2.6   Numerical Representation

2.6.1   Intro practice

Question 1

The set of digits in base-10 (decimal) number system is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Use this information to help you figure out the following.

   a. Write out the set of digits in the octal (base-8) number system.

   b. Write out the set of digits in the binary (base-2) number system.

The hexadecimal (base-16) number system is

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}.$$

Why do we use letters? To keep numbers 10 through 15 as one-character representations.

Specifying base

When we need to specify the base when writing out numbers, write it within parentheses, with a subscript of its base number.

- $(123)_{10} = 123$, base-10

- $(1337)_8 = 1337$, base-8

- $(C47)_{16} = $ C47, base-16

- $(1011)_2 = 1011$, base-2

2.6.2   Digits

For the decimal number $2,368$, we can write this as its individual digits:

Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Ones (10^0)
2	3	6	8

And then we can build out $2,368$ as the mathematical equation:

$$2 \cdot 10^3 + 3 \cdot 10^2 + 6 \cdot 10^1 + 8 \cdot 10^0$$

Likewise, for the binary number 0101 1001, we can write it as:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	1	0	0	1

And into the equation:

$$1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^0$$

Question 2

Expand each of the following numbers as a mathematical equation. Make sure to pay attention to the *base* value.

a. Write out the equation for $(19)_{10}$

10^1	10^0

b. Write out the equation for $(0010\ 1101)_2$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

c. Write out the equation for $(FFAA66)_{16}$

16^5	16^4	16^3	16^2	16^1	16^0

2.6.3   Converting between bases

Algorithm for converting a decimal number to base b:

1. Input a natural number n

2. While $n > 0$, do the following:

   (a) Divide n by b and get a quotient q and remainder r.

   (b) Write r as the next (right-to-left) digit.

   (c) Replace the value of n with q, and repeat.

Question 3
Convert the following between bases:

   a. Convert $(35)_{10}$ to binary (base-2) $n = 35,\ b = 2$

   b. Convert $(125)_{10}$ to binary (base-2) $n = 125,\ b = 16$

Hexadecimal to Binary

Often in computers, we write binary strings as hexadecimal to save space and make it easier to read.

Hex	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hex	8	9	A	B	C	D	E	F
Binary	1000	1001	1010	1011	1100	1101	1110	1111

Example: Convert 11001 from binary to hexadecimal
1. Write out in chunks of four. Add leading 0's to the left side.
     0001 1001
2. Swap out each "nibble" with hexadecimal
     0001 = 1   1001 = 9

So, $(0001\ 1001)_2 = (19)_{16}$

Example: Convert DAD from hexadecimal to binary
1. Convert each digit back to binary.
     D = 1101   A = 1010   D = 1101

So, $(DAD)_{16} = (1101\ 1010\ 1101)_2$

Question 4
Do the following conversions

     a. Convert $(1F0B)_{16}$ to binary:

     b. Convert $(0100\ 0110)_2$ to hexadecimal: