

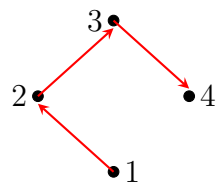
4.4 Properties of Relations

4.4.1 Relations

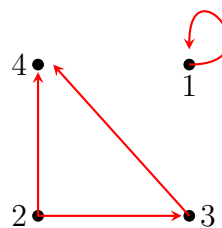
Question 1

Draw the arrows for the following relations:

R



S



Set A is $A = \{1, 2, 3, 4\}$.

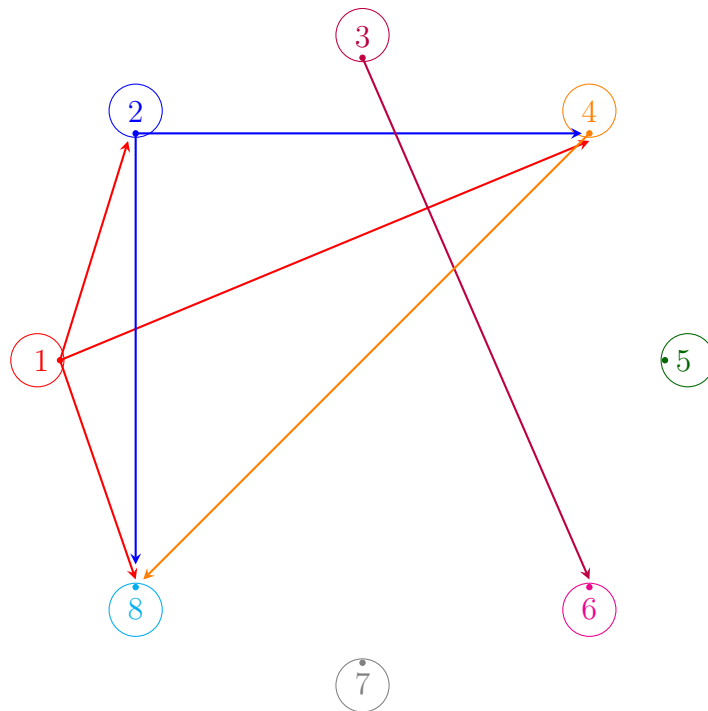
Relation R : $R : A \rightarrow A$, with the rule: $\{(1, 2), (2, 3), (3, 4)\}$

Relation S : $S : A \rightarrow A$, with the rule: $\{(1, 1), (2, 3), (2, 4), (3, 4)\}$

Question 2

Complete the arrow diagram for the relation on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Also identify the properties of each. Give reasons why the relation is transitive, intransitive, neither, symmetric, antisymmetric, neither, and/or reflexive.

$$R_1 = \{ (1,1), (1,2), (1,4), (1,8), \quad (2,2), (2,4), (2,8), \quad (3,3), (3,6), \\ (4,4), (4,8) \quad (5,5), \quad (6,6), \quad (7,7), \quad (8,8) \}$$



☐ Reflexive? ☐ Irreflexive? ☐ Neither? Why?

It is reflexive - All elements of A point back to itself (every node has a loop).

☐ Symmetric? ☐ Antisymmetric? ☐ Neither? Why?

It is antisymmetric - No arrows go in both directions.

☐ Transitive? Why?

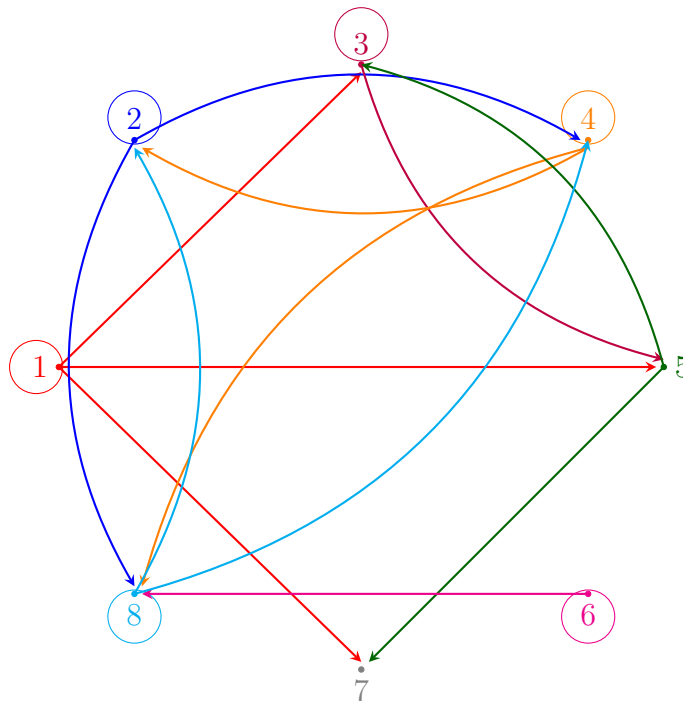
It is transitive - All two-arrow paths also have a direct arrow between the endpoints.

Question 3

Complete the arrow diagram for the relation on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Also identify the properties of each. Give reasons why the relation is transitive, intransitive, neither, symmetric, antisymmetric, neither, and/or reflexive.

FIXME: KEY SHOULD HAVE LOOPS

$$R_2 = \{ (1,1), (1,3), (1,5), (1,7), (2,2), (2,4), (2,8), (3,3), (3,5), (3,7), (4,2), (4,4), (4,8), (5,3), (5,7), (6,6), (6,8), (8,2), (8,4), (8,8) \}$$



☐ Reflexive? ☐ Irreflexive? ☐ Neither? Why?

It is neither – there are some loops, but not on all (reflexive) and not on none (irreflexive).

☐ Symmetric? ☐ Antisymmetric? ☐ Neither? Why?

It is neither – some arrows go in both directions, but not all (symmetric), and not none (antisymmetric). e.g., $(2,8)$ and $(8,2)$ exist.

☐ Transitive? Why?

It is transitive – not; 6 goes to 8, and 8 goes to other places, but 6 doesn't

go to other locations.

Recap

- **Reflexive:** $(a, a) \in R$ for all $a \in A$
- **Irreflexive:** $(a, a) \notin R$ for all $a \in A$
- **Antisymmetric:** for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \notin R$
- **Transitive:** if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$

Question 4

Given the relation, $R_1 = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a + b \text{ is even}\}$.

- a. This relation is **reflexive**. Find an example to illustrate why.

a and b are both in the set of integers. We are checking to see if the result of (a, a) is always in the relation R_1 ... so, if you plug in (a, a) into the relation, is the output still “is even”?

Since $a + a = 2a$ is always even, we know that $(a, a) \in R_1$ for all $a \in \mathbb{Z}$. Hence, R_1 is reflexive (and not irreflexive).

- b. This relation is **symmetric**. Find an example to illustrate why.

Find some (a, b) and (b, a) that are both in the relation. If you can, it's symmetric.

Since $(1, 3) \in R_1$ and $(3, 1) \in R_1$, R_1 is symmetric.

Question 5

Let C be the set of all cats who have ever lived. For each of the following relations on the set C , decide if the given is reflexive irreflexive, transitive, or antisymmetric. Some of these can satisfy more than one property. Give explanations on how you decided each of these.

a. $R_1 = \{(a, b) \in C \times C : a \text{ is a child of } b\}$

- Reflexive - Is $(a, a) \in C$ for all a valid? **no; a cat cannot be its own parent**
- Irreflexive - Is $(a, a) \notin C$ for all a valid? **yes; a cat cannot be its own parent**
- Transitive - Is there some $(a, b) \in C$ and $(b, c) \in C$? ¹ **no; if “Cat A” is a child of “Cat B”, and “Cat B” is a child of “Cat C”, then “Cat A” cannot also be a child of “Cat C”.**
- Antisymmetric - Is $(a, b) \in C$ and $(b, a) \notin C$ valid? **yes; if “Cat A” is a child of “Cat B”, then “Cat B” cannot be a child of “Cat A”.**

b. $R_2 = \{(a, b) \in C \times C : a \text{ is a descendant of } b\}$

- Reflexive - Is $(a, a) \in C$ for all a valid? **no; same as above**
- Irreflexive - Is $(a, a) \notin C$ for all a valid? **yes; same as above**
- Transitive - Is there some $(a, b) \in C$ and $(b, c) \in C$? **yes; if “Cat A” is a descendant of “Cat B” and “Cat B” is a descendant of “Cat C”, then “Cat A” is also a descendant of “Cat C”.**
- Antisymmetric - Is $(a, b) \in C$ and $(b, a) \notin C$ valid? **yes; same as above**

¹Just assume a cat isn't going to mate with its child. : |