4.2 The Composition Operation

4.2.1 Finding $f \circ g$, given f and g

Question 1

Solve the following:

a. $f(x) = 2x - 1$ and $g(x) = 3x$, what is $(f \circ g)(x)$?
$f(g(x)) = f(3x) = 2(3x) - 1 = 6x - 1$

b. $f(x) = 2x - 1$ and $g(x) = 3x$, what is $(g \circ f)(x)$?
$g(f(x)) = g(2x - 1) = 3(2x - 1) = 6x - 3$

c. $f(x) = x^2$ and $g(x) = x + 1$, what is $(f \circ g)(x)$?
$f(g(x)) = f(x + 1) = (x + 1)^2 = x^2 + 2x + 1$

d. $f(x) = x^2$ and $g(x) = x + 1$, what is $(g \circ f)(x)$?
$g(f(x)) = g(x^2) = x^2 + 1$

4.2.2 Finding g based on f and $f \circ g$

Question 2

Solve the following:

a. $f(x) = 2x - 1$ and $(f \circ g)(x) = 6x - 1$, what is $g(x)$?
$2a - 1 = 6x - 1 \qquad 2a = 6x \qquad a = 3x$
$g(x) = 3x$

b. $f(x) = x^2$ and $(f \circ g)(x) = x^2 + 2x + 1$, what is $g(x)$?
$a^2 = x^2 + 2x + 1 \qquad a^2 = (x + 1)^2 \qquad a = x + 1$
$g(x) = x + 1$

c. $f(x) = 3x - 2$ and $(f \circ g)(x) = 12x + 7$, what is $g(x)$?
$3a-2 = 12x+7 \qquad 3a = 12x+7+2 \qquad 3a = 12x+9 \qquad a = 4x+3$
$g(x) = 4x + 3$

4.2.3 Finding f based on g and $f \circ g$

Question 3

Solve the following:

a. $g(x) = 3x$ and $(f \circ g)(x) = 6x - 1$. What is $f(x)$?
$a = 3x$. Solve for x: $x = \frac{1}{3}(a)$.
$f(g(x)) = 6x - 1$, plug in x: $f(a) = 6(\frac{1}{3}a) - 1$
Simplify: $f(a) = 2a - 1$.
So $f(x) = 2x - 1$.

b. $g(x) = x + 1$ and $(f \circ g)(x) = x^2 + 2x + 1$. What is $f(x)$?
$a = x + 1$ $x = a - 1$
$f(g(x)) = f(a)$; $f(a) = (a-1)^2 + 2(a-1) + 1$
$f(a) = a^2 - 2a + 1 + 2a - 2 + 1$
$f(a) = a^2$
So $f(x) = x^2$.

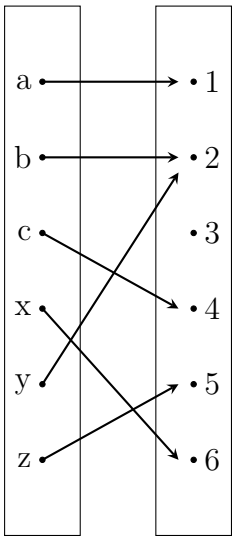
c. $g(x) = 2x - 1$ and $(f \circ g)(x) = 6x - 1$. What is $f(x)$?
$a = 2x - 1$ $x = \frac{a+1}{2}$
$f(g(x)) = f(a)$; $f(a) = 6(\frac{a+1}{2}) - 1$
$f(a) = 3(a+1) - 1$ $f(a) = 3a + 3 - 1$ $f(a) = 3a + 2$
So $f(x) = 3x + 2$.

4.2.4 More arrow diagrams

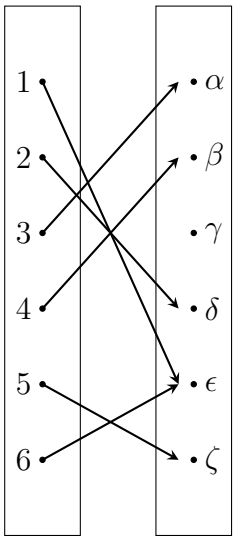
Question 4

Finish the following diagrams:

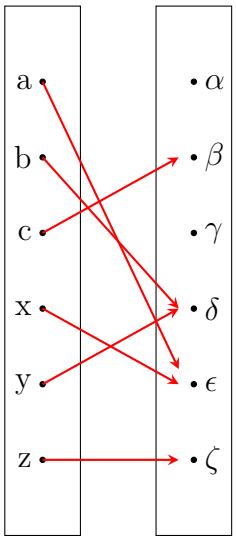
a.



$f : A \to B$

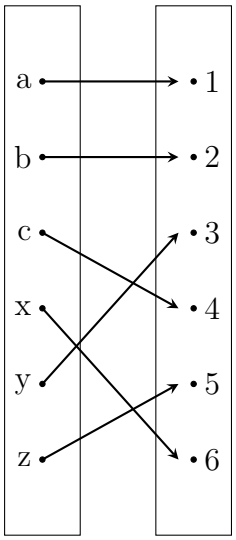


$g : B \to C$

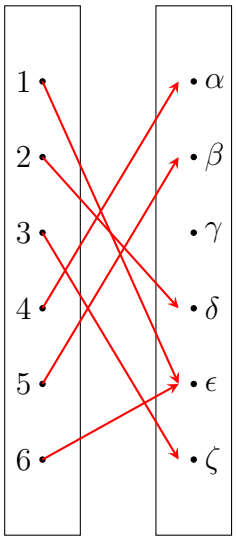


$(f \circ g) : A \to C$

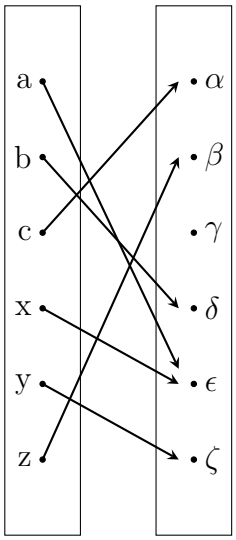
b.



$f : A \to B$



$g : B \to C$



$(f \circ g) : A \to C$