

**2.2 Exercise:** In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

## 1. More definitions

### Modulus

“In computing, the modulo operation finds the remainder after division of one number by another (sometimes called modulus).

Given two positive numbers,  $a$  (the dividend) and  $n$  (the divisor),  $a \bmod n$  (abbreviated as  $a \bmod n$ ) is the remainder of the Euclidean division of  $a$  by  $n$ .<sup>a</sup>

$$\begin{array}{r} 4r1 \\ 2 \overline{)9} \\ \underline{-8} \\ 1 \end{array}$$

$$9 \bmod 2 = 1$$

If we’re dividing  $a$  by  $b$ , the result is a quotient  $q$ . If we’re calculating  $a \bmod b$ , the result is the remainder  $r$ .

We can also write this out as:

$a = b \cdot q + r$ , where  $0 \leq r < b$ , and  $q$  and  $r$  are the only two integers that will satisfy the equation.

<sup>a</sup>From [https://en.wikipedia.org/wiki/Modulo\\_operation](https://en.wikipedia.org/wiki/Modulo_operation)

### Rational numbers

In mathematics, a rational number is any number that can be expressed as the quotient or fraction  $p/q$  of two integers, a numerator  $p$  and a non-zero denominator  $q$ .<sup>[1]</sup> Since  $q$  may be equal to 1, every integer is a rational number.<sup>a</sup>

The set of rational numbers is written as  $\mathbb{Q}$ .

<sup>a</sup>From [https://en.wikipedia.org/wiki/Rational\\_number](https://en.wikipedia.org/wiki/Rational_number)

### Question 1

Solve the following modulus problems.

**Example:** Solve  $13 \bmod 5$

$$13 / 5 = 2, \quad 13 \bmod 5 = 3, \quad 13 = 5 \cdot 2 + 3$$

- a.  $9 \bmod 7$
- b.  $5 \bmod 2$
- c.  $15 \bmod 3$
- d.  $-7 \bmod 2$

### Question 2

Prove the following propositions:

- a. If  $a$  divides  $b$  and  $a$  divides  $c$ , then  $a$  divides  $b + c$ .<sup>1</sup>

Start with:  $b = ak$  and  $c = aj$  and calculate  $b + c$ .

- b. If  $a$  divides  $b$  and  $c$  divides  $d$ , then  $ac$  divides  $bd$ .<sup>2</sup>

Start with:  $b = ak, d = cj$  and calculate  $bd$ .

<sup>1</sup>From Discrete Mathematics by Ensley and Crawley

<sup>2</sup>From Discrete Mathematics by Ensley and Crawley