Review of structures

Ordered lists of length r with items from $\{1, ..., n\}$

Repetitions allowed: An item from our input set $\{1, ..., n\}$ can be re-used multiple times for each selection.

Order matters: There is a difference between choosing item A then B, and choosing item B then A.

Example
You are filling in your name in the high-score list, and there are 3 slots to fill in your name. You can repeat letters, and the order you enter them matters ("RJM" is different from "MRJ"). Assuming 26 letters, the result is 26^3.

Unordered lists of length r with items from $\{1, ..., n\}$

Repetitions allowed: An item from our input set $\{1, ..., n\}$ can be re-used multiple times for each selection.

Order doesn't matter: Any grouping of selections from the set are considered the same, such as {a, b, c} and {b, c, a}.

Example
The number of bags of r pieces of fruit that can be bought at a store with n types of fruit available is $C(r + n - 1, r)$.

Permutations of length r with items from $\{1, ..., n\}$

No repetitions: Once one item is selected from the set $\{1, ..., n\}$, it is no longer an option for subsequent items.

Example
Pulling cards from a deck... First you have 52 options, then 51 options, then 50 options...

Order matters: The order that you select something matters, so in a way, different "slots" represent different things.

Example
Electing President, VP, and Secretary

Combinations (Sets) of length r with items from $\{1, ..., n\}$

No repetitions: Once one item is selected from the set $\{1, ..., n\}$, it is no longer an option for subsequent items.

Order doesn't matter: If any two items are selected, the order doesn't matter. Combinations deal with sets, and with sets, {a, b} and {b, a} are considered equivalent.

Example
If there are 5 different dinners, and we need to feed 3 people, then there are $C(5, 3)$ possible dinner combinations.

Types of structures

Type	Repeats allowed?	Order matters?	Formula
Ordered list of length r	yes	yes	n^r
Unordered list of length r	yes	no	$C(r+n-1, r)$
Permutations of length r	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$

Question 1 _____ / 1

How many arrangements are there of the letters in the word MATCH?

Are repetitions allowed? no

Does order matter? yes

What is n? 5

What is r? 5

Equation to use? $P(n,r)$ / $C(n,r)$ / n^r / $C(r+n-1,r)$ $P(n,r)$

Solution: $P(5,5) = 120$

Question 2 _____ / 1

There are five red, three green, and eight blue marbles in a box. In how many ways can a sample of four be selected?

Are repetitions allowed? yes

Does order matter? no

What is n? $5+3+8=16$

What is r? 4

Equation to use? $P(n,r)$ / $C(n,r)$ / n^r / $C(r+n-1,r)$ $C(n,r)$

Solution: $C(16,4) = 1820$

Question 3 ____ / 1

We can choose from four types of muffins: Blueberry, Orange, Chocolate Chip, or Cream Cheese. You're going to select muffins in this order: First for yourself, second for your sister, and third for your brother. It is OK if several people have the same muffin type.

Are repetitions allowed? yes

Does order matter? yes

What is n? 4

What is r? 3

Equation to use? $P(n,r)$ / $C(n,r)$ / n^r / $C(r+n-1,r)$ n^r

Solution: 4^3

Question 4 ____ / 1

How many bags of 20 pieces of candy can one buy from a store that sells four types of candy?

Are repetitions allowed? yes

Does order matter? no

What is n? 20

What is r? 4

Equation to use? $P(n,r)$ / $C(n,r)$ / n^r / $C(r+n-1,r)$ $C(r+n-1,r)$

Solution: $C(20+4-1,20) = C(23,20)$

6.1 Introduction

6.1.1 Experiments, Outcomes, and Events

Vocabulary

For this chapter, we will be talking about experiments and their outcomes. For any given experiment, we will have a **sample space** of possible outcomes. This will be written as the set S.

Within an experiment, we want to see if some **event** occurs, and how often it does. In cases where the event occurs, we call it a **success**.

Definition Given an experiment with a sample space S of equally likely outcomes and an event E, the *probability of the event* (denoted by $Prob(E)$) is the ratio of the number of successful outcomes to the total number of outcomes: [a]

$$Prob(E) = \frac{n(E)}{n(S)}$$

(Recall that $n(S)$ is how we symbolically write, "the amount of elements of the set S".)

[a] From Discrete Mathematics by Ensley and Crawley

Question 5 ____ / 3

Finish the following table to log all possible equally-likely outcomes for rolling a red four-sided die and a green four-sided die.

	Green 1	**Green 2**	**Green 3**	**Green 4**
Red 1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
Red 2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
Red 3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
Red 4	(4, 1)	(4, 2)	(4, 1)	(4, 4)

Using the definition above describe the following:

a. Both the red and green dice have the same values.
$n(E) =$ 4 $n(S) =$ 16 $Prob(E) = \frac{4}{16} = \frac{1}{4}$

b. The sum of both dice values is 4.
$n(E) =$ 3 $n(S) =$ 16 $Prob(E) = \frac{3}{16}$

Question 6 ____ / 3

Consider the experiment of drawing two cards from the top of a standard deck of 52 cards, and the event E of the two cards having the same value. [1]

a. Describe the set S of all outcomes, represented so that they are equally likely.

> **Hint**
> This means what structure type is this? What kind of formula are we using to choose 2 items from a deck of 52?

$n(S) = P(52, 2).$

b. Describe the event E in terms of your representation.

> **Hint**
> We're interested in the event where both our selections have the same value. This can be broken down as:
> 1. Choose any card (52 possible)
> 2. Choose a card with the same value (3 possible)
> 3. Combine with "AND" (The Rule of Product)

$n(E) = (52)(3)$

c. Compute $Prob(E) = \frac{n(E)}{n(S)}$.

$Prob(E) = \frac{n(E)}{n(S)} = \frac{(52)(3)}{(52)(51)}$

Question 7 ____ / 3

Consider the experiment of tossing a coin five successive times, and the event E that the last two tosses have the same result.

(___ ___ ___ Heads Heads) OR (___ ___ ___ Tails Tails)

a. Describe the set S of all outcomes, represented so they are equally likely
Ordered lists of length 5 with entries from $\{H, T\}$

b. Describe the event E in terms of your representation.
$n(S) = 2^5 = 32$ $\quad$ $n(E) = 2^3 \cdot 1 \cdot 1 + 2^3 \cdot 1 \cdot 1 = 16$

c. $Prob(E) = \frac{n(E)}{n(S)} = \frac{16}{32} = 0.5$

[1] From Discrete Mathematics by Ensley and Crawley

6.1.2 The complement of the Event

Proposition 1
Given an event E,
$$Prob(E) + Prob(\bar{E}) = 1$$
Where $\bar{E}$ is the complement of the event E.

Question 8 ____ / 3

What is the probability that for a six-sided die rolled three times the same result comes up more than once?

a. What is the sample space S? $\{1, 2, 3, 4, 5, 6\}$

b. What is the event E (in English)?
The set of outcomes that... use the same # more than once.

c. What is the complement of $\bar{E}$ (in English)?
The set of outcomes that... are all different numbers.

d. What *structure type* is $\bar{E}$? What is n and r?
Permutation, $n = 6$, $r = 3$

e. Calculate $Prob(\bar{E})$
$Prob(\bar{E}) = n(\bar{E})/n(S) = \frac{P(6,3)}{6^3} = \frac{5}{9} = 0.\bar{5}$

f. Calculate the probability for the Event $Prob(E)$ using the proposition.
$1 - Prob(\bar{E}) = 1 - 0.55 \approx 0.44$

Team: Please write down all people in your team.

1.
2.
3.
4.

Grading

Question	Weight	0-4	Adjusted score
1	5%		
2	6%		
3	12%		
4	15%		
5	25%		