Chapter 3.1: Set definitions and operations

14% Question 1: Set operations (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Using these sets:

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{6, 9\}$ $B = \{9, 10\}$
$C = \{3, 7\}$ $D = \{4, 9\}$

Find the results of the following set operations:

1. $C - A$ { 3, 7 }
2. $A \cap B$ { 9 }
3. $D \cup B$ { 4, 9, 10 }
4. $C \cup A$ { 3, 6, 7, 9 }

8% Question 2: Set operations (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

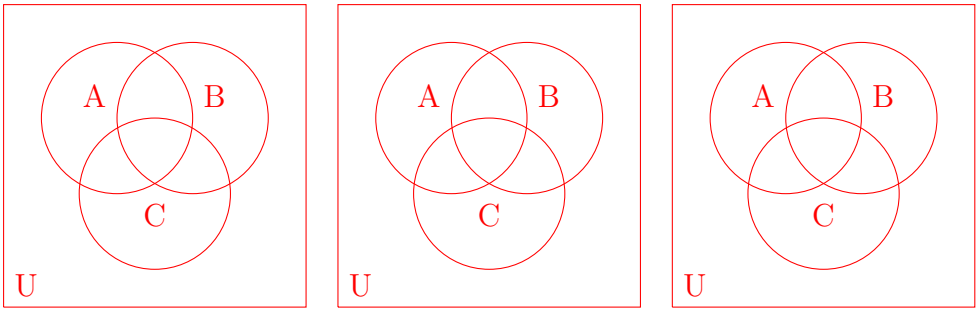
Using the sets from Question 1, find the results of the following set operations:

1. $(D \cup B) - (A \cap B)$ { 4, 10 }
2. $(D \cup B)'$ { 1, 2, 3, 5, 6, 7, 8 }
3. $(C - A)'$ { 1, 2, 4, 5, 6, 8, 9, 10 }
4. $(D \cup B) \cap (C \cup A)$ { 9 }

6% Question 3: Venn diagrams (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Fill in the Venn diagrams for each of the following statements. Remember to include the Universe.

a. $U - A$ b. $B' - A$ c. $(A - B) \cup C$



2% Question 4: Set-builder notation (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Write the following using the form description set-builder notation:

a. The set of even natural numbers. $\{2k : k \in \mathbb{N}\}$

b. The set of integers that are divisible by 5. $\{5k : k \in \mathbb{Z}\}$

Chapter 3.2: More operations on sets

15% Question 5: Additional operations (3.2) □ 0 □ 1 □ 2 □ 3 □ 4

Using the given sets, find the result of the following. Proper notation for sets is required.

$$A = \{red, green, blue\} \qquad B = \{shoes, hat\}$$

a. $A \times B =$ { (red, shoes), (red, hat), (green, shoes), (green, hat), (blue, shoes), (blue, hat) }

b. $\wp(B) =$ { ∅, { shoes }, { hat }, { shoes, hat } }

c. List out all 5 partitions of A.
{ { red, green, blue } },
{ { red }, { green, blue } },
{ { red, green }, { blue } },
{ { red, blue }, { green } },
{ { red }, { green }, { blue } }

7% Question 6: Additional operations (3.2) □ 0 □ 1 □ 2 □ 3 □ 4

Using the sets $A = \{p\}$ $B = \{T, F\}$
Find the result of the following:

a. $A \times B =$ $\{(p, T), (q, T)\}$

b. $\wp(A) =$ $\{\emptyset, \{p\}\}$

c. $\wp(B) =$ $\{\emptyset, \{T\}, \{F\}, \{T, F\}\}$

d. $\wp(A \times B) =$ $\{\emptyset, \{(p, T)\}, \{(p, F)\}, \{(p, T), (p, F)\}\}$

3% Question 7: Powersets (3.2) □ 0 □ 1 □ 2 □ 3 □ 4

Using the set $C = \{p, q, r\}$ Find $\wp(C)$.

One 0-element set:
$\emptyset$
Three 1-element sets:
$\{p\}, \{q\}, \{r\}$
Three 2-element-sets:
$\{p, q\}, \{p, r\}, \{q, r\}$
One 3-element set:
$\{p, q, r\}$
$\wp(C) =$ $\{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, \{p, q, r\}\}$

Chapter 3.4: Boolean algebra

3% Question 8: Boolean Algebra (3.4) □ 0 □ 1 □ 2 □ 3 □ 4

Answer the following:

a. The Boolean Algebra operation for "and" is: $\cdot$

b. The Boolean Algebra operation for "or" is: $+$

c. The Boolean Algebra operation for "not" is: $'$

6% Question 9: Boolean Algebra (3.4) □ 0 □ 1 □ 2 □ 3 □ 4

Rewrite each of the following with the equivalent Boolean Algebra version. Convert upper-case Set names to lower-case variables ($A \to a$) but keep the same letters for propositional variables ($p \to p$).

a. $(A \cup B) \cap (C \cup D)$ $(a + b) \cdot (c + d)$

b. $A - B$ $a \cdot b'$

c. $p \vee (q \wedge r)$ $p + (q \cdot r)$

d. $p \wedge \neg q$ $p \cdot q'$

e. $(A \cap B)' \cup C$ $(a \cdot b)' + c$

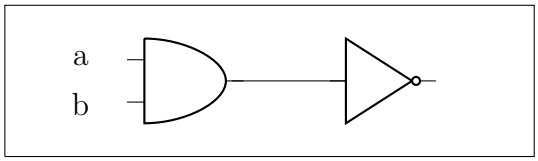
f. $\neg(p \vee q)$ $(p + q)'$

Chapter 3.5: Logic circuits

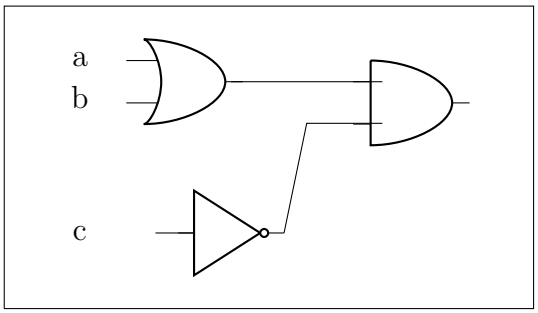
6% Question 10: Identify expression (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

Identify the Boolean Expression for the following diagrams. You do not need to simplify it.

a. $(a \cdot b)'$

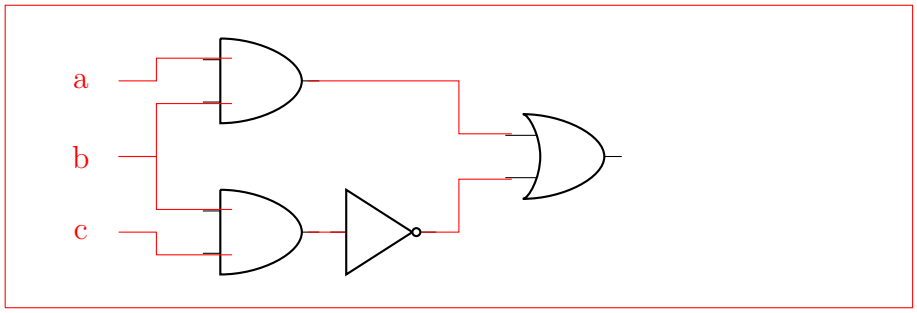


b. $(a + b) \cdot c'$



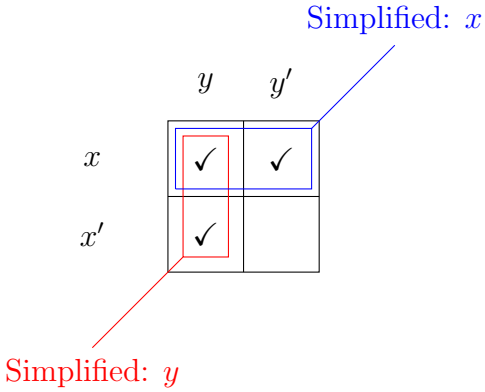
10% Question 11: Draw circuit diagram (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

Draw a circuit diagram for the following expression: $(a \cdot b) + (b \cdot c)'$



10% Question 12: 2x2 Karnaugh map (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

For the following Karnaugh map:



Identify the following:

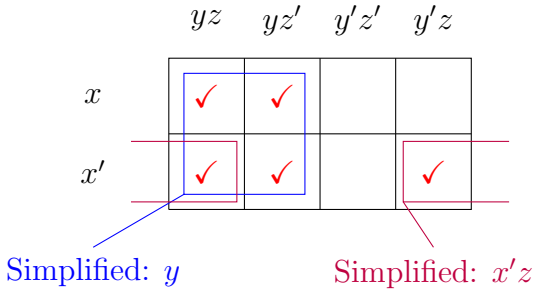
a. All 3 terms: xy, xy', and $x'y$.

b. Simplified equation: $x + y$

10% Question 13: 2x4 Karnaugh map (3.5) □ 0 □ 1 □ 2 □ 3 □ 4

Simplify the following Boolean Expression. Mark the terms in the Karnaugh map, then build out rectangles to come up with a simplified expression.

$$xyz + xyz' + x'yz + x'yz' + x'y'z$$



There are two regions: a 2x2 region and a 2x1 region. One region wraps around horizontally, which is allowed. Regions should be the largest possible regions available, so the 2x1 is used instead of just a 1x1 at $x'y'z$.

The resulting simplified expression is $y + x'z$

Extra credit

+1% Question 14: Extra credit □ 0 □ 1 □ 2 □ 3 □ 4

Convert the following from Binary to Decimal: `( 0101 1010 )`

90

+1% Question 15: Extra credit □ 0 □ 1 □ 2 □ 3 □ 4

Evaluate the following sum:

$$\sum_{k=1}^{4}(3k + 2)$$

$(3 \cdot 1 + 2) + (3 \cdot 2 + 2) + (3 \cdot 3 + 2) + (3 \cdot 4 + 2)$
$= 5 + 8 + 11 + 14 = 38$