2.6 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

2.6 Numerical Representation

2.6.1 Intro practice

Question 1

The set of digits in base-10 (decimal) number system is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Use this information to help you figure out the following.

a. Write out the set of digits in the octal (base-8) number system.

b. Write out the set of digits in the binary (base-2) number system.

he hexadecimal (base-16) number system is

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}.$$

Why do we use letters? To keep numbers 10 through 15 as one-character representations.

Specifying base
When we need to specify the base when writing out numbers, write it within parentheses, with a subscript of its base number.

- $(123)_{10}$ = 123, base-10
- $(1337)_8$ = 1337, base-8
- $(C47)_{16}$ = C47, base-16

- $(1011)_2 = 1011$, base-2

2.6.2 Digits

or the decimal number $2,368$, we can write this as its individual digits:

Thousands (10^3)	**Hundreds** (10^2)	**Tens** (10^1)	**Ones** (10^0)
2	3	6	8

And then we can build out $2,368$ as the mathematical equation:

$$2 \cdot 10^3 + 3 \cdot 10^2 + 6 \cdot 10^1 + 8 \cdot 10^0$$

Likewise, for the binary number 0101 1001, we can write it as:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	1	0	0	1

And into the equation:

$$1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^0$$

Question 2

Expand each of the following numbers as a mathematical equation. Make sure to pay attention to the *base* value.

a. Write out the equation for $(19)_{10}$

10^1	10^0

b. Write out the equation for $(0010\ 1101)_2$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

c. Write out the equation for $(FFAA66)_{16}$

16^5	16^4	16^3	16^2	16^1	16^0

2.6.3 Converting between bases

Algorithm for converting a decimal number to base b:

1. Input a natural number n
2. While $n > 0$, do the following:
   (a) Divide n by b and get a quotient q and remainder r.
   (b) Write r as the next (right-to-left) digit.
   (c) Replace the value of n with q, and repeat.

Question 3
Convert the following between bases:

a. Convert $(35)_{10}$ to binary (base-2) $n = 35$, $b = 2$

b. Convert $(125)_{10}$ to binary (base-2) $n = 125$, $b = 2$

Hexadecimal to Binary

Often in computers, we write binary strings as hexadecimal to save space and make it easier to read.

Hex	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hex	8	9	A	B	C	D	E	F
Binary	1000	1001	1010	1011	1100	1101	1110	1111

Example: Convert 11001 from binary to hexadecimal

1. Write out in chunks of four. Add leading 0's to the left side.
   0001 1001
2. Swap out each "nibble" with hexadecimal
   0001 = 1 1001 = 9

So, $(0001\ 1001)_2 = (19)_{16}$

Example: Convert DAD from hexadecimal to binary

1. Convert each digit back to binary.
   D = 1101 A = 1010 D = 1101

So, $(\text{DAD})_{16} = (1101\ 1010\ 1101)_2$

Question 4

Do the following conversions

a. Convert $(1F0B)_{16}$ to binary:

b. Convert $(0100\ 0110)_2$ to hexadecimal: