

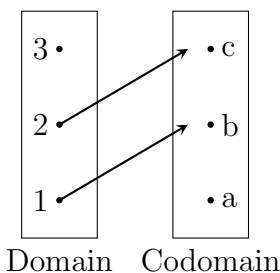
## 4.3 Properties of Functions and Set Cardinality

### 4.3.1 Review: Inverses of functions

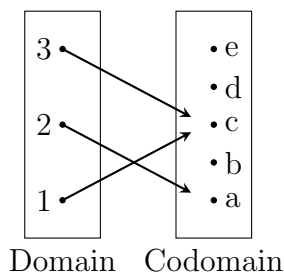
#### Question 1

Draw the inverse of each diagram. Identify whether the original diagram and/or the inverse of that diagram are functions.

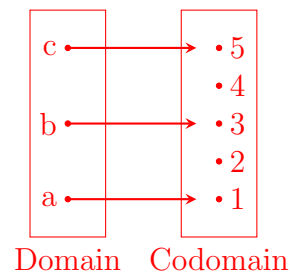
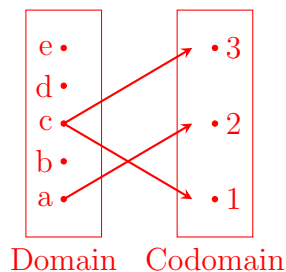
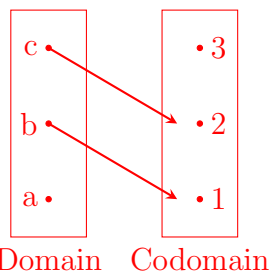
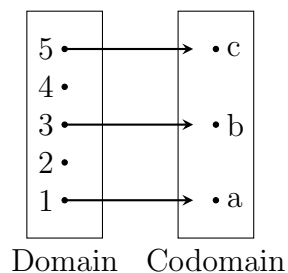
a.



b.



c.



- a. Original: Not a function; all inputs must have an output.
- a. Inverse: Not a function; all inputs must have an output.
- b. Original: Function
- b. Inverse: Not a function; “c” points to two different outputs.
- c. Original: Not a function; “2” and “4” don’t have any outputs.
- c. Inverse: Function

### 4.3.2 Functions that are invertible

#### Question 2

Draw two functions: One where the function is one-to-one but not onto, and one where the function is onto but not one-to-one. Make sure to label your domain and codomain for each.

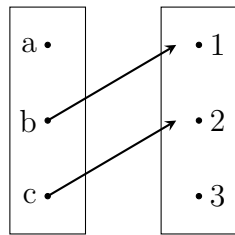
Multiple solutions

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#### Question 3

Determine whether these functions are one-to-one, onto, and/or invertible. If not, state why not.

a.

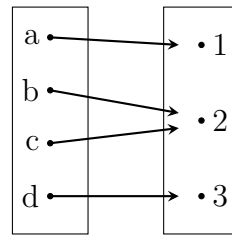


Domain Codomain

It is  
not a function.

☐ Onto      ☐ One-to-one  
☐ Invertible

b.

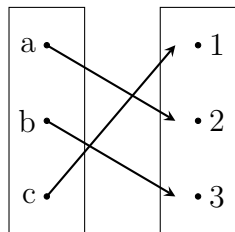


Domain Codomain

Onto, not one-to-one, not invertible

☐ Onto      ☐ One-to-one  
☐ Invertible

c.

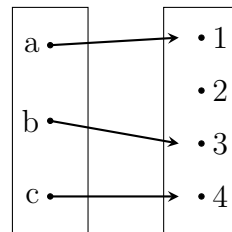


Domain Codomain

Onto, One-to-one, Invertible

☐ Onto      ☐ One-to-one  
☐ Invertible

d.

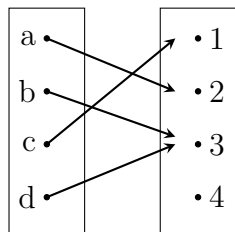


Domain Codomain

One-to-one, not onto, not invertible

☐ Onto      ☐ One-to-one  
☐ Invertible

e.

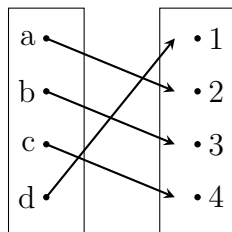


Domain Codomain

Not  
onto, not one-to-one, not invertible

☐ Onto      ☐ One-to-one  
☐ Invertible

f.



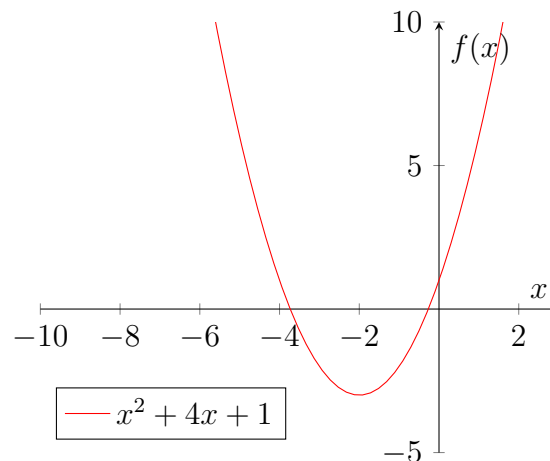
Domain Codomain

Onto, one-to-one, invertible

☐ Onto      ☐ One-to-one  
☐ Invertible

**Question 4**

The function  $f : \mathbb{R} \rightarrow \mathbb{R}$ , with the rule  $f(x) = x^2 + 4x + 1$  is not onto and not one-to-one.



- Give an example of an element in the codomain that has no element in the domain associated with it. **There is no  $x \in \mathbb{R}$  for which  $f(x) = -4$  since the equation  $x^2 + 4x + 1 = -4$  has no **real** solutions (by using the quadratic formula).**
- Given an example of two elements in the domain that are both associated with the same output in the codomain.  **$f(-1) = 1 - 4 + 1 = -2$  and  $f(-3) = 9 - 12 + 1 = -2$ .**