1. Introductory Practice

For the statement, "If n mod 3 = 1, then n mod $9 \neq 5$."

- a) What is the **hypothesis** p? (__/1)
- b) What is the **conclusion** q? (__/1)
- c) Remember that the negation of an implication is: $\neg(p \rightarrow q) \equiv p \land \neg q$ (__/1) Write out the original statement (in English terms) as a contradiction.

2. Proof by contradiction

Example: "Prove by contradiction: If n^2 is even, then n is even."

Hypothesis: n^2 is even Conclusion: n is even

A contradiction would be $\neg (p \rightarrow q) \equiv p \land \neg q$, or in English:

Hypothesis: n^2 is even Conclusion: n is odd

Write in math terms: $n^2 = 2K$ n = 2L+1

Make equation: $(2L+1)^2 = 2K$ Simplify: $4L^2 + 4L + 1 = 2K$ $1 = 2K - 4L^2 - 4L$

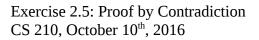
 $\frac{1}{2} = K - 2L^2 - 2L$

Result: As K and L are both integers, and through the closure property of

integers (addition, subtraction, and multiplication of integers result in an integer), as $K-2L^2-2L$ results in something that is *not an*

integer, it shows that our counterexample is false and no

counterexample can exist.



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Practice 1 (__/1)

"Prove by contradiction: If n^2 is odd, then n is odd."

Practice 2 (__/1)

"Use proof by contradiction to explain why it is impossible for a number n to be of form 5K+3 and of the form 5L+1 for integers K and L."

(Hint: n=5K+3 and n=5L+1 , so 5K+3=5L+1 is your starting point, and keep in mind the *closure principle of integers!*)