7.2 Proofs about Graphs and Trees

Although this section is named "Proofs", we are actually going to focus on Trees for this section.

7.2.1 Introduction to Trees

Question 1 ____ / 5

[1]

a. How many vertices does each graph have?

G_1 6 G_2 6 G_3 6 G_4 6

b. How many edges does each graph have?

G_1 5 G_2 5 G_3 6 G_4 4

c. Which graph is NOT a connected graph? G_4

d. Which of the graphs has at least one cycle? G_3

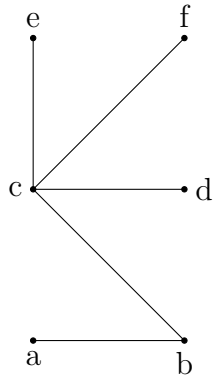
e. Which of the graphs is a tree? G_1 and G_2.

> A simple connected graph with no cycles is a **tree**.

Question 2 ____ / 2

[2]

[1] From Jim Van Horn's POGIL Activity 16
[2] From Jim Van Horn's POGIL Activity 16



G_1

a. What is the degree of each of the vertices in G_1?

$deg(a)$ 1 $deg(b)$ 2 $deg(c)$ 4

$deg(d)$ 1 $deg(e)$ 1 $deg(f)$ 1

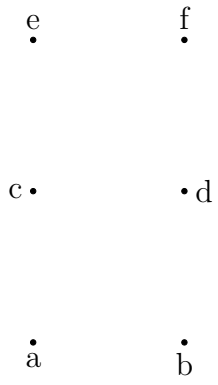
b. List the leaves for G_1. a, d, e, f

> Vertices of degree 1 in a tree are called **leaves** of the tree.

Question 3 ____ / 4

[3]

Given these 6 vertices, draw a tree other than G_1 or G_2.



[3]From Jim Van Horn's POGIL Activity 16

Multiple solutions

a. How many edges are in your new tree? 5

b. How many leaves on your new tree? 3

c. If you removed one edge, would the graph still be connected? no

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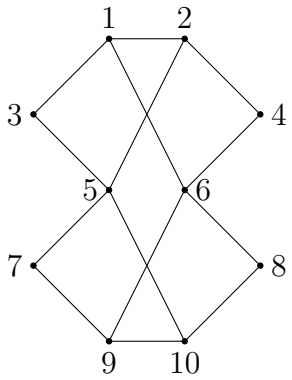
7.2.2 Subgraphs and Trees

Question 4 ____ / 4

[4]



G G_1 G_2

a. Draw a graph G_1 above using vertices and edges from G...
Vertices: 1, 2, 5, Edges: {1, 2} and {2, 5}.

Is this a **subgraph**? Yes
Are all the vertices of G_1 also nodes of G? Yes
Are all the edges of G_1 also edges of G? Yes

b. Draw a graph G_2 above using vertices and edges from G...
Vertices: 1, 3, 4, Edges: {1, 3} and {3, 4}

Is this a **subgraph**? No
Are all the vertices of G_2 also nodes of G? Yes
Are all the edges of G_2 also edges of G? No

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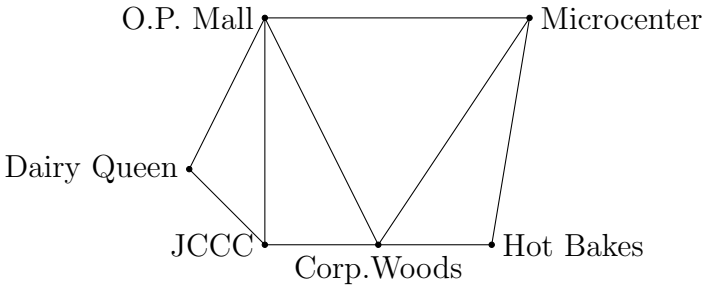
[4] From Jim Van Horn's POGIL Activity 16

7.2.3 Spanning Trees

Question 5 ____ / 2

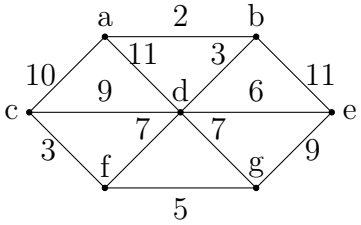
Follow the algorithm to create a Spanning Tree from this map. "x" out edges that you choose to delete as you go. Draw your spanning tree below.



Multiple solutions

7.2.4 Minimal Spanning Trees

Question 6 ____ / 2

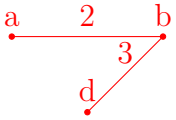


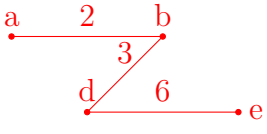
Use Prim's algorithm to find a minimal spanning tree for the graph.

Multiple solutions depending on which node you start at, but for example...

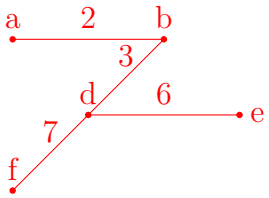
1. 

2. 

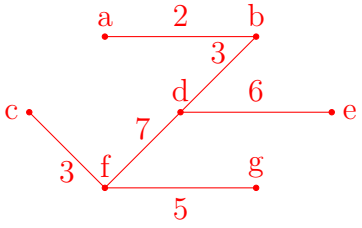
3. 

4. 

5.



6.



7.