6.5 Review

Complement

Given an event E,

$$Prob(E) + Prob(\bar{E}) = 1$$

Where $\bar{E}$ is the complement of the event E.

Example question What is the probability that for a six-sided die rolled three times the same result comes up more than once?

a. What is the sample space S?
 $\{1, 2, 3, 4, 5, 6\}$

b. What is the event E (in English)?
 The set of outcomes that use the same # more than once.

c. What is the complement of $\bar{E}$ (in English)?
 The set of outcomes that are all different numbers.

d. What *structure type* is $\bar{E}$? What is n and r?
 Permutation, $n = 6$, $r = 3$

e. Calculate $Prob(\bar{E})$
 $Prob(\bar{E}) = n(\bar{E})/n(S) = \frac{P(6,3)}{6^3} = \frac{5}{9} = 0.\bar{5}$

f. Calculate the probability for the Event $Prob(E)$ using the proposition.
 $1 - Prob(\bar{E}) = 1 - 0.55 \approx 0.44$

Expected (average) value

For a given probability experiment, let X be a random variable whose possible values come from the set of numbers $x_1, ..., x_n$. Then the **expected value of** X, denoted by $E[X]$, is the sum

$$E[X] = (x_1) \cdot Prob(X = x_1) + ... + (x_n) \cdot Prob(X = x_n)$$

6.6 Recursion revisited

Average trials until getting first value
A common type of problem in this section is to find the average amount of trials run until you get some value for the first time... For example, rolling a die until you get a "1" for the first time.

Let's say there's some probability p of success, and X is the amount of trials (rolls, flips, etc.) until the first value is received, and $E[X]$ is the average amount of trials that will run.

We start with this formula:

$$E[X] = p(1) + (1 - p)(1 + E[X])$$

The probability should be known, so by simplifying you can solve for $E[X]$ to find the result.

Example 1 Find the average number of tosses of a fair coin that it takes to get a result of heads for the first time.

The probability of getting a heads is $p = (1/2)$, so we can write this out as:

$$E[X] = \frac{1}{2}(1) + (1 - \frac{1}{2})(1 + E[X])$$

and simplify...

$$E[X] = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}(E[X])$$

$$E[X] - \frac{1}{2}(E[X]) = 1$$

$$\frac{1}{2}(E[X]) = 1$$

$$E[X] = 2$$

Question 1 ____ / 2

What is the expected number of rolls of a six-sided die that is rolled until a 1 appears?

$E[X] = \frac{1}{6}(1) + \frac{5}{6}(1 + E[X])$
$E[X] = 6$

Question 2 ____ / 2

A pair of dice are thrown until at least one of the die comes up 1 for the first time. How many tosses, on average, are required?

We are rolling two die, which comes out to:

(1,1)	**(1,2)**	**(1,3)**	**(1,4)**	**(1,5)**	**(1,6)**
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

a. What is the sample size? 36

b. How many of these rules have at least one 1? 11

c. What is the probability of getting at least one? $\frac{11}{36}$
($Prob(E) = n(E)/n(S)$)

d. Use the formula to find the expected value (average trials). $E[X] = \frac{1}{6}(1) + \frac{5}{6}(1 + E[X])$
$E[X] = 6$