1. Sums as Recursive Sequences

EXAMPLE 1 **Example 1 from the textbook**

Consider the sum $\sum_{i=1}^{n}(2i-1)$, which is the same as 1 + 3 + 5 + … + (2n-1). Use the notation s_n to denote this sum. Find a recursive description of s_n .

Step 1: Find the first term, s_1 :	Plug into the summation: $\sum_{i=1}^{1}(2i-1)=(2\cdot1-1)=1$, so $s_1=1$
Step 2: Restate the result of s_n as $s_{(n-1)}$ plus the final term	$s_n=s_{(n-1)}+(2n-1)$

So, for $\sum_{i=1}^{n}(2i-1)$, the recursive formula is: $s_1=1$, $s_n=s_{(n-1)}+(2n-1)$.

2. More proofs by induction

EXAMPLE 2 **Example 6 from the book**

Show that n^3+2n is divisible by 3 for all positive integers *n*. ($D(n)=n^3+2n$)

Step 1: Check for *D(1):*	$D(1)=1^3+2\cdot1=3$ ✓
Step 2: Acknowledge that "Show that n^3+2n is divisible by 3 for all positive integers n." has been proven for *D(1)* through *D(m-1).*	
Step 3: Write out *D(m-1)* and simplify:	$D(m-1)=(m-1)^3+2(m-1)$ $D(m-1)=m^3-3m^2+3m-1+2m-2$
Step 4: Rewrite simplified version so that *D(m)* is part of the equation:	$D(m-1)=(m^3+2m)-3m^2+3m-3$
Step 5: Rewrite with *D(m):*	$D(m-1)=D(m)-3m^2+3m-3$
Step 6: Solve for *D(m):*	$D(m)=D(m-1)+3m^2-3m+3$
Step 7: Remember that *divisibility by 3* has been proven true for *D(1)* through *D(m-1)* (from Step 2). Replace *D(m-1)* with "*3K*".	$D(m)=3K+3m^2-3m+3$
Step 8: Factor out common terms to get final proof that n^3+2n is divisible by 3:	$D(m)=3(K+m^2-m+1)$

Practice 1: Follow the steps from Example 1

Consider the sum $\sum_{i=1}^{n}\left(3n^2\right)$. Use the notation s_n to denote this sum. Find a recursive description of s_n .

Practice 2: Follow the steps from Example 1

Consider the sum $\sum_{i=1}^{n}\left(2^{(i-1)}+1\right)$. Use the notation s_n to denote this sum. Find a recursive description of s_n .

Practice 3: Follow the steps from Example 1

Consider the sum $\sum_{i=1}^{n}\left(i^3-i\right)$. Use the notation s_n to denote this sum. Find a recursive description of s_n .

Practice 4: Follow the steps from Example 2

Use induction to prove that for each integer $n\geq 1$, $2n$ is even.

Practice 5: Follow the steps from Example 2

Use induction to prove that for each integer $n\geq 1$, $4n+1$ is odd.

Practice 6: Follow the steps from Example 2

Use induction to prove that for each integer $n\geq 1$, n^2-n is even.