

Chapter 1.4 In-class Exercise

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Chapter 1.4 In-class Exercise Instructions

This is the instruction page, make sure to fill out your answers in the worksheet later in the document

1. Predicates

Predicates

In mathematical logic, a **predicate** is commonly understood to be a Boolean-valued function. ^a

We write a predicate as a function, such as $P(x)$, for example:

$P(x)$ is the predicate, “x is less than 2”.

Once some value is plugged in for x , the result is a proposition - something either unambiguously **true** or **false**, but until we have some input for x , we don't know whether it is true or false.

$$P(0) = \text{true} \quad P(2) = \text{false} \quad P(10) = \text{false}$$

Additionally, predicates can also be combined with the logical operators AND \wedge OR \vee and NOT \neg .

^aFrom [https://en.wikipedia.org/wiki/Predicate_\(mathematical_logic\)](https://en.wikipedia.org/wiki/Predicate_(mathematical_logic))

Question 1

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For the following predicates given, plug in **2**, **23**, **-5**, and **15** as inputs and write out whether the result is true or false.

- a. $P(x)$ is the predicate " $x > 15$ "
- b. $Q(x)$ is the predicate " $x \leq 15$ "
- c. $R(x)$ is the predicate " $(x > 5) \wedge (x < 20)$ "

Domain

When we're working with predicates, we will also define the domain.

The **domain** is the set of all possible inputs for our predicate. In other words, x must be chosen from the domain.

Question 2

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For the following predicates and domains given, specify whether the predicate is true for **all members of the domain**, **some members of the domain**, or **no members of the domain**.

- a. $P(x)$ is the predicate " $x > 15$ ", the domain is $\{10, 12, 14, 16, 18\}$.
- b. $Q(x)$ is the predicate " $x \leq 15$ ", the domain is $\{0, 1, 2, 3\}$.
- c. $R(x)$ is the predicate " $(x > 5) \wedge (x < 20)$ ", the domain is $\{0, 1, 2\}$.
- d. $S(x)$ is the predicate " $(x > 1) \wedge (x < 5)$ ", domain is $\{2, 3, 4\}$.

2. Quantifiers

Quantifiers

Symbolically, we can specify that the input of our predicate, x , belongs in some domain set D with the notation: $x \in D$. This is read as, “ x exists in the domain D .”

Additionally, we can also specify whether a predicate is true **for all inputs x from the domain D** using the “for all” symbol \forall , or we can specify that the predicate is true **for *some* inputs x from the domain D** using the “there exists” symbol \exists .

Example: Rewrite the predicate symbolically. $P(x)$ is “ $x > 15$ ”, the domain D is $\{16, 17, 18\}$. Here we can see that all inputs from the domain will result in the predicate evaluating to true, so we can write:

$$\forall x \in D, P(x) \text{ (“For all } x \text{ in } D, x \text{ is greater than 15.”)}$$

- The symbol \in (“in”) indicates membership in a set.
- The symbol \forall (“for all”) means “for all”, or “every”.
- The symbol \exists (“there exists”) means “there is (at least one)”, or “there exists (at least one)”.
- The symbols \forall and \exists are called **quantifiers**. When used with predicates, the statement is called a **quantified predicate**.

Question 3

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For the following predicates, rewrite the sentence symbolically, as in the example above. Use either \forall or \exists , based on whether the predicate is true for the domain given.

Hint If a predicate $P(x)$ is false for all elements in the domain, you can phrase it as: “ $\forall x \in D, \neg P(x)$ ”.

- $P(x)$ is the predicate “ $x > 15$ ”, the domain is $\{10, 12, 14, 16, 18\}$.
- $Q(x)$ is the predicate “ $x \leq 15$ ”, the domain is $\{0, 1, 2, 3\}$.
- $R(x)$ is the predicate “ $(x > 5) \wedge (x < 20)$ ”, the domain is $\{0, 1, 2\}$.
- $S(x)$ is the predicate “ $(x > 1) \wedge (x < 5)$ ”, domain is $\{2, 3, 4\}$.

Question 4

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For the following predicates, rewrite the sentence symbolically using the domain $D = \{ 3, 4, 5, 10, 20, 25 \}$. Make sure to define your predicates (state that “ $P(x)$ is the predicate...”, and afterwards specify whether the quantified predicate is true or false. (If it states “there exists” but none exist, then the quantified predicate is false.)

- a. There is (at least one) k in the set D with the property that k^2 is also in the set D .

Hint How can you specify the predicate, “ k^2 is in the set D ” symbolically?

- b. There exists some m that is a member of D , such that $m \geq 3$.

3. Negating quantifiers

Proposition 1

For any predicates P and Q over a domain D ,

- The negation of $\forall x \in D, P(x)$ is $\exists x \in D, \neg P(x)$.
- The negation of $\exists x \in D, P(x)$ is $\forall x \in D, \neg P(x)$.

When negating a predicate that uses an equal sign, the negation would be “not equals”.

Example: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \cdot y = 0$.

1. $\neg(\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \cdot y = 0)$
2. $\equiv \quad \exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \cdot y \neq 0$

Question 5

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Write the negation of each of these statements. Simplify as much as possible.

- a. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, 2x + y = 3$
- b. $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x \cdot y < x$
- c. $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, (x + y = 13) \wedge (x \cdot y = 36)$

Hint:Negating propositions From DeMorgan's laws,

$$\neg(p \wedge q) \equiv \neg p \vee \neg q.$$

Sets of numbers Some sets we will be using often in this class are...

\mathbb{Z} = "The set of all integers".

\mathbb{N} = "The set of all natural numbers".

\mathbb{Q} = "The set of all rational numbers".

\mathbb{R} = "The set of all real numbers".

Question 6

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Which elements of the set $D = \{2, 4, 6, 8, 10, 12\}$ make the **negation** of each of these predicates true?

- a. $Q(n)$ is the predicate, " $n > 10$ ".
- b. $R(n)$ is the predicate, " n is even".
- c. $S(n)$ is the predicate, " $n^2 < 1$ ".
- d. $T(n)$ is the predicate, " $n - 2$ is an element of D ".

Chapter 1.4 In-class Exercise Worksheet

ANSWER KEY

Team: Please write down all people in your team.

- | | |
|----|----|
| 1. | 2. |
| 3. | 4. |

Section:

☐ MW 4:30 - 5:45 pm ☐ M 6:00 - 8:50 pm ☐ TR 2:00 - 3:15 pm

Rules:

- **Only one worksheet will be turned in per team.** Each member of the team will receive the same score.
- You can collaborate on the exercise together, or you can work separately and then compare your answers with your team as you fill out the turn-in worksheet.
- Fill out your answers on this worksheet.
- **Write cleanly and linearly** - if I can't make sense of your solution then you won't get credit.
- **Don't scribble out cancellations** - I can't read that. For example, if a numerator/denominator cancel out, or a $+/-$ cancels out, don't scribble out the numbers - just use a single slash!

Grading

The total amount of points for an in-class exercise is 5 points each. Each question will have a weight assigned to it, and will be given a point value between 0 and 4:

0	Nothing written
1	Something written, but incorrect
2	Partially correct, but multiple errors
3	Mostly correct, with one or two errors
4	Perfect; correct answer and notation

Question	Weight	0-4	Adjusted score
1	15%		
2	20%		
3	12%		
4	14%		
5	15%		
6	24%		

Answer sheet

15% Question 1: True or false?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

$P(2) :$	False	$P(23) :$	True	$P(-5) :$	False	$P(15) :$	False
$Q(2) :$	True	$Q(23) :$	False	$Q(-5) :$	True	$Q(15) :$	True
$R(2) :$	False	$R(23) :$	False	$R(-5) :$	False	$R(15) :$	True

20% Question 2: All, some, or none?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

- a. ☐ True for all ☐ True for some ☐ True for none
 True for some
- b. ☐ True for all ☐ True for some ☐ True for none
 True for all
- c. ☐ True for all ☐ True for some ☐ True for none
 True for none
- d. ☐ True for all ☐ True for some ☐ True for none
 True for all

12% Question 3: Write symbolically

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

- a. $P(x)$ is the predicate “ $x > 15$ ”, the domain is $\{10, 12, 14, 16, 18\}$.
 $\exists x \text{ in } D, P(x)$
- b. $Q(x)$ is the predicate “ $x \leq 15$ ”, the domain is $\{0, 1, 2, 3\}$. $\forall x \text{ in } D, Q(x)$
- c. $R(x)$ is the predicate “ $(x > 5) \wedge (x < 20)$ ”, the domain is $\{0, 1, 2\}$.
 $\forall x \text{ in } D, \neg R(x)$
- d. $S(x)$ is the predicate “ $(x > 1) \wedge (x < 5)$ ”, domain is $\{2, 3, 4\}$.
 $\forall x \text{ in } D, S(x)$

14% Question 4: Write symbolically

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

- a. $\exists k \in D, P(x)$, where $P(x)$ is the predicate “ $k^2 \in D$ ”.
- b. $\exists m \in D, Q(x)$, where $Q(m)$ is the predicate “ $m \geq 3$ ”.

15% Question 5: Negations☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

- a. Negate $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, 2x + y = 3$
 $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, 2x + y \neq 3$
- b. Negate $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x \cdot y < x$
 $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x \cdot y \geq x$
- c. Negate $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, (x + y = 13) \wedge (x \cdot y = 36)$
 $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, (x + y \neq 13) \vee (x \cdot y \neq 36)$

24% Question 6: True or false?☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4Given the domain $D = \{2, 4, 6, 8, 10, 12\}$

- a. $Q(n)$ is the predicate, “ $n > 10$ ”; negation is true for inputs:
 $\neg Q(n)$ is $n \leq 10$, so $\{2, 4, 6, 8, \text{ and } 10\}$.
- b. $R(n)$ is the predicate, “ n is even”; negation is true for inputs:
 $\neg R(n)$ is n is odd, so NONE.
- c. $S(n)$ is the predicate, “ $n^2 < 1$ ”; negation is true for inputs:
 $\neg S(n)$ is $n^2 \geq 1$, so ALL.
- d. $T(n)$ is the predicate, “ $n - 2$ is an element of D ”; negation is true for inputs:
 $\neg T(n)$ is $n - 1$ is not an element of D , so $\{2\}$.