3.1 Set Definitions and Operations

3.1.1 Common Sets

Common sets we will see in this chapter:

$\mathbb{N}$, the set of natural numbers	These numbers are "counting numbers". This set contains 0 and positive integers.
$\mathbb{Z}$, the set of integers	This set contains all integers: positive, negative, and zero.
$\mathbb{Q}$, the set of rational numbers	This set contains all numbers that can be characterized as ratios, such as $\frac{1}{2}$, $\frac{-17}{4}$, or even $\frac{3}{1}$.
$\mathbb{R}$, the set of all real numbers	These can be thought of as decimal numbers with possibly unending strings of digits after the decimal point.

Question 1

For the following numbers, which set(s) do they belong to?

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
10				
-5				
12/6				
π				
2.40				

Question 2
Give examples for each of the following types of sets:

a. List three numbers that are in the set of all integers, $\mathbb{Z}$, but are NOT in the set of natural numbers, $\mathbb{N}$.

b. List three numbers that are in the set of rational numbers, $\mathbb{Q}$, but are NOT in the set of integers, $\mathbb{Z}$.

c. List three numbers that are in the set of all real numbers $\mathbb{R}$, but are NOT in the set of rational numbers, $\mathbb{Q}$.

Writing out sets

When we are building a discrete (finite) set, we usually give the set a capital letter as its identifier. Then, the elements of the set are written within curly-braces, like this:

$$A = \{2, 4, 6, 8\}$$

The elements here are 2, 4, 6, and 8. The index of the element 2 is 1 - it is at position 1 of the set - so $A_1 = 2$.

Question 3

Create sets that meet the following criteria. Give the sets any letter identifier that you want.

a. All elements of the set are odd integers.

b. All elements of the set are fractions such that, when divided, they result in an infinite string of numbers to the right of the decimal place (e.g., $3.333333\bar{3}$...)

c. Create two sets of integers, where the two sets have exactly two elements in common.

d. Create two sets of natural numbers, where the two sets have NO elements in common.

e. Create a set that is empty.

3.1.2 Subsets

Subsets and existence within sets:

x exists in A	The notation $x \in A$ means "x is an element of A" which means that x is one of the member elements of A.
A is a subset of B	A is a subset of B (written as $A \subseteq B$) if every element in A is also an element in B. Formally, this means that for every x, if $x \in A$, then $x \in B$.
A is equal to B	A is equal to B (written $A = B$) means that A and B have exactly the same members. This is expressed formally by saying, $A \subseteq B$ and $B \subseteq A$.
An Empty set	A set that contains no elements is called an empty set, and it is denoted by $\{\}$ or $\emptyset$.
The Universal set	For any given discussion, all the sets will be subsets of a larger set called the universal set (or universe) We commonly use the letter U to denote this set.

Question 4

Given theset sets:

$U = \{-2, -1, 1, 2, 3, 4, 5, 6\}$ $A = \{1, 1, 2, 2, 2, 4, 4\}$ $B = \{-2, 2\}$

$C = \{1, 2, 4, 5, 6\}$ $D = \{6, 5, 4, 2, 1\}$ $E = \{1, 4\}$

a. Which of these statements are true? Mark with a ✓

a. $B \subseteq A$ ____ b. $B \subseteq E$ ____ c. $E \subseteq A$ ____

d. $A \subseteq U$ ____ e. $D \subseteq C$ ____ f. $C \subseteq D$ ____

g. $B \subseteq \mathbb{N}$ ____ h. $E \subseteq \mathbb{Z}$ ____ i. $A \subseteq C$ ____

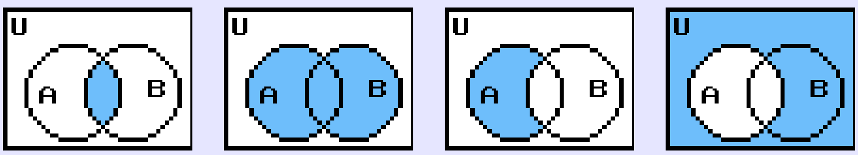
b. Fill in the blanks with either $\subseteq$ (is a subset of), or $\not\subseteq$ (is not a subset of), or $=$ (is equal to) for the following:

a. C ____ D b. B ____ U c. A ____ E

3.1.3 Intersections, unions, and differences

Intersection of A and B, $A \cap B$	Is the set that contains those elements common to both A and B. In set-builder notation, we write: $A \cap B = \{x \in U : x \in A \wedge x \in B\}$
Union of A and B, $A \cup B$	Is the set that contains those elements in either set A or B. In set-builder notation, we write: $A \cup B = \{x \in U : x \in A \vee x \in B\}$
Difference of A and B, $A - B$	Is the set that contains those elements in A which are NOT in B. In set-buidler notation, we write: $A - B = \{x \in U : x \in A \wedge x \notin B\}$
Disjoint sets	Sets A and B are disjoint if $A \cap B = \emptyset$.
Complement of A, A'	Given a set A with elements from the universe U, the complement of A (written A') is the set that contains those elemnets of the universal set U which are not in A. That is, $A' = U - A$.

Venn diagrams are used to visually represent relationships between sets. Set A and set B (or more) are drawn as overalpping circles, and the shaded-in region is the resulting set based on the *intersection*, *union*, *complement*, or *difference* operations.

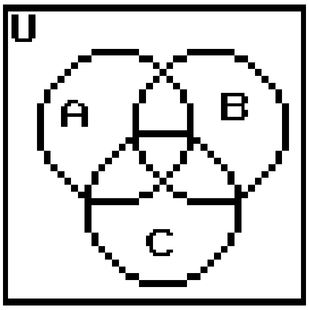


$A \cap B$ $A \cup B$ $A - B$ A'

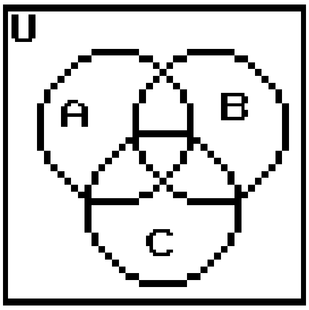
Question 5

For the following set operations, color in the Venn diagrams.

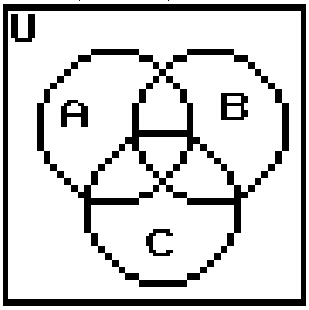
$A \cap B$



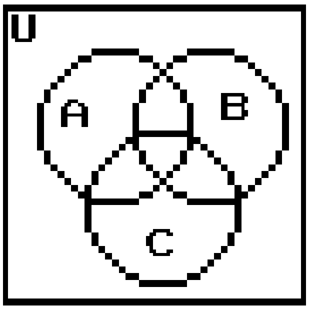
$A \cap C$



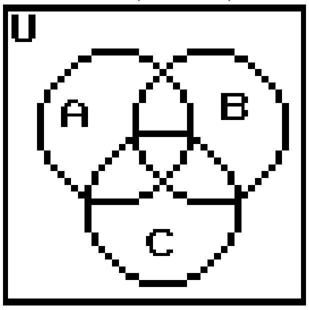
$(A \cap B) \cup C$



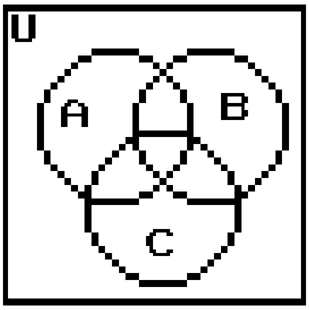
$B \cup C$



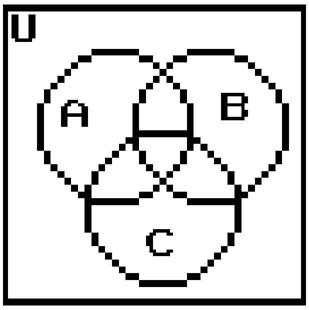
$A \cap (B \cup C)$



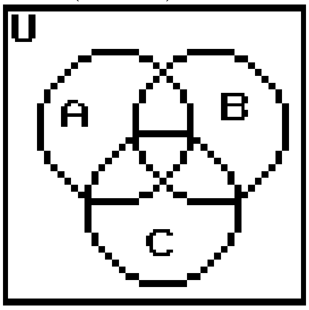
$B - C$



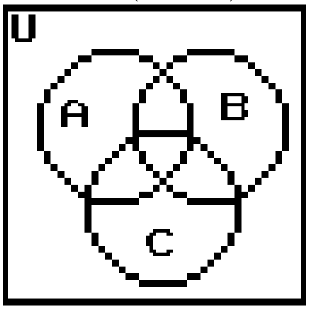
A'



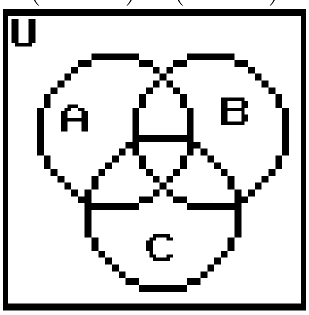
$(A \cup B) - B$



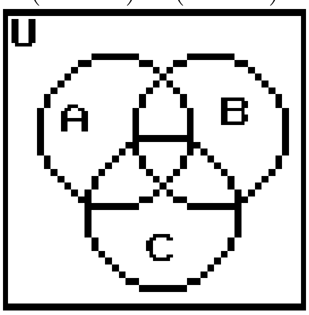
$A \cup (B - C)$



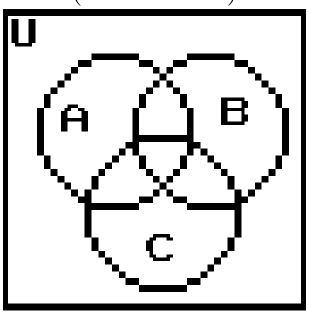
$(A \cap B) \cup (A \cap C)$



$(A \cup B) \cap (A \cup C)$



$(A \cup B \cup C)$



Question 6

Given the following sets, compute the set operations and prove the following statements.

$U = \{1, 2, 3, 4, 5, 6, 7, 8\} \quad A = \{1, 3, 5\} \quad B = \{1, 2, 3, 4\} \quad C = \{1, 2, 5, 6, 8\}$

a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

b. $(A \cup B)' = A' \cap B'$

c. $A \cap (A \cup B) = A$

3.1.4 Set-builder notation

It is impractical to try to list every element of a set. We use set-builder notation to describe most sets. There are two different forms of set-builder notation:

A **Property Description** is of the form, "The set of all x in u, such that x is ____ ." The blank is some *property* of x, which determines whether an element of U is or is not in the set.

- The set of even integers: $\{x \in \mathbb{Z} : x = 2y \text{ for some } y \in \mathbb{Z}\}$
- The set of real numbers bigger than 10: $\{x \in \mathbb{R} : x > 10\}$

A **Form Description** is of the form, "All numbers of the form ____ , where x is in the set D." The first part will be some equation (like "$2x$" for even).

- The set of even integers: $\{2k : k \in \mathbb{Z}\}$
- The set of perfect square integers: $\{m^2 : m \in \mathbb{Z}\}$

Question 7

Write the following in **property description** set-builder notation, using the steps given to help you figure it out.

"The set of all odd integers"

Step 1. Using x as the variable, what set does x belong in? $x \in$ ____

Step 2. In English, how would you describe x? x is ____________

Step 3. How would you write Step 2 symbolically? $x =$ ________

Step 4. For the **Property Description**,
it should be in the form *({ set : property })*. Fill out the following:

$\{x \in$ ____ (Step 1 set) : ________ (Step 3) for some ____ (2^{nd} var) $\in$ ____ (Step 1 set) $\}$

(The 2nd variable is part of the equation in Step 3.)

Question 8

Write the following in **form description** set-builder notation, using the steps given to help you figure it out.

"The set of all integers divisible by 3"

Step 1. Using x as the variable, what set does x belong in? $x \in$ ____

Step 2. In English, how would you describe x? x is ____________

Step 3. How would you write Step 2 symbolically? $x =$ ________

Step 4. For the **Form Description**,
it should be in the form *({ form : set })* . Fill out the following:

$\{$ ________ : ____ $\in$ ____ $\}$

Step 3 RHS   Step 3 RHS variable   Step 1 set

(Here, you don't use the full equation from Step 3; you remove the x.)