

# Chapter 1.1 In-class Exercise

## ANSWER KEY

**Info:** In-class exercises are meant to introduce you to the new topics of this chapter of the book. Each part will have an introductory description of the content and example(s), followed by practice problems for you to work on.

These assignments are **team assignments** - your team will turn in *one* copy of the exercise. It is up to your team how to approach the assignments; you can work separately and then check your work together, or you can collaborate on the assignment together.

Work must be clean; points may be deducted if the instructor cannot read the work.

**This is the instruction page,  
make sure to fill out your answers to turn in on the worksheet**

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## 1. Coin toss

### Coin toss

When we're flipping a coin, there are two possible outcomes: *heads* or *tails*. If we flip more than one coin, then we end up with more possible outcomes. For example, when flipping two coins, we have four possible outcomes:

1.	HEADS	HEADS
2.	HEADS	TAILS
3.	TAILS	HEADS
4.	TAILS	TAILS

### Question 1

\_\_\_\_\_ / 10%

Draw a table of all possible outcomes if someone flips three coins.

### Question 2

\_\_\_\_\_ / 30%

Write out how many outcomes there are for each of the following.

1. Flipping one coin
2. Flipping two coins
3. Flipping three coins
4. Rolling one 6-sided die
5. Rolling two 6-sided dice
6. Rolling three 6-sided dice

### Question 3

\_\_\_\_\_ / 10%

So if we have  $n$  possible outcomes per event (roll die, flip coin), and we “run” the event  $m$  times (one coin, two dice), how many total outcomes  $o$  will there be? (Write as a simple equation).

## 2. Josephus game

### The Josephus game

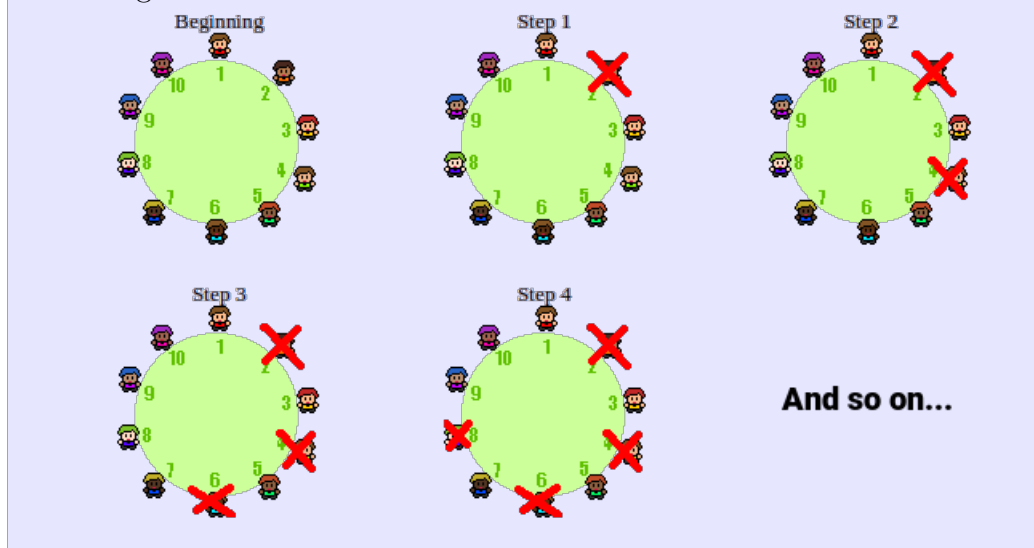
The Josephus game is a theoretical problem, and for now we will just solve it systematically by stepping through the instructions given.

**Setup:** People are sitting in a circle, each with an assigned number (their location in the circle).

**Step:** People are eliminated at every  $n$ th step, with counting beginning at the  $n$ th person.

**Result:** After stepping through, figure out the position of the *last* and *second-to-last* person left (not eliminated).

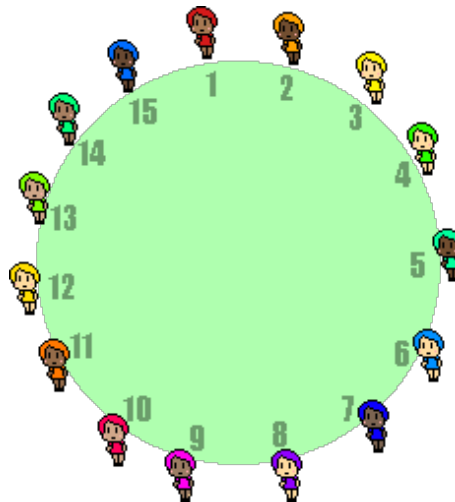
**Example:** Let's say that we begin with a circle of 10 people, numbered from 1 to 10. We will eliminate people at an interval of 2 - so, every-other-person, beginning with person #2. Who will be the last person standing?



**Question 4**

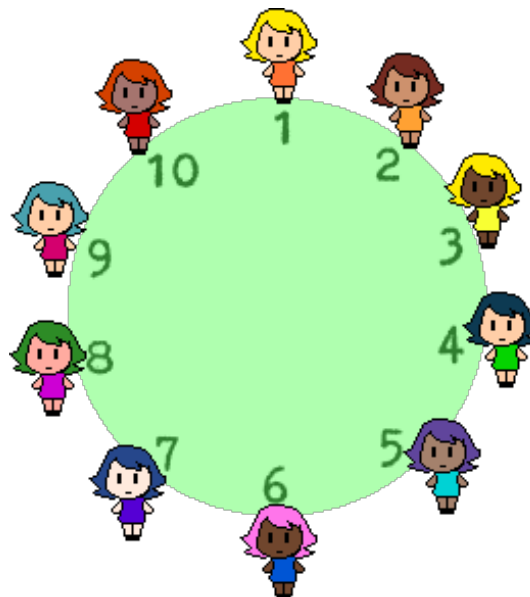
\_\_\_\_\_ / 10%

Given a Josephus circle of 15 people, if we are eliminating every 3rd person (starting with person 3), who is the last to be tagged – and the 2nd to last to be tagged?

**Question 5**

\_\_\_\_\_ / 10%

Given a Josephus circle of 10 people, if we are eliminating every 4th person (starting with person 4), who is the last to be tagged – and the 2nd to last to be tagged?



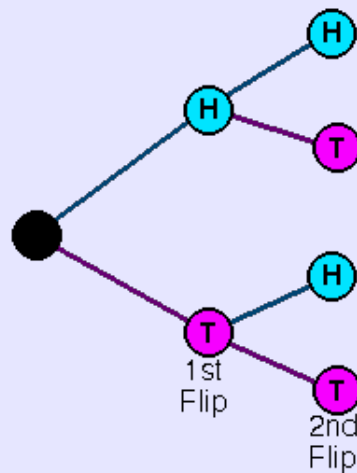
### 3. Game trees

#### Game trees

In section 1.1, we will also be looking at events that have multiple outcomes – such as flipping one coin, two coins, or three coins, or who of two people win one, two, or three tennis matches. With small amounts of “variables”, we can list out all the possible outcomes, and we can build a game tree based on this.

#### Example:

If you flip one coin, the result will be HEADS (H) or TAILS (T). If you flip two coins, what are all the outcomes?



1. HH    2. HT
3. TH    4. TT

**Question 6**

\_\_\_\_\_ / 30%

Suppose you toss three coins – a nickel, a dime, and a quarter, and you record the results in that order. For example, “HTH” means nickel = heads, dime = tails, quarter = heads.

1. In a systematic way, list all the different results you could record.
2. Draw a game tree for recording the results.
3. On the game tree, label each possible result either 0, 1, 2, or 3, to indicate how many *heads* there are.
4. Do you think a person who tosses three coins is more likely to get all three heads, or to get exactly two heads?

## Chapter 1.1 In-class Exercise Worksheet

### ANSWER KEY

**Team:** Please write down all people in your team.

- |    |    |
|----|----|
| 1. | 2. |
| 3. | 4. |

**Section:**

☐ MW 4:30 - 5:45 pm      ☐ M 6:00 - 8:50 pm      ☐ TR 2:00 - 3:15 pm

**Team rules:**

- **Only one worksheet will be turned in per team.** Each member of the team will receive the same score.
- You can collaborate on the exercise together, or you can work separately and then compare your answers with your team as you fill out the turn-in worksheet.

**Work rules:**

- Fill out your answers on this answer sheet.
- Write cleanly and linearly - if I can't make sense of your solution then you won't get credit.
- Write out each step (within reason) - if I can't see the logic you followed to get to your answer, you may get points taken off.
- Don't scribble out cancellations - I can't read that. For example, if a numerator/denominator cancel out, or a  $+/ -$  cancels out, don't scribble out the numbers - just use a single slash!

**Grading:** The total amount of points for an in-class exercise is 5 points each. Each question will have a weight assigned to it, and will be given a point value between 0 and 4:

0	Nothing written
1	Something written, but incorrect
2	Partially correct, but multiple errors
3	Mostly correct, with one or two errors
4	Perfect; correct answer and notation

## Answer sheet

10% Question 1: Draw a table of 3 coins tossed

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

1.	HEADS	HEADS	HEADS
2.	HEADS	HEADS	tails
3.	HEADS	tails	HEADS
4.	HEADS	tails	tails
5.	tails	HEADS	HEADS
6.	tails	HEADS	tails
7.	tails	tails	HEADS
8.	tails	tails	tails

30% Question 2: How many outcomes?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

1. Flipping one coin: \_\_\_\_\_ ( $2^1 = 2$ )
2. Flipping two coins: \_\_\_\_\_ ( $2^2 = 4$ )
3. Flipping three coins: \_\_\_\_\_ ( $2^3 = 8$ )
4. Rolling one 6-sided die: \_\_\_\_\_ ( $6^1 = 6$ )
5. Rolling two 6-sided dice: \_\_\_\_\_ ( $6^2 = 36$ )
6. Rolling three 6-sided dice: \_\_\_\_\_ ( $6^3 = 216$ )

10% Question 3: Equation for outcomes

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

$o = n^m$

10% Question 4: Josphehus game A

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

2nd-to-last: \_\_\_\_\_ Person 15 Last: \_\_\_\_\_ Person 5

10% Question 5: Josphehus game B

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

2nd-to-last: \_\_\_\_\_ Person 15 Last: \_\_\_\_\_ Person 5

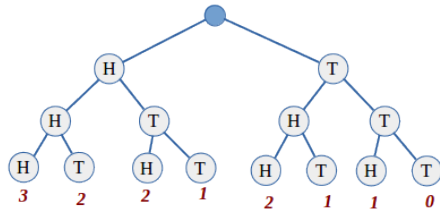
30% Question 6: Game tree

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

1. In a systematic way, list all the different results you could record.  
 1. HHH, 2. HHT, 3. HTH, 4. HTT,  
 5. THH, 6. THT, 7. TTH, 8. TTT



2. Draw a game tree for recording the results



3. On the game tree, label each possible result either 0, 1, 2, or 3, to indicate how many *heads* there are.
4. Do you think a person who tosses three coins is more likely to get all three heads, or to get exactly two heads?

More likely to get exactly 2 heads

## Grading

Question	Weight	0-4	Adjusted score
1	10%		
2	30%		
3	10%		
4	10%		
5	10%		
6	30%		