

4.1 Definitions, Diagrams, and Inverses

4.1.1 Function Terminology

Function: We use the notation $f : A \rightarrow B$ to specify a function f , which has inputs from the set A , and outputs from the set B . The function associates each input in A to one and only one output in B .^a The notation $f : A \rightarrow B$ can be read as “ f is a function from A to B ”.

Domain: The Domain is the set of inputs (A).

Codomain: The Codomain is the set of outputs (B).

^aDiscrete Mathematics, Ensley and Crawley

Question 1

Given the function:

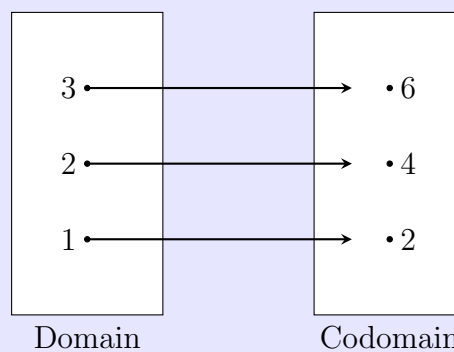
$$g : \mathbb{Z} \rightarrow \mathbb{N} \quad \dots \text{with the rule} \dots \quad g(x) = x^2$$

- What is the function name?
- What is the domain?
- What is the codomain?
- Is 2 a valid domain value?
- Is -2 a valid domain value?
- Is 4 a valid codomain value?
- Is -4 a valid codomain value?

To describe a function, you need four items: ^a (1) Give the function a name, such as f , g , and h , (2) Describe the **domain**, (3) Describe the **codomain**, (4) Describe the **rule**.

Example: Function f , with Domain: $\{1, 2, 3\}$ and Codomain: $\{2, 4, 6\}$ and the Rule: $f(x) = 2x$.

Function diagram: With a diagram, arrows start at an element in the domain, and point to an element in the codomain.



^aDiscrete Mathematics, Ensley and Crawley

Question 2

- Define a function where the inputs and outputs are integers, and the relationship is that the output is the *square* of the input provided to the function.
- Draw a diagram of the function. Include 5 values in the domain and in the co-domain.

Question 4

Finish the arrow diagram for the following Binary Relation.

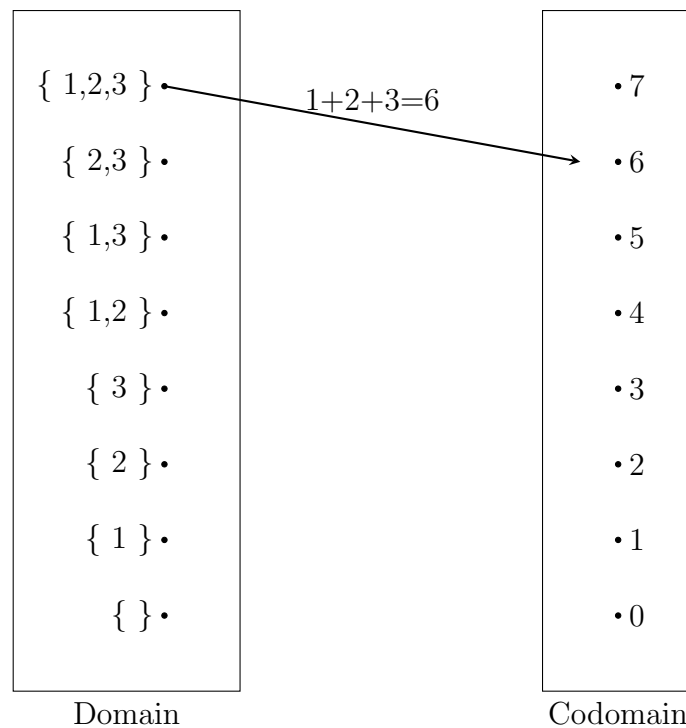
Domain: $\wp(\{1, 2, 3\})$, the Power Set of $\{1, 2, 3\}$.

Codomain: The set $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

Rule: $(S, n) \in \mathbb{R}$

This means that n is the **sum** of elements in the the set S given as an input.

For example, with the input set $\{1, 2\}$, the output will be $1 + 2$, or 3 .



A function $f : A \rightarrow B$ is a binary relation with domain A and codomain B with the property that for every $x \in A$, there is **exactly one** element $y \in B$ for which $(x, y) \in f$.^a

Bluntly, the pair (x, y) denotes that a line begins at element x from the domain, and points to the element y in the codomain.

^aDiscrete Mathematics, Ensley and Crawley

Question 5

Identify which of the following relations are also functions. Explain why not, if the relation is not a function. Also complete the diagrams given.

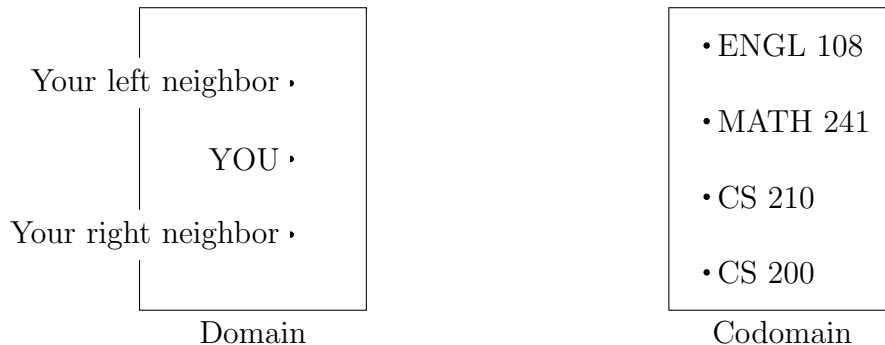
a. Relation R_1

Domain: The set \mathbb{S} of all students at your college this semester.

Codomain: The set \mathbb{C} of all classes offered at your college this semester.

Rule: (x, y) is in R_1 if student x is enrolled in class y this semester.

Let's use a small sample set. Fill it out to help you figure out if this is a function.

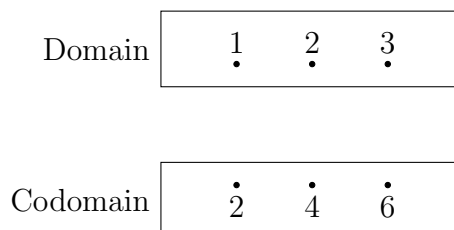


b. Relation R_2

Domain: The set $A = \{1, 2, 3\}$.

Codomain: The set $B = \{2, 4, 6\}$.

Rule: (x, y) is in R_2 if $2x = y$.



Question 6

Identify which of the following relations are also functions. Explain why not, if the relation is not a function. Also complete the diagrams given.

a. Relation R_3

Domain: The set $A = \{1, 2, 3\}$.

Codomain: The set $B = \{2, 4, 6\}$.

Rule: $\{ (1,6), (2,2), (3,4) \}$

Let's use a small sample set. Fill it out to help you figure out if this is a function.

Domain

$\overset{\cdot}{1}$	$\overset{\cdot}{2}$	$\overset{\cdot}{3}$
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Codomain

$\overset{\cdot}{2}$	$\overset{\cdot}{4}$	$\overset{\cdot}{6}$
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b. Relation R_4

Domain: The set $A = \{1, 2, 3, 4, 5, 6\}$.

Codomain: The same set A .

Rule: (x, y) is in R_3 if $x - 1 = y$.

Domain

$\overset{\cdot}{1}$	$\overset{\cdot}{2}$	$\overset{\cdot}{3}$	$\overset{\cdot}{4}$	$\overset{\cdot}{5}$	$\overset{\cdot}{6}$
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Codomain

$\overset{\cdot}{1}$	$\overset{\cdot}{2}$	$\overset{\cdot}{3}$	$\overset{\cdot}{4}$	$\overset{\cdot}{5}$	$\overset{\cdot}{6}$
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4.1.3 Inverse Relations

Given a relation R with domain A and codomain B , the relation R^{-1} (read “ R inverse”) with domain B and codomain A is called the **inverse** of R , and is defined so that

$$(x, y) \in R \quad \text{if and only if} \quad (y, x) \in R^{-1}$$

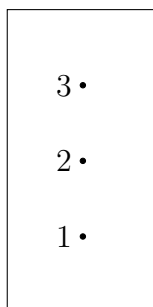
Also note that the inverse of R^{-1} is R .^a

^aDiscrete Mathematics, Ensley and Crawley

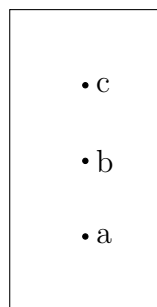
Question 7

Draw the inverse of each diagram. Identify if the original, and/or the inverse, are functions.

a.

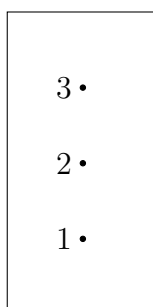


Domain

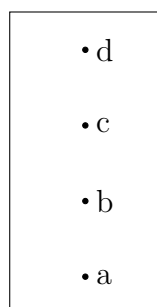


Codomain

b.



Domain



Codomain