Name: ____________________

**Chapter 2 EXAM:**

This exam covers concepts from Chapter 2. Questions will be labelled by the section from which they are based.

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Nothing written | Attempted, but incorrect | Partially correct; multiple errors | Mostly correct, one or two errors | Perfect; correct answer & notation |

**Grading breakdown:**

| Question | Weight | Points Received | Weighted Score |
|---|---|---|---|
| 1 | 2% | | |
| 2 | 2% | | |
| 3 | 2% | | |
| 4 | 12% | | |
| 5 | 14% | | |
| 6 | 10% | | |
| 7 | 5% | | |
| 8 | 14% | | |
| 9 | 12% | | |
| 10 | 6% | | |
| 11 | 9% | | |
| 12 | 12% | | |
| 13 | +4% | | |
| | | | |

Scratch page

# Exam

## 2.1 Definitions

Give the definitions for the following:

**2%** Question 1: Even number (2.1) □ 0 □ 1 □ 2 □ 3 □ 4

An integer $n$ is even if it can be written in the form...:

**2%** Question 2: Odd number (2.1) □ 0 □ 1 □ 2 □ 3 □ 4

An integer $n$ is odd if it can be written in the form...:

**2%** Question 3: Divisible by 4 (2.1) □ 0 □ 1 □ 2 □ 3 □ 4

An integer $n$ is divisible by 4 if it can be written in the form...:

# 2.1 Direct proofs

For the following problems, prove the following statement using a **direct proof.** Make sure to write the final answer in terms of the appropriate definition (i.e., if a number is even, it should be written as 2-times-some-number.)

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**12%** Question 4: Direct proof (2.1) □ 0 □ 1 □ 2 □ 3 □ 4

If $n$ is even, then $n + 8$ is even.

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**14%** Question 5: Direct proof (2.1) □ 0 □ 1 □ 2 □ 3 □ 4

If $n$ is even, then the product of $n$ and its successor is even. [1]

[1] The successor of some integer $n$ is $n + 1$.

# 2.2 Division Theorem

For the following problems, apply the division theorem to fill in the blanks. There is only one solution for each problem.

**The Division Theorem**
For all integers $a$ and $b$ (with $b > 0$), there is an integer $q$ and an integer $r$, such that: (1) $a = b \cdot q + r$, and (2) $0 \leq r < b$.

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**10%** Question 6: Division Theorem (2.2) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Solve by filling in the blanks, which represent $q$ and $r$.

a. $10 =$__________$\cdot 3+$__________

b. $13 =$__________$\cdot 6+$__________

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**5%** Question 7: Division Theorem (2.2) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Given the statement $(9k^2 + 5) =$_____$\cdot 3+$_____, where $a = (9k^2 + 5)$, $b = 3$, and $q$ and $r$ are unknown. which of the following is correct, and fits with the Division Theorem?

☐ $(9k^2 + 5) = (3k^2) \cdot 3 + 5,$ with $q = k^2$ and $r = 5$

☐ $(9k^2 + 5) = (3k^2 + 1) \cdot 3 + 2,$ with $q = (3k^2 + 1)$ and $r = 2$

# 2.3 Proof by Induction

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**14%** Question 8: Proof by Induction - Recursive (2.3) □ 0 □ 1 □ 2 □ 3 □ 4

Show that the sequence defined by the recursive formula ...

$$a_n = a_{n-1} + 2, \text{ with } a_1 = 1$$

... is equivalently described by the closed formula ...

$$a_n = 2n - 1$$

**Clearly label $a_m$ and $a_{m-1}$ in each step for full credit.** [2]

[2]Steps:
(1) Show that both formulas are equivalent for $a_1$.
(2) Write the recursive formula in terms of $m$.
(3) Find the equation for $a_{m-1}$ using the closed formula.
(4) Plug in the equation for $a_{m-1}$ back into the recursive formula and simplify.

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**12%** Question 9: Proof by Induction - Sums (2.3) □ 0 □ 1 □ 2 □ 3 □ 4

Use induction to prove that for each $n \geq 1$, [3]

$$\sum_{i=1}^{n}(2i-1) = n^2$$

.

[3] Steps:
(1) Show that they match for $n = 1$, $n = 2$, and $n = 3$.
(2) Rewrite the summation from $i = 1$ to $n$ as equal to the sum from $i = 0$ to $m - 1$, plus the final term.
(3) Find an equation for $\sum_{i=1}^{m-1}$ via the original proposition.
(4) Plug $\sum_{i=1}^{m-1}$ into the equation from step (2) and simplify.

# 2.6 Numerical Representation

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**6%** Question 10: Numerical representation (2.6) □ 0 □ 1 □ 2 □ 3 □ 4

Write the following numbers as the sum of multiples of powers of the base. For example, $(135)_{10} = 1 \cdot 10^2 + 3 \cdot 10^1 + 5 \cdot 10^0$. **Do not simplify!**

a. $(\texttt{1001 0010})_2$ =

b. $(\texttt{E37F00D})_{16}$ =

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**9%** Question 11: Decimal → Binary (2.6) □ 0 □ 1 □ 2 □ 3 □ 4

Convert $(70)_{10}$ to binary. Use the algorithm if needed. [4]

[4]Algorithm:

1. Input a natural number $n$.
2. While $n > 0$, do the following:
   (a) Divide $n$ by 2 and get a quotient $q$ and a remainder $r$.
   (b) Write $r$ as the next (right-to-left) digit.
   (c) Replace the value of $n$ with $q$, and repeat.

**12%** Question 12: Binary ↔ Hex (2.6) □ 0 □ 1 □ 2 □ 3 □ 4

Convert between Binary and Hexadecimal using the following table:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **DECIMAL** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **HEX** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **BINARY** | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 |
| | | | | | | | | |
| **DECIMAL** | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| **HEX** | 8 | 9 | A | B | C | D | E | F |
| **BINARY** | 1000 | 1001 | 1010 | 1011 | 1100 | 1101 | 1110 | 1111 |

a. Convert $(\texttt{1011 0011 1010 0111 0101})_2$ to base-16

b. Convert $(\texttt{D065})_{16}$ to base-2

**+4%** Question 13: Extra credit (1.5) □ 0 □ 1 □ 2 □ 3 □ 4

For the following statement, write out the **converse**, **inverse**, and **contrapositive** in English. $p$ : The animal has pointy ears, $q$ : the animal is a cat. $p \to q$: If the animal has pointy ears, then the animal is a cat.

a. Negation $p \wedge \neg q$:

b. Converse $q \to p$:

c. Inverse $\neg p \to \neg q$:

d. Contrapositive $\neg q \to \neg p$: