Chapter 5.2 In-class Exercise

This is the instruction page, make sure to fill out your answers in the worksheet later in the document

1. The rule of sums

> **The rule of sums**   In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.
>
> More formally, the rule of sum is a fact about set theory. It states that sum of the sizes of a finite collection of pairwise disjoint sets is the size of the union of these sets. That is, if $S_1, S_2, ..., S_n$ are pairwise disjoint sets, then we have:
> $$|S_1| + |S_2| + ... + |S_n| = |S_1 \cup S_2 \cup ... \cup S_n| \ ^a$$
> _____
> [a]From https://en.wikipedia.org/wiki/Rule_of_sum

Question 1   _____ / 10%

Uttam wants to read a new book series. He can either pick up the whole series of "Native Tongue", which is *3 books*, or the whole series of "Seed to Harvest", which is *4 books*. How many books are there total that Uttam could read?

_____

Question 2 _____ / 10%

Jennifer is trying to set up their class schedule so it won't interfere with work. They can fit in the following:

- Morning classes: 3 per week

- Night classes: 6 per week

- Weekend classes: 4 per week

Jennifer cannot take a combination of morning, night, and weekend classes. How many different classes does she have to choose from?

> **Hint**   When questions are phrased as "can choose **either ... OR ...**", this usually points to **adding** the size of both sets.

> **The rule of sums with overlap**   If the list to count can be split into two pieces of size z and y, and the pieces have z objects in common, then the original list has $x + y - z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B. [a]
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>
> [a]From Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

Question 3 _____ / 10%

Ryan has 107 games in his Steam library. Of those, 32 have the category "action", 14 have the category "RPG", and 8 are categorized as both "action" AND "RPG", so they end up getting double-counted.

a. How many games are action or RPG? (Use the rule of sums with overlap)

b. How many games are NOT action or RPG? (The total, minus *a*.)

2. The rule of products

> **The rule of products**   In combinatorics, the rule of product or multiplication principle is a basic counting principle (a.k.a. the fundamental principle of counting). Stated simply, it is the idea that if there are a ways of doing something and b ways of doing another thing, then there are $a \cdot b$ ways of performing both actions. [a]
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> ──────────
> [a]From https://en.wikipedia.org/wiki/Rule_of_product

Question 4   _____ / 10%
Cristine is painting her nails, and can choose a color and a top coat. For colors, she has *Red and Purple*, and for top coats, she has *Holo, Matte, and Gloss*. She will choose one color and one top coat.

  a. List out all possible combinations of color+top coat

  b. How many total combinations are there?

> **Hint**   When questions are phrased as "choose ... AND ...", you will usually **multiply** the size of both setes.

Question 5   _____ / 10%
On an old arcade machine, the highscore entries allow for 3 letters for a player to sign their name.

  a. How many options are there for the first, second, and third letter?

  b. If you're choosing a first letter **and** a second letter **and** a third letter, how many possible combinations are there?

3. More questions?!

Question 6   ____ / 15%

In C++, a variable name can have any letters, with lowercase and uppercase counting as different characters. The variable name can also have numbers 0 through 9, except a number cannot be the first character of the name. [1]

    a. For a single-character variable name, how many options are there? (Remember: can't use a number, but can use an uppercase or a lowercase letter).

        1.

       ————————
      `[a-zA-Z]`

    b. For a two-character variable name, how many options are there? (Counting uppercase, lowercase, and numbers.)

        1.   2.

      ————————   ————————
      `[a-zA-Z]`   `[a-zA-Z0-9]`

Question 7   ____ / 15%

Suppose that we are rolling two dice... [2]

    a. How many possible outcomes are there for rolling two 6-sided dice?

    b. How many outcomes are there where you get the same number twice?

    c. How many ways can you roll two dice and NOT get the same number on both?

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[1]Based on a similar problem from Jim Van Horn's POGIL worksheet
[2]Based on a similar problem from Jim Van Horn's POGIL worksheet

Question 8   _____ / 20%

Moneybank requires users to create a password. The password restrictions are that it must be 2 to 4 characters long, and each character can be either an uppercase letter [A-Z] or a digit [0-9]. Answer the following questions to figure out how many possible passwords there are. [3]

  a. How many possibilities are there for a 2-character password? (Take into account letters AND numbers!)

An additional restriction: Each password must contain at least one digit. Subtract all possible ways to have all six characters be a letter... Continue answering the questions to solve.

  b. How many possibilities with just 26 letters?

  c. How many six-character passwords? (Take **a.** and subtract **b.**)

Now follow the above to figure out for...

  d. How many possible passwords with 3 characters?

  e. How many possible passwords with 4 characters?

  f. How many possible passwords, with either 2 characters, 3 characters, or 4 characters. (Are we using the rule of sums or the rule of products here? Notice the wording...)

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[3]Based on a similar problem from Jim Van Horn's POGIL worksheet

Chapter 5.2 In-class Exercise Worksheet

Team: Please write down all people in your team.

1.   2.
3.   4.

Grading

Question	Weight	0-4	Adjusted score
1	10%		
2	10%		
3	10%		
4	10%		
5	10%		
6	15%		
7	15%		
8	20%		

Answer sheet

10% Question 1: Books   □ 0 □ 1 □ 2 □ 3 □ 4

$3 + 4 = 7$ books

10% Question 2: Classes   □ 0 □ 1 □ 2 □ 3 □ 4

$3 + 6 + 4 = 13$

10% Question 3: Video games   □ 0 □ 1 □ 2 □ 3 □ 4

     a. $32 + 14 - 8 = 38$ b. $107 - 69$

10% Question 4: Nail polish   □ 0 □ 1 □ 2 □ 3 □ 4

  a.   {RH, RM, RG, PH, PM,   b. $2 \times 3 = 6$
  PG}

10% Question 5: Highscore   □ 0 □ 1 □ 2 □ 3 □ 4

  a.   26, 26, and 26   b. $26 \times 26 \times 26 = 17,576$

15% Question 6: Variables   □ 0 □ 1 □ 2 □ 3 □ 4

  a.   52   b. $(52) \times (62) = 3,224$

15% Question 7: Dice   □ 0 □ 1 □ 2 □ 3 □ 4

  a. $6 \times 6 = 36$

  b. $6 - \{11, 22, 33, 44, 55, 66\}$.

  c. $36 - 6 = 30$

20% Question 8: Passwords   □ 0 □ 1 □ 2 □ 3 □ 4

  a. $(26 + 10)^2 = 1,296$

  b. $26^2 = 676$

  c. $1,296 - 676 = 620; or : (26 + 10)^2 - (26)^2$

  d. $(26 + 10)^3 - (26)^3 = 29,080$

  e. $(26 + 10)^4 - (26)^4 = 1,222,640$

  f. $620 + 29,080 + 1,222,640 = 1,252,340$