3.4 Boolean Algebra

3.4.1 Logic, Sets, and Boolean Algebra

There are a lot of similarities between the operations we have been doing on sets and with the logic operators we used back in Chapter 1. Now, we're also going to introduce the idea of Boolean Algebra, which also has similarities to Sets and Logic.

	Logic	**Sets**	**Boolean Algebra**
Variables	p, q, r	A, B, C	a, b, c
"and" operation	$\wedge$	$\cap$	$\cdot$
"or" operation	$\vee$	$\cup$	$+$
"not" operation	$\neg$	$'$	$'$
"-" operation	$a \wedge \neg b$	$A - B$	$a \cdot b'$
Special	Tautology	Universal set U	1
	Contradiction	Empty set $\emptyset$	0

Example: Rephrase the following Logic operation using Set and Boolean Algebra notations: $(p \wedge q) \vee r$

Sets: $(P \cap Q) \cup R$

Boolean algebra: $(p \cdot q) + r$

Question 1

Rewrite the following using Boolean Algebra notation:

a. $p \wedge q$

b. $p \vee q$

c. $\neg p$

d. $(p \wedge \neg q) \vee p$

e. $\neg(\neg p)$

f. $(p \wedge \neg q) \vee p \equiv p$

Question 2

Rewrite the following using Boolean Algebra notation:

a. $A \cap B$

b. $A \cup B$

c. A'

d. $(A - B)$

e. $A' \cup (A \cap B)$

f. $(A - B)' = A' \cup (A \cap B)$

3.4.2 Boolean Algebra properties

Commutative	$a \cdot b = b \cdot a$	$a + b = b + a$
Associative	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(a + b) + c = a + (b + c)$
Distributive	$a \cdot (b + c)$ $= (a \cdot b) + (a \cdot c)$	$a + (b \cdot c)$ $= (a + b) \cdot (a + c)$
Identity	$a \cdot 1 = a$	$a + 0 = a$
Negation	$a + a' = 1$	$a \cdot a' = 0$
Double negative	$(a')' = a$	
Idempotent	$a \cdot a = a$	$a + a = a$
DeMorgan's laws	$(a \cdot b)' = a' + b'$	$(a + b)' = a' \cdot b'$
Universal bound	$a + 1 = 1$	$a \cdot 0 = 0$
Absorption	$a \cdot (a + b) = a$	$a + (a \cdot b) = a$
Complements of 1 and 0	$1' = 0$	$0' = 1$

[a]

[a] From Discrete Mathematics, Ensley and Crawley

Question 3

Simplify the following equations using the Boolean Algebra properties.

a. $xy + xy'$

b. $x \cdot (x' + y)$

Question 4

Using the Boolean Algebra properties, transform the left-hand side of each equation to the right-hand side.

a. $x'yz + x'y'z + xyz' + xy'z' = xz' + x'z$
(Group up two terms at a time: $xyz' + xy'z'$ and $x'yz + x'y'z$.)

b. $xyz + xyz' + x'yz + x'yz' = y$