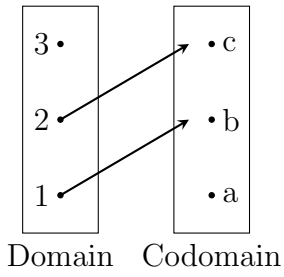
4.3 Properties of Functions and Set Cardinality

4.3.1 Review: Inverses of functions

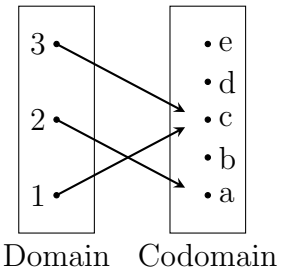
Question 1

Draw the inverse of each diagram. Identify whether the original diagram and/or the inverse of that diagram are functions.

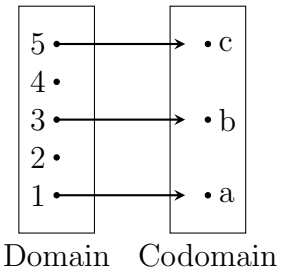
a.

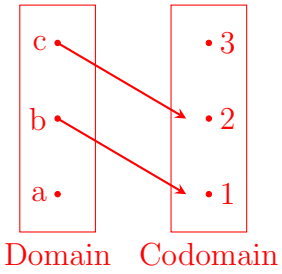


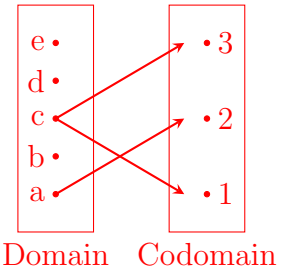
b.

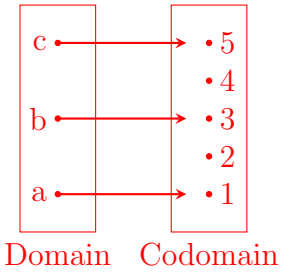


c.









a. Original: Not a function; all inputs must have an output.

a. Inverse: Not a function; all inputs must have an output.

b. Original: Function

b. Inverse: Not a function; "c" points to two different outputs.

c. Original: Not a function; "2" and "4" don't have any outputs.

c. Inverse: Function

4.3.2 Functions that are invertible

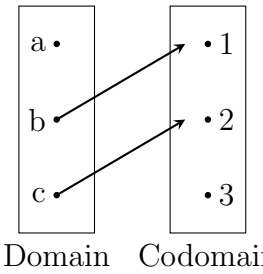
Question 2

Draw two functions: One where the function is one-to-one but not onto, and one where the function is onto but not one-to-one. Make sure to label your domain and codomain for each.

Multiple solutions

Question 3
Determine whether these functions are one-to-one, onto, and/or invertible. If not, state why not.

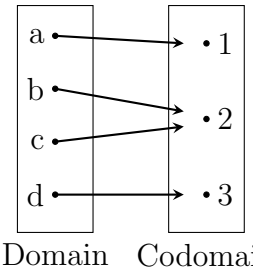
a.



It is not a function, but it is one-to-one; no output has more than one input.

☐ Onto ☐ One-to-one
☐ Invertible

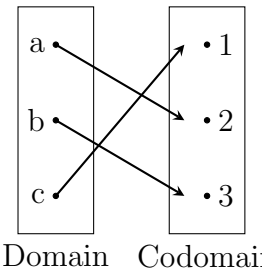
b.



Onto, not one-to-one, not invertible

☐ Onto ☐ One-to-one
☐ Invertible

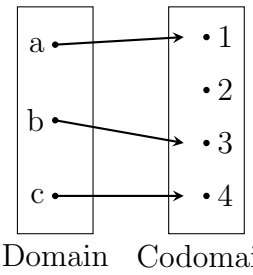
c.



Onto, One-to-one, Invertible

☐ Onto ☐ One-to-one
☐ Invertible

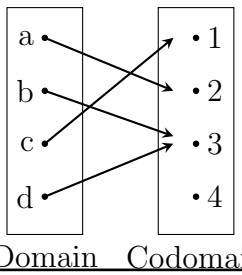
d.



One-to-one, not onto, not invertible

☐ Onto ☐ One-to-one
☐ Invertible

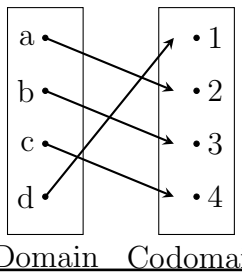
e.



Not onto, not one-to-one, not invertible

☐ Onto ☐ One-to-one
☐ Invertible

f.

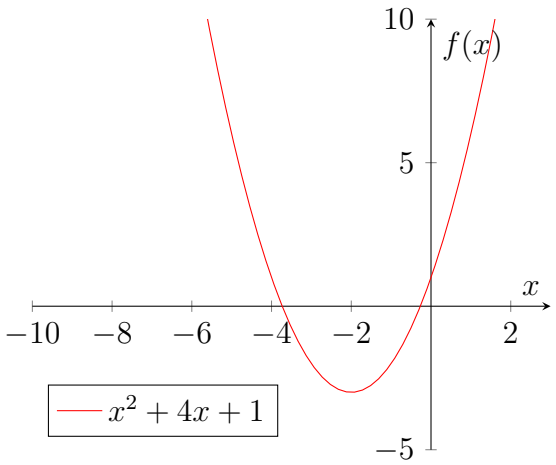


Onto, one-to-one, invertible

☐ Onto ☐ One-to-one
☐ Invertible

Question 4

The function $f : \mathbb{R} \to \mathbb{R}$, with the rule $f(x) = x^2 + 4x + 1$ is not onto and not one-to-one.



a. Give an example of an element in the codomain that has no element in the domain associated with it. There is no $x \in \mathbb{R}$ for which $f(x) = -4$ since the equation $x^2 + 4x + 1 = -4$ has no **real** solutions (by using the quadratic formula).

b. Given an example of two elements in the domain that are both associated with the same output in the codomain. $f(-1) = 1 - 4 + 1 = -2$ and $f(-3) = 9 - 12 + 1 = -2$.