Answer Key

1. a. If n is odd, then $n+1$ is even.

   b. If s is a square, then the length of every side is l.

   c. If n is a prime number, then n is odd.

2. a. This is false for all even numbers. For example, 2 and 2+1 = 3!

   b. This is false for 0!, because 0! = 1.

   c. We can find an example for $n = 9$: $9^2+4 = 85$, and this is divisible by 5 and 17.

3. a. $15 = 2 \cdot 3 + 1$

   b. $15 = 2 \cdot 4 + 1$

   c. $15 = 2 \cdot 7 + 1$

   d. $8 = 2 \cdot 4$

   e. $16 = 2 \cdot 8$

   f. $20 = 2 \cdot 10$

4. a. 100 is even.
   $100 = 2(50)$

   b. 13 is odd.
   $13 = 2(6) + 1$

   c. -13 is odd.
   $13 = 2(-7) + 1$

   d. 20 is divisible by 5.
   $20 = 5(4)$

   e. 20 is divisible by 4.
   $20 = 4(5)$

   f. $6n$ is even.
   $2(3n)$

   g. $8n^2 + 8n + 4$ is divisible by 4.
   $8n^2 + 8n + 4 = 4(2n^2 + 2n + 1)$

5. a. 2 + 8
   $10 \in \mathbb{Z}$

   b. 12 - 4
   $8 \in \mathbb{Z}$

c. 5 * 3
$15 \in \mathbb{Z}$

d. 6 / 3
$2 \in \mathbb{Z}$

e. 5 / 2
$2.5 \notin \mathbb{Z}$

6. a. For all integers $n > 0$, if n is even, then n^2 is also even.
$n = 2k$
n^2 => $(2k)(2k)$ => $2(2k^2)$

b. For all integers $n > 0$, if n is odd, then $n^2 + n$ is even.
$n = 2k + 1$
$(2k+1)^2 + (2k+1)$ => $(2k+1)(2k+1) + (2k+1)$
$(4k^2 + 4k + 1) + (2k + 1)$ => $4k^2 + 4k + 2k + 1 + 1$
$4k^2 + 6k + 2$ => $2(2k^2 + 3k + 1)$