Chapter 2.1 Exercise, Mathematical writing

CS 210: Discrete Structures I
April 13, 2017

Rules

- In-class exercises are given each class period for every chapter.
- Work with you groupmate(s) on this exercise.
- Make sure your attendance is counted for you to get credit for this assignment.
- You will not turn in this exercise, but it will be useful to keep for studying for exams.
- Solutions are given at the end of class.

Section 1: Review implications

Implications are "if-then" statements, which utilize propositional variables (like p and q). An implication is generally written as: $p \rightarrow q$, which can be read aloud as "if p is true, then q is true", or "p implies q", or "if p, then q".

In an implication $p \rightarrow q$, p is called the **hypothesis** and q is called the **conclusion**.

The hypothesis and conclusion can each be more sophisticated propositional statements than just p and q, such as:
$(p \land q) \rightarrow r$ (just as an example.)

Truth values for implications

For a statement of the form, ***if HYPOTHESIS, then CONCLUSION*** to be **false**, it must be the case that the ***hypothesis*** is true, while the ***conclusion*** is false. Otherwise, the statement is **true**.

The truth-table is as follows:

p	q		$p \rightarrow q$
T	T		T
T	F		F
F	T		T
F	F		T

The negation of the implication $p \rightarrow q$ is the statement $p \land (\neg q)$.
Notice that the negation is **not also an implication**.

Exercise 1

For each statement, assign variables to the ***`hypothesis`*** and the ***`conclusion`***, and write the statement symbolically as an implication.

a. If I were a rich man, then I wouldn't have to work hard.

b. Write the negation of (a), symbolically.

c. If your friends ***`don't`*** dance, then they're no friends of mine.

(Write with negations)

c. Write the negation of (a), symbolically.

Section 2: Mathematical writing

Writing a statement as an implication

As we get into doing proofs to prove (or disprove) statements, we need to be able to define our statements as a ***implication*** (hypothesis → conclusion).

For example, if we make a statement like

> For every even number ***n***, ***n + 1*** is odd.

We need to translate this to an "if, then" statement, like:

> If ***n*** is an even number, then ***n + 1*** is odd.

Or, for a less-mathy statement...

> **Original:**
>
> All good things in life are free.
>
> **If, then form:**
>
> If ***t*** is a good thing, then ***t*** is free.

Exercise 2

For each statement, rewrite it as an "if, then" statement. (Not symbolically, but in English)

a. All odd integers have an even integer immediately after it.

b. All triangles have three sides.

c. All Computer Science students must take Discrete Math.

Section 3: Counterexamples

Writing a ***counterexample*** is one way to disprove a proposition. With our implications, if we can come up with some scenario where ***the hypothesis is true*** but the ***conclusion is false***, then we can disprove a statement.

Example problem:

Original:

For every even number n, $n + 2$ is odd.

If, then form:

If n is an even number, then $n + 2$ is odd.

If we can find **at least one example** that causes the hypothesis to be true, and the conclusion to be false, then the ***statement is disproven*** (if it is talking about "for all"!)

For this example, it is false for all cases, but with other statements, the statement might be true part of the time!

Exercise 3

Disprove the following statements with a counterexample.

a. For all integers $n \geq 2$, $n / 2$ is also an integer.

b. For all integers $n \geq 1$, if n is odd, then n^2+4 is a prime number.

c. All math textbooks are less than $40.

Section 4: Proving an implication is true

How do we prove that an implication is true for ***all cases***?

Example statement:

The result of summing any ***odd integer*** with any ***even integer*** is an odd integer.

First, we need to translate this to mathematical language.

This problem is talking about two integers that are added together, so let's define ***m as the odd integer*** and ***n as the even integer***, so mathematically ***m + n*** should be odd.

But how do we describe "even" and "odd" mathematically?
2, 4, 6, and 8 are all divisible by 2... or in other words, ***2 times some integer***.
3, 5, 7, and 9 are even integers plus one, so, ***2 times some integer, plus 1***.

Definition of an even integer *n*
n = 2k, where ***k*** is an integer.

Definition of an even integer *m*
m = 2j + 1, where ***j*** is an integer.

So, writing it in terms of math...

- m + n = (2j + 1) + (2k)

And we simplify to get back to the definition of an ***odd integer...***

- = 2j + 1 + 2k
- = 2j + 2k + 1
- = 2(j + k) + 1

The definition of an odd integer is ***2 times some other integer, plus 1.***, and the addition of two integers ***j and k*** will also result in an integer, so we have rewritten our problem in a form that shows it will always result in an odd integer.

Exercise 4

Prove the following statement, using the definitions of even and odd integers.

For all integers $n > 0$, if n is odd, then $n^2 + n$ is even.

Definitions

The closure property of integers

If ***a*** is an integer and ***b*** is an integer...

- $a + b$ is always an integer
- $a - b$ is always an integer
- $a \times b$ is always an integer

The closure property does not hold for division!

Definition of an integer divisible by 4

An integer ***a*** is divisible by 4 if it can be written in the form $a = 4b$ for some integer ***b***.
(And similar for divisibile by other integers...)

Definition of an even integer *n*
$n = 2k$, where ***k*** is an integer.

Definition of an even integer *m*
$m = 2j + 1$, where ***j*** is an integer.

Exercise 5

Using the definitions of even, odd, and divisible by (some integer), show that the following statements are true:

a. 12 is even
(12 is some integer times 2)

b. 12 is divisible by 4
(12 is some integer times 4)

c. -13 is odd.

d. -8 is divisible by 4.

e. $8n^2 + 8n + 4$ is divisible by 4.
(Hint: You don't need to foil this, just factor out the common term...!)

f. $2n + 2n$ is an even number