1. Introductory Practice

For the statement, "If n mod 3 = 1, then n mod 9 $\neq$ 5."

a) What is the **hypothesis *p*?** (__/1)

b) What is the **conclusion *q*?** (__/1)

c) Remember that the negation of an implication is: $\neg(p \rightarrow q) \equiv p \wedge \neg q$ (__/1)
Write out the original statement (in English terms) as a contradiction.

2. Proof by contradiction

Example: "Prove by contradiction: If n^2 is even, then n is even."

Hypothesis: n^2 is even Conclusion: n is even

A contradiction would be $\neg(p \rightarrow q) \equiv p \wedge \neg q$, or in English:
Hypothesis: n^2 is even Conclusion: n is odd

Write in math terms: $n^2 = 2K$ $n = 2L + 1$

Make equation: $(2L+1)^2 = 2K$
Simplify: $4L^2 + 4L + 1 = 2K$
$1 = 2K - 4L^2 - 4L$
$\frac{1}{2} = K - 2L^2 - 2L$

Result: As K and L are both integers, and through the closure property of integers (addition, subtraction, and multiplication of integers result in an integer), as $K - 2L^2 - 2L$ results in something that is *not an integer*, it shows that our counterexample is false and no counterexample can exist.

Practice 1 (__/1)

"Prove by contradiction: If n^2 is odd, then n is odd."

Practice 2 (__/1)

"Use proof by contradiction to explain why it is impossible for a number *n* to be of form 5K+3 and of the form 5L+1 for integers *K* and *L*."

(Hint: $n=5K+3$ and $n=5L+1$, so $5K+3=5L+1$ is your starting point, and keep in mind the *closure principle of integers*!)