

Review of structures

Ordered lists of length r with items from $\{1, \dots, n\}$

Repetitions allowed: An item from our input set $\{1, \dots, n\}$ can be re-used multiple times for each selection.

Order matters: There is a difference between choosing item A then B , and choosing item B then A .

Example

You are filling in your name in the high-score list, and there are 3 slots to fill in your name. You can repeat letters, and the order you enter them matters (“RJM” is different from “MRJ”). Assuming 26 letters, the result is 26^3 .

Unordered lists of length r with items from $\{1, \dots, n\}$

Repetitions allowed: An item from our input set $\{1, \dots, n\}$ can be re-used multiple times for each selection.

Order doesn't matter: Any grouping of selections from the set are considered the same, such as $\{a, b, c\}$ and $\{b, c, a\}$.

Example

The number of bags of r pieces of fruit that can be bought at a store with n types of fruit available is $C(r + n - 1, r)$.

Permutations of length r with items from $\{1, \dots, n\}$

No repetitions: Once one item is selected from the set $\{1, \dots, n\}$, it is no longer an option for subsequent items.

Example

Pulling cards from a deck... First you have 52 options, then 51 options, then 50 options...

Order matters: The order that you select something matters, so in a way, different “slots” represent different things.

Example

Electing President, VP, and Secretary

Combinations (Sets) of length r with items from $\{1, \dots, n\}$

No repetitions: Once one item is selected from the set $\{1, \dots, n\}$, it is no longer an option for subsequent items.

Order doesn't matter: If any two items are selected, the order doesn't matter. Combinations deal with sets, and with sets, $\{a, b\}$ and $\{b, a\}$ are considered equivalent.

Example

If there are 5 different dinners, and we need to feed 3 people, then there are $C(5, 3)$ possible dinner combinations.

Types of structures

Type	Repeats allowed?	Order matters?	Formula
Ordered list of length r	yes	yes	n^r
Unordered list of length r	yes	no	$C(r + n - 1, r)$
Permutations of length r	no	yes	$P(n, r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n, r) = \frac{n!}{r!(n-r)!}$

Question 1

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How many arrangements are there of the letters in the word MATCH?

Are repetitions allowed? **no**Does order matter? **yes**What is n ? **5**What is r ? **5**Equation to use? $P(n, r)$ / $C(n, r)$ / n^r / $C(r + n - 1, r)$ **$P(n, r)$** Solution: **$P(5, 5) = 120$** **Question 2**

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There are five red, three green, and eight blue marbles in a box. In how many ways can a sample of four be selected?

Are repetitions allowed? **yes**Does order matter? **no**What is n ? **$5 + 3 + 8 = 16$** What is r ? **4**Equation to use? $P(n, r)$ / $C(n, r)$ / n^r / $C(r + n - 1, r)$ **$C(n, r)$** Solution: **$C(16, 4) = 1820$**

Question 3

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We can choose from four types of muffins: Blueberry, Orange, Chocolate Chip, or Cream Cheese. You're going to select muffins in this order: First for yourself, second for your sister, and third for your brother. It is OK if several people have the same muffin type.

Are repetitions allowed? **yes**

Does order matter? **yes**

What is n ? **4**

What is r ? **3**

Equation to use? $P(n, r)$ / $C(n, r)$ / n^r / $C(r + n - 1, r)$ **n^r**

Solution: **4^3**

Question 4

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How many bags of 20 pieces of candy can one buy from a store that sells four types of candy?

Are repetitions allowed? **yes**

Does order matter? **no**

What is n ? **20**

What is r ? **4**

Equation to use? $P(n, r)$ / $C(n, r)$ / n^r / $C(r + n - 1, r)$ **$C(r + n - 1, r)$**

Solution: **$C(20 + 4 - 1, 20) = C(23, 20)$**

6.1 Introduction

6.1.1 Experiments, Outcomes, and Events

Vocabulary

For this chapter, we will be talking about experiments and their outcomes. For any given experiment, we will have a **sample space** of possible outcomes. This will be written as the set S .

Within an experiment, we want to see if some **event** occurs, and how often it does. In cases where the event occurs, we call it a **success**.

Definition Given an experiment with a sample space S of equally likely outcomes and an event E , the *probability of the event* (denoted by $Prob(E)$) is the ratio of the number of successful outcomes to the total number of outcomes: ^a

$$Prob(E) = \frac{n(E)}{n(S)}$$

(Recall that $n(S)$ is how we symbolically write, “the amount of elements of the set S ”.)

^aFrom Discrete Mathematics by Ensley and Crawley

Question 5

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Finish the following table to log all possible equally-likely outcomes for rolling a red four-sided die and a green four-sided die.

	Green 1	Green 2	Green 3	Green 4
Red 1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
Red 2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
Red 3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
Red 4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

Using the definition above describe the following:

- a. Both the red and green dice have the same values.

$$n(E) = 4 \quad n(S) = 16 \quad Prob(E) = \frac{4}{16} = \frac{1}{4}$$

- b. The sum of both dice values is 4.

$$n(E) = 3 \quad n(S) = 16 \quad Prob(E) = \frac{3}{16}$$

Question 6

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Consider the experiment of drawing two cards from the top of a standard deck of 52 cards, and the event E of the two cards having the same value.¹

- a. Describe the set S of all outcomes, represented so that they are equally likely.

Hint

This means what structure type is this? What kind of formula are we using to choose 2 items from a deck of 52?

$$n(S) = P(52, 2).$$

- b. Describe the event E in terms of your representation.

Hint

We're interested in the event where both our selections have the same value. This can be broken down as:

1. Choose any card (52 possible)
2. Choose a card with the same value (3 possible)
3. Combine with "AND" (The Rule of Product)

$$n(E) = (52)(3)$$

- c. Compute $Prob(E) = \frac{n(E)}{n(S)}$.

$$Prob(E) = \frac{n(E)}{n(S)} = \frac{(52)(3)}{(52)(51)}$$

Question 7

_____ / 3

Consider the experiment of tossing a coin five successive times, and the event E that the last two tosses have the same result.

(_ _ _ Heads Heads) OR (_ _ _ Tails Tails)

- a. Describe the set S of all outcomes, represented so they are equally likely
Ordered lists of length 5 with entries from $\{H, T\}$

- b. Describe the event E in terms of your representation.

$$n(S) = 2^5 = 32 \qquad n(E) = 2^3 \cdot 1 \cdot 1 + 2^3 \cdot 1 \cdot 1 = 16$$

- c. $Prob(E) = \frac{n(E)}{n(S)} = \frac{16}{32} = 0.5$

¹From Discrete Mathematics by Ensley and Crawley

6.1.2 The complement of the Event

Proposition 1

Given an event E ,

$$Prob(E) + Prob(\bar{E}) = 1$$

Where \bar{E} is the complement of the event E .

Question 8

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What is the probability that for a six-sided die rolled three times the same result comes up more than once?

- What is the sample space S ? $\{1, 2, 3, 4, 5, 6\}$
- What is the event E (in English)?
The set of outcomes that... **use the same # more than once.**
- What is the complement of \bar{E} (in English)?
The set of outcomes that... **are all different numbers.**
- What *structure type* is \bar{E} ? What is n and r ?
Permutation, $n = 6$, $r = 3$
- Calculate $Prob(\bar{E})$
 $Prob(\bar{E}) = n(\bar{E})/n(S) = \frac{P(6,3)}{6^3} = \frac{5}{9} = 0.\bar{5}$
- Calculate the probability for the Event $Prob(E)$ using the proposition.
 $1 - Prob(\bar{E}) = 1 - 0.55 \approx 0.44$

Team: Please write down all people in your team.

- | | |
|----|----|
| 1. | 2. |
| 3. | 4. |
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Grading

Question	Weight	0-4	Adjusted score
1	5%		
2	6%		
3	12%		
4	15%		
5	25%		