## 7.4 Connections to Matrices and Relations

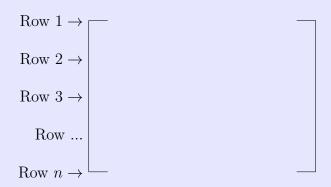
Please write down all people in your team.

- 1. 2.
- 3. 4.

## 7.4.1 Adjacency matrix

Given a graph G with vertex set  $V = \{v_1, v_2, ..., v_n\}$  and edge set E, we define the **adjacency matrix** of G as follows. The matrix M is an  $n \times n$  array of natural numbers, which we imagine having rows and columns labelled as follows:

Columns 
$$1, 2, ..., n$$



The entry in row i, column j (referred to as the (i, j) - entry of M or, more concisely,  $M_{ij}$ ) is defined as

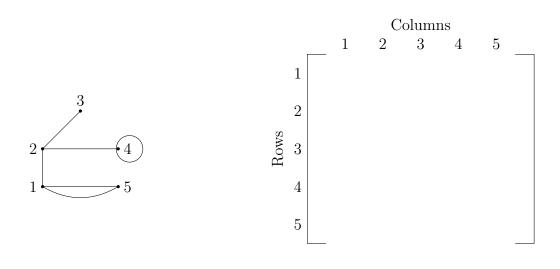
 $M_{ij}$  = the number of edges connecting  $v_i$  and  $v_j$  in G.

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<sup>&</sup>lt;sup>a</sup>Discrete Mathematics, by Ensley and Crawley

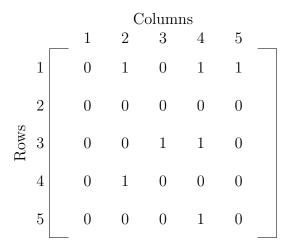
Question 1 \_\_\_\_ / 2

Fill out the adjacency matrix for the following graph.



Question 2 \_\_\_\_ / 2

Draw a graph that corresponds to the adjacency matrix



## 7.4.2 Directed graphs

- 1. A **directed graph**, like a graph, consists of a set *V* of vertices and a set *E* of edges. Each edge is associated with an ordered pair of vertices called its **endpoints**. In other words, a directed graph is the same as a graph, but the edges are described as *ordered pairs* rather than unordered pairs;
- 2. If the endpoints for edge e are a and b in that order, we say e is an edge **from** a **to** b, and in the diagram we draw the edge as a straight or curved arrow from a to b.
- 3. For a directed graph, we use (a, b) rather than [a, b] to indicate an edge from a to b. This emphasizes that the edge is an **ordered** pair, by utilizing the usual notation for ordered pairs.
- 4. A walk in a directed graph in a sequence  $v_1e_1v_2e_2...v_ne_nv_{n+1}$  of alternating vertices and edges that begins and ends with a vertex, and where each edge in the list between its endpoints in the proper order. (That is,  $e_1$  is an edge from  $v_1$  to  $v_2$ ,  $e_2$  is an edge from  $v_2$  to  $v_3$ , and so on.) If there is no chance of confusion, we omit the edges when we describe a walk.
- 5. The **adjacency matrix** for a directed graph with vertices  $\{v_1, v_2, ..., v_n\}$  is the  $n \times n$  matrix where  $M_{ij}$  (the entry in row i, column j) is the number of edges from vertex  $v_i$  to vertex  $v_j$ .

Question 3 \_\_\_\_\_ / 2

Draw a directed graph with vertices  $V = \{1, 2, 3, 4, 5\}$  and edges  $E = \{(1, 4), (1, 5), (2, 1), (3, 4), (4, 3), (5, 2)\}.$ 

<sup>&</sup>lt;sup>a</sup>Discrete Mathematics, Ensley and Crawley