

Please write down all people in your team.

1.

2.

3.

4.

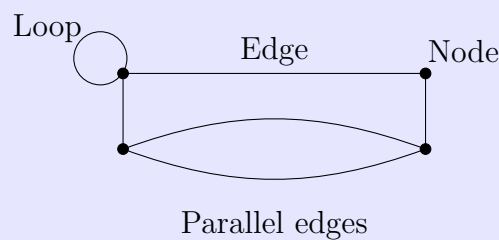
---

## 7.1 Graph Theory

### 7.1.1 Terminology

Since we're introducing a new concept, Graph Theory, we need to go over the various terms so that we can communicate about these graphs properly.

- **Graph:** A graph is a type of diagram that contains *vertices* (aka nodes) and *edges*.

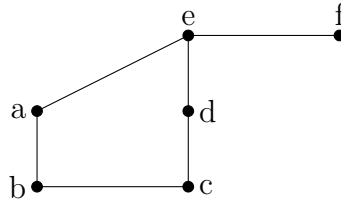


- **Node:** A vertex of the graph, drawn as a dot.
  - **Adjacent nodes:** Two nodes that are connected by an edge.
  - **Node degree:** The amount of edges that are connected to a node. Loops are counted twice.
- **Edge:** A line that connects two nodes together.
  - **Parallel edges:** Two edges that have the same two endpoints.
  - **Loop:** An edge that begins and ends at the same node, creating a loop.
  - $[a, b]$  is used to indicate an edge with  $a$  and  $b$  as endpoints, though direction can be either way.

**Question 1**

\_\_\_\_ / 6

Identify each item for the graph  $G$  given.



- a. How many nodes (vertices) are there?
- b. How many edges are there?

- c. Write down the degree of each node:

Vertex $v$	$\deg(v)$
$a$	
$b$	
$c$	
$d$	
$e$	
$f$	

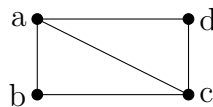
- d. The **maximum degree** of a graph is the highest  $\deg(v)$  value. What is this graph's maximum degree?
- e. The **minimum degree** of a graph is the lowest  $\deg(v)$  value. What is this graph's minimum degree?

- **Walk:** A series of alternating nodes and edges, traversing between adjacent nodes.
  - **Closed walk:** When the beginning and ending node of a walk are the same.
  - **Length of a walk:** The amount of edges in the walk.
  - **Trivial walk:** A walk of length 0.
  - **Path:** A walk with no repeated vertices.
  - **Trail:** A walk with no repeated edges.
    - \* **Circuit:** A closed trail.
      - **Trivial circuit:** A circuit with one vertex and no edges.
      - **Cycle:** A nontrivial circuit where the only repeated node is the first/last one.
    - \* **Eulerian:** A trail or circuit where every edge is traversed.

**Question 2**

\_\_\_\_\_ / 3

Answer the following questions, using the graph  $H$  given.

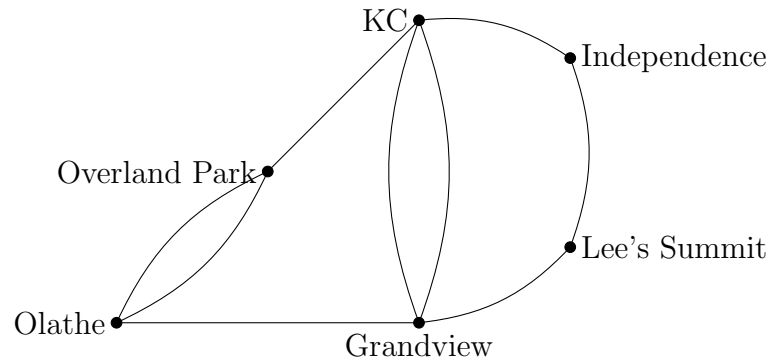


- a. Come up with several walks from  $a$  to  $c$ . Write all steps (each node visited). Also label the **length** of each walk.
- b. Come up with a **closed walk**, beginning and ending at  $a$ . You can choose to visit all nodes or not.
- c. Come up with a **path**, where no vertices are repeated.

**Question 3**

\_\_\_\_ / 5

Answer the following questions, using the graph  $I$  given.



- Come up with a **trail**, a walk with no repeated edges.
- Come up with a **circuit**, a closed trail.
- Come up with a **cycle**, a circuit where the only repeated node is the first/last one..
- Identify: Did you come up with any **Eulerian Trails**?  
If not, create one.
- Identify: Are there any **parallel edges**?

- **Simple graph:** A graph that has no loops or parallel edges.
- **Directed graph:** The edges in the graph are given a direction, which can only be traversed in that way.
  - Edges are denoted with parentheses  $(a, b)$ , showing that it goes from  $a$  to  $b$ .

**Question 4**

\_\_\_\_\_ / 1

Draw a **Directed Graph** using the following list of edges:

$$(1, 2), (2, 1), (3, 3), (4, 2)$$

(Don't confuse these for points on an  $x, y$  plane that are interconnected, each ordered pair is its own set of information - beginning and end nodes.)

- A graph is **connected** if there is a walk between any pair of distinct nodes.
- A graph  $H$  is a **subgraph** of a graph  $G$  if all nodes and edges in  $H$  are also nodes and edges in  $G$ .
- A **connected component** of a graph  $G$  is a connected subgraph  $H$  of  $G$  such that no other connected subgraph of  $G$  containing  $H$  exists.

$$a$$

---

<sup>a</sup>Discrete Mathematics, Ensley and Crawley**Question 5**

\_\_\_\_\_ / 2

Draw a graph that is **not connected**, and draw a **subgraph** of your graph.