

## 7.5 Graphs in Puzzles and Games

Please write down all people in your team.

- 1.
- 2.
- 3.
- 4.

### 7.5.1 Wolves, Goats, and Cabbages

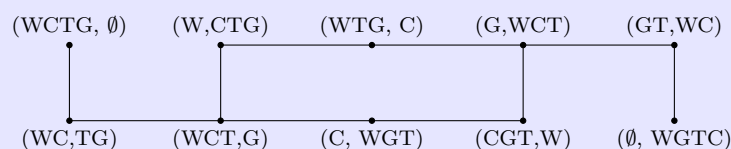


A traveler has three possessions: a wolf, a goat, and a cabbage. They must transport them across a river.

The catch is that, if left alone, the wolf will eat the goat, or the goat will eat the cabbage. The boat can only hold the traveller and one possession at a time. <sup>a</sup>

For this problem, we are concerned with what valid states are. We can draw a diagram to represent all possible moves between the starting point (everything at the starting location) and the ending point (everything at the ending location) to help us solve it.

Let's use  $(WC, TG)$  to mean that the Wolf and Cabbage are left on the departing shore, and the Traveller and the Goat are on the arriving shore. If we write  $(WCTG, \emptyset)$ , then all four are on the departing shore, and nothing is on the arriving shore.



<sup>a</sup>Discrete Mathematics, Ensley and Crawley

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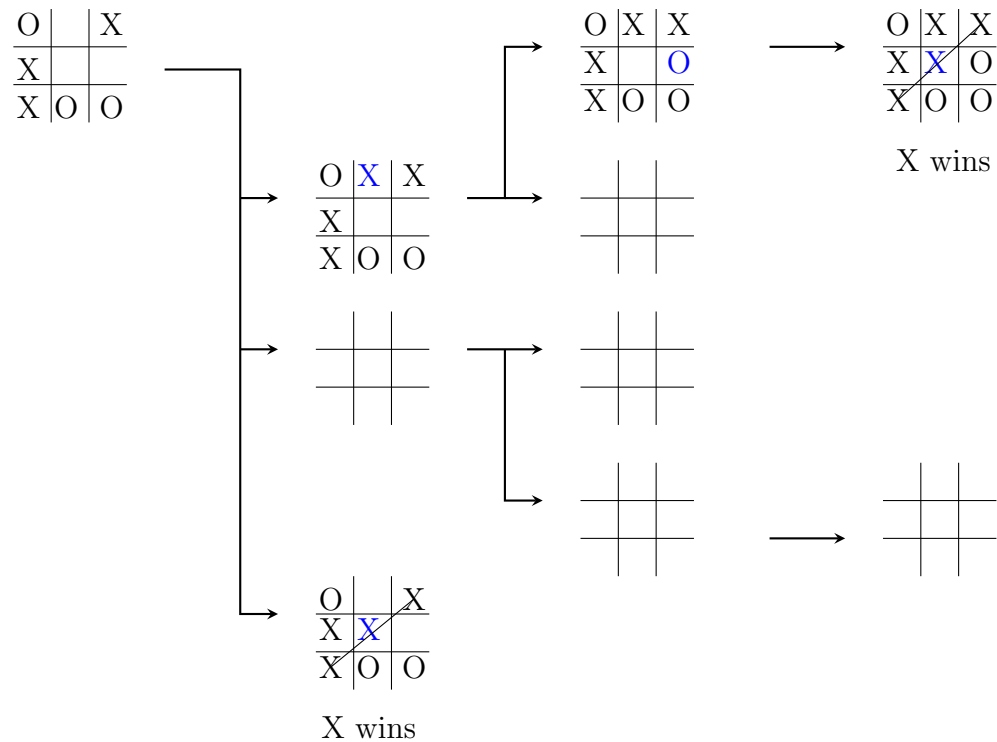
$(\text{WCTG}, \emptyset)$     $(\text{W}, \text{CTG})$     $(\text{WTG}, \text{C})$     $(\text{G}, \text{WCT})$     $(\text{GT}, \text{WC})$   
 $(\text{WC}, \text{TG})$     $(\text{WCT}, \text{G})$     $(\text{C}, \text{WGT})$     $(\text{CGT}, \text{W})$     $(\emptyset, \text{WGTC})$

- Path:
- In  $(WC, TG)$ , what does the item to the left of the comma represent?
- In  $(WC, TG)$ , what does the item to the right of the comma represent?
- What does  $W$  represent?
- What does  $C$  represent?
- What does  $T$  represent?
- What does  $G$  represent?
- What does  $\emptyset$  represent?

**Question 2**

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Given a tic-tac-toe board with the given starting state, draw a state diagram for all following valid moves until a win. Assume that it is X's turn next.<sup>1</sup>



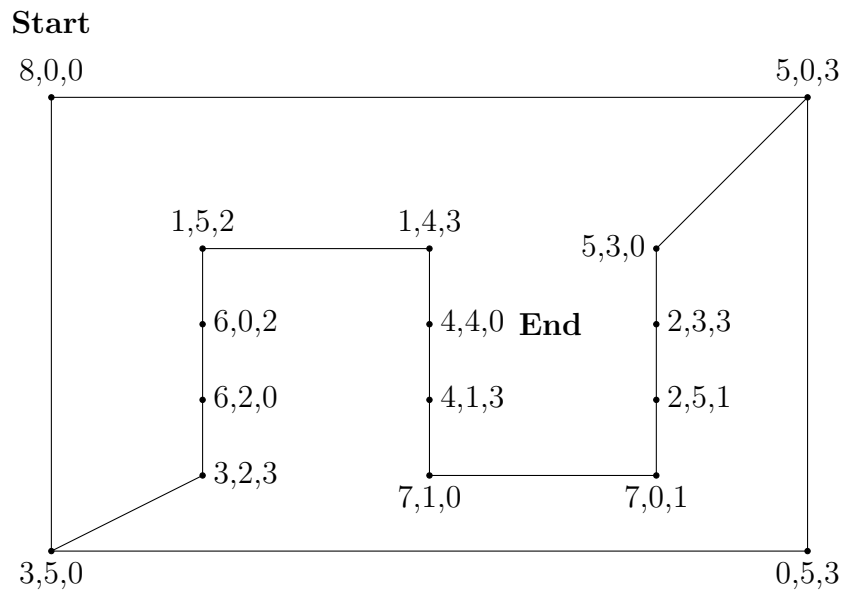
<sup>1</sup>Diagram from <http://neverstopbuilding.com/minimax>

**Question 3**

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Two friends have 2 gallons (8 quarts) of water in a pail. They also have two (empty) jars, one holding 5 quarts and the other 3. Using just these measuring devices, how can they split the water so that 4 quarts are in the larger jar and 4 quarts remain in the pail? <sup>2</sup>

Use the graph below to come up with a walk to the solution.



- Path:
- What does the 1st position in the vertex label represent?
- What does the 2nd position in the vertex label represent?
- What does the 3rd position in the vertex label represent?

<sup>2</sup>Every node within the square also has edges pointing to two of the four corners of the square, but those have been left off.