Cheat sheet

The Rule of Product

If each entry in a list can be created by firt selecting one of x objects and then one of y objects, then the list has a total of $(x)(y)$ entries. In terms of sets, this means that $n(A \times B) = n(A) \cdot n(B)$ for all finite sets A and B.

The Rule of Sum

If the list to count can be split into two disjoint pieces of size x and y, then the original list has $x + y$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B)$, provided that A and B are disjoint.

The Rule of Sum with Overlap

If the list to count can be split into two pieces of size x and y, and the pieces have z objects in common, then the original list has $x+y-z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B.

The Rule of Complements

If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$. In terms of sets, using U for the universal set, we can write this as $n(A') = n(U) - n(A)$ for all sets A with elements from U.

Cheat sheet

What?	How many?
Ordered lists of length r	n^r
Permutations of length r	$P(n,r)$
Unordered lists of size r	$C(r+n-1,r)$
Sets of size r	$C(n,r)$

Permutations

A permutation is just an ordered list in whic no element is repeated. The number of permutations from $\{1,...,n\}$ of length r is denoted by $P(n,r)$.

$$P(n,r) = \frac{n!}{(n-r)!}$$

Combinations

We will write $C(n,r)$ for the number of sets from $\{1,...,n\}$ of size r.

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Binary sequences

The number of binary sequences with r 1's and $n-r$ 0's is $C(n,r)$ or $C(n,n-r)$.

Exam 1, Chapter 5: Combinatorics

CS 210 , Fall 2017

Name: ____

Exam Format: This exam covers topics from Chapter 5. Questions are split into sections 5.1, 5.2, 5.3, 5.4, 5.5, and 5.6, and are presented in order as we would have seen them in the book. There are several versions of this exam, intended to dissuade cheating between neighbors and between class sections.
Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

Additionally, a cheat-sheet is provided.

Rules: Students may use a scientific calculator on the exam, but not a graphing calculator. Exam work must be solo-work.

15% Question 1: Basic structures (5.1) □ 0 □ 1 □ 2 □ 3 □ 4

Fill in the following table for the 4 basic structures.

		Did the order of selection matter?	
		YES	NO
Are repetitions allowed?	YES		
	NO		

5% Question 2: Club election (5.2) □ 0 □ 1 □ 2 □ 3 □ 4

In how many ways can a club with 13 members elect a president, a vice-president, and a secretary?

10% Question 3: Six-sided die (5.2) □ 0 □ 1 □ 2 □ 3 □ 4

We will represent the results of three tosses of a six-sided die as an ordered list of length 3 with entries from $\{1, ..., 6\}$. There are $6^3 = 216$ total possible outcomes. Of these, how many will all three tosses be different?

5% Question 4: Employee ID (5.3) □ 0 □ 1 □ 2 □ 3 □ 4

Each employee is assigned an alphanumeric ID of 5 characters, built from any of the following 16 characters: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

How many possible employee ID combinations are there?

5% Question 5: Pizzas (5.2) □ 0 □ 1 □ 2 □ 3 □ 4

Let's say that there are 120 total possible pizzas you could order at local pizza shop. There are 3 types of cheeses, 5 types of veggies, 4 types of meats, but the amount of sauces aren't specified. How many total sauces are there?

10% Question 6: Coin toss (5.2) □ 0 □ 1 □ 2 □ 3 □ 4

If we toss a coin 10 times in succession and record the outcomes as an ordered list of length 10 using entries from $\{H, T\}$, how many of the $2^{10}(1024)$ possibilities satisfies the following condition?

Begin with two "heads" in a row, OR end with two "tails" in a row.

15% Question 7: Rotten oranges (5.3) □ 0 □ 1 □ 2 □ 3 □ 4

A bag contains a dozen oranges, two of which are rotten. A sample of three oranges is taken from the bag.

a. In how many ways can the sample be taken?

b. Of these, how many contain exactly one rotten orange?

c. How many contain exactly two rotten oranges?

d. How many contain no rotten oranges?

7% Question 8: License plates (5.2) □ 0 □ 1 □ 2 □ 3 □ 4

In a certain state there is a rule where the license plates can only be in the form: 3 letters, then 4 digits. How many possible combinations are there?

15% Question 9: 10-digit numbers (5.4) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

How many ordered lists of length 10 with repetitions allowed are there satisfying the each condition?

a. Using the set $\{a, b, c, d\}$

b. Using the set $\{x, y, z\}$, and having exactly three x's

c. Using the set $\{0, 1, 2, 3, 4\}$, having exactly two 0's and three 2's, and NOT beginning with 0.

7% Question 10: Candy store (5.4) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

How many bags of 20 pieces of candy can one buy from a store that sells four types of candy?

7% Question 11: Ten letters (5.4) □ 0 □ 1 □ 2 □ 3 □ 4

How many ordered lists of 10 letters chosen from $\{m, a, t\}$ have exactly three m's?

Anonymous survey

Tear off this page and turn it in separately from the exam.

1. How do you think you did?

2. What parts were easy?

3. What parts were hard?

4. What did you use to prepare for the exam?

5. Any suggestions for what resources would help you do better for the next exam?