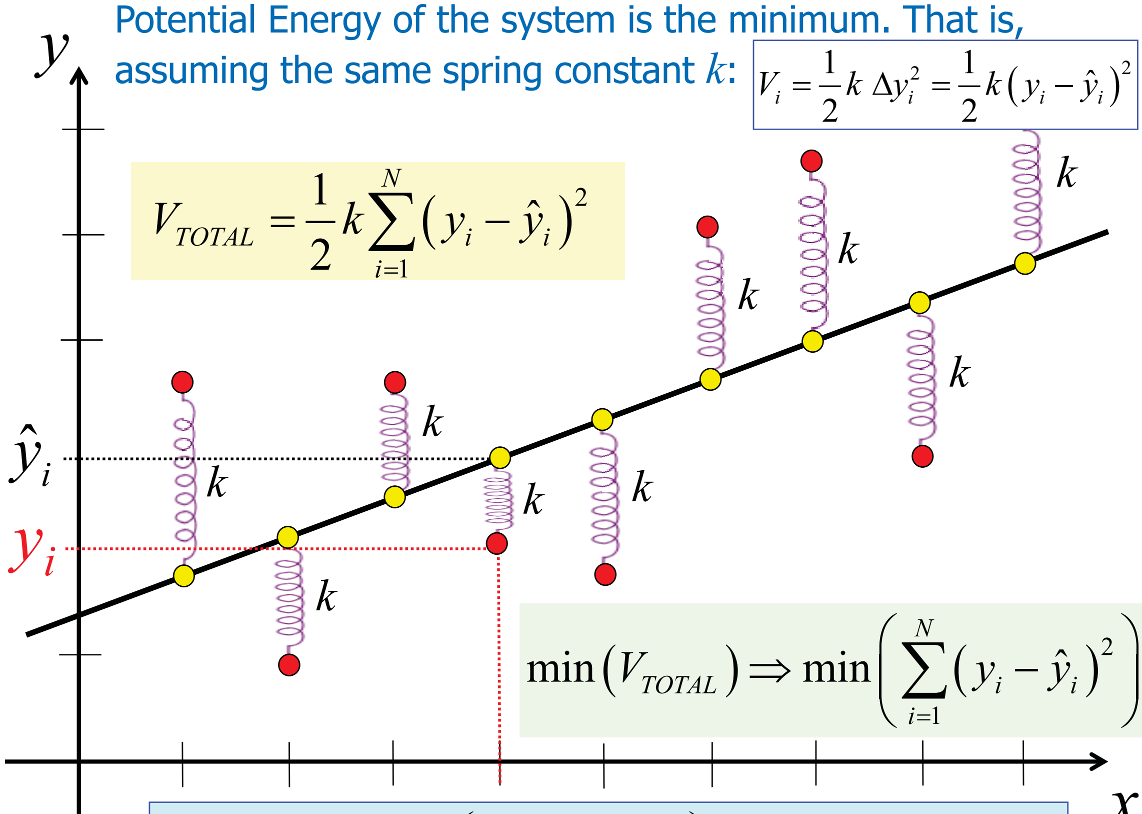
Indeed, the equilibrium solution is reached when the Total Potential Energy of the system is the minimum. That is, assuming the same spring constant k: $V_i=\frac{1}{2}k\,\Delta y_i^2=\frac{1}{2}k\left(y_i-\hat{y}_i\right)^2$



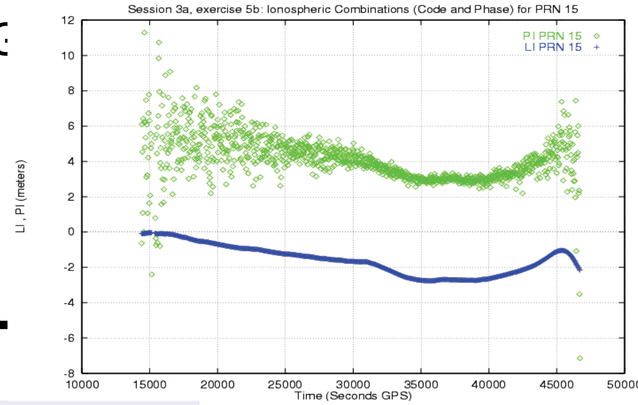
$$V_{TOTAL}=\frac{1}{2}k\sum_{i=1}^{N}\left(y_i-\hat{y}_i\right)^2$$

$$\min\left(V_{TOTAL}\right)\Rightarrow\min\left(\sum_{i=1}^{N}\left(y_i-\hat{y}_i\right)^2\right)$$

$$\min\|\mathbf{y}-\hat{\mathbf{y}}\|=\min\left(\sum_{i=1}^{N}\left(y_i-\hat{y}_i\right)^2\right)\Leftrightarrow\hat{\mathbf{p}}=(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{y}$$

Let be the basic linearized G

$$\mathbf{y} = \mathbf{G}\,\mathbf{x}$$



- Least Squares solution:

$$\hat{\mathbf{x}} = \left(\mathbf{G}^t\mathbf{G}\right)^{-1}\mathbf{G}^t\mathbf{y}$$

The **same error** is assumed in all measurements

- Weighted Least Squares solution

If the measurements have **different errors**, the equations can be weighted by matrix $\mathbf{W}$:

$$\mathbf{W} = \begin{bmatrix} w_{y_1} & & 0 \\ & \ddots & \\ 0 & & w_{y_n} \end{bmatrix}$$

Uncorrelated errors are assumed

And the weighted least squares solution is:

$$\hat{\mathbf{x}} = \left(\mathbf{G}^t\mathbf{W}\,\mathbf{G}\right)^{-1}\mathbf{G}^t\;\mathbf{W}\mathbf{y} \longleftrightarrow \min\|\mathbf{y}-\hat{\mathbf{y}}\|^2_{\mathbf{W}} = \min\left[\sum_i w_i\left(y_i-\hat{y}_i\right)^2\right]$$

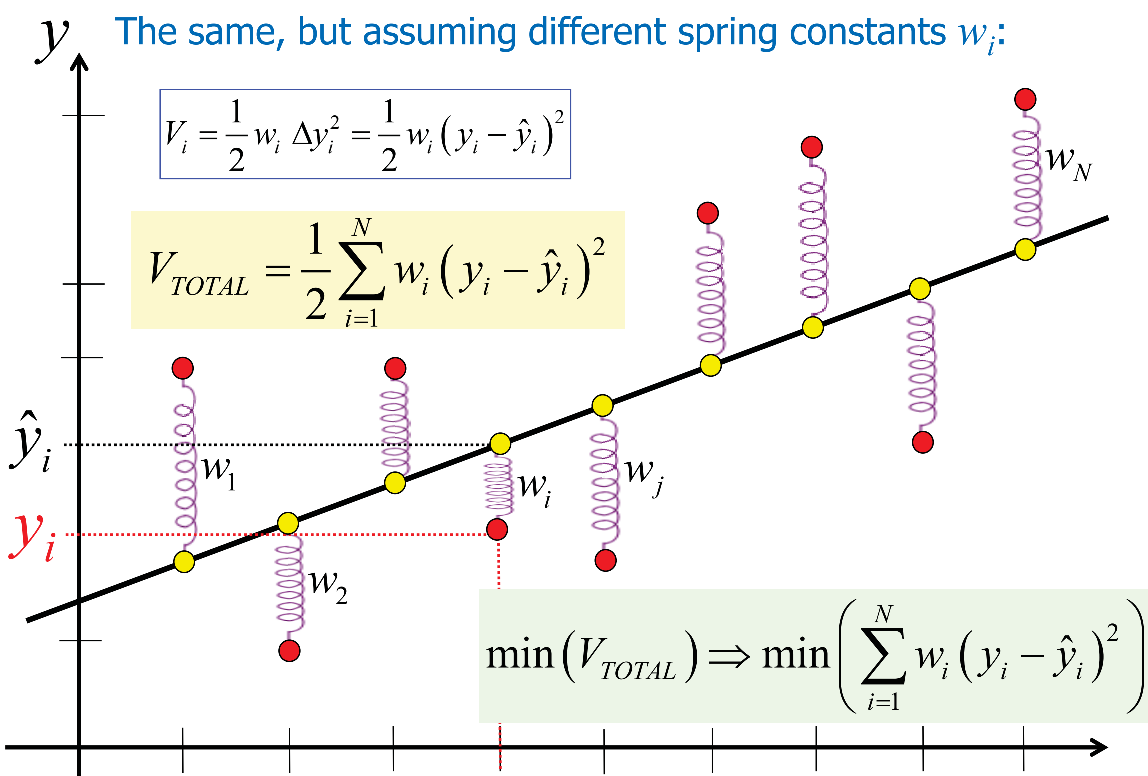
$$\hat{\mathbf{y}} = \mathbf{G}\,\hat{\mathbf{x}}$$

Weighted Least Squares solution:

The same, but assuming different spring constants w_i:

$$V_i = \frac{1}{2} w_i\,\Delta y_i^2 = \frac{1}{2} w_i\left(y_i-\hat{y}_i\right)^2$$

$$V_{TOTAL} = \frac{1}{2}\sum_{i=1}^{N} w_i\left(y_i-\hat{y}_i\right)^2$$



$$\min\left(V_{TOTAL}\right) \Rightarrow \min\left(\sum_{i=1}^{N} w_i\left(y_i-\hat{y}_i\right)^2\right)$$

$$\min\|\mathbf{y}-\hat{\mathbf{y}}\|_W = \min\left(\sum_{i=1}^{N} w_i\left(y_i-\hat{y}_i\right)^2\right) \Leftrightarrow \hat{\mathbf{p}} = (\mathbf{G}^T\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{W}\mathbf{y}$$

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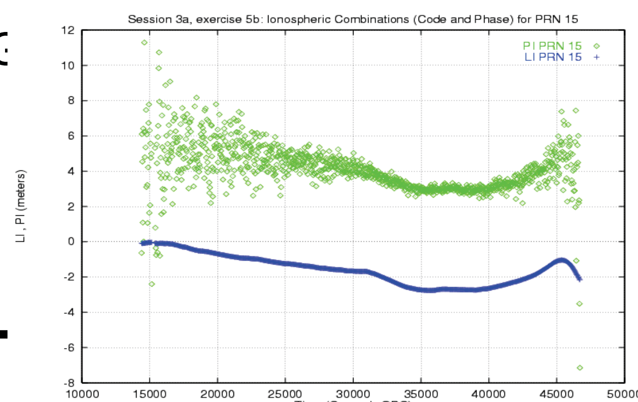
Let be the basic linearized G

$$\boxed{\mathbf{y} = \mathbf{G}\,\mathbf{x}}$$



- Least Squares solution:

$$\boxed{\hat{\mathbf{x}} = \left(\mathbf{G}^t\mathbf{G}\right)^{-1}\mathbf{G}^t\mathbf{y}}$$

The **same error** is assumed in all measurements

- Weighted Least Squares solution

If the measurements have **different errors**, the equations can be weighted by matrix $\mathbf{W}$:

$$\mathbf{W} = \begin{bmatrix} w_{y_1} & & 0 \\ & \ddots & \\ 0 & & w_{y_n} \end{bmatrix}$$

Uncorrelated errors are assumed

And the weighted least squares solution is:

$$\boxed{\hat{\mathbf{x}} = \left(\mathbf{G}^t\mathbf{W}\,\mathbf{G}\right)^{-1}\mathbf{G}^t\;\mathbf{W}\mathbf{y}} \longleftrightarrow \min\|\mathbf{y}-\hat{\mathbf{y}}\|^2_{\mathbf{W}} = \min\left[\sum_i w_i\left(y_i - \hat{y}_i\right)^2\right]$$

$$\hat{\mathbf{y}} = \mathbf{G}\,\hat{\mathbf{x}}$$

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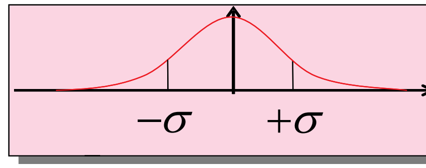
Assuming that measurements $\mathbf{Y}$ have **random errors with zero mean and variance σ^2**, and assuming that error sources for each satellite are **uncorrelated** with error sources for any other satellite, the following weighted matrix may be used:

$$\mathbf{W} = \begin{bmatrix} 1/\sigma_{y_1}^2 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_{y_n}^2 \end{bmatrix}$$

$$w_i = \frac{1}{\sigma_{y_i}^2} \rightarrow \boxed{\sigma_{y_i}^2 \uparrow \;\Rightarrow\; w_i \downarrow}$$

greater error → less weight



Best Linear Unbiased **Minimum Variance Estimator** (**BLUE**):

Let be "$\mathbf{P_y}$" the error covariance matrix for measurements $\mathbf{y}$.

If the weighting matrix is taken as $\boxed{\mathbf{W} = \mathbf{P_y}^{-1}}$, thence the Minimum Variance Solution is found:

$$\boxed{\hat{x} = \left(\mathbf{G}^t \mathbf{P_y}^{-1}\,\mathbf{G}\right)^{-1} \mathbf{G}^t\;\mathbf{P_y}^{-1}\;\mathbf{y}}$$

And the error covariance matrix for the estimation X is:

$$\boxed{\mathbf{P_{\hat{x}}} = \left(\mathbf{G}^t \mathbf{P_y}^{-1}\,\mathbf{G}\right)^{-1}}$$

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7. Navigation equations system and LS solution (XYZ)

Repeat the previous exercise, but writing the system and computing the solution in (XYZ) coordinates. Also, compute GDOP, Precision Dilution Of Precision (PDOP) and TDOP.

Complete the following steps:

(a) The matrix $\mathbf{G}$ is now

$$\mathbf{G}_i = \left[\frac{x_0 - x^i}{\rho_0^i}, \frac{y_0 - y^i}{\rho_0^i}, \frac{z_0 - z^i}{\rho_0^i}, 1\right]$$

where $\mathbf{r}_0 = (x_0, y_0, y_0)$ is the 'a priori' receiver coordinates at reception time, $\mathbf{r}^i = (x^i, y^i, y^i)$ are the satellite coordinates at transmission time, and $\rho_0^i = ||\mathbf{r}^i - \mathbf{r}_0||$.

Hint: Matrix $\mathbf{G}$ and prefit residual vector $\mathbf{y}$ can be generated directly from the `gLAB.out` output file as follows:[73]

```
grep MODEL gLAB.out | grep C1 | gawk 'BEGIN{x=4789032.6277;
     y=176595.0498;z=4195013.2503} {if ($4==300 && $6!=21)
     {r1=x-$11;r2=y-$12;r3=z-$13;r=sqrt(r1*r1+r2*r2+r3*r3);
     print $9-$10,r1/r,r2/r,r3/r,1}}' > M.dat
```

Vector $\mathbf{y}$ corresponds to the first column of file `M.dat` and matrix $\mathbf{G}$ to the last four columns.

See exercises 6 and 7, Session 5.2 in [RD-2]

The matrix $\mathbf{G}$ and vector $\mathbf{y}$ values computed by gLAB can be found by:

```
grep PREFIT gLAB.out | grep -v INFO |
 gawk '{if ($4==300 && $6!=21) print $8,$11,$12,$13,$14}'
```

(b) Compute the LS solution of the navigation system. Using Octave or MATLAB, upload the contents of file `M.dat` and execute the following instructions, as well:

```
y=M(:,1)
G=M(:,2:5)
x=inv(G'*G)*G'*y
```

The values computed by gLAB can be found by:

(X,Y,Z) coordinates:

```
grep OUTPUT gLAB.out | grep -v INFO |
               gawk '{if ($4==300) print $9,$10,$11}'
```

Receiver clock:

```
grep FILTER gLAB.out | grep -v INFO |
                        gawk '{if ($4==300) print $8}'
```

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Kalman filtering:

It is based on computing the **weighted average** between:

- the measurement $\boxed{\mathbf{y}(n)}$ (i.e., at $t = t_n$)
- the prediction of the state $\boxed{\hat{\mathbf{x}}^{-}(n)}$ from previous estimation $\boxed{\hat{\mathbf{x}}(n-1)}$

1. Weighted average:

$$\begin{cases} \mathbf{y}(n) = \mathbf{G}(n)\mathbf{x}(n) \\ \boxed{\hat{\mathbf{x}}^{-}(n) = \mathbf{x}(n)} \end{cases}$$

Let's assume, that we have the prediction $\boxed{\hat{\mathbf{x}}^{-}(n)}$, with $\boxed{\mathbf{P}_{\hat{\mathbf{x}}^{-}(n)}}$ ce, it can be used to **add an additional set of equations** to the measurement equation **y= G x**

$$\begin{bmatrix} \mathbf{y}(n) \\ \hat{\mathbf{x}}^{-}(n) \end{bmatrix} = \begin{pmatrix} \mathbf{G}(n) \\ \mathbf{I} \end{pmatrix} \mathbf{x}(n) \qquad \mathbf{W} = \begin{pmatrix} \mathbf{P}_{\mathbf{y}(n)} & 0 \\ 0 & \boxed{\mathbf{P}_{\hat{\mathbf{x}}^{-}(n)}} \end{pmatrix}^{-1}$$

$$\begin{bmatrix} \mathbf{y}(n) \\ \hat{\mathbf{x}}^-(n) \end{bmatrix} = \left(\boxed{\begin{matrix} \mathbf{G}(n) \\ \mathbf{I} \end{matrix}} \right) \mathbf{x}(n)$$

$$\boxed{\mathbf{W}} = \begin{pmatrix} \mathbf{P}_{\mathbf{y}(n)} & 0 \\ 0 & \mathbf{P}_{\hat{\mathbf{x}}^-(n)} \end{pmatrix}^{-1}$$

And the following solution of the previous equation system can be found with some elemental algebraic manipulations:

$$\hat{\mathbf{x}} = \left(\mathbf{G}^t \mathbf{P}_{\mathbf{y}}^{-1} \boxed{\mathbf{G}}\right)^{-1} \mathbf{G}^t \boxed{\mathbf{P}_{\mathbf{y}}^{-1}} \mathbf{y}$$

$$\mathbf{P}_{\hat{\mathbf{x}}} = \left(\mathbf{G}^t \mathbf{P}_{\mathbf{y}}^{-1} \mathbf{G}\right)^{-1}$$

$$\hat{\mathbf{x}}(n) = \mathbf{P}_{\hat{\mathbf{x}}(n)} \left[\mathbf{G}^t(n) \mathbf{P}_{\mathbf{y}(n)}^{-1} \mathbf{y}(n) + \mathbf{P}_{\hat{\mathbf{x}}^-(n)}^{-1} \hat{\mathbf{x}}^-(n) \right]$$

$$\mathbf{P}_{\hat{\mathbf{x}}(n)} = \left[\mathbf{G}^t(n) \mathbf{P}_{\mathbf{y}(n)}^{-1} \mathbf{G}(n) + \mathbf{P}_{\hat{\mathbf{x}}^-(n)}^{-1} \right]^{-1}$$

2.- Prediction

Scalar case:

Let's $\hat{x}_{n-1}$ be the state at epoch $n\text{-}1$ with variance $\sigma^2_{\hat{x}_{n-1}}$

The *simplest prediction model* is to assume that the prediction at epoch n is proportional to the state at epoch $n\text{-}1$. That is:

$$\boxed{\hat{x}_n^- = \phi\, \hat{x}_{n-1}}$$

Thence, existing a linear relation between $\hat{x}_{n-1}$ and $\hat{x}_n^-$, the variance of the prediction will be: $\boxed{\sigma^2_{\hat{x}_n^-} = \phi^2\, \sigma^2_{\hat{x}_{n-1}} + q^2}$

An additional term is added to account for modeling error!

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Generalization to the vector case:

$$\hat{x}_n^- = \phi\,\hat{x}_{n-1}$$
$$\sigma^2_{\hat{x}_n^-} = \phi^2\,\sigma^2_{\hat{x}_{n-1}} + q^2$$

x_n	$\rightarrow$	$\mathbf{x}(n)$
ϕ	$\rightarrow$	$\mathbf{\Phi}(n)$
$\sigma^2_{x_n}$	$\rightarrow$	$\mathbf{P}_{\mathbf{x}(n)}$
q^2	$\rightarrow$	$\mathbf{Q}(n)$

$\mathbf{\Phi}(n):$ *transition matrix*

$\mathbf{Q}(n):$ *process noise matrix*

$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi}(n-1)\cdot\hat{\mathbf{x}}(n-1)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \mathbf{\Phi}(n-1)\cdot\mathbf{P}_{\hat{\mathbf{x}}(n-1)}\cdot\mathbf{\Phi}^t(n-1)+\mathbf{Q}(n-1)$$

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Kalman filter (see kalman.f)

Measurements

$$\mathbf{y}(n)$$
$$\mathbf{P}_{\mathbf{y}(n)}$$

$$\hat{\mathbf{x}}^-(n)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)}$$

Prediction

$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi}\,\hat{\mathbf{x}}(n-1)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \mathbf{\Phi}\,\mathbf{P}_{\hat{\mathbf{x}}(n-1)}\mathbf{\Phi}^t + \mathbf{Q}$$

Estimation

$$\hat{\mathbf{x}}(n) = \mathbf{P}_{\hat{\mathbf{x}}(n)}\cdot\left[\mathbf{G}^t\,\mathbf{P}^{-1}_{\mathbf{y}(n)}\,\mathbf{y}(n)+\mathbf{P}^{-1}_{\hat{\mathbf{x}}^-(n)}\hat{\mathbf{x}}^-(n)\right]$$
$$\mathbf{P}_{\hat{\mathbf{x}}(n)} = \left[\mathbf{G}^t\,\mathbf{P}^{-1}_{\mathbf{y}(n)}\,\mathbf{G}+\mathbf{P}^{-1}_{\hat{\mathbf{x}}^-(n)}\right]^{-1}$$

$$\hat{\mathbf{x}}(0)$$
$$\mathbf{P}_{\hat{\mathbf{x}}(0)}$$

Initialization

$$\hat{\mathbf{x}}(n)$$
$$\mathbf{P}_{\hat{\mathbf{x}}(n)}$$

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Kalman filter (classical version)

Measurements

$$\mathbf{y}(n)$$
$$\mathbf{P}_{\mathbf{y}(n)}$$

$$\hat{\mathbf{x}}^-(n)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)}$$

Prediction

$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi}\,\hat{\mathbf{x}}(n-1)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \mathbf{\Phi}\,\mathbf{P}_{\hat{\mathbf{x}}(n-1)}\mathbf{\Phi}^t + \mathbf{Q}$$

Estimation

$$\hat{\mathbf{x}}(n) = \hat{\mathbf{x}}^-(n) + \mathbf{K}(n)\cdot\left[\mathbf{y}(n) - \mathbf{G}(n)\cdot\hat{\mathbf{x}}^-(n)\right]^{-1}$$
$$\mathbf{K}(n) = \mathbf{P}_{\hat{\mathbf{x}}^-(n)}\cdot\mathbf{G}^t(n)\cdot\left[\mathbf{G}(n)\cdot\mathbf{P}_{\hat{\mathbf{x}}^-(n)}\cdot\mathbf{G}^t(n) + \mathbf{P}_{\mathbf{y}(n)}\right]^{-1}$$
$$\mathbf{P}_{\hat{\mathbf{x}}(n)} = \left[\mathbf{I} - \mathbf{K}(n)\cdot\mathbf{G}(n)\right]\cdot\mathbf{P}_{\hat{\mathbf{x}}^-(n)}$$

$$\hat{\mathbf{x}}(0)$$
$$\mathbf{P}_{\hat{\mathbf{x}}(0)}$$

Initialization

$$\hat{\mathbf{x}}(n)$$
$$\mathbf{P}_{\hat{\mathbf{x}}(n)}$$

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Some simple examples to define matrices Φ and Q

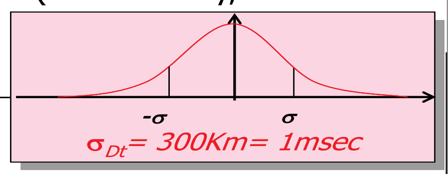
$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi}(n-1)\cdot\hat{\mathbf{x}}(n-1)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \Phi(n-1)\cdot P_{\hat{\mathbf{x}}(n-1)}\cdot\mathbf{\Phi}^t(n-1)+\mathbf{Q}(n-1)$$

a) Static positioning:

State vector to be determined is $X = (x_{rec}, y_{rec}, z_{rec}, dt_{rec})$ where coordinates $(x_{rec}, y_{rec}, z_{rec})$ are considered **constant** (because receiver is fixed) and clock offset dT_{rec} is treated as **white noise** with zero mean and variance σ^2_{Dt}. In these conditions, matrices have the form:

$$\mathbf{\Phi}(n) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \qquad \mathbf{Q}(n) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & \sigma^2_{DT} \end{pmatrix}$$

Being σ^2_{DT} process noise associated to clock offset (in some way, the uncertainty in clock value).



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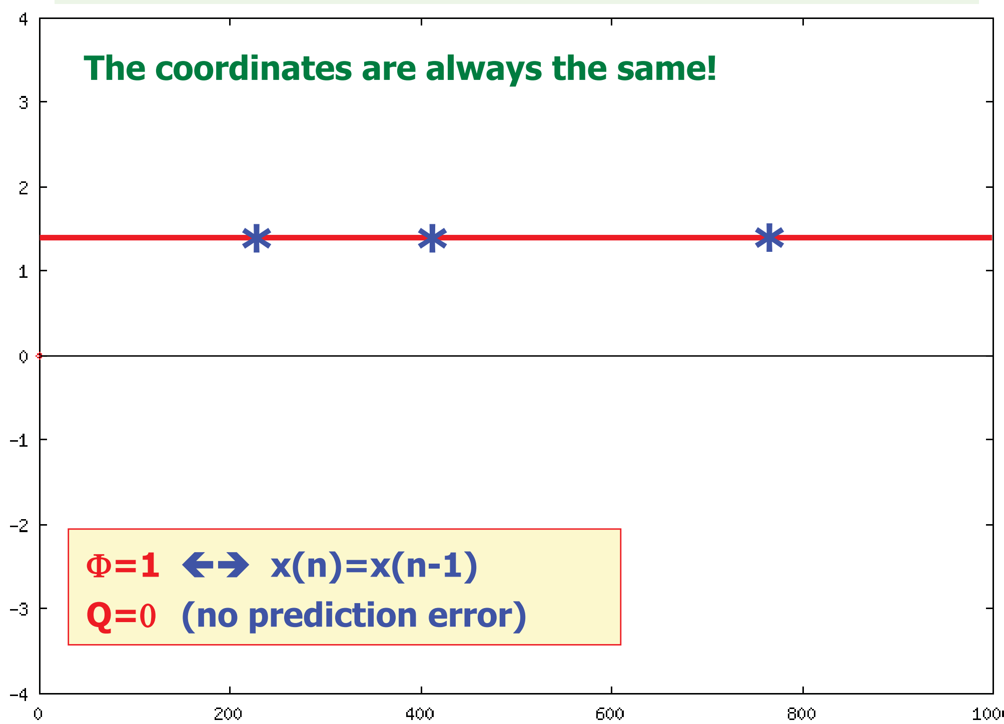
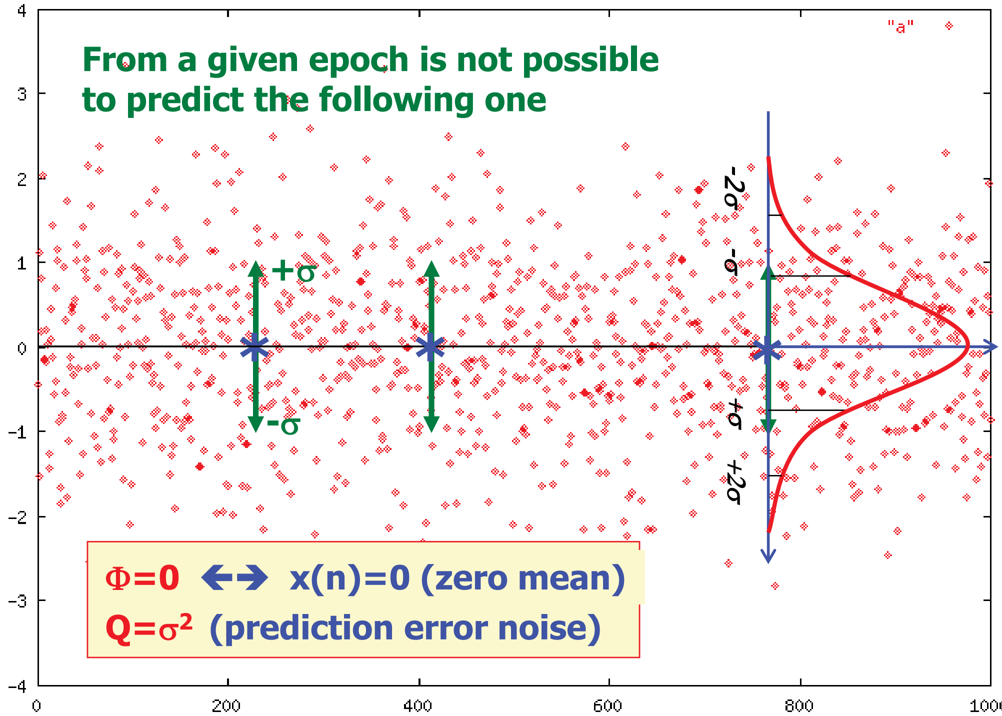
Constant



We can assure that, the next $\mathbf{x}(n)$ will be **the same as** $\mathbf{x}(n-1)$.

$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi}(\mathbf{n-1})\cdot\hat{\mathbf{x}}(n-1)$$
$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \mathbf{\Phi}(\mathbf{n-1})\cdot\mathbf{P}_{\hat{\mathbf{x}}(n-1)}\cdot\mathbf{\Phi}^t(\mathbf{n-1})+\mathbf{Q}(\mathbf{n-1})$$

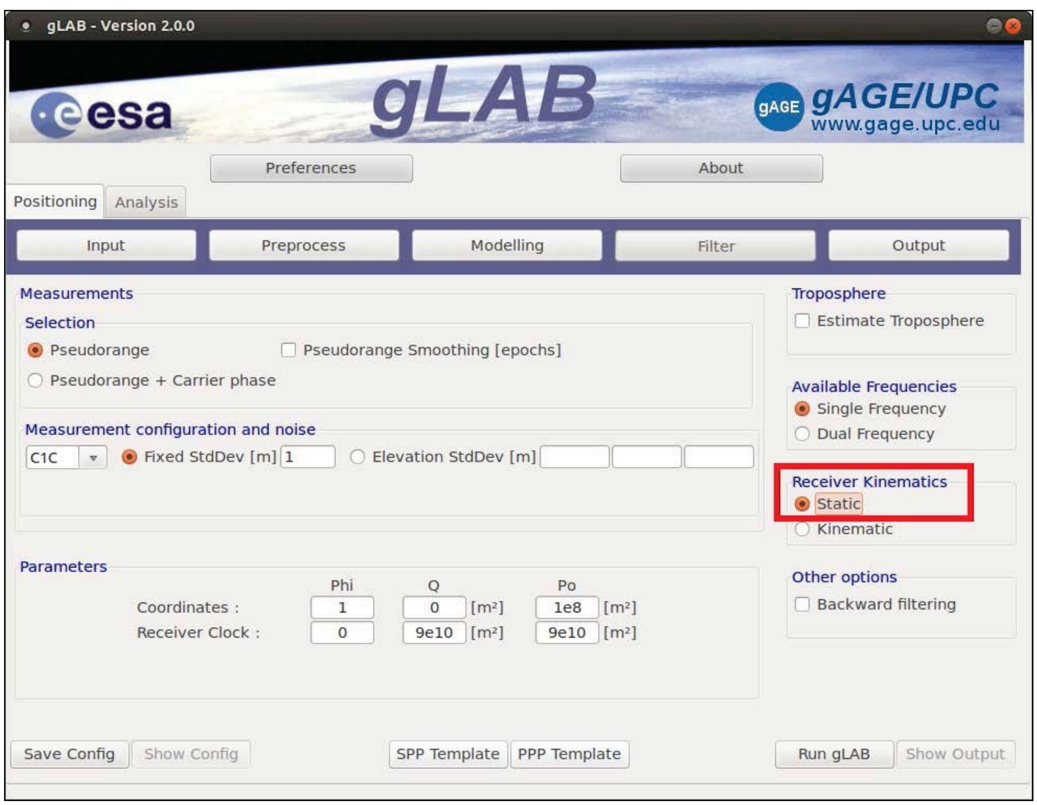
White Noise process N(0, σ)



We only can assume that, the next **x(n)** can be **x(n)=0 with a confidence σ.**

$$\hat{\mathbf{x}}^{-}(n) = \mathbf{\Phi(n-1)} \cdot \hat{\mathbf{x}}(n-1)$$

$$\mathbf{P}_{\hat{\mathbf{x}}^{-}(n)} = \mathbf{\Phi(n-1)} \cdot \mathbf{P}_{\hat{\mathbf{x}}(n-1)} \cdot \mathbf{\Phi^{t}(n-1)} + \mathbf{Q(n-1)}$$



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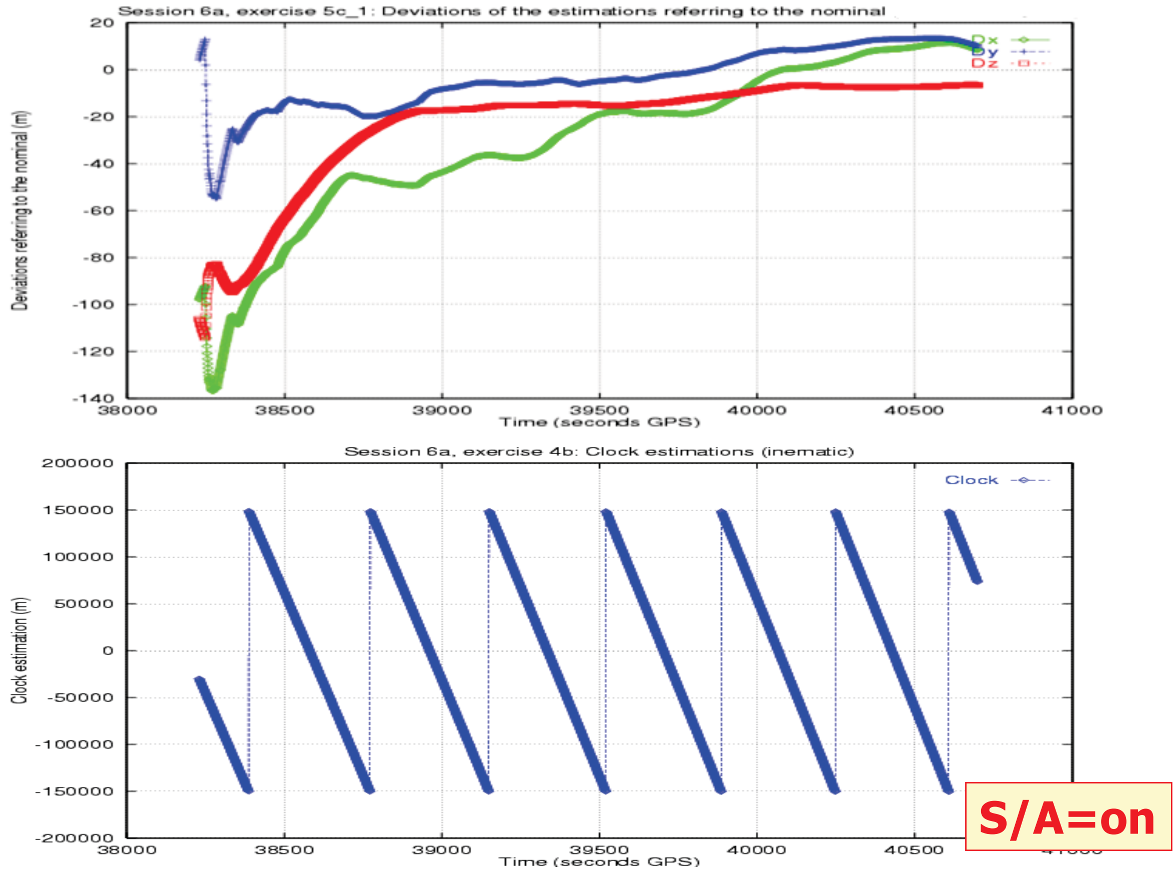
Static positioning: constant coordinates and white noise clock



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b) Kinematic positioning

1) In case of a fast moving vehicle, coordinates will be modeled as **white noise** with zero mean, and the same rationale applies for clock offset:

$$\mathbf{\Phi}(n) = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \qquad \mathbf{Q}(n) = \begin{pmatrix} \sigma_{dx}^2 & & & \\ & \sigma_{dy}^2 & & \\ & & \sigma_{dz}^2 & \\ & & & \sigma_{DT}^2 \end{pmatrix}$$

2) In case of a slow moving vehicle, coordinates may be modeled as **random walk**, process' spectral density $\dot{q} = \frac{d\sigma^2}{dt}$, and the clock as a **white noise**:

$$\mathbf{\Phi}(n) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \qquad \mathbf{Q}(n) = \begin{pmatrix} \dot{q}_{dx}\Delta t & & & \\ & \dot{q}_{dy}\Delta t & & \\ & & \dot{q}_{dz}\Delta t & \\ & & & \sigma_{DT}^2 \end{pmatrix}$$

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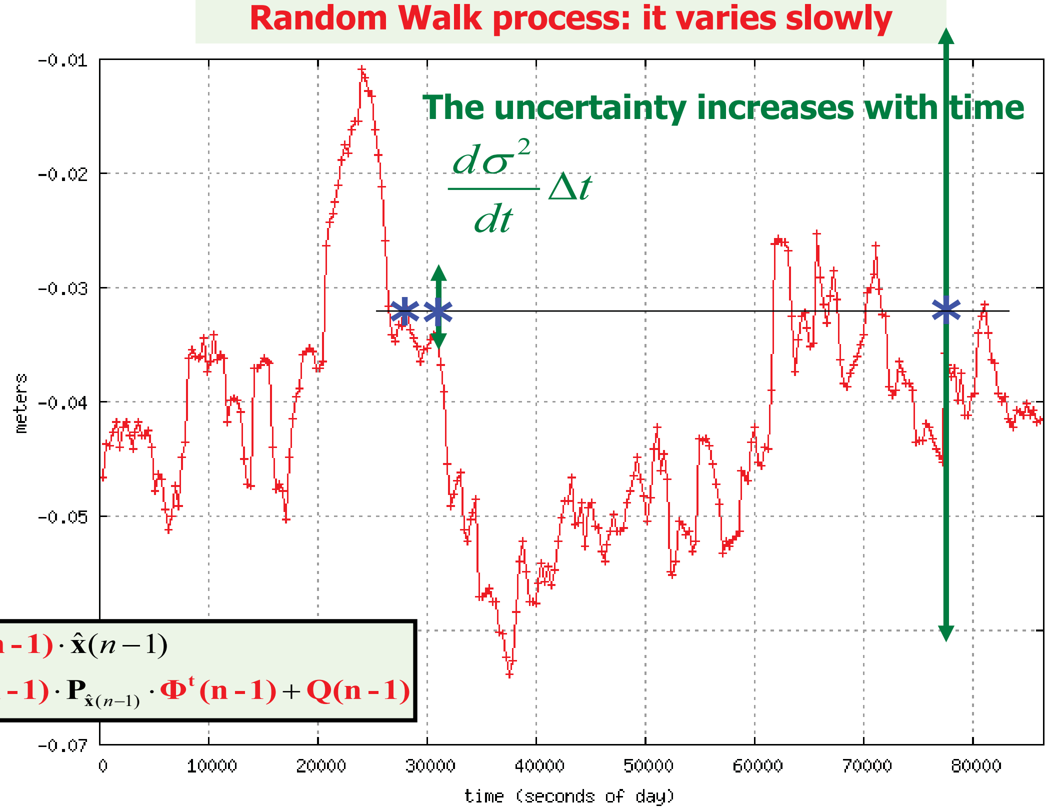
Random Walk process: it varies slowly



$$\hat{\mathbf{x}}^-(n) = \mathbf{\Phi(n-1)} \cdot \hat{\mathbf{x}}(n-1)$$

$$\mathbf{P}_{\hat{\mathbf{x}}^-(n)} = \mathbf{\Phi(n-1)} \cdot \mathbf{P}_{\hat{\mathbf{x}}(n-1)} \cdot \mathbf{\Phi}^t\mathbf{(n-1)} + \mathbf{Q(n-1)}$$

Φ=1 ↔ x(n)=x(n-1) (the same value is assumed)

Q=(dσ²/dt)*Δt (but, with prediction error noise increasing with time)

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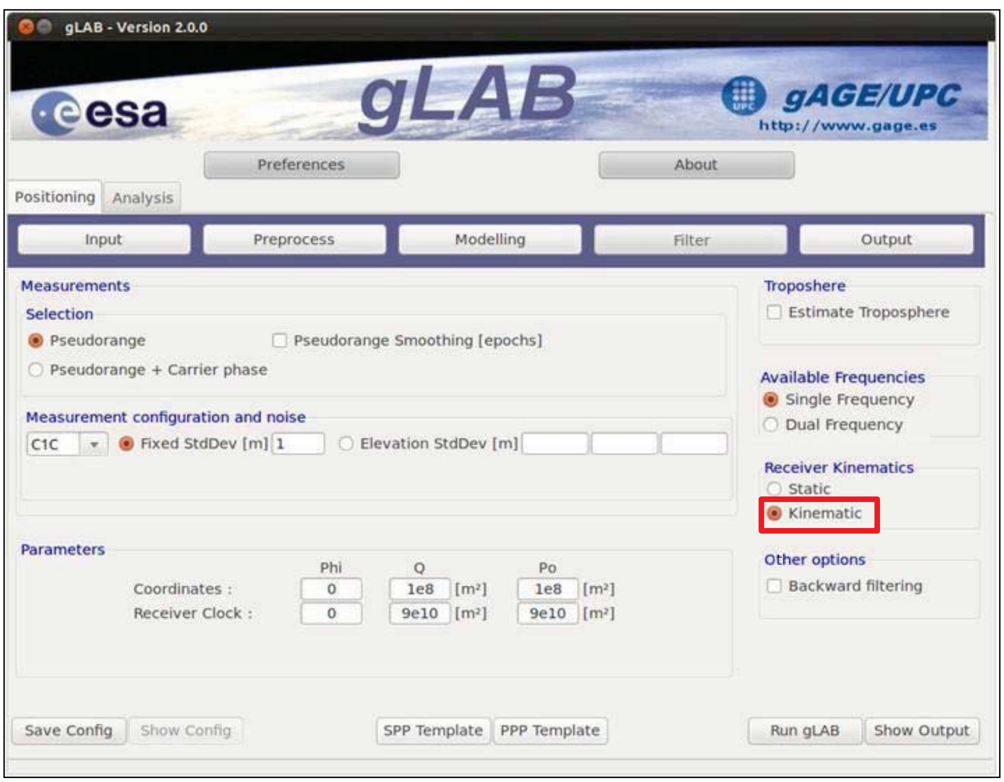

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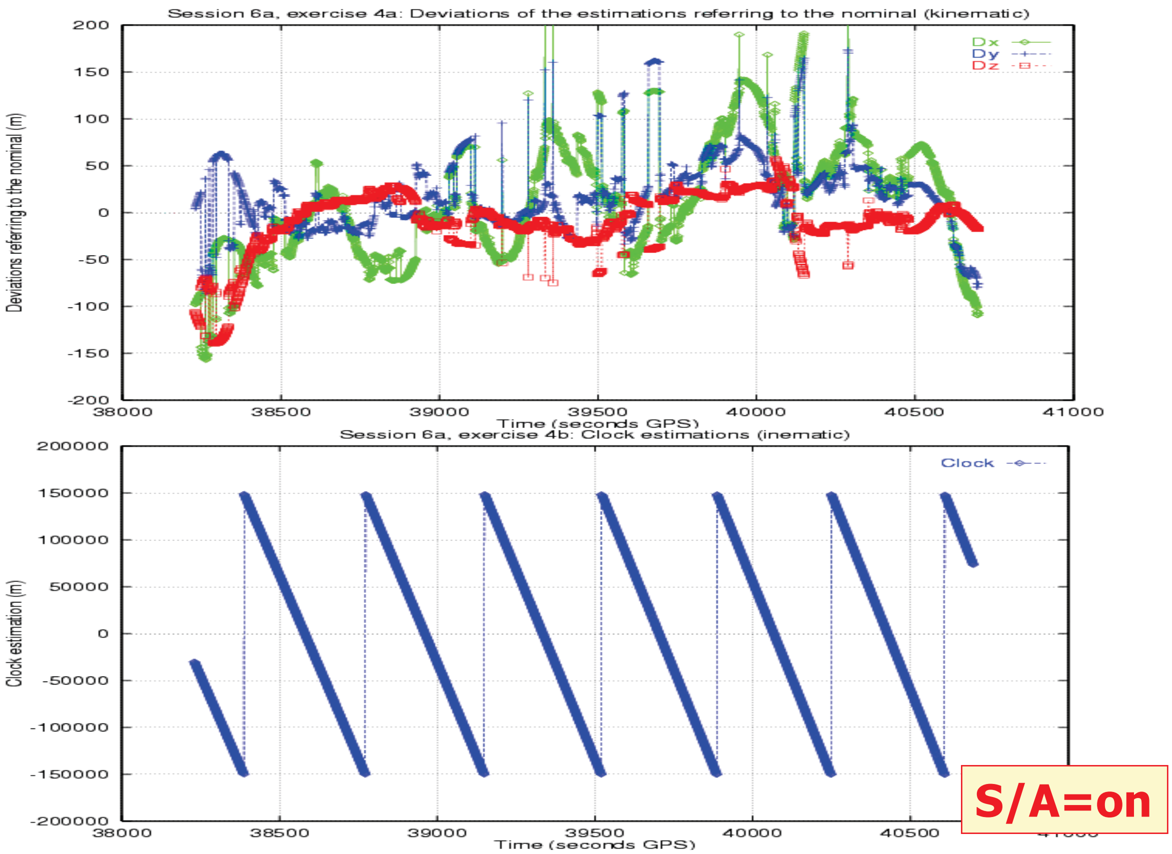
Pure Kinematic positioning: white noise coordinates and clock



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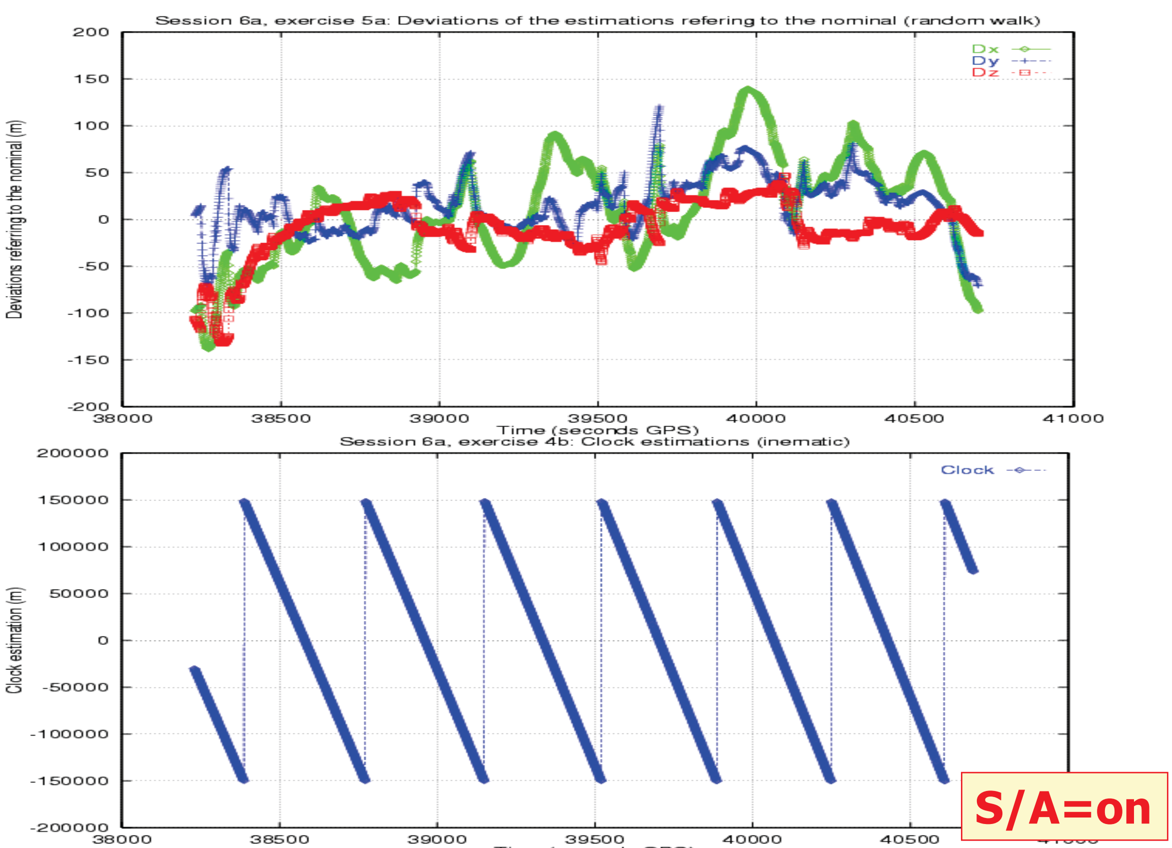
Kinematic positioning: random walk noise coordinates and white noise clock



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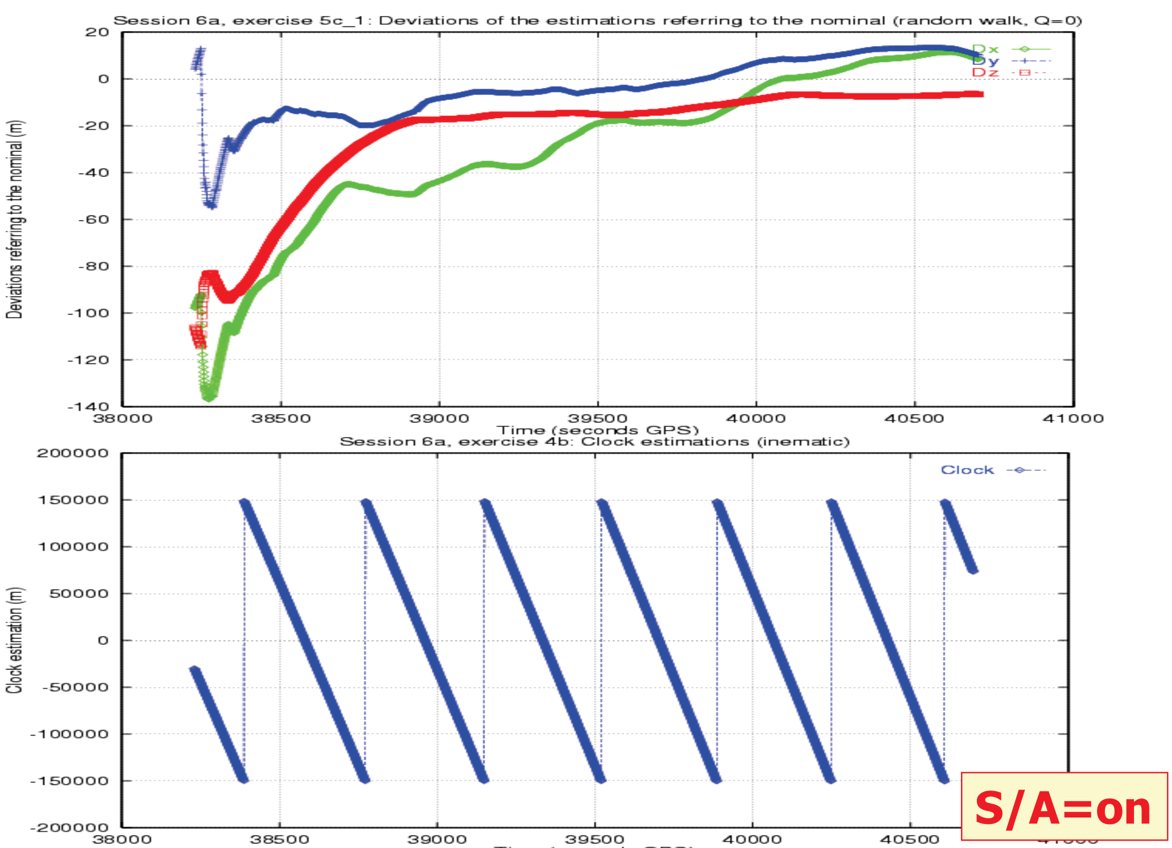
Positioning: Random walk coordinates with Q=0 and white noise clock



8. Solving with the Kalman filter

The measurement file UPC11490.05O has been collected by a receiver with fixed coordinates. Using navigation file UPC11490.05N, compute the SPP solution in static mode[74] and check *by hand* the computation of the solution for the first three epochs (i.e. $t = 300$, $t = 600$ and $t = 900$ seconds).

Complete the following steps:

(a) Set the default configuration of gLAB for the SPP mode. Then, in section [Filter], select [⊙ Static] in the Receiver Kinematics option. To process the data click Run gLAB.

Solving with the kalman filter (by hand):
See exercise 8, Session 5.2 in [RD-2]

(c) Using the previous equations and the configuration parametres applied by gLAB compute by hand the solution for the first three epochs[75] (i.e. $t = 300$, $t = 600$ and $t = 900$ s).

Note: Use the prefit residual vector $\mathbf{y}(k)$ *and design matrix* $\mathbf{G}(k)$ *computed by* gLAB.

Hint:

i. Filter configuration (according to gLAB):

- Initialisation:

  x0 $\equiv \hat{x}(0) = (0, 0, 0, 0)$,

  P0 $\equiv \mathbf{P}(0) = \sigma_0^2\,\mathbf{I}$, with $\sigma_0 = 3 \cdot 10^5$m.

- Process noise $\mathbf{Q}$ and transition matrices $\mathbf{\Phi}$:

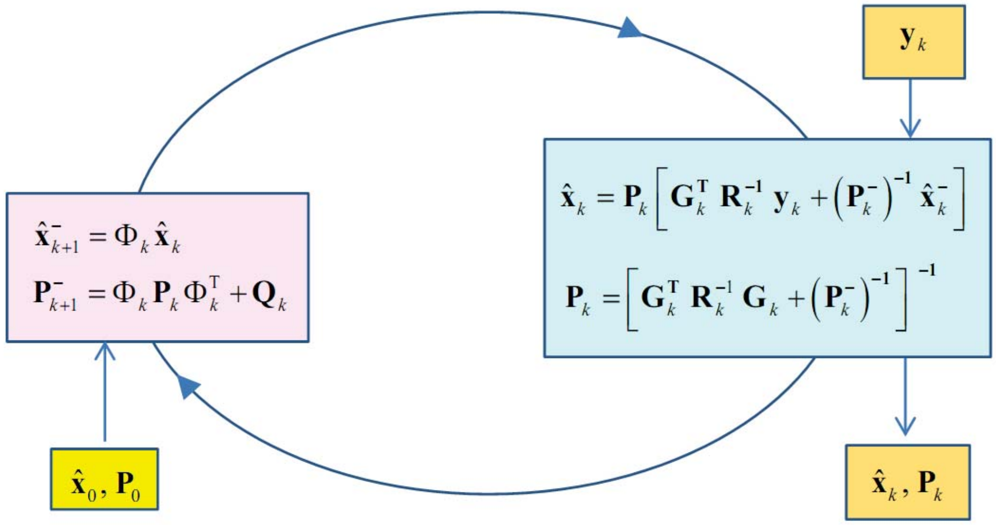
$$\mathtt{Q} \equiv \boldsymbol{Q}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{dt}^2 \end{bmatrix}, \quad \Phi \equiv \mathbf{\Phi}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

  with $\sigma_{dt} = 3 \cdot 10^5$m.

- Measurement covariance matrix:

  Rk $\equiv \mathbf{R}(k) = \sigma_y^2\,\mathbf{I}$, with $\sigma_y = 1\,$m.

(b) Write the Kalman filter equations according to Fig. 6.2, in section 6.1.2 of Volume I.



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ii. Kalman filter iterations:

k=1:

Predict:

$$\mathtt{x1}^- = \Phi \cdot \widehat{\mathtt{x}}\mathtt{0}$$
$$\mathtt{P1}^- = \Phi \cdot \mathtt{P0} \cdot \Phi^T + \mathtt{Q}$$

Update:

$$\mathtt{P1} = [\mathtt{G1}^T \cdot \mathtt{R1}^{-1} \cdot \mathtt{G1} + (\mathtt{P1}^-)^{-1}]^{-1}$$
$$\widehat{\mathtt{x}}\mathtt{1} = \mathtt{P1} \cdot [\mathtt{G1}^T \cdot \mathtt{R1}^{-1} \cdot \mathtt{y1} + (\mathtt{P1}^-)^{-1} \cdot \mathtt{x1}^-]$$

k=2:

Predict:

$$\mathtt{x2}^- = \Phi \cdot \widehat{\mathtt{x}}\mathtt{1}$$
$$\mathtt{P2}^- = \Phi \cdot \mathtt{P1} \cdot \Phi^T + \mathtt{Q}$$

Update:

$$\mathtt{P2} = [\mathtt{G2}^T \cdot \mathtt{R2}^{-1} \cdot \mathtt{G2} + (\mathtt{P2}^-)^{-1}]^{-1}$$
$$\widehat{\mathtt{x}}\mathtt{2} = \mathtt{P2} \cdot [\mathtt{G2}^T \cdot \mathtt{R2}^{-1} \cdot \mathtt{y2} + (\mathtt{P2}^-)^{-1} \cdot \mathtt{x2}^-]$$

k=3:

...

iii. Data vectors and matrices: Vectors $\mathtt{yk} \equiv \mathbf{y}(k)$ and design matrices $\mathtt{Gk} \equiv \mathbf{G}(k)$ are generated from the `gLAB.out` file.

Execute for instance:[76]

```
grep "PREFIT" gLAB.out | grep -v INFO |
            gawk '{if ($6!=21 )print $0}'|
  gawk '{if ($4==300) print $8,$11,$12,$13,$14}'
                                        > M300.dat

grep "PREFIT" gLAB.out | grep -v INFO |
  gawk '{if ($4==600) print $8,$11,$12,$13,$14}'
                                        > M600.dat

grep "PREFIT" gLAB.out | grep -v INFO |
  gawk '{if ($4==900) print $8,$11,$12,$13,$14}'
                                        > M900.dat
```

Then using Octave or MATLAB:

```
y1=M300(:,1)
G1=M300(:,2:5)

y2=M600(:,1)
G2=M600(:,2:5)

y3=M900(:,1)
G3=M900(:,2:5)
```


iv. Results computed by gLAB:

A. (X,Y,Z) coordinates:

```
grep OUTPUT gLAB.out | grep -v INFO |
         gawk '{if ($4==300) print $9,$10,$11}'

grep OUTPUT gLAB.out | grep -v INFO |
         gawk '{if ($4==600) print $9,$10,$11}'

grep OUTPUT gLAB.out | grep -v INFO |
         gawk '{if ($4==900) print $9,$10,$11}'
```

B. Receiver clock

```
grep FILTER gLAB.out | grep -v INFO |
                  gawk '{if ($4==300) print $8}'

grep FILTER gLAB.out | grep -v INFO |
                  gawk '{if ($4==600) print $8}'

grep FILTER gLAB.out | grep -v INFO |
                  gawk '{if ($4==900) print $8}'
```

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References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

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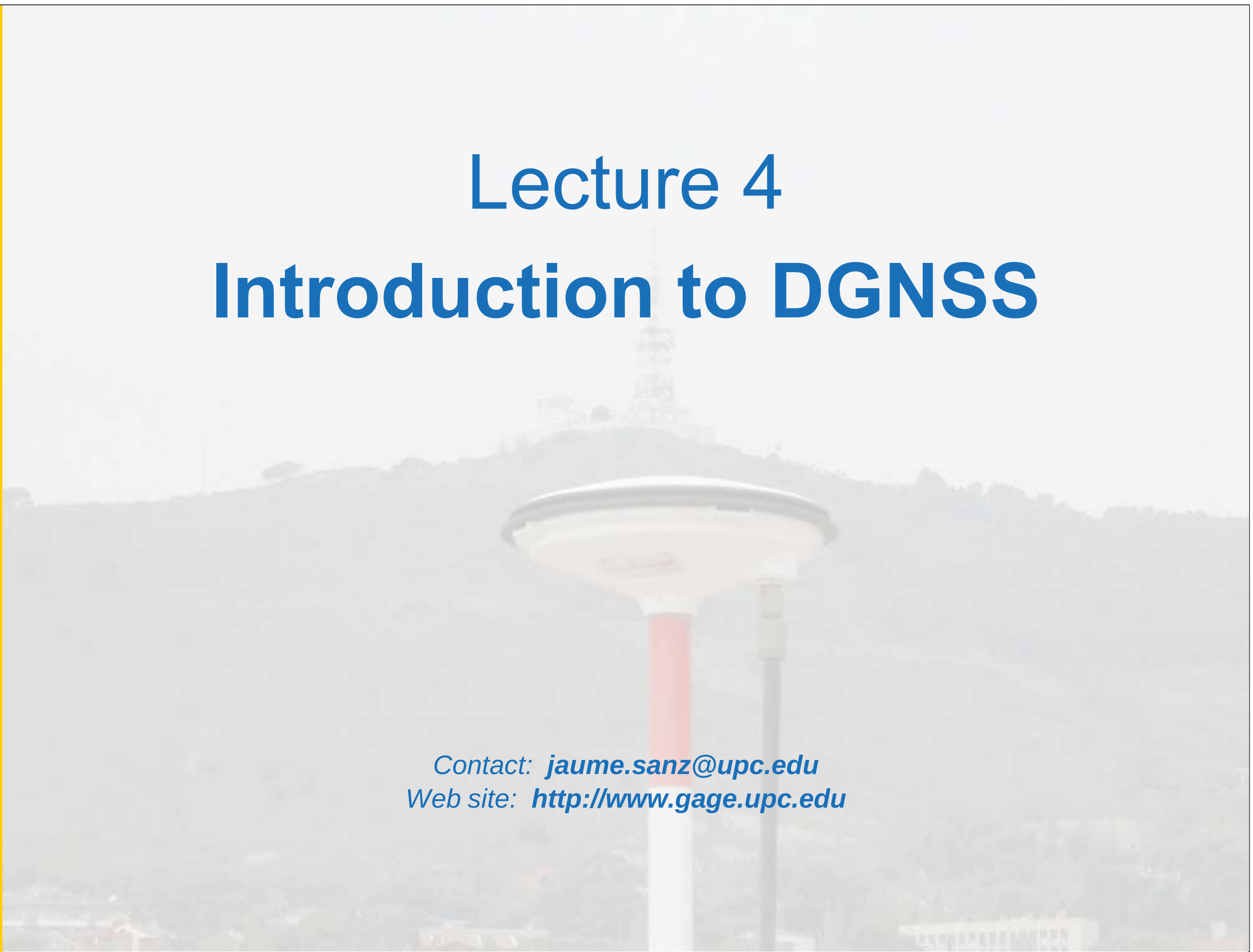
Thank you



Lecture 4
Introduction to DGNSS

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24 April 2014

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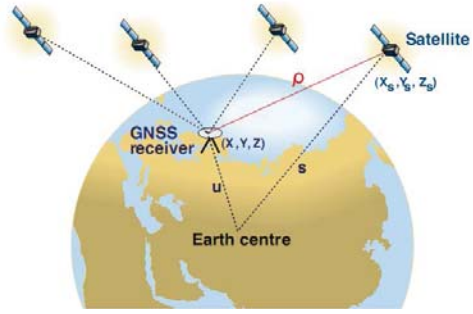
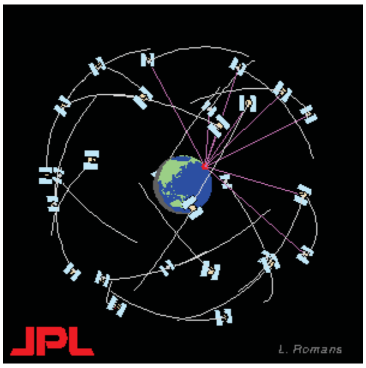
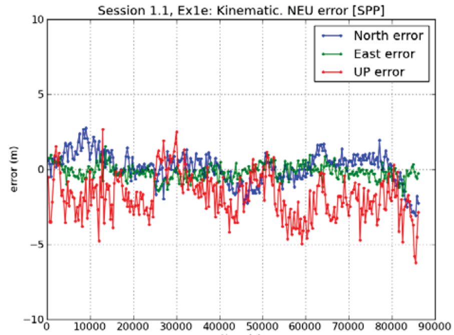
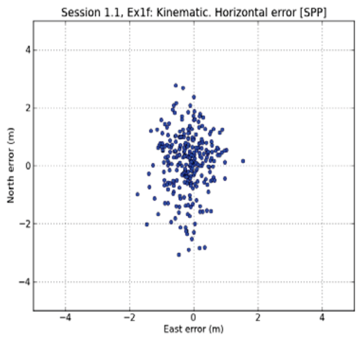
GNSS Positioning

Standalone Positioning: GNSS receiver autonomous positioning using broadcast orbits and clocks (SPS, PPS).









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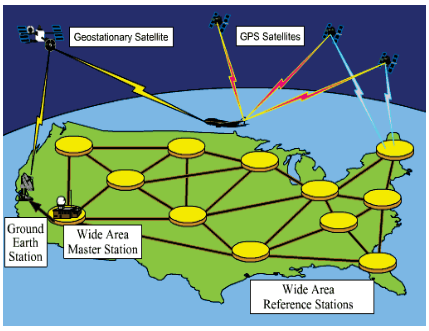
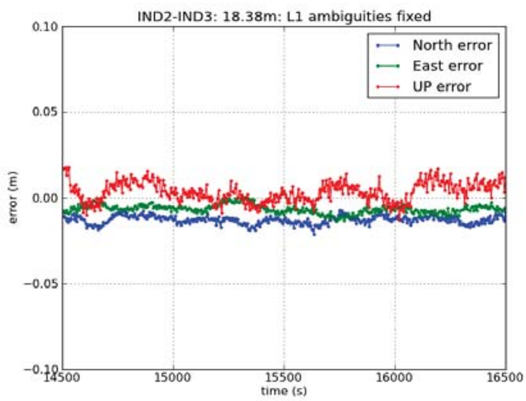
GNSS Positioning

Differential Positioning: GNSS augmented with data (differential corrections or measurements) from a single reference station or a reference station network.





Errors are similar for users separated tens, even hundred of kilometres, and these errors are removed/mitigated in differential mode, improving positioning.

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ERRORS on the Signal

- Space Segment Errors:
  - Clock errors — Common
  - Ephemeris errors
- Propagation Errors
  - Ionospheric delay
  - Tropospheric delay — Weak spatial correlation
- Local Errors
  - Multipath
  - Receiver noise

Ephemeris errors, Ionospheric delay — Strong spatial correlation

Multipath, Receiver noise — No spatial correlation

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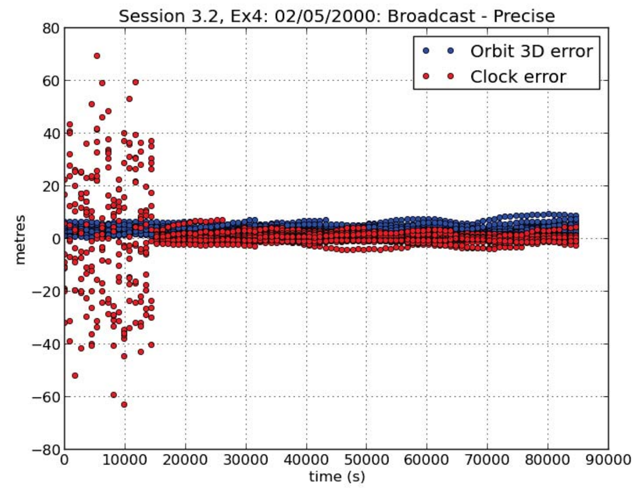

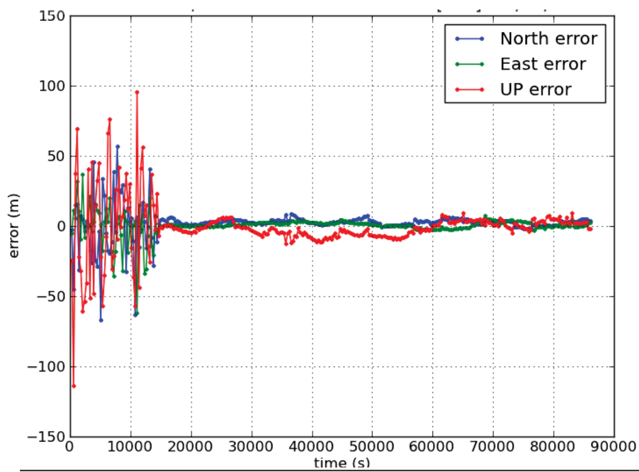

Selective Availability (S/A)

was an intentional degradation of public GPS signals implemented for US national security reasons.

S/A was turned off at May 2nd 2000 (Day-Of-Year 123).

It was permanently removed in 2008, and not included in the next generations of GPS satellites.

In the 1990s, the S/A motivated the development of DGPS.

-These systems typically computed PseudoRange Corrections (PRC) and Range-Rate Corrections (RRC) every 5-10 seconds.

- With S/A=off the life of the corrections was increased to more than one minute.

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1. Introduction: GNSS positioning and measurement errors.
2. Differential positioning concept and differential corrections.
3. Error mitigation in differential positioning.
4. DGNSS implementations: RTK, LADGNSS, WADGNSS.
5. DGNSS commercial services.

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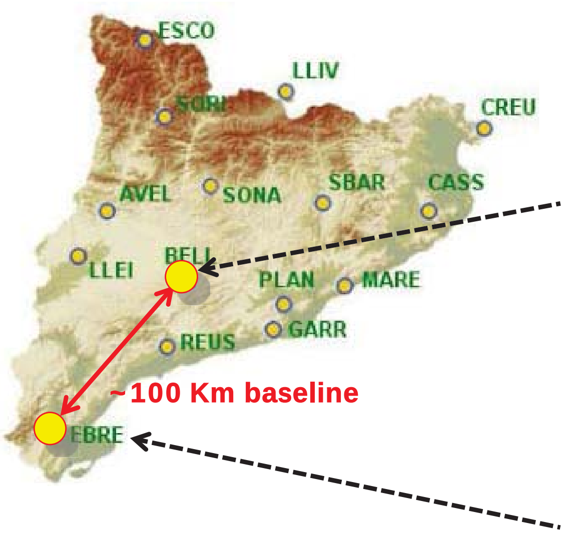
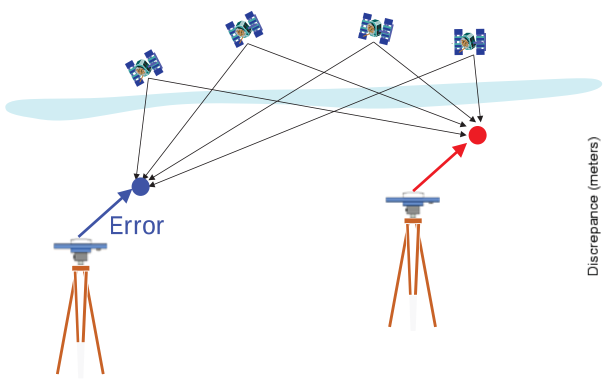
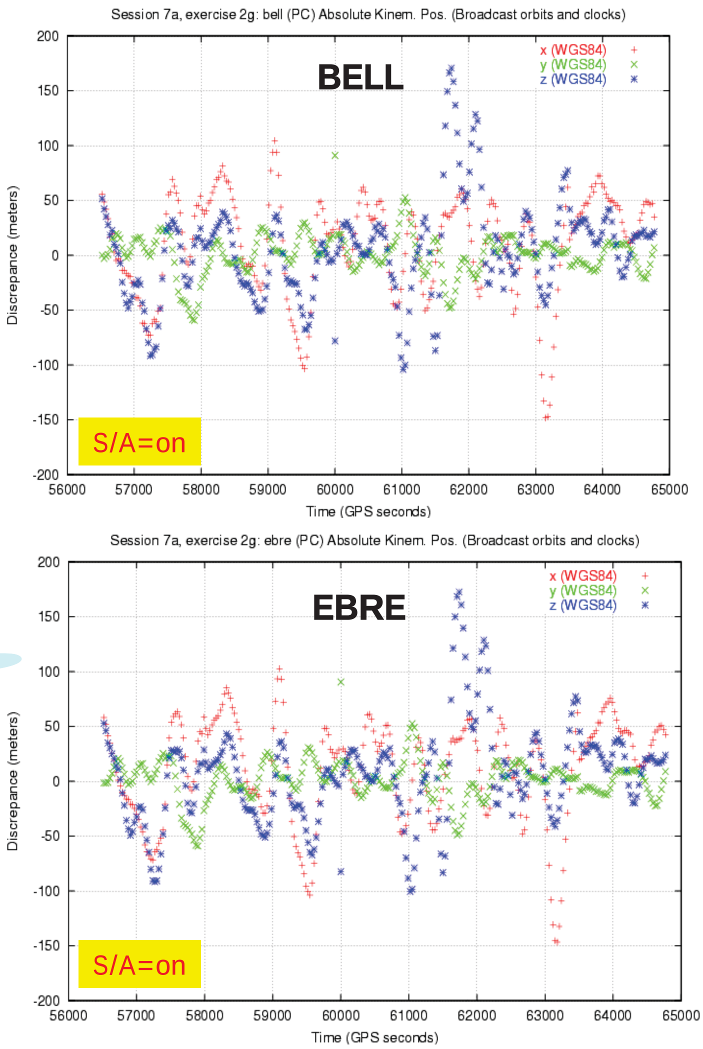
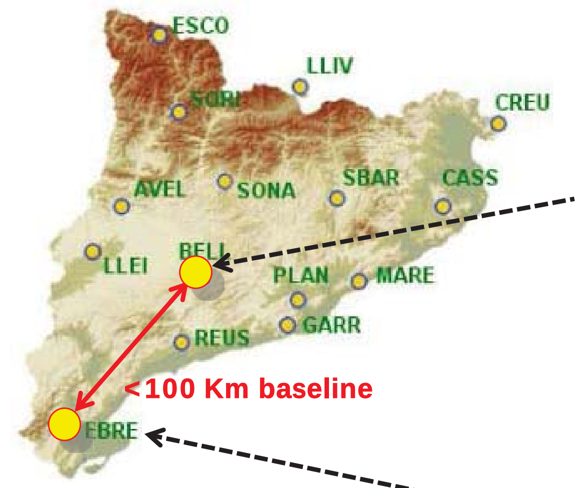
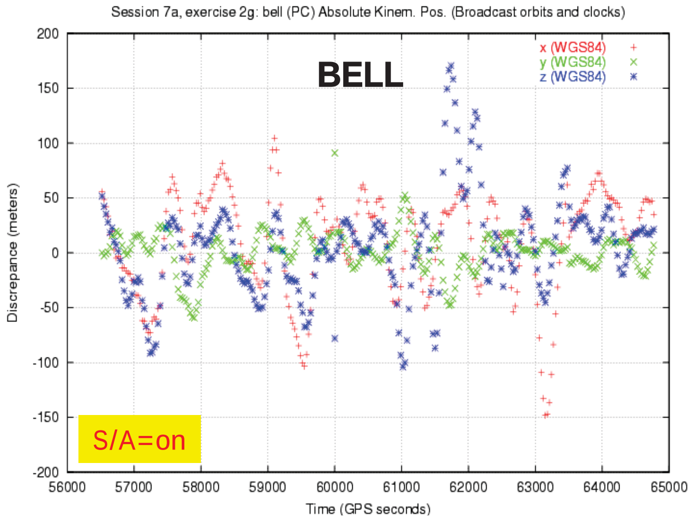
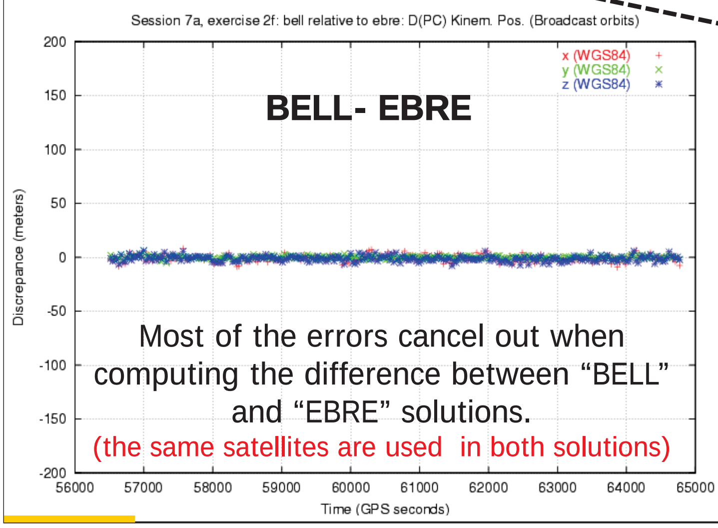


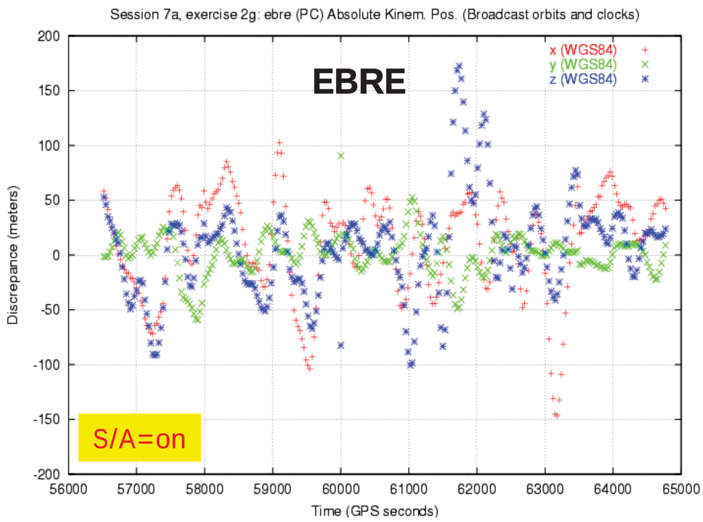


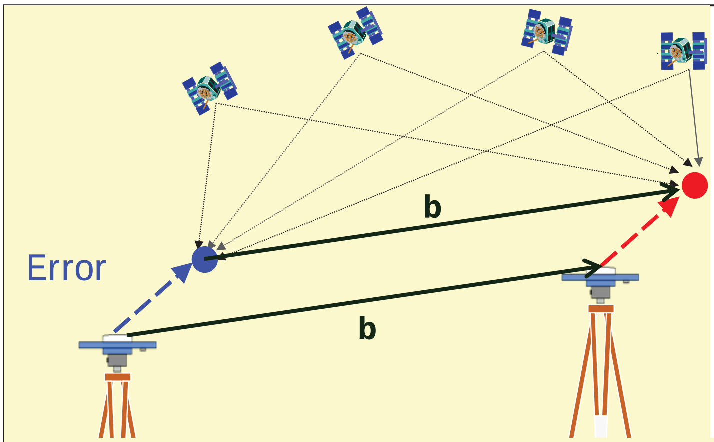




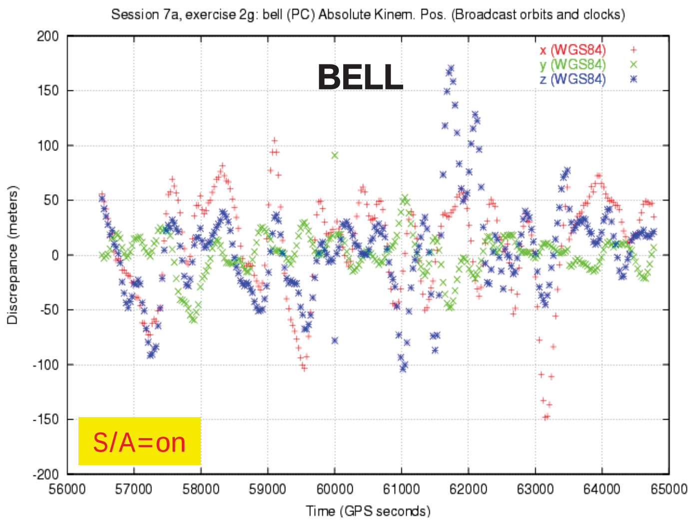


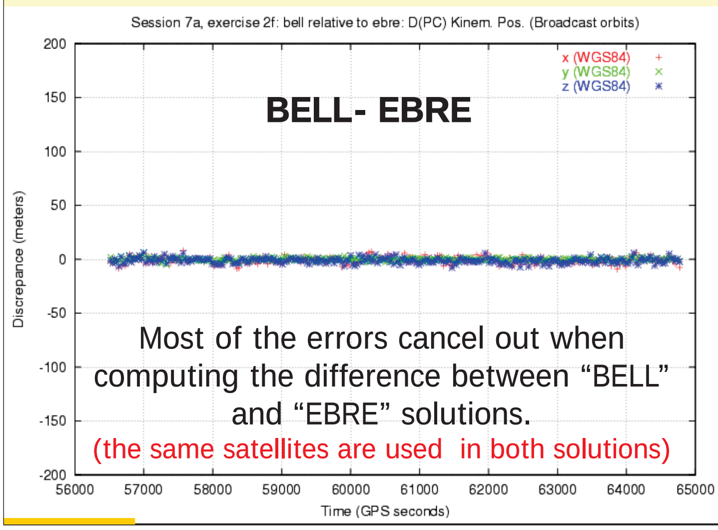


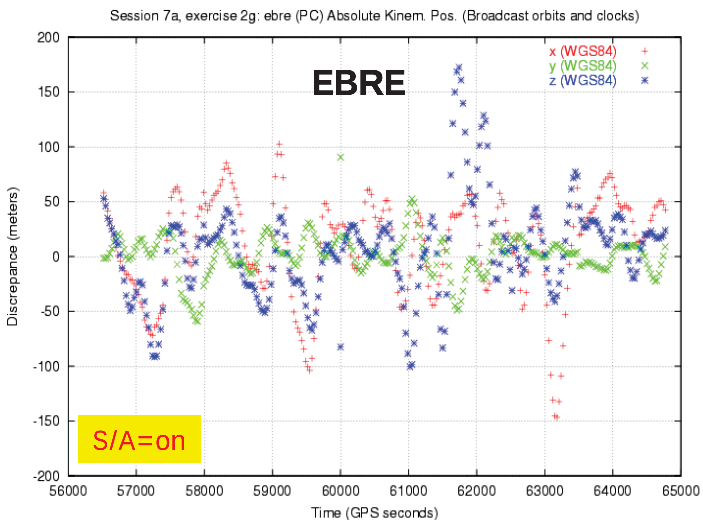




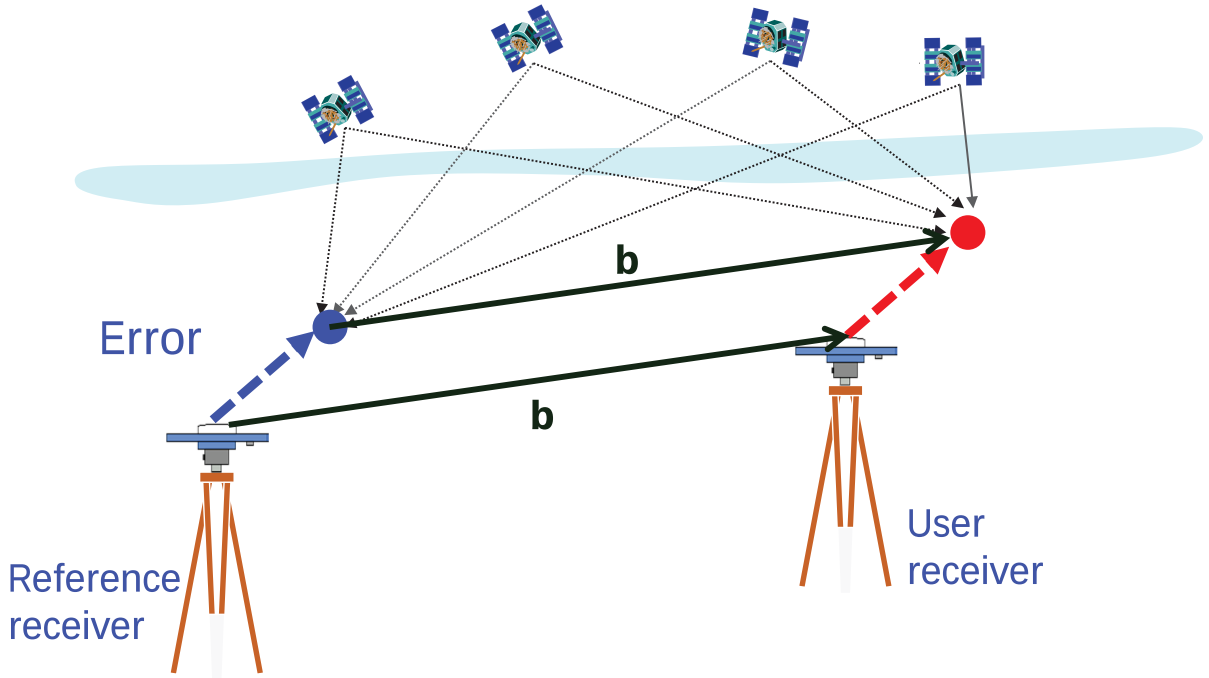
The determination of the vector between the receivers APCs (i.e. the baseline “*b*”) is more accurate than the single receiver solution, because common errors cancel







Differential GNSS (DGNSS): Relative positioning



The determination of the vector between the receivers APCs (i.e. the baseline "b") is more accurate than the single receiver solution, because common errors cancel.

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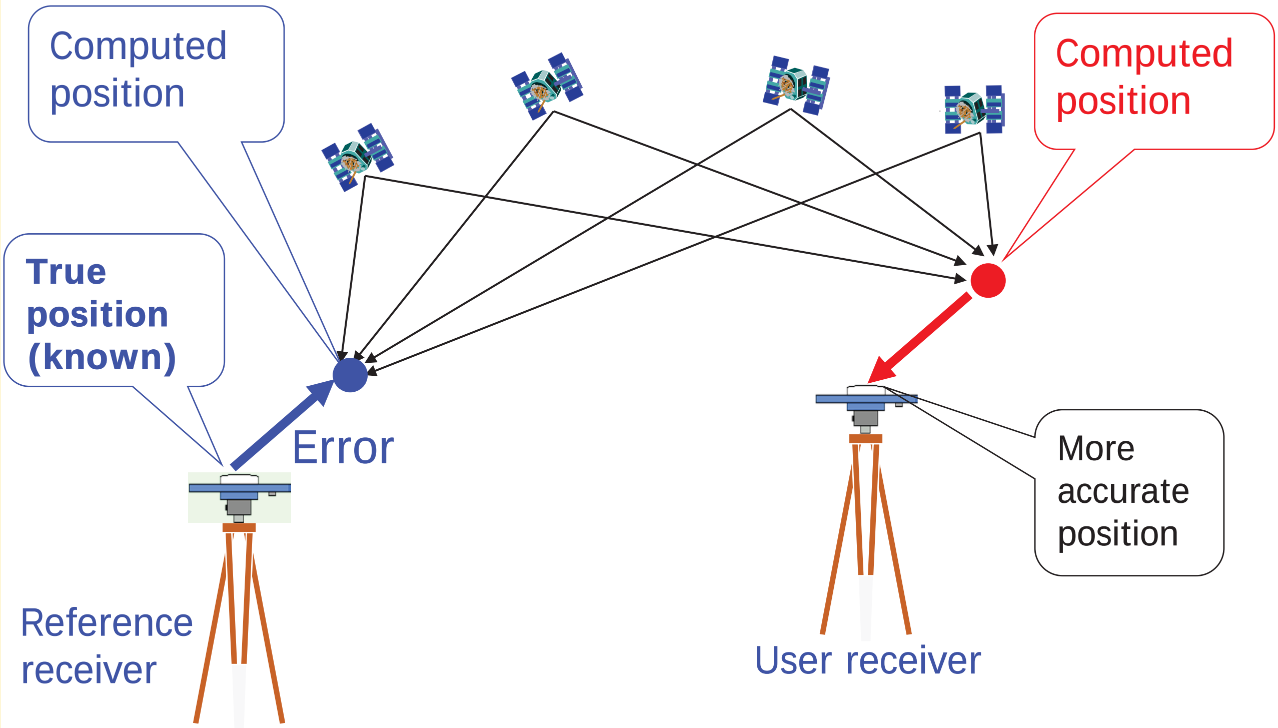
Differential GNSS (DGNSS): absolute position



If the coordinates of the reference receiver are known, thence the reference receiver can estimate its positioning error, which can be transmitted to the user. Then, the user can apply these corrections to improve the positioning

Note: Actually the corrections are computed in range domain (i.e. for each satellite) instead of in the position domain.

In the previous example, the differential error has been cancelled in the “position” domain *(i.e. solution domain approach*).

But:

It requires to use the same satellites in both stations.

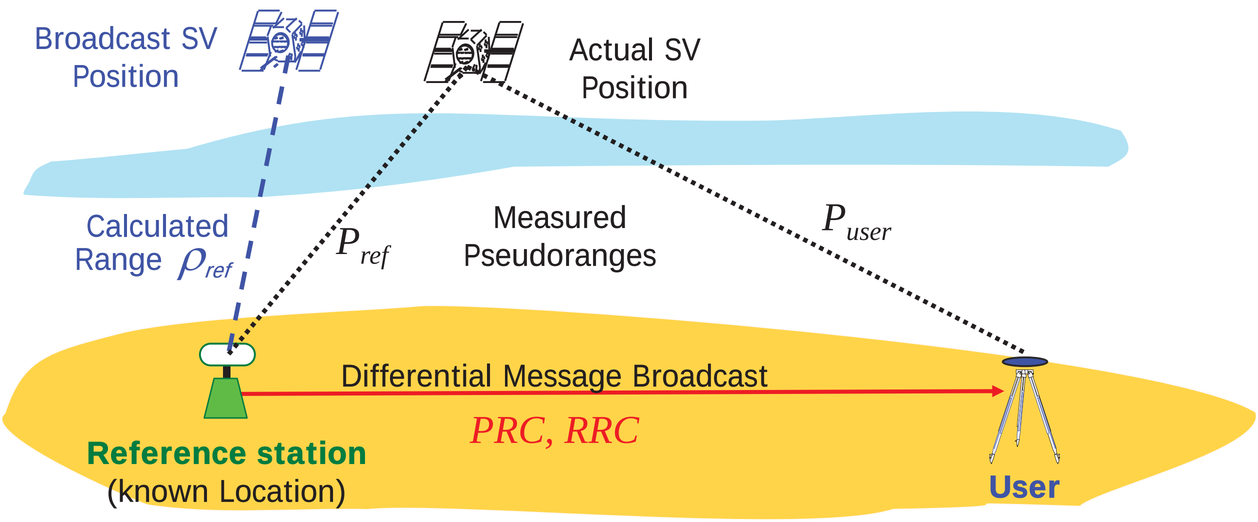
Thence, is much better to solve the problem in the “range” domain than in the “position” domain. That is, to provide corrections for each satellite in view *(i.e. measurement domain approach*)

Two implementations can be considered:

1.- The reference station, with known coordinates , computes range corrections for each satellite in view. These corrections are broadcasted to the user. The user applies these corrections to compute its “absolute position”.

2.- The reference receiver (not necessarily at rest) broadcast its time-tagged measurements to the user. The user applies these measurements to compute its “relative position” to the reference station. Note: if the reference station coordinates are known, the user can estimate its absolute position, as well.

1.- Range Differential Correction Calculation



– The **reference station** with known coordinates, computes pseudorange and range-rate corrections: $PRC= \rho_{ref,} - P_{ref}$, $RRC= \Delta PRC/\Delta t$.

– The **user** receiver applies the PRC and RRC to correct its own measurements, $P_{user} + (PRC + RRC\,(t-t_0))$, removing SIS errors and improving the positioning accuracy.

DGNSS with code ranges: users within a hundred of kilometres can obtain **one-meter-level** positioning accuracy using such pseudorange corrections.

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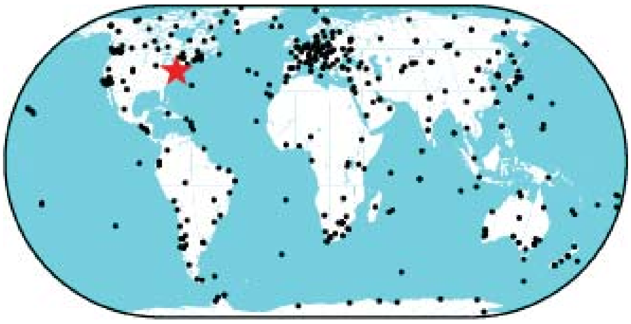

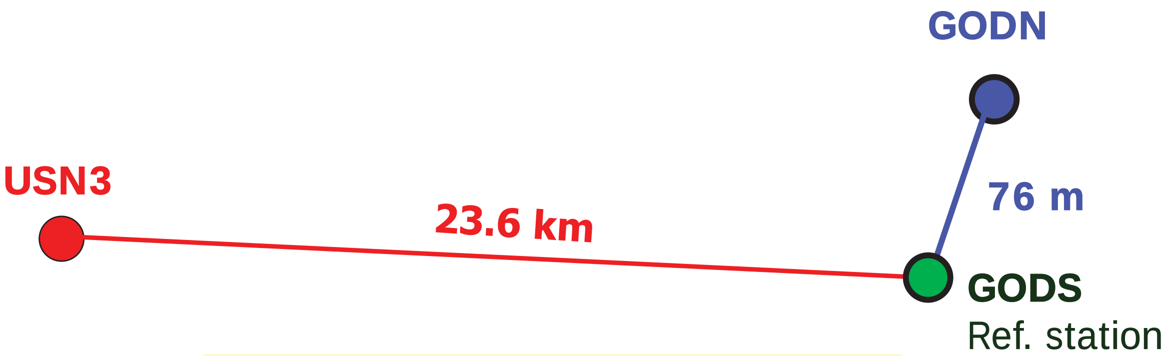

ftp://cddis.gsfc.nasa.gov/highrate/2013/

1130752.3120 -4831349.1180 3994098.9450 gods
1130760.8760 -4831298.6880 3994155.1860 godn
1112162.1400 -4842853.6280 3985496.0840 usn3

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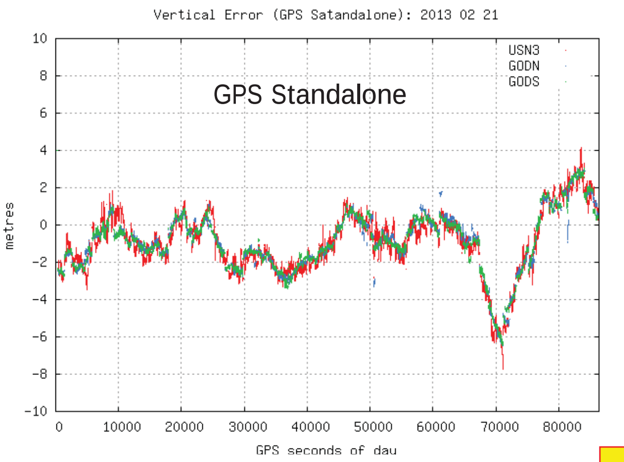
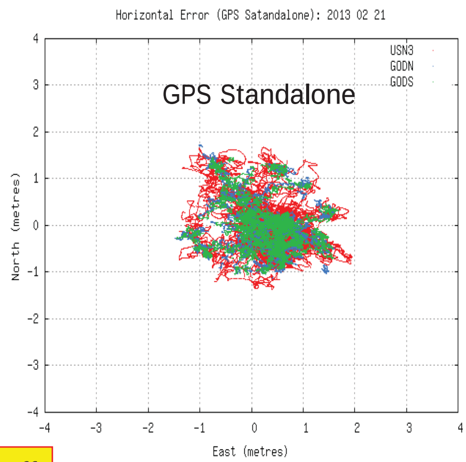
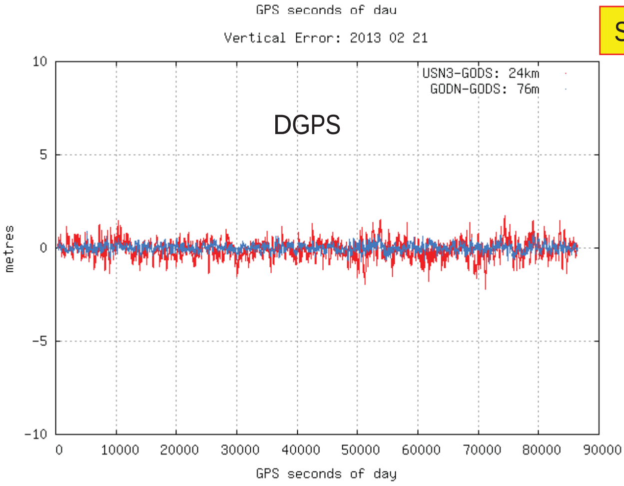
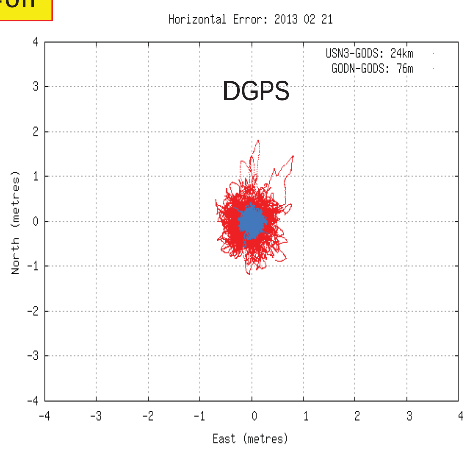
Differential Positioning Performance



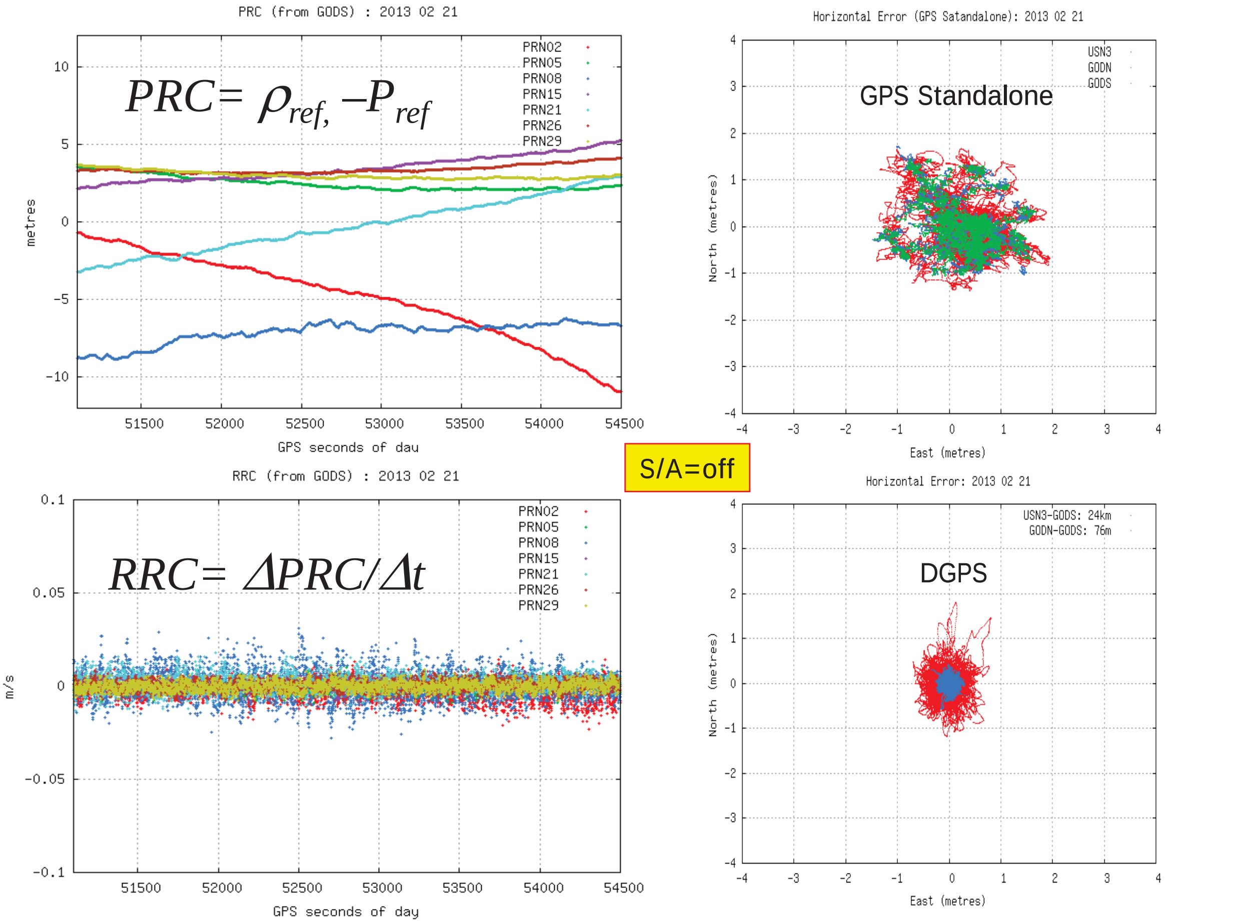


S/A=off





Differential Corrections



In the previous example, the differential error has been cancelled in the "position" domain *(i.e. solution domain approach)*.

But:

It requires to use the same satellites in both stations.

Thence, is much better to solve the problem in the "range" domain than in the "position" domain. That is, to provide corrections for each satellite in view *(i.e. measurement domain approach)*

Two implementations can be considered:

1.- The reference station, with known coordinates , computes range corrections for each satellite in view. These corrections are broadcasted to the user. The user applies these corrections to compute its "absolute position".

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2.- Differential GNSS (DGNSS): Relative position

This concept of DGNSS can be applied even if the position of the reference station is not known accurately or is moving, as well. In this case, the user estimates its relative position vector with the reference receiver.

In this **implementation of DGPS**, the reference station broadcast its time-tagged measurements rather than the computed differential corrections. The user receiver form differences of its own measurements with those at the reference receiver, (satellite by satellite) and estimate its position relative to the reference receiver.

Real-Time Kinematics (RTK) is and example of this DGNSS.
Users within some ten of kilometres can obtain **centimetre level positioning**. The baseline is **limited by the differential ionospheric error** that can reach up to 10cm, or more, in 10km, depending of the ionospheric activity.

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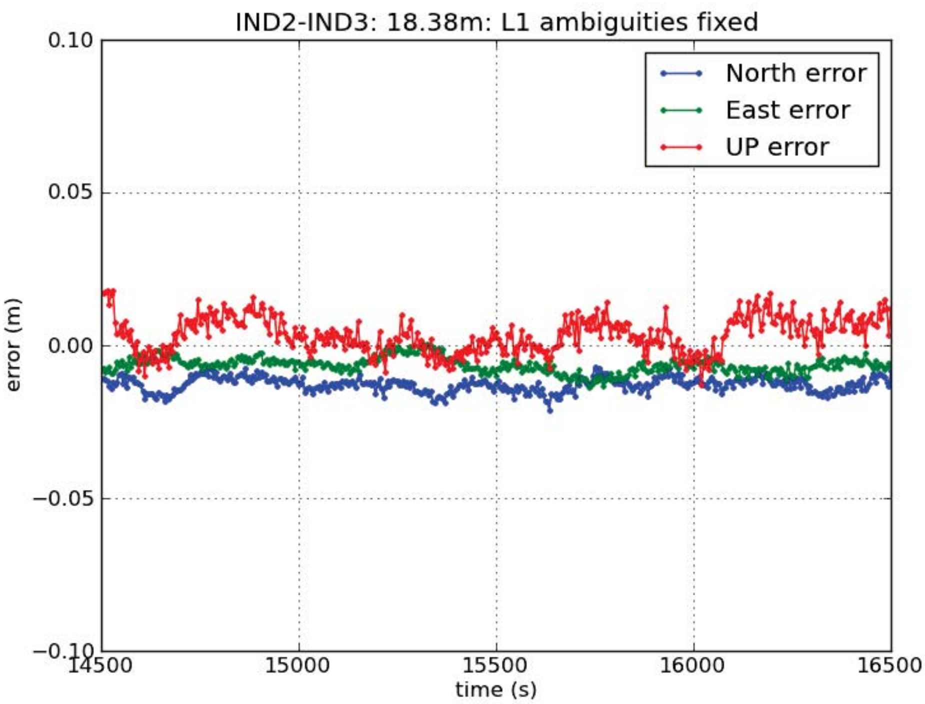

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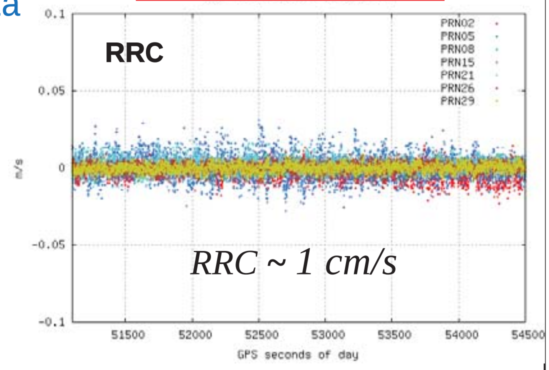
COMMENTS

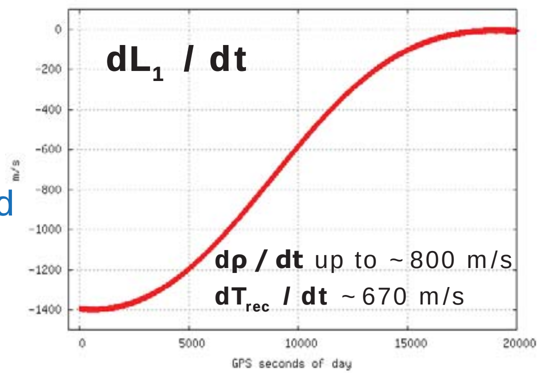
Real-Time implementation entails delays in data transmission, which can reach up to 1 or 2 s.

- Differential corrections vary slowly and its useful life is of several minutes (S/A=off)
- But, the measurements change much faster:
  - The range rate $d\rho/dt$ can be up to 800m/s and, therefore, the range can change by more than half a meter in 1 millisecond. Moreover the receiver clock offset can be up to 1 millisecond (depending on the receiver configuration).
  - Thence, the reference station measurements must be :
    - **Synchronized** to reduce station clock mismatch: station clock can be estimated to within 1μs ➔ $\varepsilon_{dt_{sta}} < 1\,\mathrm{mm}$
    - **Extrapolated** to reduce error due to latency: carrier can be extrapolated with error < 1cm.

$RRC = \Delta PRC/\Delta t$



$dL_1 \approx d\rho + dT_{rec}$



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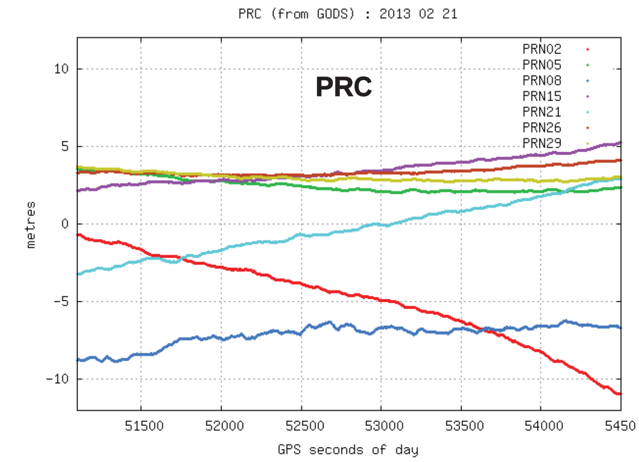
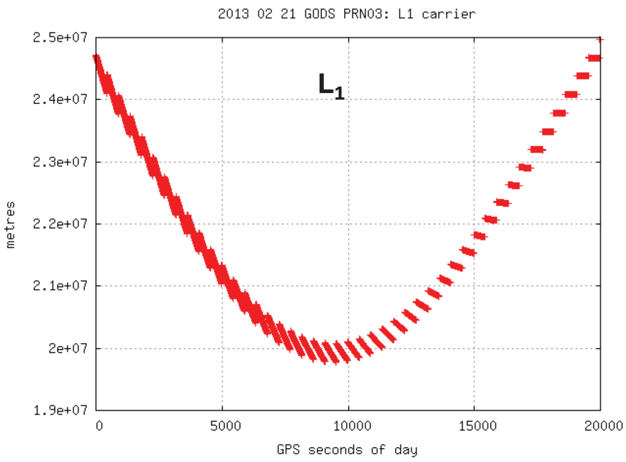
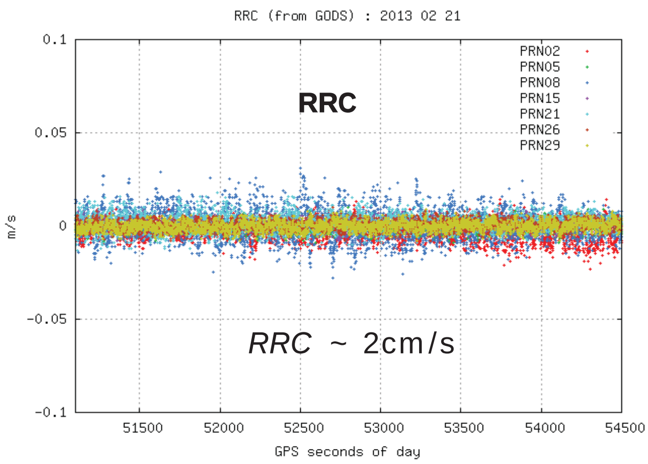
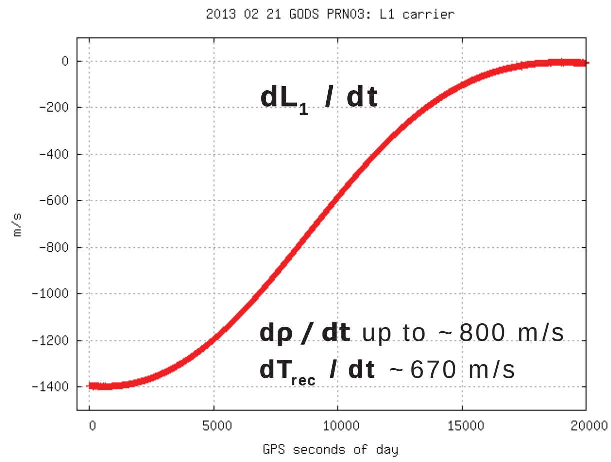
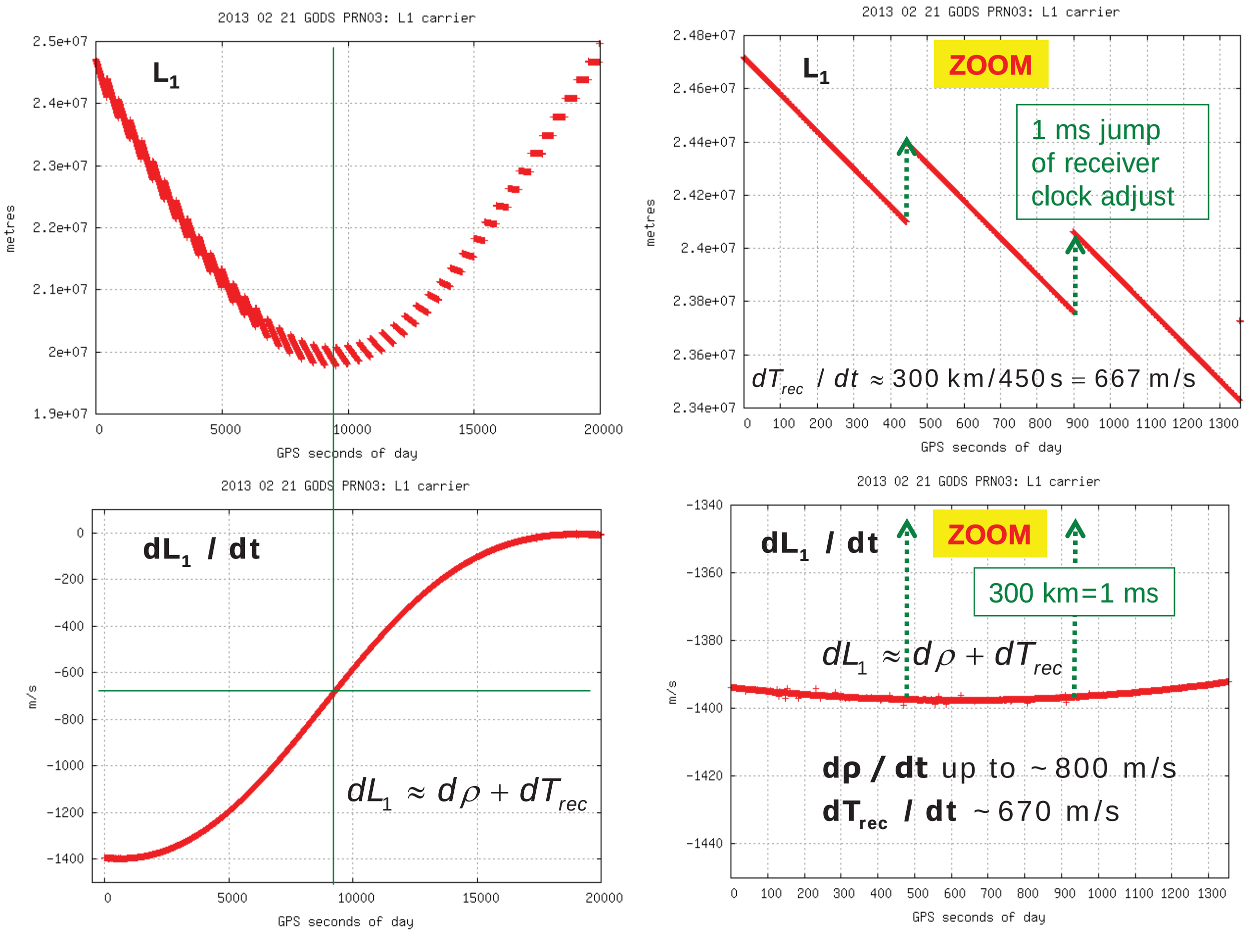








Receiver: JAVAD TRE_G3TH DELTA3.3.12



Receiver: JAVAD TRE_G3TH DELTA3.3.12

Contents

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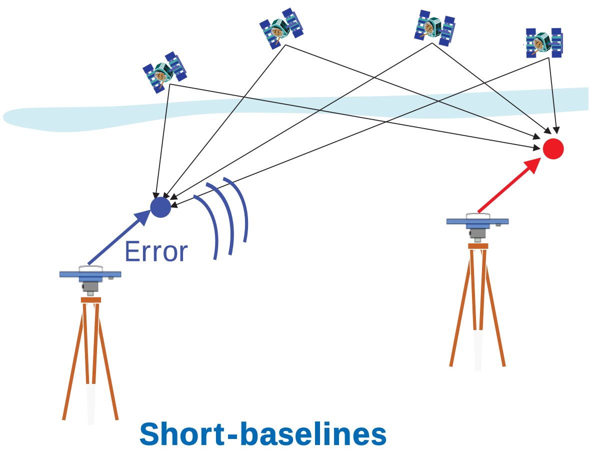
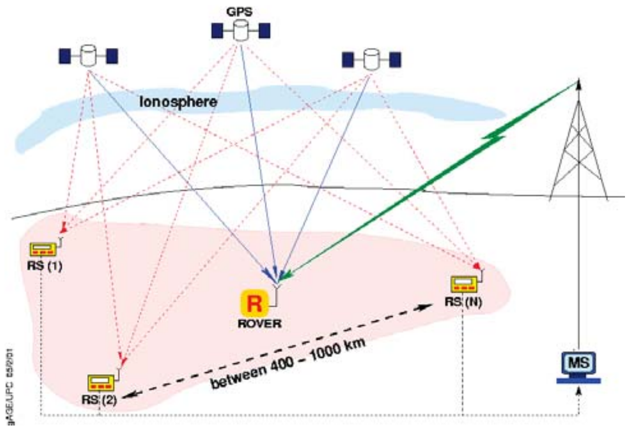
Error mitigation: DGNSS residual error

Errors are similar for users separated tens, even hundred of kilometres, and these errors vary ‘slowly’ with time. That is, the errors are correlated on space and time.

The spatial decorrelation depends on the error component (e.g. clocks are common, ionosphere ~100km...). Thence, a reference stations network is needed to cover a wide-area.



Short-baselines



Long-baselines

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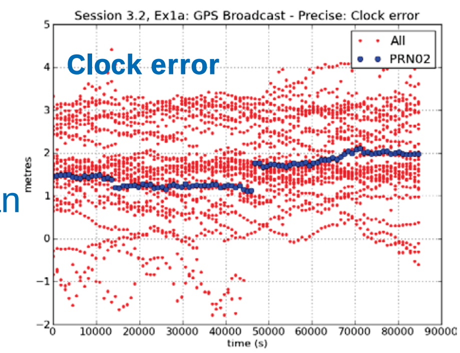
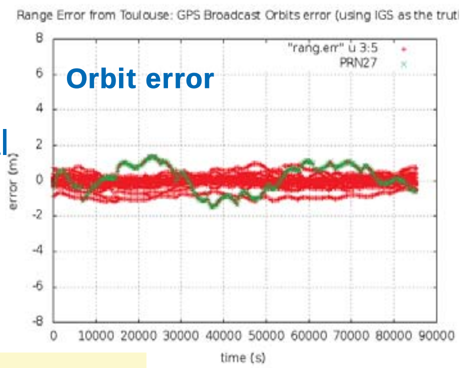
Space Segment Errors

- **Satellite clock error:**
  - Clock modelling error is small (~2m RMS) and changes slowly over hours.
  - Does not depend on user location, thence, it can be eliminated in differential mode.



- **Satellite ephemeris:**
  - Only the Line-Of-Sight (LOS) of error affects positioning. This error is small (~2m RMS) and changes slowly over minutes.
  - The residual error, after applying the differential corrections depends upon the separation between the LOS from user and reference station.
  - A conservative bound is given by:

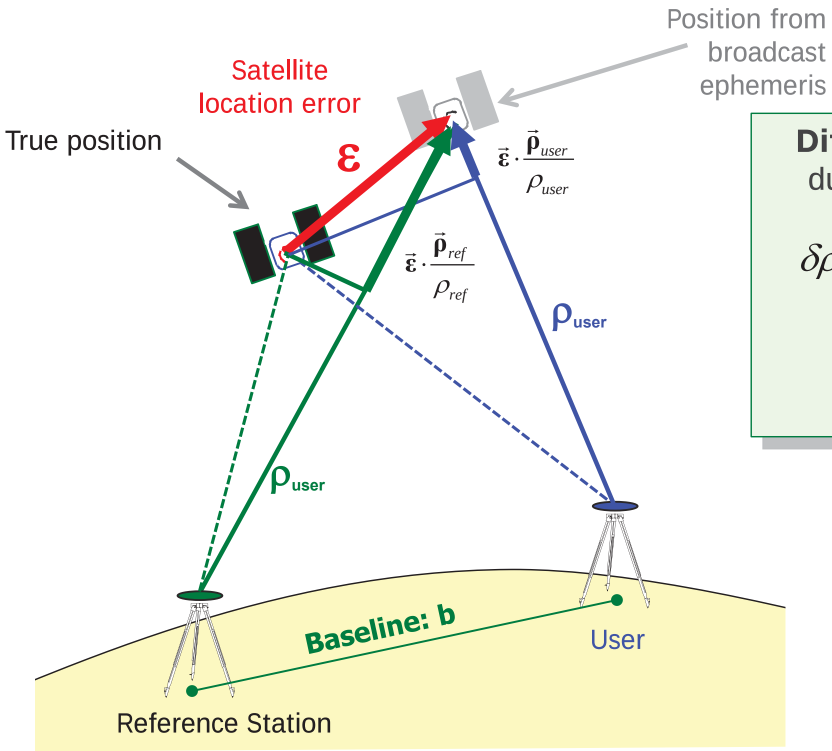


$$\boxed{\delta\rho < \frac{b}{\rho}\delta\varepsilon}$$

with a baseline $b = 20km$

$$\delta\rho < \frac{20km}{20000km}\delta\varepsilon = \frac{1}{1000}\delta\varepsilon$$

Ephemeris Errors and Geographic decorrelation



Differential range error due to satellite obit error

$$\delta\rho = \vec{\boldsymbol{\varepsilon}}\cdot\frac{\vec{\boldsymbol{\rho}}_{user}}{\rho_{user}} - \vec{\boldsymbol{\varepsilon}}\cdot\frac{\vec{\boldsymbol{\rho}}_{ref}}{\rho_{ref}}$$

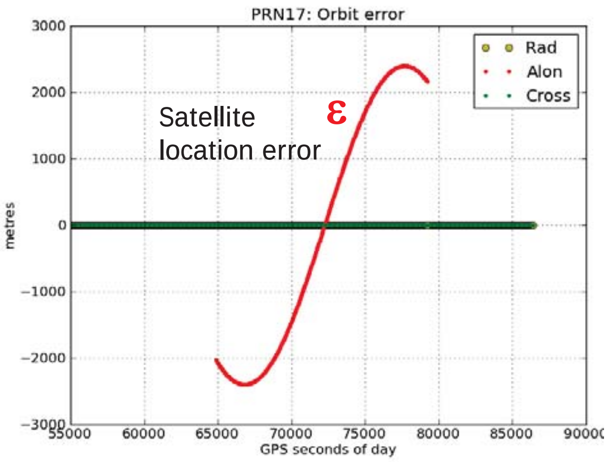
$$= -\,\vec{\boldsymbol{\varepsilon}}^T\left(\mathbf{I}-\hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{\rho}}^T\right)\frac{\mathbf{b}}{\rho}$$

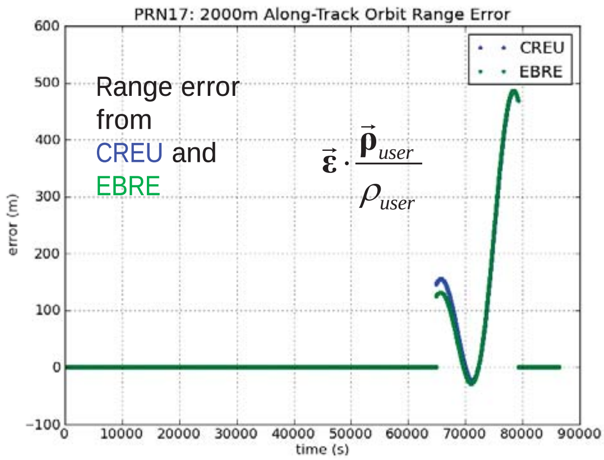
A conservative bound:

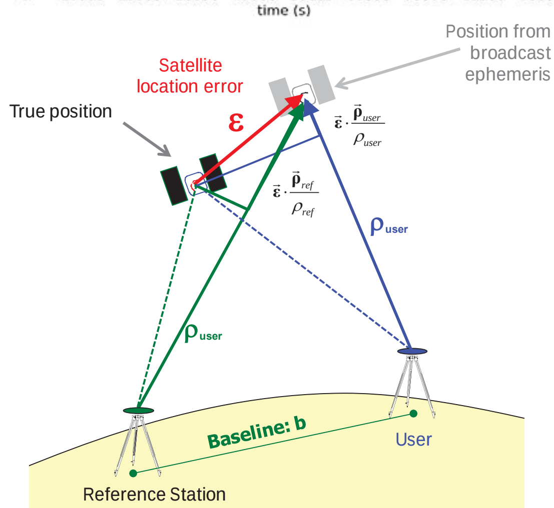
$$\boxed{\delta\rho < \frac{b}{\rho}\varepsilon}$$

with a baseline $b = 20km$

$$\delta\rho < \frac{20}{20000}\varepsilon = \frac{1}{1000}\varepsilon$$







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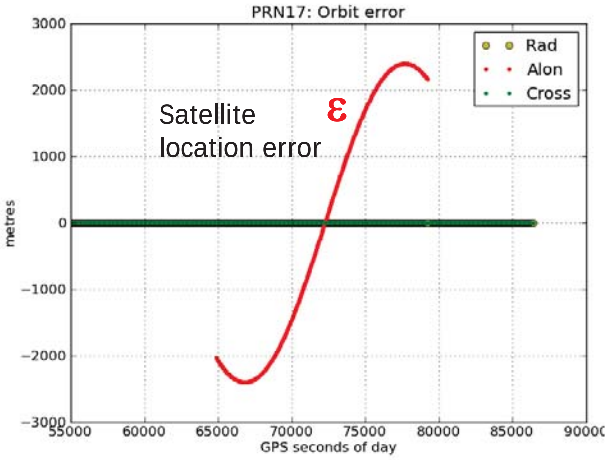

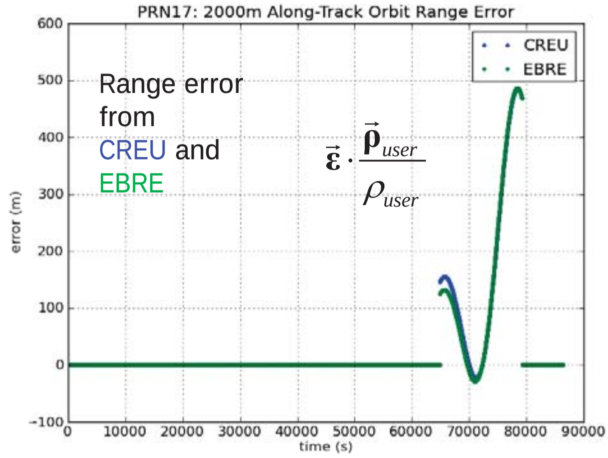

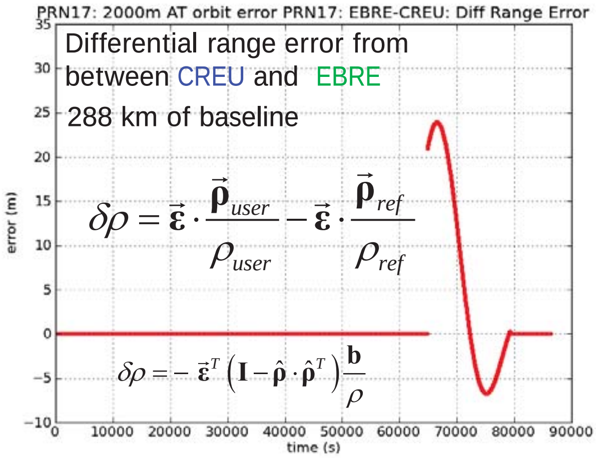

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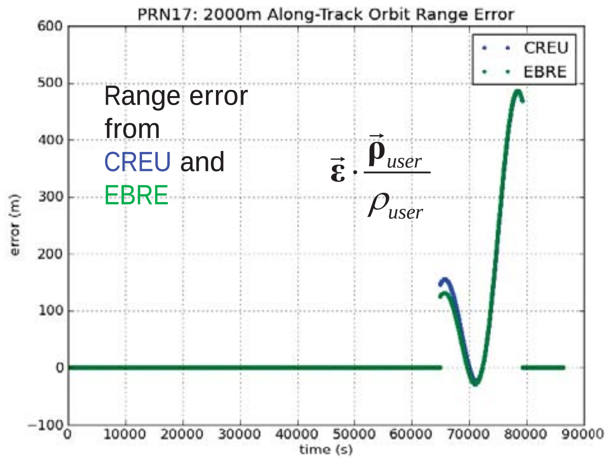

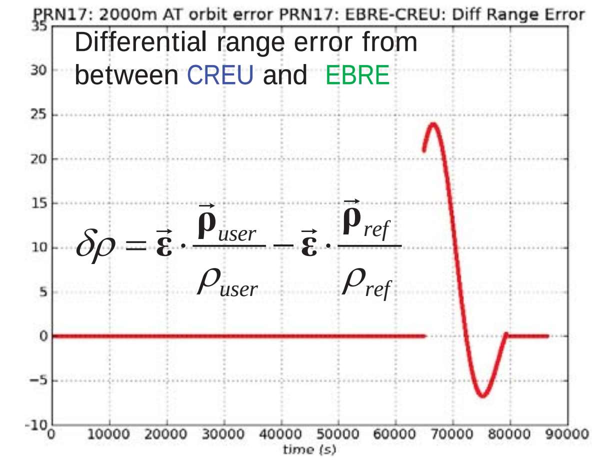

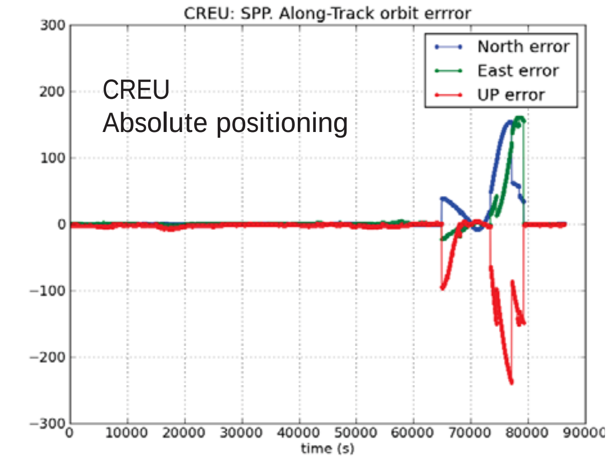

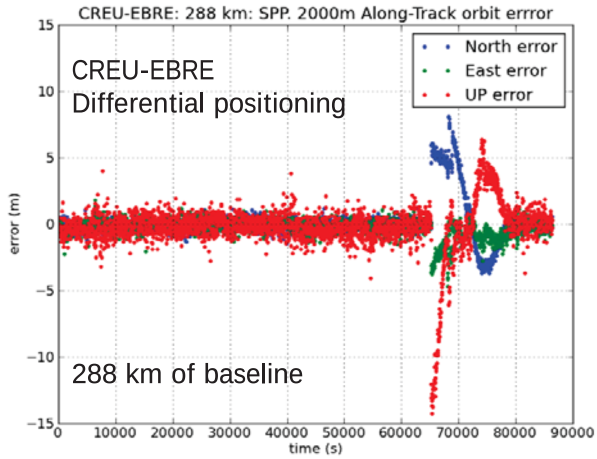

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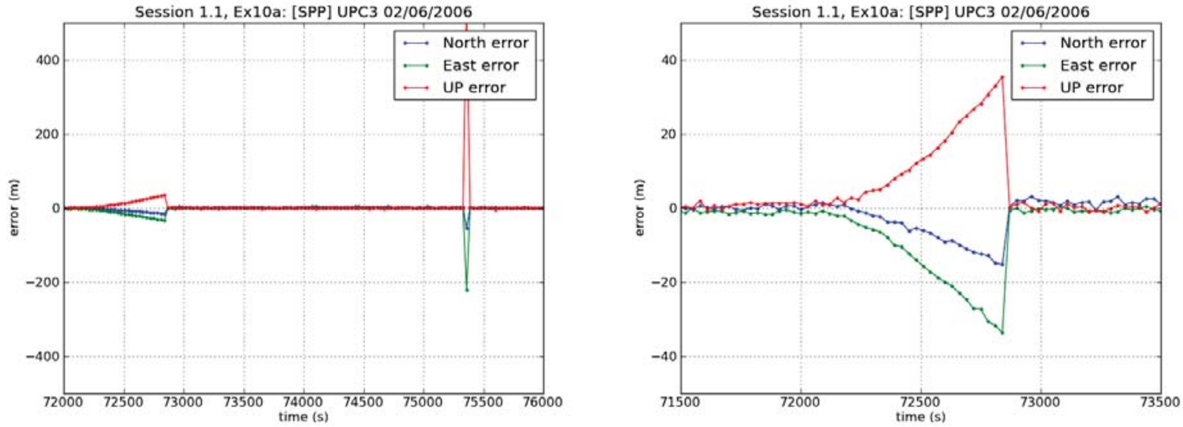
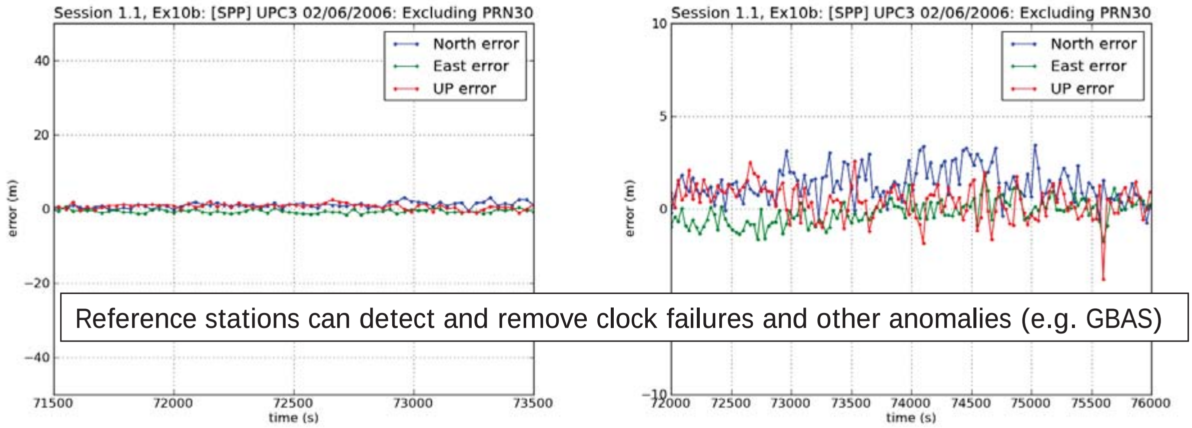


Satellite clock anomaly



Reference stations can detect and remove clock failures and other anomalies (e.g. GBAS)

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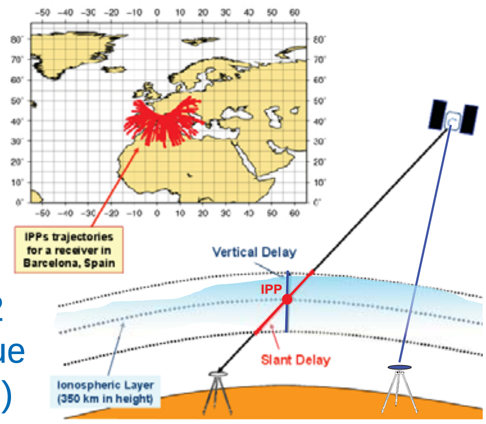
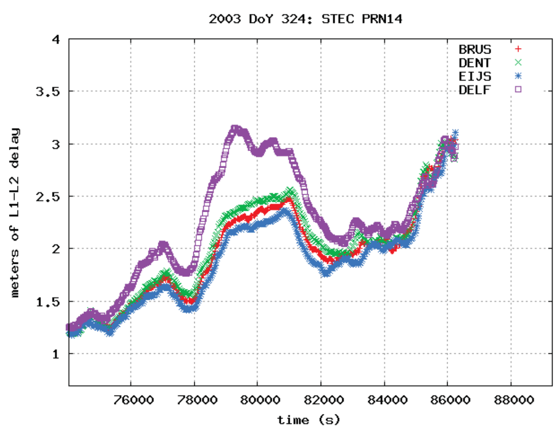
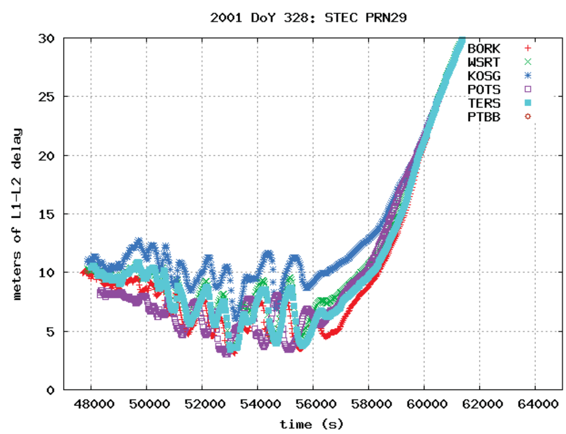
Atmosphere Propagation Errors

- **Ionospheric propagation delay:**
  - Ionospheric delay depends on the STEC (integrated electron density along ray path).
  - Reference and user receiver locations (i.e. Baselines) are mapped to Ionospheric Pierce Points (IPPs) associated to each satellite.
  - Typical spatial gradients of ionosphere are 1-2 mm/km (1σ) ➔ 0.1-0.2 m in 100km. This value can reach up to **300 mm/km** (6-7 April 2000)







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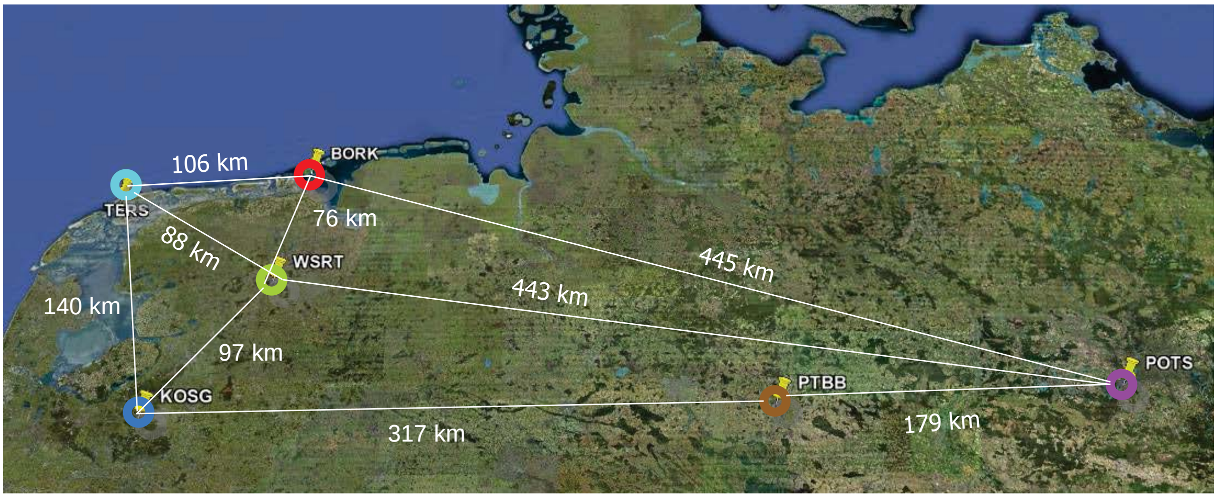

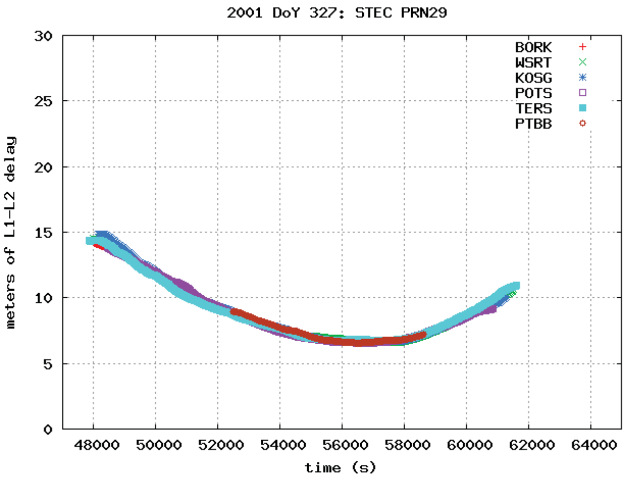

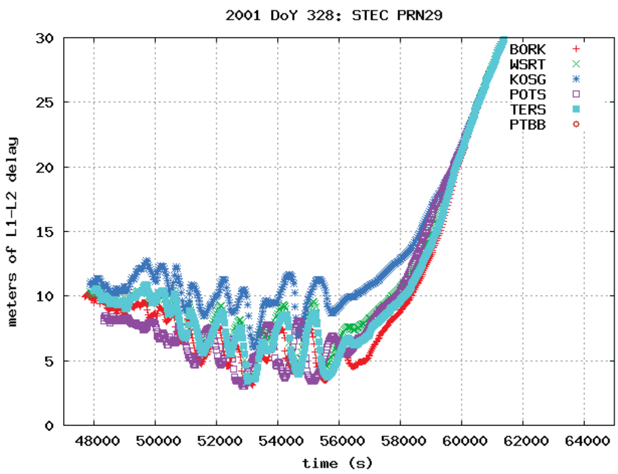

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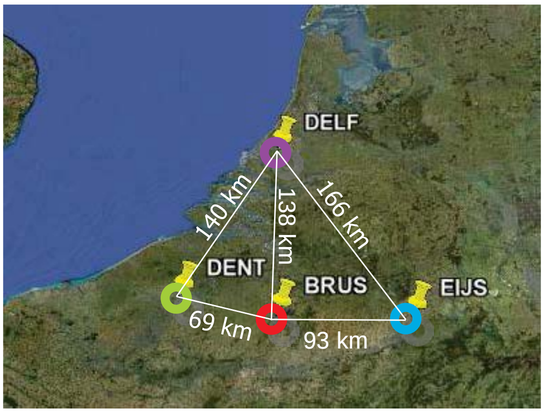

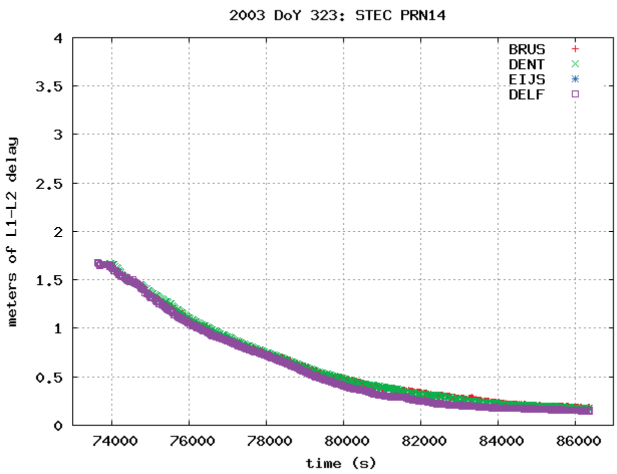

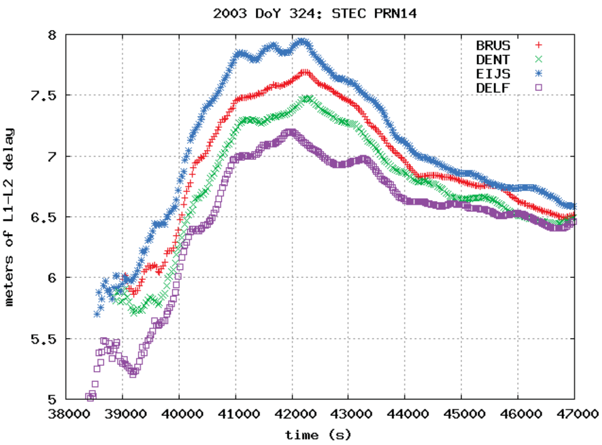

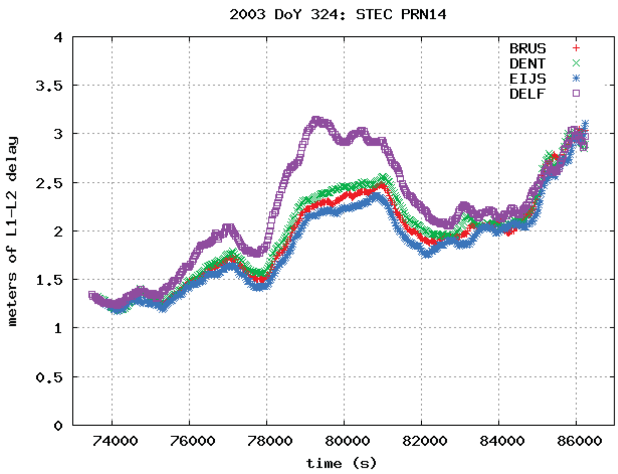

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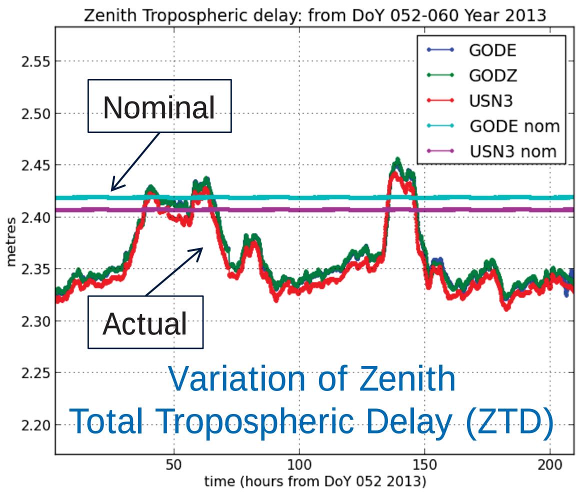
Atmosphere Propagation Errors

- **Tropospheric propagation delay:**
  - Tropospheric delay depends upon the air density profile along the signal path.
  - Most of the tropospheric delay (~90%) comes from the predictable hydrostatic component.
  - Wet component delay can vary considerably, both spatially and temporally.
  - With 10km separation between receivers, the residual range error can be 0.1-0.2m
  - For long distance or significant altitude difference it is preferable to correct for the tropospheric delay at both reference and user receivers. For a low elevation satellite, the residual range error can be 2-7 mm per meter of altitude difference.



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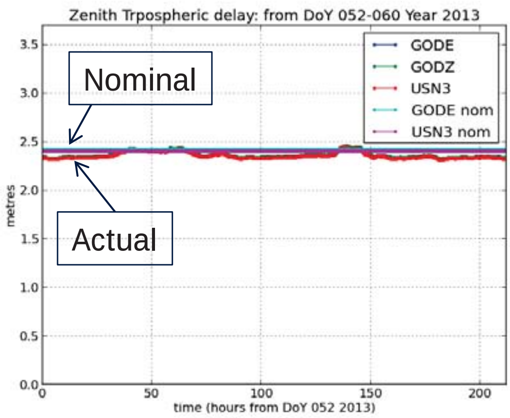

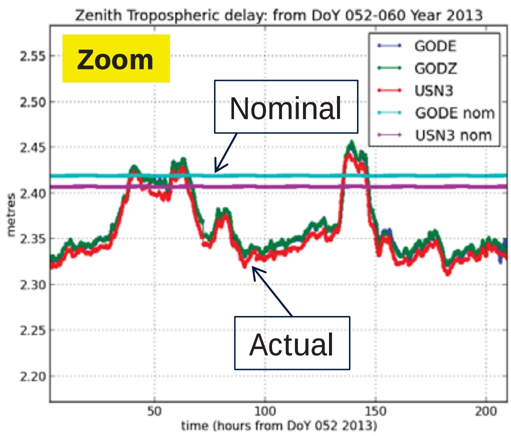

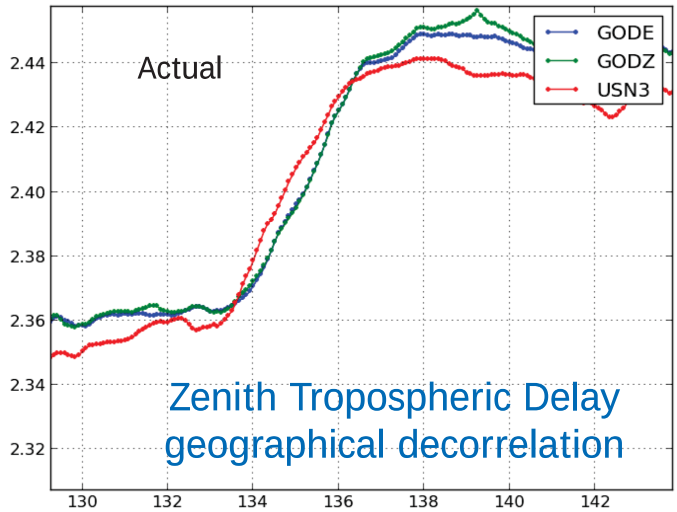

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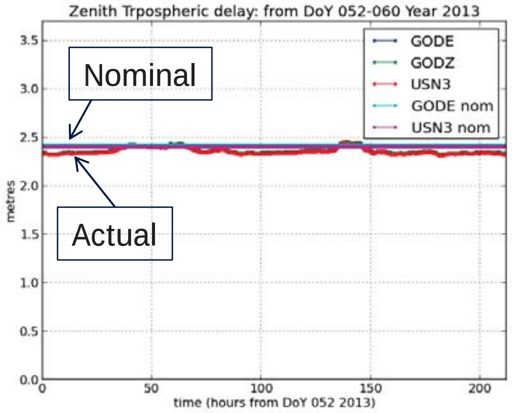

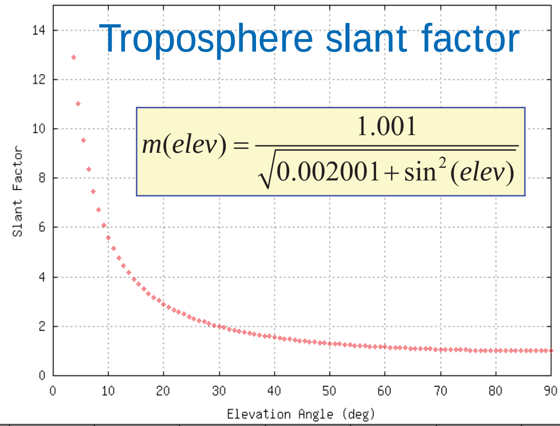

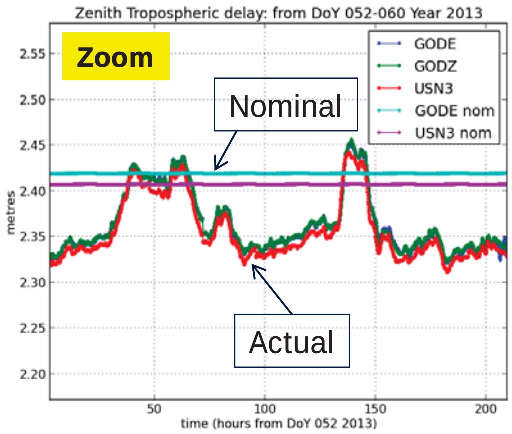

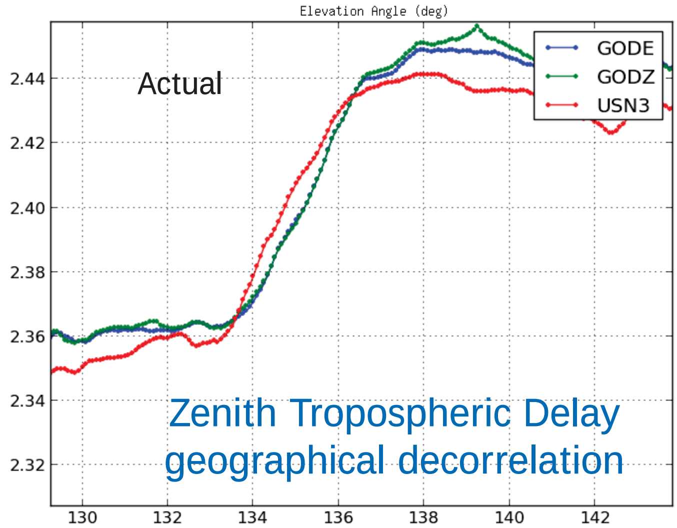

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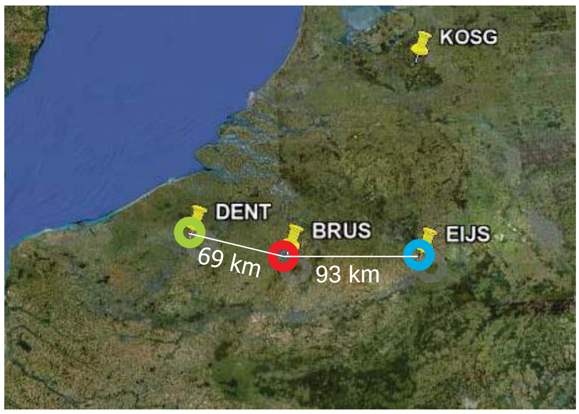
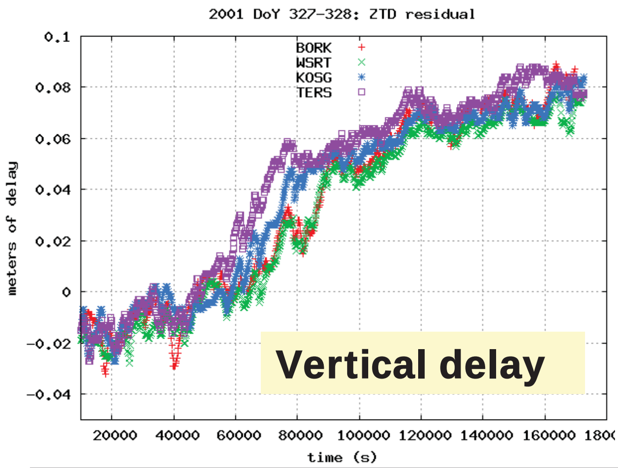
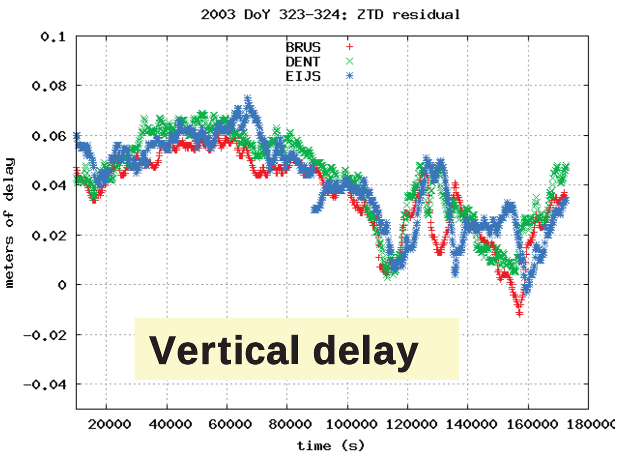


Zenith Wet Tropospheric Geographical decorrelation





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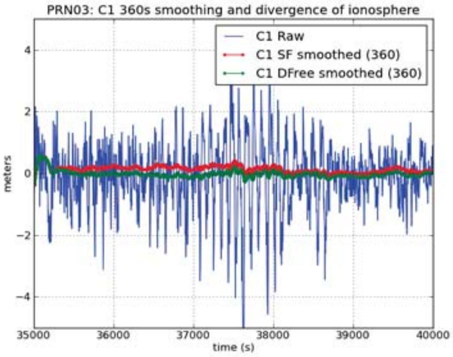
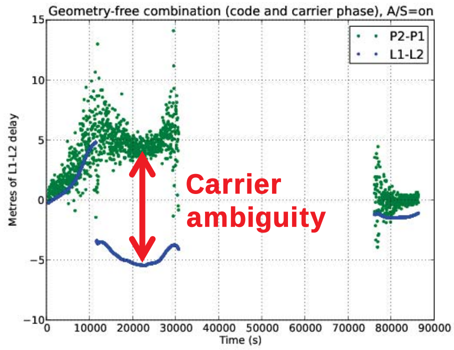
Local Errors

- **Receiver noise and multipath:**
  - These errors are uncorrelated at the reference and user receivers an cannot be corrected by DGPS.
  - In fact any error incurred in the reference station affects the user. Thence, it is important to minimize errors at the reference station.
  - Code noise can be reduced by smoothing with carrier (at the level of 0.25-0.50m). But single frequency smoothing is affected by code-carrier ionosphere divergence.
  - High accuracy applications use carrier measurements, about two orders of magnitude more precise than code measurements, but the unknown ambiguities must be fixed.





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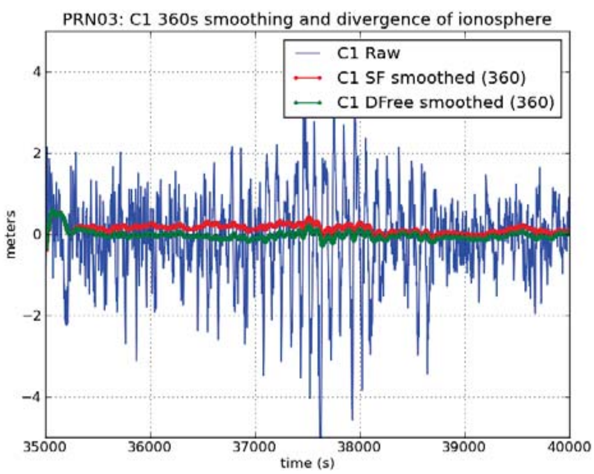
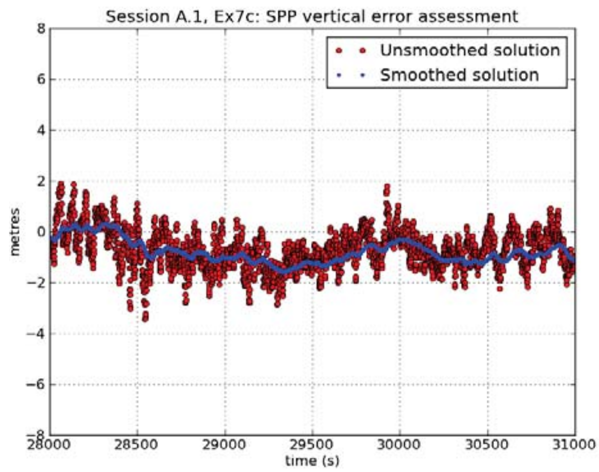
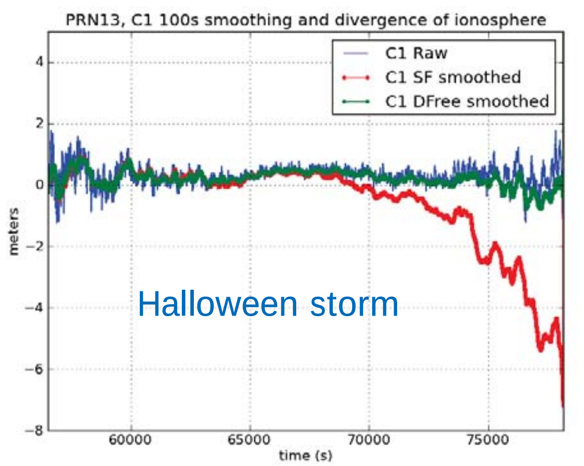
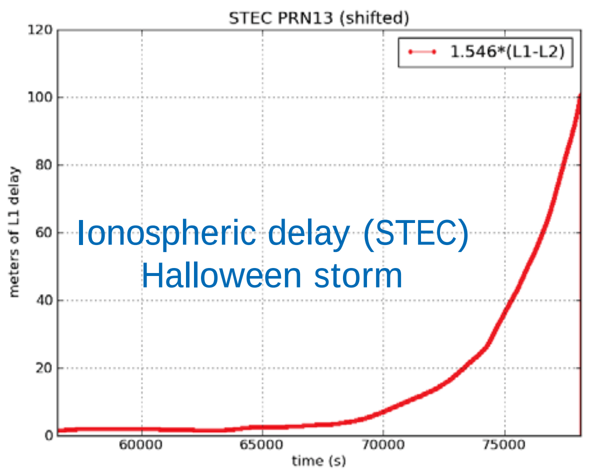
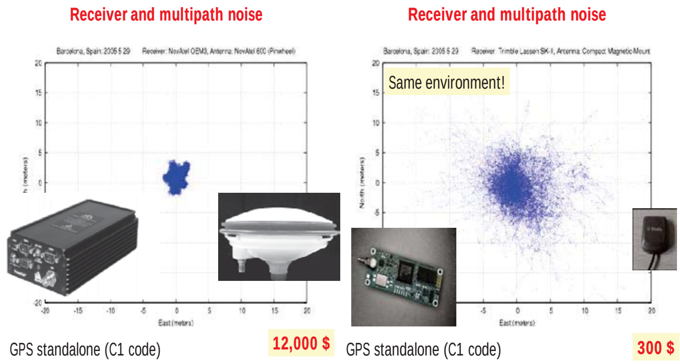


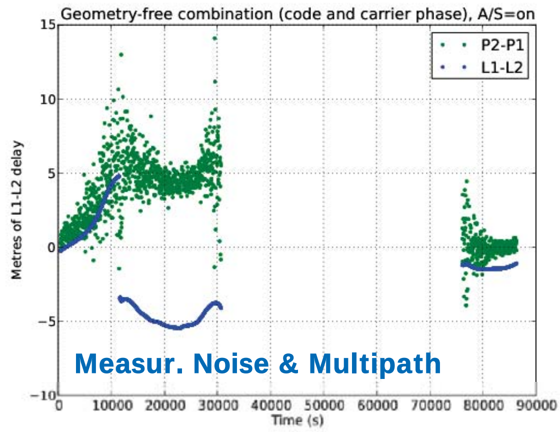


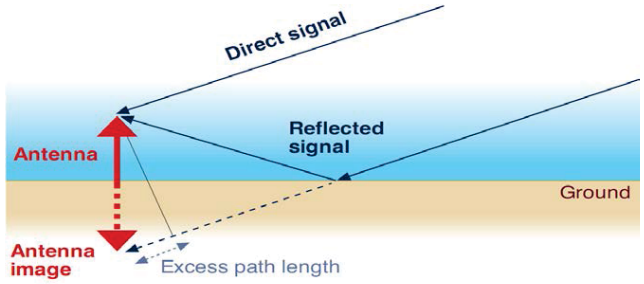


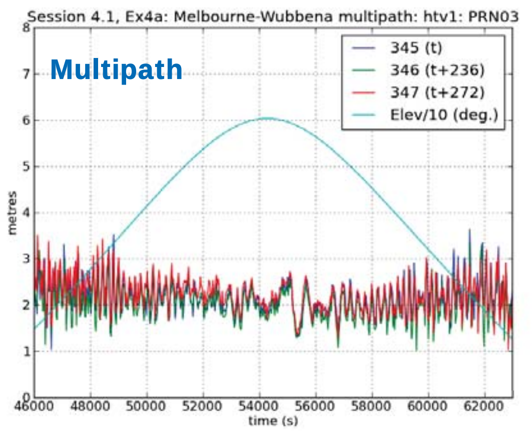


GNSS Positioning: Local errors



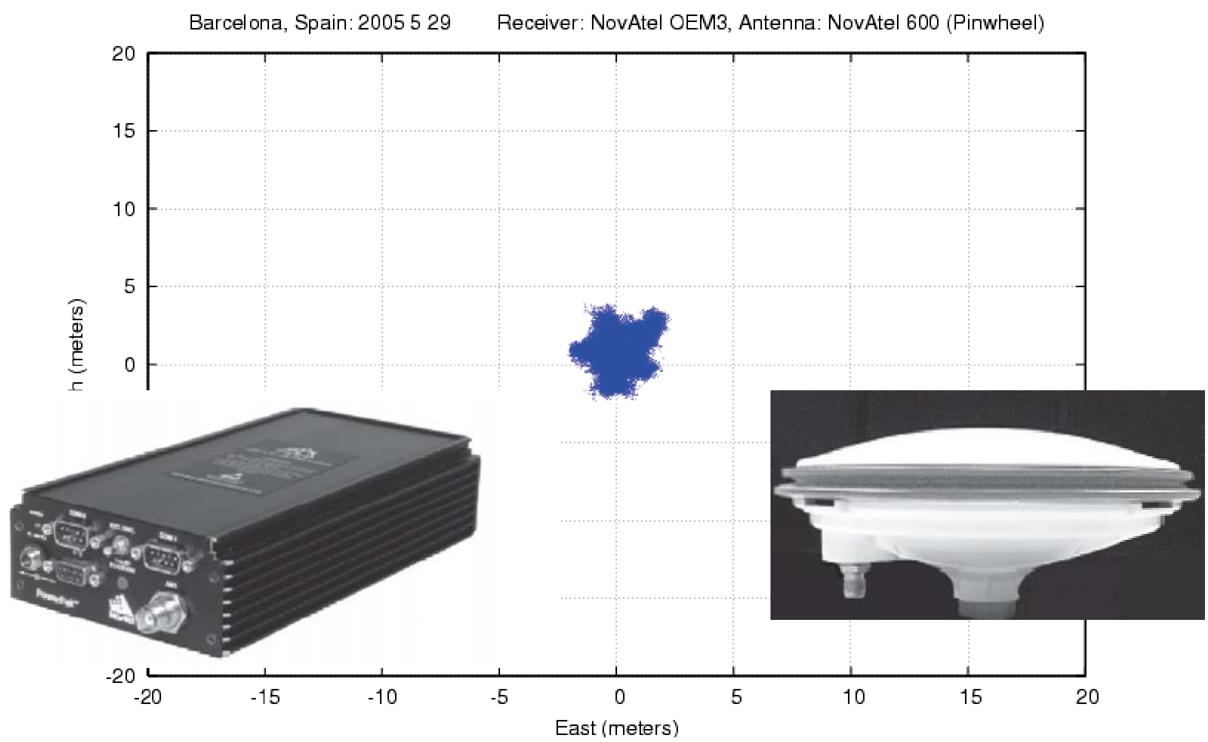






Error in carrier measurement due to multipath (cm level) or thermal noise (mm level) is typically 2 orders of magnitude lower than in code, but carrier has an unknown ambiguity.

Receiver and multipath noise



GPS standalone (C1 code)

10,000 €

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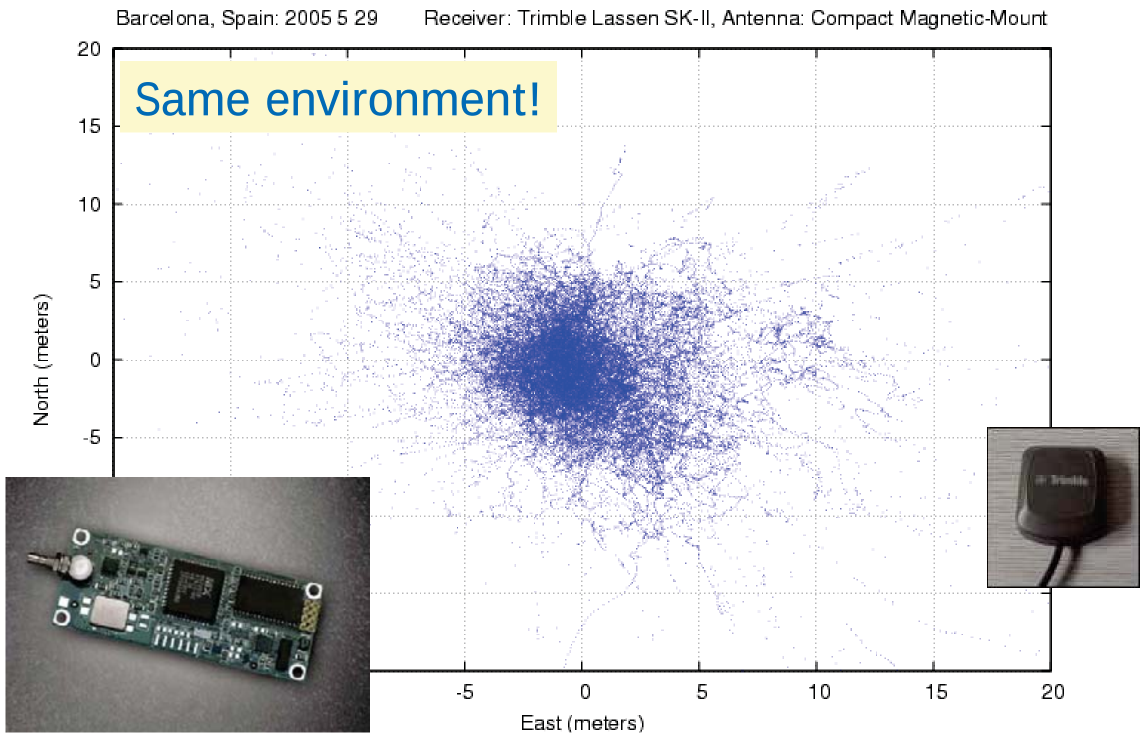
Receiver and multipath noise



GPS standalone (C1 code)

100 €

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ERRORS on the Signal

- Space Segment Errors:
  - Clock errors
  - Ephemeris errors
- Propagation Errors
  - Ionospheric delay
  - Tropospheric delay
- Local Errors
  - Multipath
  - Receiver noise

Common

Strong spatial correlation

Weak spatial correlation

No spatial correlation

This table is from the book: Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.

Source	Potential Error size	Error mitigation & Residual error
Satellite clock	Clock modelling error: 2 m (RMS)	**DGPS: 0.0m**
Ephemeris prediction	Line-Of-Sight error: 2 m (RMS)	**DGPS: 0.1m** (RMS)
Ionospheric Delay	Vertical delay: ~ 2-10 m (depending upon user location, time of day & solar activity) Obliquity factor: 1 at zenith, 1.8 at 30º , 3 at 5º.	Single-freq. using Klobuchar: 1-5m. **DGPS: 0.2m** (RMS)
Tropospheric Delay	Vertical delay ~ 2.3-2.5 m at sea level. (lower at a higher altitudes) Obliquity factor: 1 at zenith, 2 at 30º , 4 at 15º and 10 at 5º.	Model based on average meteorolog. Conditions: 0.1 -1 m **DGPS: 0.2m** (RMS) plus altitude effect.
Multipath	In clean environment: Code : 0.5 – 1 m Carrier: 0.5 -1 cm	Uncorrelated between antennas. Mitigation trough antenna design and sitting and carrier smoothing of code.
Receiver noise	Code : 0.25 – 0.50m (RMS) Carrier: 1-2 mm (RMS)	Uncorrelated between receivers

DGPS is based assuming baselines of tens of km and signal latency of tens of seconds.

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Differential GNSS (DGNSS) IMPLEMENTATIONS

Differential Positioning: GNSS augmented with data (differential corrections or measurements) from a single reference station or a reference station network.

The differential corrections can be broadcast as an:

- "Scalar" correction (**Local Area**), where all corrections are lumped together.
  Examples: GBAS, LAAS, RASAN, RTK, VRS.

- "Vector" correction (**Wide Area**), where the corrections are given for each error source separately *(state-space approach)*
  Examples: SBAS (WAAS, EGNOS...), DGPS (JPL), Fast-PPP.

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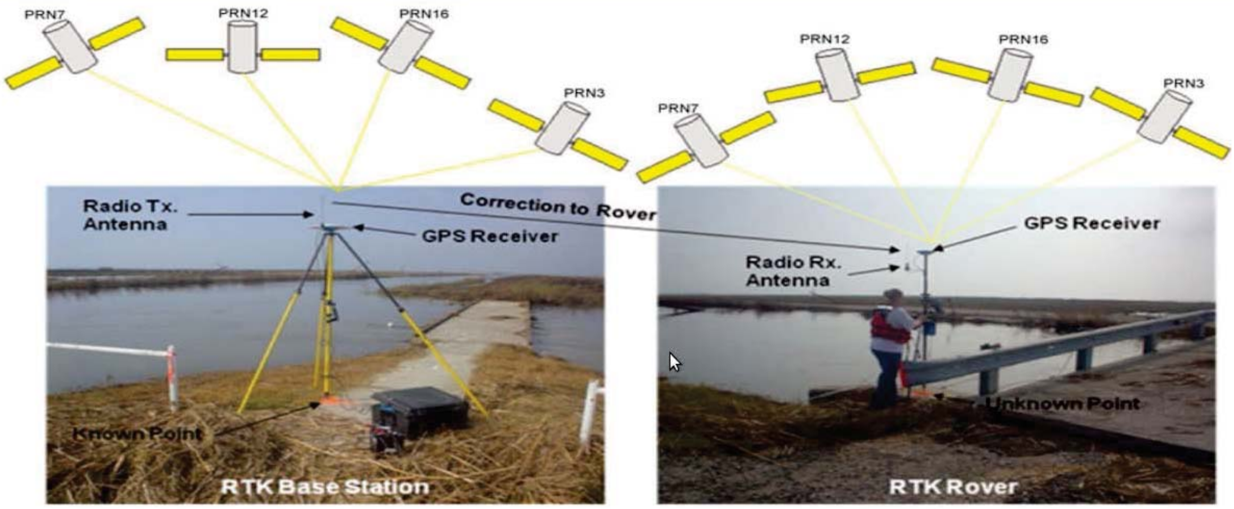
Real Time Kinematics (RTK)

In this implementation of DGPS, the reference station broadcast its time-tagged measurements rather than the computed differential corrections. The user receiver form differences of its own measurements with those at the reference receiver, satellite by satellite and estimate its position relative to the reference receiver.

- Real-Time-Kinematics (**RTK**) is a technique for relative precise positioning based on carrier phase data. The key feature of RTK is the ability to fix the carrier ambiguities On-The-Flight (OTF), i.e. while on the move. Major receivers manufacturers offer RTK solution packages consisting on a pair or receivers, a radio link, and software.

The performance of RTK is measured by (i) initialization time, and (ii) reliability (or, correctness) of the ambiguity fixing. There is an obvious trade-off between getting the answer quickly and getting it right.

For typical baselines of several kilometres, integer ambiguity resolution in thirty or sixty seconds is common, achieving centimetre error level of accuracy



(This picture is from http://water.usgs.gov/osw/gps/index.html)

Note: In Carrier phase Differential positioning, with a single reference station, the accuracy decreases as a function of the distance from the reference station by a rate of about 0.5-1cm per km.

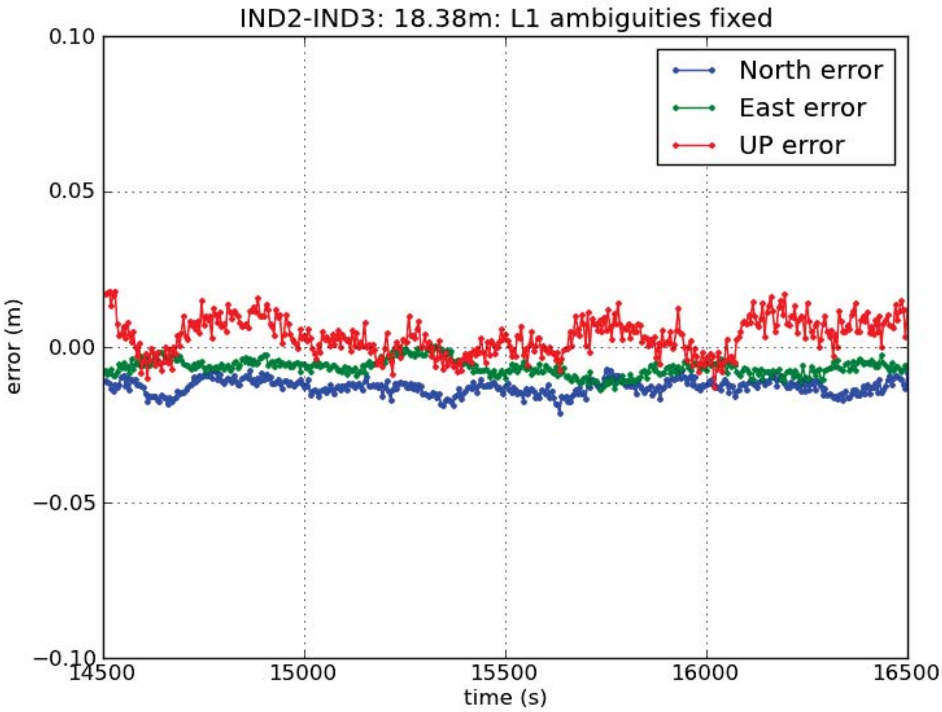
Mode	Horizontal accuracy
static	5 mm + 0.5 ppm
Kinematic	5 cm + 5 ppm

Typical baselines up to 10-15km, depending on the ionosphere conditions

RTCM format

Message Type	Title
1	Differential GPS corrections
2	Delta Differential corrections
3	GPS reference station parameters
9	GPS partial satellite set
10	P-code differential corrections
11	GPS C/A code L1, L2 delta corrections
15	Ionospheric delay message
17	GPS ephemeris
18	RTK uncorrected carrier phases
19	RTK uncorrected code pseudoranges
20	RTK carrier phase corrections
21	RTK code pseudorange corrections
59	Proprietary message

The NTRIP protocol has been defined For transmission RTCM data over internet



Local Area DGNSS (LADGNSS): VRS

LADGNSS includes a Master station and several monitor stations. The master station collects the range measurements of the monitor stations and process the data to generate the range corrections, which are broadcasted to users

- An example for High Accuracy Positioning, is the **Virtual Reference Station (VRS)** technique, which is based on generating measurements of a **virtual** (non existing) **reference station**, close to the user, from real measurements of a multiple reference station network. These virtual measurements are transmitted to the user **to compute the RTK solution**. The NRTK yields accuracies at the level of 5cm for baselines up to 40km.

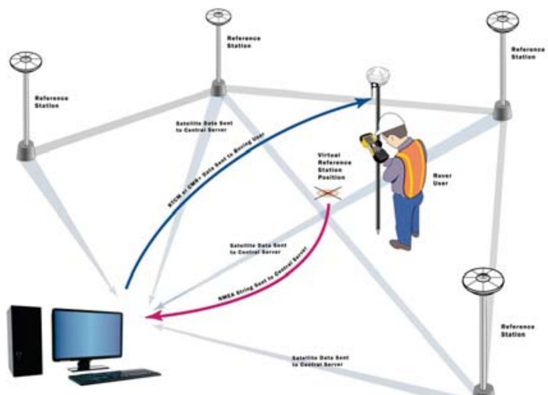
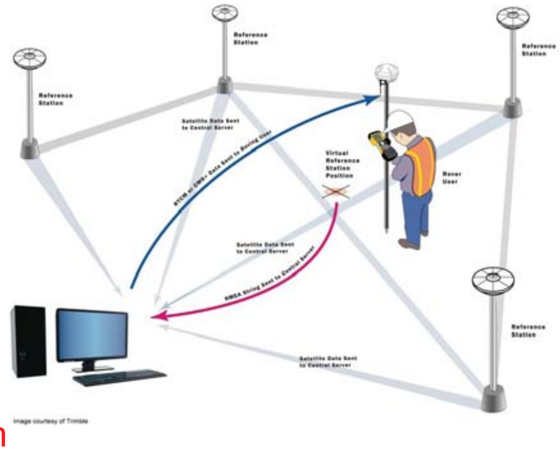


Image courtesy of Trimble

Virtual Reference Station

The basic scenario for VRS surveying is as follows:

- The user goes to a work site. The work site preferably is, as with any other style of GPS surveying, physically surrounded by the reference station network.
- The user sets up a surveying job and logs into the Real-Time Network (RTN) system using a cell phone (or other communication method). As part of the login process, the rover sends its position (via a NMEA sting) to the RTN system.



- The RTN system computes a virtual reference station, in close proximity to the rover based on the position sent. Using input from the closest surrounding reference stations, the RTN system then computes and sends corrections as if a real base station were broadcasting from the location of the virtual reference station.
- Using the cell phone, the receiver then obtains and applies the corrections in real time.

After initialization, the survey proceeds in exactly the same manner as an RTK survey.

http://water.usgs.gov/osw/gps/real-time_network.html

Benefits of VRS surveying over traditional RTK surveying include:

- No need for a user to establish a permanent/semi-permanent base station. This eliminates the time for initial site selection and (daily) set-up, any issues of security and power supply, and the possibility of set-up errors.
- The RTN can monitor its own integrity and can detect if there is a problem with a particular reference station. With a single-base RTK setup it can be difficult to tell if a problem exists or occurs with a base station while conducting a survey.
- Since the reference stations are part of a network, a loss of one station does not result in failure of the entire network or the resulting survey. Whereas the loss of a reference station with single-base RTK setup results in the end of data collection, with RTN surveying the system accuracy degrades gradually.
- A sufficiently dense reference station network can result in shorter baselines. As with any other style of GPS surveying, shorter baselines result in improved accuracy because of reduced effects of atmospheric interference.
- The RTN reference stations allow for network atmospheric modeling resulting in improved accuracy. With RTK, atmospheric effects are computed using (usually) one location.
- All users of the system are using a common, established ref. coordinate frame.

Limitations of VRS surveying include:

- There is a high cost of setting up and maintaining the RTN and to use an RTN setup by other organizations there is typically a yearly subscription fee that must be paid for network access.

- Use of the RTN can be limited by cell phone coverage and system down times.

- Availability is dependent on network extent and accuracy can be affected by the network density.

http://water.usgs.gov/osw/gps/real-time_network.html

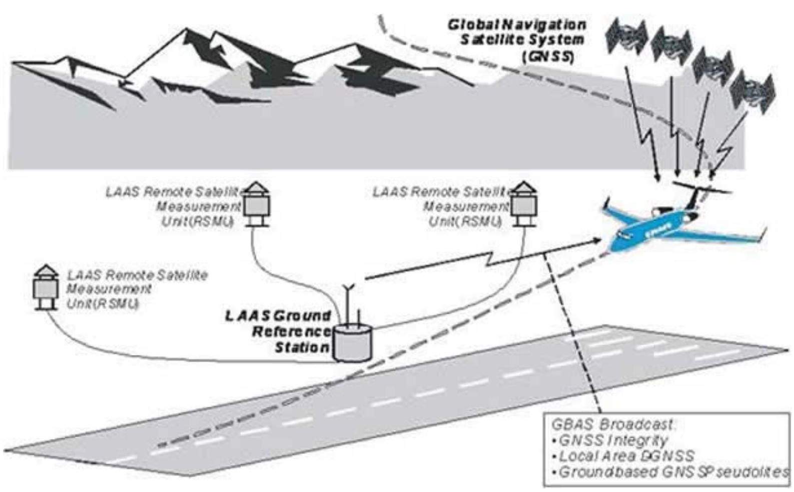
Comment:
VRS is the most widely used implementation method of **Network RTK (NRTK).** Other possible implementations are briefly described in:

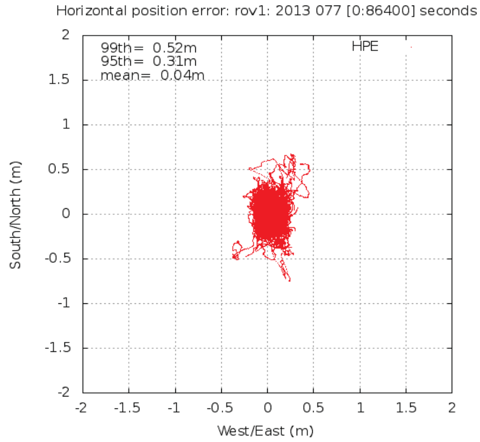
http://www.wasoft.de/e/iagwg451/intro/introduction.html

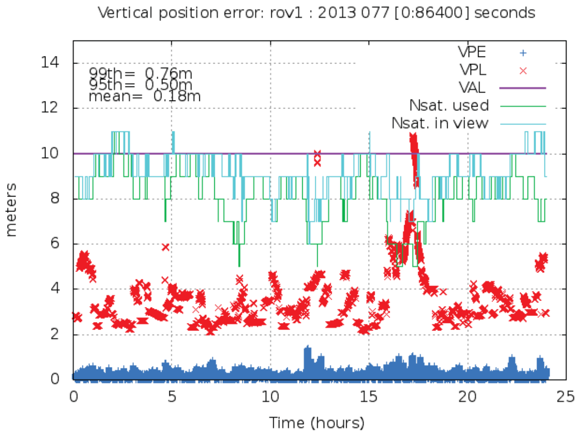
Local Area DGNSS (LADGNSS): GBAS

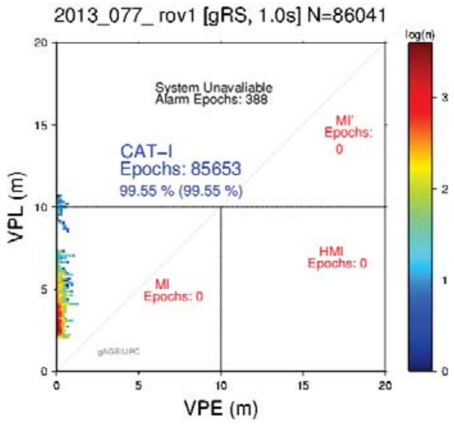
LADGNSS includes a Master station and several monitor stations. The master station collects the range measurements of the monitor stations and process the data to generate the range corrections, which are broadcasted to users

- Examples using L1 carrier smoothed code are the Local Area Augmentation System (**LAAS**) or the Ground Based Augmentation System (**GBAS**), where a ground facility computes differential corrections and integrity data from measurements collected by several redundant receivers. This system is used to support aircraft operations during approach and landing. The differential corrections are transmitted on a VHF channel, up to about 40km. Meter level accuracies with integrity fulfilling the stringent requirements of **Civil Aviation** are met.









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- "Vector" correction (**Wide Area**), where the corrections are given for each error source separately *(state-space approach)*
  Examples: SBAS (WAAS, EGNOS...), DGPS (JPL), Fast-PPP.

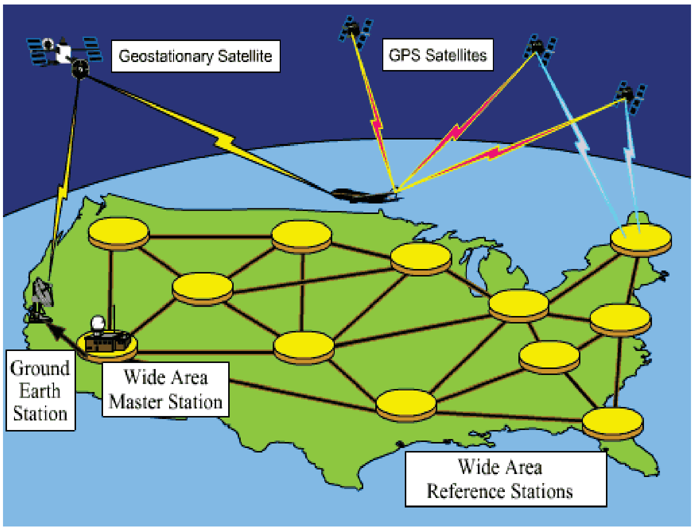
Wide Area DGNSS (WADGNSS)

Differential Corrections to cover Continent-wide (or world-wide) users must be broadcast for each error source separately: Satellite clocks, ephemeris and ionosphere.

These corrections are computed by a Central processing Facility (CPF) from the range measurements of the monitor stations network with baselines of several hundreds up to thousand of kilometres.

- Examples using L1 carrier smoothed code are the **Satellite Based Augmentation Systems (SBAS),** e.g. WAAS, EGNOS, MSASS, for Civil Aviation, where differential corrections and integrity data fulfilling the Civil aviation requirements are broadcast over continental areas by a GEO satellite. Meter level accuracies with integrity are met.

Evolution to a dual frequency (L1,L5) system.



Error Mitigation

Error component	Local Area (GBAS)	Wide Area (SBAS)
Satellite clock	Common Mode Differencing	Estimation and Removal each error component
Ephemeris		
Ionosphere		
Troposphere		Fixed Model
Multipath and Receiver Noise	Carrier Smoothing by user	

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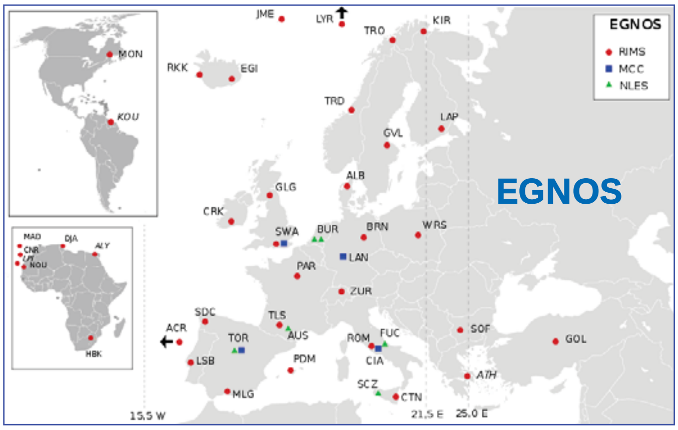

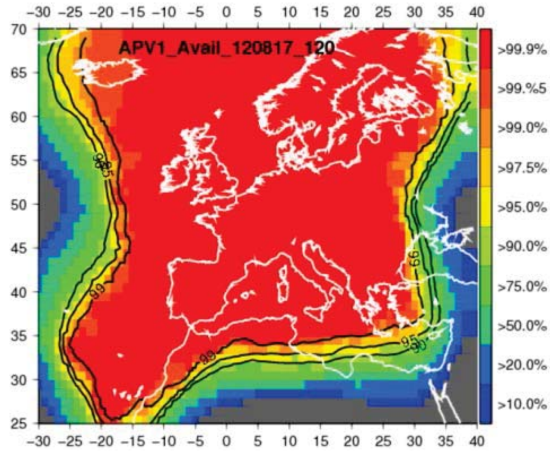

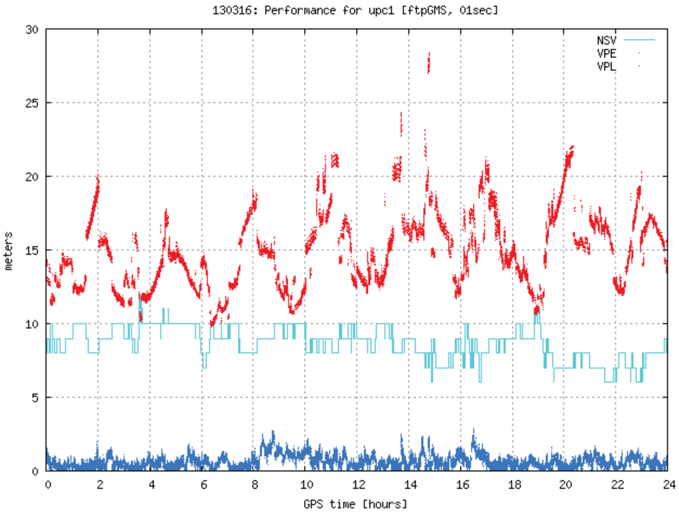

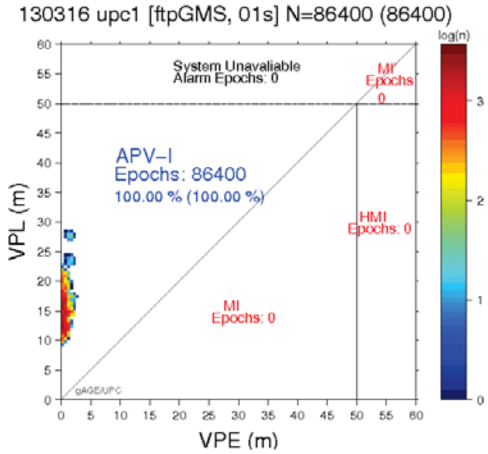

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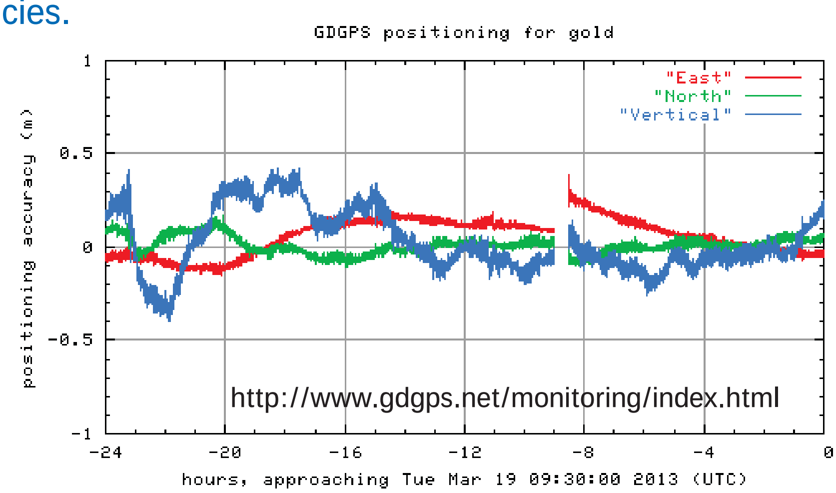
Wide Area DGNSS (WADGNSS)

For dual frequency users, the JPL-NASA provides Real-Time Global Differential GPS (**GDGPS**) to world wide users to achieve decimetre/centimetre level of accuracy, after the best part of one hour, using the Precise Point Positioning (PPP) technique.

Indeed, precise orbits and clocks are computed from a global sparse reference stations network. The ionospheric error is eliminated from the combination the two frequencies.



Current research: The Fast-PPP

Fast-PPP have prove that the high accuracy positioning of PPP users, can be reached very quickly (almost instantaneously) over continental areas with the 3-frequency Galileo signals. Moreover:

- World-wide users can perform **undifferenced ambiguity fixing**, thanks to the broadcasting of the fractional part of carrier ambiguities.
- Continent-wide area users (e.g. Europe) can achieve the accuracy quickly (few minutes, with **2-frequency** <u>or single epoch with **3-frequency**</u>, instead the 1/2-1 hour of PPP), thanks to the broadcasting of highly accurate ionospheric corrections.
- **Single frequency** users can take also benefit of the highly accurate ionospheric corrections achieving submetre positioning since the beginning and decimetre level positioning after the best pat of one hour.

As in any WADGNSS, the differential corrections (satellite clocks, ephemeris, ionosphere, fractional part of ambiguities and DCBs) are computed by CPF from the measurements collected by an sparse network (few tens world wide distributed plus 30-40 for the continental enhancement of ionosphere model)

The Fast-PPP technique has been developed gAGE/UPC and is protected by several ESA-funded patents.

CPF Corrections

Correction	Coverage	Time Update	Content	Service + Added capability
Fast	Global	~5s	Satellite Clocks	Classic PPP
Slow	Global	300s	Orbit Corrections	
			Satellite DCBs Fractional part of ambiguities (B1, BW)	+ Ambiguity Fixing
Ionospheric	Continental	300s	Iono. corrections	+ Fast PPP + Single Frequency
Integrity	Global/ Continental	~5/300s	Confidence bounds	+ Integrity

The additional required bandwidth of F-PPP is about 10% of the classical PPP bandwidth

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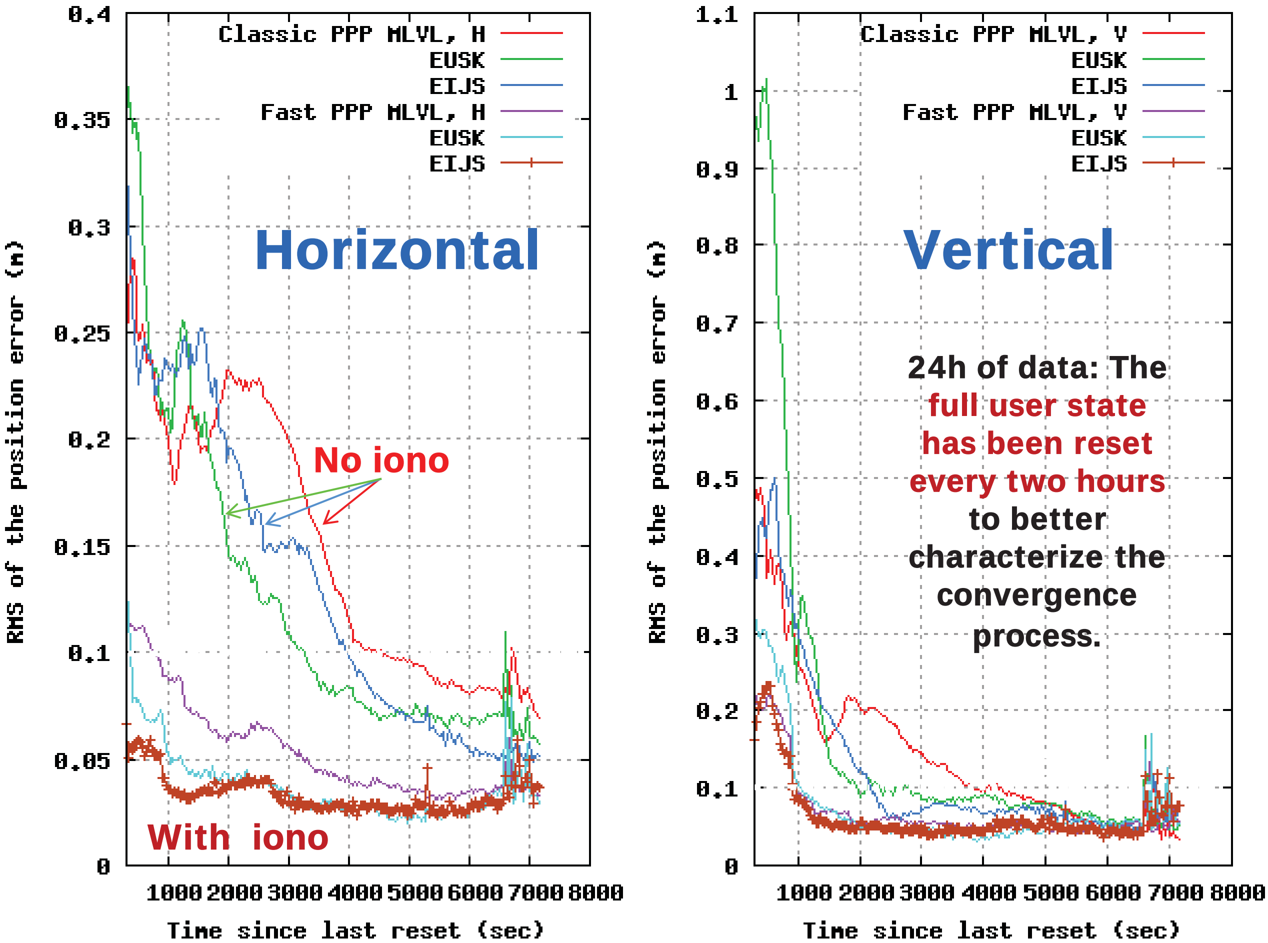

MLVL (violet), EUSK (light blue) and EIJS (brown), at 252, 170 & 94 km respectively far from the nearest reference receiver BRUS.

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Contents

1. Introduction: GNSS positioning and measurement errors.
2. Differential positioning concept and differential corrections.
3. Error mitigation in differential positioning.
4. DGNSS implementations: RTK, LADGNSS, WADGNSS.
5. DGNSS commercial services.

DGNSS Commercial services

Commercial WADGNSS services are already operational and in world-wide use for different applications: agriculture (e.g. OmniSTAR or CenterPoint RTX from **Trimble**), operations at sea (e.g. Starfix and Skyfix from **Fugro**), among others

http://www.fugro.com/ http://www.trimble.com

- **OmniSTAR** provides four levels of service: http://www.omnistar.com/
  - Virtual base Station (VBS) offering sub-meter positioning,
  - World-wide service "XP" delivering better than 20 centimeter accuracy,
  - High performance (HP) service delivering greater than10 centimeter accuracy
  - OmniSTAR "G2" service combines GPS plus GLONASS-based corrections to provide decimeter level positioning.

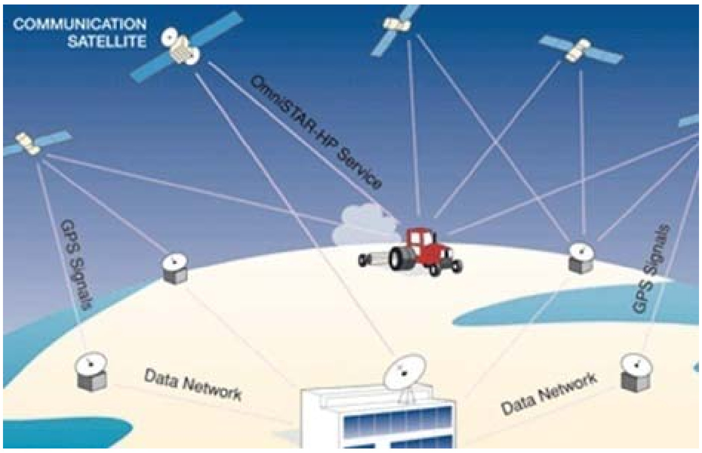
OmniSTAR services were initially introduced by Fugro company and in 2011 was acquired by Trimble company.

- Similar levels of services are provided by **Starfix**: http://www.starfix.com
  - Starfix.L1 , Starfix.XP, Starfix.HP , Starfix.G2

OmniSTAR HP

10 cm High Performance

http://www.omnistar.com/SubscriptionServices/OmniSTARHP.aspx



OmniSTAR HP (10cm) service is the **most accurate solution** available in the OmniSTAR portfolio of correction solutions. It is a L1/L2 solution requiring a **dual frequency** receiver.

OmniSTAR HP corrections are modeled on a network of reference sites using carrier phase measurement to maximize accuracy.

The expected 2-sigma (95%) accuracy of OmniSTAR HP is 10cm. It is particularly useful for Agricultural Machine guidance and many surveying tasks. It operates in real time and without the need for local Base Stations or telemetry links. OmniSTAR HP is a true advance in the use of GPS for on-the-go precise positioning.

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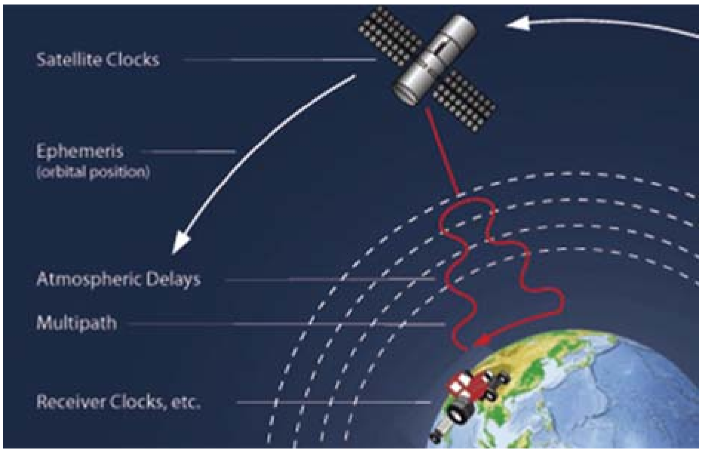
OmniSTAR XP:
15 cm Worldwide Service



OmniSTAR XP (15cm) is a **worldwide** **dual frequency** high accuracy solution. It is a L1/L2 solution requiring a dual frequency receiver.

Orbit and Clock correction data is used together with atmospheric corrections derived from the dual frequency data.

By utilizing carrier phase measurement, very high accuracy can be achieved. OmniSTAR XP service provides short term accuracy of 1-2 inches and long term repeatability of better than 10 centimeters, 95%CEP.

It is especially suited for Agricultural automatic steering systems. While it is slightly less accurate than OmniSTAR HP, it is available worldwide and its accuracy is a significant improvement over regional DGNSS such as WAAS.

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OmniSTAR VBS
Global Reliable
Sub-Meter Accuracy

OmniSTAR VBS is the foundational "**sub-meter**" level of service. It is an **L1 only**, code phase pseudo-range solution.

Pseudo-range correction data from OmniSTAR's regional reference sites is broadcast via satellite link to the user receiver.

These data are used, together with atmospheric modeling and knowledge of the receiver's location, to generate an internal RTCM SC104 correction specific to that location. This correction is then applied to the R-T solution.

A typical 24-hour sample of OmniSTAR VBS will show a 2-sigma (95%) of significantly less than 1 meter horizontal position error and the 3-sigma (99%) horizontal error will be close to 1 meter.

OmniSTAR G2

GPS + GLONASS

OmniSTAR G2 is a **worldwide dual frequency** high-accuracy solution which uses Orbit and Clock correction data.

OmniSTAR G2 includes GLONASS satellites and GLONASS correction data in the solution. The addition of GLONASS to the solution significantly **increases the number of satellites available** which is useful when faced with conditions that limit satellite visibility, such as terrain, vegetation or buildings.

OmniSTAR G2 service provides short-term accuracy of 1-2 inches and long term repeatability of better than 10 cm, 95%CEP. It is especially suited for operations in areas where trees or buildings may block the view of the sky and in areas affected by scintillation during times of high sunspot activity.

		Observable	Baseline	Broadcast message	accuracy	Init. time	Examples of Products
	DGPS (code)	Smoothed code (L1)	<100km	PRC, RRC	~ metre	Single epoch	RASANT
	RTK	Carrier L1/L2, L1	<10-15km	Carrier measurements	~ cm	~10 s	Several packages ...
LA- DGNSS	GBAS/ LAAS	Smoothed code (L1)	< 40 km	PRC, RRC	~ metre + Integrity	Single epoch	Honeywell GBAS station
	VRS	Carrier L1/L2, L1	< 50 km	Virtual Carrier measurements	few cm	~10 - 30 s	Omnistar HP, StarFix HP
WA- DGNSS	SBAS	Smoothed code (L1)	Continental	Orbits+ Clocks+ Ionosphere	~ metre + Integrity	Single epoch	WAAS, EGNOS, MSAS...
	GDGNSS	Iono-free code and carrier	Worldwide	Orbits + Clocks	~ dm	1/2h-1h.	Omnistar XP, StarFix XP, JPL-GDGPS
	F-PPP	L1, L2 code and carrier	Worldwide with Continental enhancement	Orbits+ Clocks+ Ionosphere+ DCBs + Frac. Ambig.	~ dm/cm (ambiguity fixing capability)	~5 m	Feasibility studies

References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, May 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, May 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga –Jamuna Press, 2004.

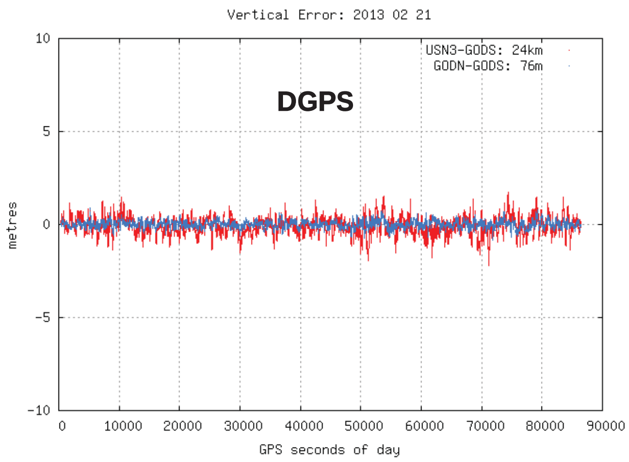
[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

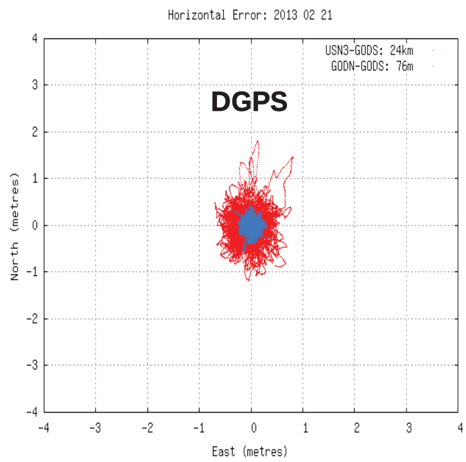
[RD-5] B. W. Parkinson and J.J. Spilker. Global Positioning System: Theory and Applications, Vol1 and Vol2. Progress in Astronautics and Aeronautics, Volume 164, Cambridge, Massachusetts, US.

Thank you

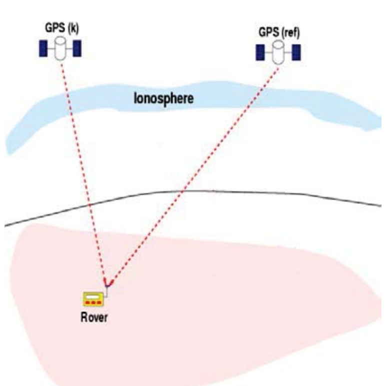
Backup slides

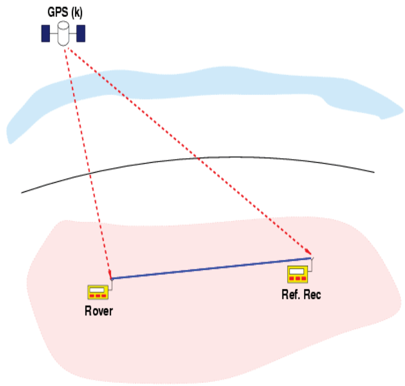
Example of Differential Atmospheric propagation effects analysis

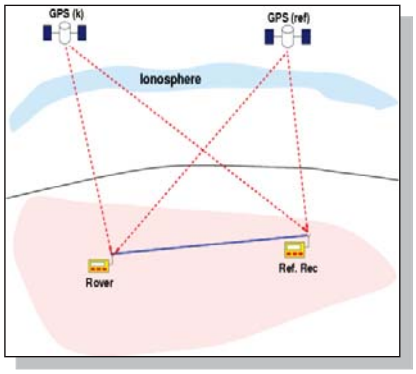




Single and double differences of receivers/satellites







$$\nabla \square_{\bullet} \equiv \square_{\bullet}^{k} - \square_{\bullet}^{R}$$

Receiver errors affecting both satellites are removed (e.g. Receiver clock)

$$\Delta \square^{\bullet} \equiv \square_{rov}^{\bullet} - \square_{ref}^{\bullet}$$

SIS errors affecting both receivers are removed (e.g. Satellite clocks,...)

$$\Delta\nabla \square \equiv \Delta \square^{k} - \Delta \square^{R} = \nabla \square_{rov} - \nabla \square_{ref}$$

Receiver errors common for all satellites do not affect positioning (as they are assimilated in the receiver clock estimate). Thence:

- Only residual errors in single differences between sat. affect absolute posit.
- Only residual errors in double differences between sat. and receivers affect relative positioning.

Exercise,
Discuss the previous sentences.

Depicting atmosphere propagation errors affecting DGNSS: Double-differences between satellites and receivers

$$L_{1rec}^{sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - \cancel{dt^{sat}}) + T_{rec}^{sat} - I_{1,rec}^{sat} + \lambda_1\omega_{rec}^{sat} + b_{1,rec} + \cancel{b_1^{sat}} + \lambda_1 N_{1,rec}^{sat} + m_{L_1} + \varepsilon_{L_1}$$

$$L_{1recR}^{sat} = \rho_{recR}^{sat} + c\cdot(dt_{recR} - \cancel{dt^{sat}}) + T_{recR}^{sat} - I_{1,recR}^{sat} + \lambda_1\omega_{recR}^{sat} + b_{1,recR} + \cancel{b_1^{sat}} + \lambda_1 N_{1,recR}^{sat} + m_{L_1} + \varepsilon_{L_1}$$

Differencing between receivers cancels satellite-only-dependent terms

$$\boxed{\Delta L_1^{sat} = \Delta\rho^{sat} - c\Delta dt + \Delta T^{sat} + \Delta I_1^{sat} + \lambda_1\Delta\omega^{sat} + \Delta b_1 + \lambda_1\Delta N_1^{sat} + m_{\Delta L_1} + \varepsilon_{\Delta L_1}}$$

$$\Delta L_1^{sat} = \Delta\rho^{sat} - \cancel{c\Delta dt} + \Delta T^{sat} - \Delta I_1^{sat} + \lambda_1\Delta\omega^{sat} + \cancel{\Delta b_1} + \lambda_1\Delta N_1^{sat} + m_{\Delta L_1} + \varepsilon_{\Delta L_1}$$

$$\Delta L_1^{satR} = \Delta\rho^{satR} - \cancel{c\Delta dt} + \Delta T^{satR} - \Delta I_1^{satR} + \lambda_1\Delta\omega^{satR} + \cancel{\Delta b_1} + \lambda_1\Delta N_1^{satR} + m_{\Delta L_1} + \varepsilon_{\Delta L_1}$$

Differencing between satellites cancels receiver-only dependent terms

$$\boxed{\nabla\Delta L_1 = \nabla\Delta\rho + \nabla\Delta T - \nabla\Delta I_1 + \lambda_1\nabla\Delta\omega + \lambda_1\nabla\Delta N_1 + m_{\nabla\Delta L_1} + \varepsilon_{\nabla\Delta L_1}}$$

Only residual errors in double differences between sat. and receivers affect relative positioning.

Double-differences between satellites and receivers

$$\nabla\Delta L_1 = \nabla\Delta\rho + \nabla\Delta T - \nabla\Delta I_1 + \lambda_1\nabla\Delta\omega + \lambda_1\nabla\Delta N_1 + m_{\nabla\Delta L_1} + \varepsilon_{\nabla\Delta L_1}$$

Satellite and receiver clocks and fractional part of ambiguities cancel.

Comments:

- The wind-up term $\Delta\nabla\omega$ can be neglected, except over long baselines.
- Double-differenced ambiguities are integer numbers of wavelengths.

Exercise,
Show that, neglecting the wind-up, the following expressions are met over a continuous carrier phase arch:

$$Lc = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2}$$

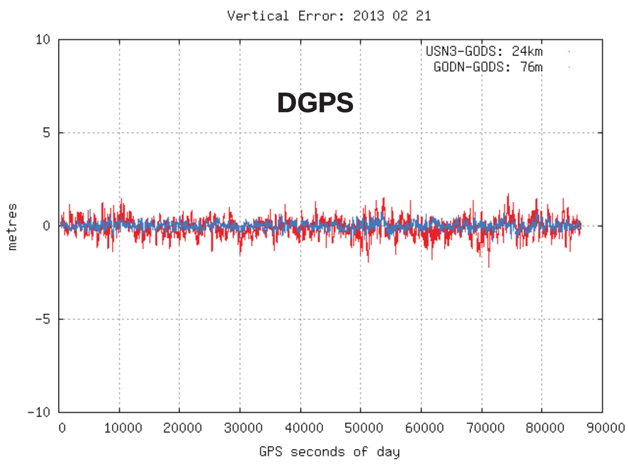
$$\gamma = \left(\frac{f_1}{f_2}\right)^2$$

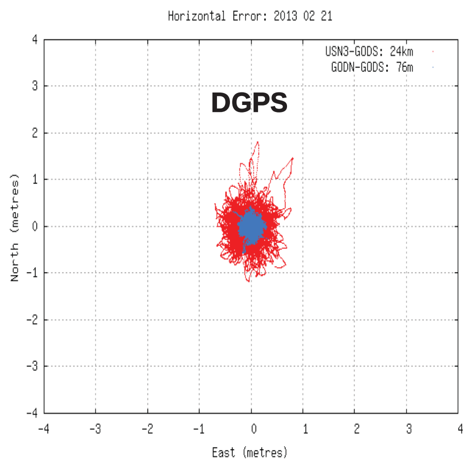
$$\nabla\Delta L_1 - \nabla\Delta\rho \simeq \nabla\Delta T - \nabla\Delta I_1 + bias_1$$

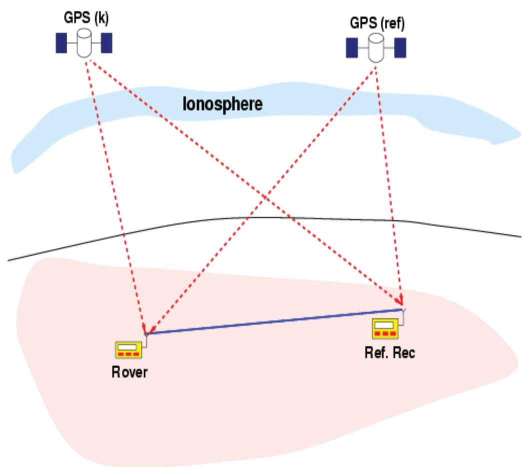
$$\nabla\Delta L_2 - \nabla\Delta\rho \simeq \nabla\Delta T - \gamma\,\nabla\Delta I_1 + bias_2$$

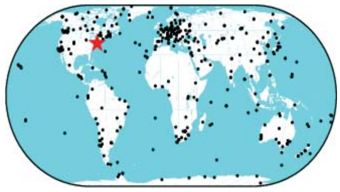
$$\nabla\Delta L_C - \nabla\Delta\rho \simeq \nabla\Delta T + bias_C \longleftarrow$$ Ionosphere-free: Only Troposphere

$$\nabla\Delta L_1 - \nabla\Delta L_2 \simeq -(1-\gamma)\nabla\Delta I_1 + bias_I \longleftarrow$$ Geometry-free: Only Ionosphere



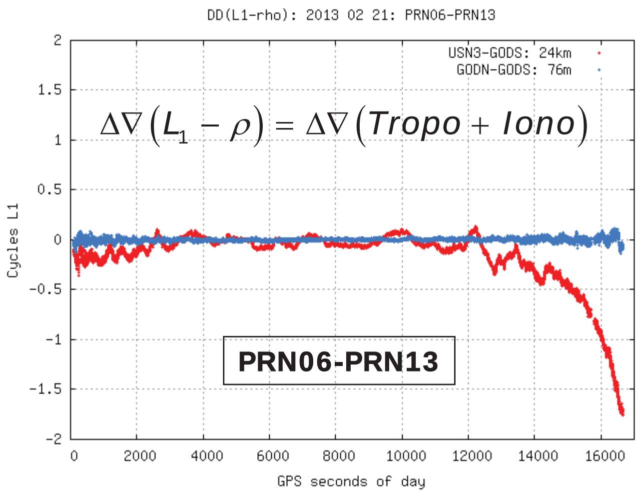


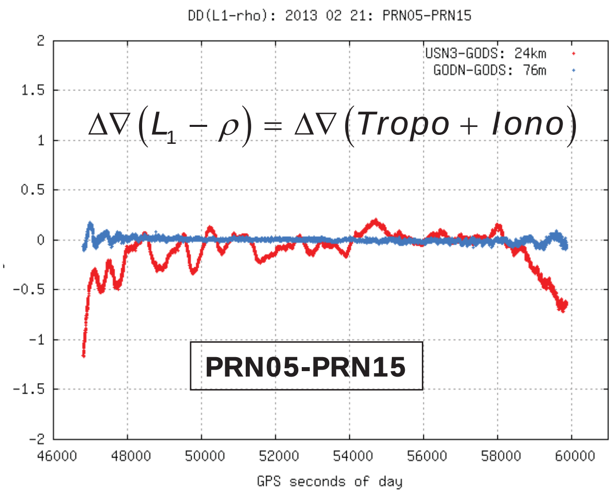

GPS (k)
GPS (ref)
Ionosphere
Rover
Ref. Rec



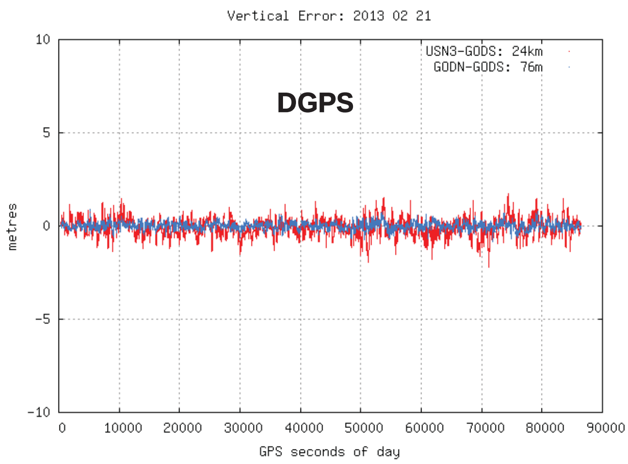

GODN
GODS
GODE

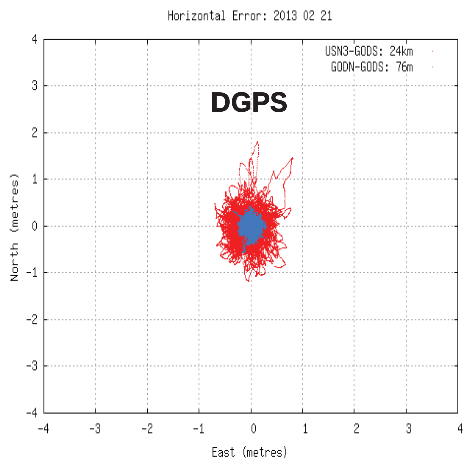

USN3
23.6 km
GODN
76 m
GODS


DD(L1-rho): 2013 02 21: PRN06-PRN13
USN3-GODS: 24km
GODN-GODS: 76m
$\Delta\nabla\left(L_1-\rho\right)=\Delta\nabla\left(Tropo+Iono\right)$
PRN06-PRN13
Cycles L1
GPS seconds of day


DD(L1-rho): 2013 02 21: PRN05-PRN15
USN3-GODS: 24km
GODN-GODS: 76m
$\Delta\nabla\left(L_1-\rho\right)=\Delta\nabla\left(Tropo+Iono\right)$
PRN05-PRN15
GPS seconds of day


USN3
23.6 km
GODN
76 m
GODS


Vertical Error: 2013 02 21
USN3-GODS: 24km
GODN-GODS: 76m
DGPS
metres
GPS seconds of day


Horizontal Error: 2013 02 21
USN3-GODS: 24km
GODN-GODS: 76m
DGPS
North (metres)
East (metres)

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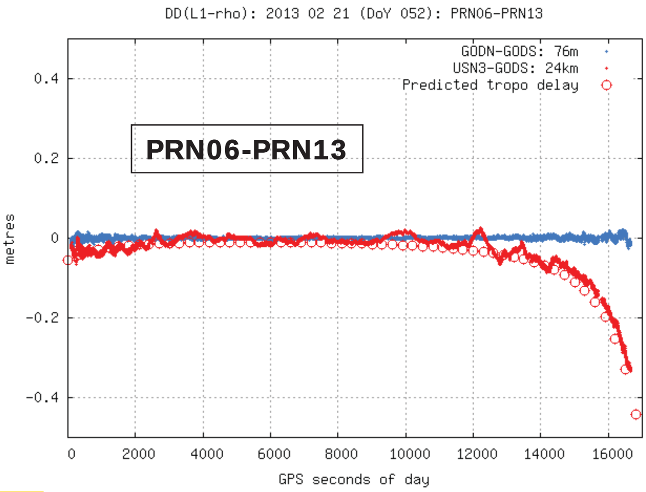

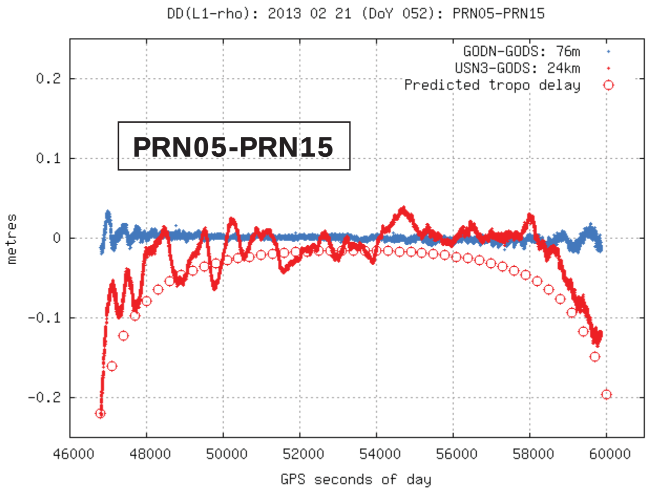

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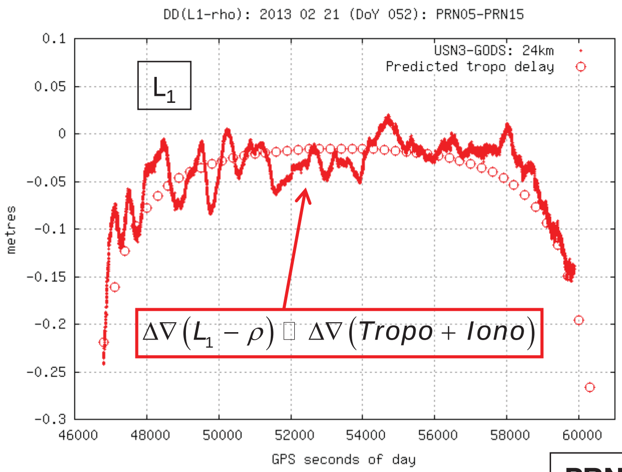
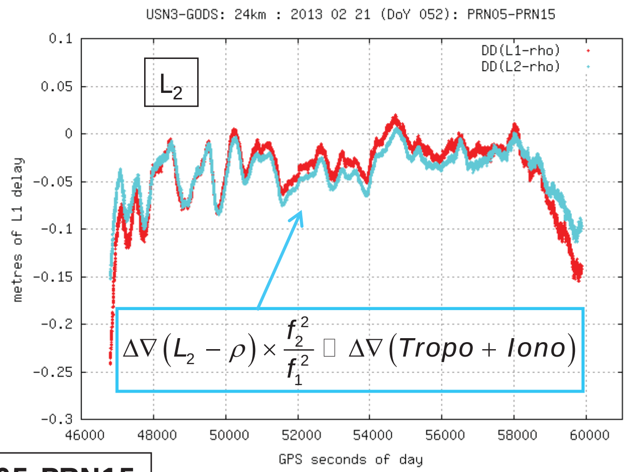
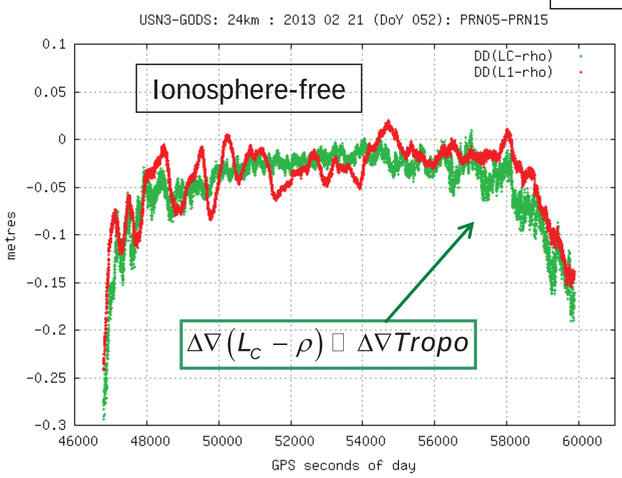
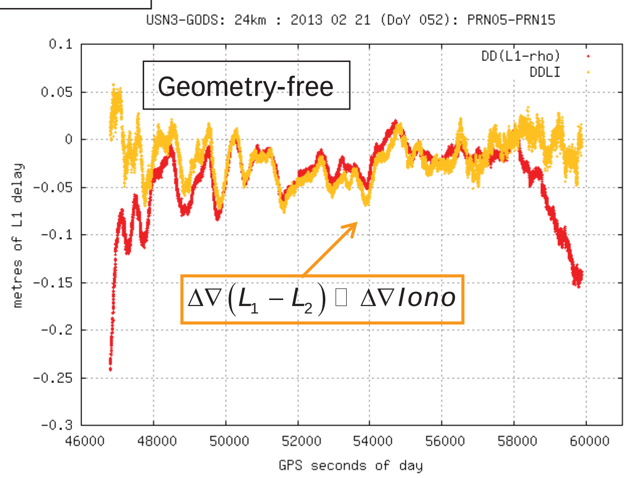




PRN05-PRN15





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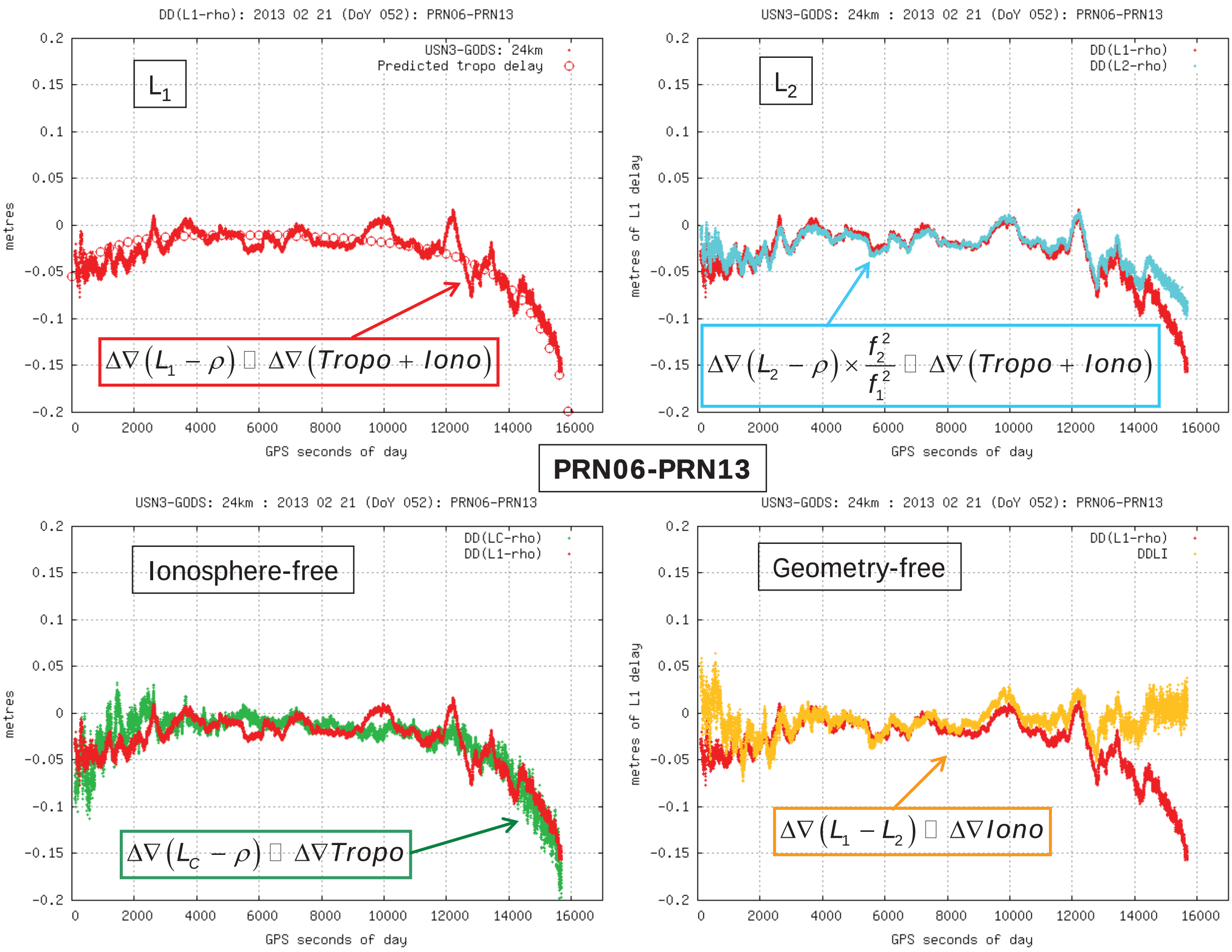

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Other
Backup slides

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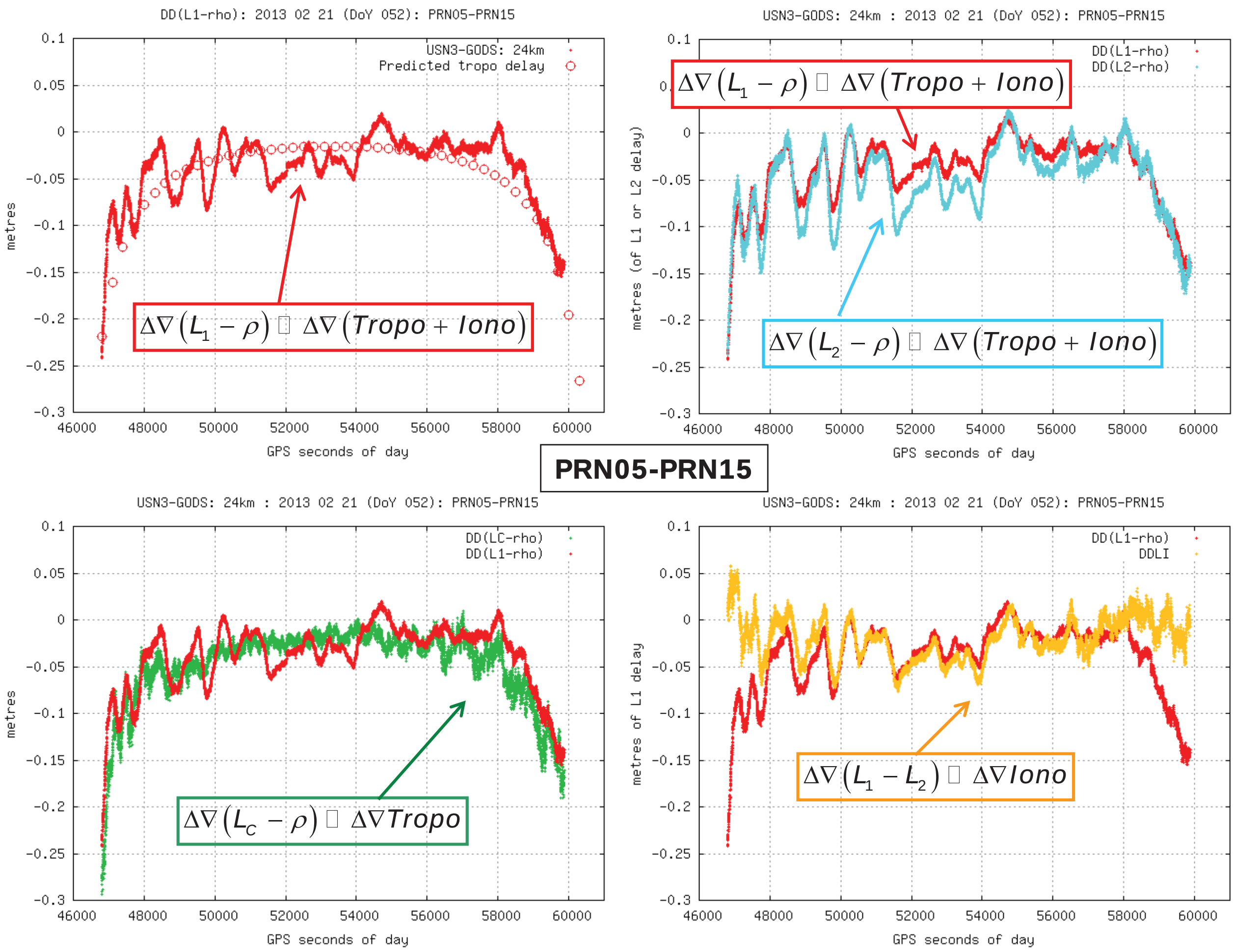

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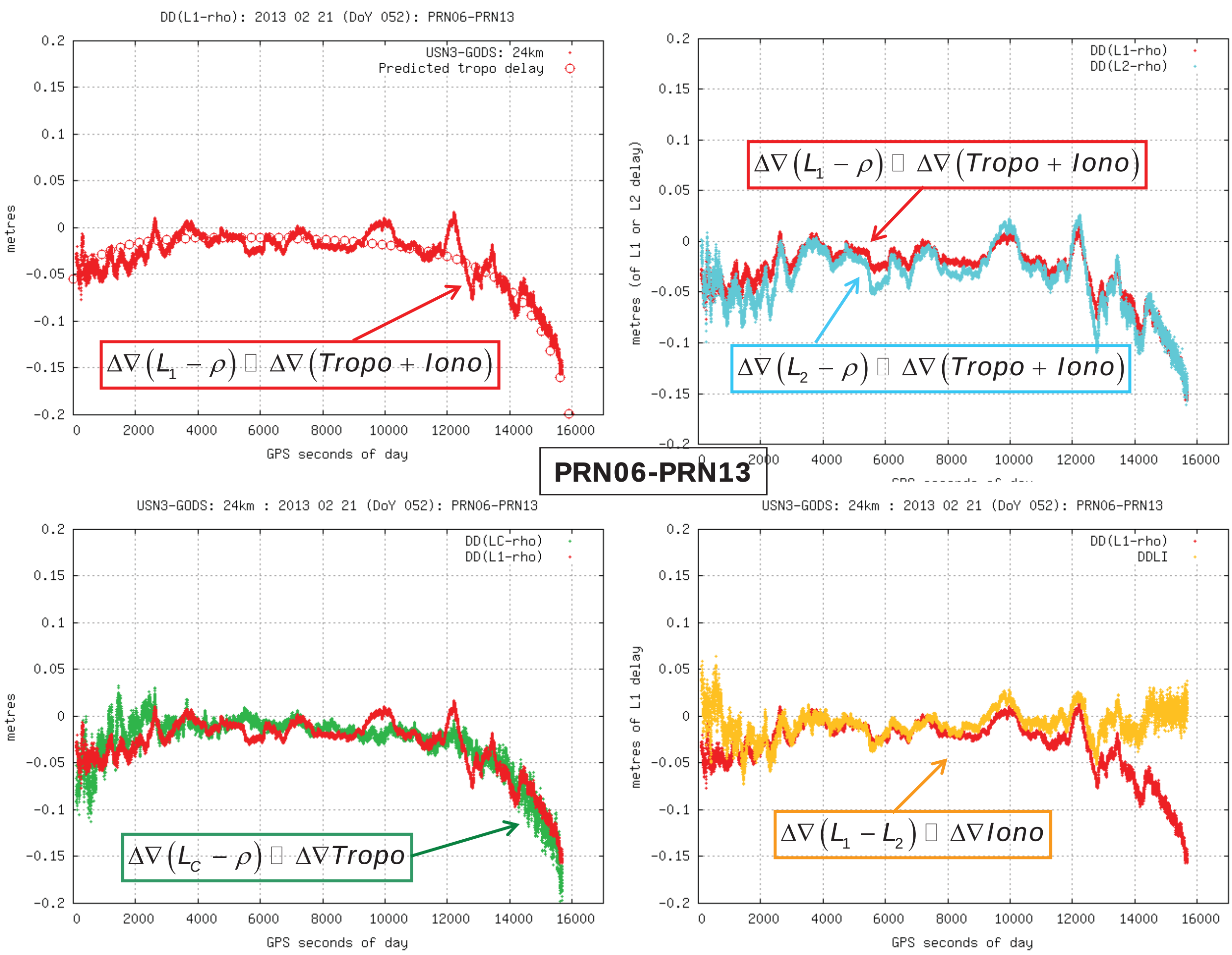

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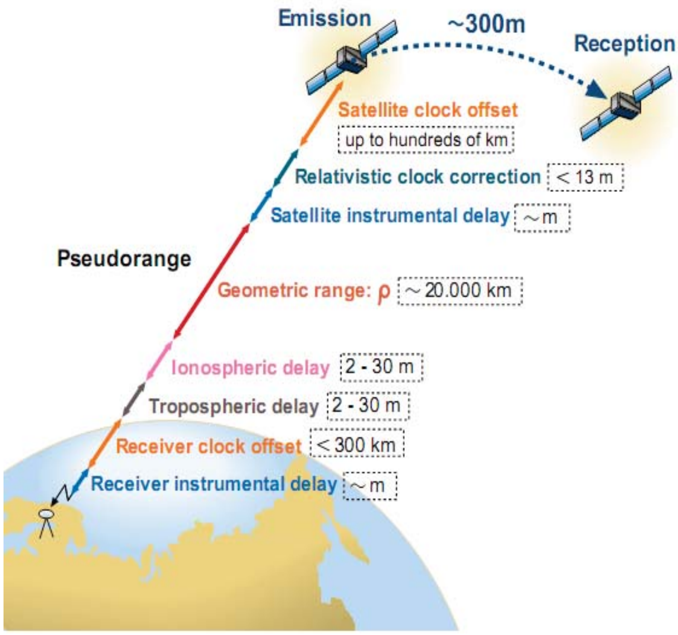
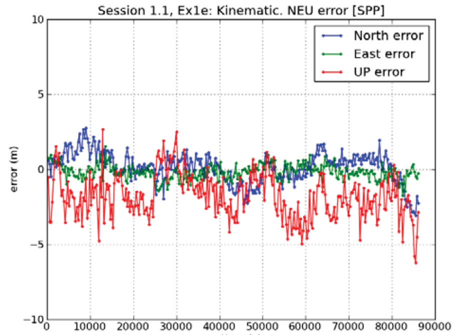
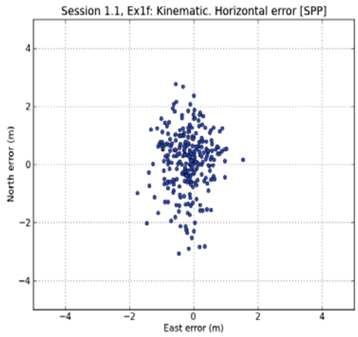
GNSS Positioning

Standalone Positioning: GNSS receiver autonomous positioning using broadcast orbits and clocks (SPS, PPS).







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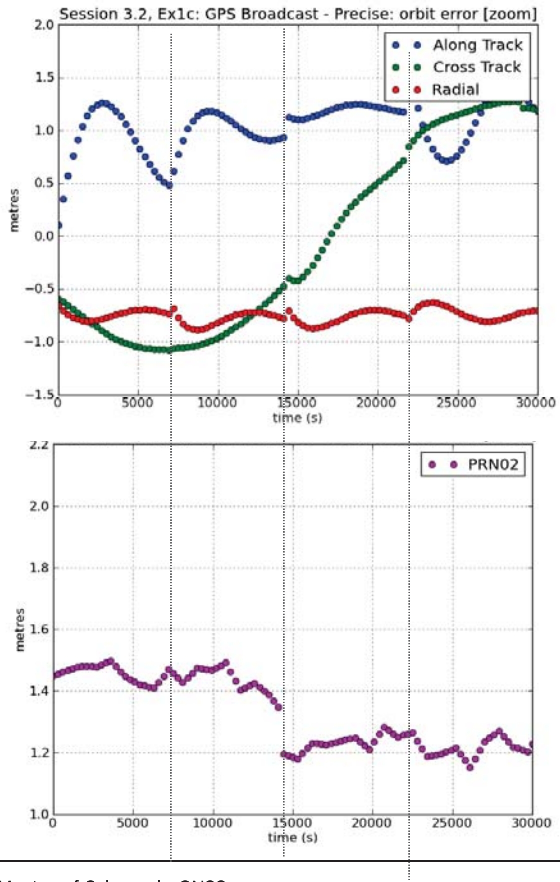
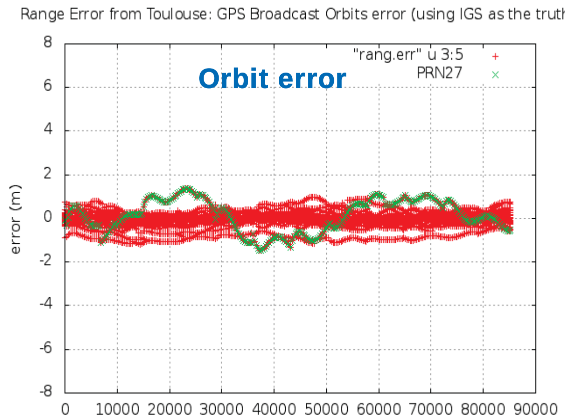
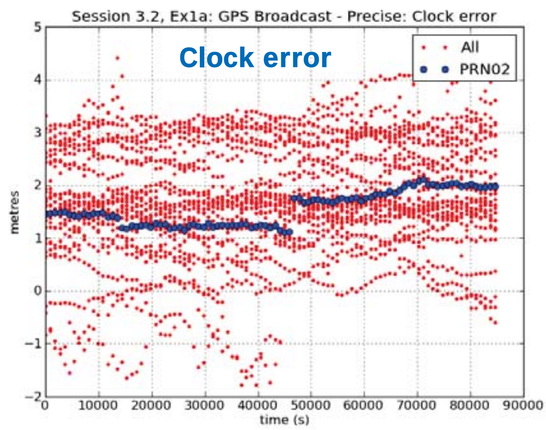
GNSS Positioning: Space Segment errors







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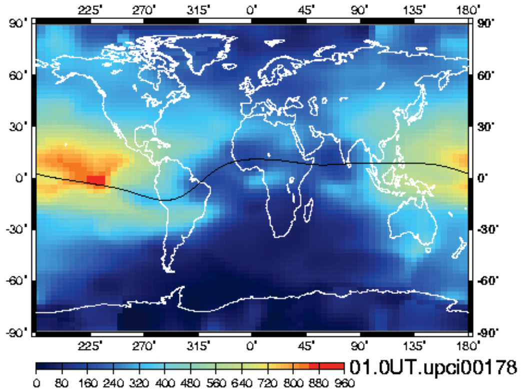
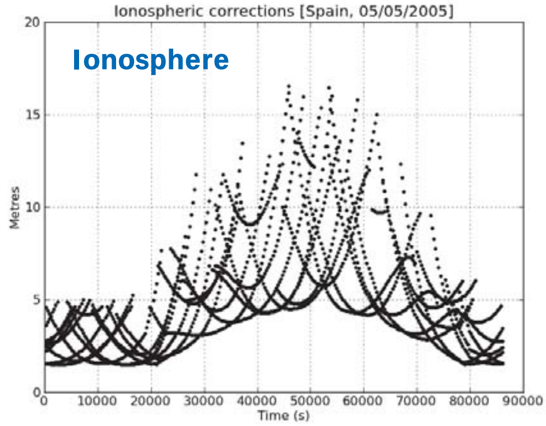
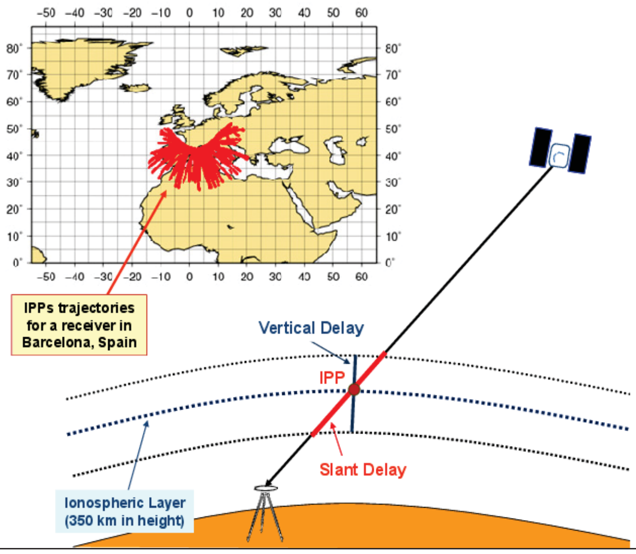
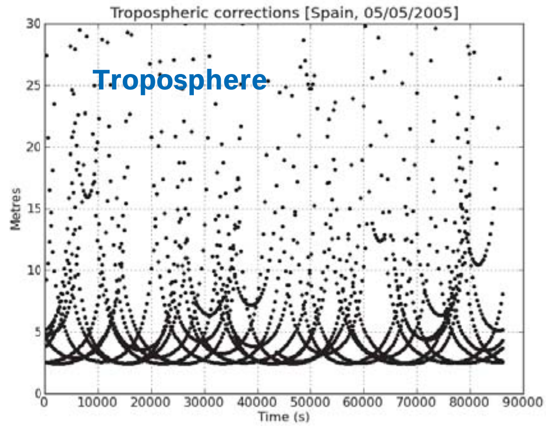
GNSS Positioning: Propagation errors









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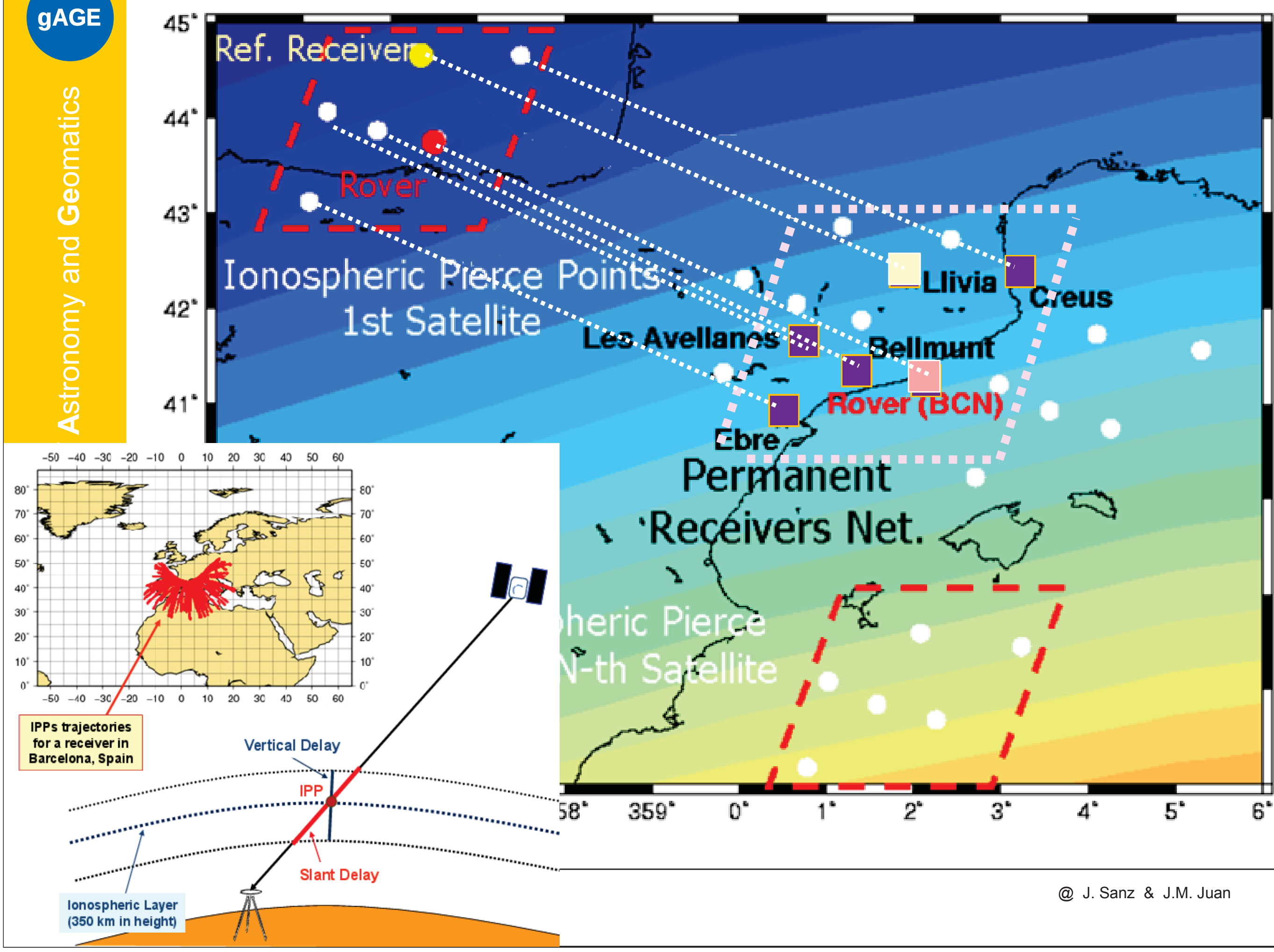
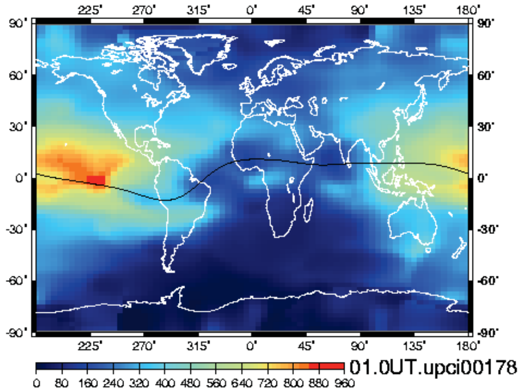
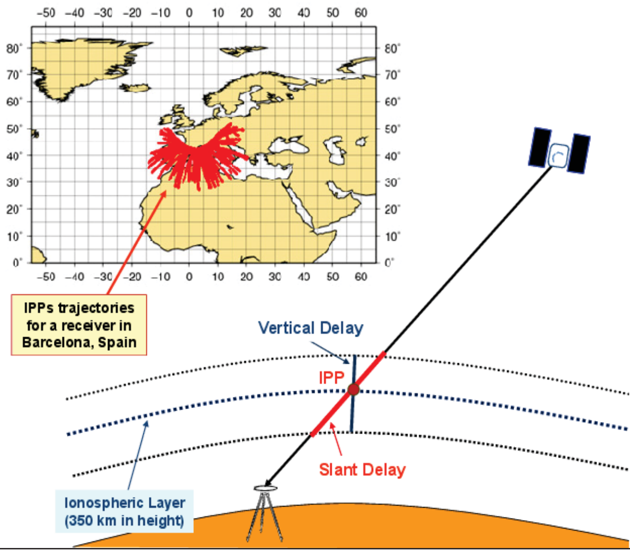
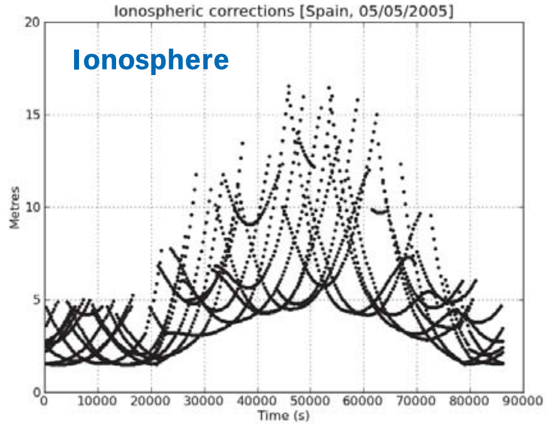
Astronomy and Geomatics

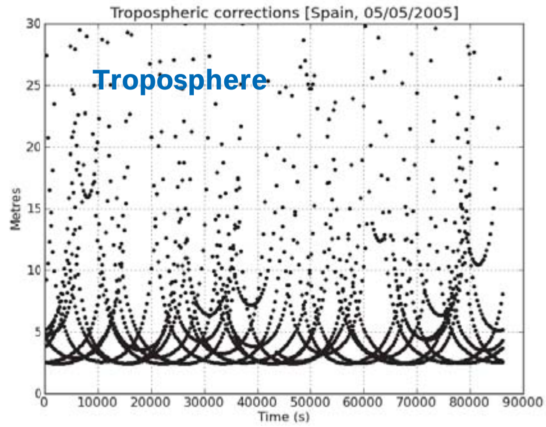


GNSS Positioning: Propagation errors









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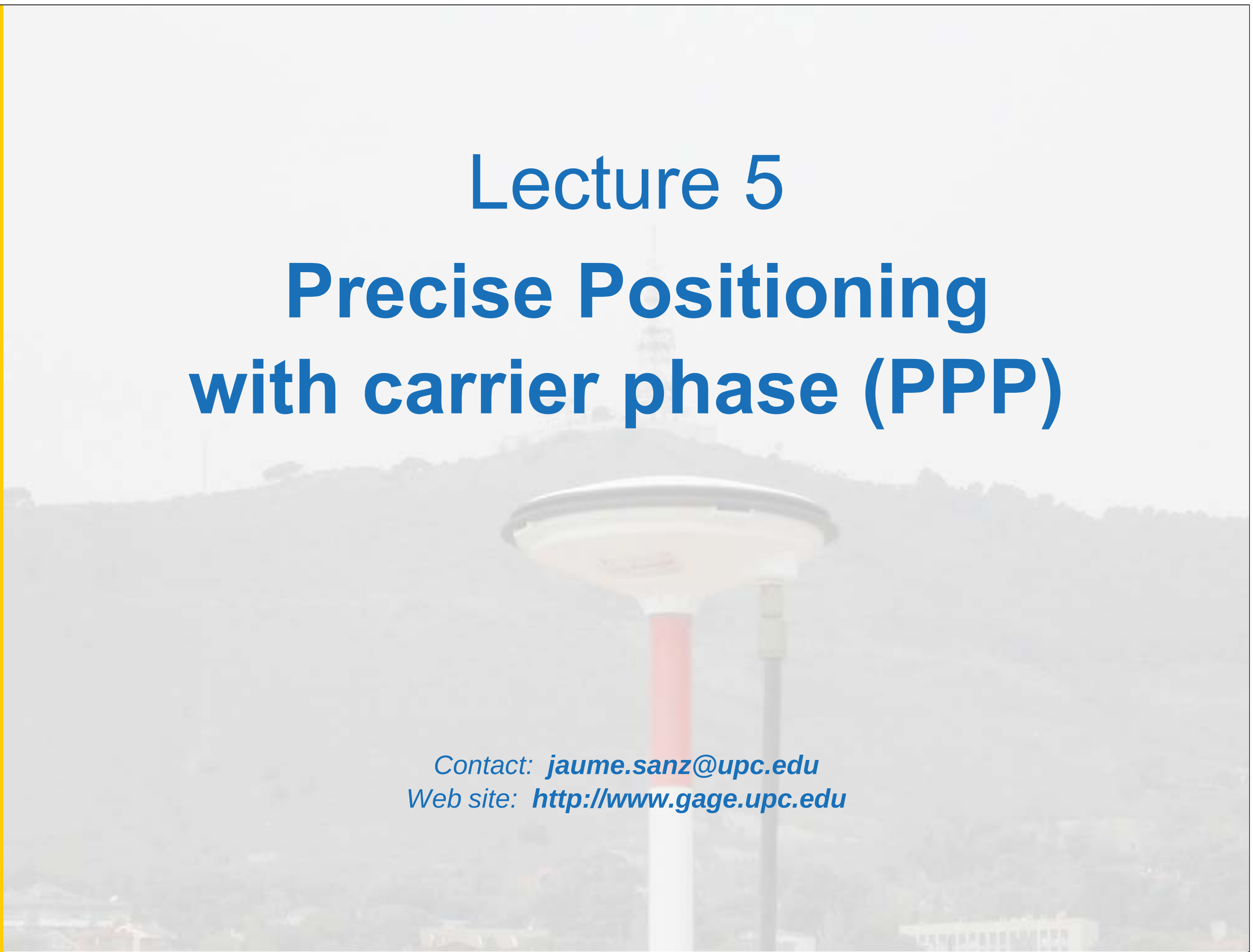

Lecture 5

Precise Positioning with carrier phase (PPP)

Contact: ***jaume.sanz@upc.edu***
Web site: ***http://www.gage.upc.edu***

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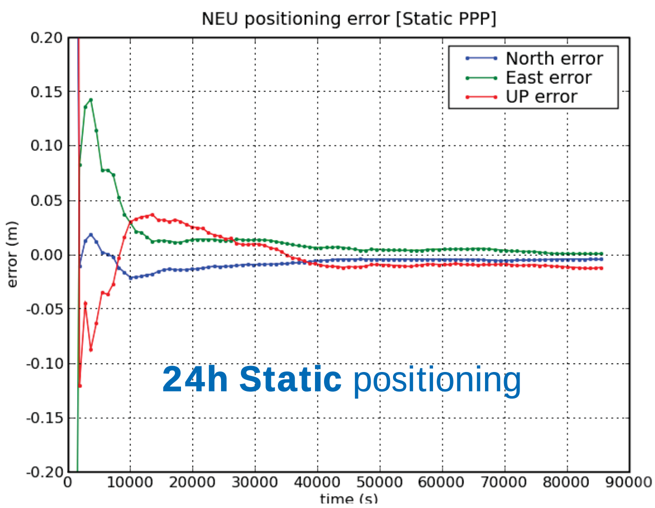
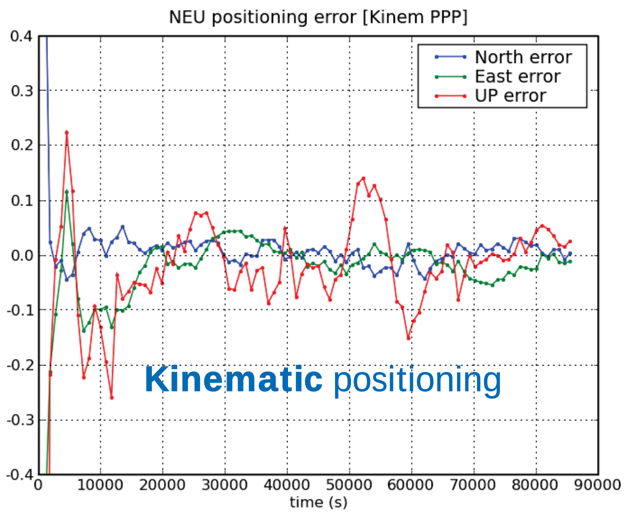
Authorship statement

22 Jan 2015

Contents

The PPP technique allows **centimetre-level** accuracy to be **achieved for static** positioning and **decimetre level, or better, for kinematic positioning, after the best part of one hour.**





This high accuracy requires the use of code and carrier measurements and an accurate measurement modelling up to centimetre level or better.

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Contents

Precise Point Positioning (PPP)

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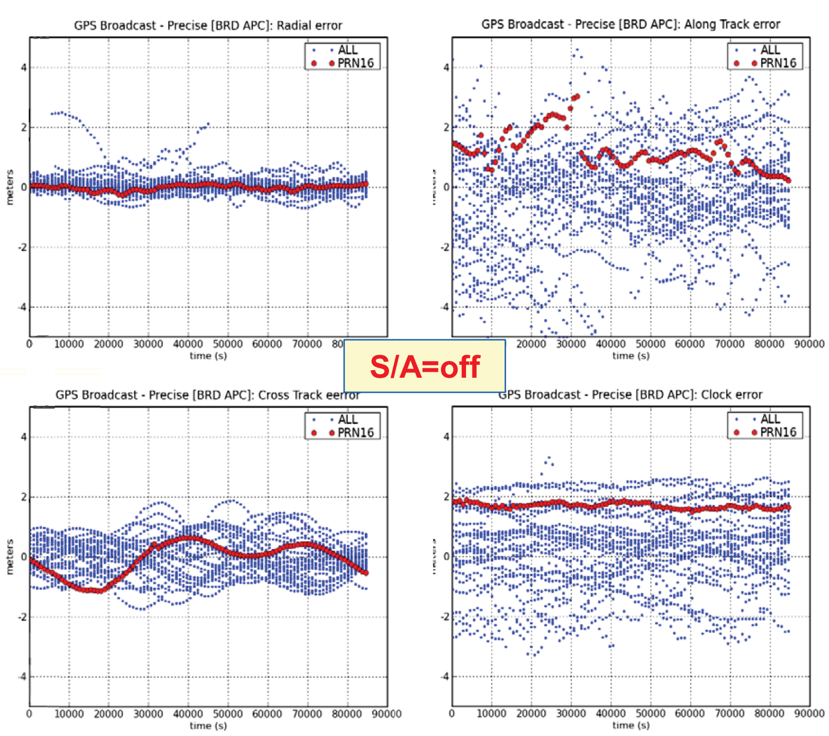
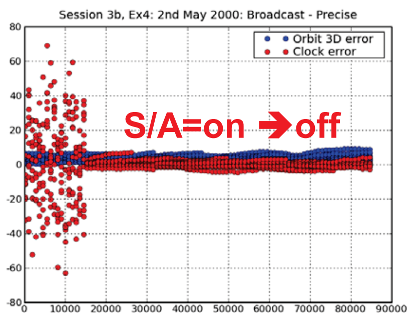
Satellite Orbits and Clocks

Broadcast versus precise



- With S/A=on, clocks were degraded several tens of meters.



- Under S/A=off , the broadcast orbits and clocks are accurate at few meters level (see plots at left).
- **IGS precise orbits & clocks are accurate at few cm level**

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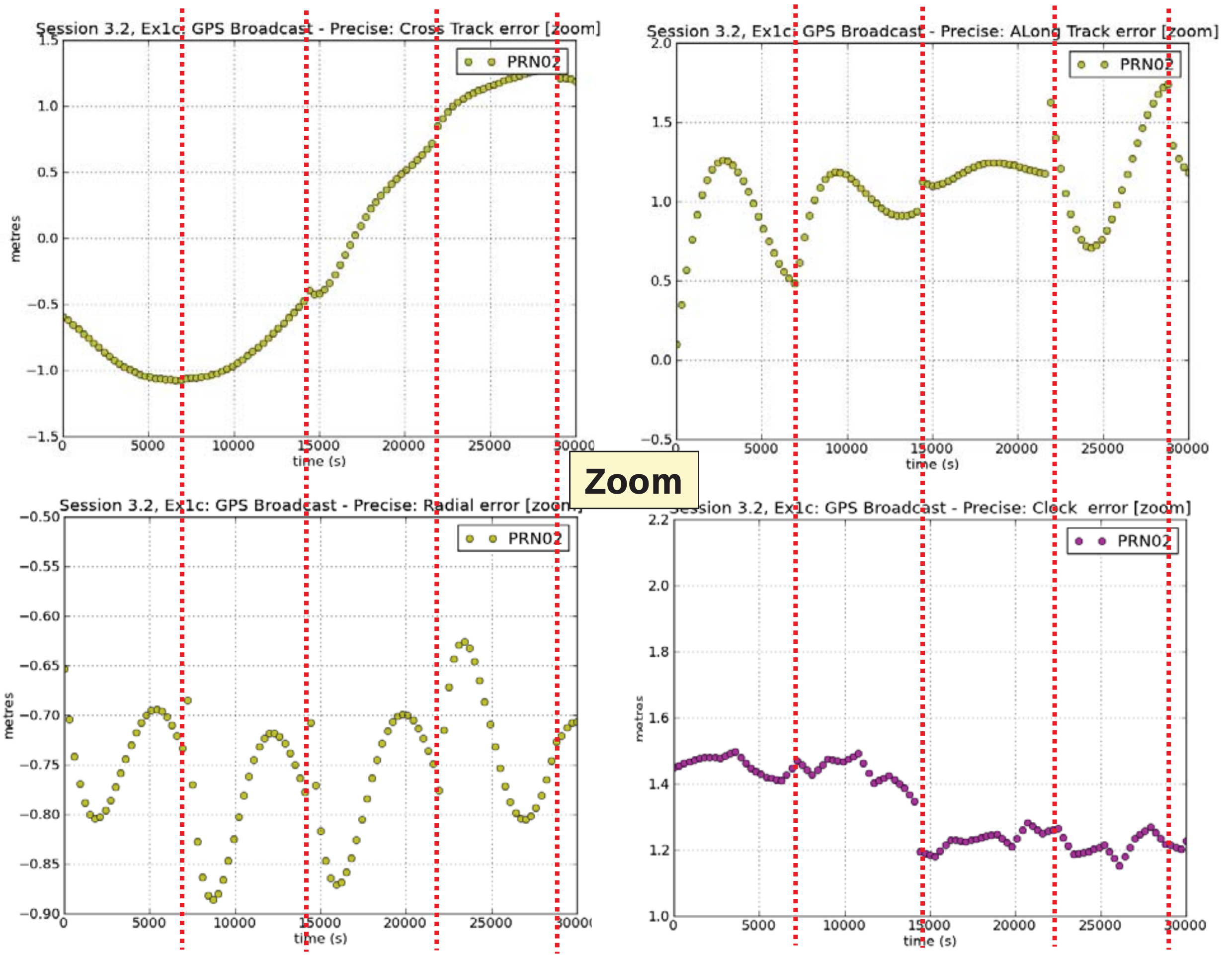

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IGS Precise orbit and clock products:

RMS accuracy, latency and sampling

Products (delay)	Broadcast (real time)	Ultra-rapid Predicted (real time)	Ultra-rapid Observed (3–9 h)	Rapid (17–41 h)	Final (12–18 d)
Orbit GPS (sampling)	~100 cm (~2 h)	~5 cm (15 min)	~3 cm (15 min)	~2.5cm (15 min)	~ 2.5 cm (15 min)
Glonass (sampling)					~5 cm (15 min)
Clock GPS (sampling)	~5 ns (daily)	~3 ns (15 min)	~150 ps (15 min)	~75 ps (5 min)	~75 ps (30 s)
Glonass (sampling)					~ TBD (15 min)

http://igscb.jpl.nasa.gov/components/prods.html

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Computation of satellite coordinates from precise products.

Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**

Orbits are given by *(x,y,z)* coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

$$
\begin{aligned}
P_n(x) &= \sum_{i=1}^{n} y_i \frac{\prod_{j\neq i}(x-x_j)}{\prod_{j\neq i}(x_i-x_j)} \\
&= y_1 \frac{x-x_2}{x_1-x_2}\cdots\frac{x-x_n}{x_1-x_n} + \cdots \\
&\quad + y_i \frac{x-x_1}{x_i-x_1}\cdots\frac{x-x_{i-1}}{x_i-x_{i-1}}\frac{x-x_{i+1}}{x_i-x_{i+1}}\cdots\frac{x-x_n}{x_i-x_n} + \cdots \\
&\quad + y_n \frac{x-x_1}{x_n-x_1}\cdots\frac{x-x_{n-1}}{x_n-x_{n-1}}
\end{aligned}
$$

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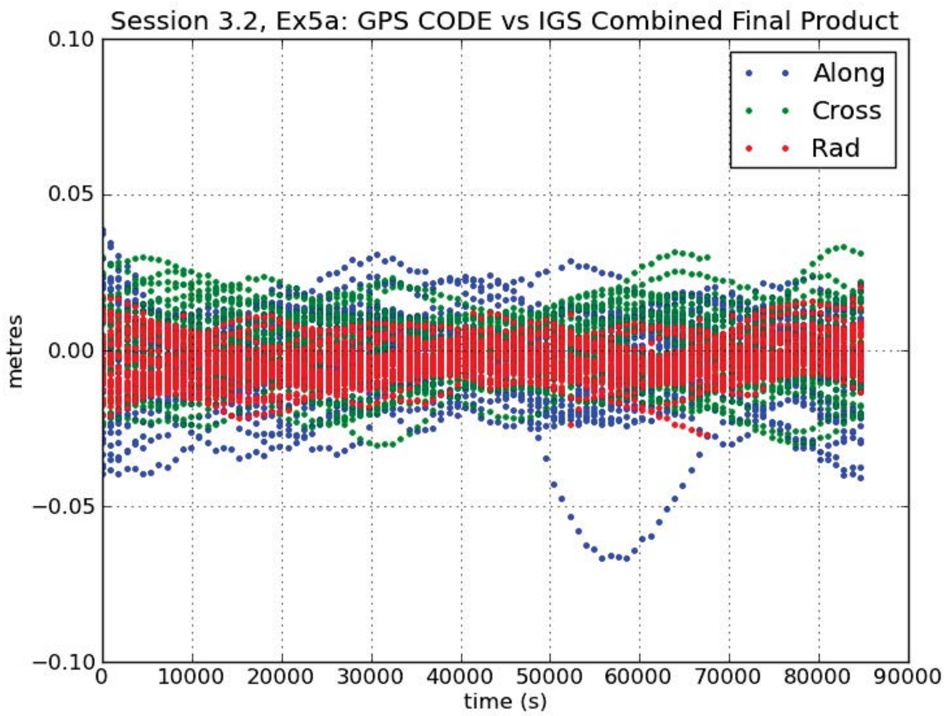
IGS orbit and clock products (for PPP):

Discrepancy between the different centres



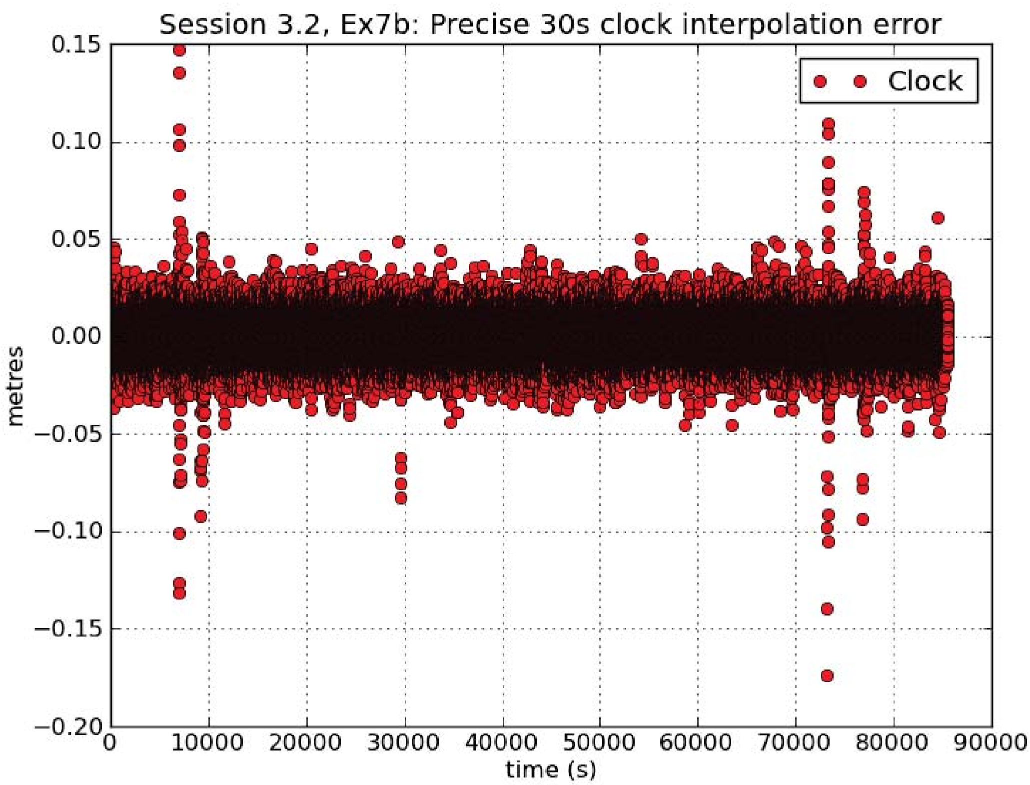
Computation of satellite clocks from precise products

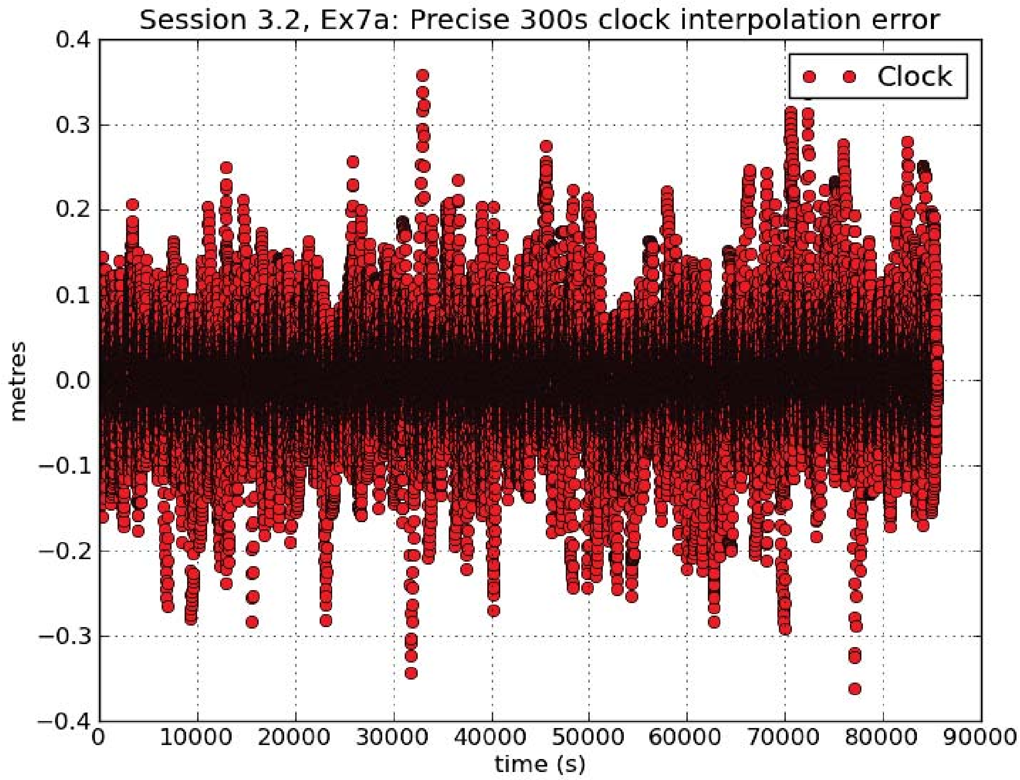
Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**

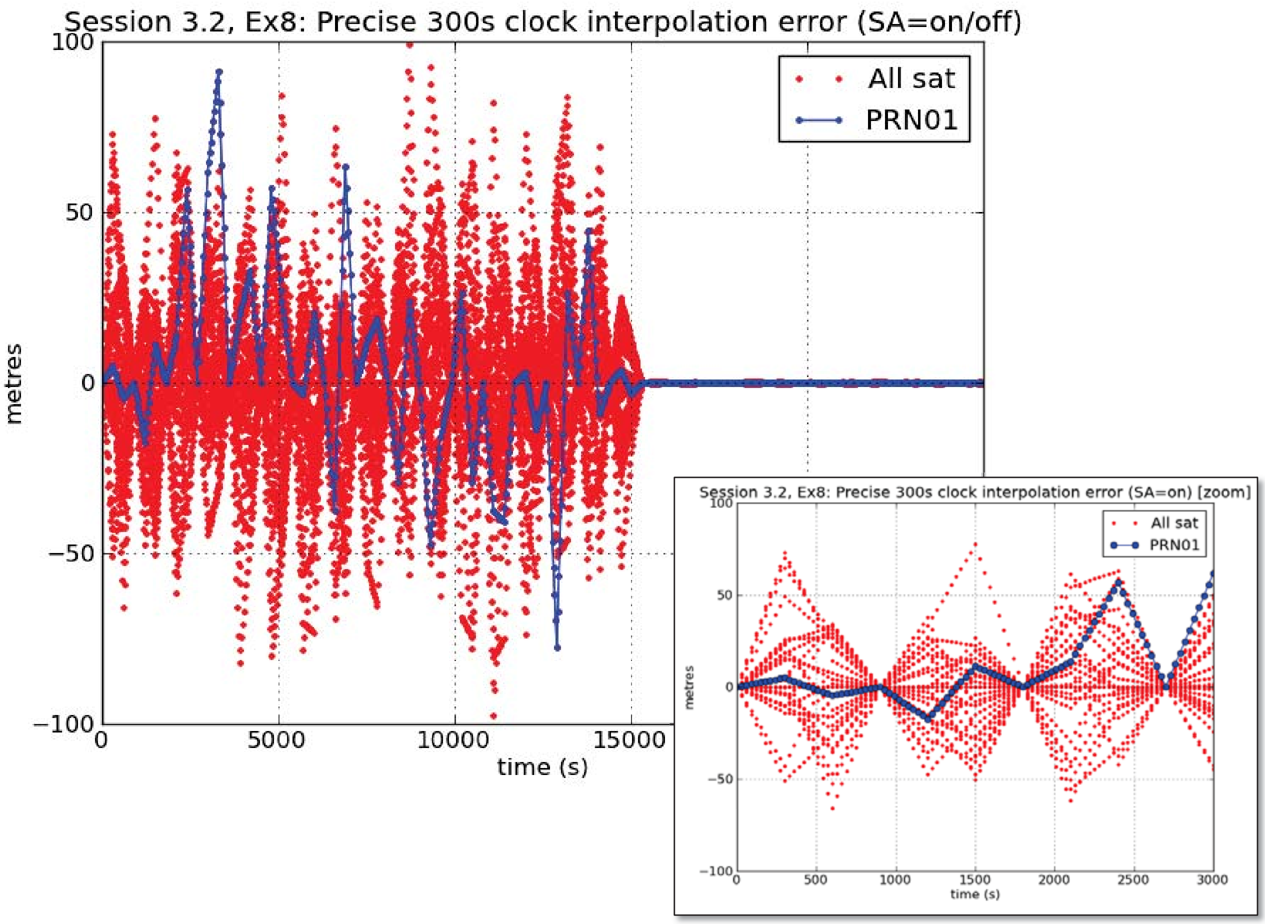
They are providing precise orbits and clock files with a sampling rate of 15 min, as well as precise clock files with a sample rate of 5 min and 30 s in SP3 format.

Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.

Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.

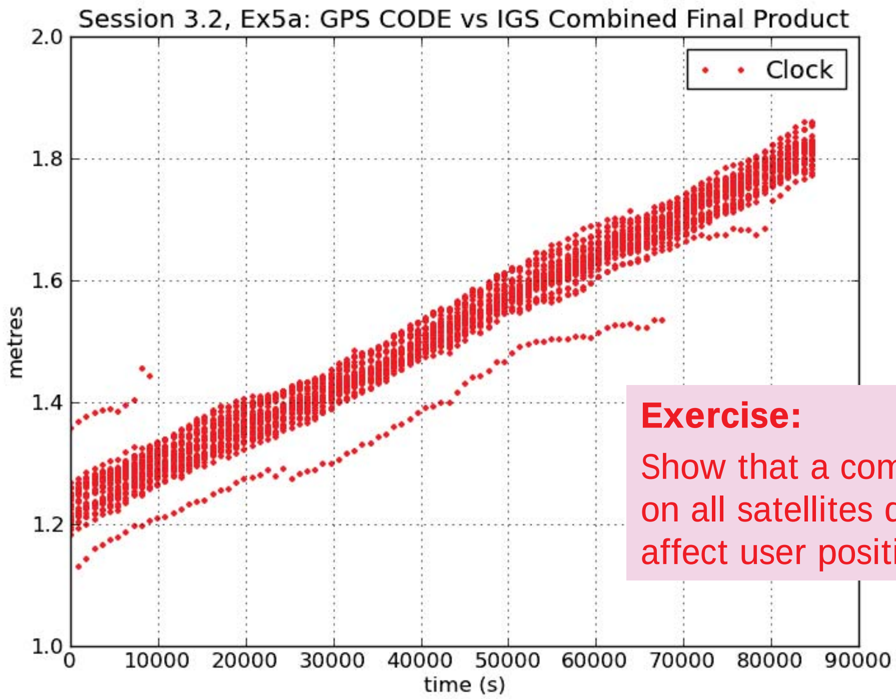



Session 3.2, Ex7a: Precise 300s clock interpolation error
Clock
metres
time (s)


Session 3.2, Ex8: Precise 300s clock interpolation error (SA=on/off)
All sat
PRN01
metres
time (s)
Session 3.2, Ex8: Precise 300s clock interpolation error (SA=on) [zoom]

IGS orbit and clock products (for PPP):

Discrepancy between the different centres



Exercise:
Show that a common error on all satellites does not affect user positioning.

Contents

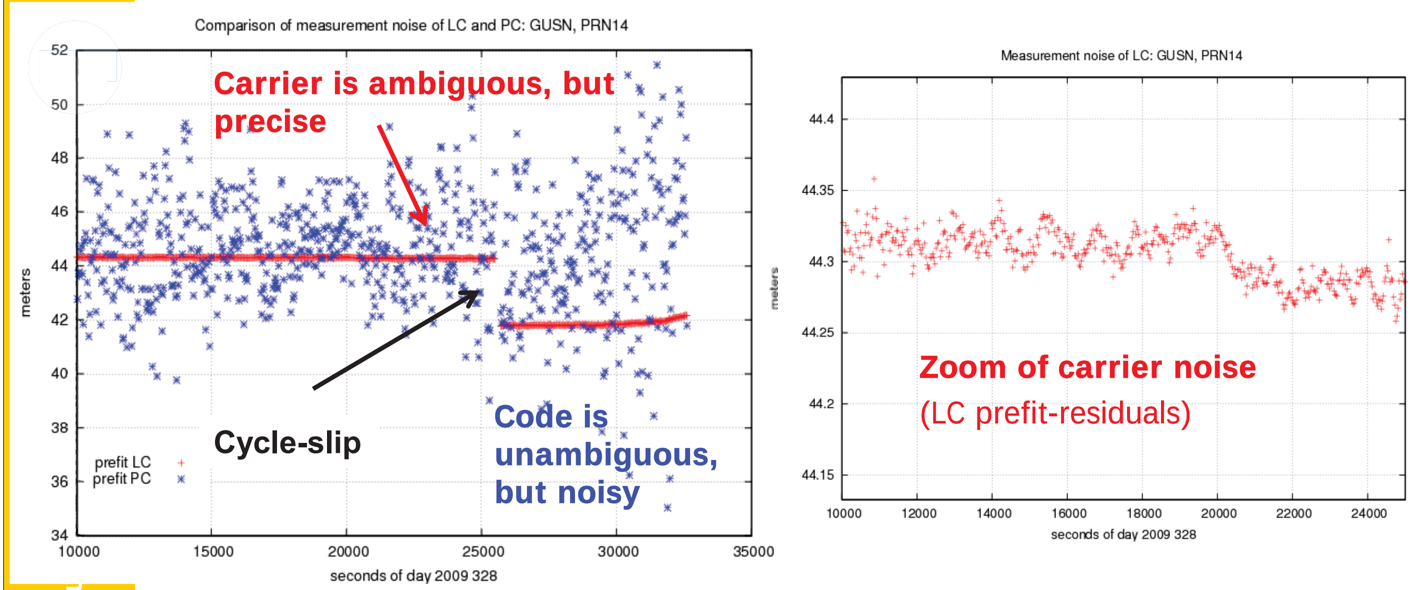
Precise Point Positioning (PPP)

Measurements: Code and carrier

For high-accuracy positioning, the carrier phase must be used, besides the code pseudorange.

As commented before, the carrier measurements are very precise, typically at the level of a few millimetres, but contain unknown ambiguities which change every time the receiver locks the signal after a cycle slip.

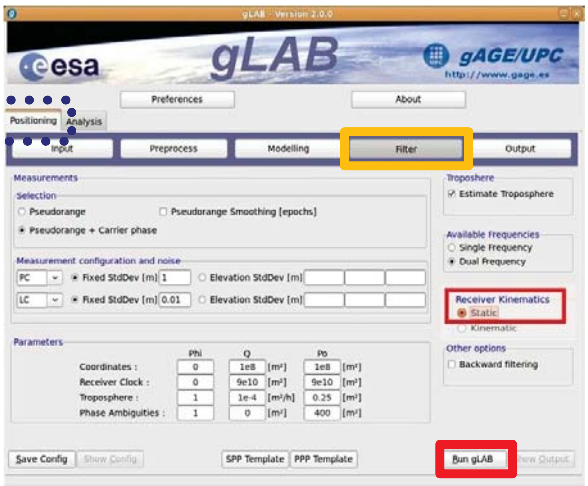
Nevertheless, such ambiguities can be estimated in the navigation solution, together with the coordinates and other parameters.



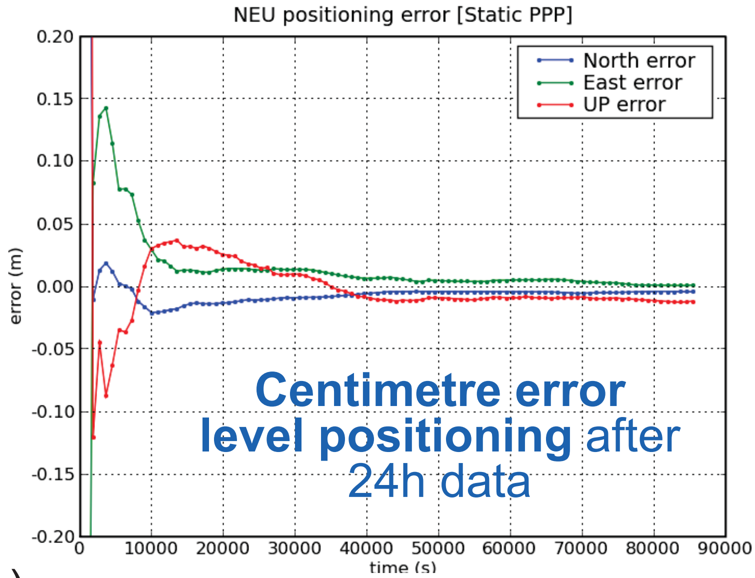
- **Code** measurements are unambiguous but noisy (meter level noise).
- **Carrier** measurements are precise (few millimetres of noise) but ambiguous (the unknown biases can reach thousands of km).
- **Carrier phase biases are estimated in the navigation filter** along with the other parameters (coordinates, clock offsets, etc.). If these biases were fixed, measurements accurate to the level of few millimetres would be available for positioning. However, some time is needed to decorrelate such biases from the other parameters in the filter, and the estimated values are not fully unbiased.

Precise Point Positioning: Static

From default configuration of **[PPP Template]**,
Select **Static** in the **[Filter]** panel. **Run *gLAB*** and plot results

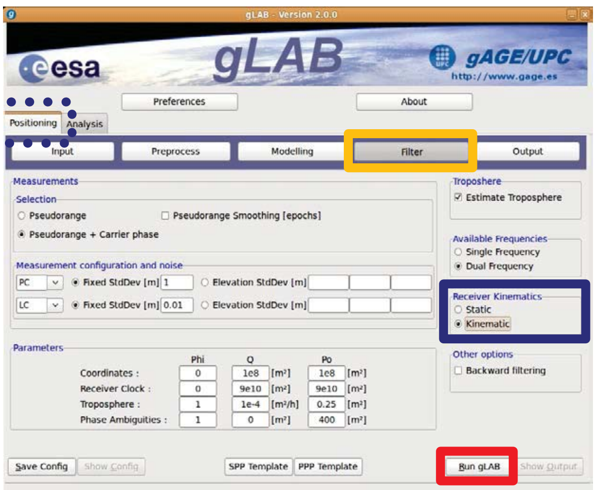


Receiver positioned as a permanent station (static mode)

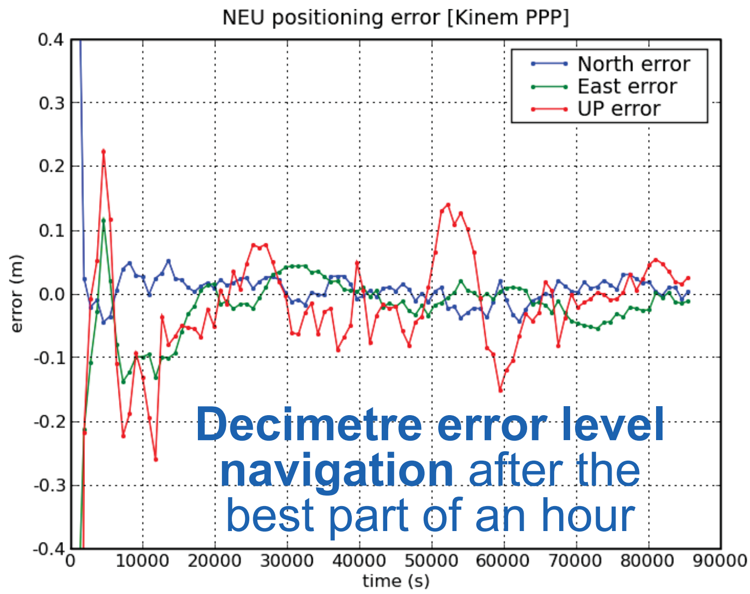


Precise Point Positioning: Kinematic

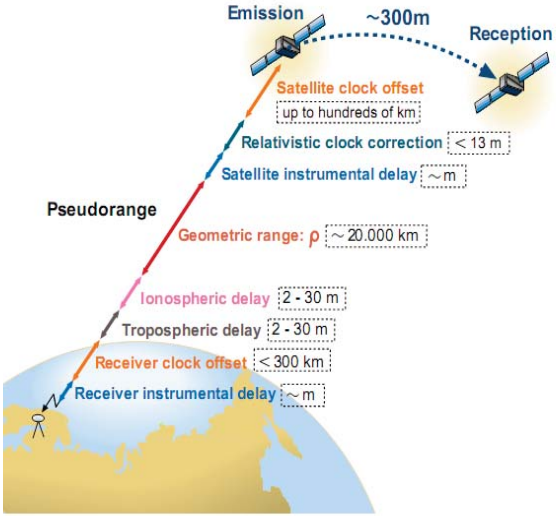
From default configuration of **[PPP Template]**,
Select **kinematics** in the **[Filter]** panel. **Run *gLAB*** and plot results



Receiver navigated as a rover in a pure kinematic mode.



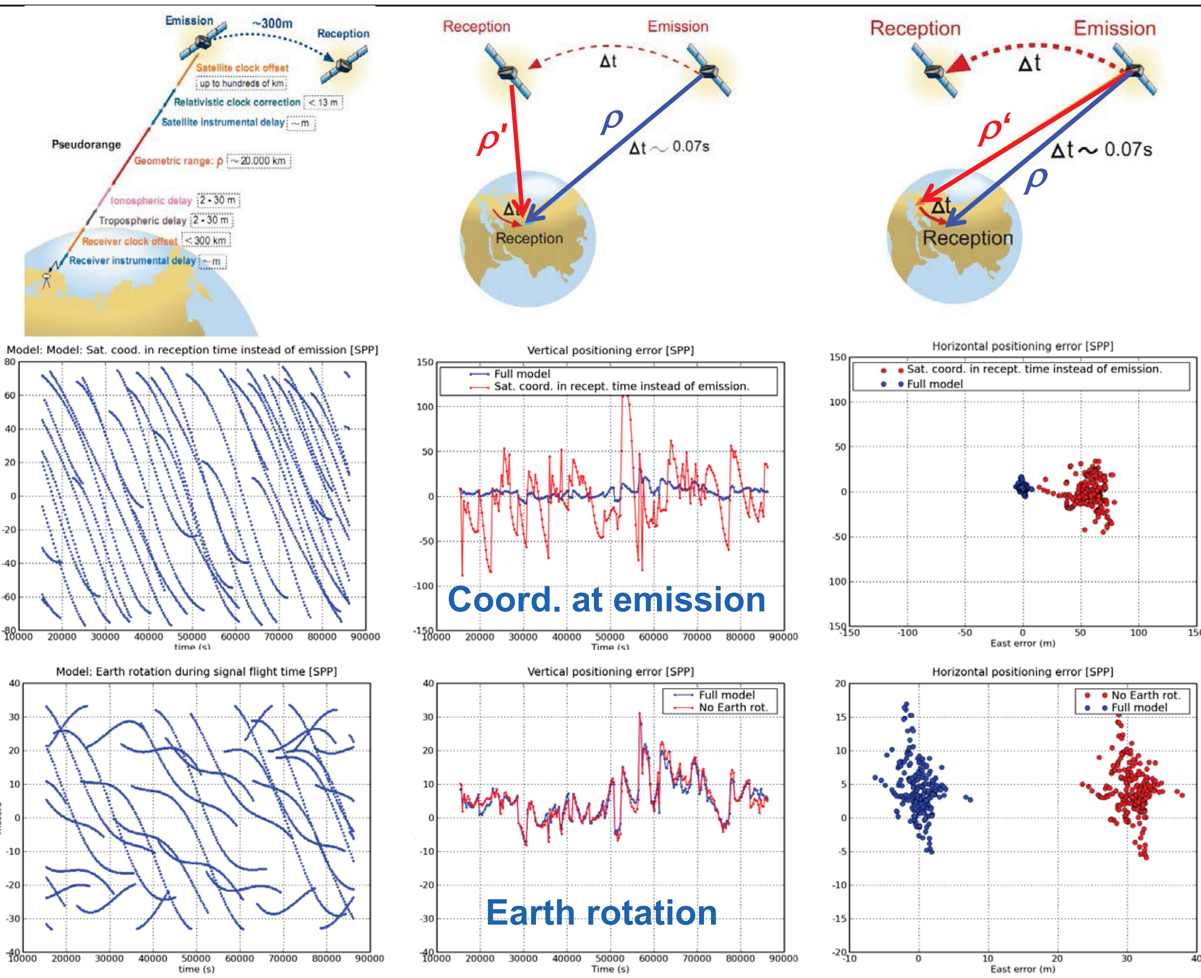
PPP Model components

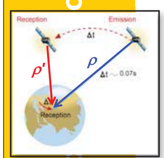


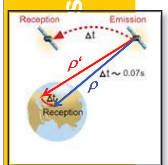
In the laboratory session (Tutorial 1) we review the measurements modelling for the Standard Point Positioning (SPP). A brief summary is given next.

After this summary, we will focus on the additional modelling need for Precise Point Positioning.

Remember that the error component most difficult to model is the ionosphere. But, in the PPP technique the ionosphere error is removed (more than 99.9%) using dual-frequency measurements in the ionosphere-free combination (Lc,Pc). This combination also removes the Differential Code Bias (or TGD).







Satellite coordinates computation at signal emission time

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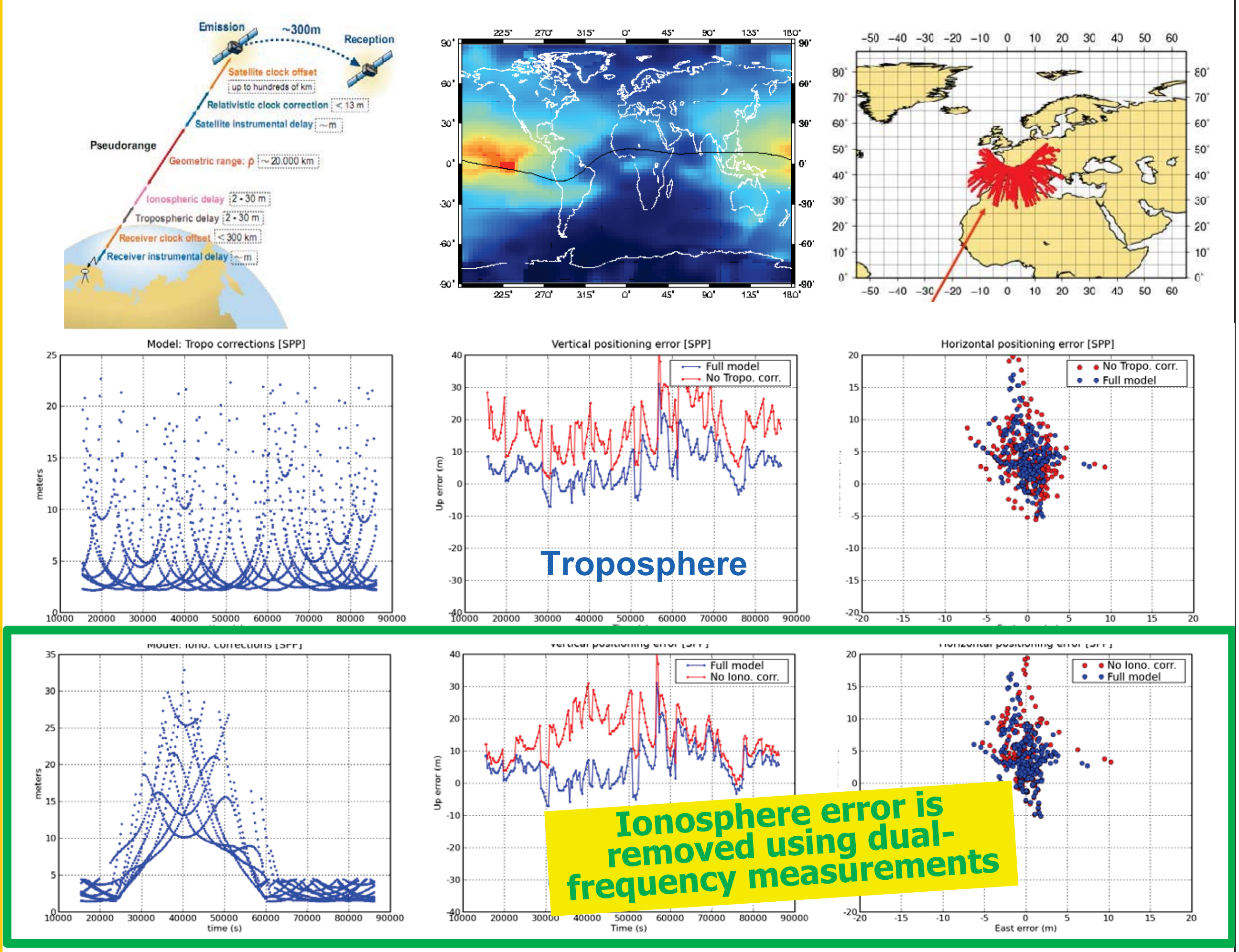

Signal propagation errors on the Atmosphere

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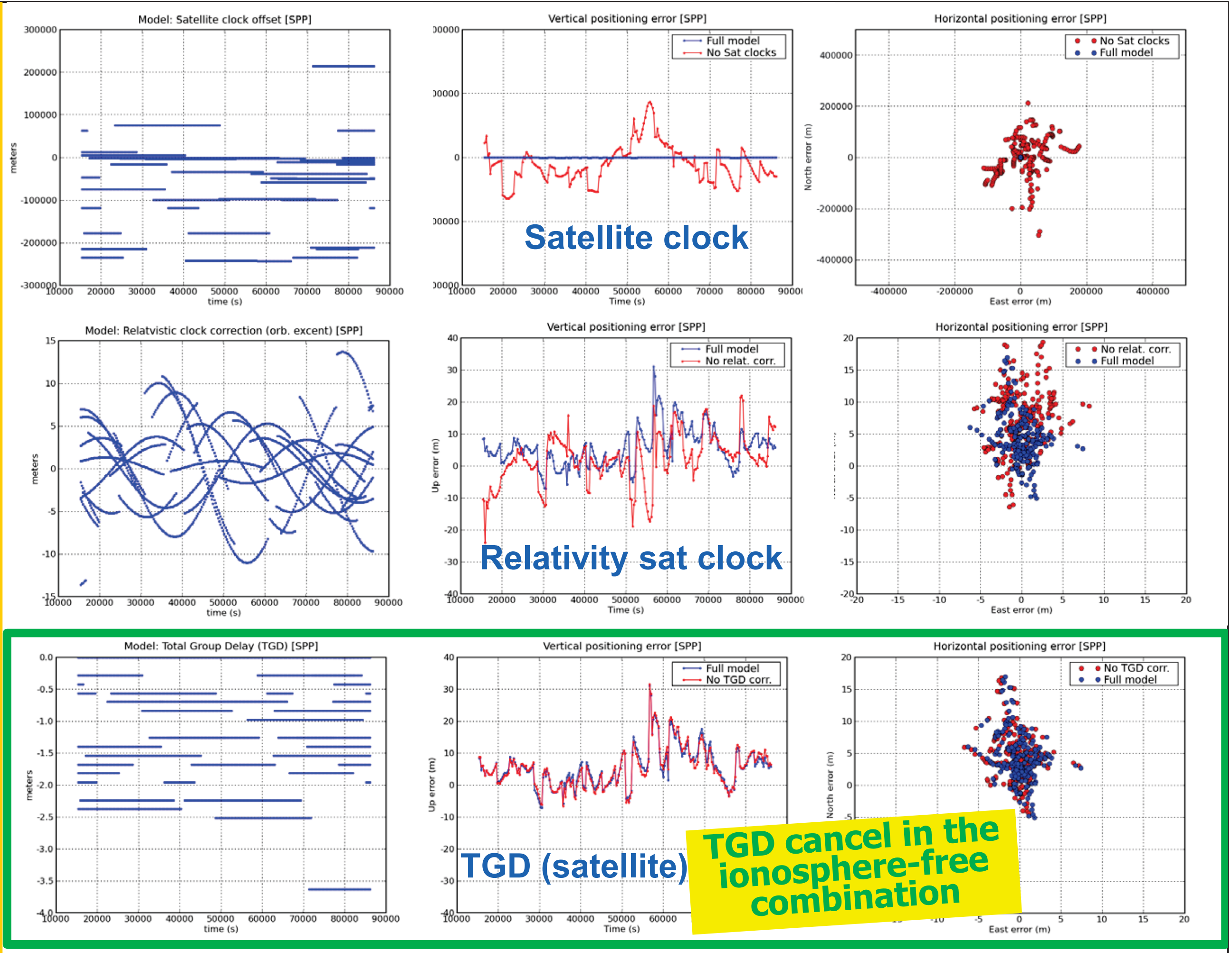
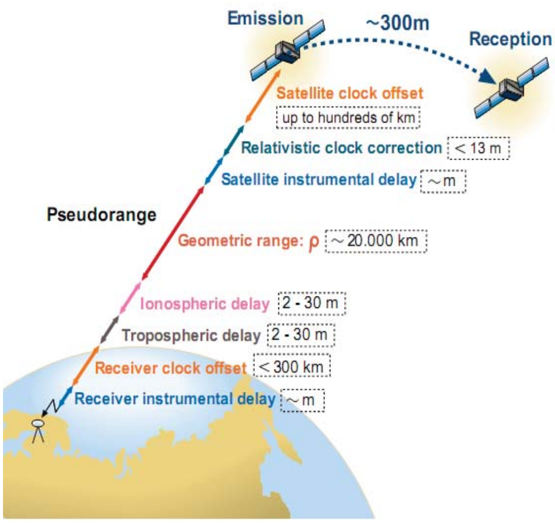


Satellite clocks and Total Group Delay (TGD)

Additional Modelling for PPP



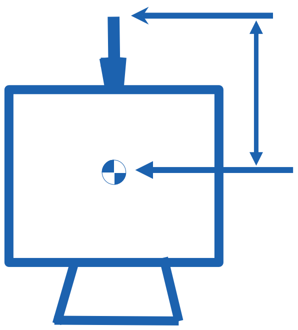
The PPP technique allows centimetre-level of accuracy to be achieved for static positioning and decimetre level, or better, for kinematic navigation.

This high accuracy requires an accurate **modelling "up to the centimetre level or better"** , where all previous model terms must be taken into account (except ionosphere and TGD [*]), **plus some additional terms given next:**

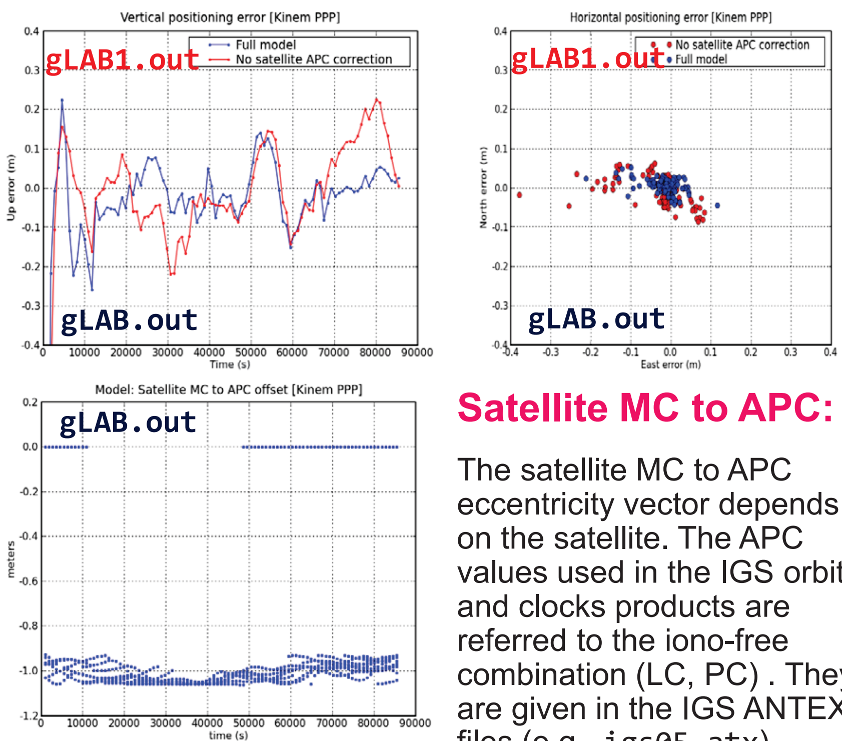
[*] Remember that in the PPP technique the ionosphere error is removed (more than 99.9%) using dual-frequency measurements in the ionosphere-free combination (Lc,Pc). This combination also removes the Differential Code Bias (or TGD).

Additional Modelling for PPP

Satellite Mass Center to Antenna Phase Center



Broadcast orbits are referred to the antenna phase center, but IGS precise orbits are referred to the satellite mass center.

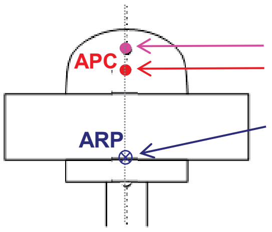


Satellite MC to APC:

The satellite MC to APC eccentricity vector depends on the satellite. The APC values used in the IGS orbits and clocks products are referred to the iono-free combination (LC, PC) . They are given in the IGS ANTEX files (e.g., `igs05.atx`).

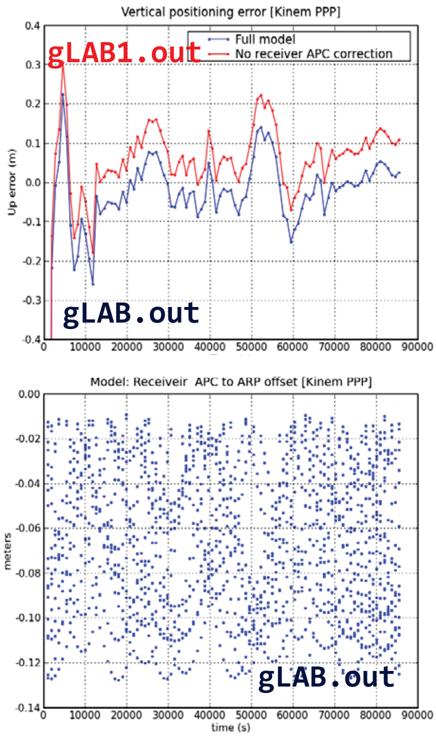
Additional Modelling for PPP

Receiver Antenna Phase center (APC)



GNSS measurements are referred to the APC. This is not necessarily the geometric center of the antenna, and it depends on the signal frequency and the incoming radio signal direction.

For geodetic positioning a reference tied to the antenna (ARP) or to monument is used



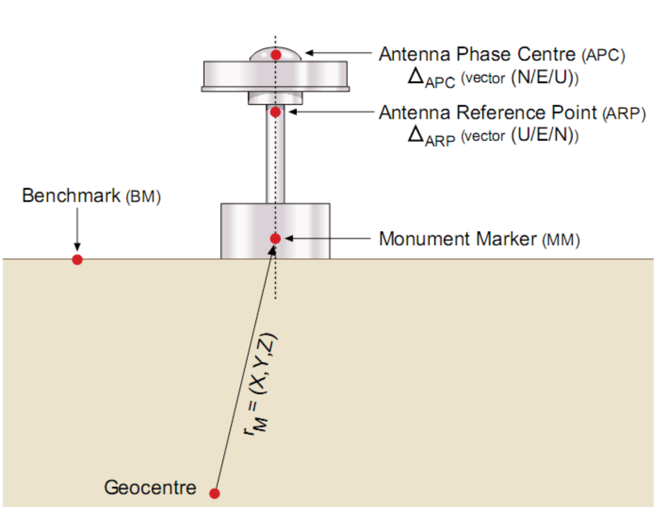
Receiver APC:

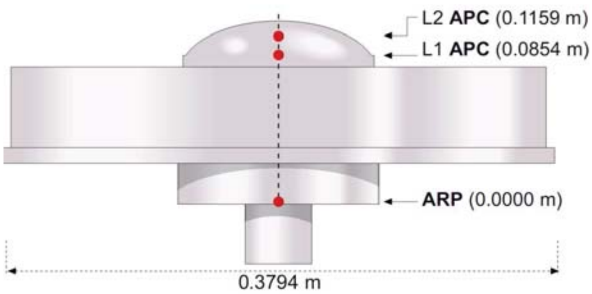
The antenna used for this experiment, has the APC position vertically shifted regarding ARP.

Thence, neglecting this correction, an error on the vertical component occurs, but not in the horizontal one.

Antenna biases and orientation:

- The satellite and receiver **antenna phase centres** can be found in the IGS ANTEX files, after GPS week 1400 (Nov. 2006)
- The **carrier phase wind-up** effect due to the satellite's motion must be taken into account.

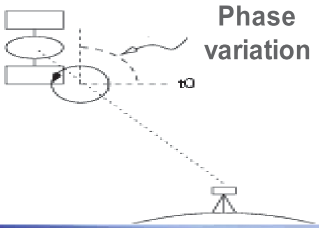


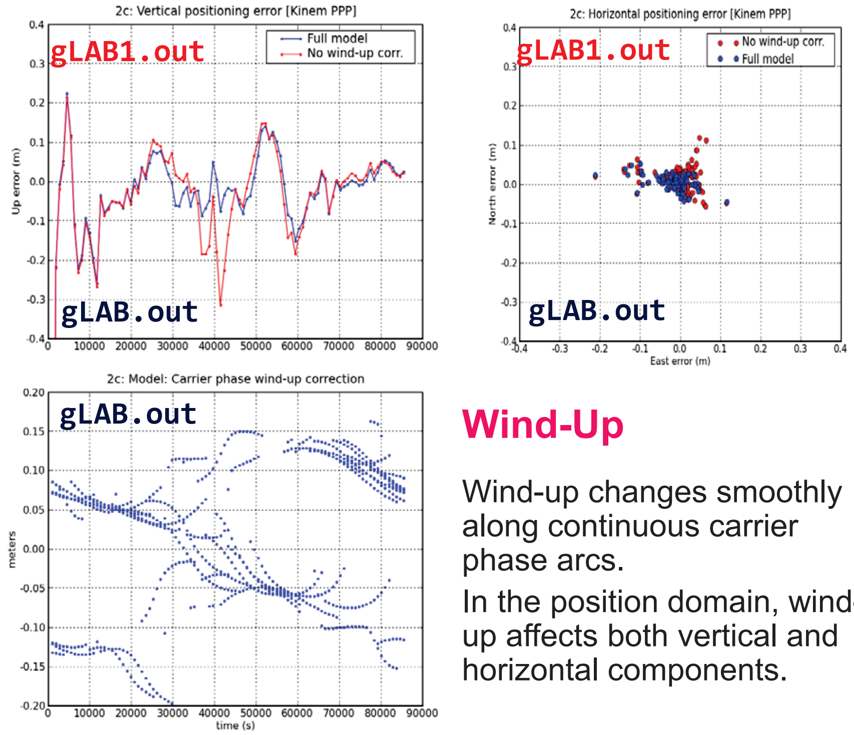


Additional Modelling for PPP

Wind-up affects only carrier phase. It is due to the electromagnetic nature of circularly polarized waves of GNSS signals.

As the satellite moves along its orbital path, it performs a rotation to keep its solar panels pointing to the Sun direction. This rotation causes a carrier variation, and thence, a range measurement variation.





Wind-Up

Wind-up changes smoothly along continuous carrier phase arcs.

In the position domain, wind-up affects both vertical and horizontal components.

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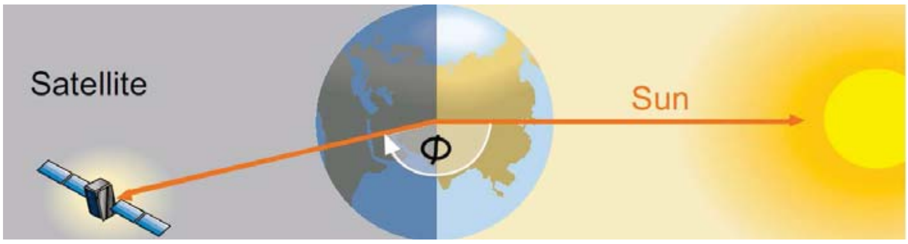
Additional Modelling for PPP: **Eclipse condition**



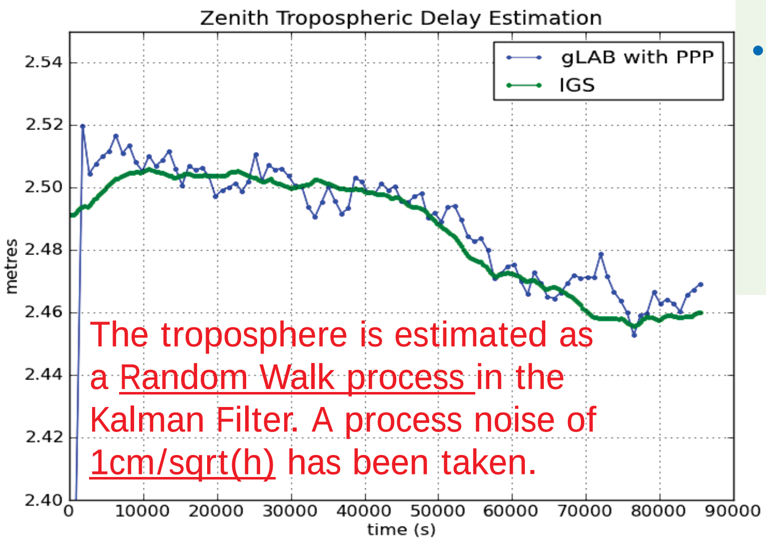
High-accuracy GNSS positioning degrades during the GNSS satellites' eclipse seasons.

- Once the satellite goes into shadow, the radiation pressure vanishes. This effect introduces errors in the satellite dynamics due to the difficulty of properly modelling the solar radiation pressure.
- On the other hand, the satellite's solar sensors lose sight of the Sun and the attitude control (trying to keep the panels facing the Sun).

As a consequence, the orbit during shadow and eclipse periods may be considerably degraded and the removal of satellites under such conditions can improve the high-precision positioning results.

Additional Modelling for PPP: **Atmospheric** Effects

- **The ionospheric refraction and TGDs** are **removed** using the ionosphere-free combination of code and carrier measurements (PC, LC).
- **The tropospheric refraction can be modelled by Dry and Wet components** (and different mappings are usually used for both components, e.g. mapping of Niell).

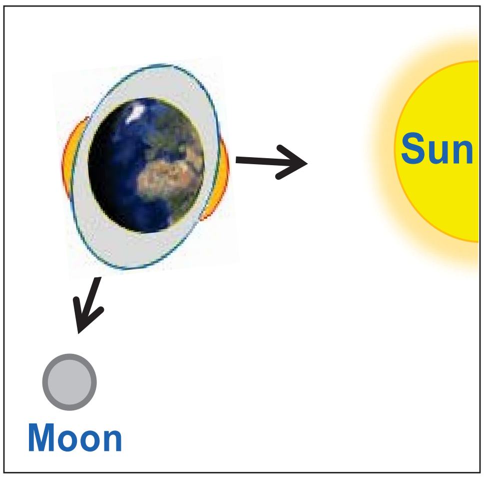


- **A residual tropospheric delay is estimated** (as wet ZTD delay) in the Kalman filter, together with the coordinates, clock and carrier phase biases.

Additional Modelling for PPP: **Earth Deformation**

Earth deformation effects:

- **Solid tides** must be modelled by equations
- **Ocean loading and pole tides** are second-order effects and can be neglected for PPP accuracies at the centimetre level



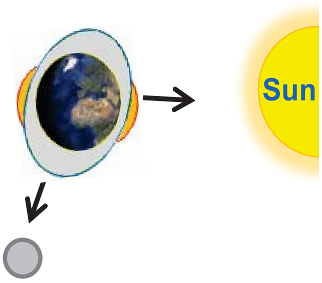
Solid Tides concern the movement of Earth's crust (and thus the variation in the receiver's location coordinates) due to gravitational attractive forces produced by external bodies, mainly the Sun and Moon.

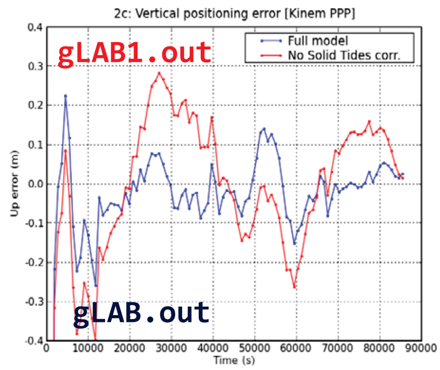
Solid tides produce vertical and horizontal displacements that can be expressed by the spherical harmonics expansion (m, n), characterised by the Love and Shida numbers h_{mn} and l_{mn}.

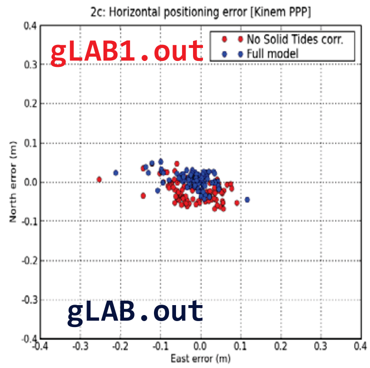
Additional Modelling for PPP

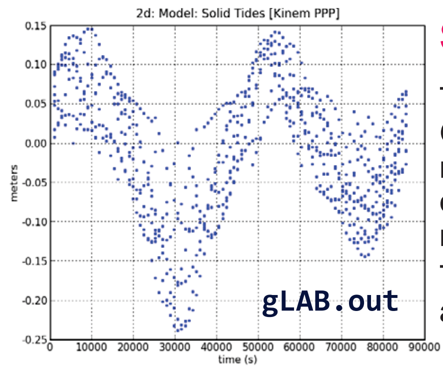
Solid Tides

It comprises the Earth's crust movement (and thence receiver coordinates variations) due to the gravitational attraction forces produced by external bodies, mainly the Sun and the Moon.









Solid Tides:

These effects do not affect the GNSS signals, but if they were not considered, the station coordinates would oscillate with relation to a mean value.

They produce vertical (mainly) and horizontal displacements.

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Contents

Precise Point Positioning (PPP)

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Linear observation model for PPP

It is based code and carrier measurements in the ionosphere-free combination (Pc, Lc), which are modelled as follows:

$$P_{C\,rec}^{\;sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + Trop_{rec}^{sat} + \mathcal{M}_{Pc} + \varepsilon_{P_C}$$

$$L_{C\,rec}^{\;sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + Trop_{rec}^{sat} + \lambda_N \omega_{rec}^{sat} + B_{C\,rec}^{\;sat} + m_{L_C} + \varepsilon_{L_C}$$

where

$$P_C = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2}; \quad L_C = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2}$$

$$B_C = \lambda_N \left(B_1 + \frac{\lambda_W}{\lambda_2} B_W \right)$$

$$B_W = B_1 - B_2$$

Ionosphere is removed

Remark, ρ is referred to the Antenna Phase Centres (APC) of satellite and receiver antennas in the ionosphere free combination.

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Linear model: For each satellite in view

$$P_{C\,rec}^{\;sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + Trop + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{rec,0}, y_{rec,0}, z_{rec,0})$

$$= \rho_{rec,o}^{sat} + \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}}\Delta x_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}}\Delta y_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}}\Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + Trop$$

where:

$$\Delta x_{rec} = x_{rec} - x_{rec,o} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{rec,o} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{rec,o}$$

and taking: $Trop_{rec}^{sat} = Trop_{0\,rec}^{\;sat} + M_{wet,rec}^{\;sat}\,\Delta Tr_{Z,wet}$

Prefit-residuals (Prefit)

$$\boxed{P_{C\,rec}^{\;sat} - \left[\rho_{rec,o}^{sat} + c\,dt^{sat} - Trop_{0rec}^{\;sat}\right]} = \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}}\Delta x_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}}\Delta y_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}}\Delta z_{rec} + c\,dt_{rec} + M_{wet}^{\;sat}\,\Delta Tr_{Z,wet}$$

measurement

computed

unknown

The same for carrier, but adding the ambiguity as an unknown

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Following the same procedure as with the code based positioning (SPP), the linear observation model $\mathbf{y} = \mathbf{G}\,\mathbf{x}$ for the code and carrier measurements can be written as follows:

$$\mathbf{y} = \mathbf{G}\,\mathbf{x}$$

$$\begin{bmatrix} Prefit(Pc)^1 \\ Prefit(Lc)^1 \\ \cdots\cdots \\ Prefit(Pc)^n \\ Prefit(Lc)^n \end{bmatrix} = \begin{bmatrix} \frac{x_{o,rec}-x^1}{\rho^1_{0,rec}} & \frac{y_{o,rec}-y^1}{\rho^1_{0,rec}} & \frac{z_{o,rec}-z^1}{\rho^1_{0,rec}} & 1 & M^1_{wet} & 0 & \cdots & \cdots & 0 \\ \frac{x_{o,rec}-x^1}{\rho^1_{0,rec}} & \frac{y_{o,rec}-y^1}{\rho^1_{0,rec}} & \frac{z_{o,rec}-z^1}{\rho^1_{0,rec}} & 1 & M^1_{wet} & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \frac{x_{o,rec}-x^n}{\rho^n_{0,rec}} & \frac{y_{o,rec}-y^n}{\rho^n_{0,rec}} & \frac{z_{o,rec}-z^n}{\rho^n_{0,rec}} & 1 & M^n_{wet} & 0 & \cdots & \cdots & 0 \\ \frac{x_{o,rec}-x^n}{\rho^n_{0,rec}} & \frac{y_{o,rec}-y^n}{\rho^n_{0,rec}} & \frac{z_{o,rec}-z^n}{\rho^n_{0,rec}} & 1 & M^n_{wet} & 0 & \cdots & \cdots & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ cdt_{rec} \\ \Delta Tr_{Z,wet} \\ B_C^{\,1} \\ \vdots \\ B_C^{\,n} \end{bmatrix}$$

$$Prefit(P_C)^k = P_C{}^k - \rho_0^k + cdt^k - Trop_0^k$$

$$Prefit(L_C)^k = L_C{}^k - \rho_0^k + cdt^k - Trop_0^k - \lambda_N \omega^k$$

Carrier ambiguities

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Equivalent notation:

Using $\rho = \rho_0 - \hat{\boldsymbol{\rho}}^T \cdot \Delta\mathbf{r}$, where $\hat{\boldsymbol{\rho}} = \dfrac{\boldsymbol{\rho}_0}{\rho_0}$

The previous system, can be written as:

$$\begin{bmatrix} Prefit(Pc)^1 \\ Prefit(Lc)^1 \\ \cdots\cdots \\ Prefit(Pc)^n \\ Prefit(Lc)^n \end{bmatrix} = \begin{bmatrix} -\hat{\boldsymbol{\rho}}^{T\,1}_{0\,rec} & 1 & M^1_{wet} & 0 & \cdots & 0 \\ -\hat{\boldsymbol{\rho}}^{T\,1}_{0\,rec} & 1 & M^1_{wet} & 1 & \cdots & 0 \\ & & \vdots & & & \\ -\hat{\boldsymbol{\rho}}^{T\,n}_{0\,rec} & 1 & M^n_{wet} & 0 & \cdots & 0 \\ -\hat{\boldsymbol{\rho}}^{T\,n}_{0\,rec} & 1 & M^n_{wet} & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{r}_{rec} \\ cdt_{rec} \\ \Delta Tr_{z,wet} \\ B^1 \\ \vdots \\ B^n \end{bmatrix}$$

$$Prefit(P_C)^k = P_C{}^k - \rho_0^k + cdt^k - Trop_0^k$$

$$Prefit(L_C)^k = L_C{}^k - \rho_0^k + cdt^k - Trop_0^k - \lambda_N \omega^k$$

Carrier ambiguities

Contents

Precise Point Positioning (PPP)

Parameter estimation PPP: Floating Ambiguities

The linear observation model $\mathbf{y} = \mathbf{G}\,\mathbf{x}$ can be solved using the **Kalman filter**. The next stochastic model can be used

- **Carrier phase biases** (B_C) are taken as `constant´ along continuous phase arcs, and as `white noise´ when a cycle slip happens ($\sigma = 10^4\ m$ can be taken, for instance) ➔ **FLOATED AMB.**
- **Wet tropospheric delay** ($Tr_{z;wet}$) is taken as a random walk process (with $d\sigma^2/dt = 1\ cm^2/h$, for instance).
- **Receiver clock** (cdt) is taken as a white-noise process (with $\sigma = 3.10^5\ m$, i.e. $1\ ms$ for instance).
- **Receiver coordinates** (Δx, Δy, Δz)
  - For **static positioning** the coordinates are taken as constants.
  - For **kinematic positioning** the coordinates are taken as white noise or a random walk process.

Comment: Floating Ambiguities

This solution procedure is called **floating ambiguities**. `Floating´ in the sense that the ambiguities are estimated by the filter `as real numbers´.

The bias estimations $\hat{B}_C$ will converge to a solution after a transition time that depends on the observation geometry, model quality and data noise.

In general, one must expect errors at the decimetre level, or better, in pure kinematic positioning (after the best part of one hour) and at the centimetre level in static PPP over 24h data.

See exercises in the laboratory session (Tutorial 1).

Contents

Carrier Ambiguity Fixing concept

In the previous formulation, the ambiguities have been estimated as real numbers in the Kalman filter, without exploiting its integer nature.

That is, the ambiguities in L_1, L_2 or L_W signals are an integer number (N) of its associated wavelength (λ) plus a fractional part associated to the satellite and to the receiver.

$$B_{1,rec}^{sat} = \lambda_1 N_{1,rec}^{sat} + b_{1,rec} + b_1^{sat}$$
$$B_{2,rec}^{sat} = \lambda_2 N_{2,rec}^{sat} + b_{2,rec} + b_2^{sat}$$
$$B_{W,rec}^{sat} = \lambda_W N_{W,rec}^{sat} + b_{W,rec} + b_W^{sat}$$

$$B_C = \lambda_N \left(N_1 + \frac{\lambda_w}{\lambda_2} N_W \right) + b_{C,rec} + b_C^{sat}$$

B_C is not an integer number of λ_N but can be related with the integers N_1, L_W

The Ambiguity Fixing techniques take benefit of this INTEGER NATURE of N_1, N_2 and N_w ambiguities to properly fix them, reducing convergence time, and thence, achieving high accuracy quickly.

Carrier Ambiguity Fixing concept

Two different approaches can be considered:

- **Double differenced Ambiguity Fixing:**

  This is the classical approach which relies in the fact that the fractional part of carrier ambiguities cancels when forming the double differences between receivers and satellites:

  ➔ That is, given: $B_{rec}^{sat} = \lambda N_{rec}^{sat} + b_{rec} + b^{sat}$

  The double differences, regarding a reference receiver and satellite, yield: $\boxed{\Delta\nabla B_{rec}^{sat}} = B_{rec}^{sat} - B_{rec,R}^{sat} - \left(B_{rec}^{sat,R} - B_{rec,R}^{sat,R} \right) = \boxed{\lambda \Delta\nabla N_{rec}^{sat}}$

  where the satellite and receiver ambiguity terms (b_{rec}, b^{sat}) cancel out.

- **Absolute (or undifferenced) ambiguity fixing:**

  This is a new approach, where the fractional part of the ambiguities are estimated from a global network of permanent stations and provided to the users. Thus, the user can remove this fractional part and fix the remaining ambiguity as an integer number.

Example of Ambiguity fixing layout

Kalman filter: → User solution (x,y,z)

$$\Delta\nabla N_W = \left[\frac{\Delta\nabla L_W - \Delta\nabla P_N}{\lambda_W}\right]_{\text{rundoff}}$$

Differential Ionosphere Error

$$\Delta\nabla N_1 = \left[\frac{\Delta\nabla L_1 - \Delta\nabla L_2 - \boxed{\Delta\nabla \hat{I}} - \lambda_2 \boxed{\Delta\nabla N_W}}{\lambda_2 - \lambda_1}\right]_{\text{rundoff}}$$

Ambiguity FIXED

$$\boxed{\Delta\nabla B_C} = \lambda_N \left(\boxed{\Delta\nabla N_1} + \frac{\lambda_W}{\lambda_2}\boxed{\Delta\nabla N_W}\right)$$

- **Quick convergence time**: Few minutes to fix $\Delta\nabla N_W$
- **Baseline** limited by the **Ionosphere**

Example of Ambiguity fixing layout

Kalman filter: → User solution (x,y,z)

Floated estimate $\Rightarrow \Delta\nabla \hat{B}_C$

$$\Delta\nabla N_W = \left[\frac{\Delta\nabla L_W - \Delta\nabla P_N}{\lambda_W}\right]_{\text{rundoff}}$$

$$\Delta\nabla N_1 = \left[\frac{1}{\lambda_N}\boxed{\left(\Delta\nabla \hat{B}_C\right)} - \frac{\lambda_W}{\lambda_2}\boxed{\Delta\nabla N_W}\right]_{\text{rundoff}}$$

Ambiguity FIXED

$$\boxed{\Delta\nabla B_C} = \lambda_N \left(\boxed{\Delta\nabla N_1} + \frac{\lambda_W}{\lambda_2}\boxed{\Delta\nabla N_W}\right)$$

- **Large convergence time**, due to the time-span need for $\Delta\nabla \hat{B}_C$ to converge
- **Immune to the ionosphere**

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[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

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[RD-5] Juan, J.M., Hernandez-Pajares, M., Sanz, J., Ramos- Bosch, P., Aragon-Angel, A., Orus, R., Ochieng, W., Feng, S., Coutinho, P., Samson, J. and Tossaint, M., 2012a. Enhanced Precise Point Positioning for GNSS Users. IEEE Transactions on Geoscience and Remote Sensing PP, issue 99, pp. 1-10.

Thank you

Lecture 6

Differential positioning with **Code pseudoranges**

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Authorship statement

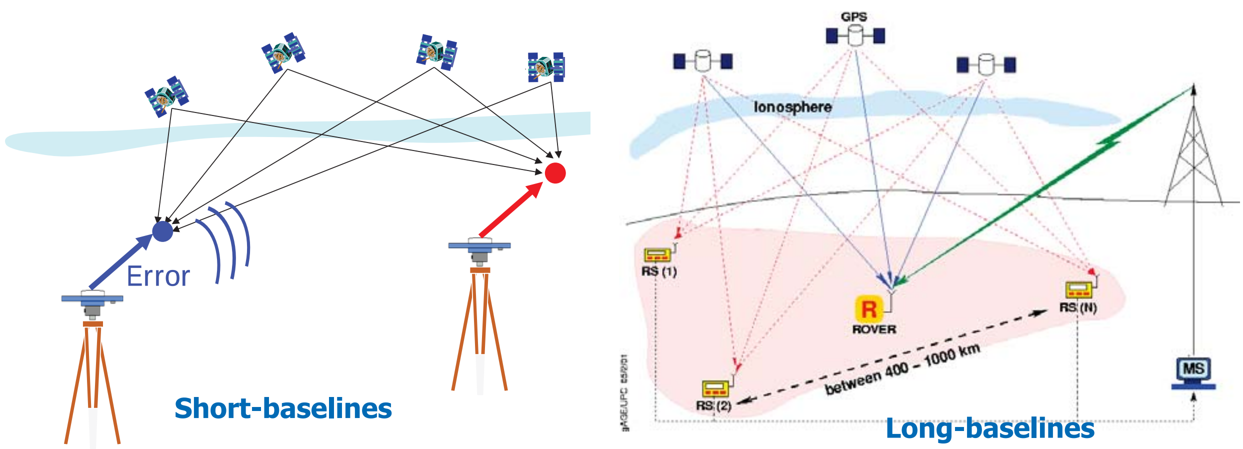
24 April 2014

Contents

Error mitigation: DGNSS residual error

Errors are similar for users separated tens, even hundred of kilometres, and these errors vary 'slowly' with time. That is, the errors are correlated on space and time.

The spatial decorrelation depends on the error component (e.g. Clocks not decorrelate, ionosphere ~100km…). Thence, long baselines need a reference stations network.



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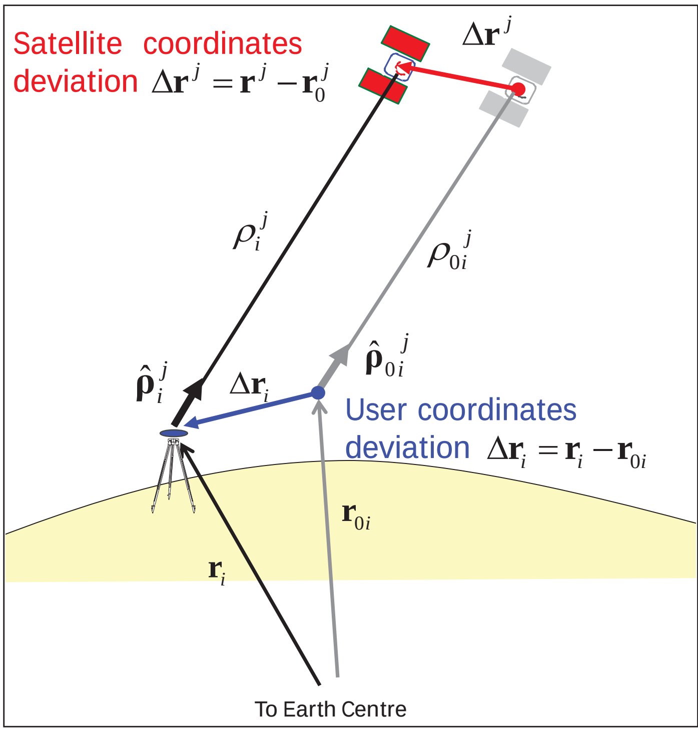
Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \mathcal{M}_i^j + \nu_{P\,i}^{\,j}$$

$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + m_i^j + \nu_{L\,i}^{\,j}$$



When approximate values of both receiver and satellite APC positions are taken, a linearization around them yields:

$$\boxed{\rho_i^j = \rho_{0i}^{\,j} - \hat{\boldsymbol{\rho}}_{0i}^{\,j} \cdot \Delta\mathbf{r}_i + \hat{\boldsymbol{\rho}}_{0i}^{\,j} \cdot \Delta\mathbf{r}^j}$$

$$\hat{\boldsymbol{\rho}}_{0i}^{\,j} = \frac{\mathbf{r}_0^j - \mathbf{r}_{oi}}{\left\|\mathbf{r}_0^j - \mathbf{r}_{oi}\right\|}$$

$\Delta\mathbf{r}_i$: Receiver coordinates error

$\Delta\mathbf{r}^j$: Satellite coordinates error

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Linear model for Differential Positioning

Code and carrier measurements

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$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + m_i^j + \nu_{L\,i}^{\,j}$$

Single difference $\boxed{(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j}$

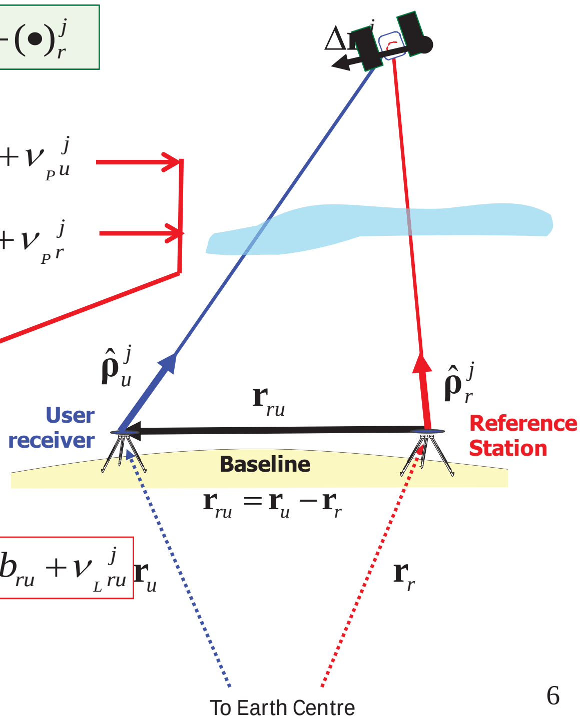
$$P_u^j = \rho_u^j + c\left(\delta t_u - \cancel{\delta t^j}\right) + T_u^j + I_u^j + K_u + \cancel{K^j} + \nu_{P\,u}^{\,j}$$

$$P_r^j = \rho_r^j + c\left(\delta t_r - \cancel{\delta t^j}\right) + T_r^j + I_r^j + K_r + \cancel{K^j} + \nu_{P\,r}^{\,j}$$

$$\boxed{P_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \nu_{P\,ru}^{\,j}}$$

The same for the carrier :

$$\boxed{L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\,j}}$$



Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \mathcal{M}_i^j + \nu_{P_i}^{\,j}$$

$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + m_i^j + \nu_{L_i}^{\,j}$$

Single difference $\boxed{(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j}$

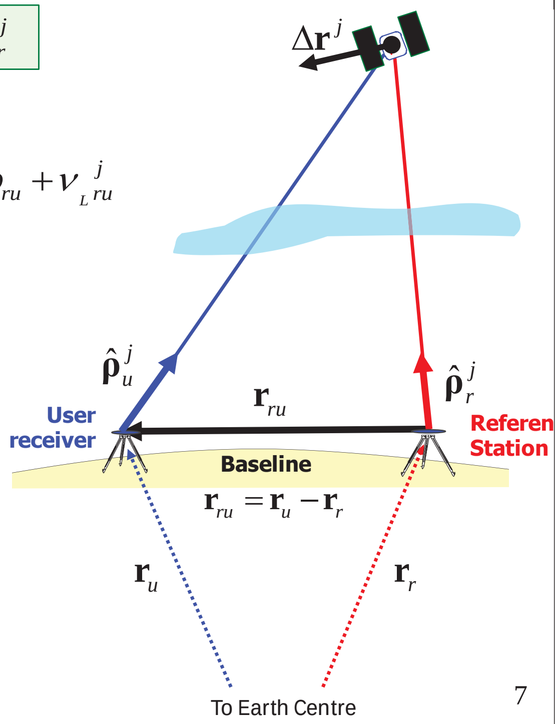
$$P_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \nu_{P\,ru}^{\,j}$$

$$L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\,j}$$

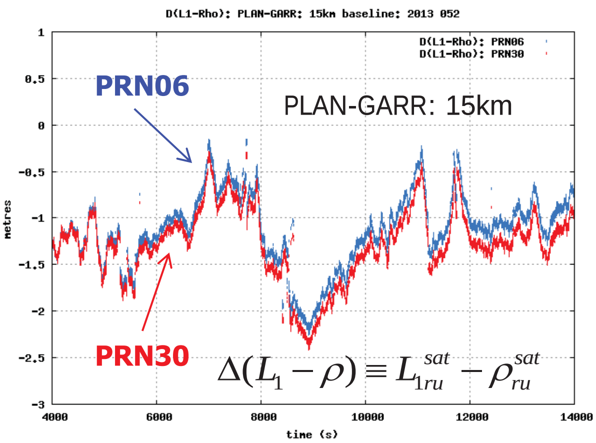
Single difference cancels:
- Satellite clock (δt^j)
- Satellite code instrumental delays (K^j)
- Satellite carrier instrumental delays (b^j)

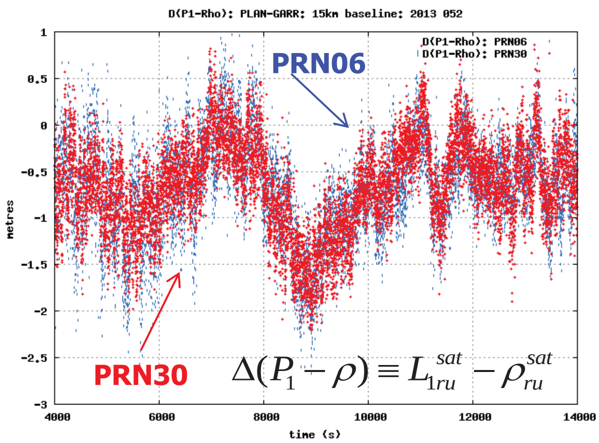
Single differences mitigate/remove errors due
- Satellite Ephemeris $(\Delta\mathbf{r}^j)$
- Ionosphere (I_i^j)
- Troposphere (T_i^j)
- Wind-up (ω_i^j)

The residual errors will depend upon the baseline length.



Single-Difference of measurements (corrected by geometric range!!)





Dif. Wind-up: Very small

$$\Delta(L_1 - \rho) \equiv L_{ru}^j - \rho_{ru}^j = \boxed{c\,\delta t_{ru}} + \boxed{T_{ru}^j} - \boxed{I_{ru}^j} + \boxed{\lambda\,\omega_{ru}^j} + \boxed{\lambda\,N_{ru}^j + b_{ru}} + \nu_{L\,ru}^{\,j}$$

$$\Delta(P_1 - \rho) \equiv P_{ru}^j - \rho_{ru}^j = \boxed{c\,\delta t_{ru}} + \boxed{T_{ru}^j} + \boxed{I_{ru}^j} + \boxed{K_{ru}} + \nu_{P\,ru}^{\,j}$$

Dif. Receiver clock: Main variations Common for all satellites

Dif. Tropo. and Iono. : Small variations

Dif. Instrumental delays and carrier ambiguities: constant

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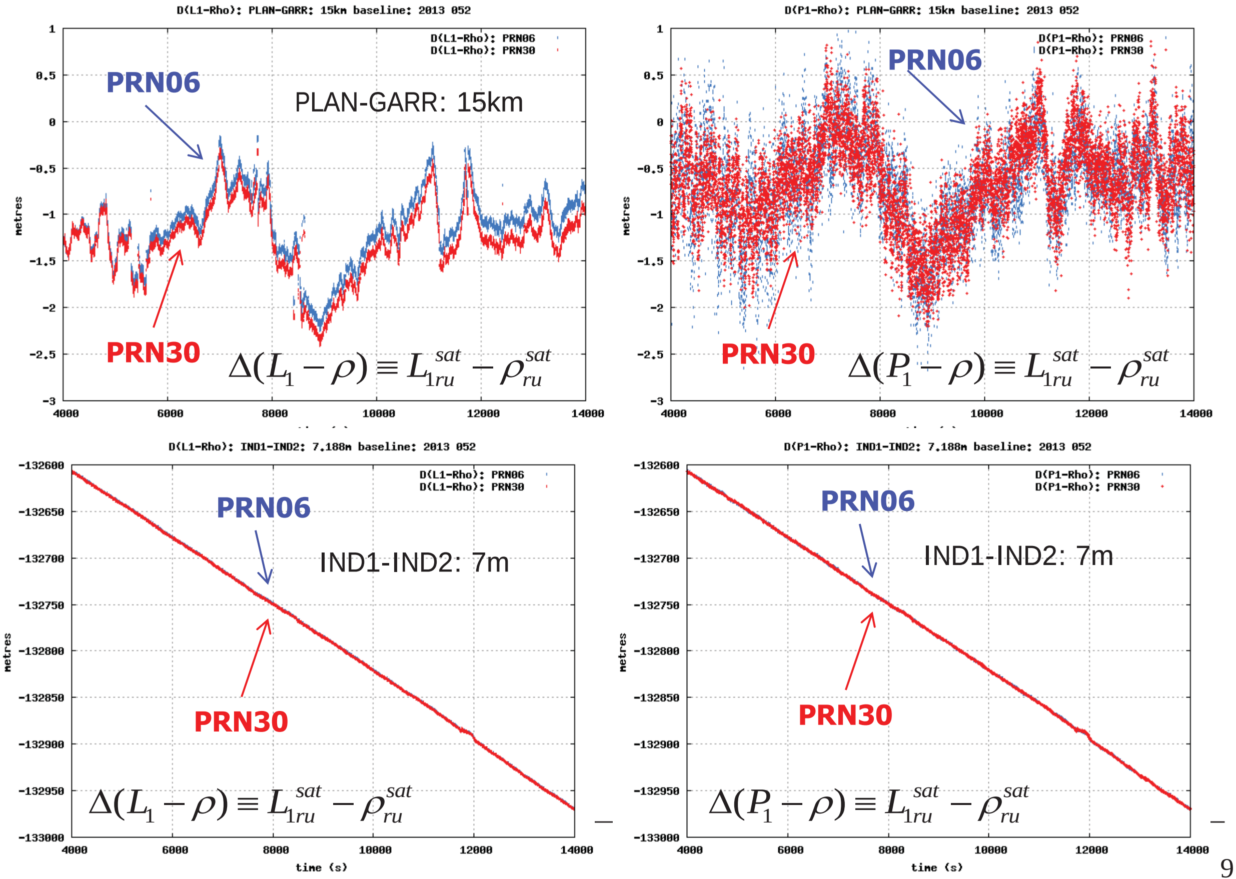
Single-Difference of measurements (corrected by geometric range!!)



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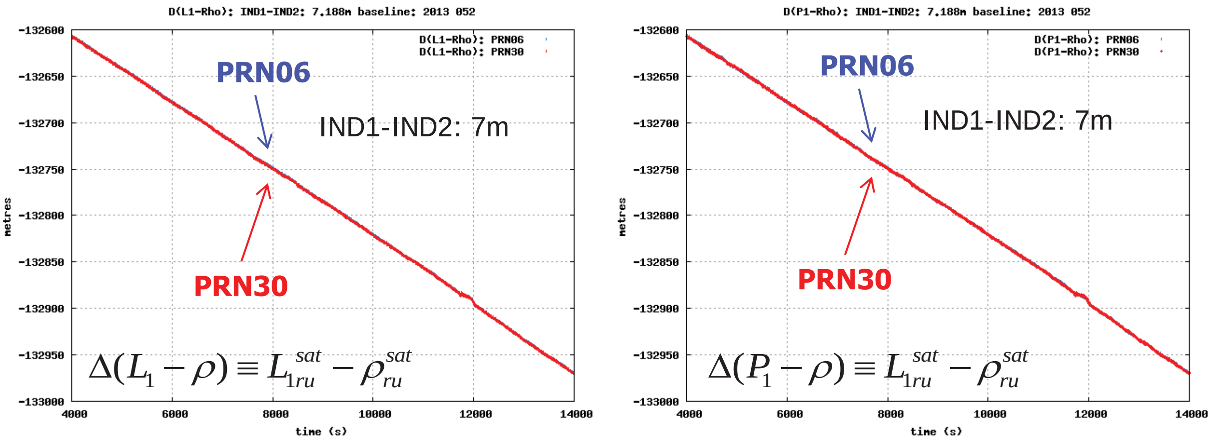
Single-Difference of measurements (corrected by geometric range!!)



Dif. Wind-up: Very small

$$\Delta(L_1-\rho)\equiv L^j_{ru}-\rho^j_{ru} = \boxed{c\,\delta t_{ru}} + \boxed{T^j_{ru}} - \boxed{I^j_{ru}} + \boxed{\lambda\,\omega^j_{ru}} + \boxed{\lambda N^j_{ru}+b_{ru}} + \nu_{L\,ru}^{\;j}$$

$$\Delta(P_1-\rho)\equiv P^j_{ru}-\rho^j_{ru} = \boxed{c\,\delta t_{ru}} + \boxed{T^j_{ru}} + \boxed{I^j_{ru}} + \boxed{K_{ru}} + \nu_{P\,ru}^{\;j}$$

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Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \nu_{P\,i}^{\;j}$$

$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + \nu_{L\,i}^{\;j}$$

where: $\rho_i^j = \rho_{0i}^j - \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}_i + \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}^j$

Single difference $(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$

$$P_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \varepsilon_{ru}^j$$

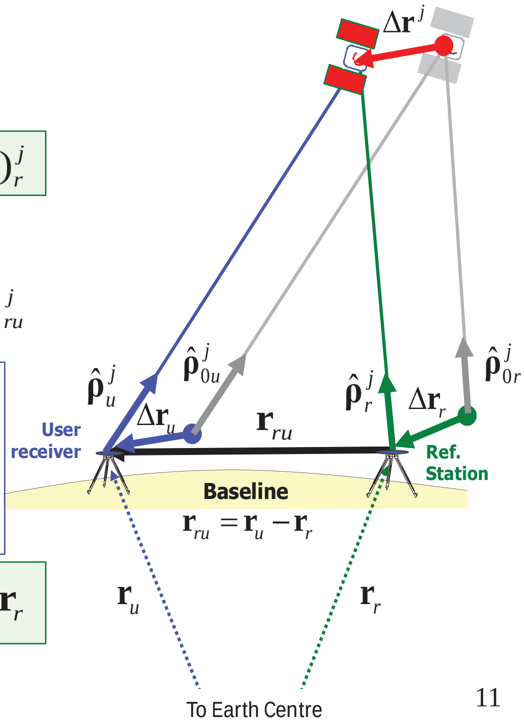
$$L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\;j}$$

where: $\rho_{ru}^j = \rho_u^j - \rho_r^j$

$$\rho_{ru}^j = \rho_{0ru}^j - \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}_u + \hat{\boldsymbol{\rho}}_{0r}^j \cdot \Delta\mathbf{r}_r + \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}^j - \hat{\boldsymbol{\rho}}_{0r}^j \cdot \Delta\mathbf{r}^j$$

$$= \rho_{0ru}^j - \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \Delta\mathbf{r}_r + \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \Delta\mathbf{r}^j$$

being: $\rho_{0ru}^j \equiv \rho_{0u}^j - \rho_{0r}^j\;;\quad \Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_u - \Delta\mathbf{r}_r$



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Exercise:

Let be: $\rho_{ru}^j = \rho_u^j - \rho_r^j$

where $\rho_i^j = \rho_{0i}^j - \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}_i + \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}^j$ (from Taylor expansion)

Show that the Single Differences are given by:

$$\rho_{ru}^j = \rho_{0ru}^j - \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \Delta\mathbf{r}_r + \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \Delta\mathbf{r}^j$$

being: $\rho_{0ru}^j \equiv \rho_{0u}^j - \rho_{0r}^j\;;\quad \Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_u - \Delta\mathbf{r}_r$

Linear model for Differential Positioning

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \Delta\mathbf{r}_{r} + \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \Delta\mathbf{r}^{j}$$

with $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_{u} - \Delta\mathbf{r}_{r}$

Let's assume that:

- The satellite coordinates are known with an uncertainty $\Delta\mathbf{r}^{j} \equiv \boldsymbol{\varepsilon}_{eph}^{j}$
- The reference station coordinates are known with an uncertainty $\Delta\mathbf{r}_{r} \equiv \boldsymbol{\varepsilon}_{site}$ (*i.e.* $\mathbf{r}_{r} = \mathbf{r}_{0r} + \boldsymbol{\varepsilon}_{site}$)

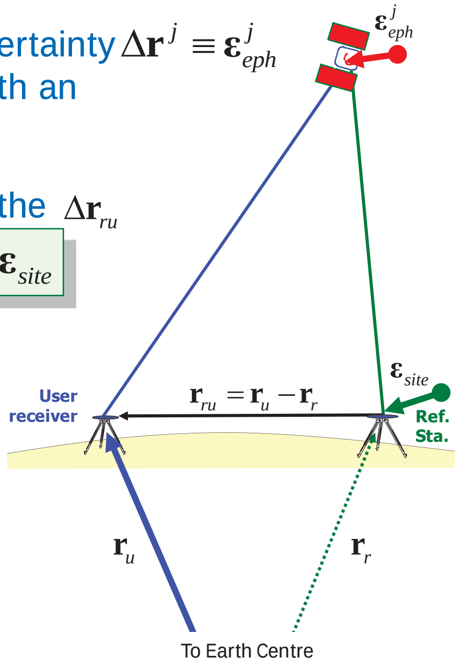
Thence, the user position can be computed from the $\Delta\mathbf{r}_{ru}$ estimate, with an error $\boldsymbol{\varepsilon}_{site}$, as: $\boxed{\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}}$

and where the $\Delta\mathbf{r}_{ru}$ estimate will be affected in turn by the ephemeris and sitting site errors as:

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \boxed{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site}} + \boxed{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}}$$

Range error due to **reference station** coordinates uncertainty

Range error due to **Sat. coordinates** uncertainty



$$\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \Delta\mathbf{r}_{r} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}$$

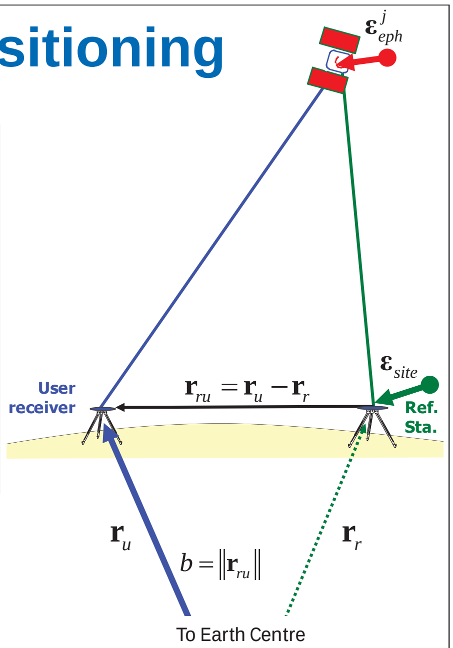
Linear model for Differential Positioning

and where the $\Delta\mathbf{r}_{ru}$ estimate will be affected in turn by the ephemeris and sitting site errors as:

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \boxed{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site}} + \boxed{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}}$$

Range error due to **reference station** coordinates uncertainty

Range error due to **Sat. coordinates** uncertainty



Thence, taking into account the relationships:

$$\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site} \le \frac{b}{\rho_{u}^{j}} \|\boldsymbol{\varepsilon}_{site}\| \quad ; \quad \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} \le \frac{b}{\rho_{u}^{j}} \|\boldsymbol{\varepsilon}_{eph}^{j}\|$$

and being $\rho_{u}^{j} \simeq 20\,000\,\mathrm{km}$ it follows that for a baseline $b = 20\,\mathrm{km}$

$$\boxed{\frac{b}{\rho_{u}^{j}} \approx \frac{1}{1000} \Rightarrow}$$

The effect of 5 metres error in orbits or in site coordinates is less than 5 mm in range for the estimation of $\Delta\mathbf{r}_{ru}$.

But, the user position estimate will be shifted by the error in the site coordinates $\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}$

Linear model for Differential Positioning

Exercise:

Demonstrate the following relationship:

$$\hat{\boldsymbol{\rho}}_{0ru}^{\,j} \cdot \boldsymbol{\varepsilon}_{site} \le \frac{b}{\rho}\left\|\boldsymbol{\varepsilon}_{site}\right\|$$

Hint:

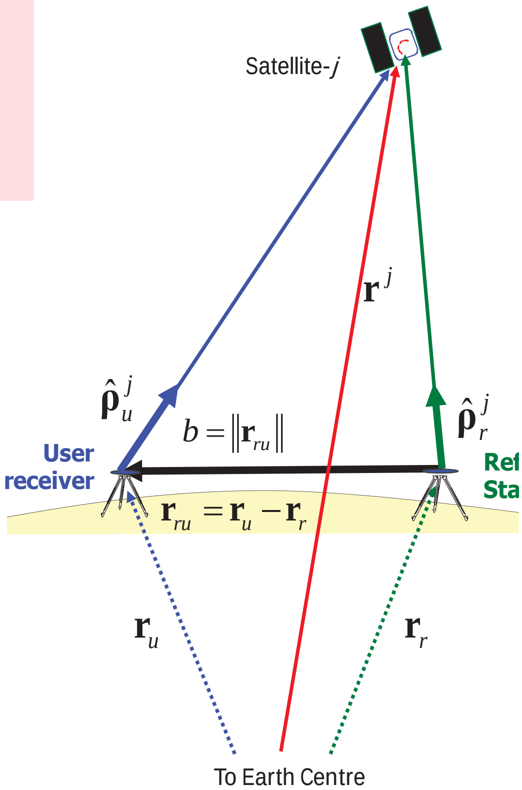
$$\hat{\boldsymbol{\rho}}_{0ru}^{\,j} \cdot \boldsymbol{\varepsilon}_{site} = \left(\hat{\boldsymbol{\rho}}_{0u}^{\,j} - \hat{\boldsymbol{\rho}}_{0r}^{\,j}\right) \cdot \boldsymbol{\varepsilon}_{site}$$

$$\simeq \left(\frac{\boldsymbol{\rho}_{0u}^{\,j} - \boldsymbol{\rho}_{0r}^{\,j}}{\rho}\right) \cdot \boldsymbol{\varepsilon}_{site} \le \frac{b}{\rho}\left\|\boldsymbol{\varepsilon}_{site}\right\|$$

Note: the following approaches have been taken:

$$\rho_{0r}^{j} \simeq \rho_{0u}^{j} \simeq \rho \quad \Rightarrow \begin{cases} \hat{\boldsymbol{\rho}}_{0r}^{\,j} = \dfrac{\boldsymbol{\rho}_{0r}^{\,j}}{\rho_{0r}^{j}} \simeq \dfrac{\boldsymbol{\rho}_{0r}^{\,j}}{\rho^{j}} \\ \hat{\boldsymbol{\rho}}_{0u}^{\,j} = \dfrac{\boldsymbol{\rho}_{0u}^{\,j}}{\rho_{0u}^{j}} \simeq \dfrac{\boldsymbol{\rho}_{0u}^{\,j}}{\rho^{j}} \end{cases}$$

$$b = \left\|\mathbf{r}_{ru}\right\| \simeq \left\|\mathbf{r}_{0ru}\right\| \qquad \mathbf{u}\cdot\mathbf{v} \le \left\|\mathbf{u}\right\|\left\|\mathbf{v}\right\|$$



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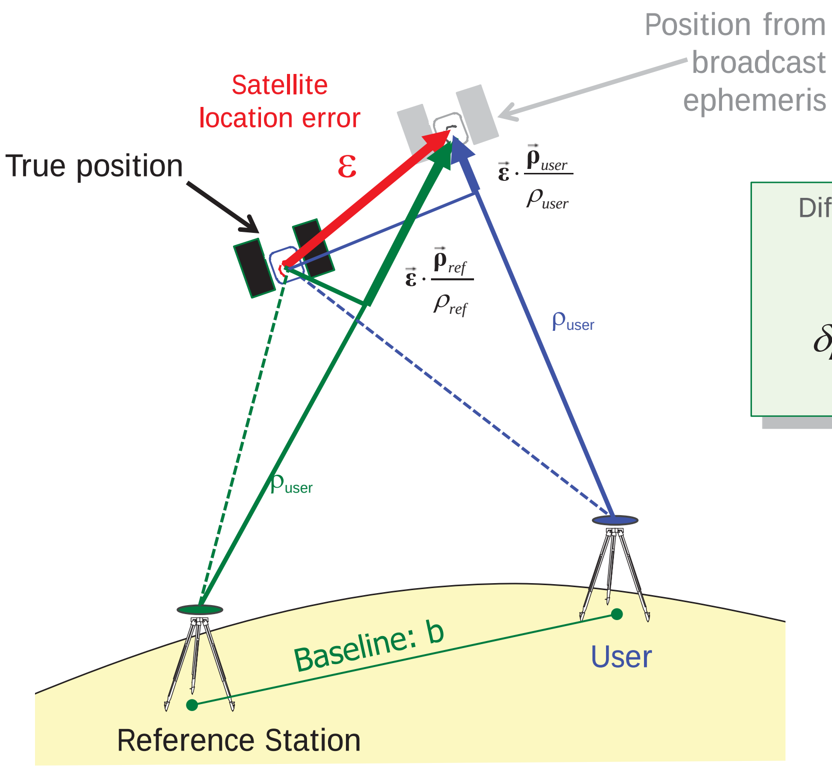
Geographic decorrelation of ephemeris errors



Differential range error due to satellite obit error

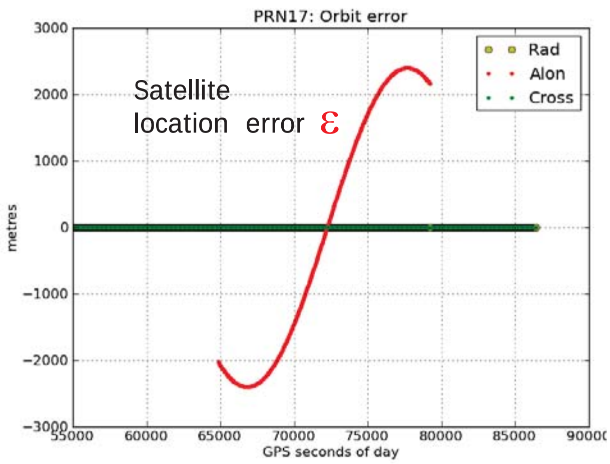
$$\delta\rho = \vec{\boldsymbol{\varepsilon}}\cdot\frac{\vec{\boldsymbol{\rho}}_{user}}{\rho_{user}} - \vec{\boldsymbol{\varepsilon}}\cdot\frac{\vec{\boldsymbol{\rho}}_{ref}}{\rho_{ref}}$$

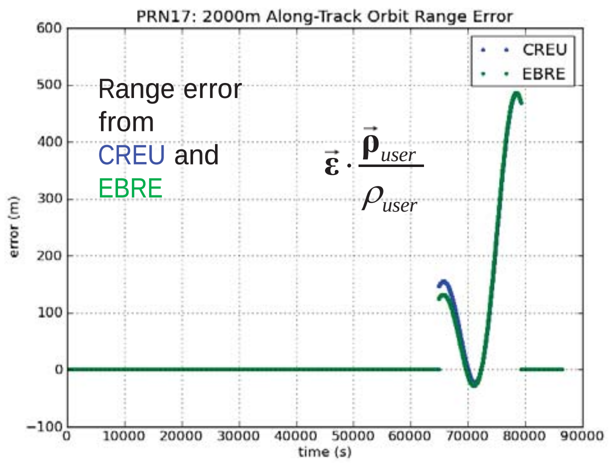
A conservative bound:

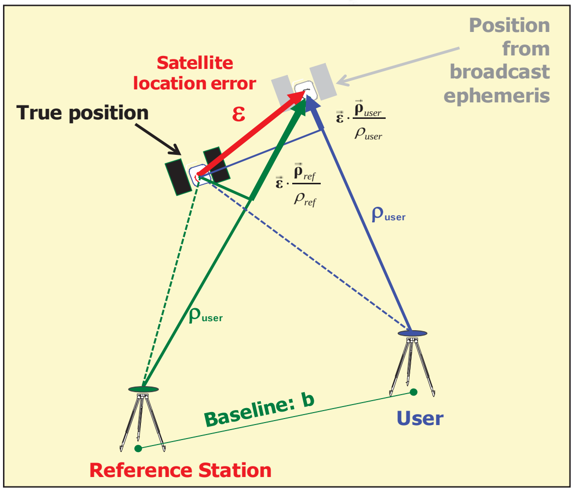
$$\boxed{\delta\rho < \frac{b}{\rho}\varepsilon}$$

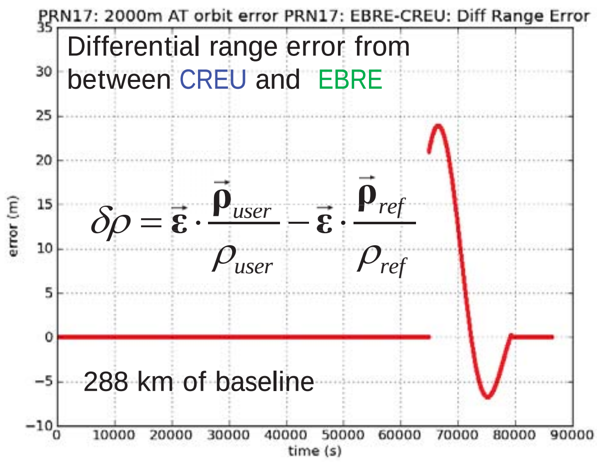
with a baseline $b = 20km$

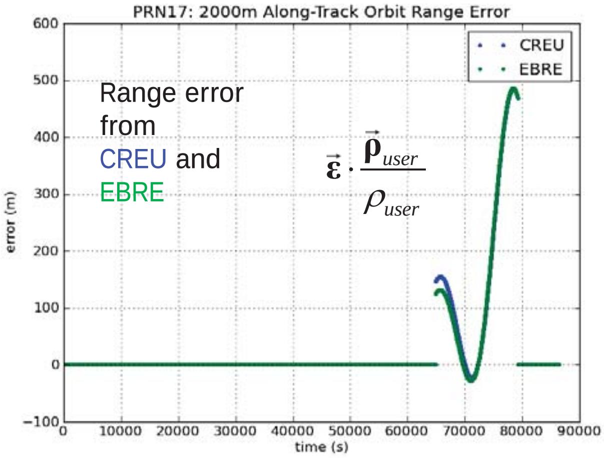
$$\delta\rho < \frac{20}{20000}\varepsilon = \frac{1}{1000}\varepsilon$$

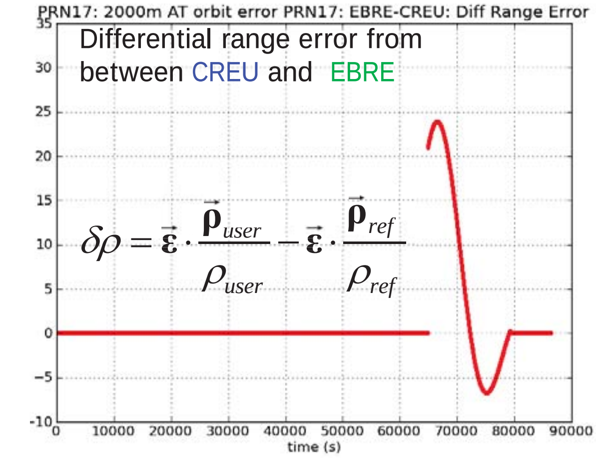


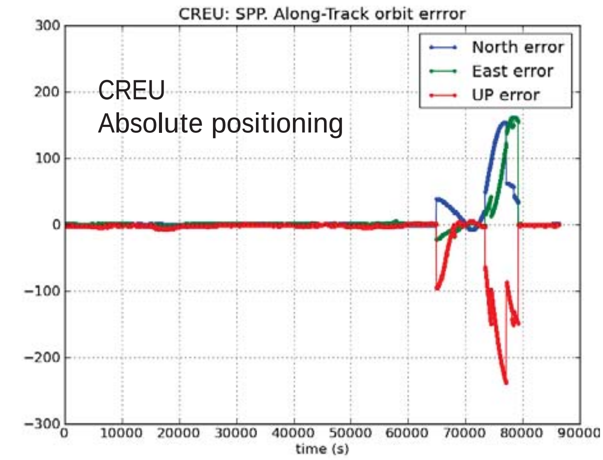


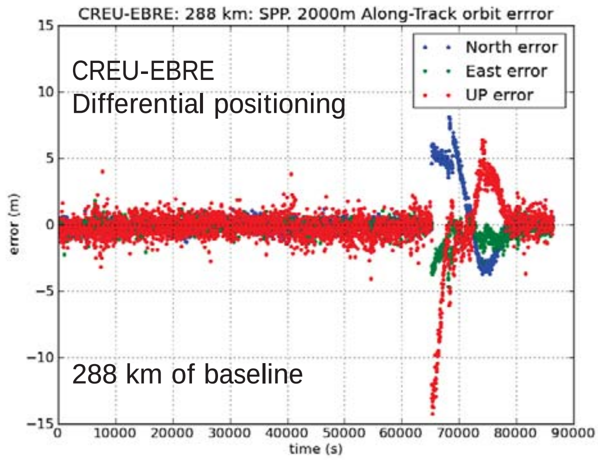


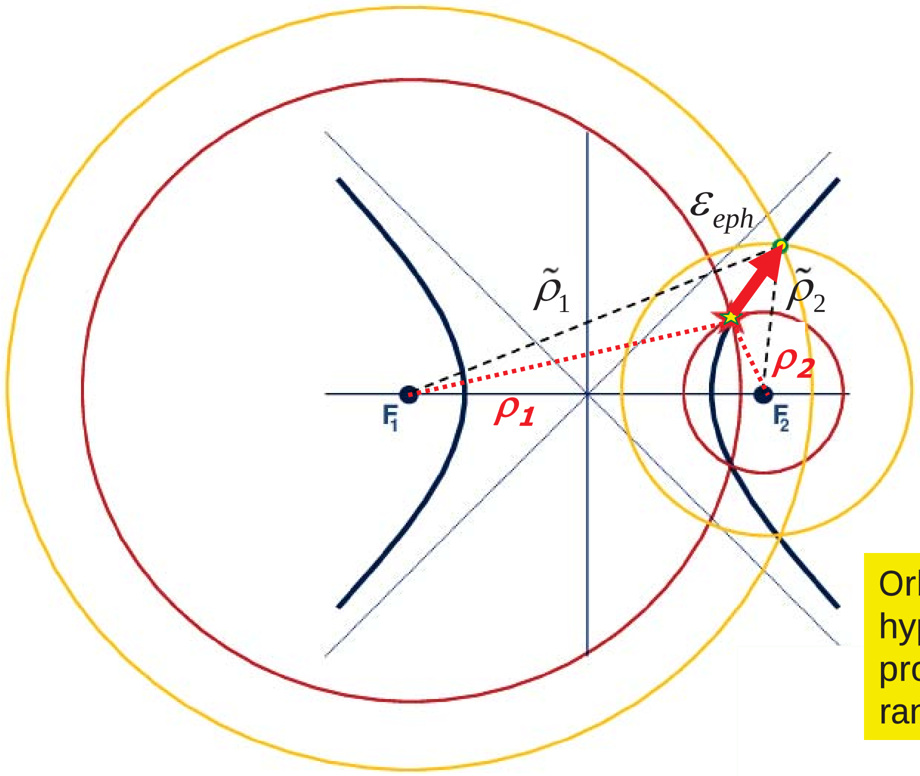






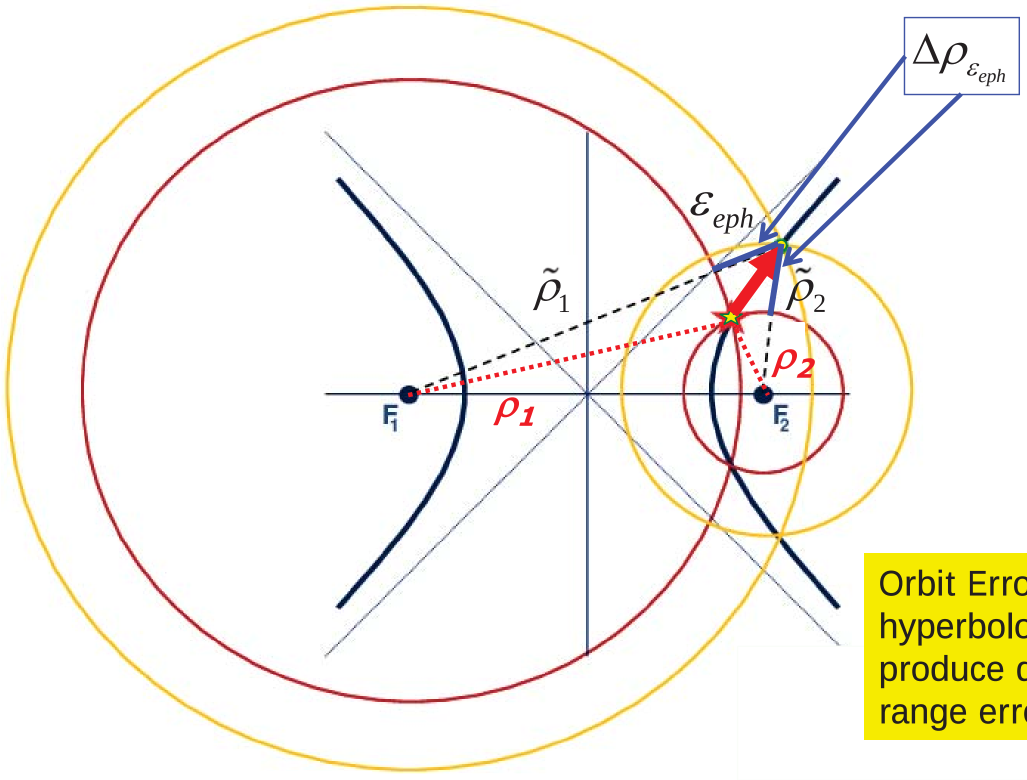






Orbit Errors over the hyperboloid will not produce differential range errors.

$$\left.\begin{array}{l}\tilde{\rho}_1 = \rho_1 + \Delta\rho_{\varepsilon_{eph}} \\ \tilde{\rho}_2 = \rho_2 + \Delta\rho_{\varepsilon_{eph}}\end{array}\right. \Rightarrow \quad \tilde{\rho}_1 - \tilde{\rho}_2 = \rho_1 - \rho_2 = \text{constant} \Rightarrow \quad \delta\rho = 0$$



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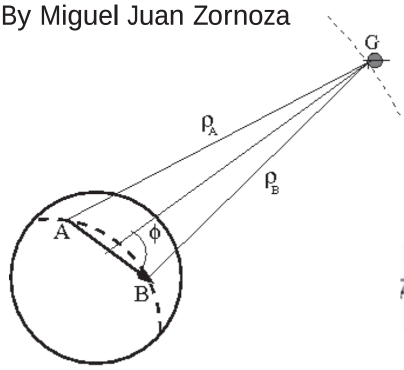
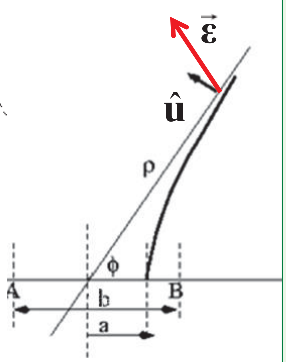
By Miguel Juan Zornoza





$a = (\rho_B - \rho_A)/2$: hyperboloid semiaxis

$b/2$: focal length

$$where \;\; a = \frac{1}{2} b \cos\phi$$

Note: in this 3D problem ϕ is NOT the elevation of ray.

- Errors over the hyperboloid (i.e. $\rho_B - \rho_A = ctt$) will not produce differential range errors.
- The highest error is given by the vector $\hat{\mathbf{u}}$, orthogonal to the hyperboloid and over the plain containing the baseline vector $\hat{\mathbf{b}}$ and the LoS vector $\hat{\boldsymbol{\rho}}$.

Note:
Being the baseline b much smaller than the distance to the satellite, we can assume that the LoS vectors from A and B receives are essentially identical to ρ.
That is, $\rho_B \cong \rho_A \cong \rho$

$$\mathbf{u} = \hat{\boldsymbol{\rho}} \times \left(\hat{\mathbf{b}} \times \hat{\boldsymbol{\rho}}\right) = \hat{\mathbf{b}}\left(\hat{\boldsymbol{\rho}}^T \cdot \hat{\boldsymbol{\rho}}\right) - \hat{\boldsymbol{\rho}}\left(\hat{\boldsymbol{\rho}}^T \cdot \hat{\mathbf{b}}\right)$$
$$= \mathbf{I}\,\hat{\mathbf{b}} - \left(\hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{\rho}}^T\right)\hat{\mathbf{b}} = \left(\mathbf{I} - \hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{\rho}}^T\right)\hat{\mathbf{b}}$$

Note: $\mathbf{u} = \sin\phi\,\hat{\mathbf{u}}$

Differential range error $\delta\rho$ produced by an orbit error $\varepsilon_{\|}$ parallel to vector $\hat{\mathbf{u}}$

Let $\delta\varepsilon \equiv \varepsilon_{\|}$

$$\delta\rho \equiv \delta(\rho_B - \rho_A) = 2\delta a =$$
$$= 2\frac{\partial a}{\partial \varepsilon}\delta\varepsilon = 2\frac{\partial a}{\partial \phi}\frac{\partial \phi}{\partial \varepsilon}\delta\varepsilon = -b\sin\phi\frac{\partial \phi}{\partial \varepsilon}\delta\varepsilon$$
$$\simeq -b\sin\phi\frac{1}{\rho}\delta\varepsilon$$

Note: $\vec{\varepsilon}_{\|} \perp \vec{\rho} \Rightarrow \delta\varepsilon \simeq \rho\,\delta\phi$

Note: being $\hat{\mathbf{u}}$ a vector orthogonal to the LoS $\hat{\boldsymbol{\rho}}$, thence, $\varepsilon_{\|} = \vec{\boldsymbol{\varepsilon}}^T \hat{\mathbf{u}}$

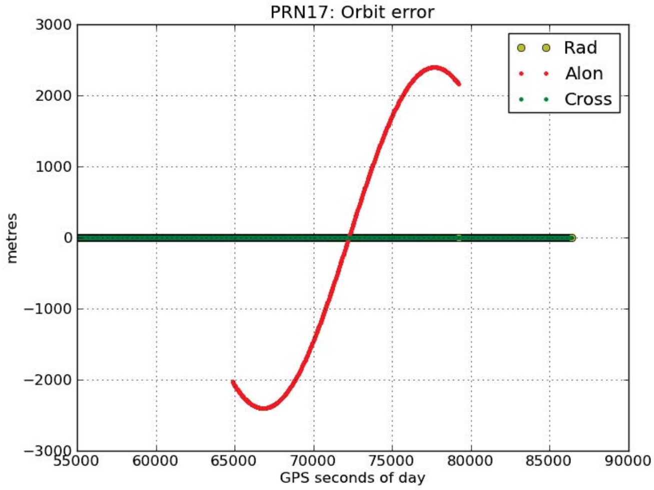
Thence:

$$\delta\rho = -\frac{b\sin\phi}{\rho}\vec{\boldsymbol{\varepsilon}}^T \cdot \hat{\mathbf{u}} = -\vec{\boldsymbol{\varepsilon}}^T \cdot \left(\sin\phi\,\hat{\mathbf{u}}\right)\frac{b}{\rho}$$
$$= -\vec{\boldsymbol{\varepsilon}}^T\left(\mathbf{I} - \hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{\rho}}^T\right)\frac{\mathbf{b}}{\rho}$$

Where: $\mathbf{b} = b\,\hat{\mathbf{b}}$ is the baseline vector

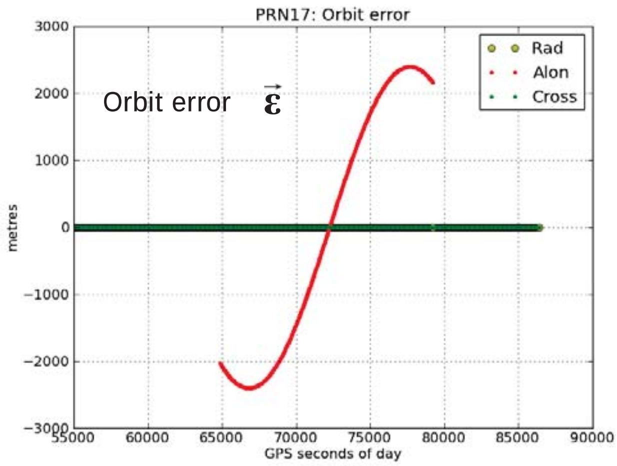
ORBIT TEST : Broadcast orbits Along-track Error (PRN17)

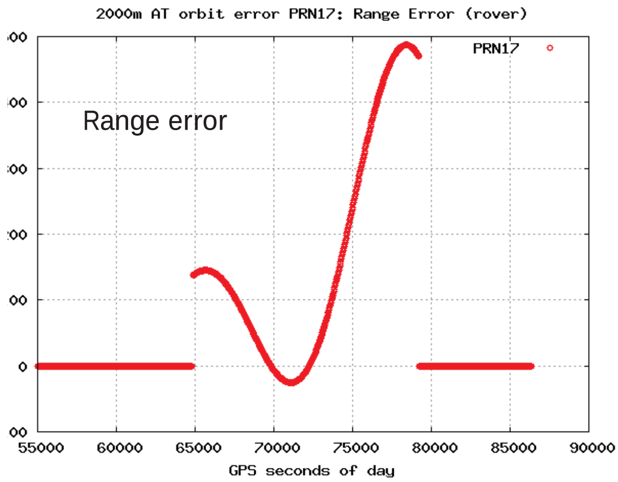
PRN17:
Doy=077, Transm. time: 64818 sec

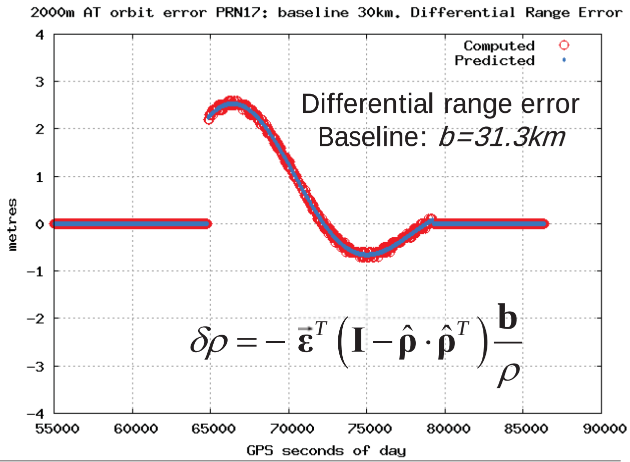



```
17 10  3 18 20  0  0.0 1.379540190101E-04 2.842170943040E-12 0.000000000000E+00
    7.800000000000E+01-5.059375000000E+01 4.506973447820E-09-2.983492318682E+00
   -9.257976353169E-05 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
    4.176000000000E+05-5.401670932770E-08-4.040348681654E-01-7.636845111847E-08
    9.603630515702E-01 2.215312500000E+02-2.547856603060E+00-7.964974630307E-09
   -3.771585673111E-10 1.000000000000E+00 1.575000000000E+03 0.000000000000E+00
    2.000000000000E+00 0.000000000000E+00-1.024454832077E-08 7.800000000000E+01
    4.104180000000E+05 4.000000000000E+00


diff EPH.dat.org EPHcuc_x0.dat ---------------------------------------------------
<     -2.579763531685E-06 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
>     -9.257976353169E-05 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
---------------------------------------------------------------------------------
```





Exercise:

Justify that clock errors completely cancel in differential positioning.

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Error mitigation and short baseline concept

If the distance between the user and the reference station is "short enough", so that the residual error ionospheric, tropospheric and ephemeris are small compared to the typical errors due to receiver noise and multipath, it can be assumed: $T_{ru}^{j} = I_{ru}^{j} = 0; \quad \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} = 0$

Note that the previous definition of "shortness" is quite fussy

Working with smoothed code, a residual error of about 0.5 metres could be tolerable, but for carrier based positioning it should be less than 1 cm to allow the carrier ambiguity fixing.

- The differential ephemeris error is at the level of few centimetres for baselines up to 100 Km (i.e. 5 cm assuming a large bound of $\varepsilon_{eph}^{j} \simeq 10\,\mathrm{m}$).
- The typical spatial gradient of the ionosphere (STEC) is 1-2 mm/km (i.e. 0.1-0.2 m in 100km), but it can be more than one order of magnitude higher when the ionosphere is active.

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Error mitigation and short baseline concept

Note that the previous definition of "shortness" is quite fussy

- The correlation radio of the troposphere is lower than for the ionosphere. At 10km of separation the residual error can be up to 0.1-0.2 m. Nevertheless, 90% of the tropospheric delay can be modelled and the remaining 10% can be estimated together with the coordinates (for high precision applications). For distances beyond a ten of kilometres or significant altitude difference it would be preferable to correct for the tropospheric delay at the reference station and user receiver.

Carrier-smoothed code:
Pseudorange code measurement errors due to receiver noise and multipath can be reduced smoothing the code with carrier measurements.
Smoothed codes of 0.5m (RMS) can be obtained with 100 seconds smoothing.
On the other hand, the ionospheric error is substantially eliminated in differential mode and the filter can be allowed for lager time smoothing windows.

Contents

Differential code based positioning

If the reference station coordinates are known at the centimetre level and the distance between reference station and user are "not too large", we can assume

$$T_{ru}^{j} \simeq 0\,;\, I_{ru}^{j} \simeq 0$$
$$\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} \simeq 0$$
$$\boldsymbol{\varepsilon}_{site} \simeq 0 \Rightarrow \boxed{\Delta\mathbf{r}_{ru} \simeq \Delta\mathbf{r}_{u}}$$

Thence,

$$P_{ru}^{j} = \rho_{ru}^{j} + c\,\delta t_{ru} + \cancel{T_{ru}^{j}} + \cancel{I_{ru}^{j}} + K_{ru} + \nu_{P\,ru}^{\;j} \longrightarrow$$
$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \cancel{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site,r}} + \cancel{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}} \rightarrow$$

$$\boxed{\begin{aligned} P_{ru}^{j} &= \rho_{ru}^{j} + c\,\delta t_{ru} + K_{ru} + \nu_{P\,ru}^{\;j} \\ \rho_{ru}^{j} &= \rho_{0\,ru}^{\;j} - \hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta\mathbf{r}_{u} \end{aligned}}$$

Note : for baselines up to 100 km *the range error of broadcast orbits is less than* 10 cm *(assuming* $\varepsilon_{eph}^{j} \simeq 10$ m *).*

Or, what is the same:

$$\boxed{P_{ru}^{j} - \rho_{0\,ru}^{\;j} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta\mathbf{r}_{u} + c\,\delta t_{ru} + K_{ru} + \nu_{P\,ru}^{\;j}}$$

The left hand side of previous equation can be spitted in two terms: one associated to the reference station and the other to the user:

$$P_{ru}^{j} - \rho_{0\,ru}^{\;j} = P_{u}^{j} - \rho_{0\,u}^{\;j} - \left(P_{r}^{j} - \rho_{0\,r}^{\;j}\right)$$

$$\boxed{\Delta\mathbf{r}_{ru} = \Delta\mathbf{r}_{u} - \Delta\mathbf{r}_{r} = \Delta\mathbf{r}_{u} - \boldsymbol{\varepsilon}_{site}}$$

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Differential code based positioning

$$\boxed{P^j_{ru} - \rho^{\,j}_{0\,ru} = -\hat{\boldsymbol{\rho}}^{\,j}_{0\,u}\cdot\Delta\mathbf{r}_u + c\,\delta t_{ru} + K_{ru} + \nu^{\,j}_{P\,ru}}$$

The left hand side of previous equation can be spitted in two terms: one associated to the reference station and the other to the user:

$$P^j_{ru} - \rho^{\,j}_{0\,ru} = P^j_u - \rho^{\,j}_{0\,u} - \left(P^j_r - \rho^{\,j}_{0\,r}\right)$$

- The term $P^j_r - \rho^{\,j}_{0\,r}$ is the error in range measured by the reference station, which can be broadcasted to the user as a differential correction:

$$\boxed{PRC^j = \rho^{\,j}_{0\,r} - P^j_r}$$

- The user applies this differential correction to remove/mitigate common errors: $$\boxed{P^j_u - \rho^{\,j}_{0\,u} + PRC^j = -\hat{\boldsymbol{\rho}}^{\,j}_{0\,u}\cdot\Delta\mathbf{r}_u + c\,\delta\tilde{t}_{ru} + \nu^{\,j}_{P\,ru}}$$

Where the receiver's instrumental delay term K_{ru} is included in the differential clock $c\,\delta\tilde{t}_{ru}$

For distances beyond a ten of kilometres, or significant altitude difference, it would be preferable to correct for the tropospheric delay at the reference station and user receiver.

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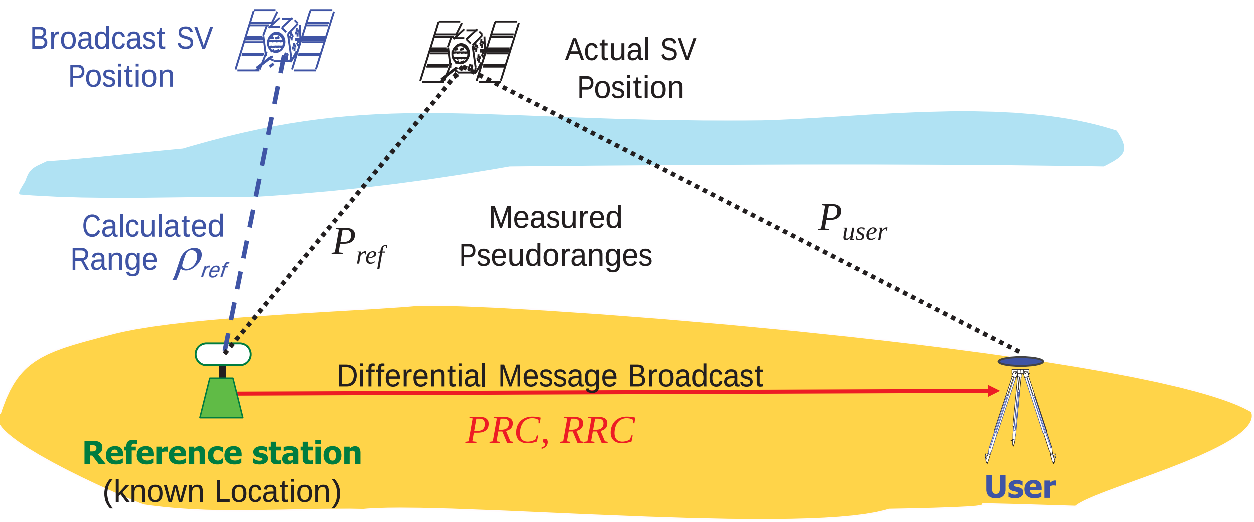
Range Differential Correction Calculation



– The **reference station** with known coordinates, computes pseudorange and range-rate corrections: $PRC = \rho_{ref} - P_{ref}$, $RRC = \Delta PRC/\Delta t$.

– The **user** receiver applies the PRC and RRC to correct its own measurements, $P_{user} + (PRC + RRC\,(t-t_0))$, removing SIS errors and improving the positioning accuracy.

DGNSS with code ranges: users within a hundred of kilometres can obtain **one-meter-level** positioning accuracy using such pseudorange corrections.

Differential code based positioning

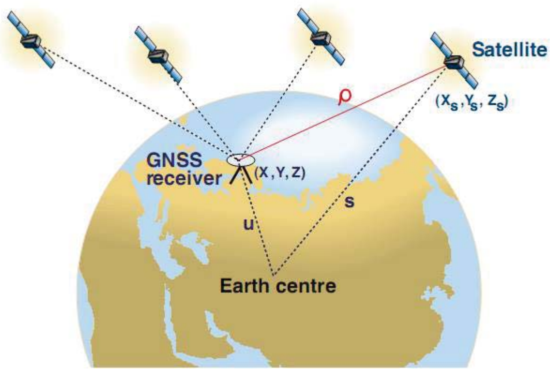
The user applies this differential correction to remove/mitigate common errors:
$$\boxed{P_u^j - \rho_{0u}^{\ j} + PRC^j = -\hat{\boldsymbol{\rho}}_{0u}^{\ j} \cdot \Delta\mathbf{r}_u + c\,\delta\tilde{t}_{ru} + \nu_{P\,ru}^{\ j}}$$

where the receiver's instrumental delay term K_{ru} is included in the differential clock $c\,\delta\tilde{t}_{ru}$

The previous system for navigation equations is written in matrix notation as:

$$\begin{bmatrix} \mathrm{Pref}^1 \\ \mathrm{Pref}^2 \\ \vdots \\ \mathrm{Pref}^n \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}_{0u}^{\ 1}\right)^T & 1 \\ -\left(\hat{\boldsymbol{\rho}}_{0u}^{\ 2}\right)^T & 1 \\ \vdots & \\ -\left(\hat{\boldsymbol{\rho}}_{0u}^{\ n}\right)^T & 1 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{r}_u \\ c\,\delta\tilde{t}_{ru} \end{bmatrix}$$

where
$$\mathrm{Pref}^j \equiv P_u^j - \rho_{0u}^{\ j} + PRC^j$$



Differential code based positioning

Time synchronization issues:

For simplicity we have dropped any reference to measurement epochs, but real-time implementations entail delays in data transmission and the time update interval can be limited by bandwidth restrictions.

- Differential corrections vary slowly and its useful life can be up to several minutes with S/A=off.
- To reduce bandwidth, the reference station computes Pseudorange Corrections (PRC) and Range-Rate Correction (RRC) for each satellite in view, which are broadcast to every several seconds, up to a minute interval with S/A=off.
- The user computes the PRC at the measurement epoch as:

$$PRC^j(t) = PRC^j(t_0) + RRC^j(t - t_0)$$

Differential code based positioning

Data handling:

Reference station and user have to coordinate how the measurements are to be processed:

- Corrections must be identified with an Issue of Data (IOD) and time-out must be considered.
- Both receivers must use the same ephemeris orbits (which are identified by the IODE).
- If reference station uses a tropospheric model the same model must be applied by the user.
- If reference station uses the broadcast ionospheric model, the user must do the same.

Note: we have considered here only code measurements. The carrier based positioning will be treated next, using double differences of measurements and targeting the ambiguity fixing.

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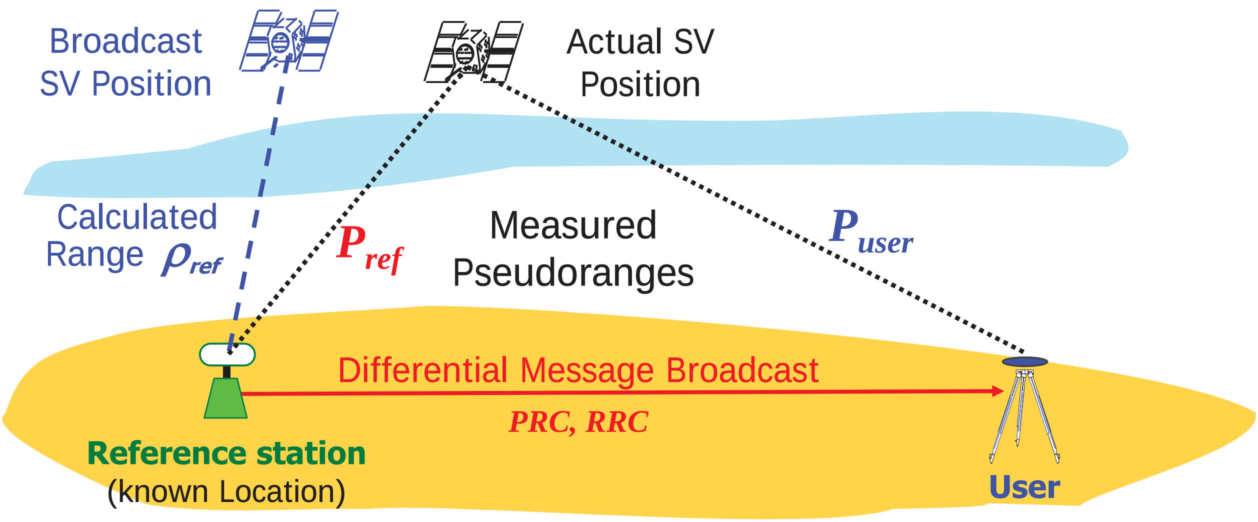
Range Differential Correction Calculation



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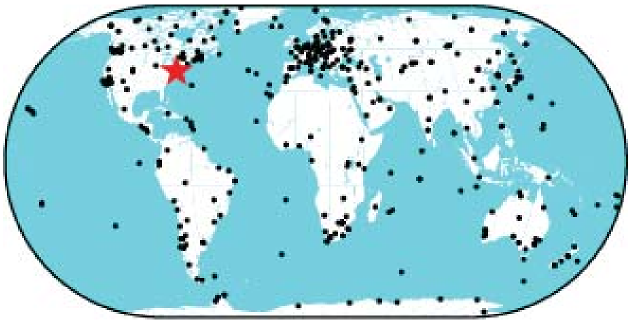

ftp://cddis.gsfc.nasa.gov/highrate/2013/

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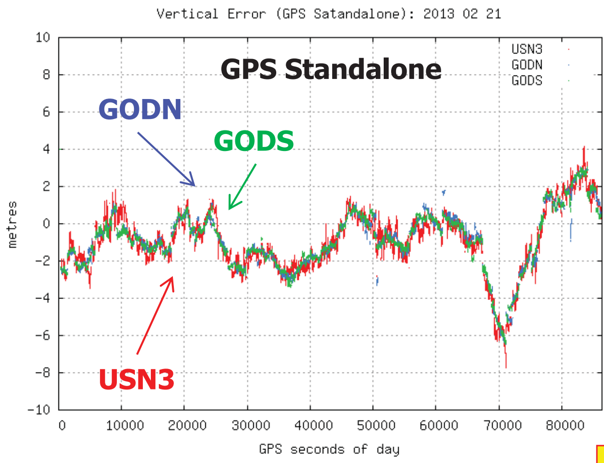
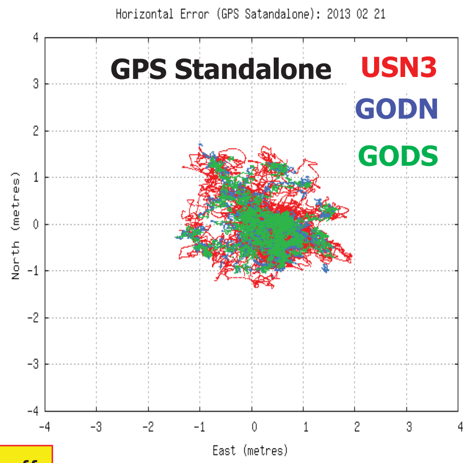
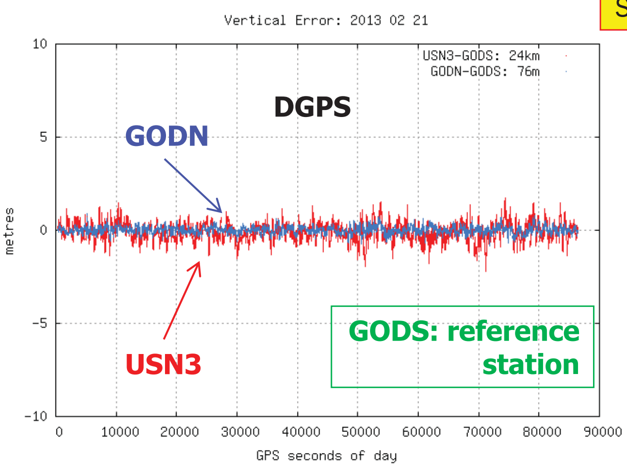
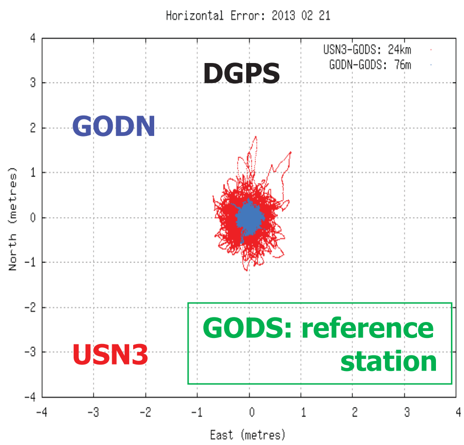




S/A=off





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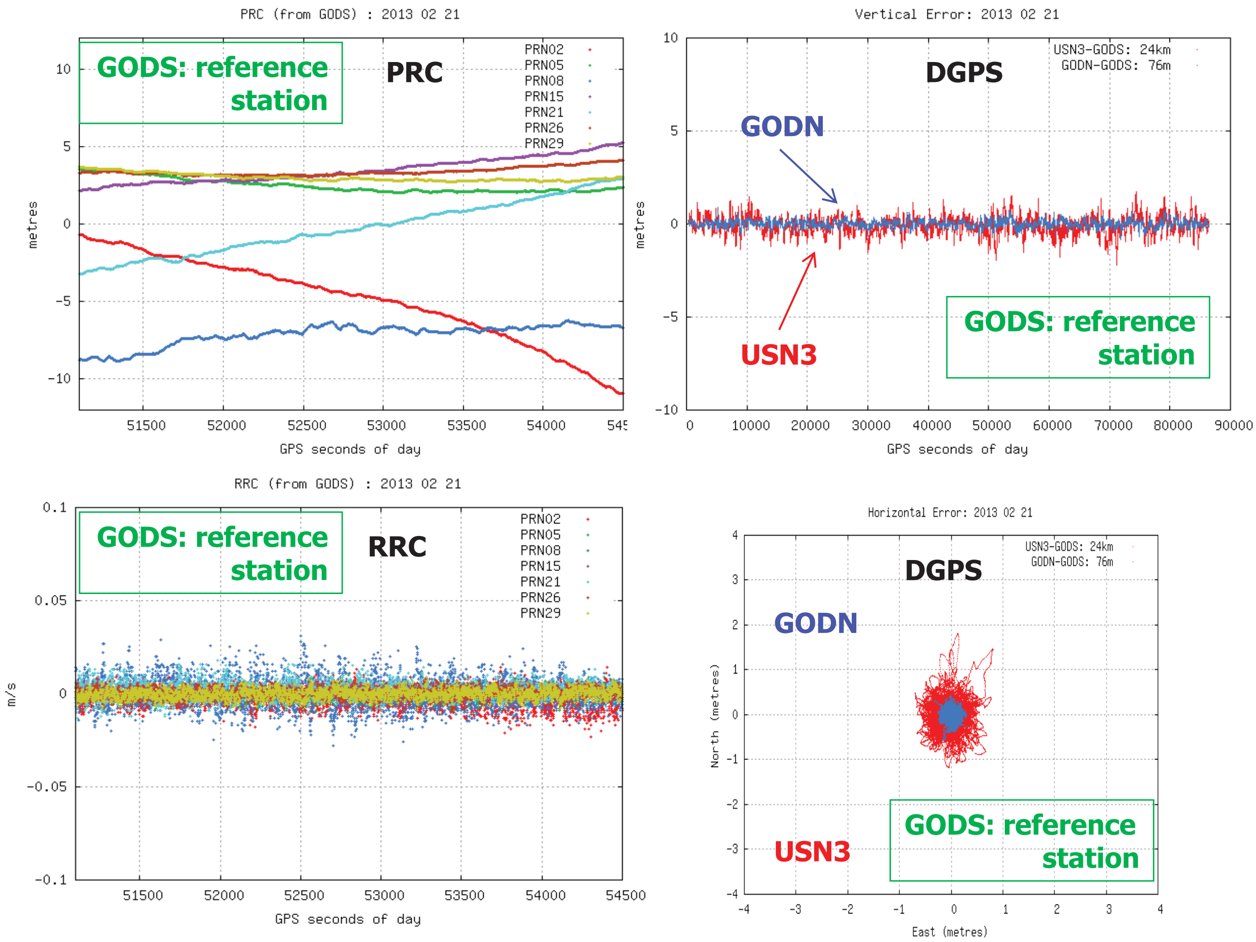

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Contents

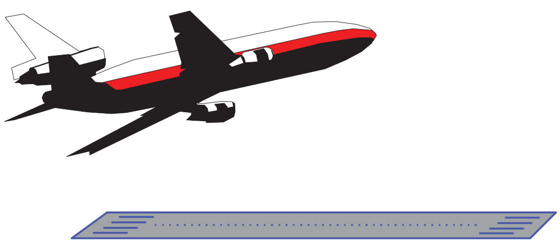
Introduction: What Augmentation is?

- To enhance the performance of the current GNSS with additional information to:
  - Improve INTEGRITY via real-time monitoring
  - Improve ACCURACY via differential corrections
  - Improve AVAILABILITY and CONTINUITY

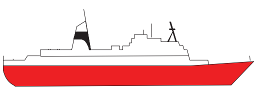
- Satellite Based Augmentation Systems (**SBAS**)
  - E.g., WAAS, **EGNOS**, MSAS
- Ground Based Augmentation Systems (**GBAS**)
  - E.g., LAAS
- Aircraft Based Augmentation (ABAS)
  - E.g., RAIM, Inertials, Baro Altimeter

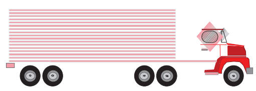
Why Augmentation Systems?

- Current GPS/GLONASS Navigation Systems cannot met the Requirements for All Phases of Flight:
  - Accuracy
  - Integrity
  - Continuity
  - Availability



- Marine and land users will also require some sort of augmentation for improving the GPS/ GLONASS performances.





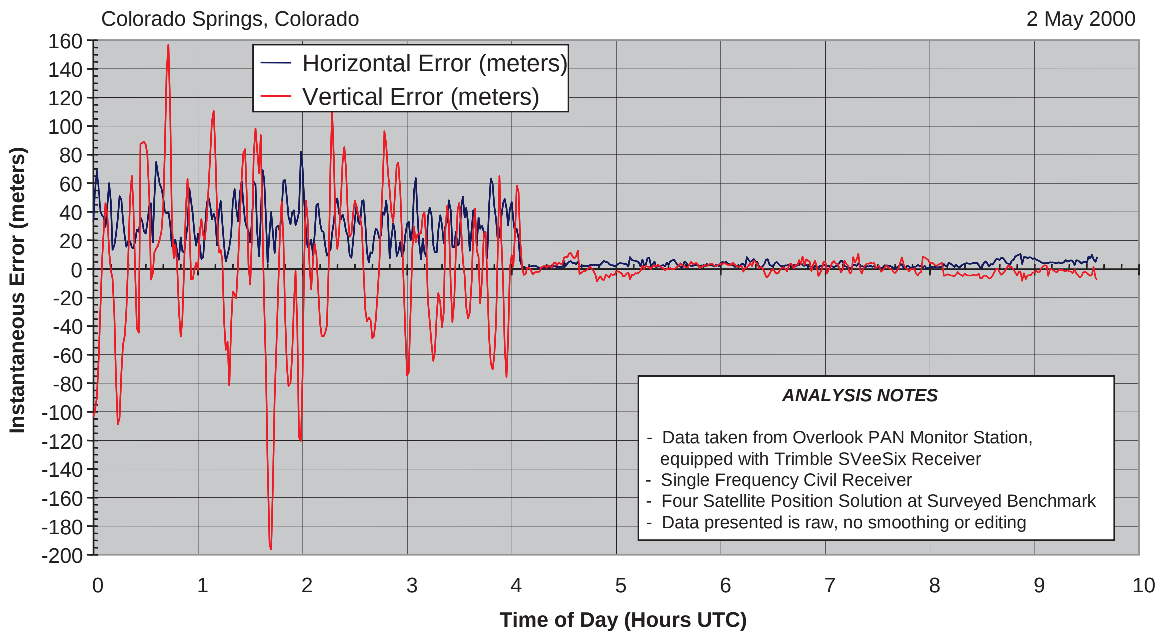
WHY GNSS NEEDS AN AUGMENTATION ?

GPS Only → ← **Civil Aviation**

PERFORMANCE		CATEGORY I Requirements
H. 13 m V. 22m	ACCURACY (95%)	H 16.0 m V 4.0 m
99% (RAIM)	AVAILABILITY	99% to 99.990%
?	INTEGRITY	2.10^{-7}/ approach Time to alarm 6 s
?	CONTINUITY OF SERVICE	10^{-5} / approach (10^{-6} / 15 s)

Accuracy: Difference between the measured position at any given time to the actual or true position.

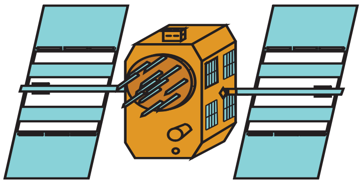
Even with S/A off a Vertical Accuracy < 4m 95% of time cannot be guaranteed with the standalone GPS.



Integrity: Ability of a system to provide timely warnings to users or to shut itself down when it should not be used for navigation.

Standalone GPS and GLONAS Integrity is Not Guaranteed

GPS/GLONASS Satellites:
Time to alarm is from minutes to hours
No indication of quality of service



Health Messages:

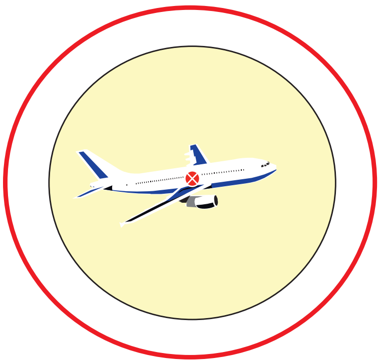
GPS up to 2 hours late

GLONASS up to 16 hours late

Continuity: Ability of a system to perform its function without (unpredicted) interruptions during the intended operation.

Availability: Ability of a system to perform its function at initiation of intended operation. System availability is the percentage of time that accuracy, integrity and continuity requirements are met.

Availability and Continuity Must meet requirements



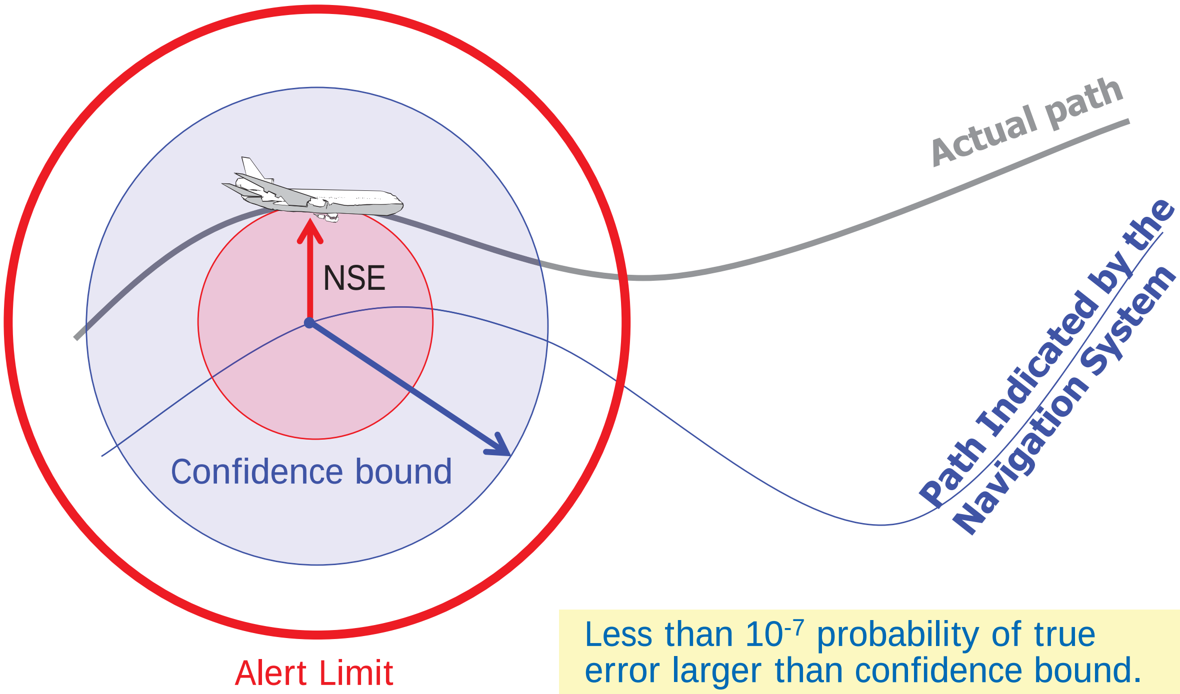
Continuity:

Less than 10^{-5} Chance of Aborting a Procedure Once it is Initiated.

Availability:

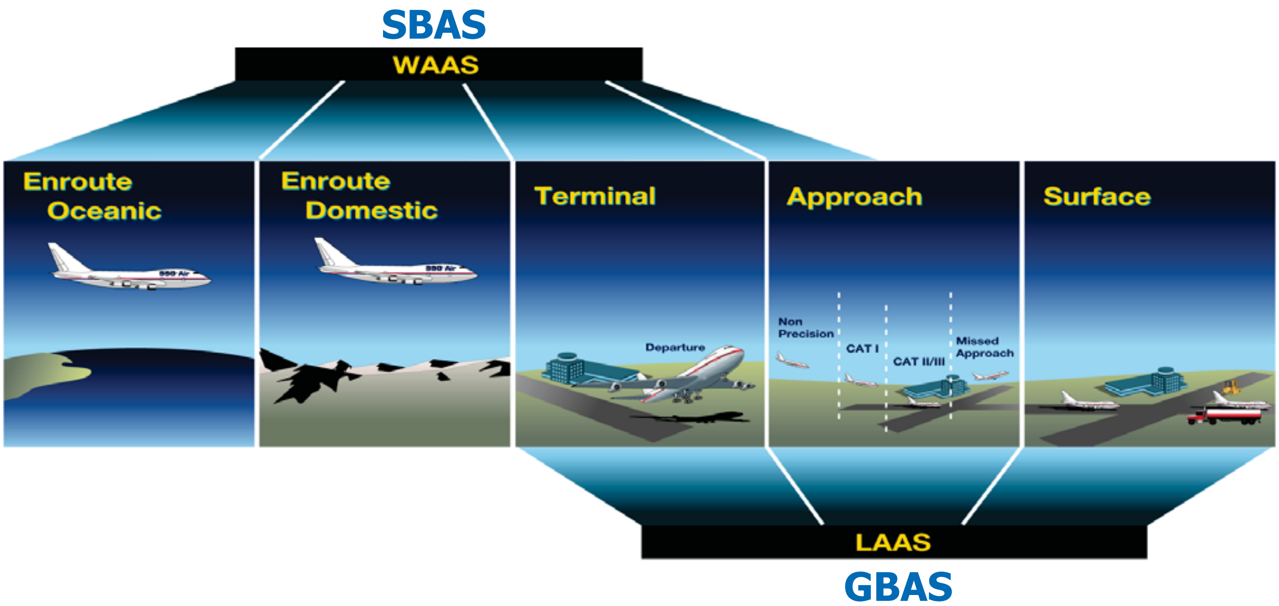
>99% for every phase of flight (SARPS).

INTEGRITY



Less than 10^{-7} probability of true error larger than confidence bound.
Time to alarm 6 s

SBAS and GBAS Navigation Modes



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Civil Aviation Signal-in-Space Performance Requirements

Aviation	Accuracy (H) 95%	Accuracy (V) 95%	Alert Limit (H)	Alert Limit (V)	Integrity	Time to alert	Continuity	Avail-ability
ENR	3.7 Km (2.0 NM)	N/A	7400 m 3700 m 1850 m	N/A	$1\text{-}10^{-7}$/h	5 min.	$1\text{-}10^{-4}$/h to $1\text{-}10^{-8}$/h	0.99 to 0.99999
TMA	0.74 Km (0.4 NM)	N/A	1850 m	N/A	$1\text{-}10^{-7}$/h	15 s	$1\text{-}10^{-4}$/h to $1\text{-}10^{-8}$/h	0.999 to 0.99999
NPA	220 m (720 ft)	N/A	600 m	N/A	1-10-7/h	10 s	1-10-4/h to 1-10-8/h	0.99 to 0.99999
APV-I	220 m (720 ft)	20 m (66 ft)	600 m	50 m	1-2x10-7 per approach	10 s	1-8x10-6 in any 15 s	0.99 to 0.99999
APV-II	16.0 m (52 ft)	8.0 m (26 ft)	40 m	20 m	1-2x10-7 per approach	6 s	1-8x10-6 in any 15 s	0.99 to 0.99999
CAT-I	16.0 m (52 ft)	6.0 - 4.0 m (20 to 13 ft)	40 m	15 -10 m	1-2x10-7 per approach	6 s	1-8x10-6 in any 15 s	0.99 to 0.99999

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Contents

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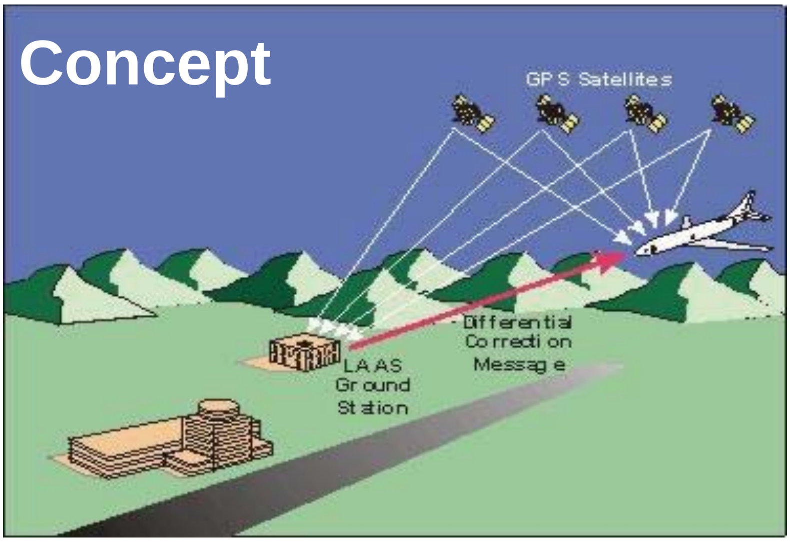
GBAS Concept



Most of the measurement errors are common:
clock, ephemeris, ionosphere and troposphere.

A common correction valid for any receiver within the LADGPS area is generated and broadcast.

The accuracy is limited by the spatial decorrelation of those error sources (1m at 100Km).

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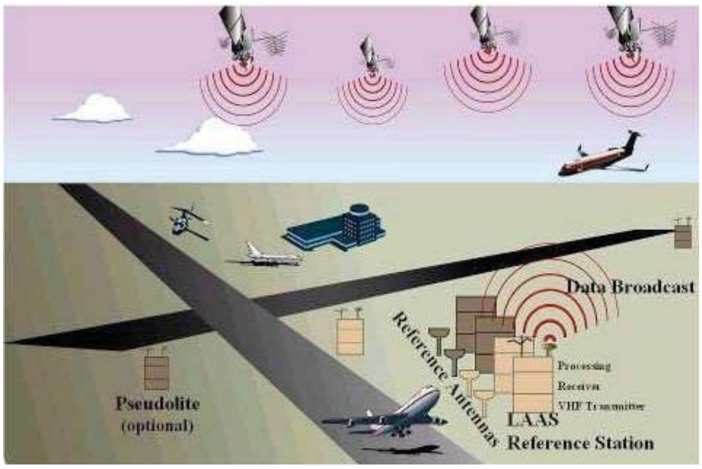
GBAS Concept



The Ground Station (GS) responsible for generating and broadcasting carrier-smoothed code differential corrections and approach-path information to user aircrafts.
This system is used to support aircraft operations during approach and landing.

- It is also responsible of detecting and alarming space-segment and ground-segment failures.
- The GS must insure that all ranging sources for which GBAS corrections are broadcast are safe to use. If a failure occurs that threatens user safety, the GS must detect and alert users (by not broadcasting corrections for the affected ranging source) within a certain time-to-alert.

This is from [RD-5]

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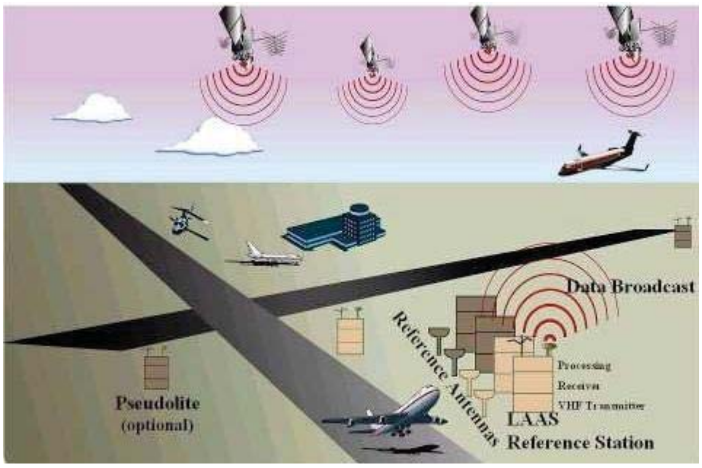
GBAS Components



The GBAS station involves redundant Reference Receivers and antennas, redundant High Frequency Data Broadcast equipment linked to a single antenna.

- The GS tracks, decodes, and monitors GPS satellite information and generates differential corrections.
- It also performs integrity checks on the generated corrections.
- The correction message, along with suitable integrity parameters and approach path information, is then broadcast to airborne users on a VHF channel, up to about 40km.

This is from [RD-5]

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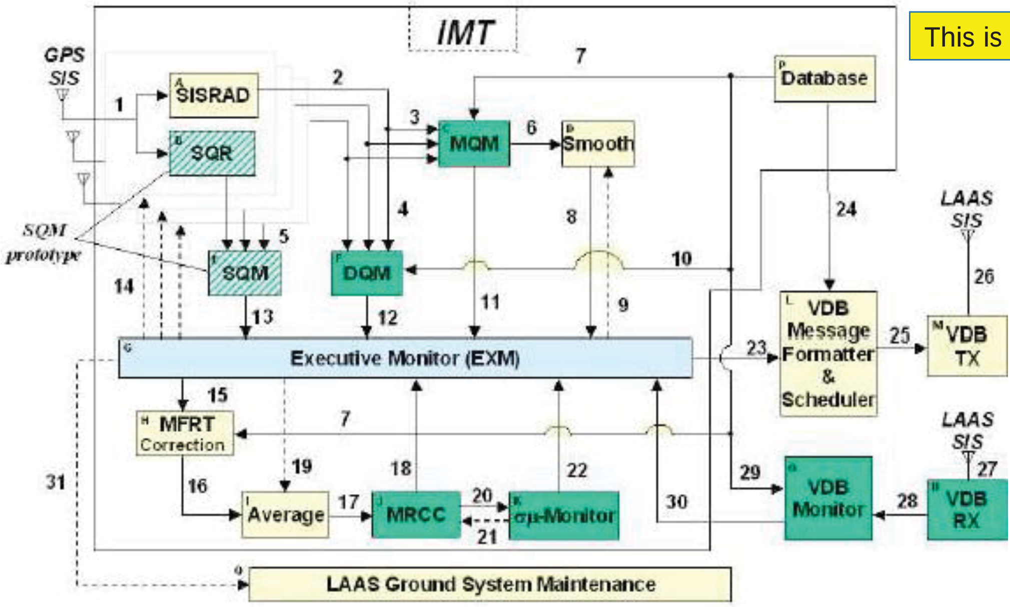
Ground System Functional Flow Diagram

This is from [RD-5]



A variety of integrity monitor algorithms that are grouped into:

- Signal Quality Monitoring (SQM)
- Data Quality Monitoring (DQM)
- Measur. Quality Monitoring (MQM)
- Multiple Reference Consist Check (MRCC)
- σμ-monitor
- Message Field Range Test (MFRT)

SQM: Targets satellite signals anomalies and local interference. Implements tests for Correlation Peak Symmetry, Receiver Signal Power and Code-Carrier Divergence.

DQM: Checks the validity of the GPS ephemeris and clock data for each satellite that rises in view of the LGF and at each time new navigation data messages are broadcast.

MQM: Confirms the consistency of the pseudorange and carrier-phase measurements over the last few epochs to detect sudden step and any other rapid errors.

MRCC: Examines the consistency of corrections for each satellite across all reference receivers.

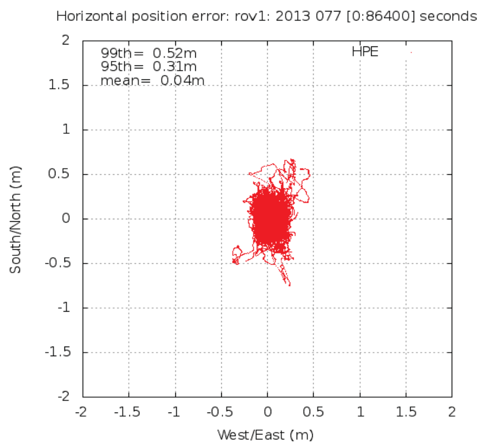
σμ-monitor: Helps ensure a Gaussian distribution for the correction error with zero mean and that the broadcast σ_{pr_gnd} overbounds the actual errors in the broadcast differential corrections.

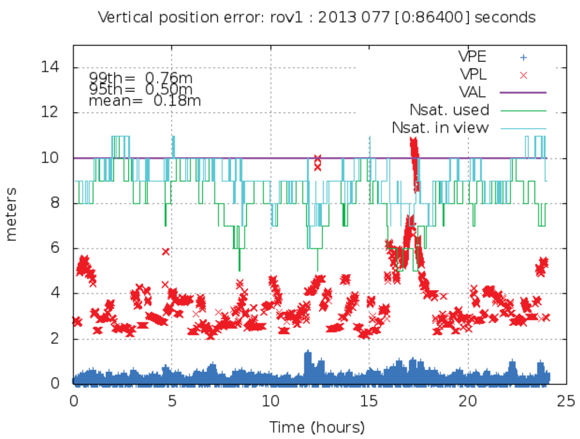
MFRT: Verifies that the computed averaged pseudorange corrections and correction rates fit within the message field bounds.

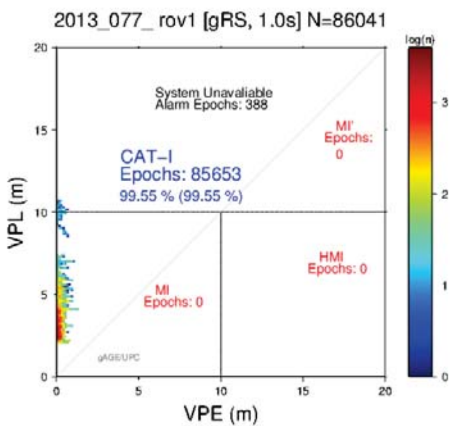
This is from [RD-5]

Executive Monitors (EXM-I and EXM-II) coordinate all previous monitors and combine failure flags.

- Signal Quality Monitoring (SQM)
- Data Quality Monitoring (DQM)
- Measur. Quality Monitoring (MQM)
- Multiple Reference Consist Check (MRCC)
- σμ-monitor
- Message Field Range Test (MFRT)







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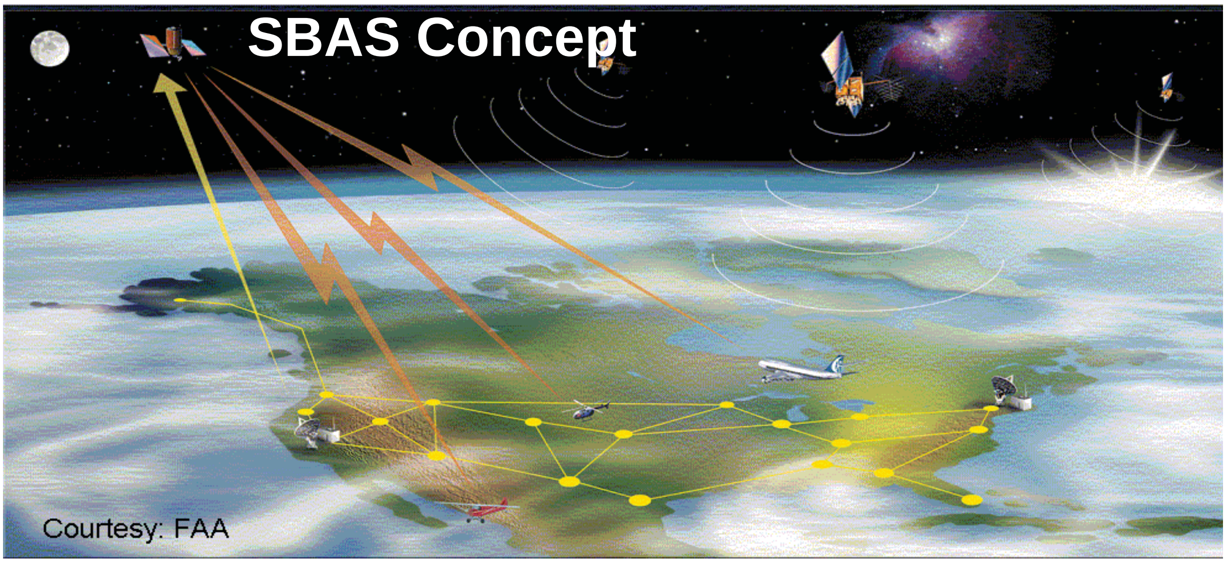


The pseurorange error is split in its components.

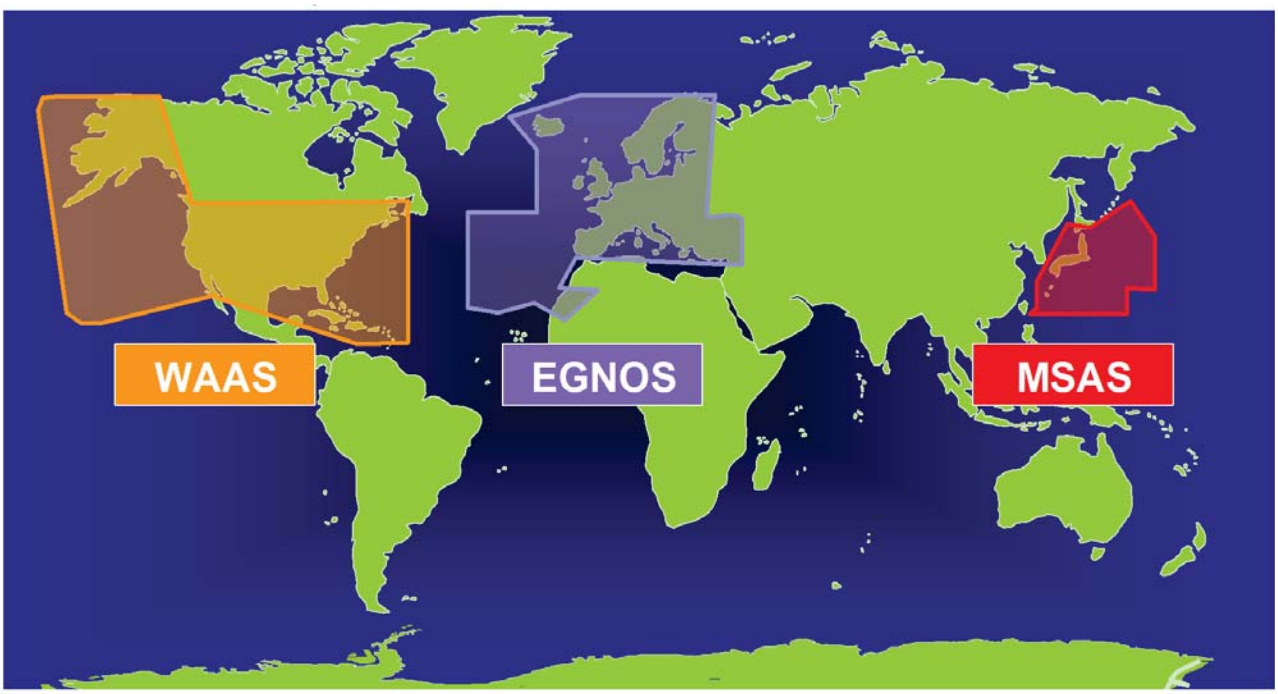
- Clock error
- Ephemeris error
- Ionospheric error
- Local errors (troposphere, multipath, receiver noise)

Uses a network of receivers to cover broad geographic area

Error Mitigation

Error component	GBAS	SBAS
Satellite clock	Common Mode Differencing	Estimation and Removal each error component
Ephemeris		
Ionosphere		
Troposphere		Fixed Model
Multipath and Receiver Noise	Carrier Smoothing by user	

Three existing SBAS Systems

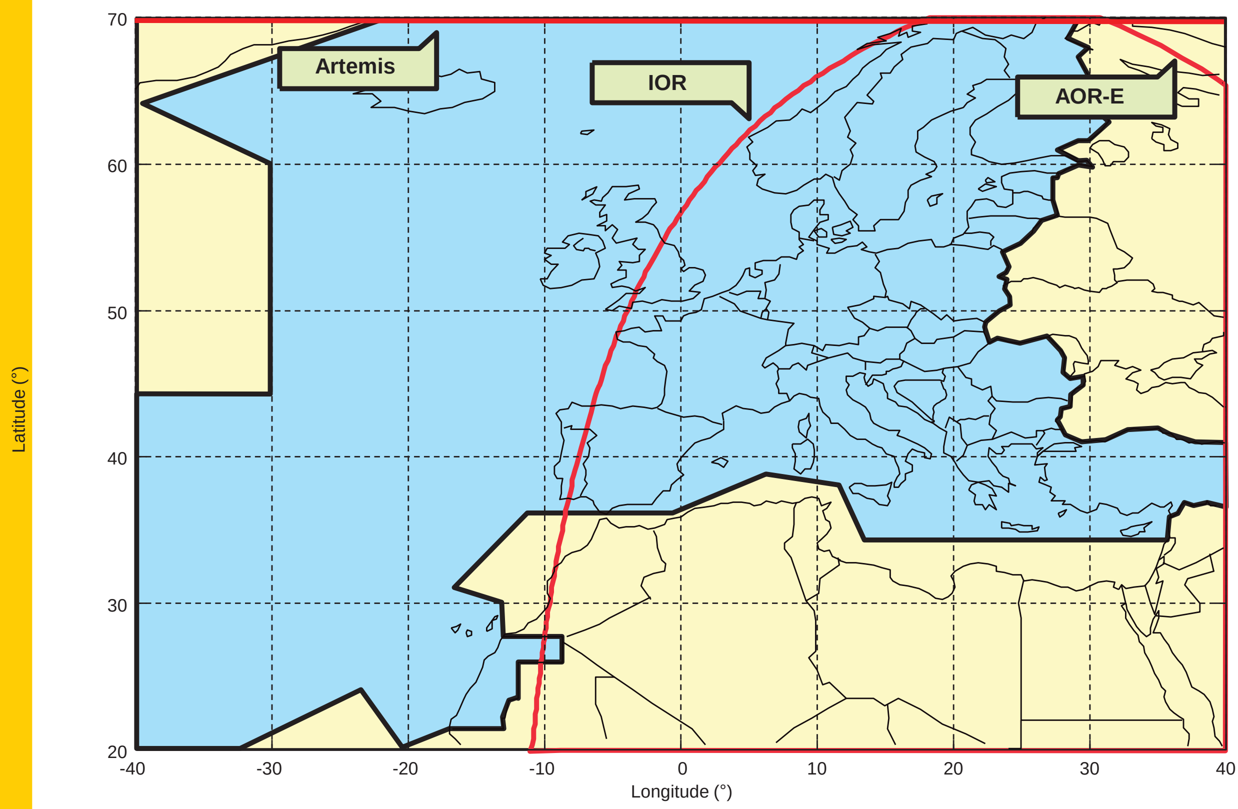


The European Geostationary Navigation Overlay SERVICE (EGNOS)

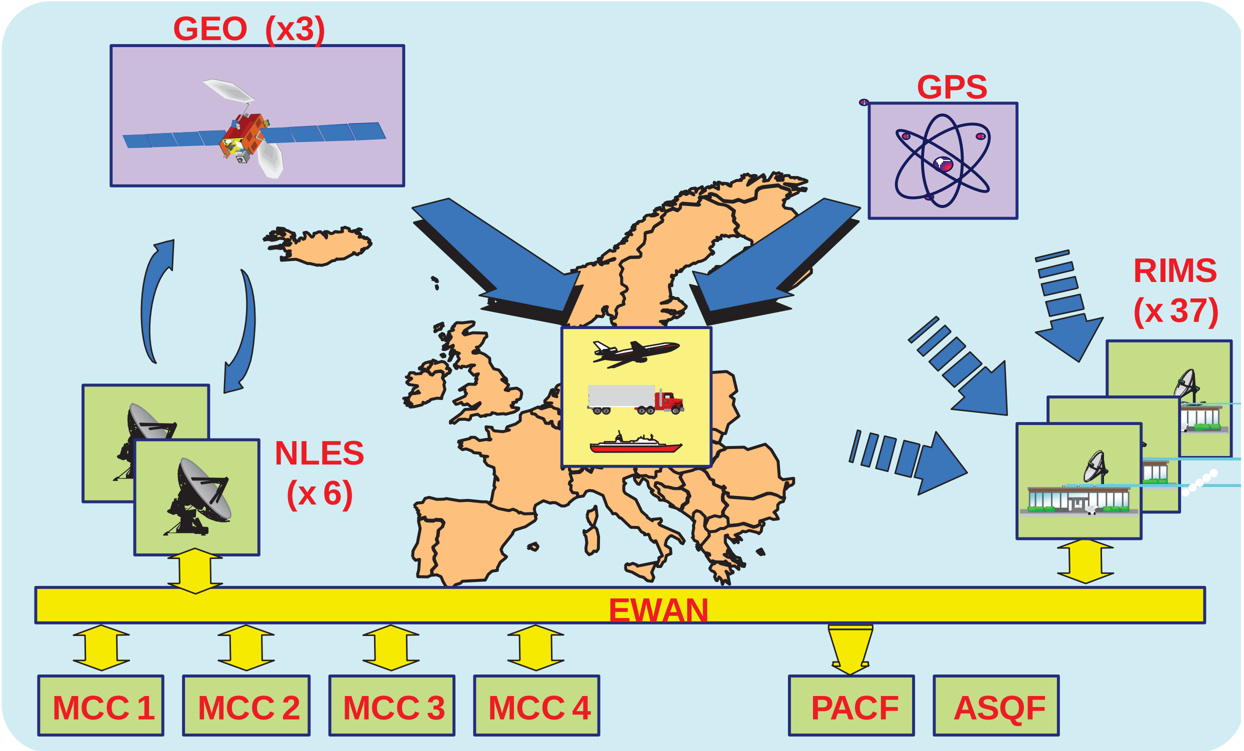
- EGNOS is the European component of a Satellite Based Augmentation to GPS.
- EGNOS is being developed under the responsibility of a tripartite group:
  - The European Space Agency (ESA)
  - The European Organization for the Safety of Air Navigation (EUROCONTROL)
  - The Commission of the European Union.

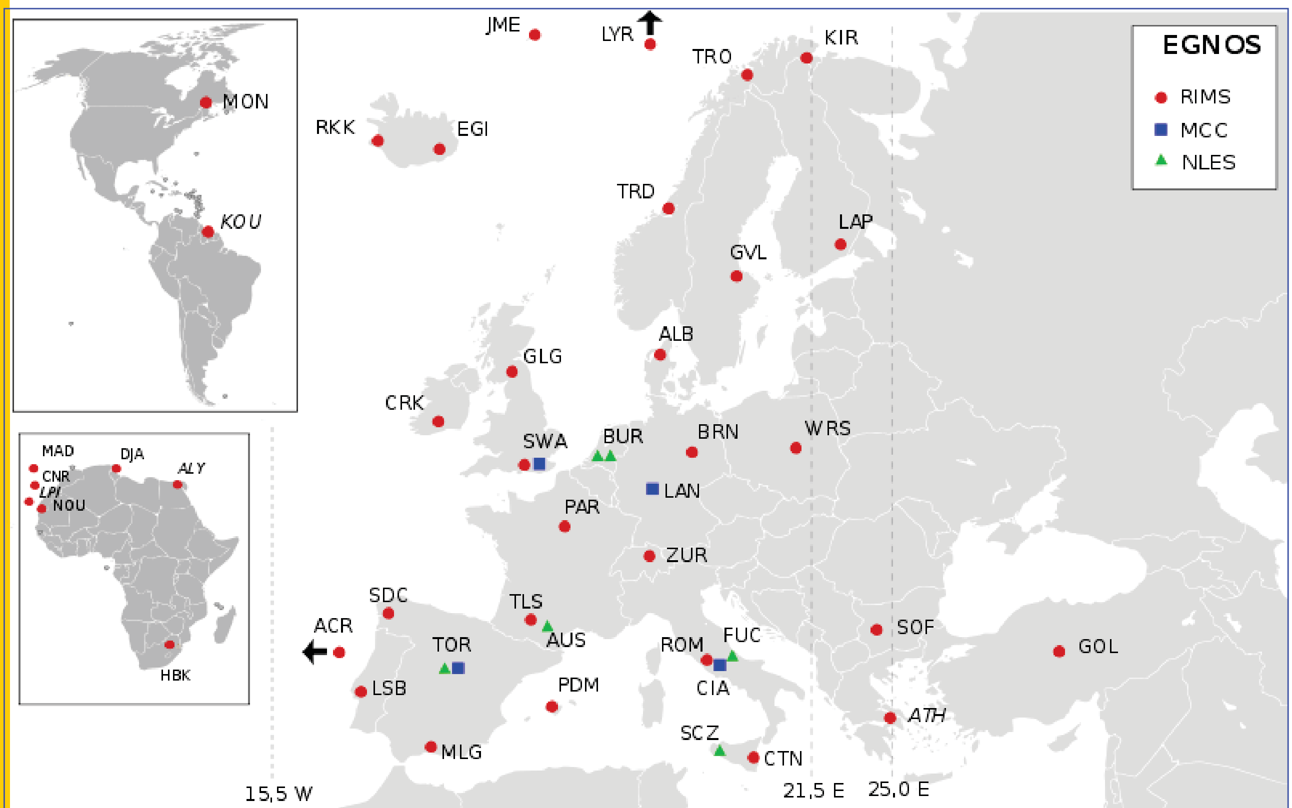
ECAC Area

(ECAC: European Civil Aviation Conference)



EGNOS Architecture





EGNOS ground segment is composed of the following stations/centres which are mainly distributed in Europe and are interconnected between themselves **through a land network**.

- **37 RIMS** (Ranging and Integrity Monitoring Stations) + seven being deployed: receive the satellite signals and send this information to the MCC centres.

- **4 MCC** (Master Control Centres) receive the information from the RIMS stations and generate correction messages to improve satellite signal accuracy and information messages on the status of the satellites (integrity). The MCC acts as the EGNOS system 'brain'.

- **6 NLES** (Navigation Land Earth Stations): they receive the correction messages from the CPFs for the upload of the data stream to the geostationary satellites and the generation of the GPS-like signal. This data is then transmitted to the European users via the geostationary Satellite

http://egnos-user-support.essp-sas.eu/egnos_ops/egnos_system/system_description/current_architecture

Master Control Center (MCC)

MCC is Subdivided into

- CCF (Central Control Facility)
  - Monitoring and control EGNOS G/S
  - Mission Monitoring and archive
  - ATC I/F
- CPF (Central Processing Facility)
  - Provides EGNOS WAD corrections
  - Ensures the Integrity of the EGNOS users
  - Utilises independent RIMS channels for checking of corrections
  - Real time software system developed to high software standards

4 MCCs are implemented in EGNOS

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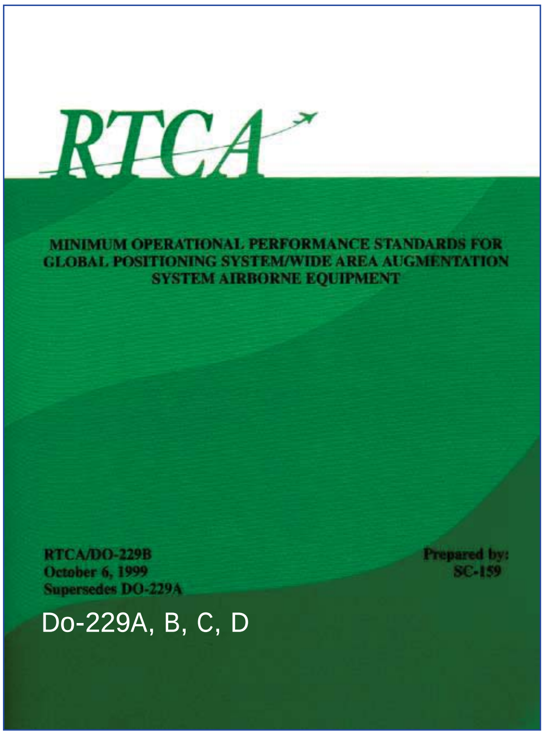
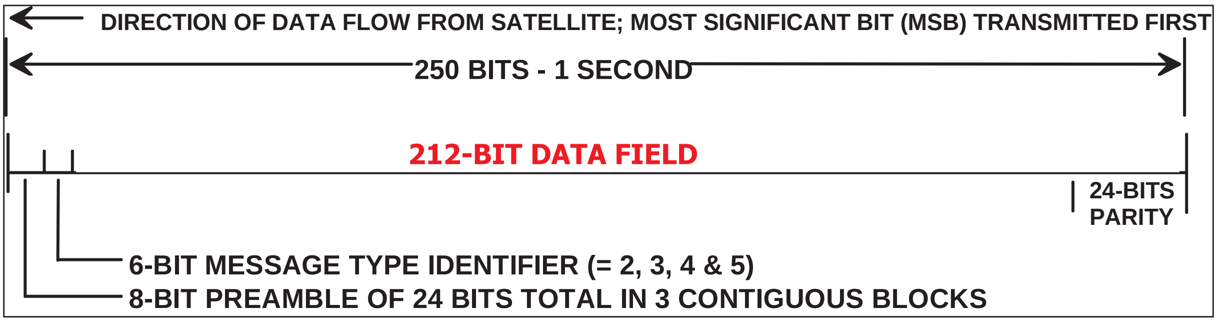
SBAS Differential Corrections and Integrity:

The RTCA/MOPS-DO 229C



Message Format



The corrections, even for individual satellites are distributed across several individual messages.

- 250 bits
- One Message per second
- All messages have identical format

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SBAS Broadcast Messages (ICAO SARPS)

MSG 0	Don't use this SBAS signal for anything (for SBAS testing)
MSG 1	PRN Mask assignments, set up to 51 of 210 bits
MSG 2 to 5	Fast corrections
MSG 6	Integrity information
MSG 7	Fast correction degradation factor
MSG 8	*Reserved for future messages*
MSG 9	GEO navigation message (*X*, *Y*, *Z*, time, etc.)
MSG 10	Degradation Parameters
MSG 11	*Reserved for future messages*
MSG 12	SBAS Network Time/UTC offset parameters
MSG 13 to 16	*Reserved for future messages*
MSG 17	GEO satellite almanacs
MSG 18	Ionospheric grid point masks
MSG 19 to 23	*Reserved for future messages*
MSG 24	Mixed fast corrections/long term satellite error corrections
MSG 25	Long term satellite error corrections
MSG 26	Ionospheric delay corrections
MSG 27	SBAS outside service volume degradation
MSG 28 to 61	*Reserved for future messages*
MSG 62	Internal Test Message
MSG 63	Null Message

Many Message Types Coordinated Through Issues Data (IOD)

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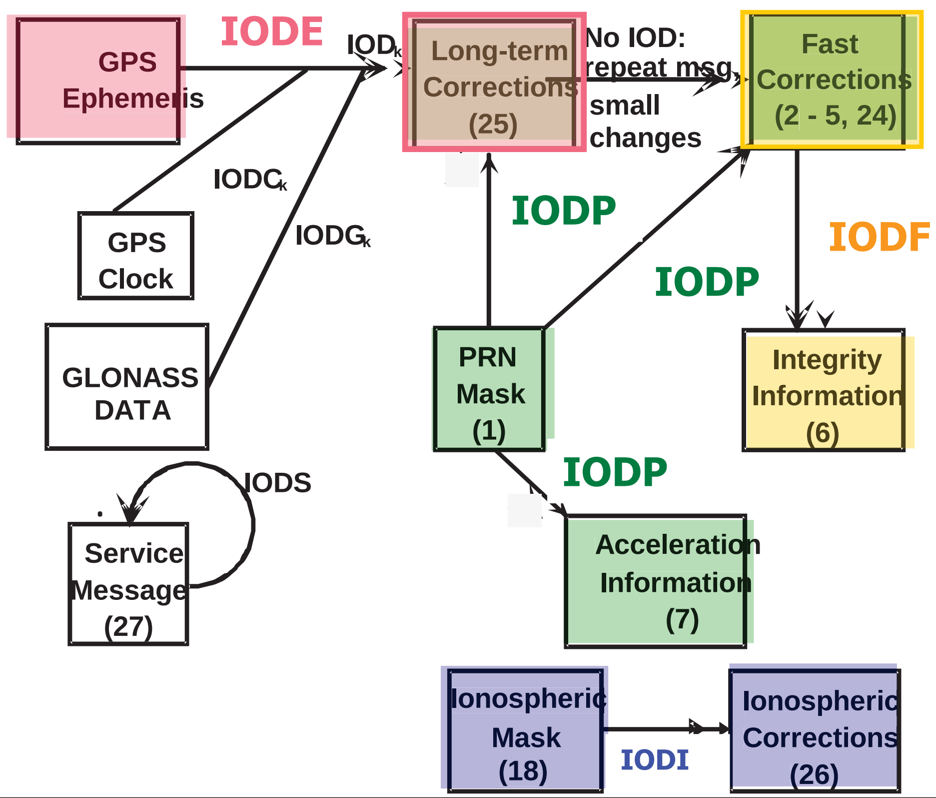
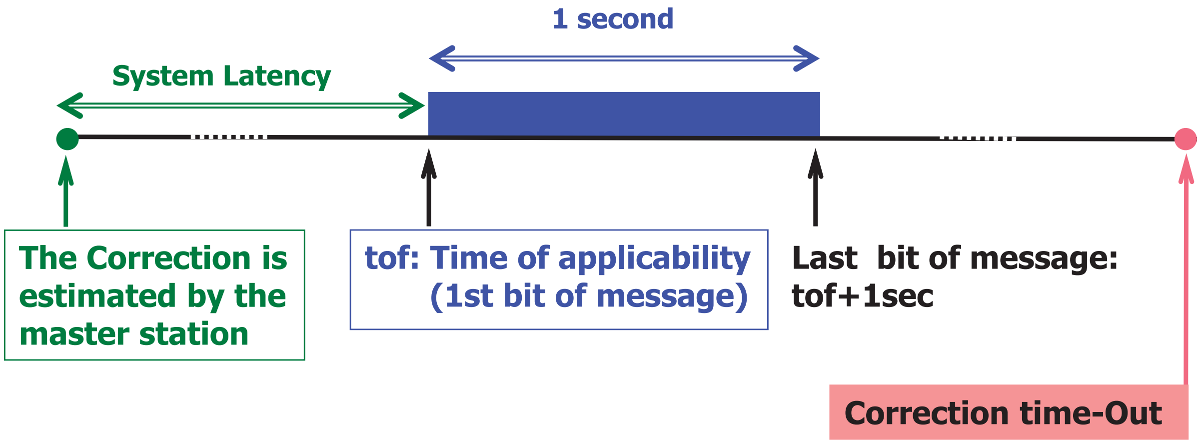
Issues of Data (IOD)



Message Time-Outs:

Users can operate even when missing Messages

- Prevents Use of Very Old Data
- Confidence Degrades When Data is Lost
- IODF: Detect Missing Fast Corrections



Data	Associated Message Types	Maximum Update Interval (seconds)	En Route, Terminal, NPA Timeout (seconds)	Precision Approach Timeout (seconds)
WAAS in Test Mode	0	6	N/A	N/A
PRN Mask	1	60	None	None
UDREI	2-6, 24	6	18	12
Fast Corrections	2-5, 24	60	(*)	(*)
Long Term Corrections	24, 25	120	360	240
GEO Nav. Data	9	120	360	240
Fast Correction Degradation	7	120	360	240
Weighting Factors	8	120	240	240
Degradation Parameters	10	120	360	240
Ionospheric Grid Mask	18	300	None	None
Ionospheric Corrections	26	300	600	600
UTC Timing Data	12	300	None	None
Almanac Data	17	300	None	None

(*) Fast Correction Time-Out intervals are given in MT7 [between 12 to 120 sec]

PRN MASK (MT01)

Bit No	1	2	3	4	5	6	.	38	.	120	.	210
Value	0	1	0	1	1	0		1		1		0
PRN		GPS PRN 2		GPS PRN 4	GPS PRN 5			GLONASS Slot 1		AORE PRN 120		
PRN mask Number		1		2	3			21		29		

Each MT01 contains its associated IODP

Up to 51 satellites in 210 slots.

Note: Each Correction set in MT 2-5,5,6,7,24,25 its characterized by its PRN-Mask number, between 1 to 51.

PRN Slot	Assignment
1-37	GPS/GPS Reserved
38-61	GLONASS
62-119	Future GNSS
120-138	GEO/SBAS
139-210	Future GNSS/GEO/SBAS/Pseudolites

Example of message: Fast Corrections (MT2-5,24)

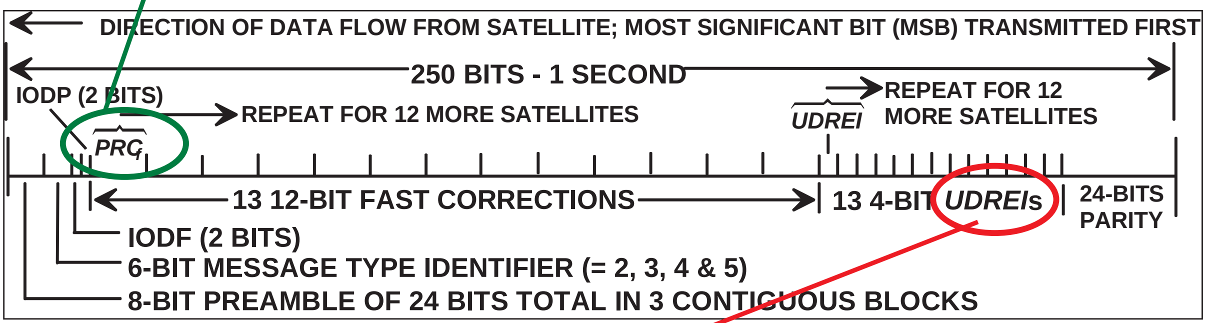
- Primarily Removed SA
  - Common to ALL users
  - Up to 13 Satellites Per Message
  - Pseudorange Correction /confidence Bound
  - Range Rate Formed by Differencing
  - UDRE degrades Over Time
    - Acceleration Term in MT 7
    - Reset when new Message Received

Example of message: Fast Corrections (MT2-5,24)

$$PRC(t) = PRC_n + RRC_n(t - t_n)$$

$$RRC_n = \frac{PRC_n - PRC_o}{t_n - t_o}$$

$$Y = C1 + PRC - \rho^* + \Delta t^{sat} + dt^{sat} - TGD + IONO + TROP$$



$(RSS_{UDRE} = 0 \quad [MT10])$

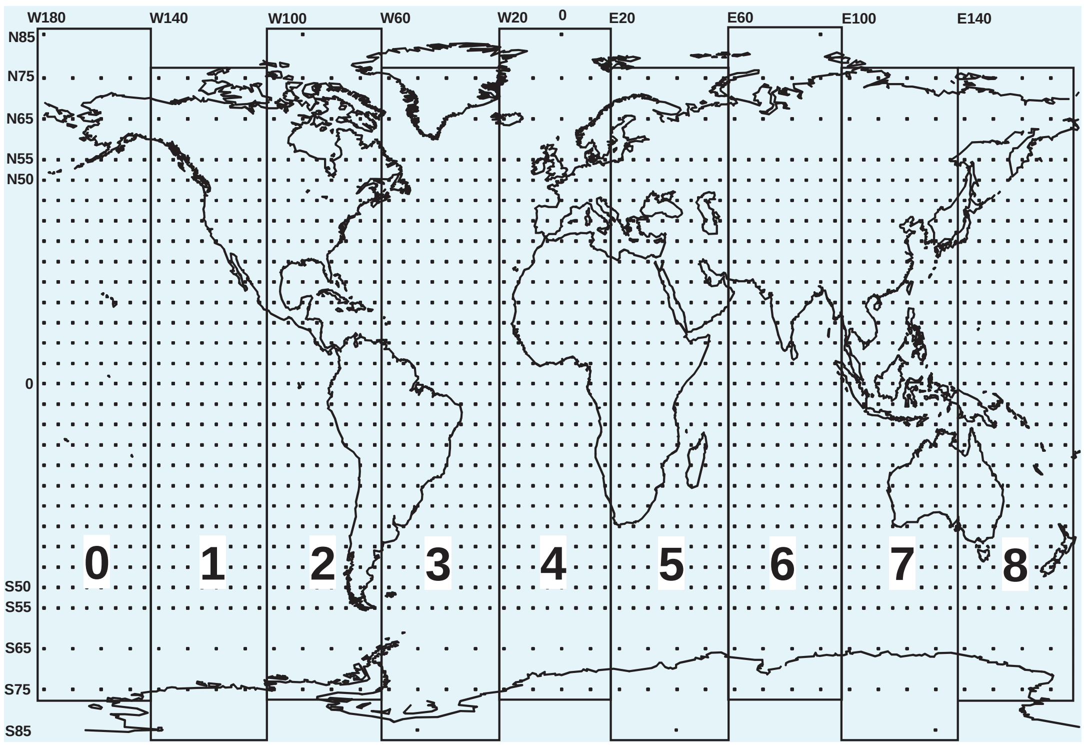
$$\sigma^2_{i,flt} = \sigma^2_{UDRE} + \varepsilon^2_{fc} + \varepsilon^2_{rrc} + \varepsilon^2_{ltc} + \varepsilon^2_{er}$$

Example of message: Ionospheric Corrections (MT26)

- Only Requiered for Precission Approach
  - Grid of Vertical Ionospheric Corrections
  - Users Select 3 o 4 IGPs that Surrounding IPP
    - 5ºx5º or 10ºx10º for 55º<Lat<55º
    - Only 10ºx10º for 55º<|Lat|<85º
    - Circular regions for |Lat|>85º
  - Vertical Correction and UIVE Interpoled to IPP
  - Both Converted to Slant by Obliquity Factor

IGP: Ionospheric Grid Point
IPP: Ionospheric Pierce Point

Global IGP Grid



Predefined Global IGP Grid

Ephemeris
+
Clocks

→

Fast Corrections
+
Long Term Corrections
+
UDRE
+
Degradation Param.

MT1, MT2,5,24, MT6, MT25, MT7, MT12, MT9

MT10

IONO

→

IONO Corrections
+
GIVE
+
Degradation Param.

MT18
MT26

$$y = C1 + PRC - \rho^{*} + \Delta t^{sat} + dt^{sat} - TGD - IONO - TROP$$

$$\sigma^2 = \sigma^2_{flt} + \sigma^2_{UIRE} + \sigma^2_{air} + \sigma^2_{tropo}$$

Navigation System Error and Protection levels

$$\mathbf{y} = \mathbf{G}\cdot\mathbf{x}$$

$$\hat{\mathbf{x}} = \left(\mathbf{G}^T\mathbf{W}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{W}\,\mathbf{y}$$

$$X = [\Delta N, \Delta E, \Delta U, cdt]$$

$$\mathbf{P}_x = \left(\mathbf{G}^T\mathbf{W}\mathbf{G}\right)^{-1} = \begin{bmatrix} d_N^2 & d_{NE} & d_{NV} & d_{NT} \\ d_{NE} & d_E^2 & d_{EV} & d_{ET} \\ d_{NV} & d_{EV} & d_V^2 & d_{VT} \\ d_{NT} & d_{ET} & d_{VT} & d_T^2 \end{bmatrix}$$

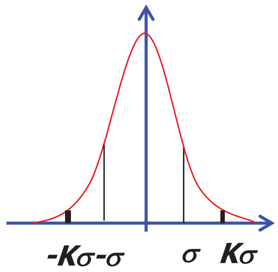
$$HPL = 6.00\sqrt{\frac{d_N^2+d_E^2}{2}+\sqrt{\left(\frac{d_N^2-d_E^2}{2}\right)^2+d_{NE}^2}}$$

$$\mathbf{W} = \mathbf{P}_Y^{-1}$$

$$VPL = 5.33\,d_V$$

$$\mathbf{P}_y = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_N^2 \end{bmatrix}$$

$$\sigma_i^2 = \boxed{\sigma_{i,flt}^2} + \boxed{\sigma_{i,UIRE}^2} + \sigma_{i,air}^2 + \sigma_{i,tropo}^2$$



$$X \sim N(0,1)$$
$$P(|X| > 5.33) = 10^{-7}$$

Fast and Long-Term Correction Degradation

$$\sigma^2_{i,flt} = \begin{cases} \left(\sigma_{UDRE} + \varepsilon_{fc} + \varepsilon_{rrc} + \varepsilon_{ltc} + \varepsilon_{er}\right)^2, & if \quad RSS_{UDRE} = 0 \quad (MT10) \\ \sigma^2_{UDRE} + \varepsilon^2_{fc} + \varepsilon^2_{rrc} + \varepsilon^2_{ltc} + \varepsilon^2_{er}, & if \quad RSS_{UDRE} = 1 \quad (MT10) \end{cases}$$

$$\varepsilon_{fc} = a\,\frac{(t - t_u + t_{lat})^2}{2}$$

$$\varepsilon_{ltc,v0} = C_{ltc,v0}\ floor\left(\frac{t - t_{ltc}}{I_{ltc,v0}}\right)$$

MT2-6,24

$IODF, UDRE_i$

$t_u = t_{of}$

MT25

$t_{ltc}, v_0\ or\ v_1$

MT7

$a_i \quad I_{fc,i} \quad t_{lat}$

(when $IODF \neq 3$)

$$\varepsilon_{ltc,v1} = \begin{cases} 0, & if \quad t_0 < t < t_0 + I_{ltc\_v1} \\ C_{lts\_lsb} + C_{ltc\_v1}\max\{0, t_0 - t, t - t_0 - I_{ltc\_v1}\}, & otherwise \end{cases}$$

$$\varepsilon_{er} = \begin{cases} 0 & \text{Neither fast nor long term corrections have time out for precision approach} \\ C_{er} & \text{Otherwise} \end{cases}$$

$IODF_{current}, IODF_{previous} \neq 3$

$$\varepsilon_{rrc} = \begin{cases} 0, & (IODF_{current} - IODF_{previous})_{\text{mod}\,2} = 1 \\ \left(\frac{a\,I_{fc}}{4} + \frac{B_{rrc}}{\Delta t}\right)(t - t_{of}), & (IODF_{current} - IODF_{previous})_{\text{mod}\,2} \neq 1 \end{cases}$$

MT10

$B_{rrc}, C_{ltc\_lsb}, C_{ltc\_v1}, I_{ltc\_v1}, C_{ltc\_v0}, I_{ltc\_v0}, C_{er}, RSS_{UDRE}, C_{iono\_ramp}, C_{iono\_step}, I_{iono}, RSS_{iono}$

Degradation of Ionospheric Corrections

$$\sigma^2_{UIRE} = F^2_{pp}\sigma^2_{UIVE}$$

$$F_{pp} = \left[1-\left(\frac{R_e\cos E}{R_e+h_I}\right)^2\right]^{-\frac{1}{2}}$$

$$\sigma^2_{UIVE} = \sum_{n=1}^{N} W_n\left(x_{pp}, y_{pp}\right)\sigma^2_{n,ionogrid}\ , \quad N = 4\, or\, 3$$

$$\sigma^2_{ionogrid} = \begin{cases} \left(\sigma_{GIVE} + \varepsilon_{iono}\right)^2, \text{if} & RSS_{iono} = 0 \quad (MT10) \\ \sigma^2_{GIVE} + \varepsilon^2_{iono} \quad, \text{if} & RSS_{iono} = 1 \quad (MT10) \end{cases}$$

$$\varepsilon_{iono} = C_{iono\_step}\ floor\left(\frac{t-t_{iono}}{I_{iono}}\right) + C_{iono\_ramp}\left(t-t_{iono}\right)$$

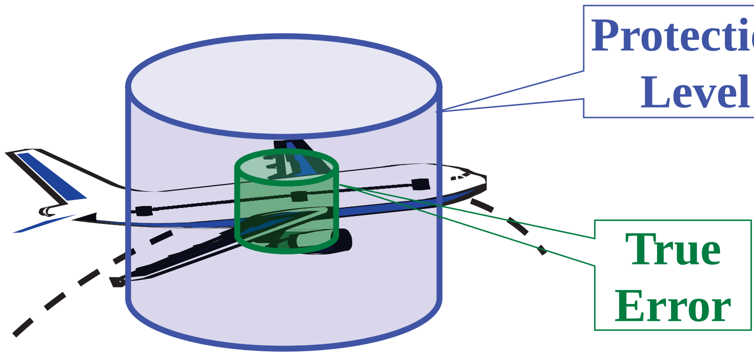
MT10

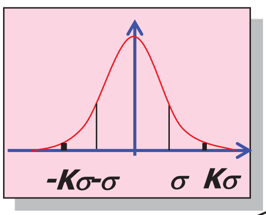
$B_{rrc}, C_{ltc\_lsb}, C_{ltc\_v1},$
$I_{ltc\_v1}, C_{ltc\_v0}, I_{ltc\_v0},$
$C_{er}, RSS_{UDRE},$
$C_{iono\_ramp}, C_{iono\_step},$
I_{iono}, RSS_{iono}

MT26

$t_{iono}, GIVE_i$

Users know the receiver-satellites geometry and can compute bounds on the horizontal and vertical position errors.
These bounds are called Protection Levels (HPL and VPL). They provide good confidence (10^{-7}/hour probability) that the true position is within a bubble around the computed position.





Tail area Probability

P(VPE>VPL) < 10^{-7} /sample

$$VPL = K_V\sqrt{\sum_{i=1}^{N} S^2_{V_i}\sigma^2_i}$$

$$\sigma^2_i = \sigma^2_{i,flt} + \sigma^2_{i,UIRE} + \sigma^2_{i,air} + \sigma^2_{i,tropo}$$

GEOMETRY

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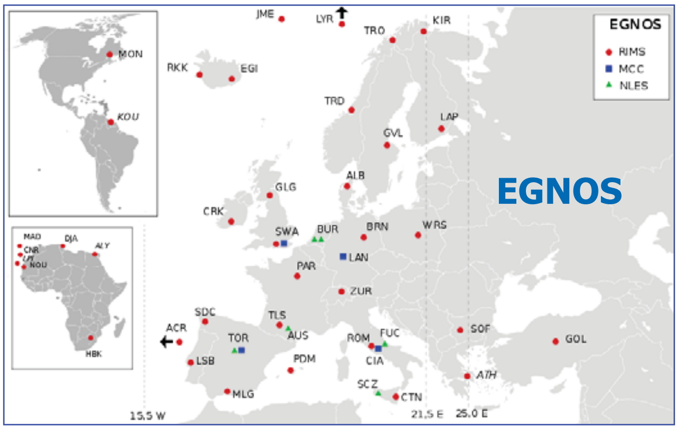

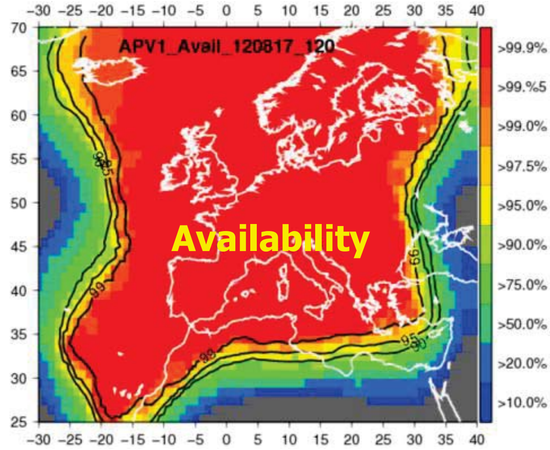

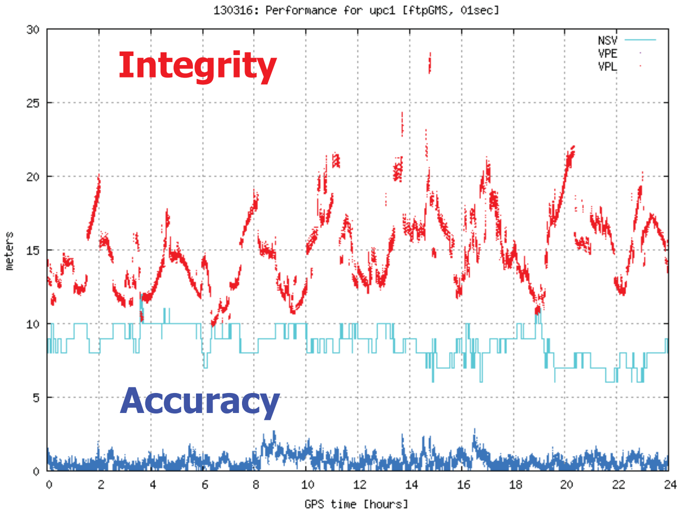

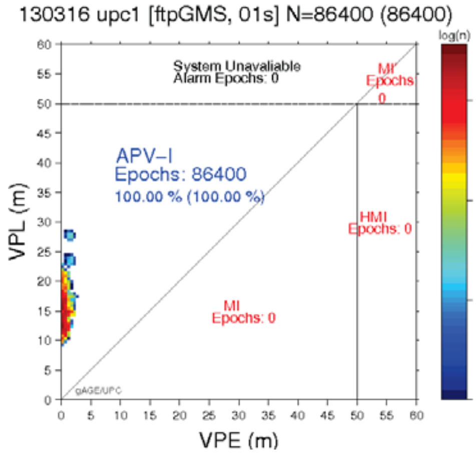

DGNSS implementations: WADGNSS (SBAS)

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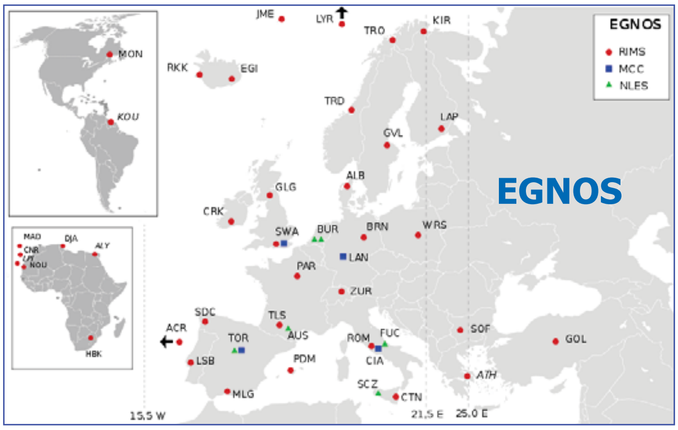

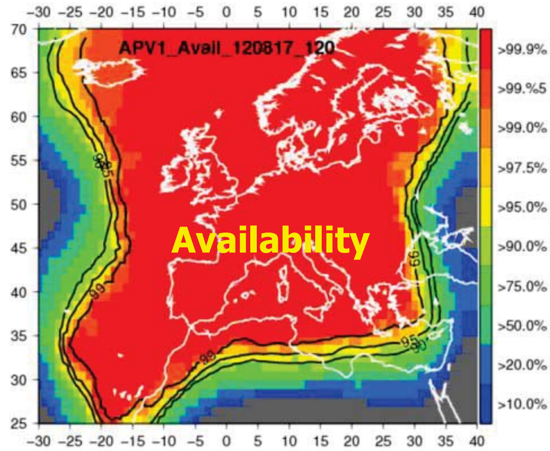

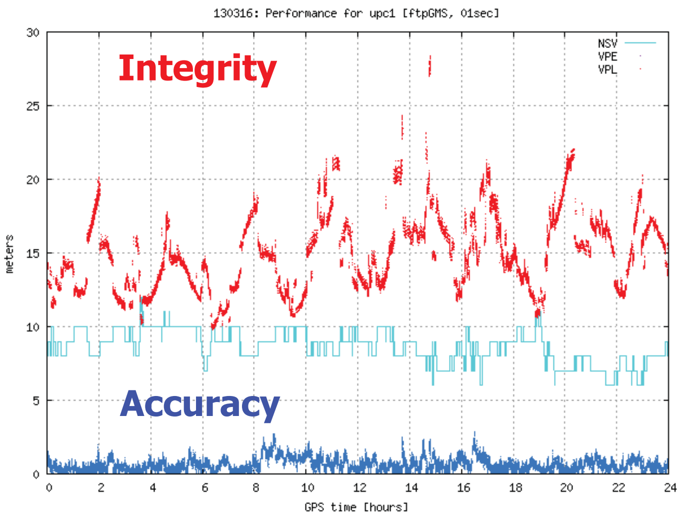

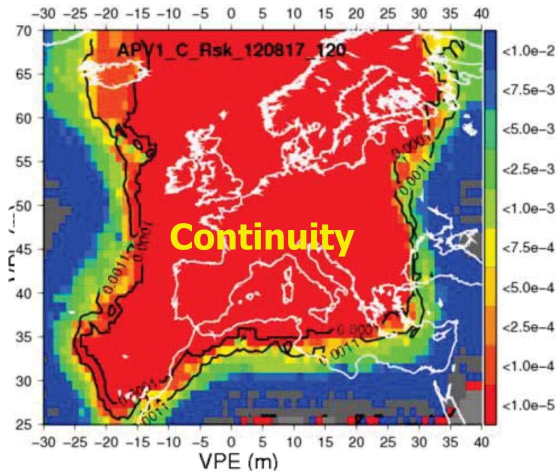

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[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

[RD-5] Gang Xie, Optimal on-airport monitoring of the integrity of GPS-based landing systems, PhD Dissertation, 2004.

EGNOS Safety of Life Service Definition Document (Ref : EGN-SDD SoL, V1.0). **European Comission**.

http://www.essp-sas.eu/downloads/vubjj/egnos_sol_sdd_in_force.pdf

Error sources (1σ)	GPS - Error Size (m)	EGNOS - Error Size (m)
GPS SREW	4.0[12]	2.3
Ionosphere (UIVD error)	2.0 to 5.0[13]	0.5
Troposphere (vertical)	*0.1*	*0.1*
GPS Receiver noise	*0.5*	*0.5*
GPS Multipath (45º elevation)	*0.2*	*0.2*
GPS UERE 5º elevation	**7.4 to 15.6**	**4.2 (after EGNOS corrections)**
GPS UERE 90º elevation	**4.5 to 6.4**	**2.4 (after EGNOS corrections)**

[12] GPS Standard Positioning Service Performance Standard [RD-3].
[13] This is the typical range of ionospheric residual errors after application of the baseline Klobuchar model broadcast by GPS for mid-latitude regions.

SREW: Satellite Residual Error for the Worst user location.
UIVD: User Ionospheric Vertical Delay.
UERE: User Equivalent Range Error

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Lecture 7

Carrier-based Differential Positioning.

Ambiguity Resolution Techniques

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Authorship statement

22 Jan 2015

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Contents

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Linear model for Differential Positioning

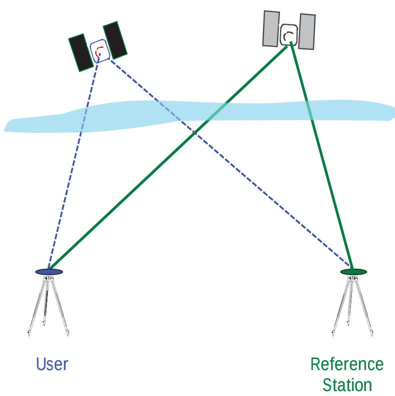
Single difference $(\bullet)^j_{ru} \equiv \Delta(\bullet)^j_{ru} = (\bullet)^j_u - (\bullet)^j_r$

$$P^j_{ru} = \rho^j_{ru} + c\,\delta t_{ru} + T^j_{ru} + I^j_{ru} + K_{ru} + \nu_{P\,ru}^{\;j}$$

$$L^j_{ru} = \rho^j_{ru} + c\,\delta t_{ru} + T^j_{ru} - I^j_{ru} + \lambda\,\omega^j_{ru} + \lambda\,N^j_{ru} + b_{ru} + \nu_{L\,ru}^{\;j}$$

Double difference

$$\boxed{(\bullet)^{jk}_{ru}} \equiv \nabla\Delta(\bullet)^{jk}_{ru} = (\bullet)^k_{ru} - (\bullet)^j_{ru} = \boxed{(\bullet)^k_u - (\bullet)^k_r - \left[(\bullet)^j_u - (\bullet)^j_r\right]}$$



$$P^k_{ru} = \rho^k_{ru} + \cancel{c\,\delta t_{ru}} + T^k_{ru} + I^k_{ru} + \cancel{K_{ru}} + \nu_{P\,ru}^{\;k}$$

$$P^j_{ru} = \rho^j_{ru} + \cancel{c\,\delta t_{ru}} + T^j_{ru} + I^j_{ru} + \cancel{K_{ru}} + \nu_{P\,ru}^{\;j}$$

$$\boxed{P^{jk}_{ru} = \rho^{jk}_{ru} + T^{jk}_{ru} + I^{jk}_{ru} + \nu_{P\,ru}^{\;jk}}$$

Now are cancelled:
- Receiver clock
- Receiver code instrumental delays
- Receiver carrier instrumental delays

The same for carrier :

$$\boxed{L^{jk}_{ru} = \rho^{jk}_{ru} + T^{j}_{ru} - I^{jk}_{ru} + \lambda\,\omega^{jk}_{ru} + \lambda\,N^{jk}_{ru} + \nu_{L\,ru}^{\;jk}}$$

→Carrier ambiguities *N* are integer

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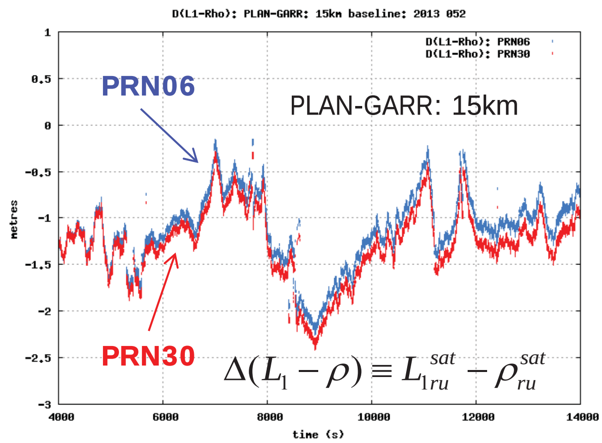
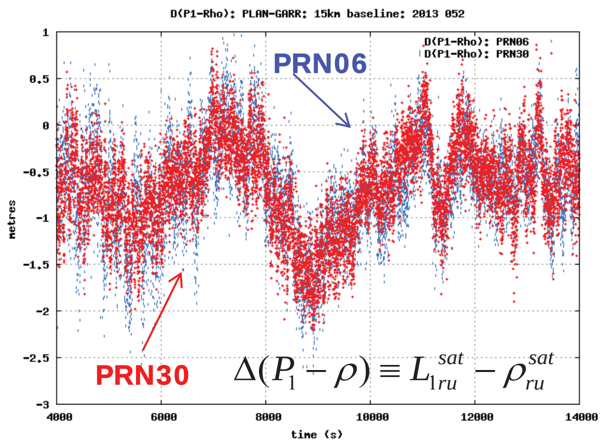
Single-Difference of measurements (corrected by geometric range!!)





Dif. Wind-up: Very small

$$\Delta(L_1-\rho)\equiv L^j_{ru}-\rho^j_{ru} = \boxed{c\,\delta t_{ru}} + \boxed{T^j_{ru}} - \boxed{I^j_{ru}} + \boxed{\lambda\,\omega^j_{ru}} + \boxed{\lambda N^j_{ru}+b_{ru}} + \nu_{L\,ru}^{\,j}$$

$$\Delta(P_1-\rho)\equiv P^j_{ru}-\rho^j_{ru} = \boxed{c\,\delta t_{ru}} + \boxed{T^j_{ru}} + \boxed{I^j_{ru}} + \boxed{K_{ru}} + \nu_{P\,ru}^{\,j}$$

Dif. Receiver clock: Main variations Common for all satellites

Dif. Tropo. and Iono. : Small variations

Dif. Instrumental delays and carrier ambiguities: constant

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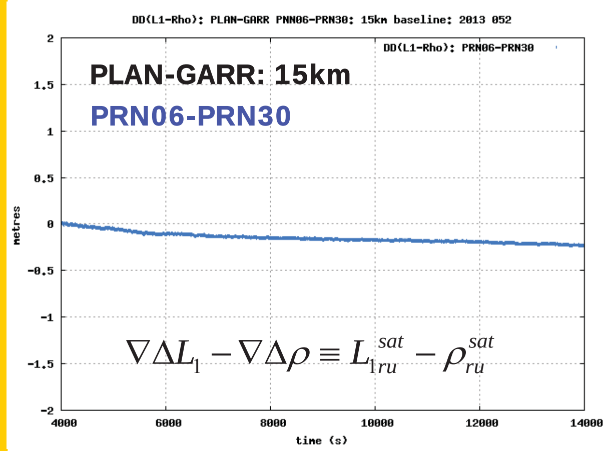
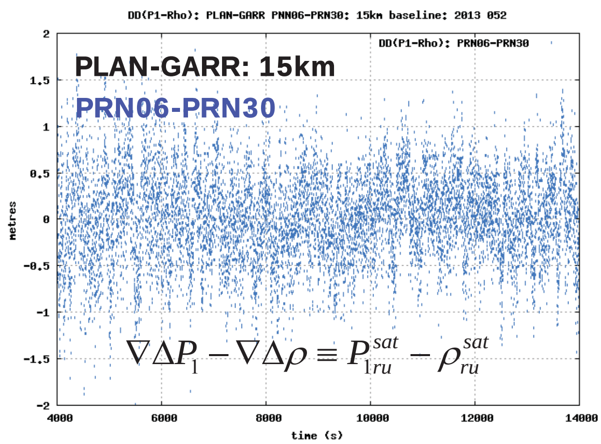
Double-Difference of measurements (corrected by geometric range!!)





Dif. wind-up: negligible

$$\nabla\Delta(L_1-\rho)\equiv L^{jk}_{ru}-\rho^{jk}_{ru} = \boxed{T^{jk}_{ru}} - \boxed{I^{jk}_{ru}} + \lambda\boxed{\omega^{jk}_{ru}} + \lambda\boxed{N^{jk}_{ru}} + \nu_{L\,ru}^{\,jk}$$

$$\nabla\Delta(P_1-\rho)\equiv P^{jk}_{ru}-\rho^{jk}_{ru} = \boxed{T^{jk}_{ru}} + \boxed{I^{jk}_{ru}} + \nu_{P\,ru}^{\,jk}$$

Carrier ambiguities: constant

Dif. Tropo. and Iono. : Small variations

Linear model for Differential Positioning

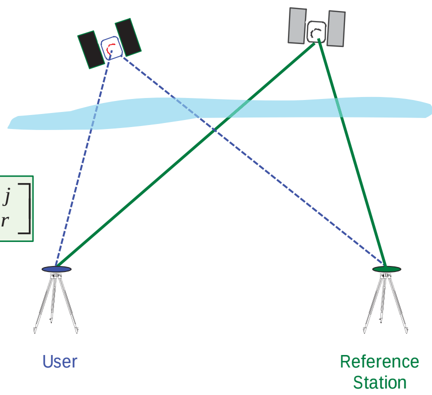
Single difference

$$(\bullet)^j_{ru} \equiv \Delta(\bullet)^j_{ru} = (\bullet)^j_u - (\bullet)^j_r$$

$$P^j_{ru} = \rho^j_{ru} + c\,\delta t_{ru} + T^j_{ru} + I^j_{ru} + K_{ru} + \nu_{P\,ru}^{\;\;j}$$

$$L^j_{ru} = \rho^j_{ru} + c\,\delta t_{ru} + T^j_{ru} - I^j_{ru} + \lambda\,\omega^j_{ru} + \lambda\,N^j_{ru} + b_{ru} + \nu_{L\,ru}^{\;\;j}$$

where: $\rho^j_{ru} = \rho^j_{0ru} - \hat{\boldsymbol{\rho}}^j_{0u}\cdot\Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}^j_{0ru}\cdot\boldsymbol{\varepsilon}_{site} + \hat{\boldsymbol{\rho}}^j_{0ru}\cdot\boldsymbol{\varepsilon}^j_{eph}$



Double difference

$$\boxed{(\bullet)^{jk}_{ru}} \equiv \nabla\Delta(\bullet)^{jk}_{ru} = (\bullet)^k_{ru} - (\bullet)^j_{ru} = \boxed{(\bullet)^k_u - (\bullet)^k_r - \left[(\bullet)^j_u - (\bullet)^j_r\right]}$$

$$P^{jk}_{ru} = \rho^{jk}_{ru} + T^{jk}_{ru} + I^{jk}_{ru} + \nu_{P\,ru}^{\;\;jk}$$

$$L^{jk}_{ru} = \rho^{jk}_{ru} + T^{j}_{ru} - I^{jk}_{ru} + \lambda\,\omega^{jk}_{ru} + \lambda\,N^{jk}_{ru} + \nu_{L\,ru}^{\;\;jk}$$

where: $\rho^{jk}_{ru} = \rho^{jk}_{0ru} - \hat{\boldsymbol{\rho}}^{jk}_{0u}\cdot\Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}^{jk}_{0ru}\cdot\boldsymbol{\varepsilon}_{site} + \hat{\boldsymbol{\rho}}^{k}_{0ru}\cdot\boldsymbol{\varepsilon}^k_{eph} - \hat{\boldsymbol{\rho}}^{j}_{0ru}\cdot\boldsymbol{\varepsilon}^j_{eph}$

being: $\rho^{jk}_{0ru} \equiv \rho^{jk}_{0u} - \rho^{jk}_{0r}$; $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_u - \Delta\mathbf{r}_r$

Exercise:

Consider the Single Differences of geometric range:

$$\rho^j_{ru} = \rho^j_{0ru} - \hat{\boldsymbol{\rho}}^j_{0u}\cdot\Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}^j_{0ru}\cdot\boldsymbol{\varepsilon}_{site} + \hat{\boldsymbol{\rho}}^j_{0ru}\cdot\boldsymbol{\varepsilon}^j_{eph}$$

where $\rho^j_{ru} = \rho^j_u - \rho^j_r$

Show that the Double Differences are given by:

$$\rho^{jk}_{ru} = \rho^{jk}_{0ru} - \hat{\boldsymbol{\rho}}^{jk}_{0u}\cdot\Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}^{jk}_{0ru}\cdot\boldsymbol{\varepsilon}_{site} + \hat{\boldsymbol{\rho}}^{k}_{0ru}\cdot\boldsymbol{\varepsilon}^k_{eph} - \hat{\boldsymbol{\rho}}^{j}_{0ru}\cdot\boldsymbol{\varepsilon}^j_{eph}$$

being: $\rho^{jk}_{0ru} \equiv \rho^{jk}_{0u} - \rho^{jk}_{0r}$; $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_u - \Delta\mathbf{r}_r$

Linear model for Differential Positioning

$$\boxed{(\bullet)_{ru}^{jk}} \equiv \nabla\Delta(\bullet)_{ru}^{jk} = (\bullet)_{ru}^{k} - (\bullet)_{ru}^{j} = \boxed{(\bullet)_{u}^{k} - (\bullet)_{r}^{k} - \left[(\bullet)_{u}^{j} - (\bullet)_{r}^{j}\right]}$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + \cancel{T_{ru}^{jk}} + \cancel{I_{ru}^{jk}} + \nu_{P\,ru}^{\;jk}$$
$$L_{ru}^{jk} = \rho_{ru}^{jk} + \cancel{T_{ru}^{j}} - \cancel{I_{ru}^{jk}} + \lambda\,\cancel{\omega_{ru}^{jk}} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

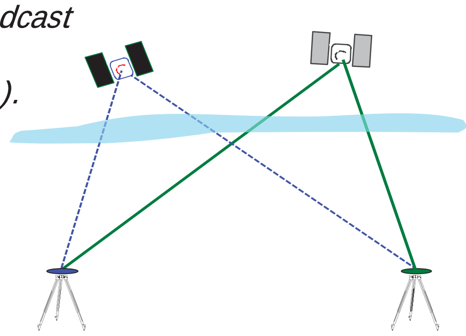
where: $\rho_{ru}^{jk} = \rho_{0\,ru}^{\;jk} - \hat{\boldsymbol{\rho}}_{0\,u}^{\;jk} \cdot \Delta\mathbf{r}_{ru} - \cancel{\hat{\boldsymbol{\rho}}_{0\,ru}^{\;jk} \cdot \boldsymbol{\varepsilon}_{site}} + \cancel{\hat{\boldsymbol{\rho}}_{0\,ru}^{\;k} \cdot \boldsymbol{\varepsilon}_{eph}^{k}} - \cancel{\hat{\boldsymbol{\rho}}_{0\,ru}^{\;j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}}$

For short baselines (e.g. up to *10 km*) and if the reference station coordinates are accurately known, we can assume:

$$\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_{u} - \Delta\mathbf{r}_{r}$$

$$T_{ru}^{jk} \approx 0 \,;\, I_{ru}^{jk} \approx 0 \,;\, \omega_{ru}^{jk} \approx 0$$
$$\hat{\boldsymbol{\rho}}_{0\,ru}^{\;j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} \approx 0$$
$$\boldsymbol{\varepsilon}_{site} \approx 0 \Rightarrow \boxed{\Delta\mathbf{r}_{ru} \approx \Delta\mathbf{r}_{u}}$$

Note for baselines up to 10 km *the range error of broadcast orbits is less than* 1cm *(assuming* $\varepsilon_{eph}^{j} \approx 10$ m *).*



With these simplifications, we have:

$$P_{ru}^{jk} - \rho_{0\,ru}^{\;jk} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;jk} \cdot \Delta\mathbf{r}_{u} + \nu_{P\,ru}^{\;jk}$$
$$L_{ru}^{jk} - \rho_{0\,ru}^{\;jk} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;jk} \cdot \Delta\mathbf{r}_{u} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

Remark: $P_{ru}^{jk} - \rho_{0\,ru}^{\;jk} = P_{u}^{jk} - \rho_{0\,u}^{\;jk} - \left(P_{r}^{jk} - \rho_{0\,r}^{\;jk}\right)$

Differential code and carrier positioning

As with the SD, the left hand side of previous equations can be spitted in two terms: one associated to the reference station and the other to the user. Then, the differential corrections can be computed for code and carrier as:

$$PRC_{P}^{\;jk} = \rho_{0\,r}^{\;jk} - P_{r}^{jk} \,; \quad PRC_{L}^{\;jk} = \rho_{0\,r}^{\;jk} - L_{r}^{jk}$$

- The user applies this differential correction to remove/mitigate common errors:

$$P_{u}^{jk} - \rho_{0\,u}^{\;jk} + PRC_{P}^{\;jk} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;jk} \cdot \Delta\mathbf{r}_{u} + \nu_{P\,ru}^{\;jk}$$
$$L_{u}^{jk} - \rho_{0\,u}^{\;jk} + PRC_{L}^{\;jk} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;jk} \cdot \Delta\mathbf{r}_{u} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

Where the carrier ambiguities *N* are integer numbers and must be estimated together with the user solution.

For larger distances, the atmospheric propagation effects (troposphere, ionosphere) must be removed with accurate modelling. Wide area users will require also orbit corrections.

Remark: $P_{ru}^{jk} - \rho_{0\,ru}^{\;jk} = P_{u}^{jk} - \rho_{0\,u}^{\;jk} - \left(P_{r}^{jk} - \rho_{0\,r}^{\;jk}\right)$

Differential code and carrier positioning

The user applies this differential correction to remove/mitigate common errors

$$P_u^{jk} - \rho_{0u}^{jk} + PRC_P^{jk} = -\hat{\boldsymbol{\rho}}_{0u}^{jk} \cdot \Delta\mathbf{r}_u + \nu_{P\,ru}^{jk}$$

$$L_u^{jk} - \rho_{0u}^{jk} + PRC_L^{jk} = -\hat{\boldsymbol{\rho}}_{0u}^{jk} \cdot \Delta\mathbf{r}_u + \lambda N_{ru}^{jk} + \nu_{L\,ru}^{jk}$$

where $\hat{\boldsymbol{\rho}}_{0u}^{jk} \equiv \hat{\boldsymbol{\rho}}_{0u}^{k} - \hat{\boldsymbol{\rho}}_{0u}^{j}$

The previous system for navigation equations is written in matrix notation as:

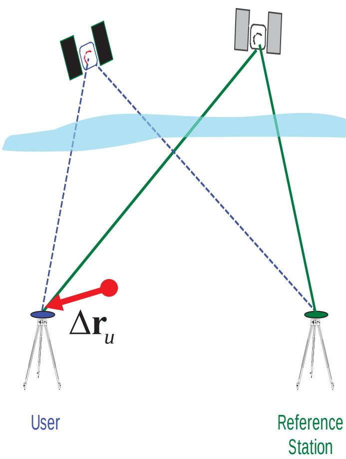
$$\begin{bmatrix} \mathrm{Pref}_P^{R,1} \\ \mathrm{Pref}_L^{R,1} \\ \vdots \\ \mathrm{Pref}_P^{R,n-1} \\ \mathrm{Pref}_L^{R,n-1} \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}_{0u}^{1} - \hat{\boldsymbol{\rho}}_{0u}^{R}\right)^T & 0 & \cdots & 0 \\ -\left(\hat{\boldsymbol{\rho}}_{0u}^{1} - \hat{\boldsymbol{\rho}}_{0u}^{R}\right)^T & 1 & \cdots & 0 \\ & \vdots & & \\ -\left(\hat{\boldsymbol{\rho}}_{0u}^{n-1} - \hat{\boldsymbol{\rho}}_{0u}^{R}\right)^T & 0 & \cdots & 0 \\ -\left(\hat{\boldsymbol{\rho}}_{0u}^{n-1} - \hat{\boldsymbol{\rho}}_{0u}^{R}\right)^T & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{r}_u \\ \lambda N_{ru}^{R,1} \\ \vdots \\ \lambda N_{ru}^{R,n-1} \end{bmatrix}$$

where

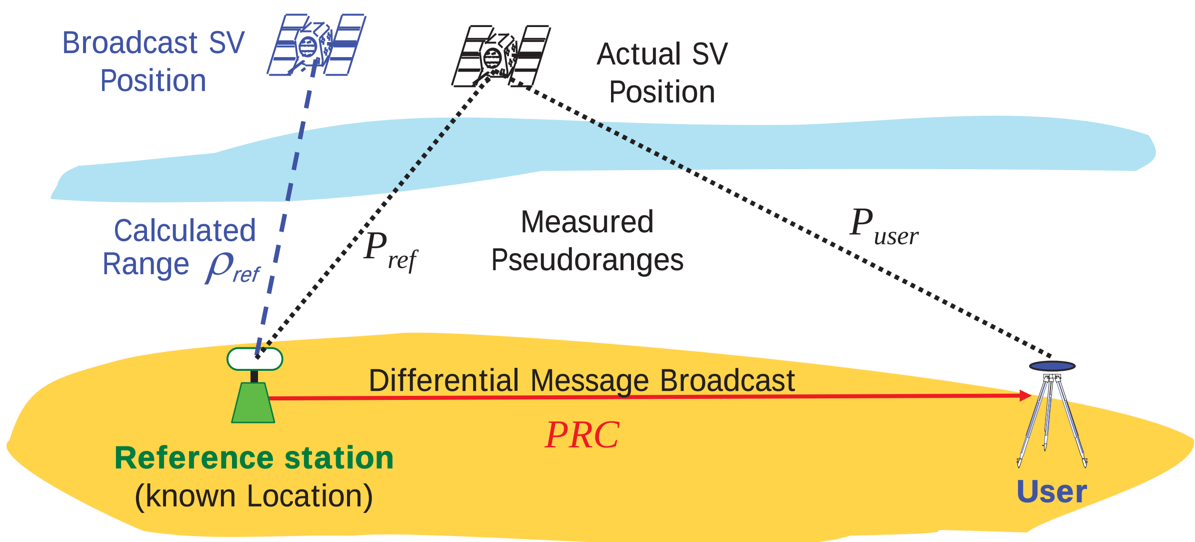
$$\mathrm{Pref}_P^{R,k} \equiv P_u^{R,k} - \rho_{0u}^{R,k} + PRC_P^{R,k}$$

$$\mathrm{Pref}_L^{R,k} \equiv L_u^{R,k} - \rho_{0u}^{R,k} + PRC_L^{R,k}$$

Carrier ambiguities



Differential positioning



– The **reference station** with known coordinates, computes differential corrections: $PRC_P^{jk} = \rho_{0r}^{jk} - P_r^{jk}$; $PRC_L^{jk} = \rho_{0r}^{jk} - L_r^{jk}$

– The **user** receiver applies these corrections to its own measurements to remove SIS errors and improve the positioning accuracy.

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Correlations among the DD Measurements

We assume uncorrelated measurements (both code and carrier). Then, the covariance matrix is diagonal:

$$\mathbf{P}_P = \sigma_P^2\ \mathbf{I} \quad \mathbf{P}_L = \sigma_L^2\ \mathbf{I}$$ where, we can assume: $\sigma_P \approx 50cm\,; \quad \sigma_L \approx 5mm$

Let X represent the code P or the Carrier L measurement.

- The **single difference (SD)** and its covariance matrix can be computed as:

$$\begin{bmatrix} X_{ru}^{k} \\ X_{ru}^{j} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} X_u^k \\ X_r^k \\ X_u^j \\ X_r^j \end{bmatrix} ;$$

Thence, if the measurements <u>are uncorrelated</u>, so are they in single differences, <u>but the noise is twice!</u>

$$\mathbf{P}_{X_{SD}} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_X^2 & 0 & 0 & 0 \\ 0 & \sigma_X^2 & 0 & 0 \\ 0 & 0 & \sigma_X^2 & 0 \\ 0 & 0 & 0 & \sigma_X^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} = 2\sigma_X^2\mathbf{I}$$

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Correlations among the DD Measurements

- Now, the **double difference (DD)** and its covariance matrix can be computed as:

$$\begin{bmatrix} X_{ru}^{jk} \\ X_{ru}^{jl} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} X_{ru}^k \\ X_{ru}^j \\ X_{ru}^l \end{bmatrix} ;$$

$$\mathbf{P}_{X_{DD}} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2\sigma_X^2 & 0 & 0 \\ 0 & 2\sigma_X^2 & 0 \\ 0 & 0 & 2\sigma_X^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \end{bmatrix} = 2\sigma_X^2 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Thence, even if the original measurements are uncorrelated, the double differences <u>are correlated</u>.

Note: The removal of the relative "User"—"Reference-station" common bias (e.g. relative receiver clock) in DD is done at the expense of one observation and the introduction of a correlation between measurements.

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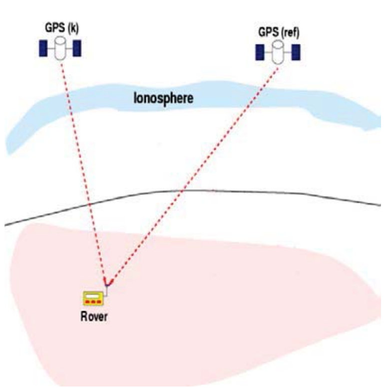
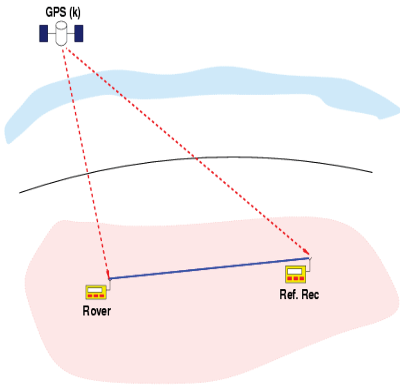
Single and double differences of receivers/satellites



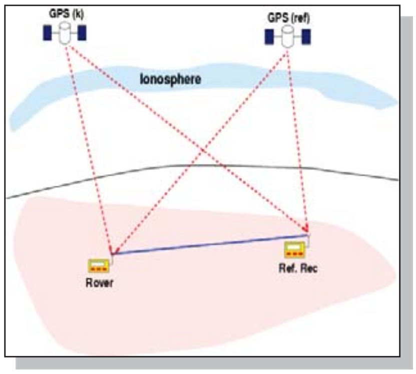
$$\nabla \square_{\bullet} \equiv \square^{k}_{\bullet} - \square^{R}_{\bullet}$$

Receiver errors affecting both satellites are removed (e.g. Receiver clock)



$$\Delta \square^{\bullet} \equiv \square^{\bullet}_{rov} - \square^{\bullet}_{ref}$$

SIS errors affecting both receivers are removed (e.g. Satellite clocks,...)



$$\Delta\nabla \square \equiv \Delta \square^{k} - \Delta \square^{R} = \\ = \nabla \square_{rov} - \nabla \square_{ref}$$

Receiver errors common for all satellites do not affect positioning (as they are assimilated in the receiver clock estimate). Thence:

- Only residual errors in single differences between sat. affect absolute posit.
- Only residual errors in double differences between sat. and receivers affect relative positioning.

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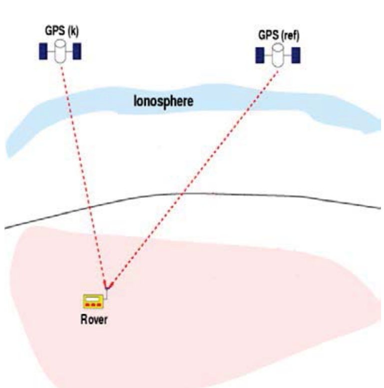
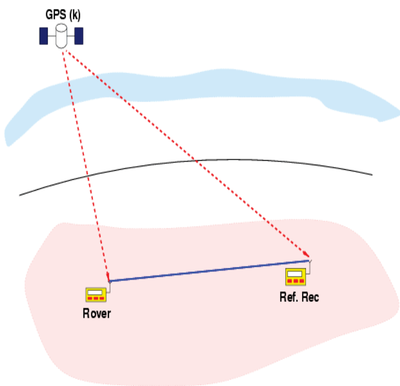
Single and double differences of receivers/satellites



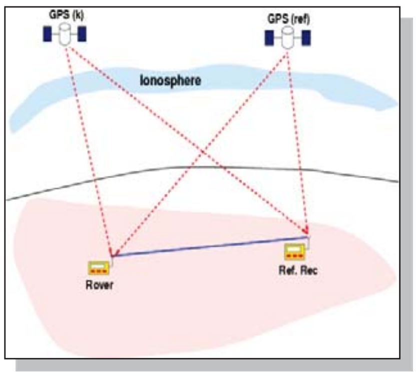
$$\nabla \square_{\bullet} \equiv \square^{k}_{\bullet} - \square^{R}_{\bullet}$$

Receiver errors affecting both satellites are removed (e.g. Receiver clock)



$$\Delta \square^{\bullet} \equiv \square^{\bullet}_{rov} - \square^{\bullet}_{ref}$$

SIS errors affecting both receivers are removed (e.g. Satellite clocks,...)



$$\Delta\nabla \square \equiv \Delta \square^{k} - \Delta \square^{R} = \\ = \nabla \square_{rov} - \nabla \square_{ref}$$

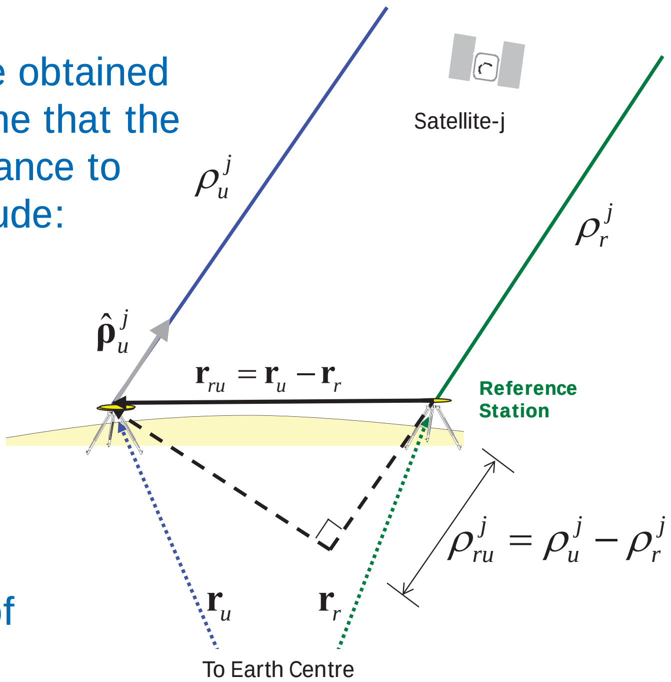
When comparing SD and DD one might suggest that in the DD formulation there is even further error reduction, positively influencing the results in positioning. This is however not true, since in the SD case the mean value of unmodelled effects is absorbed by the receiver clock. If the DD correlations are taken into account, the positioning results in both cases are the same. However the DD formulation has the advantage that it allows the direct estimation of the ambiguities.

Contents

Relative Positioning

The following relationship can be obtained from the figure, where we assume that the baseline is shorter than the distance to the satellite by orders of magnitude:

$$\rho_{ru}^{j} = \rho_{u}^{j} - \rho_{r}^{j} = -\hat{\boldsymbol{\rho}}_{u}^{j} \cdot \mathbf{r}_{ru}$$



Applying the same scheme to a second satellite "*k*"

$$\rho_{ru}^{k} = \rho_{u}^{k} - \rho_{r}^{k} = -\hat{\boldsymbol{\rho}}_{u}^{k} \cdot \mathbf{r}_{ru}$$

Thence, the double differences of ranges are:

$$\rho_{ru}^{jk} = \rho_{ru}^{k} - \rho_{ru}^{j} = -\left(\hat{\boldsymbol{\rho}}_{u}^{k} - \hat{\boldsymbol{\rho}}_{u}^{j}\right) \cdot \mathbf{r}_{ru} = -\hat{\boldsymbol{\rho}}_{u}^{jk} \cdot \mathbf{r}_{ru}$$

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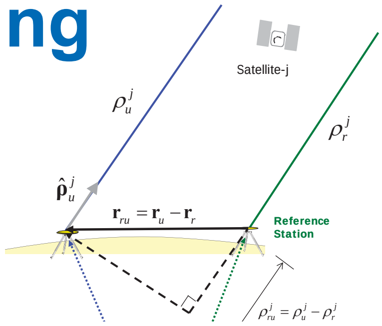
Relative Positioning



Thence, the double differences of ranges are:

$$\rho_{ru}^{jk} = \rho_{ru}^{k} - \rho_{ru}^{j} = -\left(\hat{\boldsymbol{\rho}}_u^{k} - \hat{\boldsymbol{\rho}}_u^{j}\right)\cdot \mathbf{r}_{ru} = \boxed{-\hat{\boldsymbol{\rho}}_u^{jk}\cdot \mathbf{r}_{ru}}$$

As commented before, for short baselines (e.g. less than 10km), we can assume that ephemeris and propagation errors cancel, thence:

$$T_{ru}^{jk} \approx 0\,;\, I_{ru}^{jk} \approx 0 \qquad \omega_{ru}^{jk} \approx 0$$

$$P_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + \nu_{P\,ru}^{\ jk} \qquad\qquad P_{ru}^{jk} = \rho_{ru}^{jk} + \nu_{P\,ru}^{\ jk}$$

$$L_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{j} - I_{ru}^{jk} + \lambda\,\omega_{ru}^{jk} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\ jk} \qquad\qquad L_{ru}^{jk} = \rho_{ru}^{jk} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\ jk}$$

$$\begin{aligned} P_{ru}^{jk} &= \boxed{-\hat{\boldsymbol{\rho}}_u^{jk}\cdot \mathbf{r}_{ru}} + \nu_{P\,ru}^{\ jk} \\ L_{ru}^{jk} &= \boxed{-\hat{\boldsymbol{\rho}}_u^{jk}\cdot \mathbf{r}_{ru}} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\ jk} \end{aligned}$$

Note that these equations allows a direct estimation of the baseline, **without needing an accurate knowledge** of the reference station coordinates.

Note: but, at least the approximate coordinates of reference station or user are needed to compute ρ

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Relative Positioning

$$\begin{aligned} P_{ru}^{jk} &= \boxed{-\hat{\boldsymbol{\rho}}_u^{jk}\cdot \mathbf{r}_{ru}} + \nu_{P\,ru}^{\ jk} \\ L_{ru}^{jk} &= \boxed{-\hat{\boldsymbol{\rho}}_u^{jk}\cdot \mathbf{r}_{ru}} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\ jk} \end{aligned}$$

where

$$\boxed{\hat{\boldsymbol{\rho}}_u^{jk} \equiv \hat{\boldsymbol{\rho}}_u^{k} - \hat{\boldsymbol{\rho}}_u^{j}}$$

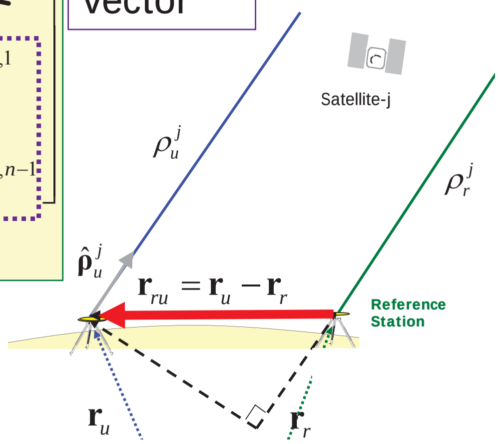
The previous system for navigation equations is written in matrix notation as:

$$\begin{bmatrix} P_{r,u}^{R,1} \\ L_{r,u}^{R,1} \\ \vdots \\ P_{r,u}^{R,n-1} \\ L_{r,u}^{R,n-1} \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}_u^{1} - \hat{\boldsymbol{\rho}}_u^{R}\right)^T & 0 & \cdots & 0 \\ -\left(\hat{\boldsymbol{\rho}}_u^{1} - \hat{\boldsymbol{\rho}}_u^{R}\right)^T & 1 & \cdots & 0 \\ \vdots & & & \\ -\left(\hat{\boldsymbol{\rho}}_u^{n-1} - \hat{\boldsymbol{\rho}}_u^{R}\right)^T & 0 & \cdots & 0 \\ -\left(\hat{\boldsymbol{\rho}}_u^{n-1} - \hat{\boldsymbol{\rho}}_u^{R}\right)^T & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_{ru} \\ \lambda N_{ru}^{R,1} \\ \vdots \\ \lambda N_{ru}^{R,n-1} \end{bmatrix}$$

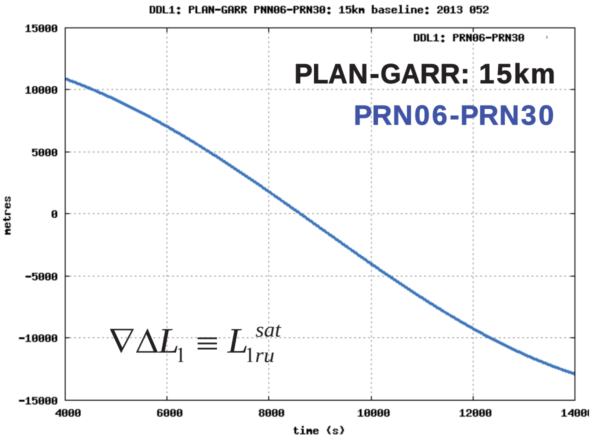
Baseline vector

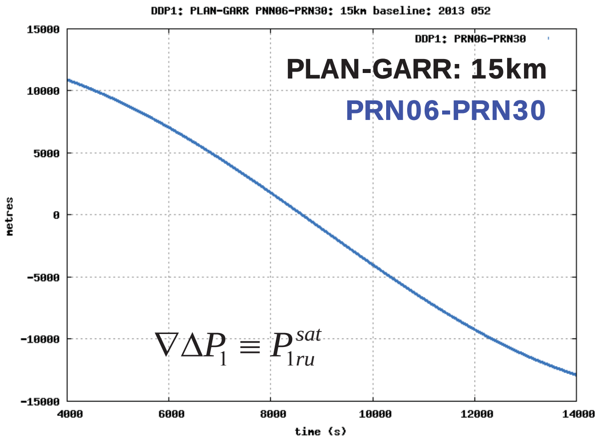
DD Code and Carrier measurements

Carrier ambiguities



Double-Difference of measurements

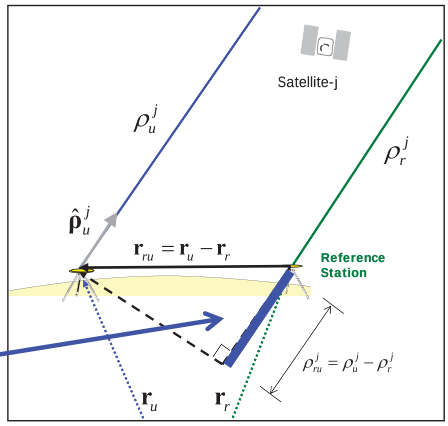




$$\nabla\Delta L_1 \equiv L_{ru}^{jk} = \boxed{\rho_{ru}^{jk}} + T_{ru}^{jk} - I_{ru}^{jk} + \lambda\,\omega_{ru}^{jk} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

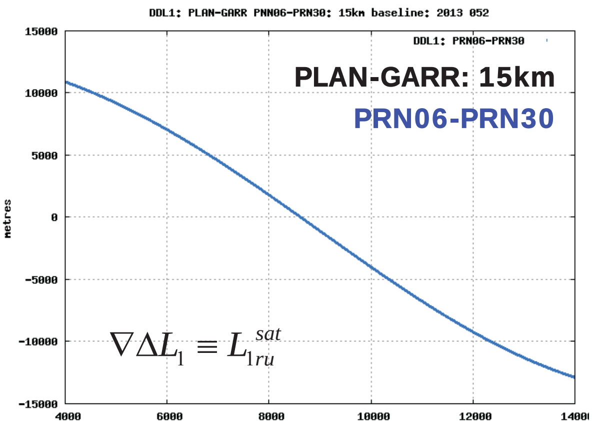
$$\nabla\Delta P_1 \equiv P_{ru}^{jk} = \boxed{\rho_{ru}^{jk}} + T_{ru}^{jk} + I_{ru}^{jk} + \nu_{P\,ru}^{\;jk}$$

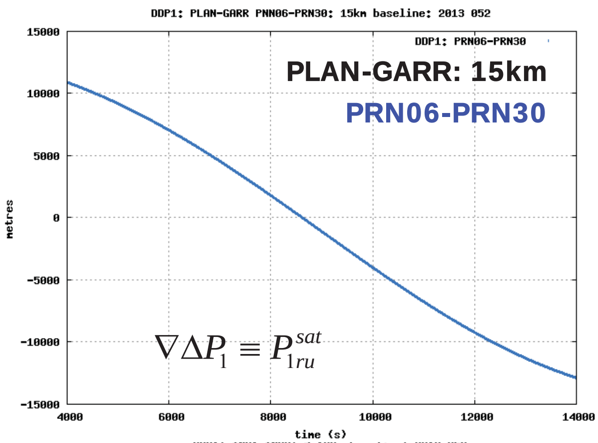
$$\boxed{\rho_{ru}^{jk} = -\hat{\boldsymbol{\rho}}_u^{jk} \cdot \mathbf{r}_{ru}}$$

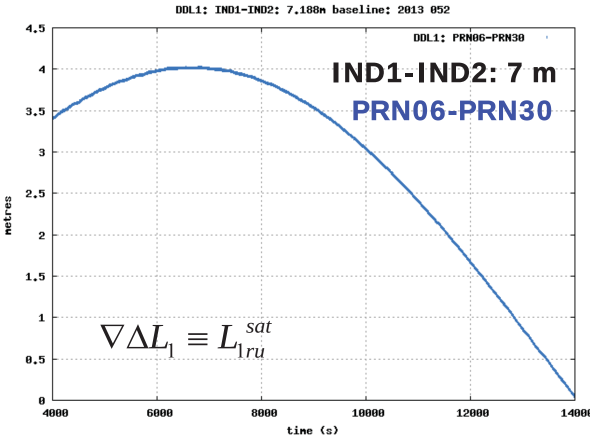


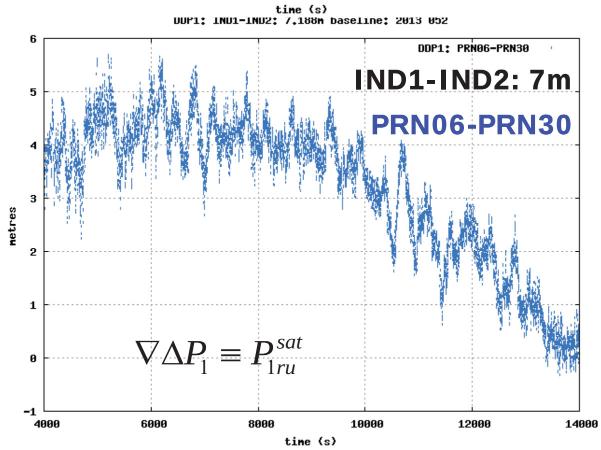
Variation of the Baseline projection over the unit Line-Of-Sight vector

Double-Difference of measurements









Relative Positioning

In this approach, the reference station broadcast its time-tagged code and carrier measurements, instead of the computed differential corrections.

Thence, the user can form the double differences of its own measurements with those of the reference receiver, satellite by satellite, and estimate its position relative to the reference receiver.

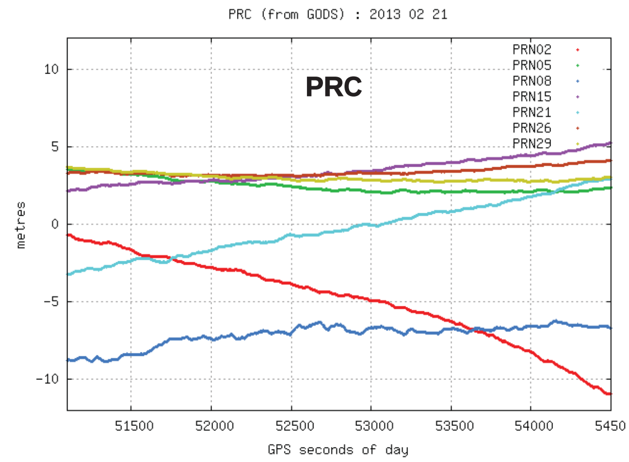
Notice that, the baseline can be estimated without needing an accurate knowledge of reference the station coordinates. Of course, the knowledge of the reference station coordinates would allow the user to compute its absolute coordinates.

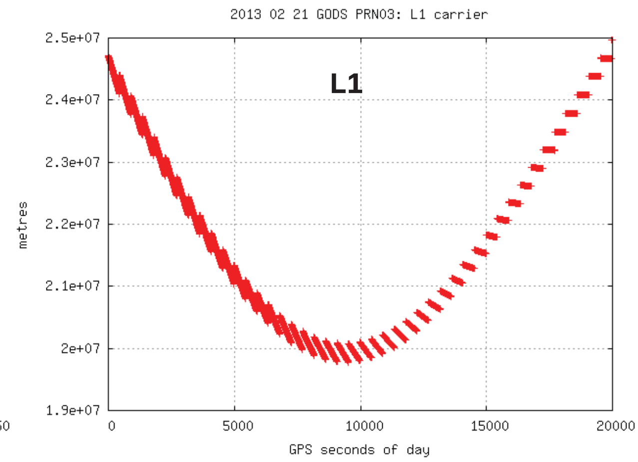
Relative Positioning

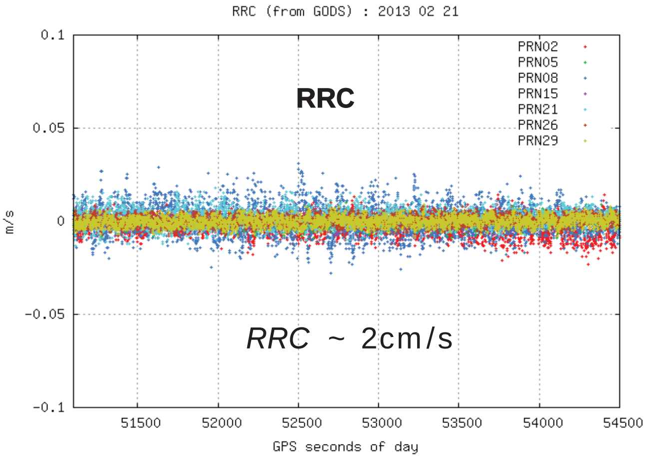
Time synchronization issues:

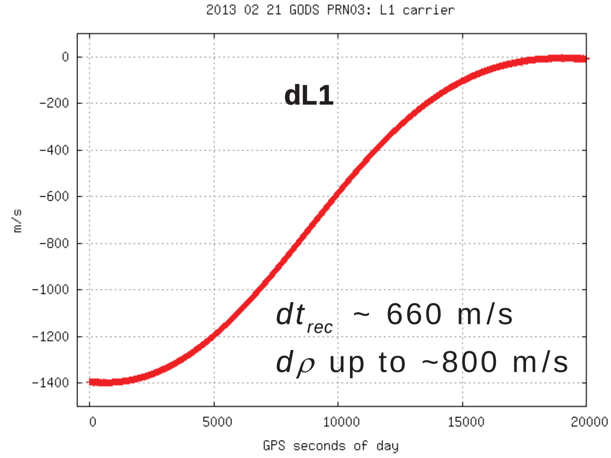
There is an important and subtitle difference between the previous approach of relative positioning (which does not need to know the reference station coordinates) and the differential positioning approach based on the knowledge of the reference station coordinates.

- The differential corrections wary slowly, and its useful life can be up to several minutes with S/A=off.
- But, the measurements change much faster. The range rate can be up to 800m/s and, thence, a synchronizer error of 1millisecond can lead up to more than 1/2 meter of error.
- As commented before, real-time implementation entails also latencies, that can be up to 2 seconds, thence, a extrapolation technique must be applied to the measurements to reduce error due to latency and epoch mismatch (to <1cm if ambiguities are intended to be fixed).

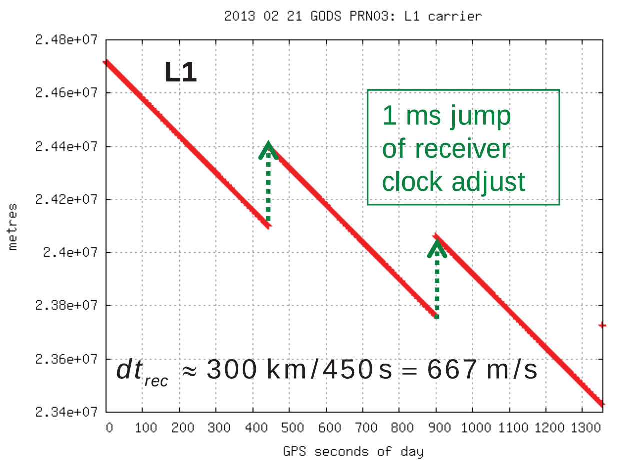


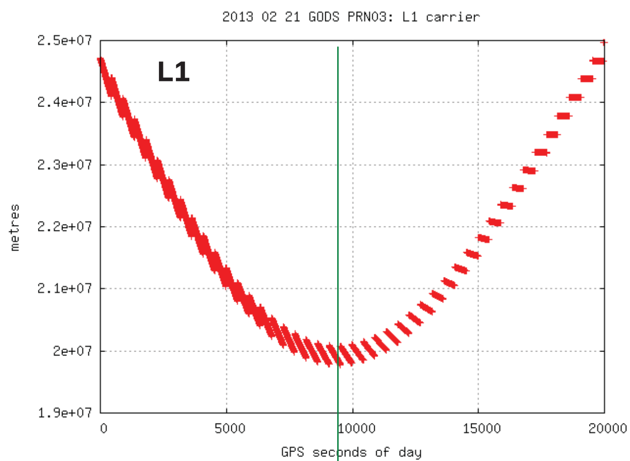


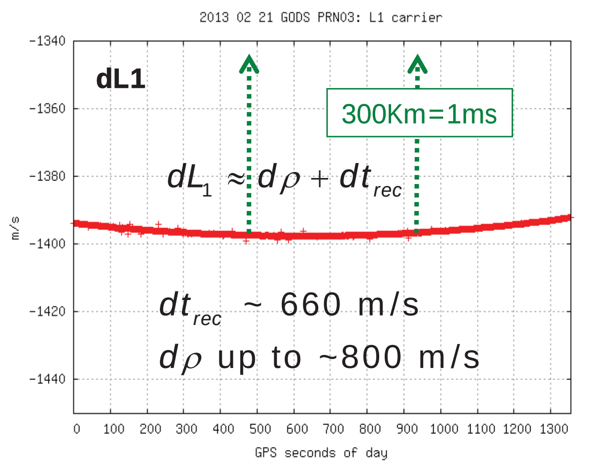


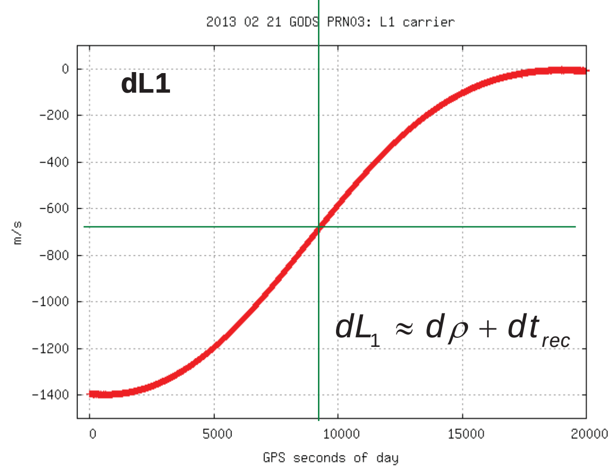


Receiver: JAVAD TRE_G3TH DELTA3.3.12







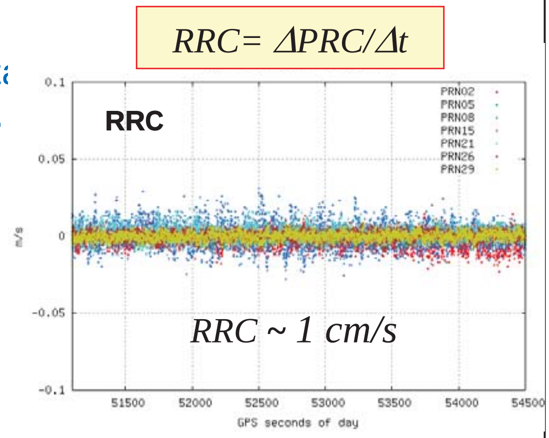


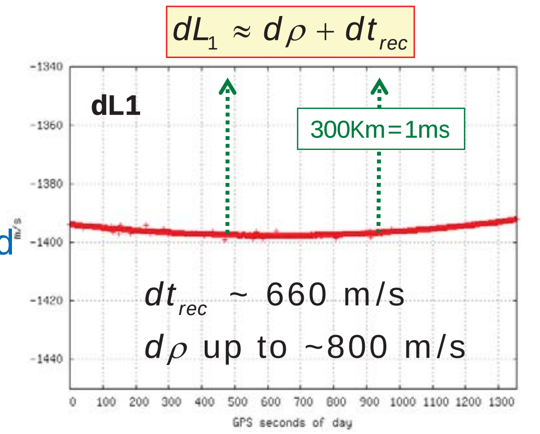
Receiver: JAVAD TRE_G3TH DELTA3.3.12

COMMENTS

Real-Time implementation entails delays in data transmission, which can reach up to 1 or 2 s.

- Differential corrections vary slowly and its useful life is of several minutes (S/A=off)
- But, the measurements change much faster:
  - The range rate $d\rho/dt$ can be up to 800m/s and, therefore, the range can change by more than half a meter in 1 millisecond. Moreover the receiver clock offset can be up to 1 millisecond (depending on the receiver configuration).
  - Thence, the reference station measurements must be :
    - **Synchronized** to reduce station clock mismatch: station clock can be estimated to within $1\mu s$ ➔ $\varepsilon_{dt_{sta}} < 1\,mm$
    - **Extrapolated** to reduce error due to latency: carrier can be extrapolated with error < 1cm.



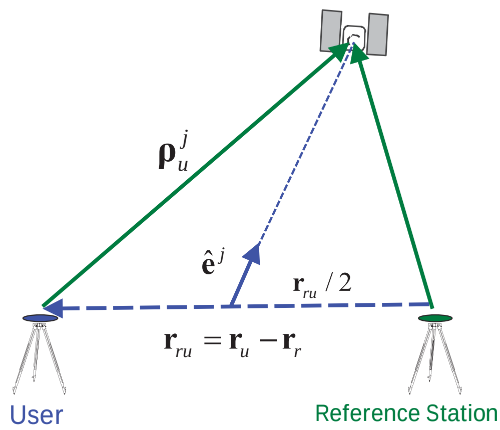


Relative Positioning

Exercise:

Demonstrate the following relationship between the baseline and the differential range [*]:

$$\rho_{ru}^j = \rho_u^j - \rho_r^j = -\left(\frac{2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}}{\|\boldsymbol{\rho}_u^j\| + \|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}\right)\cdot \mathbf{r}_{ru}$$



Comments:

The previous expression can be written as:

$$\rho_u^j - \rho_r^j = -\left(\omega^j \hat{\mathbf{e}}^j\right)\cdot \mathbf{r}_{ru} \quad \text{with} \quad \omega^j \equiv \frac{\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|}{\|\boldsymbol{\rho}_u^j\| + \|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\|} \qquad \hat{\mathbf{e}}^j = \frac{\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}/2}{\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}/2\|}$$

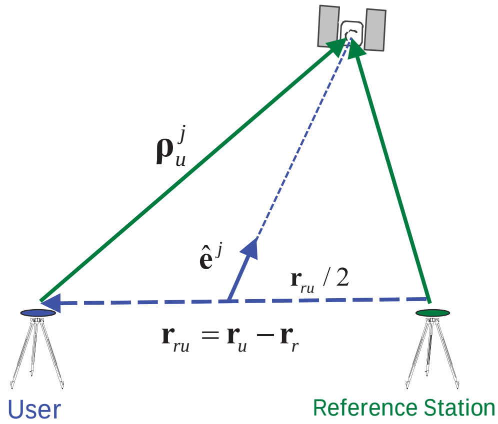
- Taking $\mathbf{r}_{ru} = 0$ in ω^j and $\hat{\mathbf{e}}^j$ leads to the approximate expression previously found.
- ω^j and $\hat{\mathbf{e}}^j$ depend on the baseline $\mathbf{r}_{ru}$, which is the vector to estimate. Nevertheless, it is not very sensitive to changes in such baseline and can be computed iteratively, computing the navigation solution starting from $\mathbf{r}_{ru} = 0$.

[*] This result is from [RD-7]

Relative Positioning

Solution

Consider the following relations:



$$\mathbf{r}_{ru} = \boldsymbol{\rho}_r^j - \boldsymbol{\rho}_u^j$$

$$\hat{\mathbf{e}}^j = \frac{\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}/2}{\left\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}/2\right\|} = \frac{\boldsymbol{\rho}_r^j + \boldsymbol{\rho}_u^j}{\left\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\right\|}$$

$$\begin{aligned}\left(\left\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\right\|\hat{\mathbf{e}}^j\right)\cdot\mathbf{r}_{ru} &= \left\|\boldsymbol{\rho}_r^j\right\|^2 - \left\|\boldsymbol{\rho}_u^j\right\|^2 = \\ &= \left(\left\|\boldsymbol{\rho}_r^j\right\| - \left\|\boldsymbol{\rho}_u^j\right\|\right)\left(\left\|\boldsymbol{\rho}_r^j\right\| + \left\|\boldsymbol{\rho}_u^j\right\|\right) \\ &= \left(\rho_r^j - \rho_u^j\right)\left(\left\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\right\| + \left\|\boldsymbol{\rho}_u^j\right\|\right)\end{aligned}$$

Then:

$$\rho_u^j - \rho_r^j = -\left(\omega^j\hat{\mathbf{e}}^j\right)\cdot\mathbf{r}_{ru}$$

with: $\omega^j \equiv \dfrac{\left\|2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\right\|}{\left\|\boldsymbol{\rho}_u^j\right\| + \left\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\right\|}$ $\qquad \omega^j\hat{\mathbf{e}}^j = \dfrac{2\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}}{\left\|\boldsymbol{\rho}_u^j\right\| + \left\|\boldsymbol{\rho}_u^j + \mathbf{r}_{ru}\right\|}$

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The Role of Geometric Diversity: **Triple differences**

Let us consider again the problem of relative positioning for short baselines. We have previously found the following equation for DD carrier measurements, assuming short baselines (e.g. < 10km)

This is from [RD-3]

$$L_{ru}^{jk} = -\left(\hat{\boldsymbol{\rho}}_u^k - \hat{\boldsymbol{\rho}}_u^j\right) \cdot \mathbf{r}_{ru} + \lambda\, N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

As the ambiguities are constant along continuous carrier phase arcs, an option could be to take differences on time. Thence, if the user and reference receiver are stationary we can write the "**triple differences**" as:

$$\delta L_{ru}^{jk} = -\left(\delta\hat{\boldsymbol{\rho}}_u^k - \delta\hat{\boldsymbol{\rho}}_u^j\right) \cdot \mathbf{r}_{ru} + \delta\nu_{L\,ru}^{\;jk} \quad \Rightarrow \quad \begin{bmatrix} \delta L_{ru}^{12} \\ \delta L_{ru}^{13} \\ \vdots \\ \delta L_{ru}^{1K} \end{bmatrix} = \begin{bmatrix} -\left(\delta\hat{\boldsymbol{\rho}}_u^2 - \delta\hat{\boldsymbol{\rho}}_u^1\right)^T \\ -\left(\delta\hat{\boldsymbol{\rho}}_u^3 - \delta\hat{\boldsymbol{\rho}}_u^1\right)^T \\ \vdots \\ -\left(\delta\hat{\boldsymbol{\rho}}_u^K - \delta\hat{\boldsymbol{\rho}}_u^1\right)^T \end{bmatrix} \mathbf{r}_{ru} + \tilde{\mathbf{v}}$$

where:

$$\delta L_{ru}^{jk} \equiv L_{ru}^{jk}(t_2) - L_{ru}^{jk}(t_1)$$

For simplicity, we assign ($j=1$) to the reference satellite

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The Role of Geometric Diversity: **Triple differences**

This is from [RD-3]

$$\delta L_{ru}^{jk} = -\left(\delta\hat{\boldsymbol{\rho}}_u^k - \delta\hat{\boldsymbol{\rho}}_u^j\right) \cdot \mathbf{r}_{ru} + \delta\nu_{L\,ru}^{\;jk} \quad \Rightarrow \quad \begin{bmatrix} \delta L_{ru}^{12} \\ \delta L_{ru}^{13} \\ \vdots \\ \delta L_{ru}^{1K} \end{bmatrix} = \begin{bmatrix} -\left(\delta\hat{\boldsymbol{\rho}}_u^2 - \delta\hat{\boldsymbol{\rho}}_u^1\right)^T \\ -\left(\delta\hat{\boldsymbol{\rho}}_u^3 - \delta\hat{\boldsymbol{\rho}}_u^1\right)^T \\ \vdots \\ -\left(\delta\hat{\boldsymbol{\rho}}_u^K - \delta\hat{\boldsymbol{\rho}}_u^1\right)^T \end{bmatrix} \mathbf{r}_{ru} + \tilde{\mathbf{v}}$$

Now, we have a "clean" equations system involving only the baseline vector to estimate. But the geometry is very weak (the associated DOP will be large number) and the position estimates will be in general worse than those from double differences.

Estimation of position and change in position: the role of Geometric Diversity

This is from [RD-3]

Let us now consider a simple model for the estimation of the relative position vector from SD carrier measurements, assuming short baselines (e.g. <10km):

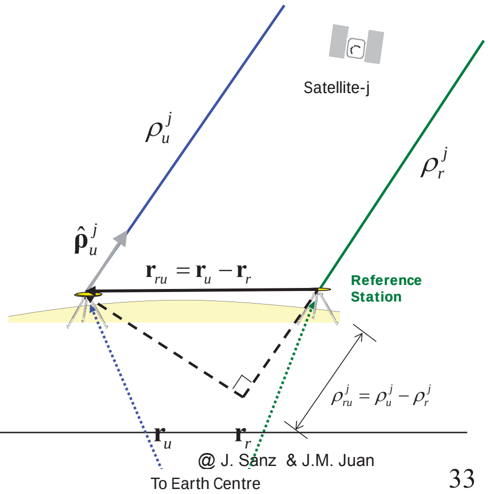
$$L_{ru}^{j} = \rho_{ru}^{j} + c\,\delta t_{ru} + \lambda N_{ru}^{j} + b_{ru} + \nu_{L\,ru}^{\ j}$$

$$\rho_{ru}^{j} = \rho_{u}^{j} - \rho_{r}^{j} = -\hat{\boldsymbol{\rho}}_{u}^{j}\cdot \mathbf{r}_{ru}$$

$$L_{ru}^{j} = -\hat{\boldsymbol{\rho}}_{u}^{j}\cdot \mathbf{r}_{ru} + d_{ru} + \lambda N_{ru}^{j} + \nu_{L\,ru}^{\ j}$$

where $d_{ru} \equiv c\,\delta t_{ru} + b_{ru}$

$$\begin{bmatrix} L_{ru}^{1} \\ L_{ru}^{2} \\ \vdots \\ L_{ru}^{K} \end{bmatrix} = \begin{bmatrix} \left(-\hat{\boldsymbol{\rho}}_{u}^{1}\right)^{T} & 1 \\ \left(-\hat{\boldsymbol{\rho}}_{u}^{2}\right)^{T} & 1 \\ & \\ \left(-\hat{\boldsymbol{\rho}}_{u}^{K}\right)^{T} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_{ru} \\ d_{ru} \end{bmatrix} + \begin{bmatrix} \lambda N_{ru}^{1} \\ \lambda N_{ru}^{2} \\ \\ \lambda N_{ru}^{K} \end{bmatrix} + \mathbf{v}$$



Estimation of position and change in position: the role of Geometric Diversity

This is from [RD-3]

Previous system can be arranged as:

$$\underbrace{\begin{bmatrix} L_{ru}^{1} \\ L_{ru}^{2} \\ \vdots \\ L_{ru}^{K} \end{bmatrix}}_{\mathbf{L}_{ru}} = \underbrace{\begin{bmatrix} \left(-\hat{\boldsymbol{\rho}}_{u}^{1}\right)^{T} & 1 \\ \left(-\hat{\boldsymbol{\rho}}_{u}^{2}\right)^{T} & 1 \\ & \\ \left(-\hat{\boldsymbol{\rho}}_{u}^{K}\right)^{T} & 1 \end{bmatrix}}_{\mathbf{G}} \begin{bmatrix} \mathbf{r}_{ru} \\ d_{ru} \end{bmatrix} + \begin{bmatrix} \lambda N_{ru}^{1} \\ \lambda N_{ru}^{2} \\ \\ \lambda N_{ru}^{K} \end{bmatrix} + \mathbf{v}$$

Considering now differences between two epochs t_0 and t_1, and assuming no cycle-slips:

$$\mathbf{L}_{ru}(t_1) - \mathbf{L}_{ru}(t_0) = \mathbf{G}(t_1)\begin{bmatrix} \mathbf{r}_{ru}(t_1) \\ d_{ru}(t_1) \end{bmatrix} - \mathbf{G}(t_0)\begin{bmatrix} \mathbf{r}_{ru}(t_0) \\ d_{ru}(t_0) \end{bmatrix} + \tilde{\mathbf{v}}$$

$\delta\mathbf{r}_{ru}(t_1) = \mathbf{r}_{ru}(t_1) - \mathbf{r}_{ru}(t_0)$

$-\mathbf{G}(t_1) + \mathbf{G}(t_1)$

$$\mathbf{L}_{ru}(t_1) - \mathbf{L}_{ru}(t_0) = \mathbf{G}(t_1)\begin{bmatrix} \delta\mathbf{r}_{ru}(t_1) \\ \delta d_{ru}(t_1) \end{bmatrix} + \left(\mathbf{G}(t_1) - \mathbf{G}(t_0)\right)\begin{bmatrix} \mathbf{r}_{ru}(t_0) \\ \boxed{d_{ru}(t_0)} \end{bmatrix} + \tilde{\mathbf{v}}$$

$$\mathbf{G}(t_1) - \mathbf{G}(t_0) = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}_{u}^{1}(t_1) - \hat{\boldsymbol{\rho}}_{u}^{1}(t_0)\right)^{T} & 0 \\ -\left(\hat{\boldsymbol{\rho}}_{u}^{1}(t_1) - \hat{\boldsymbol{\rho}}_{u}^{1}(t_0)\right)^{T} & 0 \\ \vdots & \vdots \\ -\left(\hat{\boldsymbol{\rho}}_{u}^{1}(t_1) - \hat{\boldsymbol{\rho}}_{u}^{1}(t_0)\right)^{T} & 0 \end{bmatrix}$$

Estimation of changes in baseline vector and clock bias is tied to the geometry matrix at time t_1. This can be well determined.

Estimation of absolute value of baseline vector is tied to the change in geometry matrix at time t_1. This would be poor determined if such change is not significant.

$d_{ru}(t_0)$ cannot be estimated at all!

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Ambiguity resolution Techniques

As a driven problem to study the ambiguity fixing, we will consider the problem of differential positioning in DD for **short baselines** (e.g. < 10 km).
In general we will consider that we have Code and Carrier measurements in different frequencies ($q=1,2\ldots$), i.e. $P_1, P_2, L_1, L_2\ldots$

$$P_{q,ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} + I_{q,ru}^{jk} + \nu_{P_q,ru}^{jk}\;; \quad q=1,2\ldots$$
$$L_{q,ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} - I_{q,ru}^{jk} + \lambda_q\,\omega_{ru}^{jk} + \lambda\,N_{q,ru}^{jk} + \nu_{L_q,ru}^{jk}$$

Short baseline

$$T_{ru}^{j} \simeq 0 \qquad I_{q,ru}^{jk} \simeq 0 \qquad \omega_{ru}^{jk} \simeq 0$$

$$P_{q,ru}^{jk} = \rho_{ru}^{jk} + \nu_{P_q\,ru}^{\;jk}\;; \quad q=1,2\ldots$$
$$L_{q,ru}^{jk} = \rho_{ru}^{jk} + \lambda_q\,N_{q,ru}^{jk} + \nu_{L_q\,ru}^{\;jk}$$

To simplify notation, when different frequencies are considered, we will remove the subscript "*ru*".

$$P_q^{jk} = \rho^{jk} + \nu_{P_q}^{jk}\;; \quad q=1,2\ldots$$
$$L_q^{jk} = \rho^{jk} + \lambda_q N_q^{jk} + \nu_{L_q}^{jk}$$

K sat. in view ➔ K 'SD'
➔ $K(K-1)$ 'DD',
but only K-1 DDs are linearly independent

We assume the following measurement errors:

$$\sigma_{P_q} \approx 0.5\,\text{m} \qquad \sigma_{L_q} \approx 0.5\,\text{cm} \quad \Rightarrow \quad \boxed{\sigma_{P_q^{jk}} \approx 1\,\text{m} \qquad \sigma_{L_q^{jk}} \approx 1\,\text{cm}}$$

Take the highest elevation as the reference satellite to minimize measurement error.

As commented before, the ambiguity terms are integer numbers, and we can take benefit of this property to fix such ambiguities applying integer ambiguity resolution techniques.

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Resolving ambiguities one at a Time

A simple trial would be (for instance using L1 and P1):

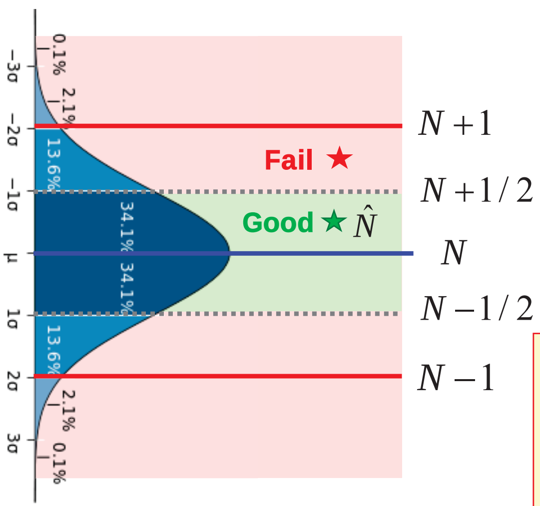
$$\begin{aligned} P_1^{jk} &= \rho^{jk} + \nu_{P_1}^{jk} \\ L_1^{jk} &= \rho^{jk} + \lambda_1 N_1^{jk} + \nu_{L_1}^{jk} \end{aligned} \rightarrow L_1^{jk} - P_1^{jk} = \lambda_1 N_1^{jk} + \nu_{P_1}^{jk} \rightarrow \hat{N}_1^{jk} = \left[\frac{L_1^{jk} - P_1^{jk}}{\lambda_1}\right]_{roundoff}$$

$$\lambda_1 \simeq 20\,\text{cm} \qquad \sigma_{P_1^{jk}} \approx 1\,\text{m} \qquad \sigma_{L_1^{jk}} \approx 1\,\text{cm}$$

$$\sigma_{\hat{N}_1^{jk}} \simeq \frac{1}{\lambda_1}\sigma_{P_1^{jk}} \approx 5$$

Too much error (5 wavelengths)!

Note that, assuming a Gaussian distribution of errors, $\sigma_{\hat{N}_1^{jk}} \simeq 1/2$ guarantee only the 68% of success



Similar results with *L2*, *P2* measurements

As the ambiguity is constant (between cycle-slips), we would try to reduce uncertainty by averaging the estimate on time, but we will need 100 epochs to reduce noise up to ½ (but measurement errors are highly correlated on time!)

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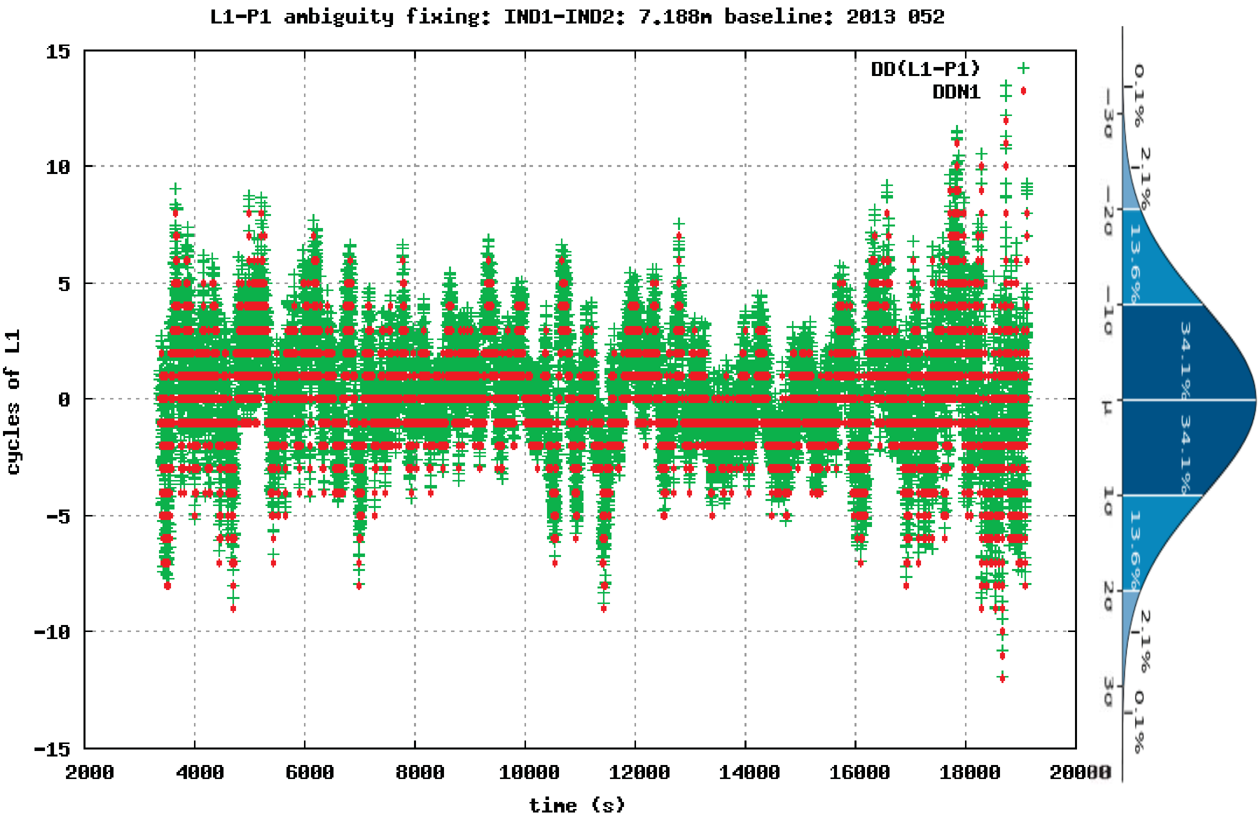

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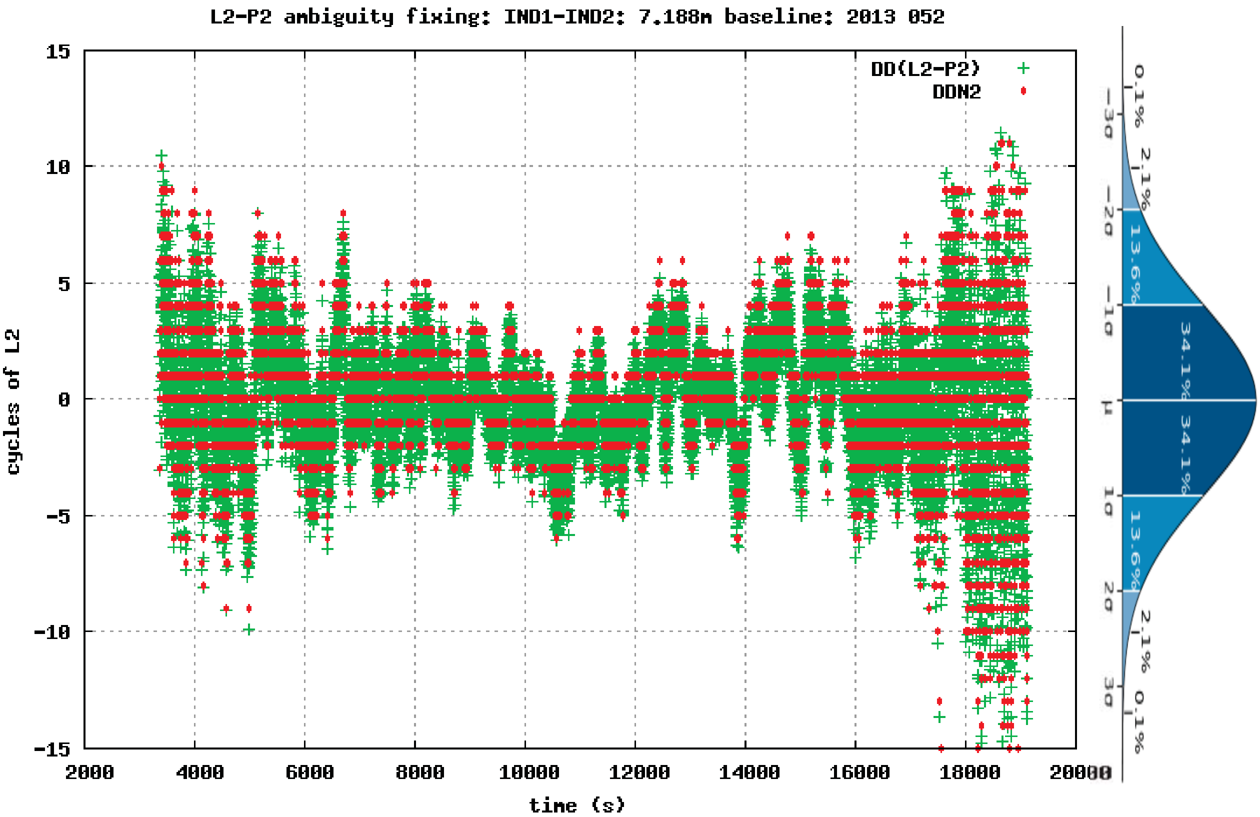

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Resolving ambiguities one at a Time

Dual frequency measurements: wide-laning with the Melbourne-Wübbena combination

$$P_1^{jk} = \rho^{jk} + \nu_{P_1}^{jk}$$
$$P_2^{jk} = \rho^{jk} + \nu_{P_2}^{jk}$$

$$L_1^{jk} = \rho^{jk} + \lambda_1\, N_1^{jk} + \nu_{L_1}^{jk}$$
$$L_2^{jk} = \rho^{jk} + \lambda_2\, N_2^{jk} + \nu_{L_2}^{jk}$$

$$P_N^{jk} = \frac{f_1 P_1^{jk} + f_2 P_2^{jk}}{f_1 + f_2} = \rho^{jk} + \nu_{P_N}^{jk}$$

$$L_W^{jk} = \frac{f_1 L_1^{jk} - f_2 L_2^{jk}}{f_1 - f_2} = \rho^{jk} + \lambda_W\, N_W^{jk} + \nu_{L_W}^{jk}$$

$$N_W = N_1 - N_2$$
$$\lambda_W = \frac{c}{f_1 - f_2} \simeq 86.2\,\text{cm}$$

$$\sigma_{P_N^{jk}} \approx \sigma_{P_1^{jk}} / \sqrt{2} \approx 71\,\text{cm}$$
$$\sigma_{L_W^{jk}} \approx 6\,\sigma_{L_1^{jk}} \approx 6\,\text{cm}$$

$$\downarrow$$

$$L_W^{jk} - P_N^{jk} = \lambda_W\, N_W^{jk} + \nu_{P_N}^{jk} \rightarrow$$

$$\hat{N}_W^{jk} = \left[\frac{L_W^{jk} - P_N^{jk}}{\lambda_W}\right]_{roundoff}$$

$$\sigma_{\hat{N}_W^{jk}} \approx \frac{1}{\lambda_W}\sigma_{P_N^{jk}} \simeq \frac{71cm}{86.2cm} \simeq 0.8$$

Now, with uncorrelated measurements from 10 epochs will reduce noise up to about ¼.

Fixing N_1 (after fixing N_W)

$$L_1^{jk} - L_2^{jk} = \lambda_1\, N_1^{jk} - \lambda_2\, N_2^{jk} + \nu_{L_1-L_2}^{jk}$$
$$= (\lambda_1 - \lambda_2)\, N_1^{jk} + \lambda_2 N_W^{jk} + \nu_{L_1-L_2}^{jk}$$

$\lambda_1 = 19.0\,\text{cm}$
$\lambda_2 = 24.4\,\text{cm}$
$\lambda_2 - \lambda_1 = 5.4\,\text{cm}$
$\sigma_{L_1^{jk}} \approx 1\,\text{cm}$

$$\hat{N}_1^{jk} = \left[\frac{L_1^{jk} - L_2^{jk} - \lambda_2\, \hat{N}_W^{jk}}{\lambda_1 - \lambda_2}\right]_{roundoff}$$

$$\hat{N}_2^{jk} = \hat{N}_1^{jk} - \hat{N}_W^{jk}$$

$$\sigma_{\hat{N}_1^{jk}} \approx \frac{1}{\lambda_1 - \lambda_2}\sqrt{2}\,\sigma_{L_1^{jk}} \simeq \frac{1.4cm}{5.4cm} \simeq 1/4$$

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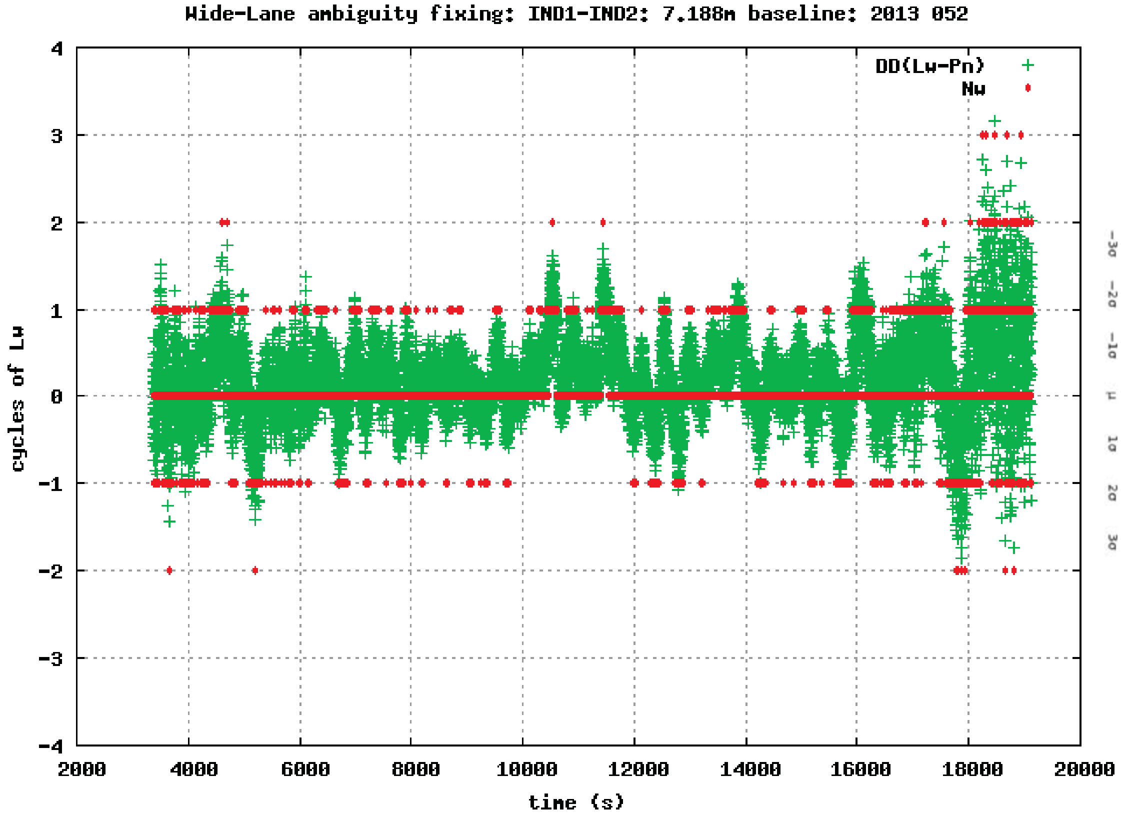

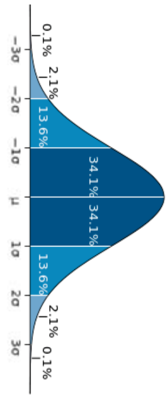

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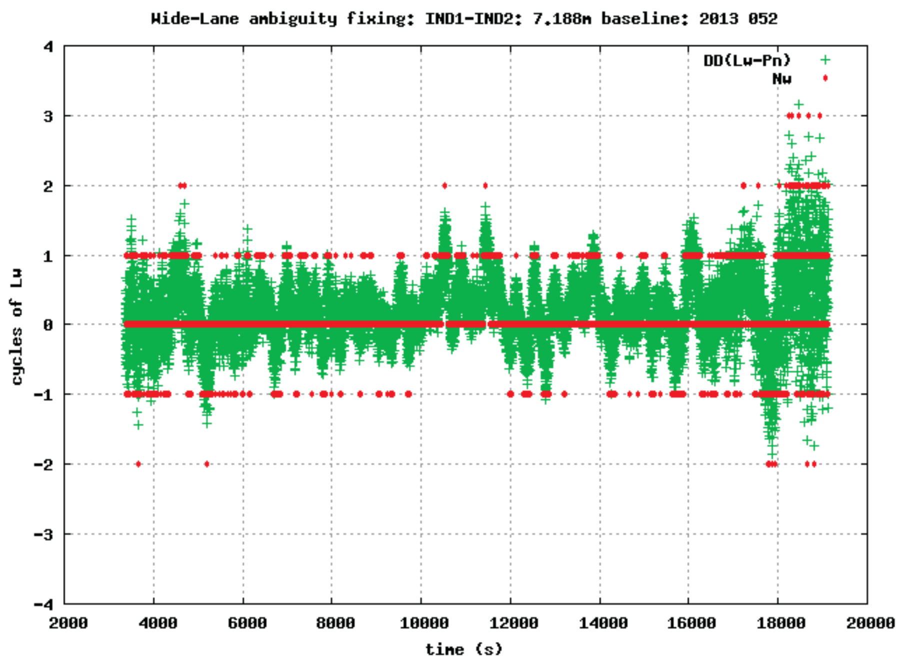

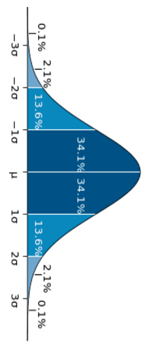

Once the integer ambiguities are known, the carrier phase measurements become unambiguous pseudoranges, accurate at the centimetre level (in DD), or better.

Thence, the estimation of the relative position vector is straightforward following the same approach as with pseudoranges.

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Exercises:

1) Consider the wide-lane combination of carrier phase measurements

$L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$, where L_W is given in length units (i.e. $L_i = \lambda_i \, \phi_i$).

Show that the corresponding wavelength is: $\lambda_W = \frac{c}{f_1 - f_2}$

Hint:

$L_W = \lambda_W \, \phi_W$; $\phi_W = \phi_1 - \phi_2$

2) Assuming L_1, L_2 uncorrelated measurements with equal noise σ_L, show that:

$$\sigma_{L_W} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}}-1}\sigma_L \quad ; \quad \gamma_{12} = \left(\frac{f_1}{f_2}\right)^2$$

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Three Frequency measurements:

We still consider the above problem of relative positioning in DD for short baselines (e.g. $<10\ \mathrm{km}$) ➔ *Ionosphere, troposphere and wind-up differential errors cancel.*

GPS	Frequency	Wavelengths	Combinations
L1	154 x 10.23 MHz	λ_1 =0.190 m	$\lambda_2-\lambda_1$ =0.054 m
L2	120 x 10.23 MHz	λ_2 =0.244 m	λ_W = 0.862 m
L5	115 x 10.23 MHz	λ_5 =0.255 m	λ_{EW} =5.861 m

With three frequency systems, having two close frequencies it is possible to generate an extra-wide-lane signal to enable the single epoch ambiguity fixing.

We drop here the superscript (*jk*) for simplicity

$$\lambda_i=\frac{c}{f_i};\quad \lambda_W=\frac{c}{f_1-f_2};\quad \lambda_{EW}=\frac{c}{f_2-f_5}$$

$$L_i=\rho+\lambda_i N_i+\nu_{L_i}\ ;i=1,2,5$$

$$L_W=\frac{f_1L_1-f_2L_2}{f_1-f_2}=\rho+\lambda_W N_W+\nu_{L_W}$$

$$L_{EW}=\frac{f_2L_2-f_5L_5}{f_2-f_5}=\rho+\lambda_{EW} N_{EW}+\nu_{L_{EW}}$$

$$P_N=\frac{f_1P_1+f_2P_2}{f_1+f_2}=\rho+\nu_{P_N}$$

$$P_{EN}=\frac{f_2P_2+f_5P_5}{f_2+f_5}=\rho+\nu_{P_{EN}}$$

$$\sigma_{L_W}=\frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}}-1}\sigma_{L_1}\simeq 5{,}7\,\mathrm{cm}$$

$$\sigma_{L_{EW}}=\frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}}-1}\sigma_{L_1}\simeq 33{,}3\,\mathrm{cm}$$

$$\sigma_{P_N}=\frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}}+1}\sigma_{P_1}\simeq 0{,}712\,\mathrm{m}$$

$$\sigma_{P_{EN}}=\frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}}+1}\sigma_{P_1}\simeq 0{,}707\,\mathrm{m}$$

$$\gamma_{12}=(f_1/f_2)^2=(77/60)^2$$

$$\gamma_{25}=(f_2/f_5)^2=(24/23)^2$$

$$N_W=N_1-N_2$$

$$N_{EW}=N_2-N_5$$

Exercise:
Justify the previous expressions for σ.

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We still consider the above problem of relative positioning in DD for short baselines (e.g. $<10\ \mathrm{km}$) ➔ *Ionosphere, troposphere and wind-up differential errors cancel .*

GPS	Frequency	Wavelengths	Combinations
L1	154 x 10.23 MHz	λ_1 =0.190 m	$\lambda_2-\lambda_1$ =0.054 m
L2	120 x 10.23 MHz	λ_2 =0.244 m	λ_W = 0.862 m
L5	115 x 10.23 MHz	λ_5 =0.255 m	λ_{EW} =5.861 m

$$L_1=\rho+\lambda_1 N_1+\nu_{L_1}$$
$$L_2=\rho+\lambda_2 N_2+\nu_{L_2}$$

$$\sigma_{L_1}\approx\sigma_{L_2}\approx 1\,\mathrm{cm}$$
$$\sigma_{P_1}\approx\sigma_{P_2}\approx 1\,\mathrm{m}$$

$$L_W=\rho+\lambda_W N_W+\nu_{L_W}\qquad P_N=\rho+\nu_{P_N}$$

$$L_{EW}=\rho+\lambda_{EW} N_{EW}+\nu_{L_{EW}}\qquad P_{EN}=\rho+\nu_{P_{EN}}$$

$$N_W=N_1-N_2\quad ;\quad N_{EW}=N_2-N_5$$

$$\hat{N}_{EW}=\left[\frac{L_{EW}-P_{EN}}{\lambda_{EW}}\right]_{roundoff}\longleftrightarrow\ \sigma_{\hat{N}_{EW}}\approx\frac{1}{\lambda_{EW}}\sigma_{P_{EN}}\simeq\frac{0.71\,\mathrm{m}}{5.861\,\mathrm{m}}\simeq 0.12$$

$$\hat{N}_W=\left[\frac{\lambda_{EW}\hat{N}_{EW}-(L_{EW}-L_W)}{\lambda_W}\right]_{roundoff}\longleftrightarrow\ \sigma_{\hat{N}_W}\approx\frac{1}{\lambda_W}\sigma_{L_{EW}}\simeq\frac{33.3\,\mathrm{cm}}{86.2\,\mathrm{cm}}\simeq 0.39$$

$$\hat{N}_1=\left[\frac{L_1-L_2-\lambda_2\hat{N}_W}{\lambda_1-\lambda_2}\right]_{roundoff}\longleftrightarrow\ \sigma_{\hat{N}_1}\approx\frac{1}{\lambda_1-\lambda_2}\sqrt{2}\sigma_{L_1}\simeq\frac{1.4cm}{5.4cm}\simeq 1/4$$

$$\gamma_{12}=(f_1/f_2)^2=(77/60)^2$$

$$\gamma_{25}=(f_2/f_5)^2=(24/23)^2$$

$$\sigma_{P_N}=\frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}}+1}\sigma_{P_1}\simeq 0{,}71\,\mathrm{m}$$

$$\sigma_{P_{EN}}=\frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}}+1}\sigma_{P_1}\simeq 0{,}71\,\mathrm{m}$$

$$\sigma_{L_W}=\frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}}-1}\sigma_{L_1}\simeq 5{,}7\,\mathrm{cm}$$

$$\sigma_{L_{EW}}=\frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}}-1}\sigma_{L_1}\simeq 33{,}3\,\mathrm{cm}$$

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Exercise:

Repeat the previous study for the Galileo signals E1, E5b and E5a

Galileo	Frequency	Wavelengths	Combinations
E1	154 x 10.23 MHz	λ_1 =0.190 m	$\lambda_2 - \lambda_1$ =0.058 m
E5b	118 x 10.23 MHz	λ_2 =0.248 m	λ_W = 0.814 m
E5a	115 x 10.23 MHz	λ_3 =0.255 m	λ_{EW} =9.768 m

$$L_1 = \rho + \lambda_1 N_1 + \nu_{L_1}$$
$$L_2 = \rho + \lambda_2 N_2 + \nu_{L_2}$$

$$\sigma_{L_1} \approx \sigma_{L_2} \approx 1\,\mathrm{cm}$$
$$\sigma_{P_1} \approx \sigma_{P_2} \approx 1\,\mathrm{m}$$

$$L_W = \rho + \lambda_W N_W + \nu_{L_W} \qquad P_N = \rho + \nu_{P_N}$$
$$L_{EW} = \rho + \lambda_{EW} N_{EW} + \nu_{L_{EW}} \qquad P_{EN} = \rho + \nu_{P_{EN}}$$

$$\hat{N}_{EW} = \left[\frac{L_{EW} - P_{EN}}{\lambda_{EW}}\right]_{roundoff} \longleftrightarrow \sigma_{\hat{N}_{EW}} \approx \frac{1}{\lambda_{EW}} \sigma_{P_{EN}} \approx [\qquad]$$

$$\hat{N}_W = \left[\frac{\lambda_{EW} \hat{N}_{EW} - (L_{EW} - L_W)}{\lambda_W}\right]_{roundoff} \longleftrightarrow \sigma_{\hat{N}_W} \approx \frac{1}{\lambda_W} \sigma_{L_{EW}} \approx [\qquad]$$

$$\hat{N}_1 = \left[\frac{L_1 - L_2 - \lambda_2 \hat{N}_W}{\lambda_1 - \lambda_2}\right]_{roundoff} \longleftrightarrow \sigma_{\hat{N}_1} \approx \frac{1}{\lambda_1 - \lambda_2} \sqrt{2} \sigma_{L_1} \approx [\qquad]$$

$$\gamma_{12} = (f_1 / f_2)^2 = (77/59)^2$$
$$\gamma_{23} = (f_2 / f_3)^2 = (118/115)^2$$

$$\sigma_{P_N} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}+1}} \sigma_{P_1} \approx [\qquad]$$
$$\sigma_{P_{EN}} = \frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}+1}} \sigma_{P_1} \approx [\qquad]$$

$$\sigma_{L_W} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}-1}} \sigma_{L_1} \approx [\qquad]$$
$$\sigma_{L_{EW}} = \frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}-1}} \sigma_{L_1} \approx [\qquad]$$

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Repeat the previous study for the Galileo signals E1, E5b and E5a

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$$L_1 = \rho + \lambda_1 N_1 + \nu_{L_1}$$
$$L_2 = \rho + \lambda_2 N_2 + \nu_{L_2}$$

$$\sigma_{L_1} \approx \sigma_{L_2} \approx 1\,\mathrm{cm}$$
$$\sigma_{P_1} \approx \sigma_{P_2} \approx 1\,\mathrm{m}$$

$$L_W = \rho + \lambda_W N_W + \nu_{L_W} \qquad P_N = \rho + \nu_{P_N}$$
$$L_{EW} = \rho + \lambda_{EW} N_{EW} + \nu_{L_{EW}} \qquad P_{EN} = \rho + \nu_{P_{EN}}$$

$$\hat{N}_{EW} = \left[\frac{L_{EW} - P_{EN}}{\lambda_{EW}}\right]_{roundoff} \longleftrightarrow \sigma_{\hat{N}_{EW}} \approx \frac{1}{\lambda_{EW}} \sigma_{P_{EN}} \approx \frac{0.71\,\mathrm{m}}{9.768\,\mathrm{m}} \approx 0.07$$

$$\hat{N}_W = \left[\frac{\lambda_{EW} \hat{N}_{EW} - (L_{EW} - L_W)}{\lambda_W}\right]_{roundoff} \longleftrightarrow \sigma_{\hat{N}_W} \approx \frac{1}{\lambda_W} \sigma_{L_{EW}} \approx \frac{54.9\,\mathrm{cm}}{81.4\,\mathrm{cm}} \approx 0.67$$

$$\hat{N}_1 = \left[\frac{L_1 - L_2 - \lambda_2 \hat{N}_W}{\lambda_1 - \lambda_2}\right]_{roundoff} \longleftrightarrow \sigma_{\hat{N}_1} \approx \frac{1}{\lambda_1 - \lambda_2} \sqrt{2} \sigma_{L_1} \approx \frac{1.4\,\mathrm{cm}}{5.8\,\mathrm{cm}} \approx 1/4$$

$$\gamma_{12} = (f_1 / f_2)^2 = (77/59)^2$$
$$\gamma_{23} = (f_2 / f_3)^2 = (118/115)^2$$

$$\sigma_{P_N} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}+1}} \sigma_{P_1} \approx 0{,}71\,\mathrm{m}$$
$$\sigma_{P_{EN}} = \frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}+1}} \sigma_{P_1} \approx 0{,}71\,\mathrm{m}$$

$$\sigma_{L_W} = \frac{\sqrt{\gamma_{12}+1}}{\sqrt{\gamma_{12}-1}} \sigma_{L_1} \approx 5.4\,\mathrm{cm}$$
$$\sigma_{L_{EW}} = \frac{\sqrt{\gamma_{25}+1}}{\sqrt{\gamma_{25}-1}} \sigma_{L_1} \approx 54.9\,\mathrm{cm}$$

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Resolving Ambiguities as a set

As a driven problem to study the ambiguity fixing, we will consider problem of differential positioning in DD for short baselines (e.g. < *10* km). To simplify, we will consider only carrier measurements at a single or dual frequency.

$$\begin{cases} L_q^{12}(t_i) = \rho^{12}(t_i) + N_q^{12} + \upsilon_{L_q}^{12}(t_i) \\ L_q^{13}(t_i) = \rho^{13}(t_i) + N_q^{13} + \upsilon_{L_q}^{13}(t_i) \\ \vdots \\ L_q^{1K-1}(t_i) = \rho^{1K-1}(t_i) + N_q^{1K-1} + \upsilon_{L_q}^{1K-1}(t_i) \end{cases} \quad \boxed{q = 1, 2 \ldots}$$

Static position	Equations	Unknowns
Single frequency	(K-1)* n_t	3+(K-1)
Dual frequency	2(K-1)*n_t	3+2(K-1)

Kin. position	Equations	Unknowns
Single frequency	(K-1)* n_t	3* n_t+(K-1)
Dual frequency	2(K-1)*n_t	3* n_t+2(K-1)

$\boxed{K \geq 4,\ n_t \geq 2}$ $\boxed{K \geq 5,\ n_t \geq 4}$

In principle, the estimation of ambiguities in this system is not a big problem **if we can wait enough time and the unmodelled errors are not so large**.

Each epoch brings a set of (K-1) equations for each frequency.
Note: n_t is the number of epochs

Linear Model:

$$\boxed{L_q^{jk}(t_i) = \rho^{jk}(t_i) + N_q^{jk} + \upsilon_{L_q}^{jk}(t_i)}$$

$$\boxed{\rho^{jk}(t_i) = \rho_0^{jk}(t_i) - \hat{\boldsymbol{\rho}}_0^{jk}(t_i) \cdot \Delta\mathbf{r}(t_i)}$$

$$\underbrace{L^{jk}(t_i) - \rho_0^{jk}(t_i)}_{\text{Prefit-residual } y(t)} = \underbrace{-\hat{\boldsymbol{\rho}}_0^{jk}(t_i)}_{G(t)} \cdot \Delta\mathbf{r}_{ru}(t_i) + \lambda N^{jk} + \nu_L^{jk}(t_i)$$

We can estimate all parameters (position and ambiguities) as a set by considering the **over-dimensioned system** of linear equations and solving it by the LS criterion.

$$\boxed{\mathbf{y}(t_i) = \mathbf{G}(t_i)\,\Delta\mathbf{r}(t_i) + \lambda\mathbf{N} + \mathbf{v}}$$

Resolving Ambiguities as a set

$$\mathbf{y}(t_i) = \mathbf{G}(t_i)\,\Delta\mathbf{r}(t_i) + \lambda\mathbf{N} + \mathbf{v}(t_i)$$

$\tilde{K}$ vector; $\tilde{K}\times 3$ matrix; 3 vector; $\tilde{K}$ vector

Single freq.: $\tilde{K}$=K-1

Dual freq.: $\tilde{K}$=2(K-1)

For static positioning, considering two epochs (for instance):

$$\begin{bmatrix}\mathbf{y}(t_i)\\ \mathbf{y}(t_{i+1})\end{bmatrix} = \begin{bmatrix}\mathbf{G}(t_i)\\ \mathbf{G}(t_{i+1})\end{bmatrix}\Delta\mathbf{r} + \lambda\begin{bmatrix}\mathbf{I}\\ \mathbf{I}\end{bmatrix}\mathbf{N} + \begin{bmatrix}\mathbf{v}(t_i)\\ \mathbf{v}(t_{i+1})\end{bmatrix}$$

In general, mixing several epochs, we will write: $\mathbf{y} = \mathbf{G}\,\Delta\mathbf{r} + \lambda\,\mathbf{A}\,\mathbf{N} + \mathbf{v}$

Using the least-squares criterion, we can look for a real valued 3-vector $\Delta\mathbf{r}$ and a $\tilde{K}$-vector of integers $\mathbf{N}$ that minimizes the cost function (sum of squared residuals):

$$c(\Delta\mathbf{r},\mathbf{N}) = \left\|\mathbf{y} - \mathbf{G}\,\Delta\mathbf{r} + \lambda\,\mathbf{A}\,\mathbf{N}\right\|$$

Weighted norm can be taken as well

The problem can be easily reformulated for the kinematic case. Kalman filtering can be applied as well.

Resolving Ambiguities as a set

Different strategies can be applied:

- **To Float the ambiguities** (i.e. treating the ambiguities as real numbers).
- **To Search ambiguities** over a limited set of integers to 'find the best solution'.
- **To solve as an Integer Least-Squares problem**.

For an observation span relatively long, e.g. one hour, the floated ambiguities would typically be very close to integers, and the change in the position solution from the float to the fixed solution should not be large.

As the observation span becomes smaller, ambiguity resolution play a more important role. But very short observation spans implies the risk of wrong ambiguity fixing, which can degrade the position solution significantly.

The performance, is thence measured by:

1. Initialization time
2. Reliability (or, correctness) of the integer estimates

Search techniques

Strategy:

- Define a volume to be searched
- Set up a grid within this volume
- Define a cost function (e.g. the sum of squared residuals)
- Evaluate the cost function at each grid point

Solution corresponds to the grid point with the lowest value of the cost function

Position domain

Ambiguity Function Method (AFM)

ARCE

....

Ambiguity domain

LSAS (Hatch, 1990)

LAMBDA (Teunissen, 1993)

MLAMBDA (Chang et al. 2005)

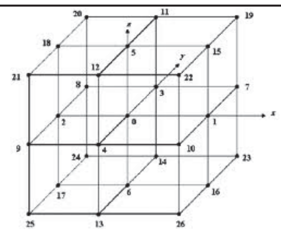
OMEGA (Kim and Langley, 2000)

FASF (Chen and Lachappelle, 1995)

IP (Xu et al., 1995)

....

Search techniques



A conceptually simpler approach would consist on:

- Estimate the floated solution N and its uncertainty (e.g. $\hat{N}=2502347.74$ cycles, $\sigma_{\hat{N}}=0.6$ cycles)
- Define as a volume to be searched $\left(\text{e.g. } \pm 3\sigma_{\hat{N}} \simeq \pm 2\,\text{cycles}\right)$ and evaluate the cost function (the RMS residuals) over the 6 ambig.: $2502345, \cdots, 2502350$

The previous search must be done for each satellite in view.

- If there are 5 satellites tracked ➔ 4 DD ambiguities ➔ 6^4 = 1 296 combinations
- If there are 8 satellites tracked ➔ 7 DD ambiguities ➔ 6^7 =279 376 combinations

The integer ambiguity solution corresponding to the smallest RMS residuals is used to select the candidate. However if two or more candidates give roughly similar values of RMS, the test can not be resolute.

➔A ratio test (of 2 or 3, depending of the algorithm) between the two smallest RMS is often used to validate the test.
If the ratio is under these values, no integer solution can be determined and is better to use the floated solution.

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LAMBDA Method

Consider again the previous problem of estimating $\Delta\mathbf{r}$, a *3*-vector of real numbers, and $\mathbf{N}$ a (*K-1*)-vector of integers, which are solution of

$$\boxed{\mathbf{y} = \mathbf{G}\,\Delta\mathbf{r} + \lambda\,\mathbf{A}\,\mathbf{N} + \mathbf{v}} \qquad \boxed{\min \left\| \mathbf{y} - \mathbf{G}\,\Delta\mathbf{r} - \lambda\,\mathbf{A}\,\mathbf{N} \right\|_{\mathbf{W_y}}}$$

To better exploit the internal correlations [*], we consider now the covariance $\mathbf{W_y} = \mathbf{P_y}^{-1}$

Let be the float solution and covariance matrix: $\begin{bmatrix} \Delta\hat{\mathbf{r}} \\ \hat{\mathbf{N}} \end{bmatrix}$; $\mathrm{Cov}\begin{bmatrix} \Delta\hat{\mathbf{r}} \\ \hat{\mathbf{N}} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{\Delta\hat{\mathbf{r}}} & \mathbf{P}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}} \\ \mathbf{P}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}} & \mathbf{P}_{\hat{\mathbf{N}}} \end{bmatrix}$

It can be shown the following orthogonal decomposition:

$$\boxed{\left\| \mathbf{y} - \mathbf{G}\,\Delta\mathbf{r} - \lambda\,\mathbf{A}\,\mathbf{N} \right\|^2_{\mathbf{W_y}} = \underbrace{\left\| \mathbf{y} - \mathbf{G}\,\Delta\hat{\mathbf{r}} - \lambda\,\mathbf{A}\,\hat{\mathbf{N}} \right\|^2_{\mathbf{W_y}}}_{\text{Residual of real-valued floated solution } (\Delta\hat{\mathbf{r}},\,\hat{\mathbf{N}})} + \left\| \Delta\mathbf{r} - \Delta\hat{\mathbf{r}}(\mathbf{N}) \right\|^2_{\mathbf{W}_{\Delta\hat{\mathbf{r}}(\mathbf{N})}} + \lambda^2 \left\| \mathbf{N} - \hat{\mathbf{N}} \right\|^2_{\mathbf{W}_{\hat{\mathbf{N}}}}}$$

[*] Remember that DD measurements are correlated, as already seen.

LAMBDA Method

Thence, we have to find $\Delta\mathbf{r}$ a *3*-vector of real numbers, and $\mathbf{N}$ a (*K-1*)-vector of integers minimizing:

$$\boxed{\left\|\mathbf{y}-\mathbf{G}\,\Delta\mathbf{r}-\lambda\,\mathbf{A}\,\mathbf{N}\right\|^2_{\mathbf{Wy}} = \underbrace{\left\|\mathbf{y}-\mathbf{G}\,\Delta\hat{\mathbf{r}}-\lambda\,\mathbf{A}\,\hat{\mathbf{N}}\right\|^2_{\mathbf{Wy}}} + \underbrace{\left\|\Delta\mathbf{r}-\Delta\hat{\mathbf{r}}(\mathbf{N})\right\|^2_{\mathbf{W}_{\Delta\hat{\mathbf{r}}(\mathbf{N})}}} + \lambda^2\underbrace{\left\|\mathbf{N}-\hat{\mathbf{N}}\right\|^2_{\mathbf{W}_{\hat{\mathbf{N}}}}}}$$

This term is irrelevant for minimization since it does not depend on $\Delta\mathbf{r}$ and $\mathbf{N}$

This term can be made zero for any $\mathbf{N}$

This term must be minimized over the integers

Float solution and covariance matrix:

$$\begin{bmatrix}\Delta\hat{\mathbf{r}}\\ \hat{\mathbf{N}}\end{bmatrix} \quad ; \quad \mathrm{Cov}\begin{bmatrix}\Delta\hat{\mathbf{r}}\\ \hat{\mathbf{N}}\end{bmatrix} = \begin{bmatrix}\mathbf{P}_{\Delta\hat{\mathbf{r}}} & \mathbf{P}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}}\\ \mathbf{P}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}} & \mathbf{P}_{\hat{\mathbf{N}}}\end{bmatrix}$$

$$\mathbf{W}^{-1}_{\hat{\mathbf{N}}} = \mathbf{P}_{\hat{\mathbf{N}}} \qquad \mathbf{W}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}} = \mathbf{P}^{-1}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}}$$

$$\boxed{\begin{array}{l}\min\left\|\mathbf{N}-\hat{\mathbf{N}}\right\|^2_{\mathbf{W}_{\hat{\mathbf{N}}}} \rightarrow \quad \breve{\mathbf{N}}\\ \Delta\breve{\mathbf{r}} = \Delta\hat{\mathbf{r}}(\breve{\mathbf{N}}) = \Delta\hat{\mathbf{r}} - \mathbf{W}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}}\mathbf{W}^{-1}_{\hat{\mathbf{N}}}\left(\breve{\mathbf{N}}-\hat{\mathbf{N}}\right)\end{array}}$$

The vectors $\Delta\breve{\mathbf{r}}$ and $\breve{\mathbf{N}}$ are often referred to as the **fixed user solution and fixed ambiguity**.

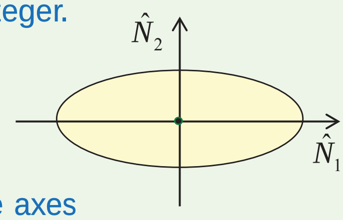
LAMBDA Method

The integer search: Finding the integer vector N that minimizes the cost function

$$\boxed{c(\mathbf{N}) = \left\|\mathbf{N}-\hat{\mathbf{N}}\right\|^2_{\mathbf{W}_{\hat{\mathbf{N}}}} = \left(\mathbf{N}-\hat{\mathbf{N}}\right)^T\mathbf{W}_{\hat{\mathbf{N}}}\left(\mathbf{N}-\hat{\mathbf{N}}\right)} \qquad \mathbf{W}_{\hat{\mathbf{N}}} = \mathbf{P}^{-1}_{\hat{\mathbf{N}}}$$

- A diagonal $\mathbf{W}_{\mathbf{N}}$ matrix would mean that the integer ambiguity estimates are uncorrelated.
- If the weight $\mathbf{W}_{\mathbf{N}}$ matrix is diagonal, the minimizing of the cost function is trivial. The best estimate is the float ambiguity rounded to the nearest integer.

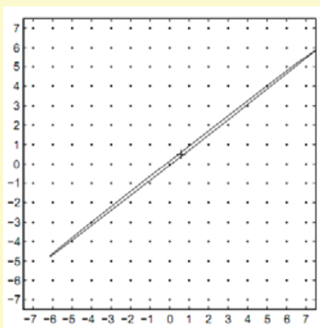
$$\mathbf{W}_{\hat{\mathbf{N}}} = \begin{bmatrix}1/\sigma^2_{\hat{N}_1\hat{N}_1} & 0\\ 0 & 1/\sigma^2_{\hat{N}_2\hat{N}_2}\end{bmatrix} \qquad c(\mathbf{N}) = \frac{\left(N_1-\hat{N}_1\right)^2}{\sigma^2_{\hat{N}_1\hat{N}_1}} + \frac{\left(N_2-\hat{N}_2\right)^2}{\sigma^2_{\hat{N}_2\hat{N}_2}}$$



Ellipse parallel to coordinate axes

In practice, the estimated (float) ambiguities are highly correlated and the ellipsoidal region stretches over a wide range of cycles. This is specially the case when the measurements are limited to a single epoch or only a few epochs.

Thence, points that appears much further away from the floated solution may have lower values of cost function than those which appear nearby. In this context, the search for integer vectors can by extremely inefficient.



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To improve the computational efficiency of the search, the float ambiguities can be transformed so that the elongated ellipsoid turns into a sphere-like. Thus, the search can be limited to the neighbours of the floated ambiguity.

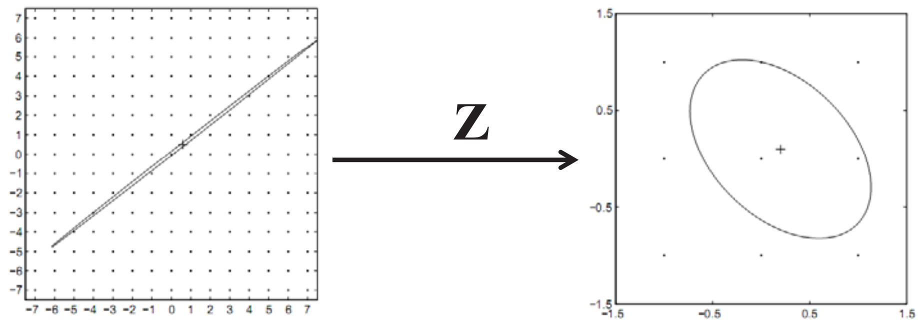
The idea is to apply a transformation that decorrelates the ambiguities so that the matrix $\mathbf{W}$ becomes diagonal. $\mathbf{W}$ is a positive definite matrix and thence, can be always diagonalized (as a real-valued matrix) with orthogonal eigenvectors. But the problem here is that the integer ambiguities $\mathbf{N}$ must be transformed preserving its integer nature!

Thence, we are looking for an "integer-valued" transformation matrix $\mathbf{Z}$ that makes the matrix $\mathbf{W}$ as close as possible to a diagonal matrix (decorrelating as much as possible the ambiguities) and with similar axes (spherical).

$$\mathbf{N}' = \mathbf{Z}\,\mathbf{N} \qquad \hat{\mathbf{N}}' = \mathbf{Z}\,\hat{\mathbf{N}} \qquad \mathbf{P}_{\hat{\mathbf{N}}'} = \mathbf{Z}\,\mathbf{P}_{\hat{\mathbf{N}}}\,\mathbf{Z}^T$$

Moreover, the inverse of transformation matrix $\mathbf{Z}^{-1}$ must be also integer, to transform back the results after finding the ambiguities

Note that $\mathbf{Z},\ \mathbf{Z}^{-1} \in \mathbb{Z} \;\Rightarrow\; |\det(\mathbf{Z})| = 1$
(i.e. it is a volume-preserving transformation)



Pictures from [RD-6]

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Exercise:

Show that:

$$\mathbf{Z},\ \mathbf{Z}^{-1} \in \mathbb{Z} \;\Rightarrow\; |\det(\mathbf{Z})| = 1$$

That is, $\mathbf{Z}$ is a volume-preserving transformation

From [RD-4]

Decorrelation: Computing the Z-transform

The following conditions must be fulfilled:

1. $\mathbf{Z}$ must have integer entries
2. $\mathbf{Z}$ must be invertible and have integer entries
3. The transformation $\mathbf{Z}$ must reduce the product of all ambiguity variances.

Note that $\mathbf{Z}, \mathbf{Z}^{-1} \in \mathbb{Z} \Rightarrow |\det(\mathbf{Z})| = 1$
(i.e. it is a volume-preserving transformation)

Gauss manipulation over matrix $\mathbf{P}=\mathbf{W}^{-1}$ can be applied to find-out the matrix $\mathbf{Z}$.

$$\mathbf{P}_{\hat{\mathrm{N}}} = \begin{bmatrix} p_{\hat{N}_1\hat{N}_1} & p_{\hat{N}_1\hat{N}_2} \\ p_{\hat{N}_1\hat{N}_2} & p_{\hat{N}_2\hat{N}_2} \end{bmatrix}$$

$$\mathbf{Z}_1 = \begin{bmatrix} 1 & 0 \\ \alpha_1 & 1 \end{bmatrix}$$

➔ Transforms N_2 (N_1 remains unchanged)

$$\alpha_i = -\mathrm{int}\left[p_{\hat{N}_1\hat{N}_2} / p_{\hat{N}_i\hat{N}_i} \right]$$

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix}$$

➔ Transforms N_1 (N_2 remains unchanged)

Note: Inverse matrices have also integer entries

$$\mathbf{Z}_1^{-1} = \begin{bmatrix} 1 & 0 \\ -\alpha_1 & 1 \end{bmatrix} \qquad \mathbf{Z}_2^{-1} = \begin{bmatrix} 1 & -\alpha_2 \\ 0 & 1 \end{bmatrix}$$

Start transforming first the element with largest variance.

Gauss manipulation over matrix $\mathbf{P}=\mathbf{W}^{-1}$ can be applied to find-out the matrix $\mathbf{Z}$

$$\mathbf{P}_{\hat{\mathrm{N}}} = \begin{bmatrix} p_{\hat{N}_1\hat{N}_1} & p_{\hat{N}_1\hat{N}_2} \\ p_{\hat{N}_1\hat{N}_2} & p_{\hat{N}_2\hat{N}_2} \end{bmatrix}$$

$$\mathbf{Z}_1 = \begin{bmatrix} 1 & 0 \\ \alpha_1 & 1 \end{bmatrix}$$

➔ Transforms N_2 (N_1 remains unchanged)

$$\alpha_i = -\mathrm{int}\left[p_{\hat{N}_1\hat{N}_2} / p_{\hat{N}_i\hat{N}_i} \right]$$

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix}$$

➔ Transforms N_1 (N_2 remains unchanged)

Example:

$$\hat{\mathbf{N}} = \begin{bmatrix} 1.05 \\ 1.30 \end{bmatrix} \qquad \mathbf{P}_{\hat{\mathrm{N}}} = \begin{bmatrix} 53.4 & 38.4 \\ 38.4 & 28.0 \end{bmatrix}$$

Example from [RD-4]

Step 1:

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad \alpha_2 = -\mathrm{int}[38.4/28.0] = -1$$

We transform first the element with largest variance (in this case N_1)

$$\mathbf{P}_{\hat{\mathrm{N}}'} = \mathbf{Z}_2\,\mathbf{P}_{\hat{\mathrm{N}}}\,\mathbf{Z}_2^T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \boxed{53.4} & 38.4 \\ 38.4 & 28.0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4.6 & \boxed{10.4} \\ 10.4 & \boxed{28.0} \end{bmatrix}$$

The half, at most!

Step 2:

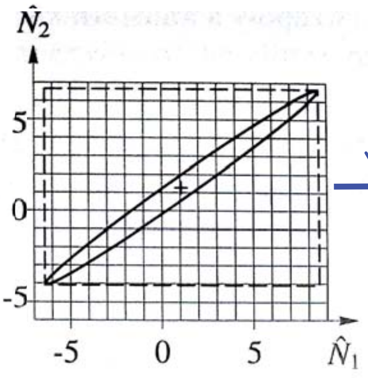
$$\mathbf{Z}_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \qquad \alpha_1 = -\mathrm{int}[10.4/4.6] = -2$$

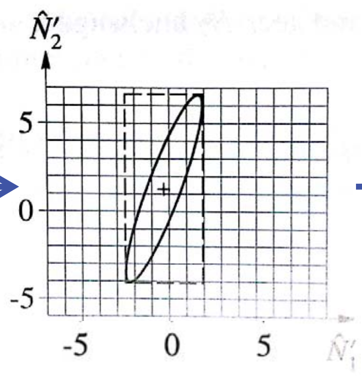
$$\mathbf{P}_{\hat{\mathrm{N}}''} = \mathbf{Z}_1\,\mathbf{P}_{\hat{\mathrm{N}}'}\,\mathbf{Z}_1^T = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 4.6 & 10.4 \\ 10.4 & 28.0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{4.6} & 1.2 \\ \boxed{1.2} & 4.8 \end{bmatrix}$$

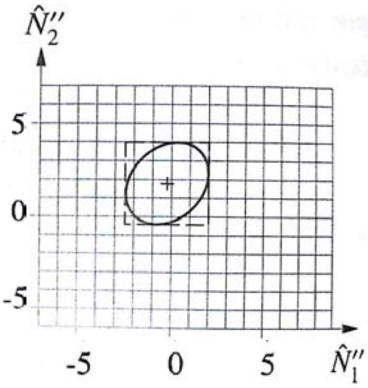
In general, to increase the number of small off-diagonal elements, we have to transform first the elements with largest variance

Example and pictures from [RD-4]

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{Z}_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$







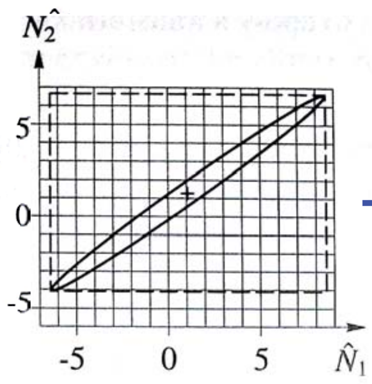
$$\mathbf{P}_{\hat{\mathrm{N}}} = \begin{bmatrix} 53.4 & 38.4 \\ 38.4 & 28.0 \end{bmatrix} \qquad \mathbf{P}_{\hat{\mathrm{N}}'} = \begin{bmatrix} 4.6 & 10.4 \\ 10.4 & 28.0 \end{bmatrix} \qquad \mathbf{P}_{\hat{\mathrm{N}}''} = \begin{bmatrix} 4.6 & 1.2 \\ 1.2 & 4.8 \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{Z}_1 \mathbf{Z}_2 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

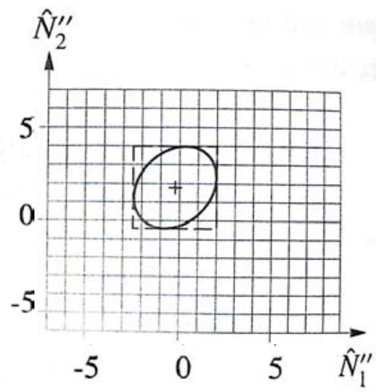
$$\mathbf{P}_{\hat{\mathrm{N}}''} = \mathbf{Z}\,\mathbf{P}_{\mathrm{N}}\,\mathbf{Z}^T = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 53.4 & 38.4 \\ 38.4 & 28.0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4.6 & 1.2 \\ 1.2 & 4.8 \end{bmatrix}$$

Example:

$$\hat{\mathbf{N}} = \begin{bmatrix} 1.05 \\ 1.30 \end{bmatrix} \qquad \mathbf{P}_{\hat{\mathrm{N}}} = \begin{bmatrix} 53.4 & 38.4 \\ 38.4 & 28.0 \end{bmatrix} \qquad \mathbf{P}_{\hat{\mathrm{N}}''} = \begin{bmatrix} 4.6 & 1.2 \\ 1.2 & 4.8 \end{bmatrix}$$



$$\mathbf{Z} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$



$$\hat{\mathbf{N}}'' = \mathbf{Z}\,\hat{\mathbf{N}} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1.05 \\ 1.30 \end{bmatrix} = \begin{bmatrix} -0.25 \\ 1.80 \end{bmatrix}$$

$$\breve{\mathbf{N}}'' = \mathrm{int} \begin{bmatrix} -0.25 \\ 1.80 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\breve{\mathbf{N}} = \mathbf{Z}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Let $\mathbf{P}$ be a symmetric and positive-definite matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$$

$$\downarrow$$

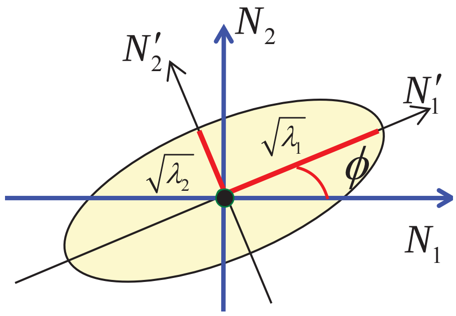
$$\mathbf{P}' = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\lambda_1 = \frac{1}{2}\left(p_{11} + p_{22} + w\right)$$

$$\lambda_2 = \frac{1}{2}\left(p_{11} + p_{22} - w\right)$$

$$w = \sqrt{(p_{11} - p_{22})^2 + 4 p_{12}^2}$$

$$\tan 2\phi = \frac{2 p_{12}}{p_{11} - p_{22}}$$



Example:

$$\mathbf{P}_{\hat{\mathbf{N}}} = \begin{bmatrix} 53.4 & 38.4 \\ 38.4 & 28.0 \end{bmatrix} \qquad \hat{\mathbf{N}} = \begin{bmatrix} 1.05 \\ 1.30 \end{bmatrix}$$

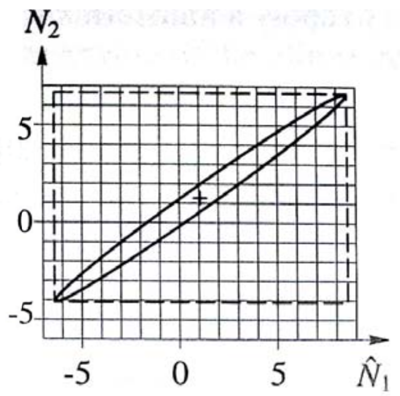
$$\downarrow$$

$$\mathbf{P}'_{\hat{\mathbf{N}}} = \begin{bmatrix} 81.14 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\sqrt{\lambda_1} = \sqrt{81.14} = 9.0$$

$$\sqrt{\lambda_2} = \sqrt{0.25} = 0.5$$

$$\tan 2\phi = 3.02 \Rightarrow \phi = 35^\circ 85$$



Consider again the previous problem of estimating $\Delta\mathbf{r}$, a *3*-vector of real numbers, and $\mathbf{N}$ a *(K-1)*-vector of integers, which are solution of

$$\boxed{\mathbf{y} = \mathbf{G}\,\Delta\mathbf{r} + \lambda\,\mathbf{A}\,\mathbf{N} + \mathbf{v}}$$

The solution comprises the following steps:

1. Obtain the float solution and its covariance matrix: $\begin{bmatrix} \Delta\hat{\mathbf{r}} \\ \hat{\mathbf{N}} \end{bmatrix}$; $\begin{bmatrix} \mathbf{P}_{\Delta\hat{\mathbf{r}}} & \mathbf{P}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}} \\ \mathbf{P}_{\Delta\hat{\mathbf{r}},\hat{\mathbf{N}}} & \mathbf{P}_{\hat{\mathbf{N}}} \end{bmatrix}$

2. Find the integer vector $\mathbf{N}$ which minimizes the cost function

$$\boxed{c(\mathbf{N}) = \left\|\mathbf{N} - \hat{\mathbf{N}}\right\|^2_{\mathbf{W}_{\hat{\mathbf{N}}}} = \left(\mathbf{N} - \hat{\mathbf{N}}\right)^T \mathbf{W}_{\hat{\mathbf{N}}} \left(\mathbf{N} - \hat{\mathbf{N}}\right)} \qquad \mathbf{W}_{\hat{\mathbf{N}}} = \mathbf{P}_{\hat{\mathbf{N}}}^{-1}$$

   a) Decorrelation: Using the $\mathbf{Z}$ transform, the ambiguity search space is re-parametrized to decorrelate the float ambiguities.
   b) Integer ambiguities estimation (e.g. using sequential conditional least-squares adjustment, together with a discrete search strategy).
   c) Using the $\mathbf{Z}^{-1}$ transform, the ambiguities are transformed to the original ambiguity space.

3. Obtain the 'fixed' solution $\Delta\mathbf{r}$, from the fixed ambiguities $\mathbf{N}$.

$$\boxed{\mathbf{y} - \lambda\,\mathbf{A}\,\mathbf{N} = \mathbf{G}\,\Delta\mathbf{r} + \mathbf{v}}$$

b) Integer ambiguities estimation

Several approach can be applied:

- Integer rounding
- Integer bootstrapping
- Integer Least-Squares
- .....

Comment:
In principle, the previous transformation $\mathbf{Z}$ is not required by the estimation concept; it is only to achieve considerable gain in speed in the computation process [RD-5].

b1) Integer rounding

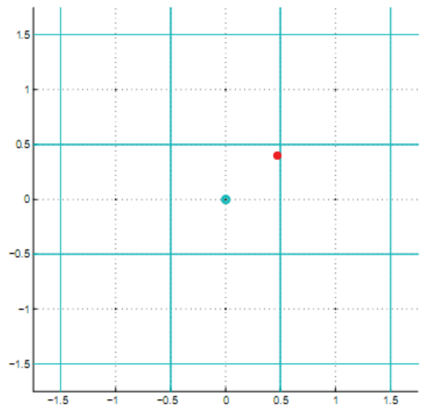
This is the simplest way.

Just to round-up the ambiguity vector entries to its nearest integer

$$\breve{\mathbf{N}} = \left(\mathrm{int}(N_1),\ldots,\mathrm{int}(N_K)\right)$$

For instance, in the previous example:

$$\breve{\mathbf{N}}'' = \mathrm{int}\begin{bmatrix} -0.25 \\ 1.80 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$



b2) Integer bootstrapping (from [RD-6])

It makes use of integer rounding, but it takes some of the correlations between the ambiguities into account.

1. We start with the most precise ambiguity (here we will assume N_n)
2. Then, the remaining float ambiguities are corrected taking into account their correlation with the last ambiguity.

$$\breve{N}_n = \mathrm{int}\left[\hat{N}_n\right]$$

$$\breve{N}_{n-1} = \mathrm{int}\left[\hat{N}_{n-1|n}\right] = \mathrm{int}\left[\hat{N}_{n-1} - \sigma_{\hat{N}_{n-1},\hat{N}_n}\sigma_{\hat{N}_n}^{-2}\left(\hat{N}_n - \breve{N}_n\right)\right]$$

$$\vdots$$

$$\breve{N}_1 = \mathrm{int}\left[\hat{N}_{n|I}\right] = \mathrm{int}\left[\hat{N}_1 - \sum_{i=2}^{n} \sigma_{\hat{N}_1,\hat{N}_{i|I}}\sigma_{\hat{N}_{i|I}}^{-2}\left(\hat{N}_{i|I} - \breve{N}_i\right)\right]$$

Using the triangular decomposition

$$\mathbf{P}_{\hat{\mathbf{N}}} = \mathbf{L}^T\,\mathbf{D}\mathbf{L}$$

$$l_{ij} = \sigma_{\hat{N}_j,\hat{N}_{i|I}}\sigma_{\hat{N}_{i|I}}^{-2}$$

$\hat{N}_{i|I}$ Stands for the i-th ambiguity obtained through a conditioning of the previous $I=\{i+1,\ldots,n\}$ sequentially rounded ambiguities.

Example and pictures from [RD-6]

$$N_1 = \hat{N}_1 - \sigma_{\hat{N}_2,\hat{N}_1}\sigma_{\hat{N}_2}^{-2}\left(\hat{N}_2 - N_2\right)$$

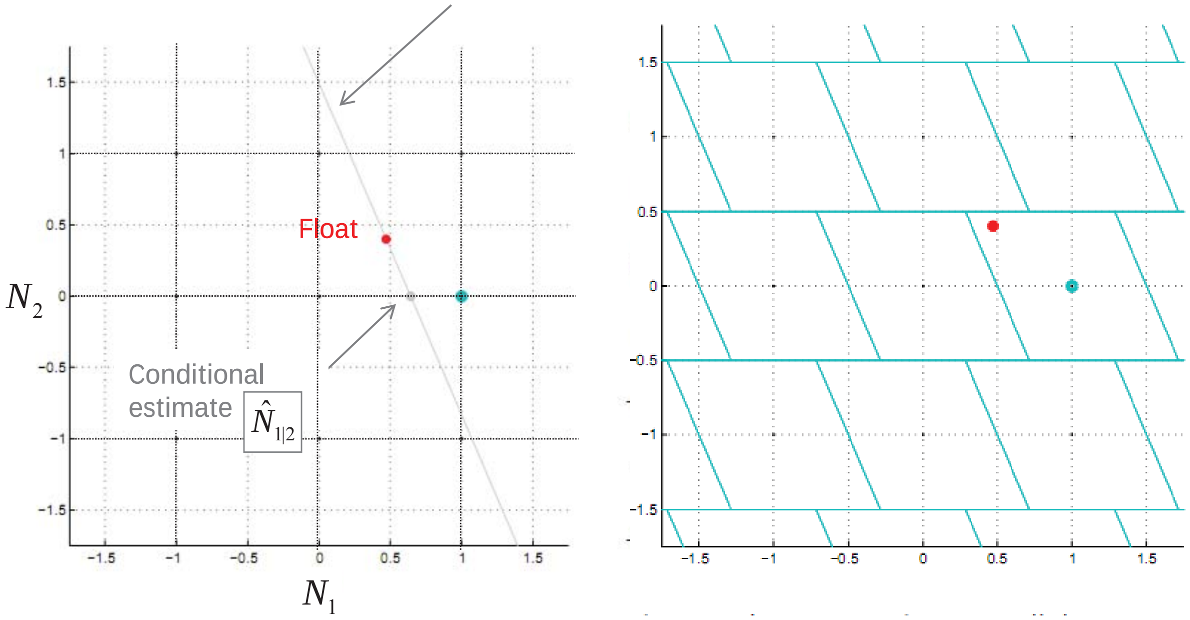


Figure 2.2: Principle and 2D pull-in regions for integer bootstrapping: parallelograms.

$$\breve{N}_2 = \mathrm{nint}\left[\hat{N}_2\right] = 0$$

$$\breve{N}_1 = \mathrm{nint}\left[\hat{N}_{1|2}\right] = \mathrm{nint}\left[\hat{N}_1 - \sigma_{\hat{N}_2,\hat{N}_1}\sigma_{\hat{N}_2}^{-2}\left(\hat{N}_2 - \breve{N}_2\right)\right] = 1$$

b3) Integer Least Squares (ISL) (from [RD-6])

1. The target to find the integer vector $\mathbf{N}$ which minimizes the cost function

$$c(\mathbf{N}) = \left\|\mathbf{N} - \hat{\mathbf{N}}\right\|^2_{\mathbf{P}_{\hat{\mathbf{N}}}^{-1}} = \left(\mathbf{N} - \hat{\mathbf{N}}\right)^T \mathbf{P}_{\hat{\mathbf{N}}}^{-1}\left(\mathbf{N} - \hat{\mathbf{N}}\right) \qquad \mathbf{W}_{\hat{\mathbf{N}}} = \mathbf{P}_{\hat{\mathbf{N}}}^{-1}$$

2. The integer minimiser is obtained through a search over the integer grid points on the n-dimensional hyper-ellipsoid: $\left(\mathbf{N} - \hat{\mathbf{N}}\right)^T \mathbf{P}_{\hat{\mathbf{N}}}^{-1}\left(\mathbf{N} - \hat{\mathbf{N}}\right) \le \chi^2$

   - Where χ^2 determines the size of search region.
   - The solution is the integer grid point $\mathbf{N}$, inside the ellipsoid, giving the minimum value of cost function $c(\mathbf{N})$.

Using the triangular decomposition: $\mathbf{P}_{\hat{\mathbf{N}}} = \mathbf{L}^T\,\mathbf{D}\,\mathbf{L}$

where: $d_i = \sigma_{\hat{N}_{i|I}}^{-2}$

$l_{ij} = \sigma_{\hat{N}_j,\hat{N}_{i|I}}\,\sigma_{\hat{N}_{i|I}}^{-2}$

$$\left(\mathbf{N} - \hat{\mathbf{N}}\right)^T \mathbf{L}^{-1}\mathbf{D}^{-1}\mathbf{L}^{-T}\left(\mathbf{N} - \hat{\mathbf{N}}\right) \le \chi^2 \rightarrow \left(\mathbf{N} - \tilde{\mathbf{N}}\right)^T \mathbf{D}^{-1}\left(\mathbf{N} - \tilde{\mathbf{N}}\right) \le \chi^2$$

Defining:

$$\tilde{\mathbf{N}} = \mathbf{N} - \mathbf{L}^{-T}(\mathbf{N} - \hat{\mathbf{N}}) \rightarrow \quad \mathbf{L}^T\left(\tilde{\mathbf{N}} - \mathbf{N}\right) = (\hat{\mathbf{N}} - \mathbf{N})$$

$$c(N) = \frac{\left(N_1 - \tilde{N}_1\right)^2}{d_1} + \frac{\left(N_2 - \tilde{N}_2\right)^2}{d_2} + \cdots + \frac{\left(N_n - \tilde{N}_n\right)^2}{d_n} \le \chi^2$$

$$\left(\mathbf{N}-\tilde{\mathbf{N}}\right)^T \mathbf{D}^{-1}\left(\mathbf{N}-\tilde{\mathbf{N}}\right) \le \chi^2 \longrightarrow \boxed{c(N)=\frac{\left(N_1-\tilde{N}_1\right)^2}{d_1}+\frac{\left(N_2-\tilde{N}_2\right)^2}{d_2}+\cdots+\frac{\left(N_n-\tilde{N}_n\right)^2}{d_n}\le\chi^2}$$

But $\tilde{N}_i$ depends on $\hat{N}_{i+1},\cdots,\hat{N}_n$.

$$\mathbf{L}^T\left(\tilde{\mathbf{N}}-\mathbf{N}\right)=\left(\hat{\mathbf{N}}-\mathbf{N}\right) \longrightarrow \boxed{\begin{array}{l}\tilde{N}_n=\hat{N}_n\\ \tilde{N}_i=\hat{N}_i+\sum_{j=i+1}^{n}\left(N_j-\hat{N}_i\right)l_{ji};\quad i=n-1,n-2,\ldots,1\end{array}}$$

Search region bounds:

$$\tilde{N}_n-d_n^{1/2}\chi \;\le\; N_n \;\le\; \tilde{N}_n+d_n^{1/2}\chi$$

$$\tilde{N}_{n-1}-d_{n-1}^{1/2}\left(\chi^2-\left(N_n-\hat{N}_n\right)^2 d_n\right)^{1/2} \;\le\; N_{n-1} \;\le\; \tilde{N}_{n-1}+d_{n-1}^{1/2}\left(\chi^2-\left(N_n-\hat{N}_n\right)^2 d_n\right)^{1/2}$$

$$\vdots$$

$$\tilde{N}_1-d_1^{1/2}\left(\chi^2-\sum_{j=2}^{n}\left(N_j-\hat{N}_j\right)^2 d_j\right)^{1/2} \;\le\; N_1 \;\le\; \tilde{N}_n+d_1^{1/2}\left(\chi^2-\sum_{j=2}^{n}\left(N_j-\hat{N}_j\right)^2 d_j\right)^{1/2}$$

Acceptance test: The integer ambiguity solution corresponding to the smallest RMS residuals is used to select the candidate. However if two or more candidates give roughly similar values of RMS, the test can not be resolute.
➔A ratio test (of 2 or 3, depending of the algorithm) between the two smallest RMS is often used to validate the test.

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Ellipsoid size: selecting the candidates for the acceptance test

The size of the ellipsoidal search region $\left(\mathbf{N}-\tilde{\mathbf{N}}\right)^T \mathbf{D}^{-1}\left(\mathbf{N}-\tilde{\mathbf{N}}\right)\le\chi^2$ is controlled by χ^2

Therefore, the performance of the search process is highly dependent on χ^2:

- A small χ^2 may result in a ellipsoidal region that fails to contain the solution.
- A too large value for χ^2 may result in high time-consuming for the search process.

Search with enumeration: When the number of required candidates is at most $n+1$ (with $n=dim(\mathbf{N})$), the following procedure can be applied to set the value χ^2:

- The best determined ambiguity is rounded to its nearest integer. The remaining ambiguities are then rounded using their correlations with the first ambiguity:

$$\begin{array}{l}\breve{N}_n=\mathrm{nint}\left[\hat{N}_n\right]\\ \breve{N}_{n-1}=\mathrm{nint}\left[\hat{N}_{n-1|n}\right]=\mathrm{nint}\left[\hat{N}_{n-1}-\sigma_{\hat{N}_{n-1},\hat{N}_n}\sigma_{\hat{N}_n}^{-2}\left(\hat{N}_n-\breve{N}_n\right)\right]\\ \vdots\\ \breve{N}_1=\mathrm{nint}\left[\hat{N}_{n|I}\right]=\mathrm{nint}\left[\hat{N}_1-\sum_{i=2}^{n}\sigma_{\hat{N}_1,\hat{N}_{i|I}}\sigma_{\hat{N}_{i|I}}^{-2}\left(\hat{N}_{i|I}-\breve{N}_i\right)\right]\end{array}$$

based on the bootstrapping estimator

- In each step of the conditional rounding procedure, two candidates are taken: The nearest and second-nearest, and conditional rounding is proceeded in both cases.

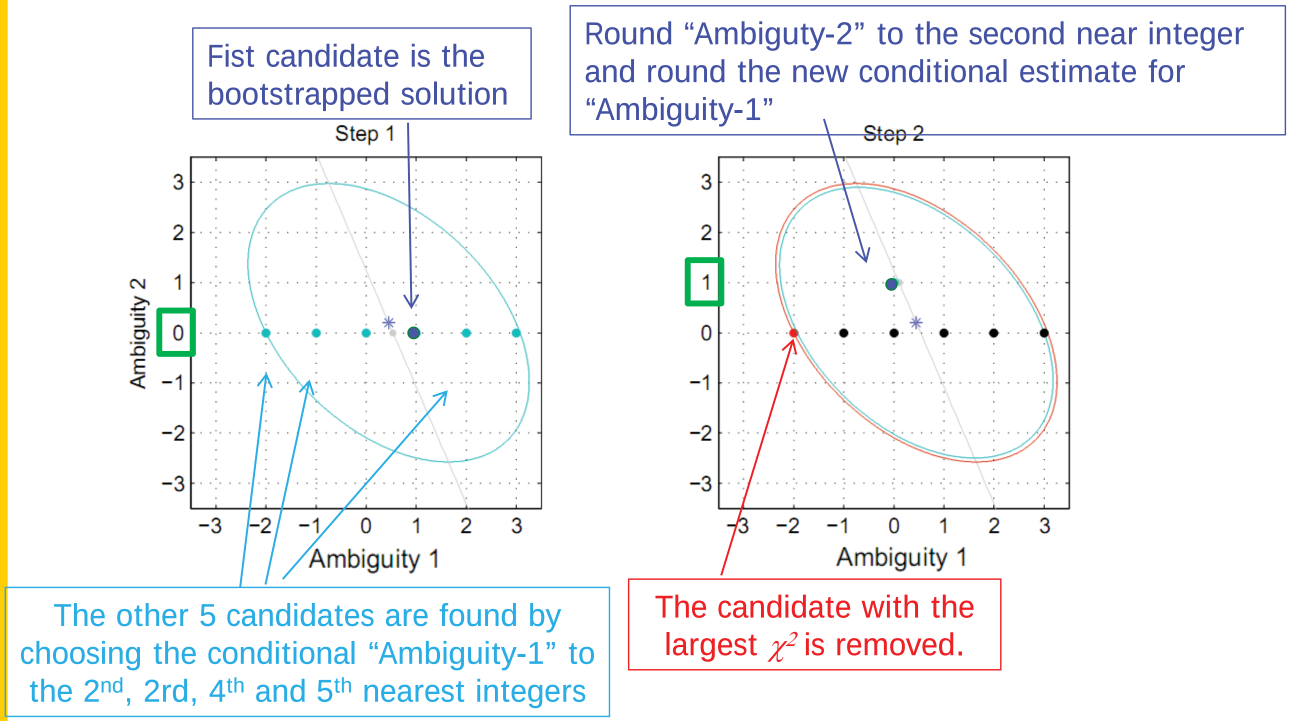
- If p candidates are requested, the values of cost function $c(\mathbf{N})$ are ordered in ascending order and χ^2 is chosen equal to the p-th value.

If more than $n+1$ candidates are requested, the volume of the search ellipsoid can be used ([RD-6]).

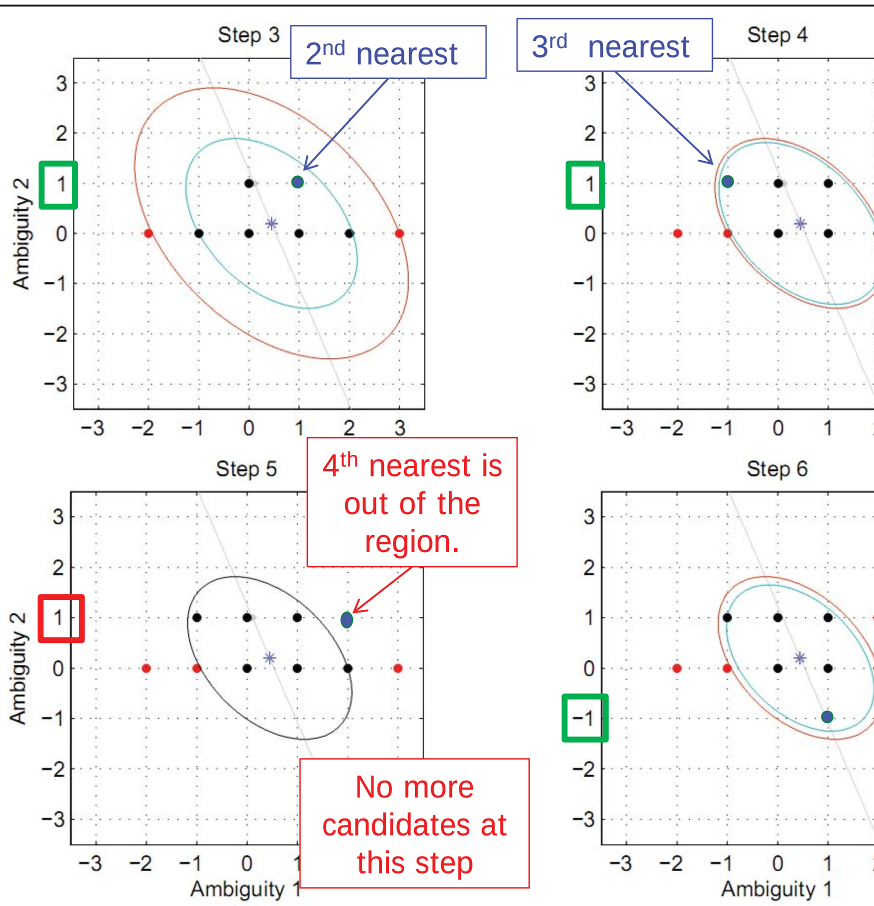
Search <u>with shrinking</u> technique: practical example

This is an alternative to the previous strategy, based on <u>shrinking</u> the search ellipsoid during the process of finding the candidates.

In the next example, we have to choose 6 candidates:



Example and pictures from [RD-6]



Example and pictures from [RD-6]

Thence, the best 6 candidates are found (in the ISL sense). The one with the smallest cost function $c(\mathbf{N})$ value is the actual ISL solution.

Acceptance Test

The integer ambiguity solution corresponding to the smallest RMS residuals is used to select the candidate.

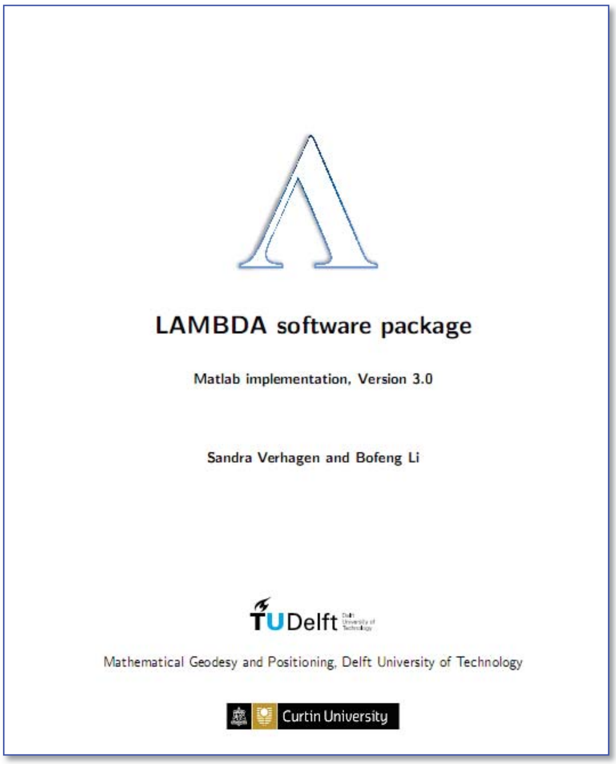
However if two or more candidates give roughly similar values of RMS, the test can not be resolutive.

➔A ratio test (of 2 or 3, depending on the algorithm) between the two smallest RMS is often used to validate the test.

If the ratio is under these values, no integer solution can be determined and is better to use the floated solution.

$$RMS = \left\| \mathbf{N} - \hat{\mathbf{N}} \right\|_{\mathbf{P}^{-1}_{\hat{\mathbf{N}}}} = \sqrt{\left(\mathbf{N} - \hat{\mathbf{N}}\right)^T \mathbf{P}^{-1}_{\hat{\mathbf{N}}} \left(\mathbf{N} - \hat{\mathbf{N}}\right)}$$

Examples with MATLAB (octave)


LAMBDA software package

Matlab implementation, Version 3.0

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Curtin University

Note:
This document uses the transposed matrix $\mathbf{Z}^{\mathbf{T}}$, but the principle is the same.

Examples with MATLAB (octave)

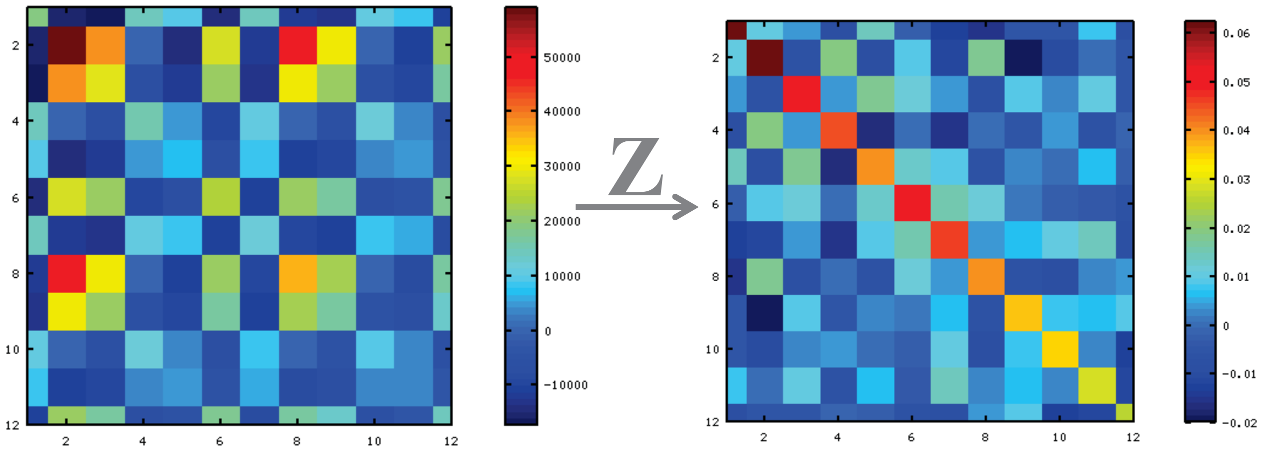
`load large` → Q, a

$$Q \equiv \mathbf{P}_{\hat{N}} = \mathbf{W}_{\hat{N}}^{-1}$$

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
```

```
imagesc(Q)
colorbar
```

```
imagesc(Qz)
colorbar
```




```
Qz=Lz'*diag(Dz)*Lz
```



```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
```

$\boxed{\texttt{Z=Zt'}}$

```
Qz=Lz'*diag(Dz)*Lz
```

```
[L,D] = ldldecom(Q)

Q=L'*diag(D)*L
```

```
az= Z*a
a= inv(Z)*az
```

```
Qz= Z*Q*Z'
Q= inv(Z)*Qz* inv(Z')
```

```
Z =
     3    0   -4   -3   -5   -4   -4    2   -2    1   -3    1
    -0   -1    1   -1   -2    4    4   -3    4    1    0    1
     3    5   -2   -2    1   -1   -2    1   -1   -4   -1   -1
    -5   -2    3    2    4    3   -3   -2   -2    1   -3   -1
     4    5    1    4    2    6    5    2   -4    1    2   -4
    -8   -4    1    0    0   -3    2    3    2   -1   -0    4
     4   -7   -0    1    0   -4   -1   -7    3   -5   -1    2
     2   -1   -8   -1    2   -4    1    2   -4    2    2   -2
    -3    2    3   10   -8   -2   -5    0   -4    1   -4    0
    -1    6    8   -1    2    1    2    7    3   -2    6    1
    -8    7   -8    3   -6   -1    1    0    0    3   -1   -1
     8    1    6   -3    5    4   -5   -3    0   -0    1   -3
```

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```
load large  → Q, a
```

Integer rounding

```
round(a)
```

[- 28491 65753 38830 5004 -29196 -298 -22201 51236 30258 3899 -22749 -159]

Decorrelation + Integer rounding

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a)
azfixed=round(az);
afixed=iZ*azfixed
```

[-28537 65473 38692 4939 -29228 -504 -22237 51018 30150 3849 -22774 -320]

Decorrelation + bootstrapping

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a)
azfixed=bootstrap(az,Lz);
afixed=iZ*azfixed
```

[-28451 65749 38814 5025 -29165 -278 -22170 51233 30245 3916 -22725 -144]

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Decorrelation + bootstrapping

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a)
azfixed=bootstrap(az,Lz);
afixed=iZ*azfixed
```

[-28451 65749 38814 5025 -29165 -278 -22170 51233 30245 3916 -22725 -144]

Decorrelation + ILS with enumeration search

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);

[azfixed,sqnorm] = lsearch (az,Lz,Dz,6);
afixed=iZ*azfixed
```

													c(N)
-28451	65749	38814	5025	-29165	-278	-22170	51233	30245	3916	-22725	-144	→	**15.0**
-28279	65862	38805	5170	-29061	-192	-22036	51321	30238	4029	-22644	-77	→	31.6
-28727	65935	39032	4844	-29337	-178	-22385	51378	30415	3775	-22859	-66	→	33.9
-28546	66062	39027	4998	-29228	-83	-22244	51477	30411	3895	-22774	8	→	34.5
-28229	65518	38583	5197	-29056	-500	-21997	51053	30065	4050	-22640	-317	→	34.7
-28365	65586	38683	5084	-29124	-418	-22103	51106	30143	3962	-22693	-253	→	35.5

Decorrelation + bootstrapping

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a)
azfixed=bootstrap(az,Lz);
afixed=iZ*azfixed
```

[-28451 65749 38814 5025 -29165 -278 -22170 51233 30245 3916 -22725 -144]

Decorrelation + ILS with search-and-shrink

```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);

[azfixed,sqnorm] = ssearch (az,Lz,Dz,6);
afixed=iZ*azfixed
```

												$c(N)$
-28451	65749	38814	5025	-29165	-278	-22170	51233	30245	3916	-22725	-144	→ **15.0**
-28279	65862	38805	5170	-29061	-192	-22036	51321	30238	4029	-22644	-77	→ 31.6
-28727	65935	39032	4844	-29337	-178	-22385	51378	30415	3775	-22859	-66	→ 33.9
-28546	66062	39027	4998	-29228	-83	-22244	51477	30411	3895	-22774	8	→ 34.5
-28229	65518	38583	5197	-29056	-500	-21997	51053	30065	4050	-22640	-317	→ 34.7
-28365	65586	38683	5084	-29124	-418	-22103	51106	30143	3962	-22693	-253	→ 35.5

References

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[RD-5] P. Joosten and Tiberius, C. LAMBDA: FAQs. GPS Solutions (2202), 6:109-114.

[RD-6] Sandra Verhagen and Bofeng L., LAMBDA software package. MATLAB implementation, Version 3.0. Mathematical Geodesy and Positioning, Delft University of Technology.

[RD-7] XW. Chang, CC. Paige, L. Yin, 2005, Code and carrier phase based short baseline GPS positioning: computational aspects. GPs Solutions, 2005 (2005) 9:72-83, DOI 10.1007/s10291-004-0112-8 Springer.

Least-Squares Ambiguity Search technique

This technique requires an approximate solution, which can be obtained from code range measurements. The search area can be defined by surrounding the approximate position by a 3σ region (i.e. $\Delta\hat{\mathbf{r}} \pm \delta$).

$$\mathbf{y} = \mathbf{G}\,\Delta\mathbf{r} + \lambda\mathbf{N} + \mathbf{v} \longrightarrow \hat{\mathbf{N}} = \frac{1}{\lambda}\left(\mathbf{y} - \mathbf{G}\,\Delta\hat{\mathbf{r}}\right)$$

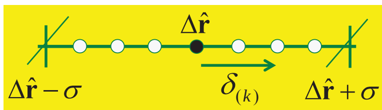
K-1 equations with *3+(K-1)* unknowns.

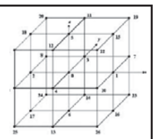
Then , given the 3-D position, the *(K-1)* integer ambiguities can be resolved automatically.

That is, the *(K-1)* integer ambiguities are constrained to three degrees of freedom.

Thence, *K-4* equations are redundant

The technique is based on exploiting the constrains on the integer ambiguities:





- The tracked satellites into two groups: 4 sat. (with good DOP) + N-4 sat.

- The primary group of 4 sat., is used determine the possible ambiguity sets $\mathbf{N}_{(k)}$ (given an initial position estimate and its associated uncertainty $\Delta\hat{\mathbf{r}} \pm \sigma$)

$$\Delta\hat{\mathbf{r}} \pm \sigma \quad \rightarrow \quad \mathbf{N}_{(k)} = \frac{1}{\lambda}\left(\boxed{\mathbf{y}} - \mathbf{G}\left(\Delta\hat{\mathbf{r}} + \delta_{(k)}\right)\right) \quad \rightarrow \quad \mathbf{N}_{(k)}$$

Then, the corresponding position estimates $\Delta\hat{\mathbf{r}}_{(k)}$ are computed.

$$\mathbf{N}_{(k)} \quad \rightarrow \quad \Delta\hat{\mathbf{r}}_{(k)} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}\left[\boxed{\mathbf{y}} - \lambda\mathbf{N}_{(k)}\right]$$

- The remaining *N-4* secondary satellites are used to eliminate candidates of the possible ambiguity sets:
  - ➔ Each position estimate is checked against measurements from the secondary group of *N-4* satellites.
  - ➔ With the correct position estimate, the difference between the measured and computed carrier phase should be "**close to an integer**" for each satellite pair.

$$\frac{1}{\lambda}\left(\boxed{\mathbf{y}} - \mathbf{G}\,\Delta\hat{\mathbf{r}}_{(k)}\right) - \mathbf{N}_{(k)} \in \mathbb{Z} \;?$$

Contents

List of Acronyms

AGW Atmospheric Gravity Waves

ANTEX ANTenna EXchange format

APC Antenna Phase Centre

ARP Antenna Reference Point

ASCII American Standard Code for Information Interchange

A/S Anti-Spoofing

C/A Coarse/Acquisition

CDDIS Crustal Dynamics Data Information System

CODE Centre for Orbit Determination in Europe

CRF Celestial Reference Frame

CRS Conventional Celestial Reference System

CS Cycle Slip

DAT Data Analysis Tool

DCB Differential Code Bias

DLL Dynamic Link Library

DLR Deutsches Zentrum für Luft- und Raumfahrt

DOP Dilution Of Precision

DoY Day of Year

DPC Data Processing Core

ECEF Earth-Centred, Earth-Fixed

ECI Earth-Centred Inertial

EMR Energy Mines and Resources

ENU East North Up

ESA European Space Agency

gAGE Research group of Astronomy and Geomatics

GAL	GALileo Satellite Identifier
GDOP	Geometric Dilution Of Precision
GEO	GEOstationary Satellite Identifier
GIPSY	GPS Inferred Positioning SYstem
gLAB	GNSS-LAB tool suite
GLO	Glonass Satellite Identifier
Glonass	GLObal NAvigation Satellite System
GNSS	Global Navigation Satellite System
GNU	GNU's Not Unix
GPS	Global Positioning System
GRACE	Gravity Recovery and Climate Experiment
GRAPHIC	Group and Phase Ionospheric Calibration
GFree	Geometry Free
GUI	Graphical User Interface
HDOP	Horizontal Dilution Of Precision
HTML	HyperText Markup Language
IAC	Information Analytical Centre
ICD	Interface Control Document
IFree	Ionosphere Free
IGL	IGS Glonass orbit products
IGRF	International Geomagnetic Reference Field
IGS	International GNSS Service
IGST	IGS Time
IODE	Issue Of Data Ephemerides
IODN	Issue Of Data Navigation
IONEX	IONosphere map EXchange format
IRI	International Reference Ionosphere
ITRF	International Terrestrial Reference Frame
JPL	Jet Propulsion Laboratory
LEO	Low Earth Orbit
LS	Least Squares

LSTIDs Large-Scale TIDs

MC Satellite Mass Centre

MCC Mission Control Centre

MJD Modified Julian Day

MM Monument Marker

MSDOS Microsoft Disk Operating System

MSTID Medium-Scale Travelling Ionospheric Disturbance

MSTIDs Medium-Scale Travelling Ionospheric Disturbances

MW Melbourne–Wübbena

NANU Notice Advisory to NAVSTAR Users

NASA National Aeronautics and Space Administration

NEU North East Up

NGA National Geospatial-Intelligence Agency

NSE Navigation System Error

OS Operating System

PCO Phase Centre Offset

PCV Phase Centre Variation

PDOP Precision Dilution Of Precision

PNG Portable Network Graphics

PPP Precise Point Positioning

PRN Pseudo-Random Noise

PVT Position, Velocity, Time

PZ-90 Parametry Zemli 1990 (Parameters of Earth 1990)

RINEX Receiver INdependent EXchange format

RK4 Fourth-order Runge–Kutta

RMS Root Mean Square

RO Radio Occultation

S/A Selective Availability

SBAS Satellite-Based Augmentation System

SED Storm Enhancement Density

SINEX Solution (Software/technique) INdependent EXchange format

SIS	Signal In Space
SISRE	Signal In Space Range Error
SOHO	SOlar Helioscopic Observatory
SP3	Standard Product #3
SPP	Standard Point Positioning
SPS	Standard Positioning Service
SSI	Signal Strength Indicator
STEC	Slant Total Electron Content
STROP	Slant TROPospheric delay
SU	Soviet Union
SV	Space Vehicle
SVN	Space Vehicle Number
TDOP	Time Dilution Of Precision
TEC	Total Electron Content
TECU	Total Electron Content Unit
TGD	Total Group Delay
TID	Travelling Ionospheric Disturbance
TRF	Terrestrial Reference Frame
TRS	Conventional Terrestrial Reference System
TUM	Technische Universität München
UPC	Technical University of Catalonia
USNO	United States Naval Observatory
UTC	Coordinated Universal Time
VDOP	Vertical Dilution Of Precision
WGS-84	World Geodetic System 84
ZTD	Zenith Tropospheric Delay