

**6.2 Exercise:** In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

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## 6.1 Sum and Product Rules for Probability

### 6.1.1 The Sum Rule

#### Disjoint events

Two events are said to be **disjoint** (or *mutually exclusive* if they cannot occur simultaneously).<sup>a</sup>

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<sup>a</sup>From Discrete Math by Ensley and Crawley, page 448

#### Question 1

For each of the experiments given below, decide if the events described are disjoint:

- a. When tossing a coin four times, let  $E_1$  be the event that there are exactly three heads and  $E_2$  be the event that there are exactly two heads.
  
  
  
  
  
  
  
  
  
  
- b. When choosing four cards, let  $E_1$  be the event that the cards have the same value and  $E_2$  be the event that the cards have the same suit.
  
  
  
  
  
  
  
  
  
  
- c. When choosing a committee of three people from a club with 8 men and 12 women, let  $E_1$  be the event that the committee has a woman and let  $E_2$  be the event that the committee has a man.

**The Sum Rule, Theorem 1**

If  $E_1$  and  $E_2$  are disjoint events in a given experiment, then the probability that  $E_1$  or  $E_2$  occurs is the sum of  $Prob(E_1)$  and  $Prob(E_2)$ . That is,

$$Prob(E_1 \text{ or } E_2) = Prob(E_1) + Prob(E_2)$$

for disjoint events. <sup>a</sup>

<sup>a</sup>From Discrete Math by Ensley and Crawley, page 449

**Question 2**

A card is drawn from an ordinary deck of 52 cards. Show how to use the basic sum rule to find the probability that the card is...

- a. An ace or a jack.

**Hint**

$E_1$  is the set of outcomes where you get an Ace...  
 $\{ \text{Ace-Heart, Ace-Diamond, Ace-Spade, Ace-Club} \}$  and the probability of getting an Ace,  $Prob(E_1)$  is 4 outcomes out of 52, or  $\frac{1}{13}$ .

- b. A diamond or a black jack, queen, or king card.

- c. An even number value or a red jack, queen, or king card.

**The General Sum Rule, Theorem 2**

If  $E_1$  and  $E_2$  are any events in a given experiment, then the probability that  $E_1$  or  $E_2$  occurs is given by

$$Prob(E_1 \text{ or } E_2) = Prob(E_1) + Prob(E_2) - Prob(E_1 \text{ and } E_2)$$

If  $E_1$  and  $E_2$  are disjoint, then  $E_1 \cap E_2 = \emptyset$ , so  $Prob(E_1 \text{ and } E_2) = 0$ .

<sup>a</sup>

<sup>a</sup>From Discrete Math by Ensley and Crawley, page 450

**Question 3**

What is the probability that when a pair of dice are rolled, either (at least) one die shows a 5 or the dice sum to 8?

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

- What is the sample size?  $n(S) =$
- How many outcomes  $n(E_1)$  are there where you get either 5 for the first die, 5 for the second die, or 5 for both dice? Write out the set of  $E_1$ .
- How many outcomes  $n(E_2)$  are there where the two dice sum to 8? Write out the set of  $E_2$ .
- What is the amount of overlap  $n(E_1 \text{ AND } E_2)$ ? Write out this set.
- Use the General Sum Rule to find the probability that you will get either at least one die showing a 5, OR the dice sum to 8.

### 6.1.2 This given that

**The probability of  $E_1$  given  $E_2$** 

Given events  $E_1$  and  $E_2$  for some experiment, we define the probability of  $E_1$  given  $E_2$ , denoted by  $Prob(E_1|E_2)$ , as the probability that  $E_1$  happens given that  $E_2$  occurs. Note that if  $E_1$  and  $E_2$  are independent, then  $Prob(E_1|E_2) = Prob(E_1)$ .<sup>a</sup>

<sup>a</sup>From Discrete Math by Ensley and Crawley, page 452

**Question 4**

You're drawing two cards from a deck, without replacement.  $E_1$  occurs first, and  $E_2$  occurs second. Find the probability of  $Prob(E_2|E_1)$  (Probability of the 2nd event occurring, given that the 1st even has occurred).

$E_1$  is that you draw a face card.

$E_2$  is that you draw an ace.

**Hint**

You don't need to calculate  $Prob(E_1)$ ; the main idea is how does  $E_1$  occurring affect the state of the sample space? The answer here is that there is one less card in the deck, so  $Prob(E_2|E_1)$  is the probability of getting an ace in a deck with 51 cards in it.

### 6.1.3 The Product Rule

**Independent events**

Two events are said to be **independent** if the occurrence of one event is not influenced by the occurred (or nonoccurrence) of the other event.

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<sup>a</sup>From Discrete Math by Ensley and Crawley, page 451

**Question 5**

For each of the following experiments given below, decide if the events described are independent:

- a. When rolling a 6-sided die four times, let  $E_1$  be the event that the first two rolls sum to 7 and let  $E_2$  be the event that the last two rolls sum to 10.
  
- b. When choosing a committee of three dogs from a club of 8 corgis and 12 labradors, let  $E_1$  be the event that the committee has a labrador and let  $E_2$  be the event that the committee has a corgi.

**The Product Rule, Theorem 3**

If  $E_1$  and  $E_2$  are independent events in a given experiment, then the probability that both  $E_1$  and  $E_2$  occur is the product of  $Prob(E_1)$  and  $Prob(E_2)$ . That is,

$$Prob(E_1 \text{ and } E_2) = Prob(E_1) \cdot Prob(E_2)$$

for independent events. <sup>a</sup>

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<sup>a</sup>From Discrete Math by Ensley and Crawley, page 452

**Question 6**

Suppose I have a “loaded” die for which the probability of a 6 appearing is  $\frac{1}{2}$ , while the probability of each of the other faces appearing is  $\frac{1}{10}$ . What is the probability of getting a 5 and then a 6 on two tosses of the loaded die?

First identify  $E_1$  and  $E_2$ . These events are independent, so you can use the Product Rule to find  $Prob(E_1 \text{ and } E_2)$ .

**The General Product Rule, Theorem 4**

If  $E_1$  and  $E_2$  are any events in a given experiment, then the probability that both  $E_1$  and  $E_2$  occur is given by

$$\begin{aligned} \text{Prob}(E_1 \text{ and } E_2) &= \text{Prob}(E_2) \cdot \text{Prob}(E_1|E_2) \\ &= \text{Prob}(E_1) \cdot \text{Prob}(E_2|E_1) \end{aligned}$$

Note that if  $E_1$  and  $E_2$  are independent, then this says the same thing as Theorem 3. <sup>a</sup>

<sup>a</sup>From Discrete Math by Ensley and Crawley, page 453

**Question 7**

Two marbles are chosen from a bag containing three red, five white, and eight green marbles, so there are 16 total marbles. What is the probability that both are red?

Here, the event  $R_1$  is “the first marble is red”, and the event  $R_2$  is “the second marble is red”.

What is  $\text{Prob}(R_1)$ ?

Since  $R_2$  depends on  $R_1$  occurring, after  $R_1$  occurs, there are 15 marbles left. One red marble has been selected, so there are 2 red marbles left.

What is  $\text{Prob}(R_2|R_1)$ ?

With this information, what is  $\text{Prob}(R_1 \text{ and } R_2)$ ?

**Question 8**

Two marbles are chosen from a bag containing three red, five white, and eight green marbles, so there are 16 total marbles. What is the probability that one is white and one is green?

Let's say we have the events  $W_1$  (White first),  $W_2$  (White second),  $G_1$  (Green first), and  $G_2$  (Green second), so we can get our result in two ways: with  $(W_1, G_2)$  **OR** with  $(G_1, W_2)$ , so you can calculate the result as

$$Prob(W_1 \text{ and } G_2) + Prob(G_1 \text{ and } W_2)$$