

## 4.3 Properties of Functions and Set Cardinality

### 4.3.1 Review: Inverses of functions

Given a relation  $R$  with domain  $A$  and codomain  $B$ , the relation  $R_{-1}$  (read “ $R$  inverse”) with domain  $B$  and codomain  $A$  is called the **inverse** of  $R$ , and is defined so that

$$(x, y) \in R \quad \text{if and only if} \quad (y, x) \in R^{-1}$$

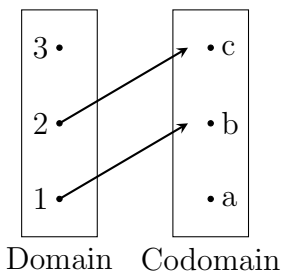
Also note that the inverse of  $R^{-1}$  is  $R$ .<sup>a</sup>

<sup>a</sup>Discrete Mathematics, Ensley and Crawley

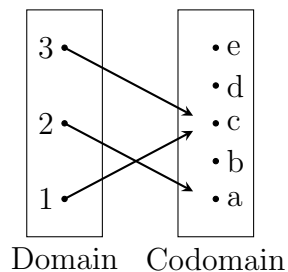
#### Question 1

Draw the inverse of each diagram. Identify whether the original diagram and/or the inverse of that diagram are functions.

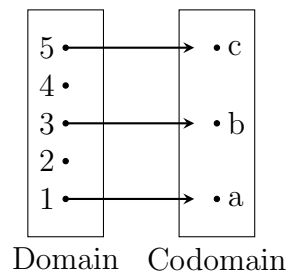
a.



b.



c.



### 4.3.2 Functions that are invertible

**Functions that are onto**

A function is **onto** if every output from the codomain has at least one input from the domain. Every output is attainable via at least one input.

For diagrams: Every point in the codomain has at least one arrow pointing at it.

**Functions that are one-to-one**

A function is **one-to-one** if every output from the codomain has no more than one input from the domain. No two inputs lead to the same output.

For diagrams: None of the points in the codomain has two or more arrows pointing at it.

**Functions that are invertible**

If a function is both one-to-one AND onto, then it is invertible. This means that the inverse of the function is ALSO a function.

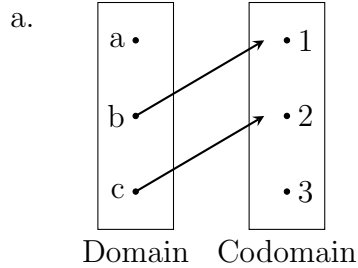
For diagrams: Every point in the codomain has exactly one arrow pointing at it.

#### Question 2

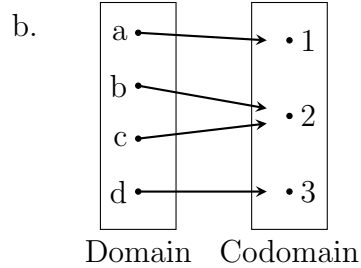
Draw two functions: One where the function is one-to-one but not onto, and one where the function is onto but not one-to-one. Make sure to label your domain and codomain for each.

**Question 3**

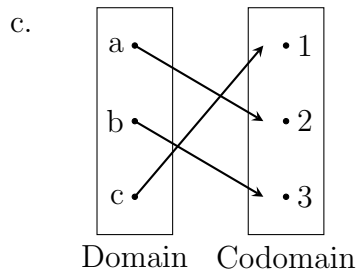
Determine whether these functions are one-to-one, onto, and/or invertible. If not, state why not.



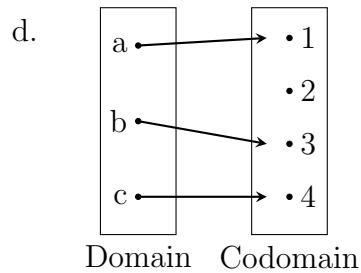
- ☐ Onto      ☐ One-to-one  
☐ Invertible



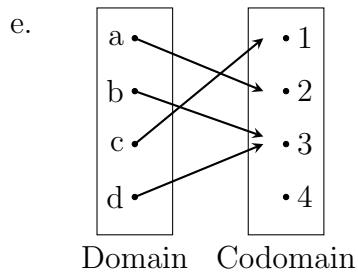
- ☐ Onto      ☐ One-to-one  
☐ Invertible



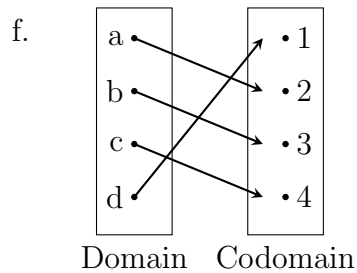
- ☐ Onto      ☐ One-to-one  
☐ Invertible



- ☐ Onto      ☐ One-to-one  
☐ Invertible



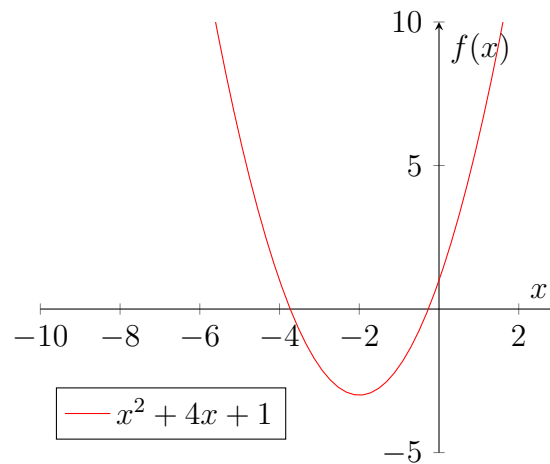
- ☐ Onto      ☐ One-to-one  
☐ Invertible



- ☐ Onto      ☐ One-to-one  
☐ Invertible

**Question 4**

The function  $f : \mathbb{R} \rightarrow \mathbb{R}$ , with the rule  $f(x) = x^2 + 4x + 1$  is not onto and not one-to-one.



- Give an example of an element in the codomain that has no element in the domain associated with it.
- Given an example of two elements in the domain that are both associated with the same output in the codomain.