4.3 Properties of Functions and Set Cardinality

4.3.1 Review: Inverses of functions

Given a relation R with domain A and codomain B, the relation R_{-1} (read "R inverse") with domain B and codomain A is called the **inverse** of R, and is defined so that

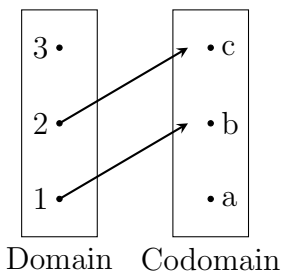$$(x, y) \in R \qquad \text{if and only if} \qquad (y, x) \in R^{-1}$$

Also note that the inverse of R^{-1} is R. [a]
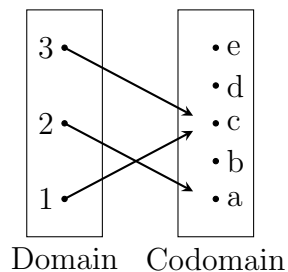
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[a]Discrete Mathematics, Ensley and Crawley

Question 1

Draw the inverse of each diagram. Identify whether the original diagram and/or the inverse of that diagram are functions.
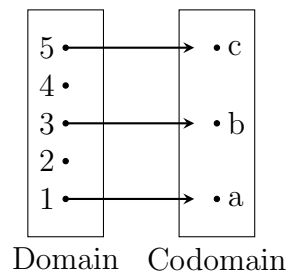
a.

Domain   Codomain

b.

Domain   Codomain

c.

Domain   Codomain

4.3.2   Functions that are invertible

Functions that are onto
A function is **onto** if every output from the codomain has at least one input from the domain. Every output is attainable via at least one input.

For diagrams: Every point in the codomain has at least one arrow pointing at it.

Functions that are one-to-one
A function is **one-to-one** if every output from the codomain has no more than one input from the domain. No two inputs lead to the same output.

For diagrams: None of the points in the codomain has two or more arrows pointing at it.

Functions that are invertible
If a function is both one-to-one AND onto, then it is invertible. This means that the inverse of the function is ALSO a function.

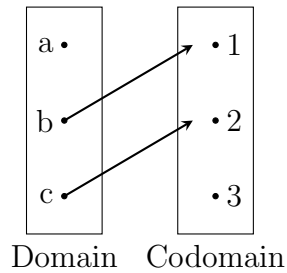For diagrams: Every point in the codomain has exactly one arrow pointing at it.

Question 2

   Draw two functions: One where the function is one-to-one but not onto, and one where the function is onto but not one-to-one. Make sure to label your domain and codomain for each.
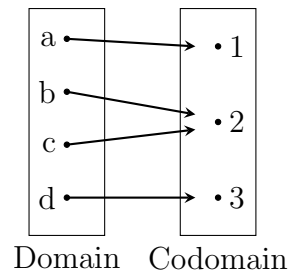
Question 3

Determine whether these functions are one-to-one, onto, and/or invertible. If not, state why not.
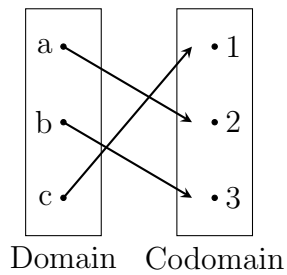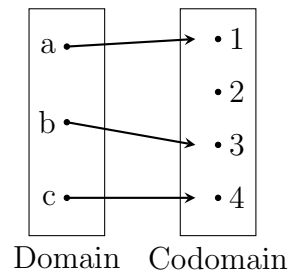
a.

a •
b •
c •
• 1
• 2
• 3

Domain   Codomain

b.

a •
b •
c •
d •
• 1
• 2
• 3

Domain   Codomain

☐ Onto   ☐ One-to-one
☐ Invertible

☐ Onto   ☐ One-to-one
☐ Invertible

c.

a •
b •
c •
• 1
• 2
• 3

Domain   Codomain

d.

a •
b •
c •
• 1
• 2
• 3
• 4

Domain   Codomain

☐ Onto   ☐ One-to-one
☐ Invertible

☐ Onto   ☐ One-to-one
☐ Invertible

e.

a •
b •
c •
d •
• 1
• 2
• 3
• 4

Domain   Codomain

f.

a •
b •
c •
d •
• 1
• 2
• 3
• 4
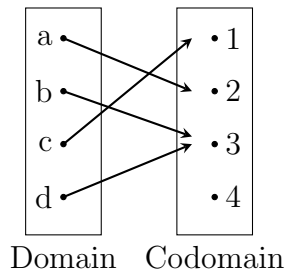
Domain   Codomain

☐ Onto   ☐ One-to-one
☐ Invertible
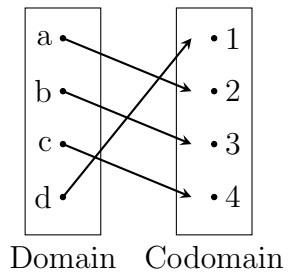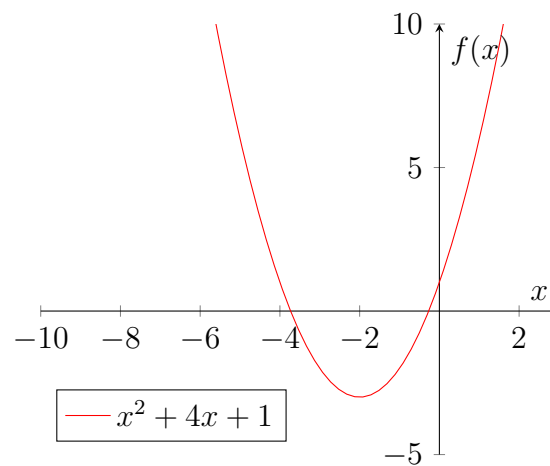
☐ Onto   ☐ One-to-one
☐ Invertible

Question 4

The function $f : \mathbb{R} \to \mathbb{R}$, with the rule $f(x) = x^2 + 4x + 1$
is not onto and not one-to-one.

a. Give an example of an element in the codomain that has no element in the domain associated with it.

b. Given an example of two elements in the domain that are both associated with the same output in the codomain.