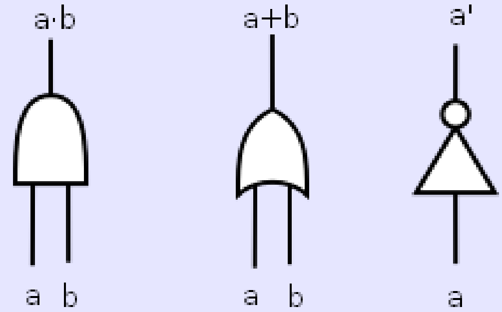
3.5 Logic Circuits

3.5.1 Logic Circuits

We are going to be using logic gates as one way to represent our Boolean Algebra expressions graphically. The gates that we will be using are:

AND $\cdot$ OR $+$ NOT $'$



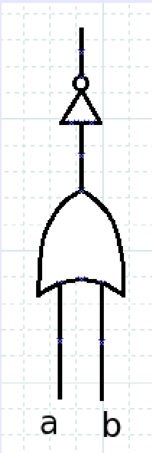
a	b	$a \cdot b$
0	0	0
0	1	0
1	0	0
1	1	1

a	b	$a + b$
0	0	0
0	1	1
1	0	1
1	1	1

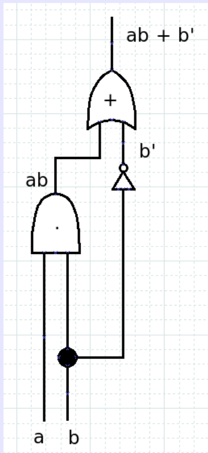
a	a'
0	1
1	0

Additionally, we can connect gates together in order to build an expression. For example:

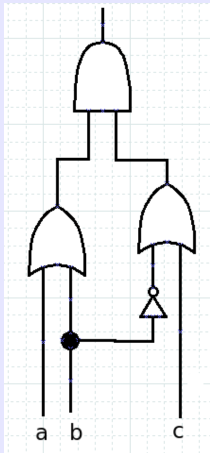
$(a + b)'$



$ab + a'b$

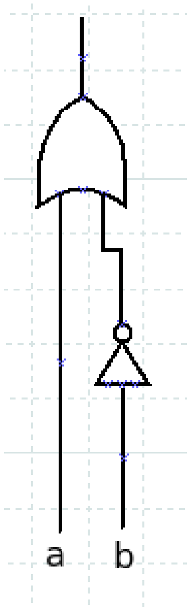


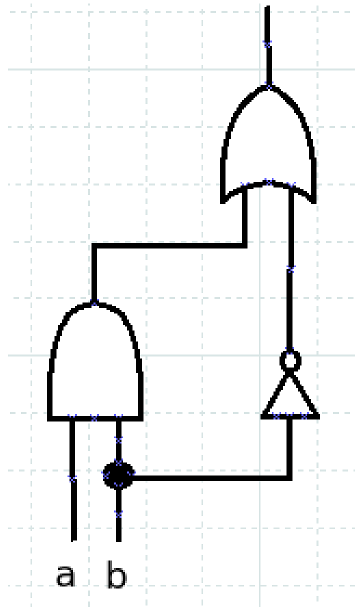
$(a + b) \cdot (b' + c)$

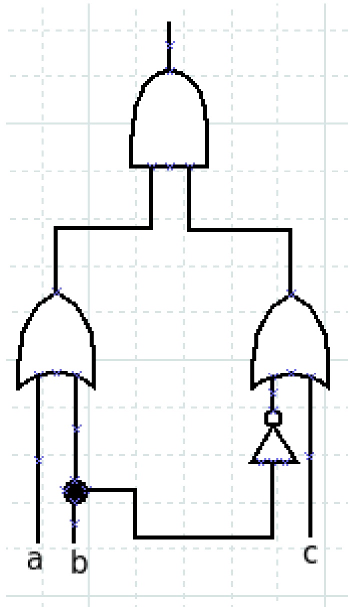


Question 1

Write out the Boolean expression that describes each diagram:







Question 2

Draw a circuit diagram for the following Boolean expressions:

a. $a + b'$	b. $a' \cdot b'$	c. $a + (b \cdot c)$

3.5.2 2-variable Karnaugh Maps

2x2 Karnaugh Map basics

We can use Karnaugh maps to visually represent Boolean expressions in order to simplify them. For an expression with two variables, our map will be a 2x2 grid like this:

	y	y'
x		
x'		

If you have one product in your equation, you check off the cell where the terms intersect:

xy:

	y	y'
x	✓	
x'		

$x'y$:

	y	y'
x		
x'	✓	

xy':

	y	y'
x		✓
x'		

Each product translates to a cell being ✓'d. When there are multiple products, they're added with +:

$xy + x'y$:

	y	y'
x	✓	
x'	✓	

$xy + xy' + x'y'$:

	y	y'
x	✓	✓
x'		✓

Question 3

Check off all appropriate term cells for the following maps.

a. xy

	y	y'
x		
x'		

b. $xy + xy'$

	y	y'
x		
x'		

c. $xy + x'y' + x'y$

	y	y'
x		
x'		

One region

After writing out all of the products, any adjacent cells can be grouped off within a rectangular region. For a 2x2 grid, our grouping rectangles can be 1x1 (no simplification), 2x1, 1x2, or 2x2.

$xy + x'y$:

	y	y'
x	✓	
x'	✓	

When you have a rectangular region, you can remove one of the variables. Whichever variable has both the **normal** and **prime** version can be removed.

$xy + x'y$:

	y	y'
x	✓	
x'	✓	

$xy + x'y \Rightarrow$ Remove the x variable, change to y

Question 4

Fill out the map, outline adjacent regions, and eliminate one variable.

a. $xy + xy'$

	y	y'
x		
x'		

b. $x'y + xy$

	y	y'
x		
x'		

c. $x'y' + x'y$

	y	y'
x		
x'		

Multiple regions

If you can highlight multiple regions, you will end up with that amount of terms, combined by Boolean addition $+$.

Example: We have the expression $xy + xy' + x'y$, our map looks like this:

	y	y'
x	✓	✓
x'	✓	

We will have two regions...

This

	y	y'
x	✓	✓
x'	✓	

and this

	y	y'
x	✓	✓
x'	✓	

So our two terms will be x and y. Therefore, we can simplify

$$xy + xy' + x'y$$
to
$$x + y$$

Question 5

Use Karnaugh maps to simplify the following expressions.

a. $xy' + x'y'$

	y	y'
x		
x'		

b. $xy + x'y + x'y'$

	y	y'
x		
x'		

3.5.3 3-variable Karnaugh Maps

Once we have three variables, we will have to change our map. It will become a $2x4$ map, where each column represents **two variables**.

	yz	yz'	$y'z'$	$y'z$
x				
x'				

Note that for each column, the difference between any two columns can be only **one**... in other words, you can go from yz to $y'z$ or yz', but you CANNOT go from yz to $y'z'$.

We can then follow the same approach to mark off cells that correspond to products of terms:

	yz	yz'	$y'z'$	$y'z$
x	✓			
x'	✓			

$$xyz + x'yz$$

We can also highlight regions in order to simplify the expression. For a $2x4$ Karnaugh map, our regions can be...: $1x1$ (no simplification), $2x1$, $1x2$, or $2x2$.

Example: Simplify $x'yz + x'y'z + xyz' + xy'z'$

	yz	yz'	$y'z'$	$y'z$
x		✓	✓	
x'	✓			✓

We can also *wrap around* horizontally; the checkmarks at $x'yz$ and $x'y'z$ can be counted as one region.

	yz	yz'	$y'z'$	$y'z$
x		✓	✓	
x'	✓			✓

We have two regions, so we can simplify it into two terms. Any variables in a region that have both the normal and prime versions stay, any variable that is the same for all cells in the region are removed.

$$x'yz + x'y'z + xyz' + xy'z' \qquad \Rightarrow \qquad xz' + x'z$$

More rules...

Choosing regions: Sometimes you can come up with multiple regions in your map. What is the best way to make your regions? In order to get the simplest expression, choose the *smallest amount of rectangles*, and such that *each rectangle is as large as possible.*

1x1 regions You can choose a 1x1 region if doing so allows you to have largest-possible-regions for other terms.

Question 6

Simplify the following equations with a Karnaugh map.

a. $xyz + xyz' + xy'z' + xy'z$

	yz	yz'	$y'z'$	$y'z$
x				
x'				

b. $xyz + xyz' + x'yz + x'yz'$

	yz	yz'	$y'z'$	$y'z$
x				
x'				

c. $xyz + xyz' + x'y'z' + x'y'z$

	yz	yz'	$y'z'$	$y'z$
x				
x'				

d. $x'y'z' + xyz + xyz' + xy'z' + xy'z$

	yz	yz'	$y'z'$	$y'z$
x				
x'				