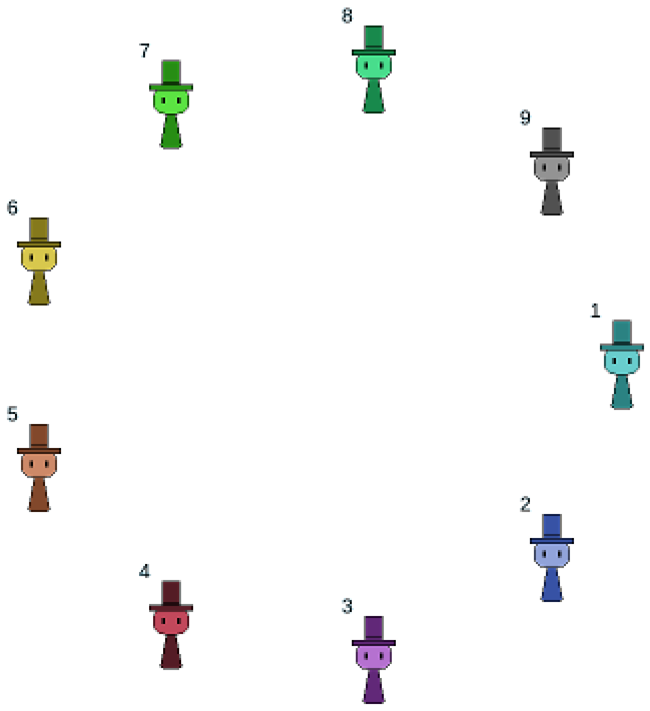
1. Thinking Logically

Written by Rachel J. Morris, last updated Jun 23, 2017

About



This part is meant to get you thinking *systematically* and *logically* about solving problems.

These problems can be solved by following some pattern, step-by-step, and we do this as an introduction, rather than coming up with some algorithm to solve the problems.

Topics

1. Flipping a coin
2. Josephus game
3. Game tree

1. Flipping a coin

Let’s start off simple.

We are going to be flipping one coin. How many possible outcomes are there?

1. Flipping a coin

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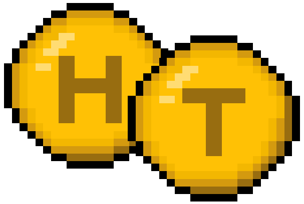
We are going to be flipping one coin.
How many possible outcomes are there?

Two possible outcomes: Heads or Tails



1 coin,
2 outcomes

1. Flipping a coin

If we have two coins, how many total outcomes are there?

1 

2 

1 coin,
2 outcomes

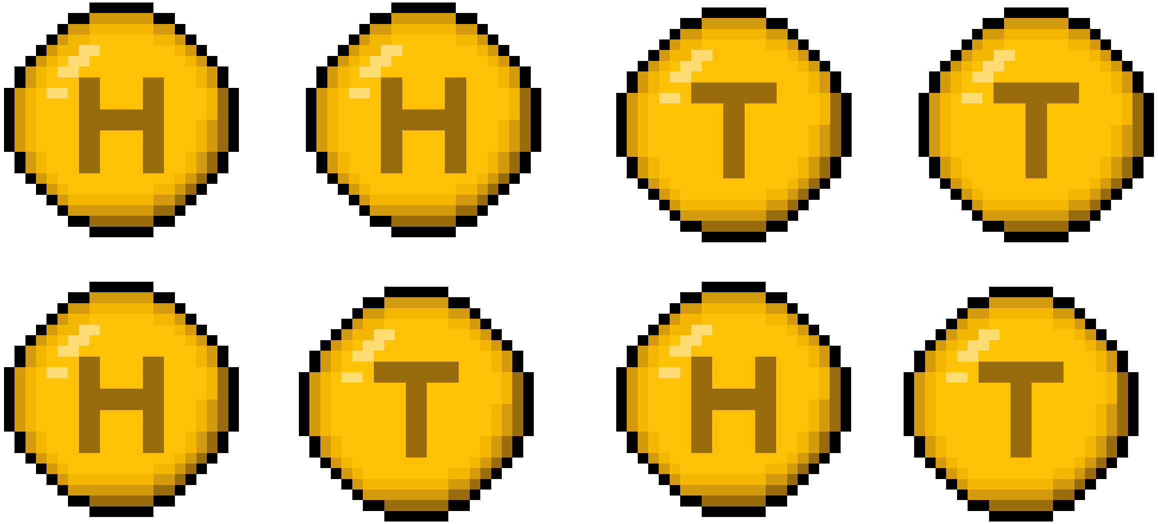
1. Flipping a coin

If we have two coins, how many total outcomes are there?

There are four possible outcomes.

1

2



1 coin,
2 outcomes

2 coins,
4 outcomes

1. Flipping a coin

If we have three coins, how many total outcomes are there?

1 

2 

3 

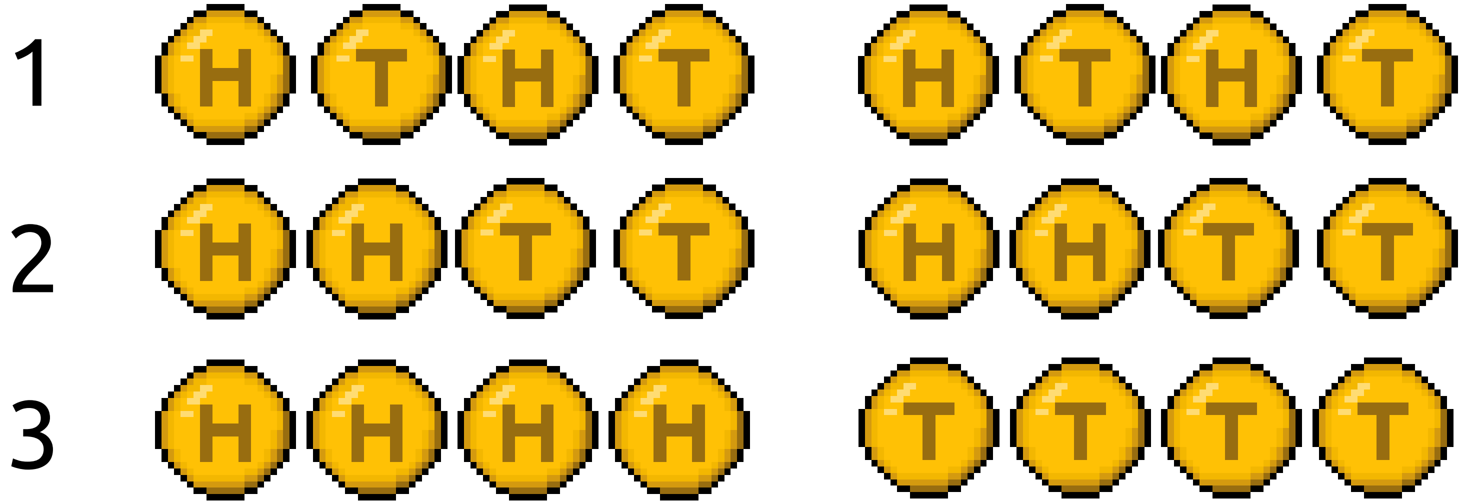
1 coin,
2 outcomes

2 coins,
4 outcomes

1. Flipping a coin

If we have three coins, how many total outcomes are there?

There are eight possible outcomes



1 coin,
2 outcomes

2 coins,
4 outcomes

3 coins,
8 outcomes

1. Flipping a coin

Can you see the pattern in these examples?

1 coin	=	2 outcomes
2 coins	=	4 outcomes
3 coins	=	8 outcomes

If we have any *arbitrary* number of coins *n*, how many outcomes will there be?

1 coin,
2 outcomes

2 coins,
4 outcomes

3 coins,
8 outcomes

1. Flipping a coin

Can you see the pattern in these examples?

1 coin	=	2 outcomes
2 coins	=	4 outcomes
3 coins	=	8 outcomes

If we have any *arbitrary* number of coins *n*, how many outcomes will there be?

For any $n > 0$, there will be 2^n outcomes.

1 coin,
2 outcomes

2 coins,
4 outcomes

3 coins,
8 outcomes

n coins,
2^n outcomes

2. Josephus Game

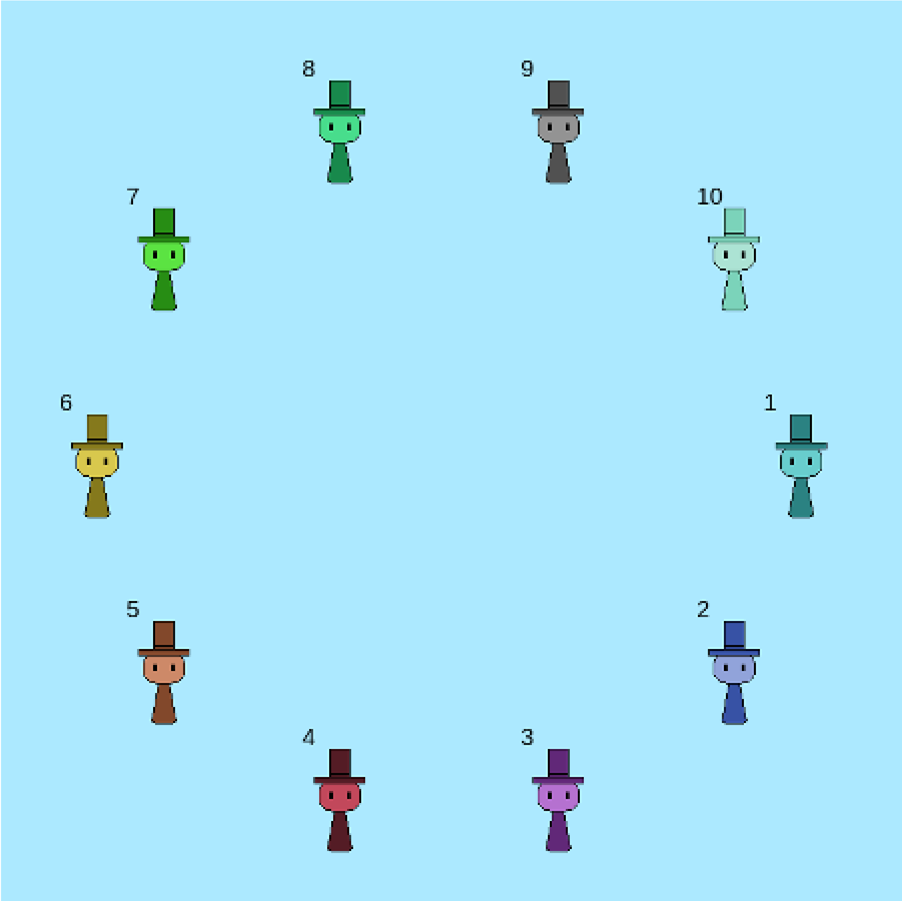
You can read about the Josephus game in the textbook, but the general overview is:

Josephus and other rebels are sitting in a circle. One-by-one, they go around the circle, and every n^{th} person dies. The last person alive can escape.

How do we figure out who is the last to go? And maybe the 2nd to last, in case Josephus wants a buddy with him.

Josephus Game:
I guess we can illustrate math with death… ok...

2. Josephus Game



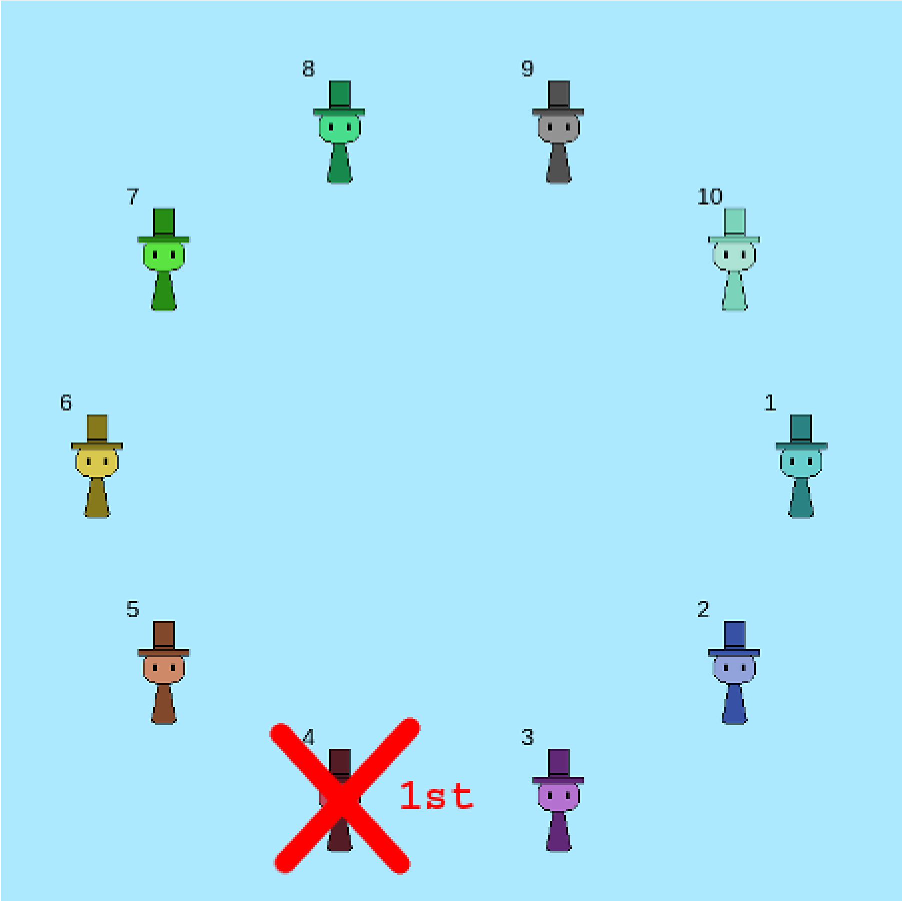
So let's say we have 10 total people sitting in a circle,

and we "remove" every 4th person,

starting with person at position 4.

Link to JavaScript illustration of the game: http://edu.moosader.com/course/cs210/illustration_ch_1.1_josephus.php

2. Josephus Game



Goodbye 4

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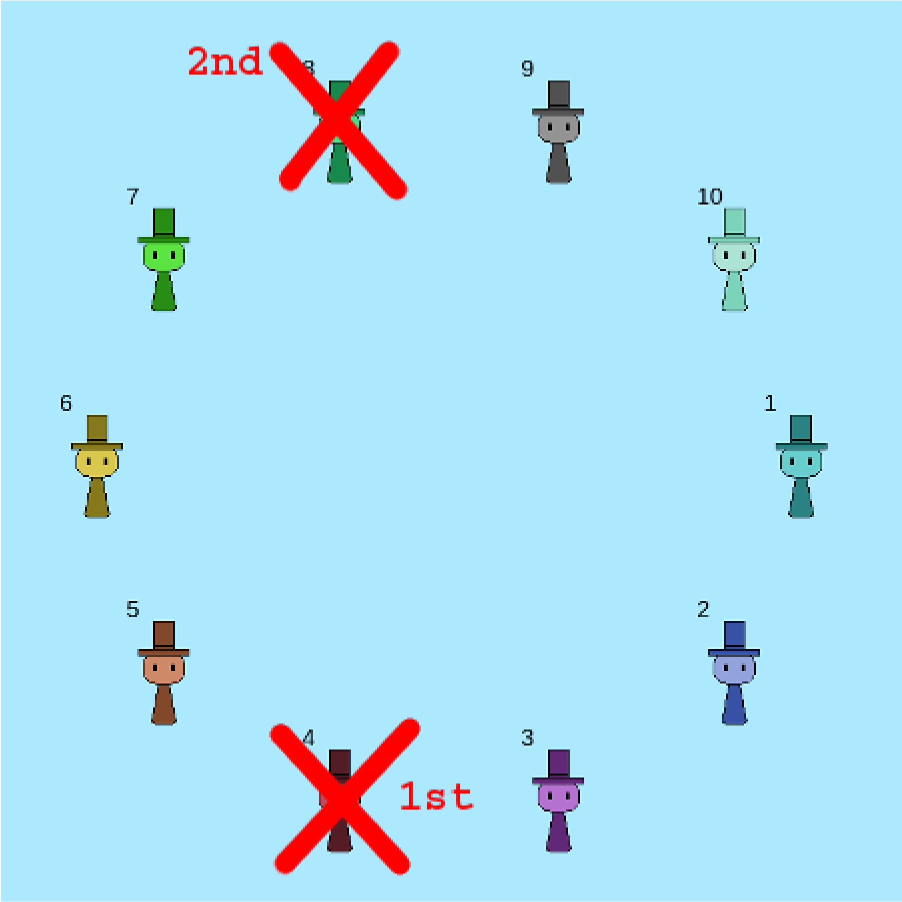
2. Josephus Game



1. Person 5…
2. Person 6…
3. Person 7…
4. Person 8…

Goodbye,
Person 8.

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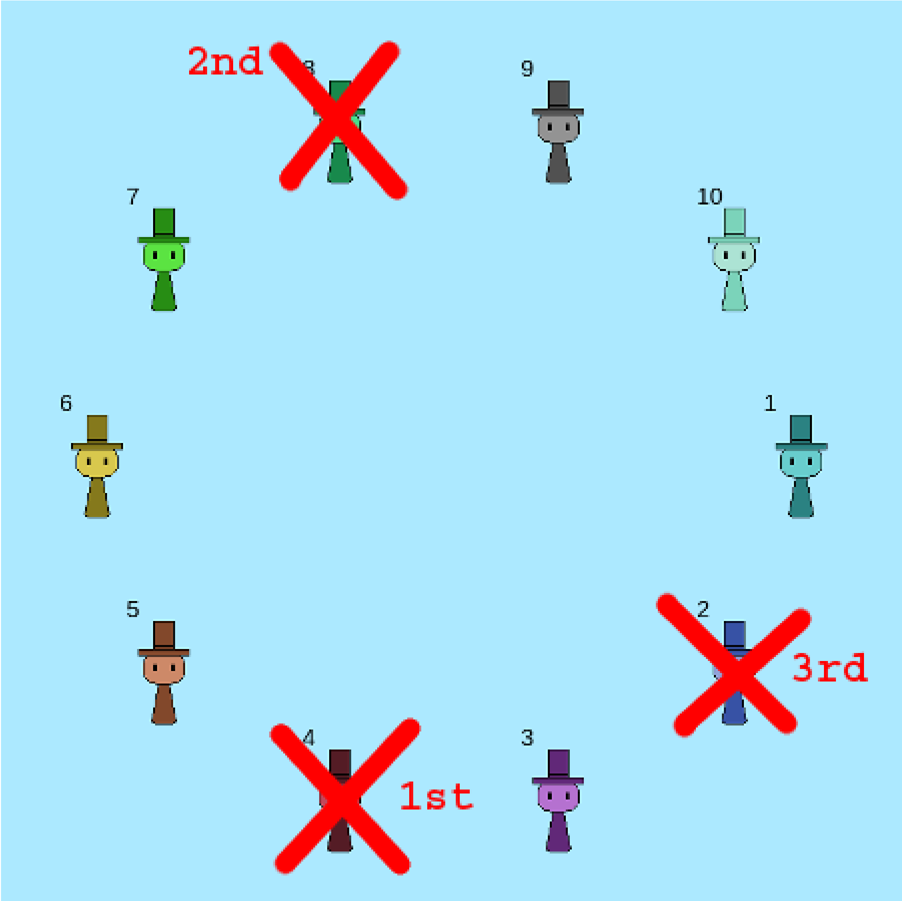
2. Josephus Game



1. Person 9...
2. Person 10...
3. Person 1...
4. Person 2...

Goodbye,
Person 2.

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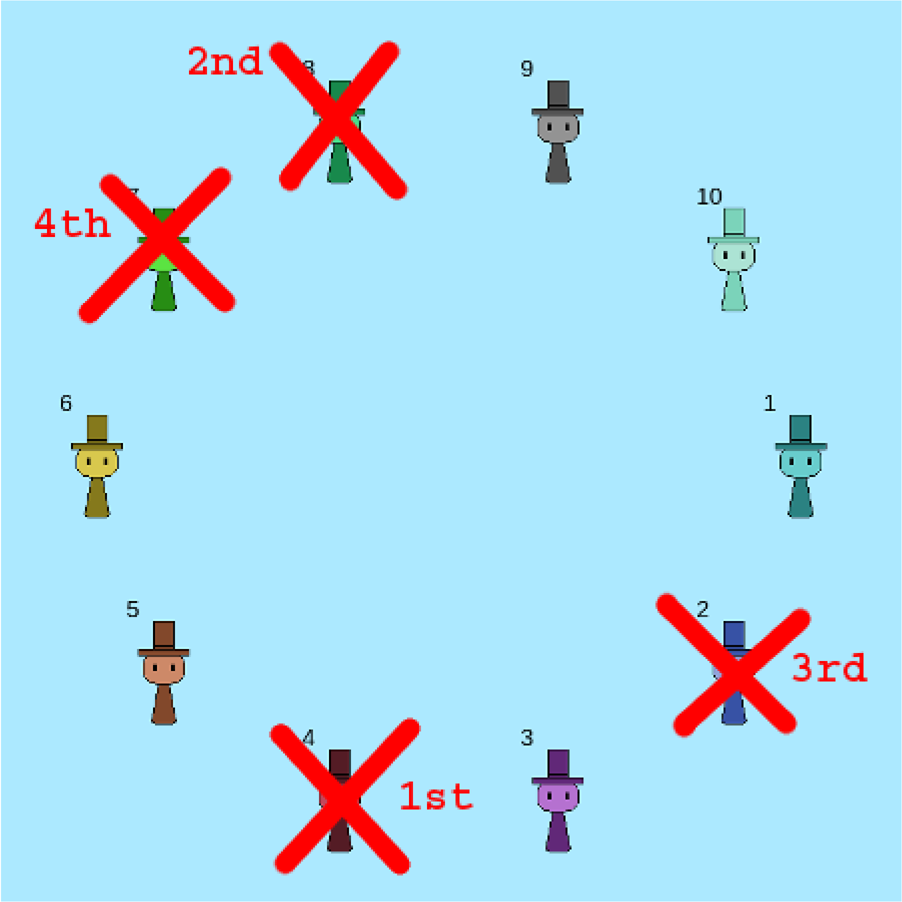
2. Josephus Game



1. Person 3…
2. Person 5…
3. Person 6…
4. Person 7…

Goodbye,
Person 7.

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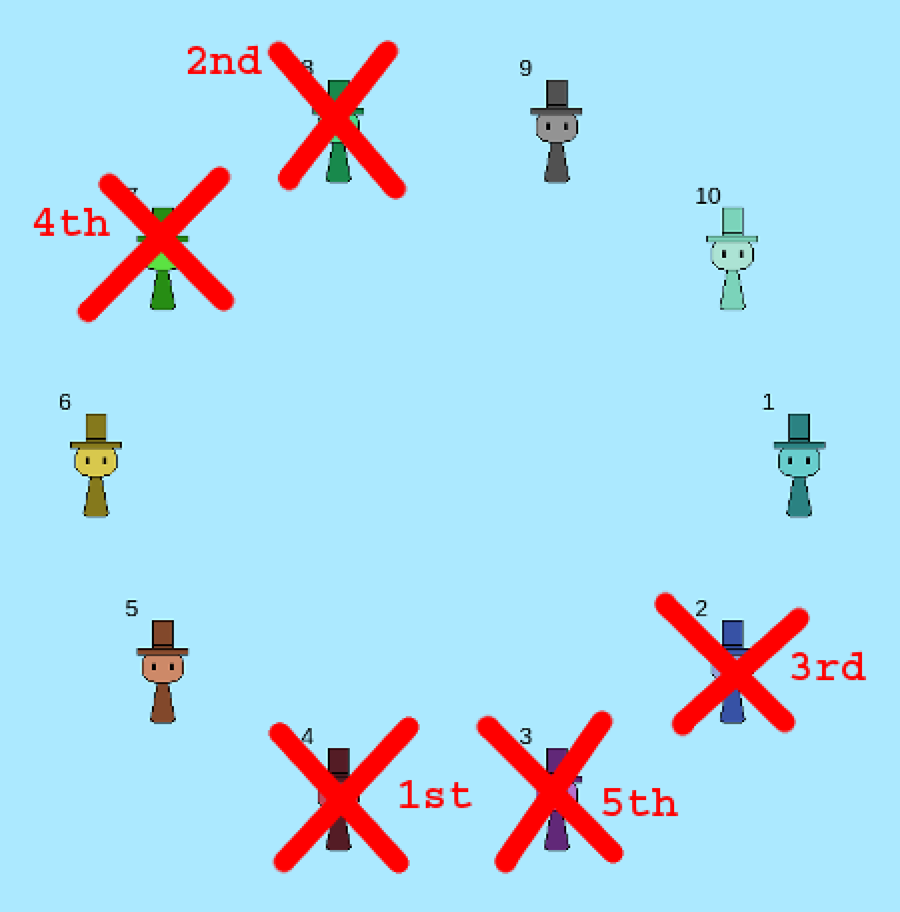
2. Josephus Game



1. Person 9…
2. Person 10…
3. Person 1…
4. Person 3…

Goodbye,
Person 3.

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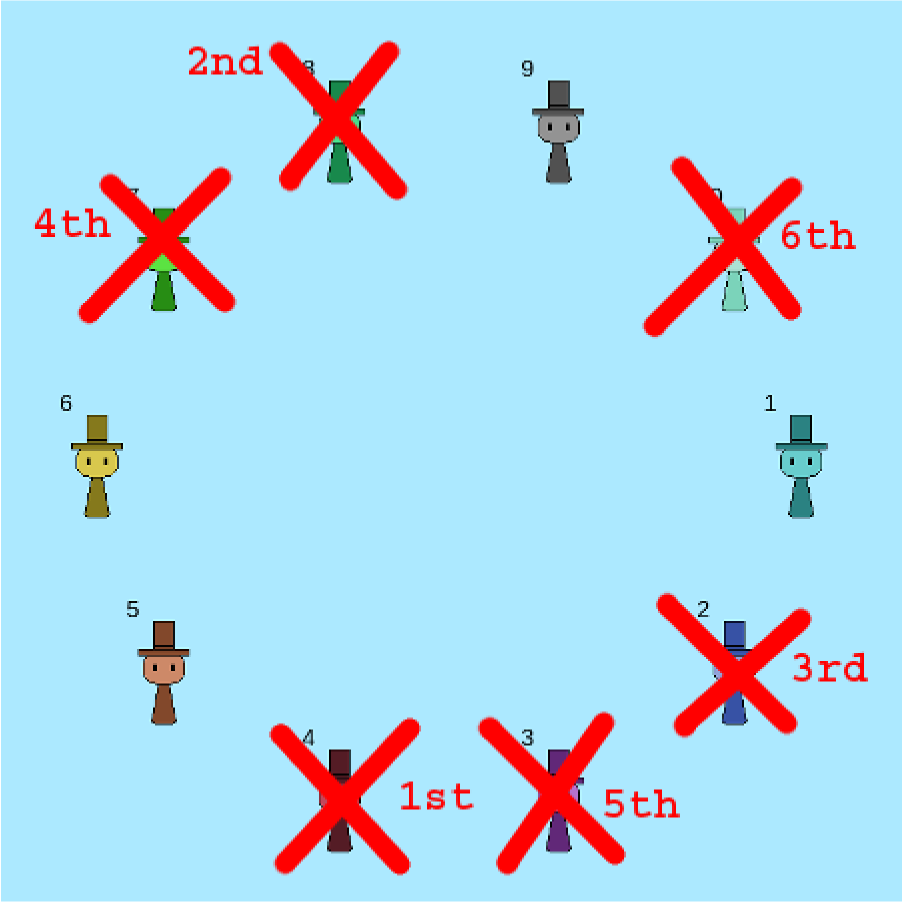
2. Josephus Game

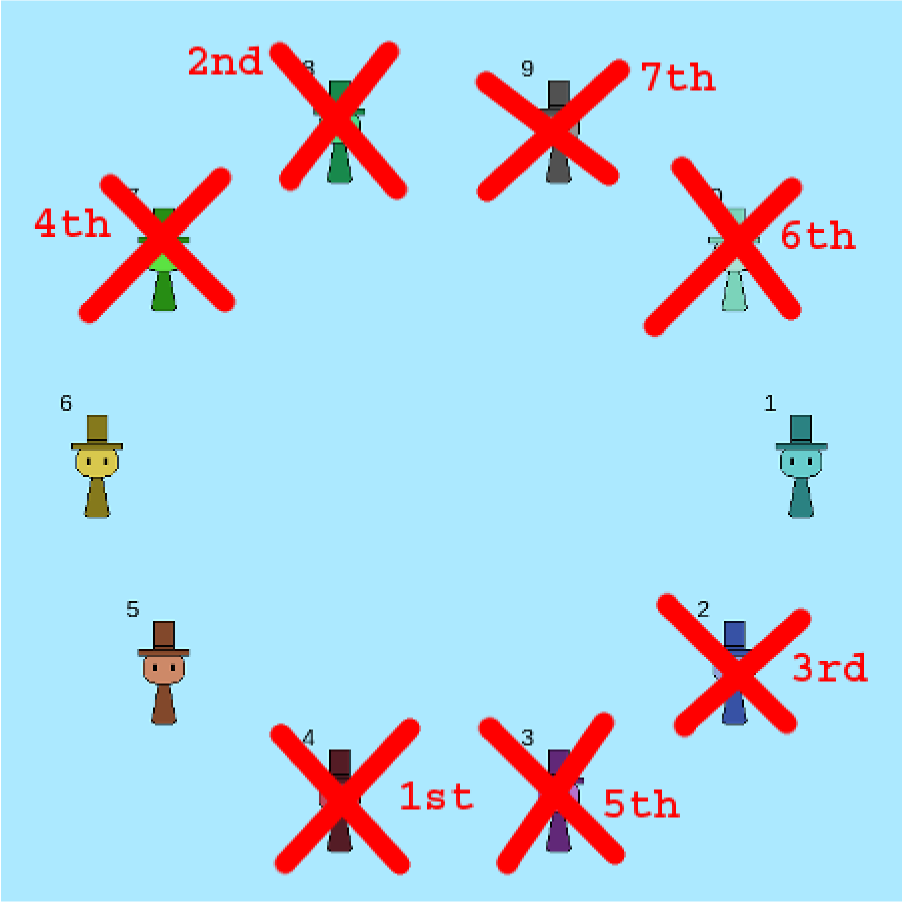


1. Person 5…
2. Person 6…
3. Person 9…
4. Person 10…

Goodbye, Person 10.

Link to JavaScript illustration of the game: http://edu.moosader.com/course/cs210/illustration_ch_1.1_josephus.php

2. Josephus Game



1. Person 1…
2. Person 5…
3. Person 6…
4. Person 9…

Goodbye,
Person 9.

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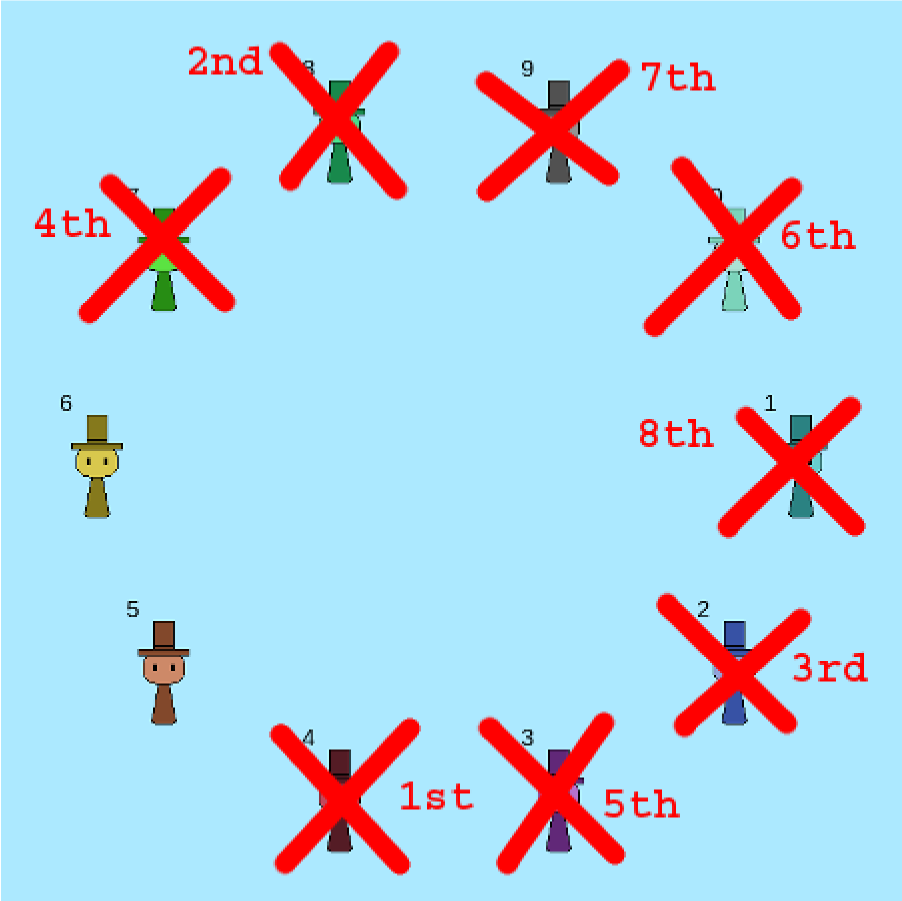
2. Josephus Game



1. Person 1...
2. Person 5...
3. Person 6...
4. Person 1...

Goodbye, Person 1.

Link to JavaScript illustration of the game: http://edu.moosader.com/course/cs210/illustration_ch_1.1_josephus.php

2. Josephus Game

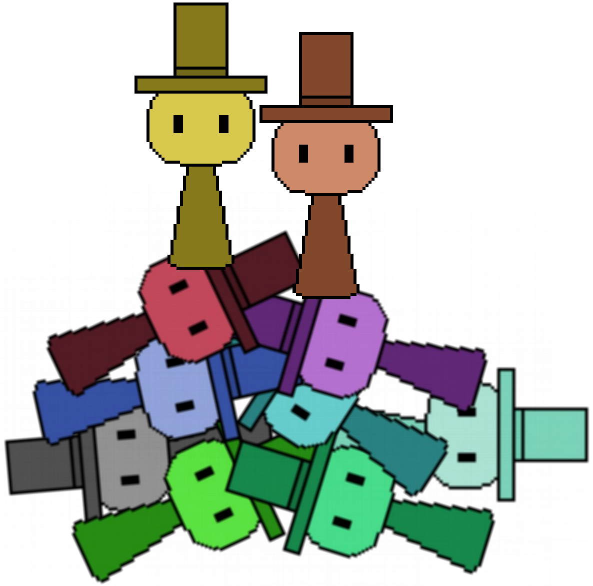
1. Person 5...
2. Person 6...
3. Person 5...
4. Person 6...

Person 6 is 2nd-to-last and Person 5 is last,

so this is where Josephus and his friend should sit.

Link to JavaScript illustration of the game: http://edu.moosader.com/course/cs210/illustration_ch_1.1_josephus.php

2. Josephus Game



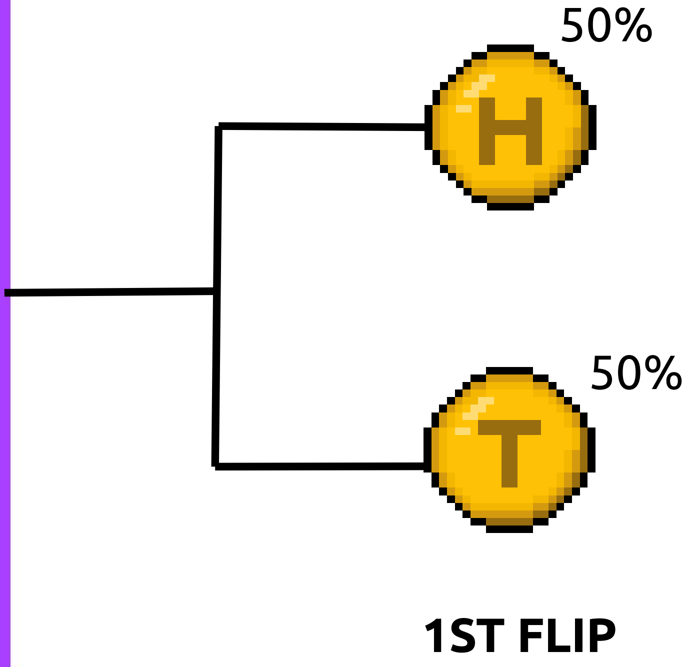
We can take a systematic approach to solving this problem by stepping through the "algorithm"

(every *n*th person, skipping already-dead ones)

Link to JavaScript illustration of the game:
http://edu.moosader.com/course/cs210/illustration_ch_1.1_josephus.php

3. Game Tree

We can use a Game Tree to help us estimate the likelihood of some outcome-set, such as if we were flipping a coin:



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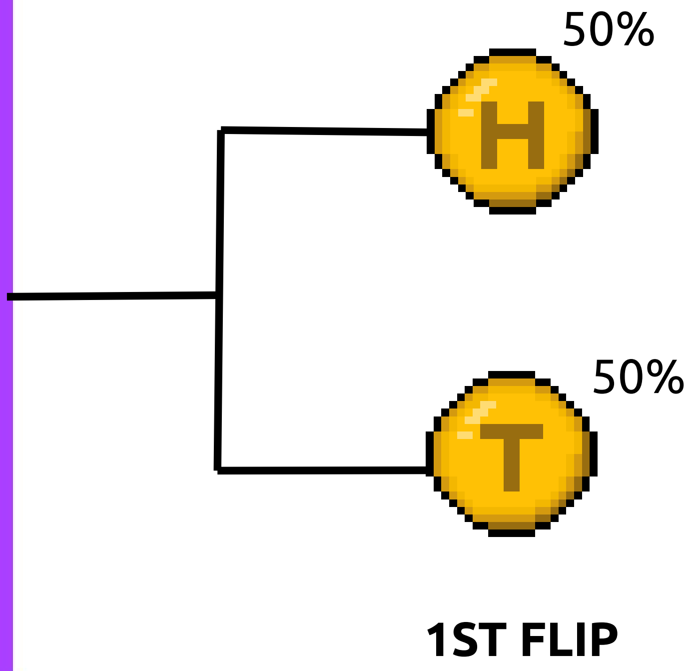

If we're just flipping one fair coin, there are two outcomes, and they are equally likely. So we can say the probability is:

- Heads: 1 out of 2 outcomes (50%)
- Tails: 1 out of 2 outcomes (50%)

3. Game Tree

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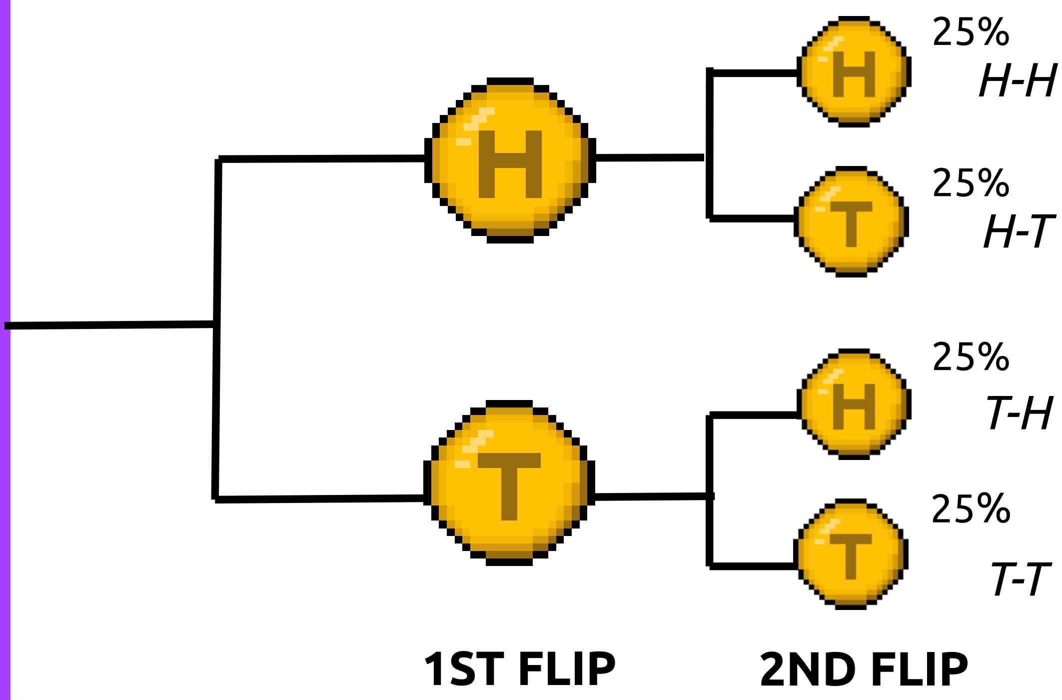

If we're flipping a fair coin twice, we have four outcomes, and the likelihood of getting the following sequences are:

- Heads-Heads: 1 out of 4 (25%)
- Heads-Tails: 1 out of 4 (25%)
- Tails-Heads: 1 out of 4 (25%)
- Tails-Tails: 1 out of 4 (25%)

3. Game Tree

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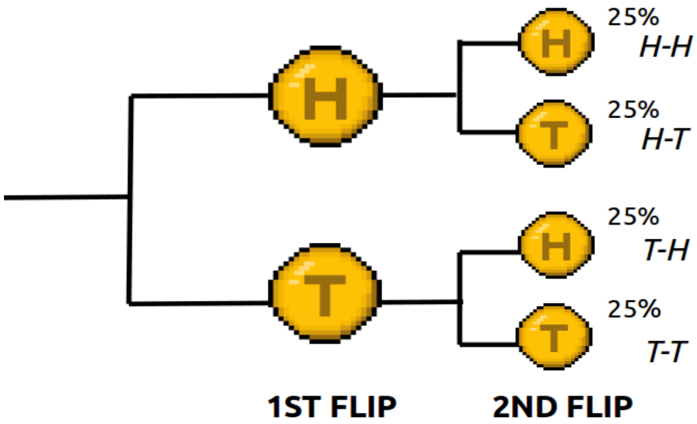
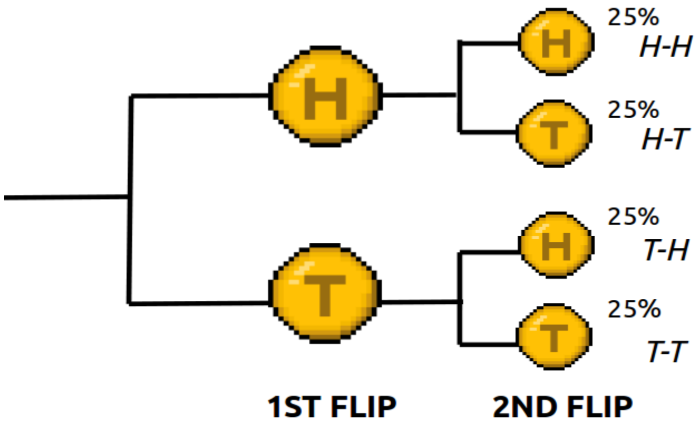


Those might be obvious, but what about these?

- What is the likelihood that the sequence will have *at least one heads* in it?
- What is the likelihood that the sequence will have *exactly one tails* in it?

3. Game Tree

We can use a Game Tree to help us estimate the likelihood of some outcome-set, such as if we were flipping a coin:



Those might be obvious, but what about these?

- What is the likelihood that the sequence will have *at least one heads* in it?
  - **Possible outcomes: H-H, H-T, T-H, so 3 out of 4 outcomes (75%)**
- What is the likelihood that the sequence will have *exactly one tails* in it?
  - **Possible outcomes: T-H, H-T, so 2 out of 4 outcomes (50%)**

Conclusion

So at this point, we're not doing much in the way of *math*, but we are getting some practice in being methodical and thinking about these things as systems.

Next we will be looking at sequences of numbers, and how to come up with a function to represent that sequence, so it can take some work analyzing your numbers to figure out what is in common between them!