

## 1. More definitions

### Modulus

“In computing, the modulo operation finds the remainder after division of one number by another (sometimes called modulus).

Given two positive numbers,  $a$  (the dividend) and  $n$  (the divisor),  $a \bmod n$  (abbreviated as  $a \bmod n$ ) is the remainder of the Euclidean division of  $a$  by  $n$ .”<sup>a</sup>

$$\begin{array}{r} 4r1 \\ 2 \overline{)9} \\ \underline{-8} \\ 1 \end{array}$$

$$9 \bmod 2 = 1$$

If we’re dividing  $a$  by  $b$ , the result is a quotient  $q$ . If we’re calculating  $a \bmod b$ , the result is the remainder  $r$ .

We can also write this out as:

$a = b \cdot q + r$ , where  $0 \leq r < b$ , and  $q$  and  $r$  are the only two integers that will satisfy the equation.

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<sup>a</sup>From [https://en.wikipedia.org/wiki/Modulo\\_operation](https://en.wikipedia.org/wiki/Modulo_operation)

### Rational numbers

In mathematics, a rational number is any number that can be expressed as the quotient or fraction  $p/q$  of two integers, a numerator  $p$  and a non-zero denominator  $q$ .<sup>[1]</sup> Since  $q$  may be equal to 1, every integer is a rational number.<sup>a</sup>

The set of rational numbers is written as  $\mathbb{Q}$ .

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<sup>a</sup>From [https://en.wikipedia.org/wiki/Rational\\_number](https://en.wikipedia.org/wiki/Rational_number)

**Question 1**

Solve the following modulus problems.

**Example:** Solve  $13 \bmod 5$

$$13 / 5 = 2, \quad 13 \bmod 5 = 3, \quad 13 = 5 \cdot 2 + 3$$

- a.  $9 \bmod 7$
  - b.  $5 \bmod 2$
  - c.  $15 \bmod 3$
  - d.  $-7 \bmod 2$
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**Question 2**

Prove the following propositions:

- a. If  $a$  divides  $b$  and  $a$  divides  $c$ , then  $a$  divides  $b + c$ .<sup>1</sup>

Start with:  $b = ak$  and  $c = aj$  and calculate  $b + c$ .

- b. If  $a$  divides  $b$  and  $c$  divides  $d$ , then  $ac$  divides  $bd$ .<sup>2</sup>

Start with:  $b = ak, d = cj$  and calculate  $bd$ .

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<sup>1</sup>From Discrete Mathematics by Ensley and Crawley

<sup>2</sup>From Discrete Mathematics by Ensley and Crawley