

3.2 More Operations on Sets

3.2.1 Cartesian Products

Question 1

Given the following sets, calculate each Cartesian Product. Write it out in a table and as a set, like above.

$$A = \{1, 2\} \quad B = \{3, 4\}$$

a. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

	$B_1 = 3$	$B_2 = 4$
$A_1 = 1$	$(1, 3)$	$(1, 4)$
$A_2 = 2$	$(2, 3)$	$(2, 4)$

b. $B \times A = \{(3, 1), (3, 2), (4, 1), (4, 2)\}$

	$A_1 = 1$	$A_2 = 2$
$B_1 = 3$	$(3, 1)$	$(3, 2)$
$B_2 = 4$	$(4, 1)$	$(4, 2)$

Question 2

Given the following sets, calculate each Cartesian Product. Write it out in a table and as a set.

$$A = \{x, y, z\} \quad B = \{1, 3\}$$

a. $A \times B = \{(x, 1), (x, 3), (y, 1), (y, 3), (z, 1), (z, 3)\}$

	$B_1 = 1$	$B_2 = 3$
$A_1 = x$	$(x, 1)$	$(x, 3)$
$A_2 = y$	$(y, 1)$	$(y, 3)$
$A_3 = z$	$(z, 1)$	$(z, 3)$

b. $B \times A = \{(1, x), (1, y), (1, z), (3, x), (3, y), (3, z)\}$

	$A_1 = x$	$A_2 = y$	$A_3 = z$
$B_1 = 1$	$(1, x)$	$(1, y)$	$(1, z)$
$B_2 = 3$	$(3, x)$	$(3, y)$	$(3, z)$

Question 3

Calculate the Cartesian Products and write out the result as a set of coordinate pairs.

$$A = \{2, 4\} \quad B = \{1, 3\} \quad C = \{3, 4, 5, 6\}$$

- a. $A \times B$ $\{2, 4\} \times \{1, 3\} = \{(2, 1), (2, 3), (4, 1), (4, 3)\}$
 - b. $A \times C$ $\{2, 4\} \times \{3, 4, 5, 6\} = \{(2, 3), (2, 4), (2, 5), (2, 6), (4, 3), (4, 4), (4, 5), (4, 6)\}$
 - c. $B \times C$ $\{1, 3\} \times \{3, 4, 5, 6\} = \{(1, 3), (1, 4), (1, 5), (1, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$
 - d. A^2 (Hint: $A \times A$) $\{2, 4\} \times \{2, 4\} = \{(2, 2), (2, 4), (4, 2), (4, 4)\}$
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Question 4

With the given sets, find the intersections, unions, and differences.

$$A = \{1\} \quad B = \{3, 5, 7\} \quad C = \{3, 5, 9, 11\}$$

$$A \times B = \{(1, 3), (1, 5), (1, 7)\} \quad A \times C = \{(1, 3), (1, 5), (1, 9), (1, 11)\}$$

- a. $(A \times B) - (A \times C)$ $\{(1, 7)\}$
 - b. $(A \times C) - (A \times B)$ $\{(1, 9), (1, 11)\}$
 - c. $A \times (B \cup C)$
 $B \cup C = \{3, 5, 7, 9, 11\}$
 $A \times (B \cup C) = \{(1, 3), (1, 5), (1, 7), (1, 9), (1, 11)\}$
 - d. $(A \times (B \cup C)) \cap (A \times B)$ $\{(1, 3), (1, 5), (1, 7)\}$
 - e. $(A \times B) \cup (A \times C)$ $\{(1, 3), (1, 5), (1, 7), (1, 9), (1, 11)\}$
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3.2.2 Partitions

Question 5

For the given set, write out all possible partitions of $A = \{1, 2\}$. There should be 2. Note that the *order* of the elements of the set does not matter.

$\{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$

Question 6

For the given set, write out all possible partitions. There should be 5.

$$B = \{1, 2, 3\}$$

1. $\{\{1\}, \{2\}, \{3\}\}$
 2. $\{\{1, 2\}, \{3\}\}$
 3. $\{\{1\}, \{2, 3\}\}$
 4. $\{\{1, 3\}, \{2\}\}$
 5. $\{\{1, 2, 3\}\}$
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Question 7

Which of the following are valid partitions of the set $A = \{1, 2, 4, 8, 16, 32, 64, 128\}$? For those that are not, explain why not.

- a. $\{1, 2, \{4, 8, 16\}, \{32, 64, 128\}\}$ Not valid - 1 and 2 aren't sets
 - b. $\{\{1, 16\}, \{32, 64, 2\}, \{8, 4, 16\}, \{128\}\}$ Not valid - 16 shows up twice.
 - c. $\{\{1, 128\}, \{8, 4, 16\}, \{64, 2\}\}$ Not valid - 32 is missing.
 - d. $\{\{8, 4, 2\}, \{16, 1, 128\}, \{32, 64\}\}$ Valid partition
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Question 8

For the set $A = \{1, 2, 3, 4, 5, 6\}$, build partitions that meet the following criteria:

- a. Find a partition where each part has the same size. **Multiple solutions**
 - b. Find a partition where no two parts have the same size. **Multiple solutions**
 - c. Find a partition that has as many parts as possible. **Multiple solutions**
 - d. Find the partition that has as few parts as possible. **Multiple solutions**
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3.2.3 Power Sets**Question 9**

Find the Power Set for each.

- a. $\wp(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}$
- b. $\wp(\{3, 4\}) = \{\emptyset, \{3\}, \{4\}, \{3, 4\}$
- c. $\wp(\{1, 2, 3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$