1.3 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

1.3   Propositional Logic

1.3.1   Propositional Logic

A **proposition** is a statement which has truth value: it is either true (T) or false (F). [a]

The statement doesn't have to necessarily be TRUE, it could also be FALSE, but it has to be unambiguously so.

Statement	Proposition?	Result?
$2 + 3 = 5$	Yes	True
$2 + 2 = 5$	Yes	False
This class has 30 students	Yes	False[b]
Is the movie good?	No	

——————
[a]From https://en.wikibooks.org/wiki/Discrete_Mathematics/Logic
[b]Probably :)

Question 1
For the following, mark whether the statement is a **proposition** and, if it is, mark whether it is **true** or **false**.

	statement	is proposition?	is true/false?
a.	$10 + 20 = 30$		
b.	2 times an integer is always even.		
c.	Is $4 < 5$?		
d.	My cat is the best cat.		
e.	$x \cdot y$ is odd.		
f.	$5 + 1$ is odd.		

1.3.2 Compound Propositions

We can also create a **compound proposition** using the logical operators for AND $\wedge$, OR $\vee$, and NOT $\neg$. When we're writing out a compound proposition, we will usually assign each proposition a **propositional variable**.

If we have the propositions p is the proposition "Paul is taking discrete math", q is the proposition, "Paul has a calculator", then we can build the compound propositions like:

$p \wedge q$: Paul is taking discrete math and Paul has a calculator

$p \vee q$: Paul is taking discrete math or Paul has a calculator

$\neg p$: Paul is NOT taking discrete math

$\neg q$: Paul does NOT have a calculator

The result of a compound proposition depends on the value of each proposition it is made up of:

1. A compound proposition $a \wedge b \wedge c$ is **only true** if all propositions are true; it will be **false** if one or more of the propositions is false.

2. A compound proposition $a \vee b \vee c$ is **true** if one or more of the propositions is true; it is **only false** if all propositions are false.

3. A compound proposition $\neg a$ is **true** only if the proposition is false; it is only **false** if the proposition is true.

Question 2

Given the following compound propositions and proposition values, write out whether the full compound proposition is **true** or **false**.

a. a AND b:

	Compound	Values	Result
a.	$a \land b$	$a = true, b = true$	TRUE
b.	$a \land b$	$a = true, b = false$	FALSE
c.	$a \land b$	$a = false, b = true$	FALSE
d.	$a \land b$	$a = false, b = false$	FALSE

b. a OR b:

	Compound	Values	Result
e.	$a \lor b$	$a = true, b = true$	
f.	$a \lor b$	$a = true, b = false$	
g.	$a \lor b$	$a = false, b = true$	
h.	$a \lor b$	$a = false, b = false$	

c. Combinations:

	Compound	Values	Result
i.	$a \land \neg b$	$a = true, b = false$	
j.	$a \lor \neg b$	$a = false, b = true$	
k.	$\neg a \land b$	$a = false, b = true$	
l.	$\neg a \lor b$	$a = false, b = false$	

Question 3

For the following, "translate" the following English statements into compound propositions.

m: There is milk in the fridge e: There are eggs in the fridge
c: There is cheese in the fridge

   a. There is milk in the fridge, or there is cheese in the fridge.

   b. There is milk in the fridge, or there is cheese in the fridge, but not both.

   c. There is milk in the fridge but there are no eggs in the fridge.

   d. There is milk in the fridge and there is either cheese or eggs in the fridge.

   e. There is not milk in the fridge and there is not cheese in the fridge.

   f. It is not true that... there is milk or cheese in the fridge.

   g. There is not milk in the fridge or there is not cheese in the fridge.

   h. It is not true that... there is milk and cheese in the fridge.

1.3.3   Truth Tables

When we're working with compound propositional statements, the result of the compound depends on the true/false values of each proposition it is built up of.

We can diagram out all possible states of a compound proposition by using a **truth table**. In a truth table, we list all propositional variables first on the left, as well as all possible combinations of their states, and then the compound statement's result on the right.

p	q		$p \wedge q$
T	T		T
T	F		F
F	T		F
F	F		F

p	q		$p \vee q$
T	T		T
T	F		T
F	T		T
F	F		F

p		$\neg p$
T		F
F		T

Question 4
Complete the following truth tables.

a.

p	q		$\neg q$	$p \wedge \neg q$
T	T		F	
T	F		T	
F	T			
F	F			

b.

p	q		$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T				
T	F				
F	T				
F	F				

When you're building out truth tables, there is a specific order you should write out the "T" and "F" states. Begin with all "T" values first, and work your way down to all "F" values first. As you go, change the right-most state from "T" to "F", working your way from right-to-left.

So with two variables:
"TT", "TF", "FT", "FF".

Question 5
Using the rules above, write out all the states for a truth table with three propositional variables.

p	q	r
T	T	T
T	T	F
T		
T	F	F
F		
F		
F		
F	F	F

> Whenever the final columns of the truth tables for two propositions p and q are the same, we say that p and q are **logically equivalent**, and we write: $p \equiv q$. [a]
>
> _____
>
> [a]From https://en.wikibooks.org/wiki/Discrete_Mathematics/Logic

Question 6

Use a truth table to show that the compound propositions,

$$\neg p \wedge \neg q \qquad \text{and} \qquad \neg(p \vee q)$$

are logically equivalent. The final two columns are the compound propositions above

p	q		$\neg p$	$\neg q$	$(p \vee q)$		$\neg p \wedge \neg q$		$\neg(p \vee q)$
T	T		F	F					
T	F		F	T					
F	T		T	F					
F	F		T	T					