3.1 Set Definitions and Operations

3.1.1 Common Sets

Question 1

For the following numbers, which set(s) do they belong to?

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
10	✓	✓	✓	✓
-5		✓	✓	✓
12/6	✓	✓	✓	✓
π				✓
2.40			✓	✓

Question 2

Give examples for each of the following types of sets:

a. List three numbers that are in the set of all integers, $\mathbb{Z}$, but are NOT in the set of natural numbers, $\mathbb{N}$. $-1, -2, -3$

b. List three numbers that are in the set of rational numbers, $\mathbb{Q}$, but are NOT in the set of integers, $\mathbb{Z}$. $3/2, 5/2, 1/2$

c. List three numbers that are in the set of all real numbers $\mathbb{R}$, but are NOT in the set of rational numbers, $\mathbb{Q}$. $\pi, e, \sqrt{2}$

Question 3

Create sets that meet the following criteria. Give the sets any letter identifier that you want.

a. All elements of the set are odd integers. $A = \{1, 3, 5\}$

b. All elements of the set are fractions such that, when the numerator and denominator are divided, they result in an infinite string of numbers to the right of the decimal place (e.g., $3.3333333\bar{3}...$) $B = \{3/2, 5/2, 7/2\}$

c. Create two sets of integers, where the two sets have exactly two elements in common. $C = \{1, 2, 3\}$ $D = \{2, 3, 4\}$

d. Create two sets of natural numbers, where the two sets have NO elements in common. $E = \{1, 2\}$ $F = \{3, 4\}$

e. Create a set that is empty. $G = \{\}$

3.1.2 Subsets

Question 4

Given theset sets:
$U = \{-2, -1, 1, 2, 3, 4, 5, 6\}$ $A = \{1, 1, 2, 2, 2, 4, 4\}$ $B = \{-2, 2\}$
$C = \{1, 2, 4, 5, 6\}$ $D = \{6, 5, 4, 2, 1\}$ $E = \{1, 4\}$

a. Which of these statements are true? Mark with a ✓

a. $B \subseteq A$ x b. $B \subseteq E$ x c. $E \subseteq A$ ✓
d. $A \subseteq U$ ✓ e. $D \subseteq C$ ✓ f. $C \subseteq D$ ✓
g. $B \subseteq \mathbb{N}$ x h. $E \subseteq \mathbb{Z}$ ✓ i. $A \subseteq C$ ✓

b. Fill in the blanks with either $\subseteq$ (is a subset of), or $\not\subseteq$ (is not a subset of), or $=$ (is equal to) for the following:

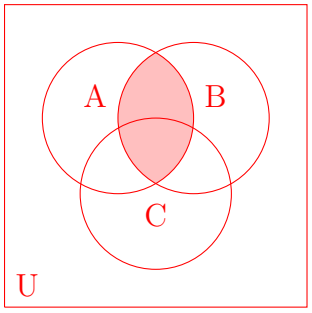
a. $C = D$ b. $B \subseteq U$ c. $A \not\subseteq E$

3.1.3 Intersections, unions, and differences

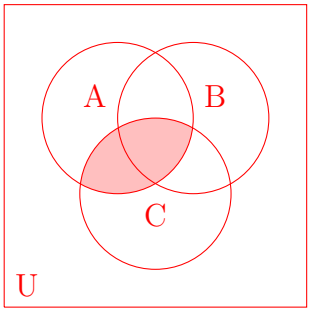
Question 5

For the following set operations, color in the Venn diagrams.

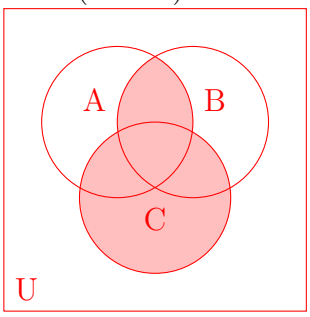
$A \cap B$



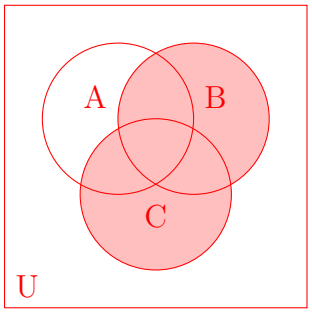
$A \cap C$



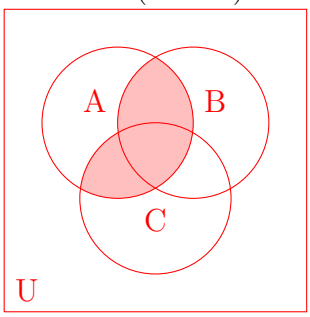
$(A \cap B) \cup C$



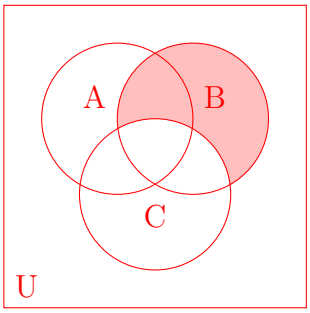
$B \cup C$



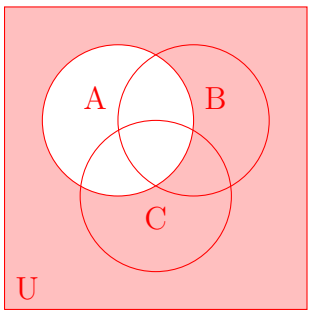
$A \cap (B \cup C)$



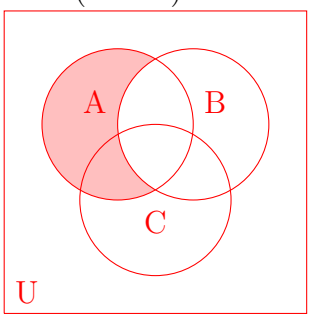
$B - C$



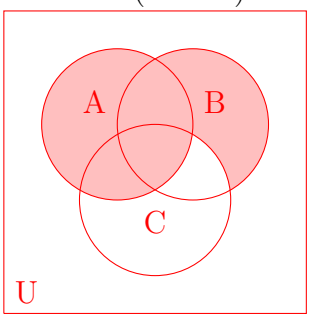
A'



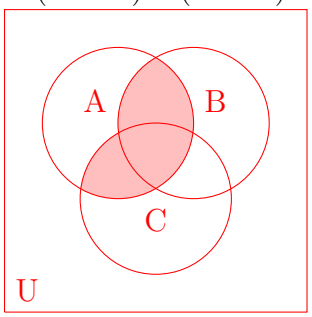
$(A \cup B) - B$



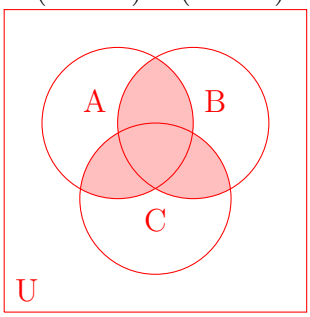
$A \cup (B - C)$



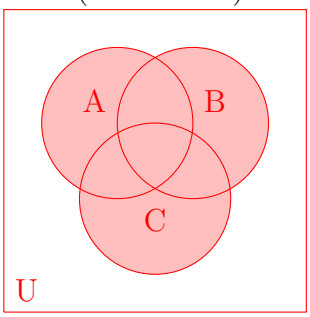
$(A \cap B) \cup (A \cap C)$



$(A \cup B) \cap (A \cup C)$



$(A \cup B \cup C)$



Question 6

Given the following sets, compute the set operations and prove the following statements.

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{1, 3, 5\}$ $B = \{1, 2, 3, 4\}$ $C = \{1, 2, 5, 6, 8\}$

a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$B \cup C = \{1, 2, 3, 4, 5, 6, 8\}$
$A \cap (B \cup C) = \{1, 3, 5\}$

b. $(A \cup B)' = A' \cap B'$
$(A \cup B) = \{1, 2, 3, 4, 5\}$
$(A \cup B)' = \{6, 7, 8\}$
$A' = \{2, 4, 6, 7, 8\}$
$B' = \{5, 6, 7, 8\}$
$A' \cap B' = \{6, 7, 8\}$

c. $A \cap (A \cup B) = A$
$(A \cup B) = \{1, 2, 3, 4, 5\}$
$A \cap (A \cup B) = \{1, 3, 5\}$

3.1.4 Set-builder notation

Question 7

Write the following in **property description** set-builder notation, using the steps given to help you figure it out.

"The set of all odd integers"

Step 1. Using x as the variable, what set does x belong in? $x \in \mathbb{Z}$

Step 2. In English, how would you describe x? x is an odd integer

Step 3. How would you write Step 2 symbolically? $x = 2k + 1$

Step 4. For the **Property Description**, it should be in the form *({ set : property })*. Fill out the following:

$\{x \in \underset{\text{Step 1 set}}{\mathbb{Z}} : \underset{\text{Step 3}}{x = 2k + 1} \text{ for some } \underset{2^{nd} \text{ var}}{k} \in \underset{\text{Step 1 set}}{\mathbb{Z}}\}$

(The 2nd variable is part of the equation in Step 3.)

Question 8

Write the following in **form description** set-builder notation, using the steps given to help you figure it out.

"The set of all integers divisible by 3"

Step 1. Using x as the variable, what set does x belong in? $x \in \mathbb{Z}$

Step 2. In English, how would you describe x? x is an odd integer

Step 3. How would you write Step 2 symbolically? $x = 2k + 1$

Step 4. For the **Form Description**,
it should be in the form $(\{\ form : set\ \})$. Fill out the following:

$\{$ $3m$: m $\in$ $\mathbb{Z}$ $\}$

Step 3 RHS | Step 3 RHS variable | Step 1 set

(Here, you don't use the full equation from Step 3; you remove the x.)