Chapter 5.5 Notes

Review: Mathematical induction (Chapter 2.3 and 2.4)

Back in CS 210, we did proofs by using induction. With induction, our goal was to show that some statement was true for the first value, a_1, and then for all values going up through a_{m-1}. Let's look at the steps for this sort of problem.

Exercise 2.3, 3a from textbook Show that the sequence defined by $a_k = a_{k-1} + 4; a_1 = 1$ for $k \geq 2$ is equivalently described by the closed formula $a_n = 4n - 3$.

Step 1: Check values for both formulas, for a_1:
Recursive: $a_1 = 1$ (provided) Closed: $a_1 = 4(1) - 3 = 1$
They match, so we can continue.

Step 2: Rewrite the recursive formula in terms of m: $a_m = a_{m-1} + 4$

Step 3: Find the equation for a_{m-1} through the recursive formula:
$a_n = 4n - 3$
$=> a_{m-1} = 4(m-1) - 3$
$=> a_{m-1} = 4m - 7$

Step 4: Plug a_{m-1} into the recursive formula from step 2, and simplify.
$a_m = a_{m-1} + 4$
$=> a_m = (4m - 7) + 4$
$=> a_m = 4m - 3$

PROOF: $a_m = 4m - 3$ and the closed formula $a_n = 4n - 3$ match, so the closed formula and recursive formula are equivalent.

Recursive counting

Question II-3 from POGIL exercise Prove by induction and the recurrence relation $P(n,r) = n \cdot P((n-1),(r-1))$ with $P(n,0) = 1$ that $P(n,n) = n!$ for all $n \geq 0$.

Step 1: Check that it works for $n = 1$:
$P(1,1) = 1 \cdot P(0,0) = \frac{0!}{0!} = \frac{1}{1} = 1$;
$1! = 1$. OK

Step 2: $P(n,n) = n \cdot P((n-1),(n-1))$,

$$
\begin{aligned}
&= n \cdot \frac{(n-1)!}{((n-1)-(n-1))!} \\
&= n \cdot \frac{(n-1)!}{(n-1-n+1)!} \\
&= n \cdot \frac{(n-1)!}{(n-n)!} \\
&= n \cdot \frac{(n-1)!}{(0)!} \\
&= n \cdot \frac{(n-1)!}{1} \\
&= n \cdot (n-1)! \\
&= n!
\end{aligned}
$$

(n times $(n-1)!$ is equivalent to just having $n!$)