2.5 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

2.5 Proof by contradiction

2.5.1 Review practice

Question 1

For the statement, "if $n\%3 = 1$, then $n\%9 \neq 5$", where % stands for "modulus"...

a. What is the hypothesis p?

b. What is the conclusion q?

c. Using $\neg(p \rightarrow q) \equiv p \wedge \neg q$, write out the negation of this implication in English.

This negation can be used to build the counter-example, which we will use for the proof by contradiction.

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2.5.2 Proof by contradiction

Prove by contradiction: If n^2 is even, then n is even

Step 1: Identify the hypothesis and conclusion: Here, our **hypothesis** is "n^2 is even", and our **conclusion** is "n is even".

Step 2: Identify the negation: The negation of an implication is **not** an implication. A counter-example of this would be if we have $p \wedge \neg q$, or "n^2 is even and n is odd".

Step 3: Build a counter-example: Our counter-example is the scenario where the hypothesis is true and the conclusion is false... or in other words, the negation. **Counter-example:** "n^2 is even and n is odd"

Step 4: Write the hypothesis & conclusion symbolically: (For our counter-example implication)
$n^2 = 2k$ (some even integer) $\quad n = 2j + 1$ (some odd integer)

Step 5: Write equation: Using the statement, we are going to turn this into an equation. n is odd, and n^2 is even, so if we square the odd n to get the even n^2, we would have... to get the even n^2, we would have... $(2j + 1)^2 = 2k$

Step 6: Simplify until we have a contradiction:
$(2j + 1)^2 = 2k$
$\Rightarrow \quad 4j^2 + 4j + 1 = 2k$

$$\Rightarrow \qquad 1 = 2k - 4j^2 - 4j$$
$$\Rightarrow \qquad \frac{1}{2} = k - 2j^2 - 2j$$

Since k and j are both integers, through the closure property of integers (+, -, and $\times$ results in an integer), we can show that $k - 2j^2 - 2j$ results in something that is *not an integer* – this is a contradiction. It shows that our counter-example is **false**, and no counter-example can exist.

Question 2

Prove by contradiction: If n^2 is odd, then n is odd.

Step 1: Identify the hypothesis and conclusion:
Hypothesis p:
Conclusion q:

Step 2: Identify the negation (counter-example):

p: ____________________ AND $\neg q$: ____________________

Step 3: Write the hypothesis & conclusion-negation symbolically:
(Make sure you use different variables for n^2 and n.)

(p) $n^2 =$

$(\neg q)$ $n =$

Step 5: Write equation: Set the equation for n-squared equal to the equation for n^2.

Step 6: Simplify until we have a contradiction:

Result:

Question 3

Use proof by contradiction to explain why it is impossible for a number n to be of the form $5k + 3$ and of $5j + 1$ for integers k and j.

Hint
$n = 5k + 3$ is one statement, and $n = 5j + 1$ is the other statement, so $5k + 3 = 5j + 1$ is your starting point.

This isn't in an implication form, so we begin at Step 5...

Step 5: Write equation: $5k + 3 = 5j + 1$

Step 6: Simplify until we have a contradiction:

Result: