Review of structures

Ordered lists of length r with items from $\{1, ..., n\}$

Repetitions allowed: An item from our input set $\{1, ..., n\}$ can be re-used multiple times for each selection.

Order matters: There is a difference between choosing item A then B, and choosing item B then A.

Example
You are filling in your name in the high-score list, and there are 3 slots to fill in your name. You can repeat letters, and the order you enter them matters ("RJM" is different from "MRJ"). Assuming 26 letters, the result is 26^3.

Unordered lists of length r with items from $\{1, ..., n\}$

Repetitions allowed: An item from our input set $\{1, ..., n\}$ can be re-used multiple times for each selection.

Order doesn't matter: Any grouping of selections from the set are considered the same, such as {a, b, c} and {b, c, a}.

Example
The number of bags of r pieces of fruit that can be bought at a store with n types of fruit available is $C(r + n - 1, r)$.

Permutations of length r with items from $\{1, ..., n\}$

No repetitions: Once one item is selected from the set $\{1, ..., n\}$, it is no longer an option for subsequent items.

Example
Pulling cards from a deck... First you have 52 options, then 51 options, then 50 options...

Order matters: The order that you select something matters, so in a way, different "slots" represent different things.

Example
Electing President, VP, and Secretary

Combinations (Sets) of length r with items from $\{1, ..., n\}$

No repetitions: Once one item is selected from the set $\{1, ..., n\}$, it is no longer an option for subsequent items.

Order doesn't matter: If any two items are selected, the order doesn't matter. Combinations deal with sets, and with sets, {a, b} and {b, a} are considered equivalent.

Example
If there are 5 different dinners, and we need to feed 3 people, then there are $C(5, 3)$ possible dinner combinations.

Types of structures

Type	Repeats allowed?	Order matters?	Formula
Ordered list of length r	yes	yes	n^r
Unordered list of length r	yes	no	$C(r+n-1, r)$
Permutations of length r	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$

Question 1 ____ / 1

How many arrangements are there of the letters in the word MATCH?

Are repetitions allowed? ____

Does order matter? ____

What is n? ____

What is r? ____

Equation to use? $P(n,r)$ / $C(n,r)$ / n^r / $C(r+n-1,r)$ ____

Solution: ____________

Question 2 ____ / 1

There are five red, three green, and eight blue marbles in a box. In how many ways can a sample of four be selected?

Are repetitions allowed? ____

Does order matter? ____

What is n? ____

What is r? ____

Equation to use? $P(n,r)$ / $C(n,r)$ / n^r / $C(r+n-1,r)$ ____

Solution: ____________

Question 3 ______ / 1

We can choose from four types of muffins: Blueberry, Orange, Chocolate Chip, or Cream Cheese. You're going to select muffins in this order: First for yourself, second for your sister, and third for your brother. It is OK if several people have the same muffin type.

Are repetitions allowed? ______

Does order matter? ______

What is n? ______

What is r? ______

Equation to use? $P(n, r)$ / $C(n, r)$ / n^r / $C(r + n - 1, r)$ ______

Solution: ____________________

Question 4 ______ / 1

How many bags of 20 pieces of candy can one buy from a store that sells four types of candy?

Are repetitions allowed? ______

Does order matter? ______

What is n? ______

What is r? ______

Equation to use? $P(n, r)$ / $C(n, r)$ / n^r / $C(r + n - 1, r)$ ______

Solution: ____________________

6.1 Introduction

6.1.1 Experiments, Outcomes, and Events

Vocabulary

For this chapter, we will be talking about experiments and their outcomes. For any given experiment, we will have a **sample space** of possible outcomes. This will be written as the set S.

Within an experiment, we want to see if some **event** occurs, and how often it does. In cases where the event occurs, we call it a **success**.

Definition Given an experiment with a sample space S of equally likely outcomes and an event E, the *probability of the event* (denoted by $Prob(E)$) is the ratio of the number of successful outcomes to the total number of outcomes: [a]

$$Prob(E) = \frac{n(E)}{n(S)}$$

(Recall that $n(S)$ is how we symbolically write, "the amount of elements of the set S".)

[a]From Discrete Mathematics by Ensley and Crawley

Question 5 _____ / 3

Finish the following table to log all possible equally-likely outcomes for rolling a red four-sided die and a green four-sided die.

	Green 1	**Green 2**	**Green 3**	**Green 4**
Red 1	(1, 1)	(1, 2)		
Red 2	(2, 1)	(2, 2)		
Red 3			(3, 3)	
Red 4				(4, 4)

Using the definition above describe the following:

a. Both the red and green dice have the same values.
$n(E) =$ _____ $n(S) =$ _____ $Prob(E) =$ _____

b. The sum of both dice values is 4.
$n(E) =$ _____ $n(S) =$ _____ $Prob(E) =$ _____

Question 6 ____ / 3

Consider the experiment of drawing two cards from the top of a standard deck of 52 cards, and the event E of the two cards having the same value. [1]

a. Describe the set S of all outcomes, represented so that they are equally likely.

> **Hint**
> This means what structure type is this? What kind of formula are we using to choose 2 items from a deck of 52?

$n(S) =$ ____________

b. Describe the event E in terms of your representation.

> **Hint**
> We're interested in the event where both our selections have the same value. This can be broken down as:
> 1. Choose any card (52 possible)
> 2. Choose a card with the same value (3 possible)
> 3. Combine with "AND" (The Rule of Product)

$n(E) =$

c. Compute $Prob(E) = \frac{n(E)}{n(S)}$.

$Prob(E) = \frac{n(E)}{n(S)} =$

Question 7 ____ / 3

Consider the experiment of tossing a coin five successive times, and the event E that the last two tosses have the same result.

(__ __ __ Heads Heads) OR (__ __ __ Tails Tails)

a. Describe the set S of all outcomes, represented so they are equally likely

b. Describe the event E in terms of your representation.

$n(S) =$ ____ $n(E) =$ ____

c. $Prob(E) = \frac{n(E)}{n(S)} =$

[1] From Discrete Mathematics by Ensley and Crawley

6.1.2 The complement of the Event

Proposition 1
Given an event E,

$$Prob(E) + Prob(\bar{E}) = 1$$

Where $\bar{E}$ is the complement of the event E.

Question 8 ______ / 3

What is the probability that for a six-sided die rolled three times the same result comes up more than once?

a. What is the sample space S?

b. What is the event E (in English)?
The set of outcomes that...

c. What is the complement of $\bar{E}$ (in English)?
The set of outcomes that...

d. What *structure type* is $\bar{E}$? What is n and r?

e. Calculate $Prob(\bar{E})$
$Prob(\bar{E}) = n(\bar{E})/n(S) =$

f. Calculate the probability for the Event $Prob(E)$ using the proposition.

Team: Please write down all people in your team.

1.
2.
3.
4.

Grading

Question	**Weight**	**0-4**	**Adjusted score**
1	5%		
2	6%		
3	12%		
4	15%		
5	25%		