

4.4 Properties of Relations

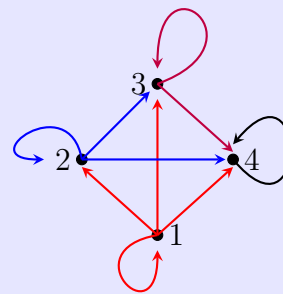
4.4.1 Relations

Relations: A Relation is a way to relate two sets of data together. The two sets are the Domain and Codomain, and there is a Rule that associates them together.

Example: $R : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$;

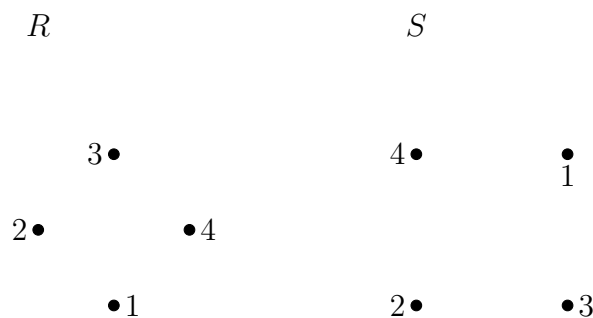
The relation R has a domain of $\{1, 2, 3\}$ and a codomain of $\{1, 2, 3\}$.

The rule is: $(x, y) \in R$ if $x \leq y$; so there is a relation (an arrow) from x to y if x is less than or equal to y . 1 points to 1, 2, 3, and 4, 2 points to 2, 3, and 4, and 3 points to 3 and 4, and 4 only points to itself.



Question 1

Draw the arrows for the following relations:



Set A is $A = \{1, 2, 3, 4\}$.

Relation R : $R : A \rightarrow A$, with the rule: $\{(1, 2), (2, 3), (3, 4)\}$

Relation S : $S : A \rightarrow A$, with the rule: $\{(1, 1), (2, 3), (2, 4), (3, 4)\}$

Properties of Binary Relations

Let R be a binary relation on set A .

Reflexive: R is said to be reflexive if $(a, a) \in R$ for all $a \in A$. In terms of the arrow diagram, this means that **every node has a loop**.

Irreflexive: A relation R on set A is irreflexive if, for all $a \in A$, $(a, a) \notin R$. On the arrow diagram, this means **there are no loops**.

Symmetric: R is called symmetric if for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \in R$. In terms of the arrow diagram, this means that **every arrow goes in both directions**.

Antisymmetric: R is called antisymmetric if for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \notin R$. In terms of the arrow diagram, this means that **arrows only go in one direction**.

Transitive: R is transitive if, whenever $(a, b) \in R$ and $(b, c) \in R$, it must also be the case that $(a, c) \in R$. In terms of the arrow diagram, this means that **whenever you can follow two arrows to get from node a to node c , you can also get there along a single arrow**.^a

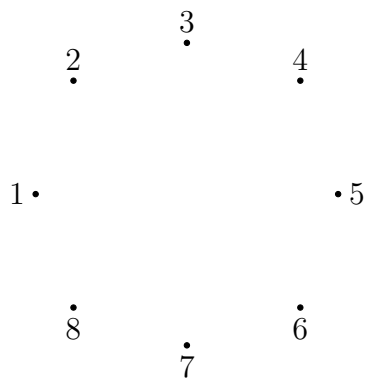
Note that relations can be Reflexive, Irreflexive, or Neither, as well as Symmetric, Antisymmetric, or Neither.

^aDiscrete Mathematics, Ensley and Crawley

Question 2

Complete the arrow diagram for the relation on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Also identify the properties of each. Give reasons why the relation is transitive, intransitive, neither, symmetric, antisymmetric, neither, and/or reflexive.

$$R_1 = \{ (1,1), (1,2), (1,4), (1,8), \quad (2,2), (2,4), (2,8), \quad (3,3), (3,6), \\ (4,4), (4,8) \quad (5,5), \quad (6,6), \quad (7,7), \quad (8,8) \}$$



☐ Reflexive? ☐ Irreflexive? ☐ Neither? Why?

☐ Symmetric? ☐ Antisymmetric? ☐ Neither? Why?

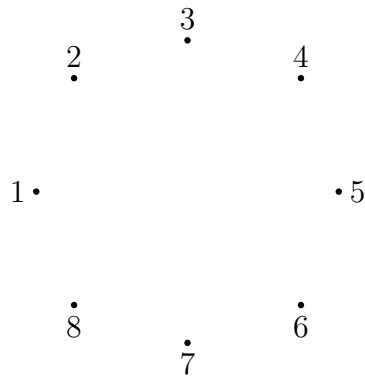
☐ Transitive? Why?

Question 3

Complete the arrow diagram for the relation on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Also identify the properties of each. Give reasons why the relation is transitive, intransitive, neither, symmetric, antisymmetric, neither, and/or reflexive.

FIXME: KEY SHOULD HAVE LOOPS

$$R_2 = \{ (1,1), (1,3), (1,5), (1,7), (2,2), (2,4), (2,8), \\ (3,3), (3,5), (3,7), (4,2), (4,4), (4,8), (5,3), (5,7), \\ (6,6), (6,8), (8,2), (8,4), (8,8) \}$$



☐ Reflexive? ☐ Irreflexive? ☐ Neither? Why?

☐ Symmetric? ☐ Antisymmetric? ☐ Neither? Why?

☐ Transitive? Why?

Recap

- **Reflexive:** $(a, a) \in R$ for all $a \in A$
- **Irreflexive:** $(a, a) \notin R$ for all $a \in A$
- **Antisymmetric:** for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \notin R$
- **Transitive:** if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$

Question 4

Given the relation, $R_1 = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a + b \text{ is even}\}$.

- a. This relation is **reflexive**. Find an example to illustrate why.

a and b are both in the set of integers. We are checking to see if the result of (a, a) is always in the relation R_1 ... so, if you plug in (a, a) into the relation, is the output still “is even”?

- b. This relation is **symmetric**. Find an example to illustrate why.

Find some (a, b) and (b, a) that are both in the relation. If you can, it's symmetric.

Question 5

Let C be the set of all cats who have ever lived. For each of the following relations on the set C , decide if the given is reflexive irreflexive, transitive, or antisymmetric. Some of these can satisfy more than one property. Give explanations on how you decided each of these.

- a. $R_1 = \{(a, b) \in C \times C : a \text{ is a child of } b\}$
- Reflexive - Is $(a, a) \in C$ for all a valid?
 - Irreflexive - Is $(a, a) \notin C$ for all a valid?
 - Transitive - Is there some $(a, b) \in C$ and $(b, c) \in C$? ¹
 - Antisymmetric - Is $(a, b) \in C$ and $(b, a) \notin C$ valid?
- b. $R_2 = \{(a, b) \in C \times C : a \text{ is a descendant of } b\}$
- Reflexive - Is $(a, a) \in C$ for all a valid?
 - Irreflexive - Is $(a, a) \notin C$ for all a valid?
 - Transitive - Is there some $(a, b) \in C$ and $(b, c) \in C$?
 - Antisymmetric - Is $(a, b) \in C$ and $(b, a) \notin C$ valid?

¹Just assume a cat isn't going to mate with its child. : |