5.4 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

5.4 Binary Sequences

5.4.1 Binary sequences

Binary sequence theorem

> The number of binary sequences with r 1's and $n - r$ 0's is $C(n, r)$ or $C(n, n - r)$. [a]

This can be used for straightforward problems, like "how many binary sequences are there with x 1's and y 0's?", but we can also use it for more practical applications...!

[a] From Discrete Mathematics, Ensley and Crawley, page 409

Question 1
How many binary sequences are there with three 1's and two 0's?

a. How many 1's? $r =$

b. How many 0's? $n - r =$

c. What is the value of r?

d. How many binary sequences are there? $C(n, r) =$

Question 2
Using the Binary Sequence theorem and the Sum Rule, find the number of binary sequences that have an odd number of 1's.

Other uses of Binary Sequences

We can also use this theorem for strings that have more than just 0's and 1's...

Example 2 from the book: How many ordered lists of 10 letters, chosen from $\{m, a, t\}$, have exactly three m's?

If we think in terms of spots to fill for each letter, we can diagram it like this:

$$\underset{1}{\_\_} \quad \underset{2}{\_\_} \quad \underset{3}{\_\_} \quad \underset{4}{\_\_} \quad \underset{5}{\_\_} \quad \underset{6}{\_\_} \quad \underset{7}{\_\_} \quad \underset{8}{\_\_} \quad \underset{9}{\_\_} \quad \underset{10}{\_\_}$$

Since we have a restriction on the m's, these should be filled in first. Selecting three m's gives us $C(10, 3) = 120$ combinations.

Then, since there are no restrictions on the rest of the letters, we can select the final 7. However, we don't have 3 to choose from anymore; we wanted **exactly** 3 m's and we have already filled those. So, instead of selecting from 3 options, we only have two: $\{a, t\}$.

Since for each of the remaining, we have two options each, it will be calculated as $2 \cdot 2 \cdot 2 \cdot ...$ seven times. These remaining 2's are an **ordered list**: Order matters, and we can have duplicates (a or t).

Our final result will be $C(10, 3) \cdot 2^7$.

Formulas for each structure type

Type	**Repeats allowed?**	**Order matters?**	**Formula**
Ordered list of length r	yes	yes	n^r
Unordered list of length r	yes	no	$C(r + n - 1, r)$
Permutations of length r	no	yes	$P(n, r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n, r) = \frac{n!}{r!(n-r)!}$

Question 3
How many distinguishable arrangements are there of the letters in the word "BANANA"?

a. How many B's? How many A's? How many N's?

b. How many ways are there to select a position for B? *(Specify the formula and the final answer)*

c. How many ways are there to place the A's? *(Hint: B has been placed, so there are 5 available spaces and 3 A's to place.)*

d. How many ways are there to place the N's? *(Hint: How many spaces are left? How many N's to place?*

e. Which Rule do you use to find the final answer? What is the final answer?

Question 4
How many distinguishable arrangements are there of the word "PENNSYLVANIA"? *(Hint: You need the total amount of letters, and a count for each letter in the word!)*