## 6.3 Probability in games of chance

**Theorem 1**
Given a simple experiment, called a **Bernoulli trial**, and an event that occurs with a probability $p$, if the trial is repeated independently $n$ times, then the probability of having exactly $k$ successes is

$$C(n,k) \cdot p^k \cdot (1-p)^{n-k}$$

[a]

**Example 1** What is the probability that in 10 successive rolls of a fair, six-sided die, we get exactly five results of 6?

Here, we have $n = 10$, $k = 5$, and $p = \frac{1}{6}$, so:

$$C(10,5) \cdot (\frac{1}{6})^5 \cdot (1 - \frac{1}{6})^{10-5}$$

$$\frac{10!}{5!(10-5)!} \cdot (\frac{1}{6})^5 \cdot (\frac{5}{6})^5$$

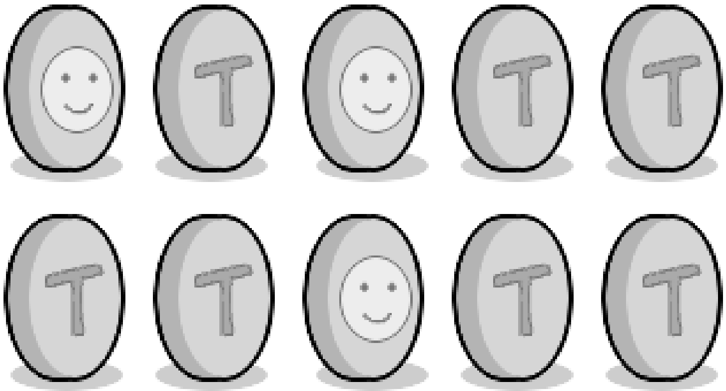
$$\frac{3628800}{14400} \cdot \frac{1}{7776} \cdot \frac{3125}{7776}$$

$$\approx 0.013$$

[a] From Discrete Math by Ensley and Crawley, page 460

**Question 1** ____ / 3

What is the probability of getting exactly 3 heads on 10 tosses of a fair coin?



$n$, the amount of trial repeats: ____

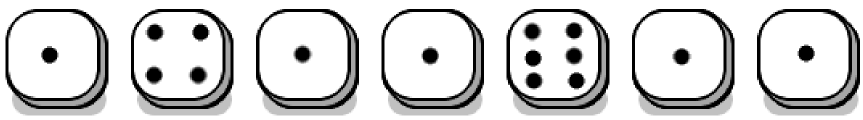
$k$, the amount of successes (heads): ____

$p$, the probability of success: ____

Use the formula of $C(n,k) \cdot p^k \cdot (1-p)^{n-k}$ to find the probability.

**Question 2** ____ / 3

What is the probability that in seven rolls of a six-sided die, the result of 1 appears *at least* five times?



**Hint**

For this one, we will need to use the **rule of sums** to combine several outcomes: Getting 5 1's, 6 1's, OR 7 1's.

| | | repeats $n$ | successes $k$ | probability $p$ |
|---|---|---|---|---|
| A | Getting five 1's | 7 | 5 | 1/6 |
| B | Getting six 1's | 7 | | |
| C | Getting seven 1's | 7 | | |

Now, using the formula $C(n,k) \cdot p^k \cdot (1-p)^{n-k}$ three different times for case (A), (B), and (C).

(A) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} =$

(B) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} =$

(C) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} =$

To find the probability of getting at least five 1's in seven rolls, add (A), (B), and (C) together. (Just write out the formula; don't solve.)

$Prob($ at least five 1's $) =$

**Question 3** ____ / 3

What is the probability of getting exactly one 6 on 10 tosses of a fair six-sided die?