Exam 3 , Chapter 3

CS 210 , Fall 2017

Name: _____

Calculations: Students may use a scientific calculator on the exam, but not a graphing calculator. Exam work must be solo-work.

Readability: Please write in a clear and linear manner. **Using a pencil is best**. If I am not able to interpret your answer, you might lose out on points.

Exam format: This exam covers topics from Chapter 3.

Grading: Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

#	Name	Weight	Points	Weighted Score
1	Set operations	14%		
2	Set operations	8%		
3	Venn diagrams	6%		
4	Set-builder notation	2%		
5	Additional operations	15%		
6	Additional operations	7%		
7	Additional operations	3%		
8	Boolean algebra	3%		
9	Boolean algebra	6%		
10	Identify expression	6%		
11	Draw circuit diagram	10%		
12	2x2 Karnaugh map	10%		
13	2x4 Karnaugh map	10%		
ex1	Bin $\rightarrow$ Dec	+1%		
ex2	Sum	+1%		

Chapter 3.1: Set definitions and operations

Set operations:

Intersection	Union	Difference	Prime
$\cap$	$\cup$	$-$	$'$

Common Sets:

$\mathbb{N}$, the set of natural numbers $\mathbb{Z}$, the set of integers
$\mathbb{Q}$, the set of rational numbers $\mathbb{R}$, the set of all real numbers

Set-builder notation: Form description
A **Form Description** is of the form,
"All numbers of the form _____ " as the first part, and
"The variables used in the first part belong to these sets" as the second part.

For the set of all even integers, we have the form $2k$, and because we specify the form with the variable k, we have to specify that it is an integer, $k \in \mathbb{Z}$, so the form description is: $\{2k : k \in \mathbb{Z}\}$

14% Question 1: Set operations (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Using these sets:

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{6, 9\}$ $B = \{9, 10\}$
$C = \{3, 7\}$ $D = \{4, 9\}$

Find the results of the following set operations:

1. $C - A$
2. $A \cap B$
3. $D \cup B$
4. $C \cup A$

8% Question 2: Set operations (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Using the sets from Question 1, find the results of the following set operations:

1. $(D \cup B) - (A \cap B)$

2. $(D \cup B)'$

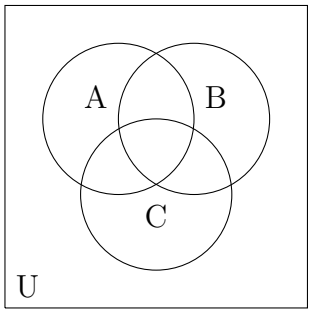
3. $(C - A)'$

4. $(D \cup B) \cap (C \cup A)$

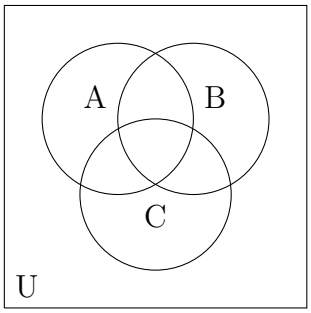
6% Question 3: Venn diagrams (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Fill in the Venn diagrams for each of the following statements. Remember to include the Universe.

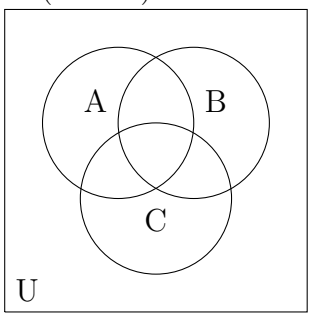
a. $U - A$



b. $B' - A$



c. $(A - B) \cup C$



2% Question 4: Set-builder notation (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Write the following using the form description set-builder notation:

a. The set of even natural numbers.

b. The set of integers that are divisible by 5.

Chapter 3.2: More operations on sets

Cartesian product: We can compute the Cartesian Product of two sets, such as A and B. The result will be a set of **ordered pairs**, such as (a, b), combining the elements of A and B together.

Power set: The Power Set of A is defined as $\wp(A) = \{S : S \subseteq A\}$. In other words, the Power Set is a set of all possible subsets you could build from A, including the empty set.

Partition: For a set A, a partition of A is a set $S = \{S_1, S_2, S_3, ...\}$ of subsets of A, where no *part* is empty, no parts have any elements in common, and all elements are A are contained in the parts.

Common error: $\{\emptyset\} \neq \{\}$ or $\emptyset$

15% Question 5: Additional operations (3.2) □ 0 □ 1 □ 2 □ 3 □ 4

Using the given sets, find the result of the following. Proper notation for sets is required.

$$A = \{red, green, blue\} \qquad B = \{shoes, hat\}$$

a. $A \times B =$

b. $\wp(B) =$

c. List out all 5 partitions of A.

1. 2.

3. 4.

5.

7% Question 6: Additional operations (3.2) □ 0 □ 1 □ 2 □ 3 □ 4

Using the sets $A = \{p\}$ $B = \{T, F\}$
Find the result of the following:

a. $A \times B =$

b. $\wp(A) =$

c. $\wp(B) =$

d. $\wp(A \times B) =$

3% Question 7: Powersets (3.2) □ 0 □ 1 □ 2 □ 3 □ 4

Using the set $C = \{p, q, r\}$ Find $\wp(C)$.

One 0-element set:

Three 1-element sets:

Three 2-element-sets:

One 3-element set:

$\wp(C) =$

Chapter 3.4: Boolean algebra

Logic	$\wedge$	$\vee$	$\neg$
Sets	$\cap$	$\cup$	
Boolean	$\cdot$	$+$	$'$

3% Question 8: Boolean Algebra (3.4) □ 0 □ 1 □ 2 □ 3 □ 4

Answer the following:

a. The Boolean Algebra operation for "and" is: _____

b. The Boolean Algebra operation for "or" is: _____

c. The Boolean Algebra operation for "not" is: _____

6% Question 9: Boolean Algebra (3.4) □ 0 □ 1 □ 2 □ 3 □ 4

Rewrite each of the following with the equivalent Boolean Algebra version. Convert upper-case Set names to lower-case variables ($A \to a$) but keep the same letters for propositional variables ($p \to p$).

a. $(A \cup B) \cap (C \cup D)$

b. $A - B$

c. $p \vee (q \wedge r)$

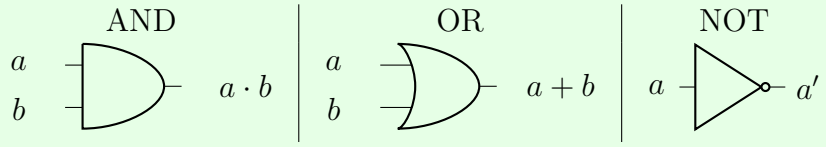
d. $p \wedge \neg q$

e. $(A \cap B)' \cup C$

f. $\neg(p \vee q)$

Chapter 3.5: Logic circuits

Circuits

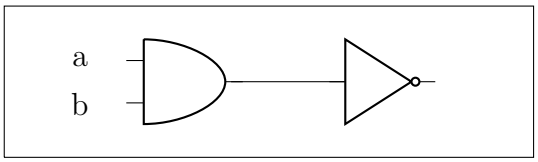


Karnaugh Regions Remember that with Karnaugh maps, your region sizes can be length 1, 2, or 4, but 3 is invalid. In 2D space terms, the region can be 1x1, 2x1, 1x2, 2x2, or 4x1.

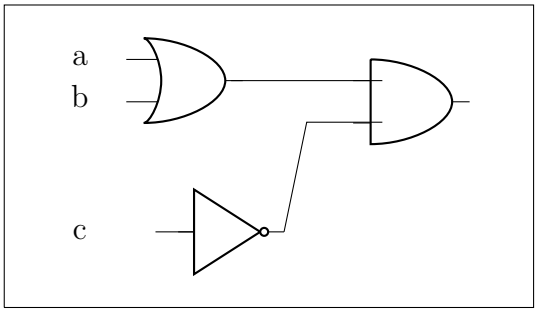
6% Question 10: Identify expression (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

Identify the Boolean Expression for the following diagrams. You do not need to simplify it.

a.



b.



10% Question 11: Draw circuit diagram (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

Draw a circuit diagram for the following expression: $(a \cdot b) + (b \cdot c)'$

10% Question 12: 2x2 Karnaugh map (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

For the following Karnaugh map:

	y	y'
x	✓	✓
x'	✓	

Identify the following:

a. All 3 terms:

b. Simplified equation:

10% Question 13: 2x4 Karnaugh map (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

Simplify the following Boolean Expression. Mark the terms in the Karnaugh map, then build out rectangles to come up with a simplified expression.

$$xyz + xyz' + x'yz + x'yz' + x'y'z$$

	yz	yz'	$y'z'$	$y'z$
x				
x'				

Extra credit

+1% Question 14: Extra credit □ 0 □ 1 □ 2 □ 3 □ 4

Convert the following from Binary to Decimal: `( 0101 1010 )`

+1% Question 15: Extra credit □ 0 □ 1 □ 2 □ 3 □ 4

Evaluate the following sum:

$$\sum_{k=1}^{4}(3k+2)$$