- You will not turn in this assignment.
- Solutions are given at the end of class.
- Make sure your attendance is counted in order to get credit for your participation.
- You can keep this exercise to help you study for future exams.

Section 1: Composition of Functions

If $f: A \to B$ and $g: B \to C$, then we can build a new function called $(g \circ f)$ that has domain A and codomain C, and that follows the rule $(g \circ f)(x) = g(f(x))$. We call $(g \circ f)$, read "g of f", the composition of g with f.

Example: $f(x) = 2x+1$ **and** $g(x) = x^2 - 1$ **. What is** $(f \circ g)(x)$ **?**

1. **Plug f(x) into g(y):** $g \circ f = f(g(x)) = 2(x^2-1)+1$
2. **Simplify:** $= 2x^2 - 2 + 1$
   $= 2x^2 - 1$

We may also need to find the function $g(x)$ **based on** $f(x)$ **and** $(f \circ g)(x)$ **:**

Example: $f(x) = 2x+1$ **and** $(f \circ g)(x) = 2x^2 - 1$ **. What is** $g(x)$ **?**

1. **Let's use the alias "a" to symbolize g(x). f(g(x)) therefore will be f(a):**
   $2a+1 = 2x^2 - 1$ **Now we can find the value of g(y) by solving for a.**
2. $2a = 2x^2 - 1 - 1$ => $2a = 2x^2 - 2$ => $a = x^2 - 1$
3. **Therefore,** $g(x) = x^2 - 1$

And finding $f(x)$ **from** $g(x)$ **and** $(f \circ g)(x)$ **:**

Example: $g(x) = x^2 - 1$ **and** $(f \circ g)(x) = 2x^2 - 1$ **. What is** $f(x)$ **?**

Method 1:

1. **Start with f(g(x)). Let's say that the alias "a" will symbolize g(x). So** $a = x^2 - 1$ **.**
2. **Solve the alias function for the other variable:** $x^2 = a+1$ **…** $x = \sqrt{a+1}$ **.**
3. **Then, for f(g(x)), we use f(x):** $f(g(x)) = 2x^2 - 1$
4. **Plug in our found value for *x*:** $2(\sqrt{a+1})^2 - 1$
5. **Simplify:** $2(a+1) - 1$ **…** $= 2a + 2 - 1$ **…** $= 2a + 1$
6. **So** $f(x) = 2x+1$

1. Find the value of $(f \circ g)(x)$ for the following $f(x)$ and $g(x)$ functions:

a. $f(x) = 2x - 1$ and $g(x) = 3x$, what is $(f \circ g)(x)$? **(___/1)**

b. $f(x) = 2x - 1$ and $g(x) = 3x$, what is $(g \circ f)(x)$? **(___/1)**

c. $f(x) = x^2$ and $g(x) = x + 1$, what is $(f \circ g)(x)$? **(___/1)**

d. $f(x) = x^2$ and $g(x) = x + 1$, what is $(g \circ f)(x)$? **(___/1)**

2. Find the value of $g(x)$ from the following $f(x)$ and $(f \circ g)(x)$ given:

a. $f(x) = 2x - 1$ and $(f \circ g)(x) = 6x - 1$. What is $g(x)$? **(___/1)**

b. $f(x) = x^2$ and $(f \circ g)(x) = x^2 + 2x + 1$. What is $g(x)$? **(___/1)**

c. $f(x) = 3x - 2$ and $(f \circ g)(y) = 12y + 7$. What is $g(x)$? **(___/1)**

3. Find the value of $f(x)$ from the following $g(x)$ and $(f \circ g)(x)$ given:

a. $g(x)=3x$ and $(f \circ g)(x)=6x-1$. What is $f(x)$? **(___/1)**

b. $g(x)=x+1$ and $(f \circ g)(x)=x^2+2x+1$. What is $f(x)$? **(___/1)**

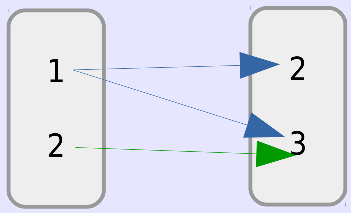
c. $g(x)=2x-1$ and $(f \circ g)(x)=6x-1$. What is $f(x)$? **(___/1)**

Section 2: More arrow diagrams

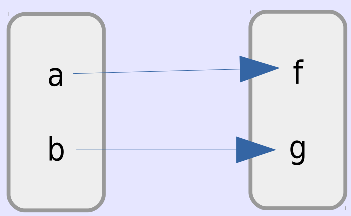
We can also use arrow diagrams to represent compositions of relational functions.

Example:

f(x): Domain A = {1, 2}, Codomain B = {2, 3}, Rule: L = { (1, 2), (1, 3), (2, 3) }.

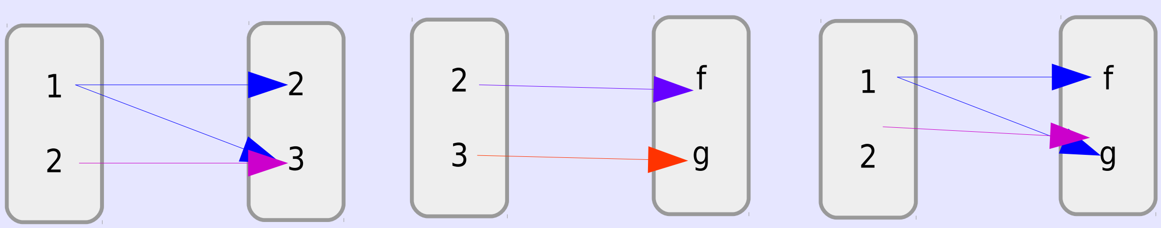


f(g): Domain: C = {a, b}, Codomain D = {f, g}, Rule: M = { (a, f), (b, g) }



So in order to draw the diagram of $(g \circ f)(x)$, we take the <u>domain</u> of *f* and the <u>codomain</u> of *g*. In this example, in *f*, 2 goes to 3. In *g*, 3 goes to g, so in $g \circ f$, 2 goes to g.

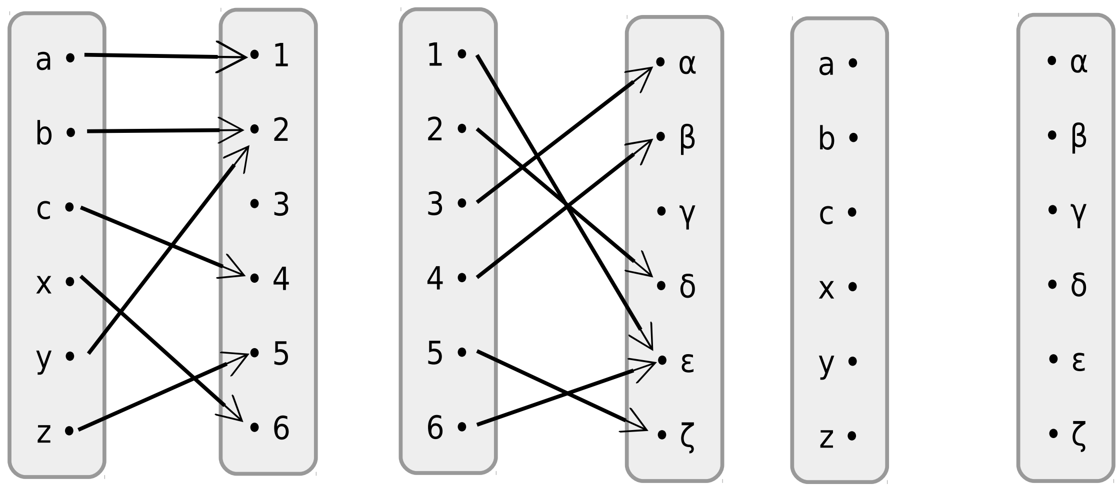
f g $g \circ f$



4. Complete the arrow diagrams

a. **(___/2)**

f g $g \circ f$



b. **(___/2)**

f g $g \circ f$