2.6 Numerical Representation

2.6.1 Intro practice

Question 1

The set of digits in base-10 (decimal) number system is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Use this information to help you figure out the following.

a. Write out the set of digits in the octal (base-8) number system. $\{0, 1, 2, 3, 4, 5, 6, 7\}$

b. Write out the set of digits in the binary (base-2) number system. $\{0, 1\}$

The hexadecimal (base-16) number system is

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}.$$

Why do we use letters? To keep numbers 10 through 15 as one-character representations.

Specifying base

When we need to specify the base when writing out numbers, write it within parentheses, with a subscript of its base number.

- $(123)_{10} = 123$, base-10
- $(1337)_8 = 1337$, base-8
- $(C47)_{16} =$ C47, base-16
- $(1011)_2 = 1011$, base-2

2.6.2 Digits

For the decimal number $2,368$, we can write this as its individual digits:

Thousands (10^3)	**Hundreds** (10^2)	**Tens** (10^1)	**Ones** (10^0)
2	3	6	8

And then we can build out $2,368$ as the mathematical equation:

$$2 \cdot 10^3 + 3 \cdot 10^2 + 6 \cdot 10^1 + 8 \cdot 10^0$$

Likewise, for the binary number 0101 1001, we can write it as:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	1	0	0	1

And into the equation:

$$1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^0$$

Question 2

Expand each of the following numbers as a mathematical equation. Make sure to pay attention to the *base* value.

a. Write out the equation for $(19)_{10}$

10^1	10^0
1	9

b. Write out the equation for $(0010\ 1101)_2$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	1	1	0	1

c. Write out the equation for $(FFAA66)_{16}$

16^5	16^4	16^3	16^2	16^1	16^0
F	F	A	A	6	6

2.6.3 Converting between bases

Algorithm for converting a decimal number to base b:

1. Input a natural number n
2. While $n > 0$, do the following:
   (a) Divide n by b and get a quotient q and remainder r.
   (b) Write r as the next (right-to-left) digit.
   (c) Replace the value of n with q, and repeat.

Question 3

Convert the following between bases:

a. Convert $(35)_{10}$ to binary (base-2) $n = 35$, $b = 2$

$35/2 = 17 + 1/2$ $(a/b = q + r/b)$ $q = 17$, $r = 1$
$17/2 = 8 + 1/2$ $q = 8$, $r = 1$
$8/2 = 4 + 0/2$ $q = 4$, $r = 0$
$4/2 = 2 + 0/2$ $q = 2$, $r = 0$
$2/2 = 1 + 0/2$ $q = 1$, $r = 0$
$1/2 = 0 + 1/2$ $q = 0$, $r = 1$
n = 0

= 0010 0011

b. Convert $(125)_{10}$ to binary (base-2) $n = 125$, $b = 16$

$125/2 = 62 + 1/2$ $(a/b = q + r/b)$ $q = 62$, $r = 1$
$62/2 = 31 + 0/2$ $q = 31$, $r = 0$
$31/2 = 15 + 1/2$ $q = 15$, $r = 1$
$15/2 = 7 + 1/2$ $q = 7$, $r = 1$
$7/2 = 3 + 1/2$ $q = 3$, $r = 1$
$3/2 = 1 + 1/2$ $q = 1$, $r = 1$
$1/2 = 0 + 1/2$ $q = 0$, $r = 1$
n = 0

= 0111 1101

Hexadecimal to Binary

Often in computers, we write binary strings as hexadecimal to save space and make it easier to read.

Hex	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hex	8	9	A	B	C	D	E	F
Binary	1000	1001	1010	1011	1100	1101	1110	1111

Example: Convert 11001 from binary to hexadecimal

1. Write out in chunks of four. Add leading 0's to the left side.
   0001 1001
2. Swap out each "nibble" with hexadecimal
   0001 = 1   1001 = 9

So, $(0001\ 1001)_2 = (19)_{16}$

Example: Convert DAD from hexadecimal to binary

1. Convert each digit back to binary.
   D = 1101   A = 1010   D = 1101

So, $(\text{DAD})_{16} = (1101\ 1010\ 1101)_2$

Question 4

Do the following conversions

a. Convert $(1F0B)_{16}$ to binary:
   1 = 0001   F = 1111   0 = 0000   B = 1011

   = 0001 1111 0000 1011

b. Convert $(0100\ 0110)_2$ to hexadecimal:
   0100 = 4   0110 = 6

   = 46