1.2 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

5.1 Number Puzzles and Sequences

5.1.1 Number sequences

For this section, we are analyzing sequences of numbers in order to build *closed* and/or *recursive* formulas to describe them.

It can be a bit challenging at first to figure out the equation based on a list of numbers, so make sure to take note of some techniques for analyzing these sequences!

Let's start off simple...

Question 1

For the given sequence of numbers: 2, 4, 6, 8, 10

a. What is the next number in the sequence? If you can tell just by looking at it, how can you tell?

b. If we assign numbers to each of these...

Item 1	Item 2	Item 3	Item 4	Item 5
2	4	6	8	10

...How can we come up with some formula to associate the item # to the value?

c. If we're describing **Item 2** in terms of **Item 1**... Item 2 = Item 1 + ?

d. If we're describing **Item 3** in terms of **Item 2**... Item 3 = ?

e. If we want to generalize this and describe any item n in terms of the previous item, $n-1$... Item n = ?

5.1.2 Sequences

Definition: Recursive formula (aka recurrence relation)
In mathematics, a recurrence relation is an equation that recursively defines a sequence [...] of values, once one or more initial terms are given: each further term of the sequence [...] is defined as a function of the preceding terms. [a]

Definition: Closed formula
A closed formula for a sequence is a formula where each term is described only in relation to its position in the list. [b]

Definition: Sequence notation Sequence notation is where we have some sequence, a, and a_n denotes the element at position n. On a computer, the subscript may be written as `a[n]`.

[a] From https://en.wikipedia.org/wiki/Recurrence_relation
[b] From Discrete Mathematics Mathematical Reasoning and Proof with Puzzles, Patterns, and Games by Douglas E Ensley

Question 2
Write out the first 5 elements of the following equations:

a. The closed formula $a_n = n + 1$

b. The closed formula $a_n = 2n + 1$

c. The recursive formula $a_1 = 1, a_n = a_{n-1} + 2$

d. The resursive formula $a_1 = 2, a_n = 2a_{n-1} + 1$

Tips for finding equations

If it isn't immediately obvious what a sequence's function is, here are a few tips:

- Write out each element with its position, like $a_1 = 2$, $a_2 = 5$, $a_3 = 10$, etc. This helps with trying to find a pattern between the **index** (position) and the **element** (value).
- Compare the difference between each element, like $5 - 2 = 3$, $10 - 5 = 5$, $17 - 10 = 7$. Can you find a pattern in the difference between the elements?
- Compare the difference between the *differences*. Above, we can see that the difference *increases* by 2 between each element.

Question 3

Figure out the **closed formula** for the following sequences.
For these sequences, n will not be multiplied by anything, but will have something added to it.

a. 3, 4, 5, 6, 7 b. 6, 7, 8, 9, 10

Question 4

Figure out the **closed formula** for the following sequences.
For these sequences, n will have something multiplied to it.

a. 2, 4, 6, 8, 10 b. 3, 6, 9, 12, 15
c. 5, 10, 15, 20, 25 d. 1, 4, 9, 16, 25

Question 5

Figure out the **closed formula** for the following sequences.
For these sequences, n will have something multipled to it and added to (or subtracted from) the product.

a. 1, 3, 5, 7, 9 b. 4, 7, 10, 13, 16
c. 7, 12, 17, 22, 27 d. 2, 5, 10, 17, 26

Question 6

Figure out the **recursive formula** for the following sequences.
For these sequences, a_{n-1} will not be multiplied by anything, but will have something added to it. Be sure to specify a_1 first. It will be the first number in the sequence.

a. 1, 3, 5, 7, 9 *($a_1 = 1$)*
b. 1, 5, 9, 13, 17
c. 2, 4, 6, 8, 10 *($a_1 = 2$)*
d. 2, 6, 10, 14, 18

Question 7

Figure out the **recursive formula** for the following sequences.
For these sequences, a_{n-1} will have something multiplied to it, but nothing added to it. Be sure to specify a_1 first.

a. 2, 4, 8, 16, 32
b. 1, 3, 9, 27, 81
c. 3, 6, 12, 24, 48
d. 2, 4, 16, 256, 65536

Question 8

Figure out the **recursive formula** for the following sequences.
For these sequences, a_{n-1} will have something multiplied to it and added to it. Be sure to specify a_1 first.

a. 1, 3, 7, 15, 31
b. 2, 5, 11, 23, 47
c. 1, 5, 17, 53, 161
d. 1, 4, 10, 22, 46

5.1.3 Summations

For a sequence of numbers (denoted a_k, where $k >= 1$, we can use the notation

$$\sum_{k=1}^{n} a_k$$

to denote the sum of the first n terms of the sequence. This is called *sigma notation*.

Example: Evaluate the sum $\sum_{k=1}^{3}(2k-1)$.
First, we need to find the elements at $k = 1$, $k = 2$, and $k = 3$:

$k = 1$	$k = 2$	$k = 3$
$a_1 = (2 \cdot 1 - 1) = 1$	$a_2 = (2 \cdot 2 - 1) = 3$	$a_3 = (2 \cdot 3 - 1) = 5$

Then, we can add the values:
$\sum_{k=1}^{3}(2k-1) \quad = a_1 + a_2 + a_3 \quad = 1 + 3 + 5 \quad \boxed{= 9}$

Question 9
Evaluate the following summations.

a.
$$\sum_{k=1}^{4}(3k)$$

b.
$$\sum_{k=1}^{5}(4)$$