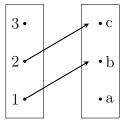
4.3 Properties of Functions and Set Cardinality

4.3.1 Review: Inverses of functions

Question 1

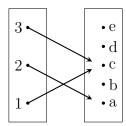
Draw the inverse of each diagram. Identify whether the original diagram and/or the inverse of that diagram are functions.

a.



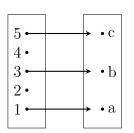
Domain Codomain

b.

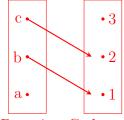


Domain Codomain

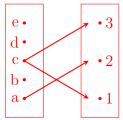
c.



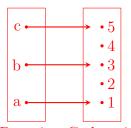
Domain Codomain



Domain Codomain



Domain Codomain



Domain Codomain

- a. Original: Not a function; all inputs must have an output.
- a. Inverse: Not a function; all inputs must have an output.
- b. Original: Function
- b. Inverse: Not a function; "c" points to two different outputs.
- c. Original: Not a function; "2" and "4" don't have any outputs.
- c. Inverse: Function

4.3.2 Functions that are invertible

Question 2

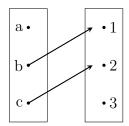
Draw two functions: One where the function is one-to-one but not onto, and one where the function is onto but not one-to-one. Make sure to label your domain and codomain for each.

Multiple solutions

Question 3

Determine whether these functions are one-to-one, onto, and/or invertible. If not, state why not.

a.

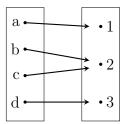


Domain Codomain It is

not a function, but it is onto

- \square Onto \square One-to-one
- □ Invertible

b.

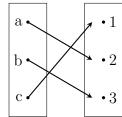


Domain Codomain

Onto, not one-to-one, not invertible

- $\hfill\Box$ Onto $\hfill\Box$ One-to-one
- $\hfill\Box$ Invertible

c.

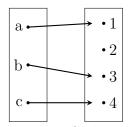


Domain Codomain

Onto, One-to-one, Invertible

- \square Onto \square One-to-one
- □ Invertible

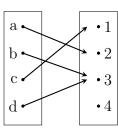
d.



Domain Codomain Or to-one, not onto, not invertible

- \square Onto \square One-to-one
- $\hfill\Box$ Invertible

e.

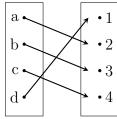


Domain Codomain Not

onto, not one-to-one, not invertible

- □ Onto □ One-to-one
- □ Invertible

f.



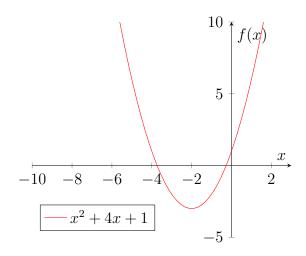
Domain Codomain

Onto, one-to-one, invertible

- □ Onto □
 - □ One-to-one
- \square Invertible

Question 4

The function $f: \mathbb{R} \to \mathbb{R}$, with the rule $f(x) = x^2 + 4x + 1$ is not onto and not one-to-one.



- a. Give an example of an element in the codomain that has no element in the domain associated with it. There is no $x \in \mathbb{R}$ for which f(x) = -4 since the equation $x^2 + 4x + 1 = -4$ has no **real** solutions (by using the quadratic formula).
- b. Given an example of two elements in the domain that are both associated with the same output in the codomain. f(-1) = 1 4 + 1 = -2 and f(-3) = 9 12 + 1 = -2.