

- You will not turn in this assignment.
- Solutions are given at the end of class.
- You can keep this exercise to help you study for future exams.
- Make sure your attendance is counted in order to get credit for your participation.

Section 1: Functions that are one-to-one, onto, and invertible

Functions that are onto

A function f is *onto* if everything in the codomain really is an output of f . That is, for every element y in the codomain, there must be (at least one) x in the domain where $f(x) = y$.

Functions that are one-to-one

A function f is *one-to-one* if nothing in the codomain is an output via two different inputs. That is, for every choice of different elements x_1 and x_2 in the domain, $f(x_1)$ and $f(x_2)$ must be different.

Functions that have a one-to-one correspondence (invertible)

A function f is a *one-to-one correspondence* if it is both *one-to-one* and *onto*. This is equivalent to saying that f is *invertible*.

Another way of stating this is:

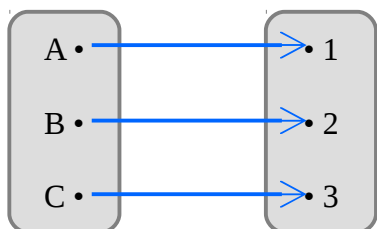
The function $f: A \rightarrow B$ is *invertible* if there is a function $f^{-1}: B \rightarrow A$ such that $f(x) = y$ if and only if $f^{-1}(y) = x$. The notation f^{-1} is read as “ f inverse” and the symmetry of the definition means that $(f^{-1})^{-1} = f$.

In diagramming terms:

- A function is *onto* if every point in the codomain has an arrow ending at that point.
- A function is *one-to-one* if no point in the codomain has two or more arrows ending at a point.
- A function is a *one-to-one correspondence (invertible)* if every point in the codomain has *exactly one arrow* ending at that point.

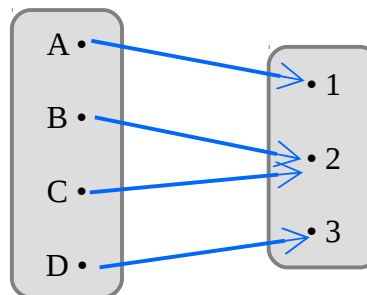
1. Determine whether these functions are one-to-one, onto, and/or invertible.
If not, state why not.

(___/1) a.



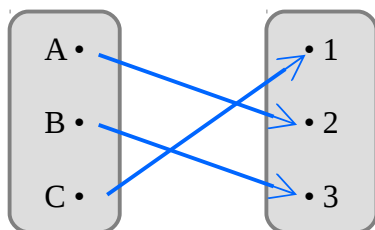
Onto?
One-to-one?
Invertible?

(___/1) b.



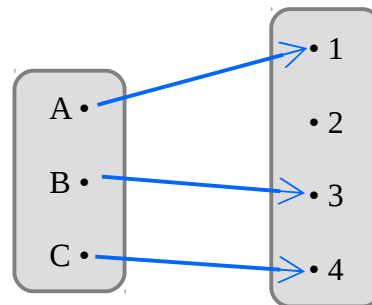
Onto?
One-to-one?
Invertible?

(___/1) c.



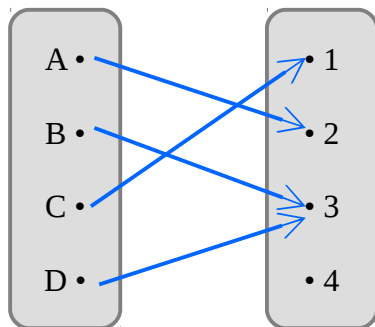
Onto?
One-to-one?
Invertible?

(___/1) d.



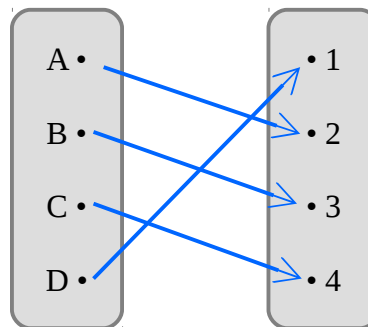
Onto?
One-to-one?
Invertible?

(___/1) e.



Onto?
One-to-one?
Invertible?

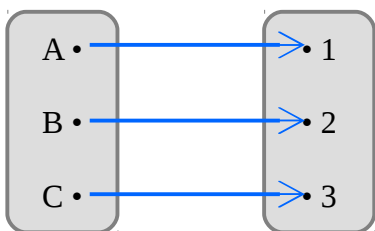
(___/1) f.



Onto?
One-to-one?
Invertible?

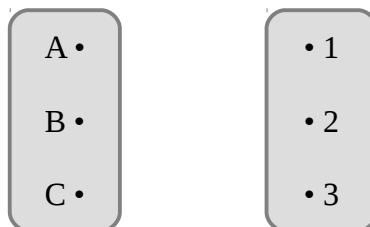
2. Draw the inversions of the following functions:

(___/1) a. $f: A \rightarrow B$

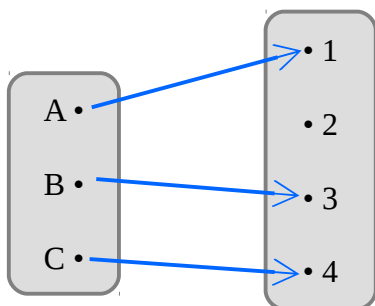


Draw $f^{-1}: B \rightarrow A$

Is this a valid function?

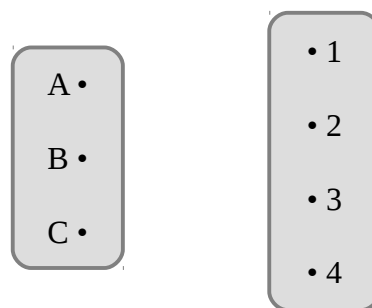


(___/1) b. $g: A \rightarrow B$

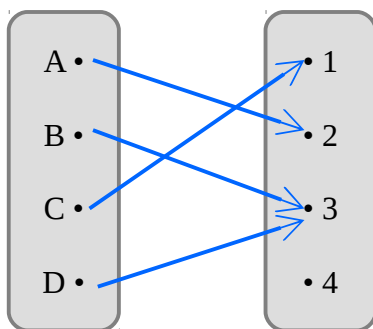


Draw $g^{-1}: B \rightarrow A$

Is this a valid function?

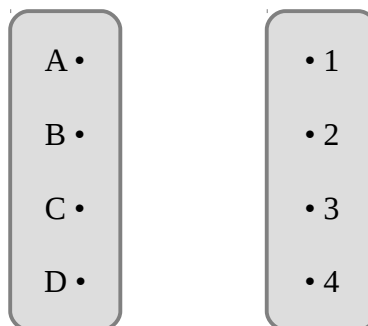


(___/1) c. $h: A \rightarrow B$



Draw $h^{-1}: B \rightarrow A$

Is this a valid function?



3. This function is not onto. Give an example of an element in the codomain and explain why no element in the domain is associated with it. (___/1)

$$f: \mathbb{R} \rightarrow \mathbb{R}, \text{ with } f(x) = x^2 + 4x + 1$$

4. This function is not one-to-one. To demonstrate this, provide an example of two elements of the domain that are associated with the same element of the codomain. (___/1)

$$f: \mathbb{R} \rightarrow \mathbb{R}, \text{ with } f(x) = x^2 + 4x + 1$$

5. Which of the following functions are invertible? For each noninvertible function, explain why it is not one-to-one or not onto. It might help to list out several mappings from domain \rightarrow codomain to see the results.

a. $c: \mathbb{Z} \rightarrow \mathbb{Z}$, with $c(x) = x^3$ for all $x \in \mathbb{Z}$ (___/1)

What result of $c(x)$ can you find where x is not in the set of all integers?

b. $s: \mathbb{N} \rightarrow \mathbb{N}$, defined so that $s(x)$ is the closest whole number to \sqrt{x} for all $x \in \mathbb{N}$ (___/1)

Are there any elements in the codomain that have multiple inputs?

c. $h: \{0, 1, 2, 3, 4\} \rightarrow \{1, 2, 4, 6, 8\}$, given so that $h(n)$ is the ones' digit of 2^n for all $n \in \{0, 1, 2, 3, 4\}$ (___/1)

d. $g: \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, given so that $g(n)$ is the ones digit of 2^n for all $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (___/1)