Answer Key

1a. p: You play, w: You win,
$\neg p \to \neg w$

1b. d: Your friends dance, f: Your friends are my friends,
$\neg d \to \neg f$

1c. e: Timmy's age is over 8, t: Timmy's age is less than 13,
m: Timmy gets Tween-priced movie tickets.
$(e \wedge t) \to m$

1d. s: I get enough sleep, c: I drink coffee, w: I go to work.
$(s \vee c) \to w$

2a. 2b.

p	q	r	a. $(p \wedge q) \to r$	b. $p \to (q \vee r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	T	F

2c.

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

3a. $P(x) \to Q(X)$

3b. $P(x) \to \neg Q(x)$

3c. $\neg P(x) \to Q(x)$

3d. $\neg P(x) \to Q(x)$

4a. $i \wedge \neg j$: You go to New Delhi and you do NOT see the Jantar Mantar.

4b. $(j \wedge c) \wedge \neg b$: Jessica gets chocolate AND Jessica gets cake AND Jessica DOES NOT have a happy birthday.

4c. $p \wedge \neg(w \wedge s) \equiv p \wedge \neg w \vee \neg s$: You have a group project AND neither you do all the work, nor someone else does all the work.

5a. $\forall x \in D, \neg B(x) \rightarrow H(x)$

- Converse: $\forall x \in D, H(x) \rightarrow \neg B(x)$
  For all people x, if x is hungry, then x didn't eat breakfast
- Inverse: $\forall x \in D, B(x) \rightarrow \neg H(x)$
  For all people x, if x ate breakfast then x is NOT hungry
- Contrapositive: $\forall x \in D, \neg H(x) \rightarrow B(x)$
  For all people x, if x is NOT hungry, then x ate breakfast

5b. $\forall x \in D, (P(x) \vee G(x)) \rightarrow M(x)$

- Converse: $\forall x \in D, M(x) \rightarrow (P(x) \vee G(x))$
  For all people x, if x is a musician, then x plays piano or x plays guitar
- Inverse: $\forall x \in D, \neg(P(x) \vee G(x)) \rightarrow \neg M(x)$
  For all people x, if x does NOT play piano and does NOT play guitar, then x is NOT a musician
- Contrapositive: $\forall x \in D, \neg M(x) \rightarrow \neg(P(x) \vee G(x))$
  For all people x, if x is NOT a musician, then x doesn't play piano and x doesn't play guitar.

5c. $\forall x \in \mathbb{R} : P(x) \rightarrow Q(x)$

- Converse: $\forall x \in D, Q(x) \rightarrow P(x)$
  For all real numbers x, if $x^2 > 4$ then $x > 2$.
- Inverse: $\forall x \in D, \neg P(x) \rightarrow \neg Q(x)$
  For all real numbers x, if $x \leq 2$ then $x^2 \leq 4$
- Contrapositive: $\forall x \in D, \neg Q(x) \rightarrow \neg P(x)$
  For all real numbers x, if $x^2 \leq 4$ then $x \leq 2$

5d. $\forall x \in \mathbb{Z} : R(x) \rightarrow S(x)$

- Converse: $\forall x \in D, S(x) \rightarrow R(x)$
  For all integers x, if $x^3 = 1$ then $x^2 = 1$
- Inverse: $\forall x \in D, \neg R(x) \rightarrow \neg S(x)$
  For all integers x, if $x^2 \neq 1$ then $x^3 \neq 1$
- Contrapositive: $\forall x \in D, \neg S(x) \rightarrow \neg R(x)$
  For all integers x, if $x^3 \neq 1$ then $x^2 \neq 1$