Answer Key

1. a. $P(1) = 1 + 1 = 2$, true
   b. $P(3) = 9 + 1 = 10$, false
   c. $P(9) = 81 + 1 = 81$, false

2. a. $a_1 = 1$
   b. $a_2 = 1 + 4 = 5$
   c. $a_3 = 5 + 4 = 9$
   d. $a_{m-1} =$ (Anywhere you see k, plug in $m - 1$.) $a_{m-2} + 4$

3. a. $a_1 = 4 - 3 = 1$
   b. $a_3 = 12 - 3 = 9$
   c. $a_5 = 20 - 3 = 27$
   d. $a_{m-1} =$ (Anywhere you see n, plug in $m - 1$.)
   $4(m - 1) - 3 = 4m - 4 - 3 = 4m - 7$

4. **Step 1:**

   Recursive: $a_1 = 5$; Closed: $a_1 = 2(1) + 3 = 5$ ✓

   Step 2:

   $a_m = 4 \cdot a_{m-1} + 2$

   Step 3:

   $a_{m-1} = 2(m - 1) + 3 \quad = 2m - 2 + 3 \quad = 2m + 1$

   Step 4:

   $a_m = a_{m-1} + 2$

   $a_m = (2m + 1) + 2$

   $a_m = 2m + 3$

5. **Step 1:**

   Recursive: $a_1 = 1$ Closed: $a_1 = 2^1 - 1 = 1$ ✓

   Step 2:

   $a_m = 2 \cdot a_{m-1} + 1$

   Step 3:

   $a_{m-1} = 2^{m-1} - 1 \quad = 2^m \cdot 2^{-1} - 1 \quad = \frac{2^m}{2^1} - 1$

Step 4:

$a_m = 2 \cdot a_{m-1} + 1$

$a_m = 2^1(\frac{2^m}{2^1} - 1) + 1$

$a_m = 2^m - 2 + 1$

$a_m = 2^m - 1$

6. **Step 1:**

   Recursive: $b_1 = 3$; Closed: $b_1 = 2^{2 \cdot 1} - 1 = 4 - 1 = 3$ ✓

   Step 2:

   $b_m = 4 \cdot b_{m-1} + 3$

   Step 3:

   $b_{m-1} = 2^{2(m-1)} - 1 \qquad = 2^{2m} \cdot 2^{-2} - 1 \qquad = \frac{2^{2m}}{2^2} - 1;$

   Step 4:

   $b_m = 4a_{m-1} + 3$

   $b_m = 4(\frac{2^{2m}}{2^2} - 1) + 3$

   $b_m = 2^2(\frac{2^{2m}}{2^2} - 1) + 3$

   $b_m = 2^{2m} - 4 + 3$

   $b_m = 2^{2m} - 1$

7. **Step 1:**

i **value**	$\sum_{i=1}^{n}(2i+4)$	$n^2 + 5n$
$i = 1$	$2(1) + 4 = 6$	$1^2 + 5(1) = 6$
$i = 2$	$6 + 2(2) + 4 = 14$	$2^2 + 5(2) = 14$
$i = 3$	$14 + 2(3) + 4 = 24$	$3^2 + 5(3) = 9 + 15 = 24$

   Step 2:

   $\sum_{i=1}^{m}(2i+4) = \sum_{i=1}^{m-1}(2i+4) + (2m+4)$

   Step 3:

   $\sum_{i=1}^{m-1}(2i+4) = (m-1)^2 + 5(m-1)$
   $\sum_{i=1}^{m-1}(2i+4) = m^2 - 2m + 1 + 5m - 5$
   $\sum_{i=1}^{m-1}(2i+4) = m^2 + 3m - 4$

   Step 4:

$\sum_{i=1}^{m}(2i+4) = (m^2+3m-4)+(2m+4)$
$\sum_{i=1}^{m}(2i+4) = m^2+5m$
This matches the original proposition.

8. **Step 1:**

i **value**	$\sum_{i=1}^{n} i$	$\frac{n(n+1)}{2}$
$i=1$	1	$\frac{1(2)}{2}=1$
$i=2$	$1+2=3$	$\frac{2(2+1)}{2}=3$
$i=3$	$1+2+3=6$	$\frac{3(3+1)}{2}=6$

Step 2:

$\sum_{i=1}^{m} i = \sum_{i=1}^{m-1}(i)+m$

Step 3:

$\sum_{i=1}^{m-1} i = \frac{(m-1)(m)}{2}$
$\sum_{i=1}^{m-1} i = \frac{m^2-m}{2}$

Step 4:

$\sum_{i=1}^{m} i = \frac{m^2-m}{2}+m$
$\sum_{i=1}^{m} i = \frac{m^2-m}{2}+\frac{2m}{2}$
$\sum_{i=1}^{m} i = \frac{m^2+m}{2}$
$\sum_{i=1}^{m} i = \frac{m(m+1)}{2}$
This matches the original proposition.