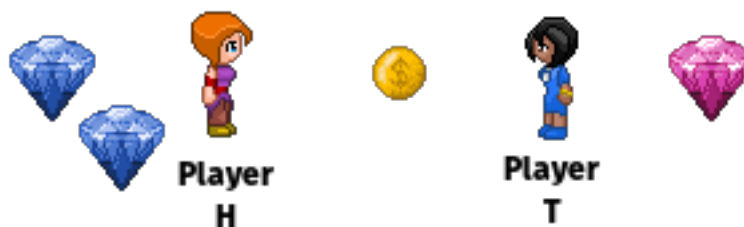


## 6.6 Matrices and Markov Chains

### Question 1

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Let's say a game starts where Player H has 2 markers and player 1 has 1 marker. This is state 3.



What is the probability of...

- Going from State 3 to State 1? 0
- Going from State 3 to State 2?  $1/2$
- Going from State 3 to State 3? 0
- Going from State 3 to State 4?  $1/2$

### Question 2

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Let's say in the game there are 2 total markers instead of three.

- What are all the states in the game? 1. H: 0, T: 2      2. H: 1, T: 1  
3. H: 2, H: 0
- Draw the transition matrix for this game.

	State 1	State 2	State 3
State 1 →	1	0	0
State 2 →	$1/2$	0	$1/2$
State 3 →	0	0	1

**Question 3**

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Calculate the product  $M \cdot M$  (aka  $M^2$ ) for our original game with 3 markers.

	Col 1	Col 2	Col 3	Col 4
Row 1	1	0	0	0
Row 2	1/2	0	1/2	0
Row 3	0	1/2	0	1/2
Row 4	0	0	0	1

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/4 & 0 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

The result ends up being the probability that the game processes from state  $i$  to state  $j$  in **two** moves.

$$M_{1,1}^2 = \begin{matrix} M_{1,1} \cdot M_{1,1} + & M_{1,2} \cdot M_{2,1} + & M_{1,3} \cdot M_{3,1} + & M_{1,4} \cdot M_{4,1} \\ 1 \cdot 1 & 0 \cdot 1/2 & 0 \cdot 0 & 0 \cdot 0 \end{matrix} = 1$$

$$M_{1,2}^2$$

$$M_{1,3}^2$$

$$M_{1,4}^2$$

$$M_{2,1}^2$$

$$M_{2,2}^2$$

$$M_{2,3}^2$$

$$M_{2,4}^2$$

$$M_{3,1}^2 = \begin{matrix} M_{3,1} \cdot M_{1,1} + & M_{3,2} \cdot M_{2,1} + & M_{3,3} \cdot M_{3,1} + & M_{3,4} \cdot M_{4,1} \\ 0 \cdot 1 & 1/2 \cdot 1/2 & 0 \cdot 0 & 1/2 \cdot 0 \end{matrix} = 1/4$$

$$M_{3,2}^2$$

$$M_{3,3}^2$$

$$M_{3,4}^2$$

$$M_{4,1}^2$$

$$M_{4,2}^2$$

$$M_{4,3}^2$$

$$M_{4,4}^2$$

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Draw the matrix  $M^2$ :