4.1 Definitions, Diagrams, and Inverses

4.1.1 Function Terminology

Function: We use the notation $f : A \to B$ to specify a function f, which has inputs from the set A, and outputs from the set B. The function associates each input in A to one and only one output in B. [a] The notation $f : A \to B$ can be read as "f is a function from A to B".

Domain: The Domain is the set of inputs (A).

Codomain: The Codomain is the set of outputs (B).

[a] Discrete Mathematics, Ensley and Crawley

Question 1

Given the function:

$$g : \mathbb{Z} \to \mathbb{N} \qquad \text{...with the rule...} \qquad g(x) = x^2$$

a. What is the function name?

b. What is the domain?

c. What is the codomain?

d. Is 2 a valid domain value?

e. Is -2 a valid domain value?

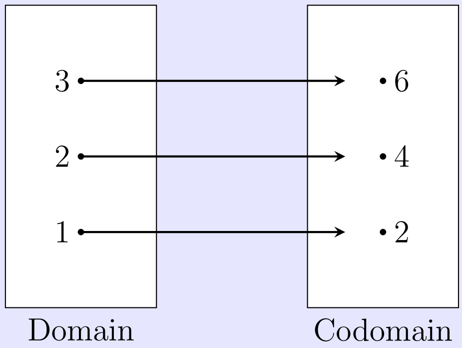
f. Is 4 a valid codomain value?

g. Is -4 a valid codomain value?

To describe a function, you need four items: [a] (1) Give the function a name, such as f, g, and h, (2) Describe the **domain**, (3) Describe the **codomain**, (4) Describe the **rule**.

Example: Function f, with Domain: $\{1, 2, 3\}$ and Codomain: $\{2, 4, 6\}$ and the Rule: $f(x) = 2x$.

Function diagram: With a diagram, arrows start at an element in the domain, and point to an element in the codomain.



[a] Discrete Mathematics, Ensley and Crawley

Question 2

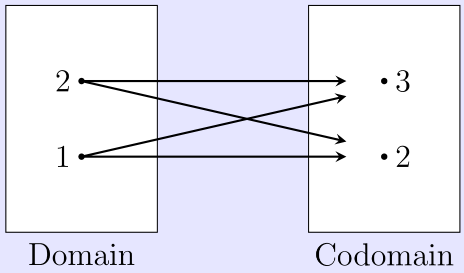
a. Define a function where the inputs and outputs are integers, and the relationship is that the output is the *square* of the input provided to the function.

b. Draw a diagram of the function. Include 5 values in the domain and in the co-domain.

4.1.2 Binary Relations

A **binary relation** R consists of three components: a domain A, a codomain B, and a subset of $A \times B$ called the “rule” for the relation. [a]

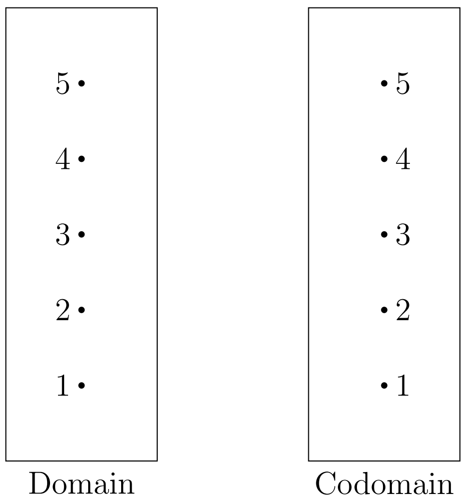
Example: Domain = $\{1, 2\}$, Codomain = $\{2, 3\}$, and the Rule is $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$.



[a] Discrete Mathematics, Ensley and Crawley

Question 3

Finish the arrow diagram for the following Binary Relation.
Domain: $\{1, 2, 3, 4, 5\}$
Codomain: $\{1, 2, 3, 4, 5\}$
Rule: { (1,5), (2,3), (3,3), (4,2), (5,1) }



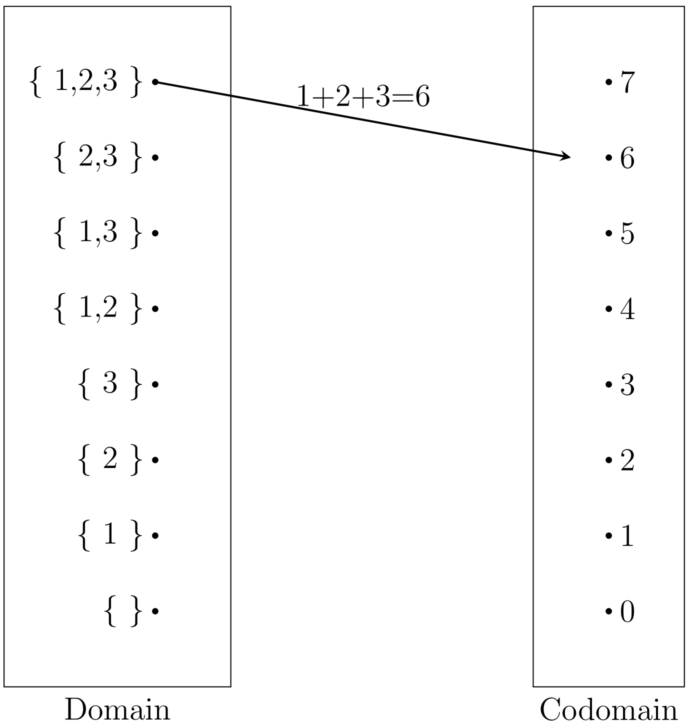
Question 4

Finish the arrow diagram for the following Binary Relation.

Domain: $\wp(\{1,2,3\})$, the Power Set of $\{1, 2, 3\}$.

Codomain: The set $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

Rule: $(S, n) \in \mathbb{R}$
This means that n is the **sum** of elements in the the set S given as an input.
For example, with the input set $\{1, 2\}$, the output will be $1 + 2$, or 3.



A function $f : A \to B$ is a binary relation with domain A and codomain B with the property that for every $x \in A$, there is **exactly one** element $y \in B$ for which $(x, y) \in f$. [a]

Bluntly, the pair (x, y) denotes that a line begins at element x from the domain, and points to the element y in the codomain.

[a] Discrete Mathematics, Ensley and Crawley

Question 5

Identify which of the following relations are also functions. Explain why not, if the relation is not a function. Also complete the diagrams given.

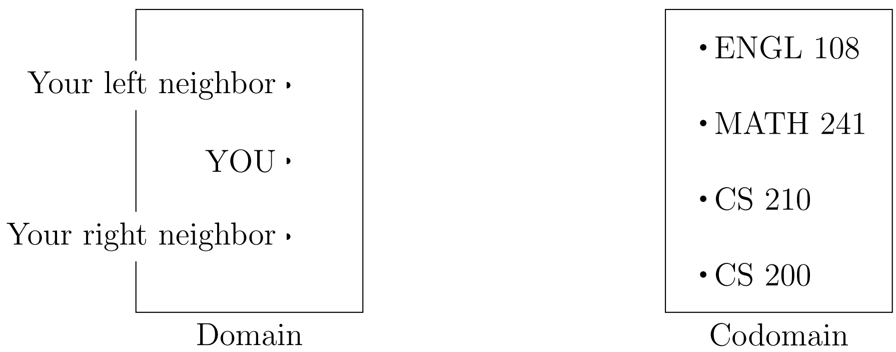
a. Relation R_1

Domain: The set $\mathbb{S}$ of all students at your college this semester.

Codomain: The set $\mathbb{C}$ of all classes offered at your college this semester.

Rule: (x, y) is in R_1 if student x is enrolled in class y this semester.

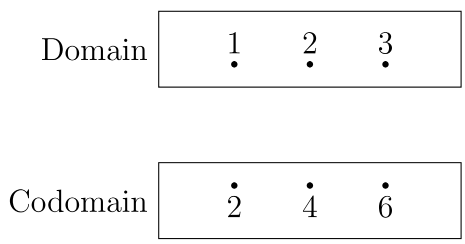
Let's use a small sample set. Fill it out to help you figure out if this is a function.



b. Relation R_2

Domain: The set $A = \{1, 2, 3\}$. Codomain: The set $B = \{2, 4, 6\}$.

Rule: (x, y) is in R_2 if $2x = y$.



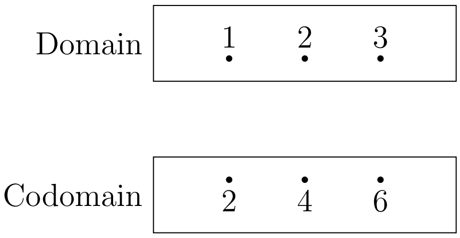
Question 6

Identify which of the following relations are also functions. Explain why not, if the relation is not a function. Also complete the diagrams given.

a. Relation R_3

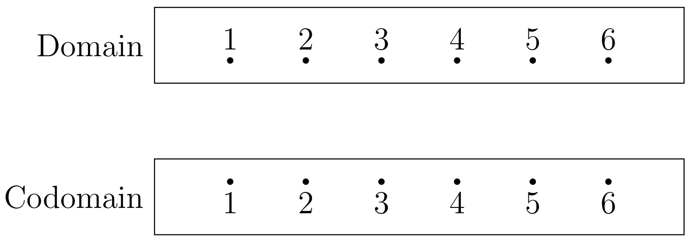
Domain: The set $A = \{1, 2, 3\}$. Codomain: The set $B = \{2, 4, 6\}$.
Rule: { (1,6), (2,2), (3,4) }

Let's use a small sample set. Fill it out to help you figure out if this is a function.



b. Relation R_4

Domain: The set $A = \{1, 2, 3, 4, 5, 6\}$. Codomain: The same set A.
Rule: (x, y) is in R_3 if $x - 1 = y$.



4.1.3 Inverse Relations

Given a relation R with domain A and codomain B, the relation R_{-1} (read "R inverse") with domain B and codomain A is called the **inverse** of R, and is defined so that

$$(x, y) \in R \qquad \text{if and only if} \qquad (y, x) \in R^{-1}$$

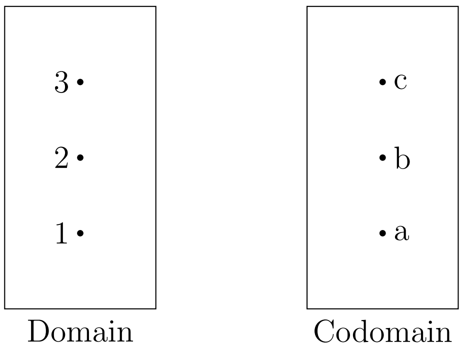
Also note that the inverse of R^{-1} is R. [a]

[a] Discrete Mathematics, Ensley and Crawley

Question 7

Draw the inverse of each diagram. Identify if the original, and/or the inverse, are functions.

a.



b.