Chapter 1.2 In-class Exercise

Info: In-class exercises are meant to introduce you to the new topics of this chapter of the book. Each part will have an introductory description of the content and example(s), followed by practice problems for you to work on.

These assignments are **team assignments** - your team will turn in *one* copy of the exericse. It is up to your team how to appropach the assignments; you can work separately and then check your work together, or you can collaborate on the assignment together.

Work must be clean; points may be deducted if the instructor cannot read the work.

Chapter 1.2 In-class Exercise Instructions

This is the instruction page, make sure to fill out your answers in the worksheet later in the document

1. Number sequences

For this section, we are analyzing sequences of numbers in order to build *closed* and/or *recursive* formulas to describe them.

It can be a bit challenging at first to figure out the equation based on a list of numbers, so make sure to take note of some techniques for analyzing these sequences!

Let's start off simple...

Question 1 _____ / 4%

For the given sequence of numbers: 2, 4, 6, 8, 10

a. What is the next number in the sequence? If you can tell just by looking at it, how can you tell?

b. If we assign numbers to each of these...

Item 1	Item 2	Item 3	Item 4	Item 5
2	4	6	8	10

...How can we come up with some formula to associate the item # to the value?

c. If we're describing **Item 2** in terms of **Item 1** as a math sequence, we can say that...
Item 2 = Item 1 + ?

d. If we're describing **Item 3** in terms of **Item 2** as a math equation, we can say that...
Item 3 = ?

e. If we want to generalize this and describe any item n in terms of the previous item, $n-1$, we can say that...
Item n = ?

2. Sequences

Definition: Recursive formula (aka recurrence relation)
In mathematics, a recurrence relation is an equation that recursively defines a sequence [...] of values, once one or more initial terms are given: each further term of the sequence [...] is defined as a function of the preceding terms. [a]

Definition: Closed formula
A closed formula for a sequence is a formula where each term is described only in relation to its position in the list. [b]

Definition: Sequence notation Sequence notation is where we have some sequence, a, and a_n denotes the element at position n. On a computer, the subscript may be written as `a[n]`.

[a] From https://en.wikipedia.org/wiki/Recurrence_relation
[b] From Discrete Mathematics Mathematical Reasoning and Proof with Puzzles, Patterns, and Games by Douglas E Ensley

Question 2 _____ / 12%

Write out the first 5 elements of the following equations:

a. The closed formula $a_n = n + 1$
b. The closed formula $a_n = 2n + 1$
c. The recursive formula $a_1 = 1, a_n = a_{n-1} + 2$
d. The resursive formula $a_1 = 2, a_n = 2a_{n-1} + 1$

Tips for finding equations If it isn't immediately obvious what a sequence's function is, here are a few tips:

- Write out each element with its position, like $a_1 = 2$, $a_2 = 5$, $a_3 = 10$, etc. This helps with trying to find a pattern between the **index** (position) and the **element** (value).
- Compare the difference between each element, like $5 - 2 = 3$, $10 - 5 = 5$, $17 - 10 = 7$. Can you find a pattern in the difference between the elements?
- Compare the difference between the *differences*. Above, we can see that the difference *increases* by 2 between each element.

Question 3 _____ / 12%

Figure out the **closed formula** for the following sequences.
For these sequences, n will not be multiplied by anything, but will have something added to it.

a. 3, 4, 5, 6, 7

b. 6, 7, 8, 9, 10

Question 4 _____ / 12%

Figure out the **closed formula** for the following sequences.
For these sequences, n will have something multiplied to it.

a. 2, 4, 6, 8, 10

b. 3, 6, 9, 12, 15

c. 5, 10, 15, 20, 25

d. 1, 4, 9, 16, 25

Question 5 _____ / 12%

Figure out the **closed formula** for the following sequences.
For these sequences, n will have something multipled to it and added to (or subtracted from) the product.

a. 1, 3, 5, 7, 9

b. 4, 7, 10, 13, 16

c. 7, 12, 17, 22, 27

d. 2, 5, 10, 17, 26

Question 6 _____ / 12%

Figure out the **recursive formula** for the following sequences.
For these sequences, a_{n-1} will not be multiplied by anything, but will have something added to it. Be sure to specify a_1 first. It will be the first number in the sequence.

a. 1, 3, 5, 7, 9 *($a_1 = 1$)*

b. 1, 5, 9, 13, 17

c. 2, 4, 6, 8, 10 *($a_1 = 2$)*

d. 2, 6, 10, 14, 18

Question 7 _____ / 12%

Figure out the **recursive formula** for the following sequences.
For these sequences, a_{n-1} will have something multiplied to it, but nothing added to it. Be sure to specify a_1 first.

a. 2, 4, 8, 16, 32

b. 1, 3, 9, 27, 81

c. 3, 6, 12, 24, 48

d. 2, 4, 16, 256, 65536

Question 8 _____ / 12%

Figure out the **recursive formula** for the following sequences.
For these sequences, a_{n-1} will have something multiplied to it and added to it. Be sure to specify a_1 first.

a. 1, 3, 7, 15, 31

b. 2, 5, 11, 23, 47

c. 1, 5, 17, 53, 161

d. 1, 4, 10, 22, 46

3. Summations

For a sequence of numbers (denoted a_k, where $k >= 1$, we can use the notation

$$\sum_{k=1}^{n} a_k$$

to denote the sum of the first n terms of the sequence. This is called *sigma notation*.

Example: Evaluate the sum $\sum_{k=1}^{3}(2k-1)$.
First, we need to find the elements at $k = 1$, $k = 2$, and $k = 3$:

$k = 1$	$k = 2$	$k = 3$
$a_1 = (2 \cdot 1 - 1) = 1$	$a_2 = (2 \cdot 2 - 1) = 3$	$a_3 = (2 \cdot 3 - 1) = 5$

Then, we can add the values:
$\sum_{k=1}^{3}(2k-1) \quad = a_1 + a_2 + a_3 \quad = 1 + 3 + 5 \quad \boxed{= 9}$

Question 9 _____ / 12%

Evaluate the following summations.

a.

$$\sum_{k=1}^{4}(3k)$$

b.

$$\sum_{k=1}^{5}(4)$$

Chapter 1.2 In-class Exercise Worksheet

Team: Please write down all people in your team.

1. 2.
3. 4.

Section:
☐ MW 4:30 - 5:45 pm ☐ M 6:00 - 8:50 pm ☐ TR 2:00 - 3:15 pm

Team rules:

- **Only one worksheet will be turned in per team.** Each member of the team will receive the same score.
- You can collaborate on the exercise together, or you can work separately and then compare your answers with your team as you fill out the turn-in worksheet.

Work rules:

- Fill out your answers on this answer sheet.
- Write cleanly and linearly - if I can't make sense of your solution then you won't get credit.
- Write out each step (within reason) - if I can't see the logic you followed to get to your answer, you may get points taken off.
- Don't scribble out cancellations - I can't read that. For example, if a numerator/denominator cancel out, or a +/- cancels out, don't scribble out the numbers - just use a single slash!

Grading: The total amount of points for an in-class exercise is 5 points each. Each question will have a weight assigned to it, and will be given a a point value between 0 and 4:

0	Nothing written
1	Something written, but incorrect
2	Partially correct, but multiple errors
3	Mostly correct, with one or two errors
4	Perfect; correct answer and notation

Answer sheet

4% Question 1: Sequence of numbers □ 0 □ 1 □ 2 □ 3 □ 4

a. What is the next number in the sequence? If you can tell just by looking at it, how can you tell?

b. What is a formula to associate the index and the value?

Item 1	Item 2	Item 3	Item 4	Item 5
2	4	6	8	10

c. Item 2 = Item 1 +

d. Item 3 =

e. Item n =

12% Question 2: First 5 elements □ 0 □ 1 □ 2 □ 3 □ 4

a. $a_n = n + 1$
$a_1 =$
$a_2 =$
$a_3 =$
$a_4 =$
$a_5 =$

b. $a_n = 2n + 1$
$a_1 =$
$a_2 =$
$a_3 =$
$a_4 =$
$a_5 =$

c. $a_1 = 1, a_n = a_{n-1} + 2$
$a_1 =$
$a_2 =$
$a_3 =$
$a_4 =$
$a_5 =$

d. $a_1 = 2, a_n = 2a_{n-1} + 1$
$a_1 =$
$a_2 =$
$a_3 =$
$a_4 =$
$a_5 =$

6% Question 3a: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 3, 4, 5, 6, 7

6% Question 3b: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 6, 7, 8, 9, 10

3% Question 4a: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 2, 4, 6, 8, 10

3% Question 4b: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 3, 6, 9, 12, 15

3% Question 4c: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 5, 10, 15, 20, 25

3% Question 4d: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 1, 4, 9, 16, 25

3% Question 5a: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 1, 3, 5, 7, 9

3% Question 5b: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 4, 7, 10, 13, 16

3% Question 5c: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 7, 12, 17, 22, 27

3% Question 5d: Find closed formula □ 0 □ 1 □ 2 □ 3 □ 4
Find closed formula for sequence: 2, 5, 10, 17, 26

3% Question 6a: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 1, 3, 5, 7, 9

3% Question 6b: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 1, 5, 9, 13, 17

3% Question 6c: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 2, 4, 6, 8, 10

3% Question 6d: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 2, 6, 10, 14, 18

3% Question 7a: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 2, 4, 8, 16, 32

3% Question 7b: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 1, 3, 9, 27, 81

3% Question 7c: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 3, 6, 12, 24, 48

3% Question 7d: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 2, 4, 16, 256, 65536

3% Question 8a: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 1, 3, 7, 15, 31

3% Question 8b: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 2, 5, 11, 23, 47

3% Question 8c: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 1, 5, 17, 53, 161

3% Question 8d: Find recursive formula □ 0 □ 1 □ 2 □ 3 □ 4
Find recursive formula for sequence: 1, 4, 10, 22, 46

6% Question 9a: Summations □ 0 □ 1 □ 2 □ 3 □ 4
Evaluate the sum: $\sum_{k=1}^{4}(3k)$

First, List out all the elements of the sequence $a_k = 3k$:
$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

Then, add each of the elements to get the result of $\sum_{k=1}^{4}(3k)$

6% Question 9b: Summations □ 0 □ 1 □ 2 □ 3 □ 4
Evaluate the sum: $\sum_{k=1}^{5}(4)$

First, List out all the elements of the sequence $a_k = 4$:
$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$ $a_5 =$

Then, add each of the elements to get the result of $\sum_{k=1}^{5}(4)$

Grading

Question	Weight	0-4	Adjusted score
1	4%		
2	12%		
3a	6%		
3b	6%		
4a	3%		
4b	3%		
4c	3%		
4d	3%		
5a	3%		
5b	3%		
5c	3%		
5d	3%		
6a	3%		
6b	3%		
6c	3%		
6d	3%		
7a	3%		
7b	3%		
7c	3%		
7d	3%		
8a	3%		
8b	3%		
8c	3%		
8d	3%		
9a	6%		
9b	6%		