- 1. a. p: You play, w: You win,  $\neg p \rightarrow \neg w$ 
  - b. d: Your friends dance, f: Your friends are my friends,  $\neg d \rightarrow \neg f$
  - c. e: Timmy's age is over 8, t: Timmy's age is less than 13, m: Timmy gets Tween-priced movie tickets.  $(e \wedge t) \to m$
  - d. s: I get enough sleep, c: I drink coffee, w: I go to work.  $(s \lor c) \to w$

|          | p              | q            | r            | a. $(p \land q) \to r$ | b. $p \to (q \lor r)$ |
|----------|----------------|--------------|--------------|------------------------|-----------------------|
|          | $\overline{T}$ | Τ            | Τ            | T                      | T                     |
|          | $\mathbf{T}$   | $\mathbf{T}$ | $\mathbf{F}$ | F                      | ${ m T}$              |
|          | $\mathbf{T}$   | F            | $\mathbf{T}$ | ${ m T}$               | ${ m T}$              |
| 2. a. b. | $\mathbf{T}$   | F            | F            | ${ m T}$               | F                     |
|          | F              | Τ            | Τ            | ${ m T}$               | F                     |
|          | F              | Τ            | F            | ${ m T}$               | F                     |
|          | F              | F            | T            | m T                    | F                     |
|          | F              | F            | F            | ${ m T}$               | F                     |

$$\begin{array}{c|ccccc} p & q & p \rightarrow q & \neg(p \rightarrow q) \\ \hline T & T & T & F \\ c. & T & F & F & T \\ F & T & T & F \\ F & F & T & F \end{array}$$

- 3. a.  $P(x) \to Q(X)$ 
  - b.  $P(x) \to \neg Q(x)$
  - c.  $\neg P(x) \to Q(x)$
  - d.  $\neg P(x) \rightarrow Q(x)$
- 4. a.  $i \wedge \neg j$ : You go to New Delhi and you do NOT see the Jantar Mantar.
  - b.  $(j \lor c) \land \neg b$ : Jessica gets chocolate OR Jessica gets cake AND Jessica DOES NOT have a happy birthday.
  - c.  $p \land \neg (w \land s) \equiv p \land \neg w \lor \neg s$ : You have a group project AND neither you do all the work, nor someone else does all the work.
- 5. a.  $\forall x \in D, \neg B(x) \to H(x)$

- Converse:  $\forall x \in D, H(x) \to \neg B(x)$ For all people x, if x is hungry, then x didn't eat breakfast
- Inverse:  $\forall x \in D, B(x) \to \neg H(x)$ For all people x, if x at breakfast then x is NOT hungry
- Contrapositive:  $\forall x \in D, \neg H(x) \to B(x)$ For all people x, if x is NOT hungry, then x at breakfast

## b. $\forall x \in D, (P(x) \vee G(x)) \to M(x)$

- Converse:  $\forall x \in D, M(x) \to (P(x) \vee G(x))$ For all people x, if x is a musician, then x plays piano or x plays guitar
- Inverse:  $\forall x \in D, \neg(P(x) \vee G(x)) \rightarrow \neg M(x)$ For all people x, if x does NOT play piano and does NOT play guitar, then x is NOT a musician
- Contrapositive:  $\forall x \in D, \neg M(x) \rightarrow \neg (P(x) \lor G(x))$ For all people x, if x is NOT a musician, then x doesn't play piano and x doesn't play guitar.

## c. $\forall x \in \mathbb{R} : P(x) \to Q(x)$

- Converse:  $\forall x \in D, Q(x) \to P(x)$ For all real numbers x, if  $x^2 > 4$  then x > 2.
- Inverse:  $\forall x \in D, \neg P(x) \rightarrow \neg Q(x)$ For all real numbers x, if  $x \leq 2$  then  $x^2 \leq 4$
- Contrapositive:  $\forall x \in D, \neg Q(x) \rightarrow \neg P(x)$ For all real numbers x, if  $x^2 < 4$  then x < 2

## d. $\forall x \in \mathbb{Z} : R(x) \to S(x)$

- Converse:  $\forall x \in D, S(x) \to R(x)$ For all integers x, if  $x^3 = 1$  then  $x^2 = 1$
- Inverse:  $\forall x \in D, \neg R(x) \rightarrow \neg S(x)$ For all integers x, if  $x^2 \neq 1$  then  $x^3 \neq 1$
- Contrapositive:  $\forall x \in D, \neg S(x) \to \neg R(x)$ For all integers x, if  $x^3 \neq 1$  then  $x^2 \neq 1$