Name: ______________________________

**Chapter 1 EXAM:**

This exam covers concepts from Chapter 1. Questions will be labelled by the section from which they are based.

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Nothing written | Attempted, but incorrect | Partially correct; multiple errors | Mostly correct, one or two errors | Perfect; correct answer & notation |

## Grading breakdown:

| Question | Weight | Points Received | Weighted Score |
|---|---|---|---|
| 1 | 8% | | |
| 2 | 8% | | |
| 3 | 10% | | |
| 4 | 10% | | |
| 5 | 12% | | |
| 6 | 12% | | |
| 7 | 5% | | |
| 8 | 5% | | |
| 9 | 8% | | |
| 10 | 10% | | |
| 11 | 12% | | |
| | | | |

Scratch page

# Exam

## 1.2 Sequences and Summations

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**8%** Question 1: Closed formula (1.2) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

For the given sequence, find the **closed formula**.

$$5, 7, 9, 11, 13$$

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**8%** Question 2: Recursive formula (1.2) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

For the given sequence, find the **recursive formula**.

$$1, 2, 4, 8, 16$$

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**10%** Question 3: Summation (1.2) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Evaluate the following summation:

$$\sum_{k=1}^{4}(3k+2)$$

$a_1 =$ $a_2 =$

$a_3 =$ $a_4 =$

$a_1 + a_2 + a_3 + a_4 =$

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## 1.3 Propositions

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**10%** Question 4: Truth table (1.3) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Complete the truth table for the following expression:

$$(p \wedge q) \vee (p \wedge \neg r)$$

You don't need to fill out every column in the table; those are there to help you. The final column is the only required part.

| $p$ | $q$ | $r$ | $\neg r$ | $p \wedge \neg r$ | $p \wedge q$ | $(p \wedge q) \vee (p \wedge \neg r)$ |
|---|---|---|---|---|---|---|
| T | T | T | | | | |
| T | T | F | | | | |
| T | F | T | | | | |
| T | F | F | | | | |
| F | T | T | | | | |
| F | T | F | | | | |
| F | F | T | | | | |
| F | F | F | | | | |

**12%** Question 5: Propositional statements (1.3) □ 0 □ 1 □ 2 □ 3 □ 4

Given three propositional variables, $p$, $q$, and $r$, write a compound statement that will meet the following criteria. Note the logic operator precedence: 1. $\neg$, 2. $\wedge$, 3. $\vee$; if you're mixing AND and OR, you might want to use parentheses to make the order explicit.

a. $p$ and $q$ are true, but not $r$.

b. $p$ and one other statement are true, but not all three.

c. Exactly two statements are true, but not only one and not all three.

**12%** Question 6: Propositional statements (1.3) □ 0 □ 1 □ 2 □ 3 □ 4

Translate the following statements into propositional logic using the given variables: $b$ : The sandwich has bacon $l$ : The sandwich has lettuce
$t$ : The sandwich has tomato $c$ : The sandwich has cheese

a. The sandwich has bacon and lettuce and tomato.

b. The sandwich has tomato and it doesn't have cheese.

c. The sandwich has tomato and it doens't have bacon, nor does it have cheese.

d. Either the sandwich has (only) lettuce and tomato or it has (only) bacon and cheese.

## 1.4 Quantifiers

**5%** Question 7: Domains (1.4) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Define a domain for the following quantified statement.
$P(n)$ is the predicate, "$n$ ends with the number 5."
Quantified statement: $\forall n \in D, P(n)$

$D = \{ \qquad \}$

**5%** Question 8: Domains (1.4) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Define a domain for the following quantified statement.
$Q(x)$ is the predicate, "$x$ is even"
Quantified statement: $\forall x \in E, \neg Q(x)$

$E = \{ \qquad \}$

**8%** Question 9: Quantified statements (1.4) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Given the domain, $D = \{2, 3, 4, 5, 6\}$, and the predicate:
$P(x)$ is "For every element $x$ that is a member of the domain $D$, $x$ is odd."

a. Write the statement symbolically, including the quantifiers.

b. Write the negation of the statement from (a) symbolically.

c. Write the negation from (b) in English.

d. Write statement ends up being true for the domain $D$? The original or the negation?

## 1.5 Implications

**10%** Question 10: Truth table (1.5) □ 0 □ 1 □ 2 □ 3 □ 4

Complete the truth table for the following expressions:

$$p \to q \qquad \neg(p \to q) \qquad p \wedge \neg q$$

Use the empty column if needed. Fill in the last 3 columns.

| $p$ | $q$ | | $p \to q$ | $\neg(p \to q)$ | $p \wedge \neg q$ |
|---|---|---|---|---|---|
| T | T | | | | |
| T | F | | | | |
| F | T | | | | |
| F | F | | | | |

**12%** Question 11: Inverse, converse, ... (1.5) □ 0 □ 1 □ 2 □ 3 □ 4

For the following statement, write out the **converse**, **inverse**, and **contrapositive** in English. $p$ : The animal has pointy ears, $q$ : the animal is a cat. $p \to q$: If the animal has pointy ears, then the animal is a cat.

a. Negation $p \wedge \neg q$:

b. Converse $q \to p$:

c. Inverse $\neg p \to \neg q$:

d. Contrapositive $\neg q \to \neg p$:

# Cheat sheet

**1.2: Sequences**

**Closed formula:** A formula written where the value is derived from its position in the list. For example, $a_n = 3n$, where the value, $3n$, changes based on the index value, $n$.

**Recursive formula:** A formula written where the value is derived from the previous item in the list. The first element $a_1$ is always given. For example, $a_1 = 3, a_n = a_{n-1} + 3$, subsequent elements $a_n$ are based on the prevoius term, $a_{n-1}$, with $a_1$ given to use as part of the formula.

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**1.3 Propositions**

**Proposition:** A statement that is unambiguously **true** or **false**. A propositional variable can be used to shorten a statement. Compound propositions can be formed using logic operators.

**Logic operators:** And: $\wedge$, Or: $\vee$, Not: $\neg$
AND: Only true if all propositions are true.
OR: Only false if all propositions are false.

**Negation of propositions:**
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$

### 1.4 Predicates

**Predicate:** A logical function that takes in some input, usually $x$, and returns either **true** or **false** based on what that predicate is defined as. For example, $P(x)$ is $x \leq 5$; this will be true for $P(2)$ and false for $P(10)$.

**Domain:** The domain given is the set of all possible input values we can use for $x$, when plugging in inputs to a predicate. Sometimes, domains may be explicitly defined (like $D = \{1, 2, 3\}$), or a number set might be used, such as $\mathbb{Z}$ for the set of all integers.
We can symbolically show that a variable $x$ is in the domain $D$ with the $\in$ operator. For example, $x \in D$.

**Quantifiers:** Quantifiers can be used to specify whether a predicate function is true for **all** possible inputs from the domain ($\forall$), or just **some** possible inputs from the domain ($\exists$).

**Negations with quantifiers:**
$\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x)$
$\neg(\exists x \in D, P(x)) \equiv \forall x \in D, \neg P(x)$

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### 1.5 Implications

**Implications:** Implications are **"if, then"** statements, where one proposition (or predicate) is the **hypothesis** and another proposition (or predicate) is the **conclusion**. For $p \to q$, $p$ is the hypothesis and $q$ is the conclusion. This is read as "if $p$ (is true), then $q$ (is true)".

**Negation of implication:** The negation of an implication, $p \to q$ is not an implication. $\neg(p \to q) \equiv p \wedge \neg q$

**Contrapositive, converse, and inverse:**

| Implication | Converse | Inverse | Contrapositive |
|---|---|---|---|
| $p \to q$ | $q \to p$ | $\neg p \to \neg q$ | $\neg q \to \neg p$ |