

4.3 Properties of Functions and Set Cardinality

4.3.1 Review: Inverses of functions

Given a relation R with domain A and codomain B , the relation R_{-1} (read “ R inverse”) with domain B and codomain A is called the **inverse** of R , and is defined so that

$$(x, y) \in R \quad \text{if and only if} \quad (y, x) \in R^{-1}$$

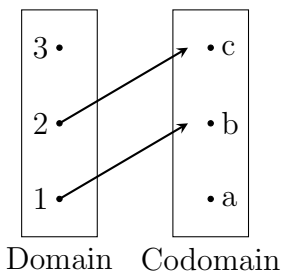
Also note that the inverse of R^{-1} is R .^a

^aDiscrete Mathematics, Ensley and Crawley

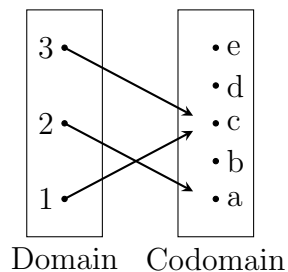
Question 1

Draw the inverse of each diagram. Identify whether the original diagram and/or the inverse of that diagram are functions.

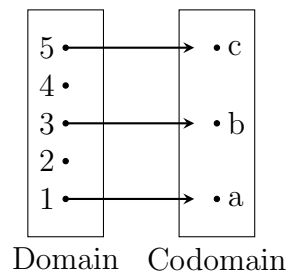
a.



b.



c.



4.3.2 Functions that are invertible

Functions that are onto

A function is **onto** if every output from the codomain has at least one input from the domain. Every output is attainable via at least one input.

For diagrams: Every point in the codomain has at least one arrow pointing at it.

Functions that are one-to-one

A function is **one-to-one** if every output from the codomain has no more than one input from the domain. No two inputs lead to the same output.

For diagrams: None of the points in the codomain has two or more arrows pointing at it.

Functions that are invertible

If a function is both one-to-one AND onto, then it is invertible. This means that the inverse of the function is ALSO a function.

For diagrams: Every point in the codomain has exactly one arrow pointing at it.

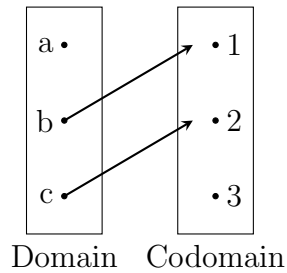
Question 2

Draw two functions: One where the function is one-to-one but not onto, and one where the function is onto but not one-to-one. Make sure to label your domain and codomain for each.

Question 3

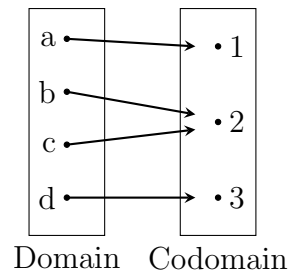
Determine whether these functions are one-to-one, onto, and/or invertible. If not, state why not.

a.



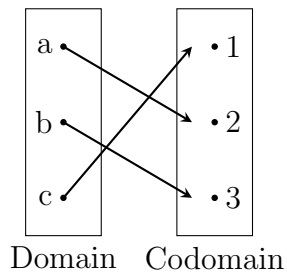
- ☐ Onto ☐ One-to-one
☐ Invertible

b.



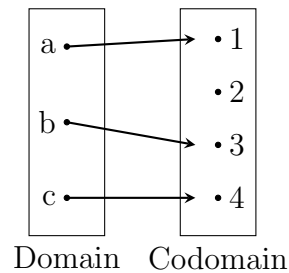
- ☐ Onto ☐ One-to-one
☐ Invertible

c.



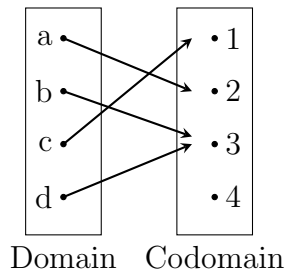
- ☐ Onto ☐ One-to-one
☐ Invertible

d.



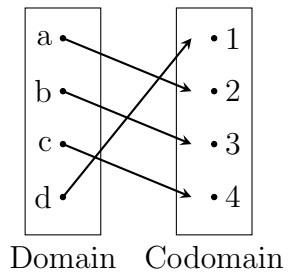
- ☐ Onto ☐ One-to-one
☐ Invertible

e.



- ☐ Onto ☐ One-to-one
☐ Invertible

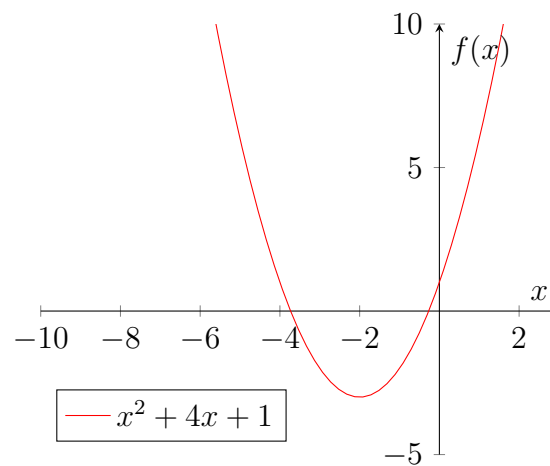
f.



- ☐ Onto ☐ One-to-one
☐ Invertible

Question 4

The function $f : \mathbb{R} \rightarrow \mathbb{R}$, with the rule $f(x) = x^2 + 4x + 1$ is not onto and not one-to-one.



- a. Give an example of an element in the codomain that has no element in the domain associated with it.

- b. Given an example of two elements in the domain that are both associated with the same output in the codomain.