7.4 Connections to Matrices and Relations

Please write down all people in your team.

1. 2.

3. 4.

7.4.1 Adjacency matrix

Given a graph G with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E, we define the **adjacency matrix** of G as follows. The matrix M is an $n \times n$ array of natural numbers, which we imagine having rows and columns labelled as follows:

$$\begin{array}{r c} & \text{Columns } 1, 2, ..., n \\ \begin{array}{r} \text{Row } 1 \rightarrow \\ \text{Row } 2 \rightarrow \\ \text{Row } 3 \rightarrow \\ \text{Row } ... \\ \text{Row } n \rightarrow \end{array} & \left[\begin{array}{ccc} \quad & \quad & \quad \\ & & \\ & & \\ & & \\ & & \end{array} \right] \end{array}$$

The entry in row i, column j (referred to as the (i, j) - entry of M or, more concisely, M_{ij}) is defined as

$$M_{ij} = \text{the number of edges connecting } v_i \text{ and } v_j \text{ in } G.$$

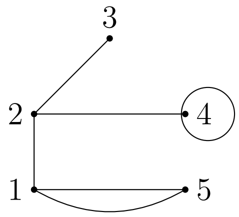
[a]

[a]Discrete Mathematics, by Ensley and Crawley

Question 1

_____ / 2

Fill out the adjacency matrix for the following graph.



$$\begin{array}{cc}
 & \text{Columns} \\
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{Rows} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \quad & \quad & \quad & \quad & \quad \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}
\end{array}$$

7.4.2 Directed graphs

1. A **directed graph**, like a graph, consists of a set V of vertices and a set E of edges. Each edge is associated with an ordered pair of vertices called its **endpoints**. In other words, a directed graph is the same as a graph, but the edges are described as *ordered pairs* rather than unordered pairs;

2. If the endpoints for edge e are a and b in that order, we say e is an edge **from** a **to** b, and in the diagram we draw the edge as a straight or curved arrow from a to b.

3. For a directed graph, we use (a, b) rather than $[a, b]$ to indicate an edge from a to b. This emphasizes that the edge is an **ordered pair**, by utilizing the usual notation for ordered pairs.

4. A **walk** in a directed graph in a sequence $v_1 e_1 v_2 e_2 ... v_n e_n v_{n+1}$ of alternating vertices and edges that begins and ends with a vertex, and where each edge in the list between its endpoints in the proper order. (That is, e_1 is an edge from v_1 to v_2, e_2 is an edge from v_2 to v_3, and so on.) If there is no chance of confusion, we omit the edges when we describe a walk.

5. The **adjacency matrix** for a directed graph with vertices $\{v_1, v_2, ..., v_n\}$ is the $n \times n$ matrix where M_{ij} (the entry in row i, column j) is the number of edges from vertex v_i to vertex v_j.

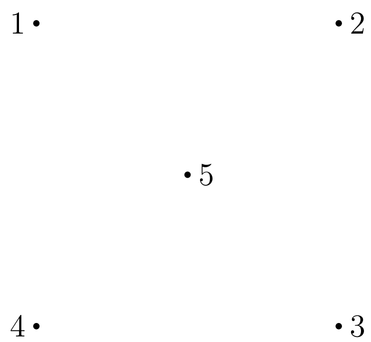
[a]

[a]Discrete Mathematics, Ensley and Crawley

Question 2 ____ / 2

Draw a graph that corresponds to the adjacency matrix. This is a directed graph, so the matrix is not symmetric. It should be read as row $i \to$ column j. For example, row 1 shows $1 \to 2$, $1 \to 4$, and $1 \to 5$.

$$\begin{array}{cc} & \text{Columns} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Rows} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



Question 3 ____ / 2

Draw a directed graph with vertices $V = \{1, 2, 3, 4, 5\}$ and edges $E = \{(1, 4), (1, 5), (2, 1), (3, 4), (4, 3), (5, 2)\}$.