1. Recursive ↔ Closed formula equivalence

Example

Show that the sequence defined by the recursive formula
$a_k = a_{k-1} + 2$ **, where** $a_1 = 2$ **, and for** $k \geq 2$
is equivalently described by the closed formula
$a_n = 2 \cdot n$

1. Check equivalence for first element
   Recursive: $a_1 = 2$, Closed: $a_1 = 2 \cdot 1 = 2$

2. Find a_{m-1} through the closed formula
   $a_{m-1} = 2(m-1) = 2m - 2$, or $a_{m-1} = 2m - 2$

3. Rewrite the recursive formula in terms of m
   $a_m = a_{m-1} + 2$

4. Plug in a_{m-1} into the recursive formula from step (3)
   $a_m = a_{m-1} + 2 \rightarrow a_m = 2m - 2 + 2$

5. Simplify to get the closed formula as the proof
   $a_m = 2m - 2 + 2 \rightarrow a_m = 2m$

3. Summation ↔ formula equivalence

Use induction to prove that $\sum_{i=1}^{n}(2i-1)=n^2$ **for each** $n\geq 1$.

1. Check the first few values:

n = 1 $\sum_{i=1}^{1}(2i-1)=(2\cdot 1-1)=1$, $1^2=1$

n = 2 $\sum_{i=1}^{2}(2i-1) \quad =(2\cdot 1-1)+(2\cdot 2-1) \quad =1+3=4$, $2^2=4$

n = 3 $\sum_{i=1}^{3}(2i-1) \quad =(2\cdot 1-1)+(2\cdot 2-1)+(2\cdot 3-1) \quad =1+3+5=9$, $3^2=9$

2. Rewrite the summation from *1 to n* as the summation from *i = 1 to m-1*, plus the final term.

$$\sum_{i=1}^{m}(2i-1)=\sum_{i=1}^{m-1}(2i-1)+(2m-1)$$

3. Redefine the original proposition in terms of *m – 1*

$$\sum_{i=1}^{m-1}(2i-1)=(m-1)^2$$

4. Plug in the summation from i = *1 to m-1* to the form in step (2), simplify to get the original form of the right-hand side.

$$\sum_{i=1}^{n}(2i-1)=(m-1)^2+(2m-1) \quad \rightarrow \quad ...=m^2-2m+1+(2m-1)$$

$$\rightarrow \quad ...=m^2-2m+2m+1-1 \quad \rightarrow \quad ...=m^2$$

$$\sum_{i=1}^{n}(2i-1)=m^2$$

Set 1: Show that the sequence defined by the recursive formula is equivalently described by the closed formula

Practice a

Recursive: $a_k = a_{k-1} + (2k+1)$, where $a_1 = 4$, and for $k \geq 2$	Closed: $a_n = (n+1)^2$

Practice b

Recursive: $a_k = a_{k-1} + (2k-1)$, where $a_1 = 2$, and for $k \geq 2$	Closed: $a_n = n^2 + 1$

Practice c

Recursive: $a_k = a_{k-1} + 3$, where $a_1 = 2$, and for $k \geq 2$	Closed: $a_n = 3 \cdot n - 1$

Practice d

Recursive: $a_k = a_{k-1} + 4$, where $a_1 = 1$, and for $k \geq 2$	Closed: $a_n = 4 \cdot n - 3$

Practice e

Recursive: $a_k = a_{k-1} + 3$, where $a_1 = 3$, and for $k \geq 2$	Closed: $a_n = 3n$

Set 2: Use induction to prove that the summation and the equation are equivalent.

Practice a

$\sum_{i=1}^{n} (2 \cdot i + 4) = n^2 + 5n$, for each $n \geq 1$

Practice b

$\sum_{i=1}^{n} (2^i) - 1 = 2^{n+1} - n - 2$, for each $n \geq 1$

Practice c

$2 \cdot (\sum_{i=1}^{n} 3^{i-1}) = 3^n - 1$, for each $n \geq 1$