6.5 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

5.1 Recursion Revisited

5.1.1 Review

Complement

Given an event E,

$$Prob(E) + Prob(\bar{E}) = 1$$

Where $\bar{E}$ is the complement of the event E.

Example question What is the probability that for a six-sided die rolled three times the same result comes up more than once?

a. What is the sample space S?
   $\{1, 2, 3, 4, 5, 6\}$

b. What is the event E (in English)?
   The set of outcomes that use the same # more than once.

c. What is the complement of $\bar{E}$ (in English)?
   The set of outcomes that are all different numbers.

d. What *structure type* is $\bar{E}$? What is n and r?
   Permutation, $n = 6$, $r = 3$

e. Calculate $Prob(\bar{E})$
   $Prob(\bar{E}) = n(\bar{E})/n(S) = \frac{P(6,3)}{6^3} = \frac{5}{9} = 0.\bar{5}$

f. Calculate the probability for the Event $Prob(E)$ using the proposition.
   $1 - Prob(\bar{E}) = 1 - 0.55 \approx 0.44$

Expected (average) value

For a given probability experiment, let X be a random variable whose possible values come from the set of numbers $x_1, ..., x_n$. Then the **expected value of** X, denoted by $E[X]$, is the sum

$$E[X] = (x_1) \cdot Prob(X = x_1) + ... + (x_n) \cdot Prob(X = x_n)$$

5.1.2 Recursion revisited

Average trials until getting first value
A common type of problem in this section is to find the average amount of trials run until you get some value for the first time... For example, rolling a die until you get a "1" for the first time.

Let's say there's some probability p of success, and X is the amount of trials (rolls, flips, etc.) until the first value is received, and $E[X]$ is the average amount of trials that will run.

We start with this formula:

$$E[X] = p(1) + (1 - p)(1 + E[X])$$

The probability should be known, so by simplifying you can solve for $E[X]$ to find the result.

Example 1 Find the average number of tosses of a fair coin that it takes to get a result of heads for the first time.

The probability of getting a heads is $p = (1/2)$, so we can write this out as:

$$E[X] = \frac{1}{2}(1) + (1 - \frac{1}{2})(1 + E[X])$$

and simplify...

$$E[X] = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}(E[X])$$

$$E[X] - \frac{1}{2}(E[X]) = 1$$

$$\frac{1}{2}(E[X]) = 1$$

$$E[X] = 2$$

Question 1
What is the expected number of rolls of a six-sided die that is rolled until a 1 appears?

Question 2
A pair of dice are thrown until at least one of the die comes up 1 for the first time. How many tosses, on average, are required?

We are rolling two die, which comes out to:

(1,1)	**(1,2)**	**(1,3)**	**(1,4)**	**(1,5)**	**(1,6)**
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

a. What is the sample size?

b. How many of these rules have at least one 1?

c. What is the probability of getting at least one? $(Prob(E) = n(E)/n(S))$

d. Use the formula to find the expected value (average trials).