6.6 Matrices and Markov Chains

Question 1 ____ / 4

Let's say a game starts where Player H has 2 markers and player 1 has 1 marker. This is state 3.

What is the probability of...

a. Going from State 3 to State 1? 0

b. Going from State 3 to State 2? 1/2

c. Going from State 3 to State 3? 0

d. Going from State 3 to State 4? 1/2

Question 2 ____ / 4

Let's say in the game there are 2 total markers instead of three.

a. What are all the states in the game? 1. H: 0, T: 2 2. H: 1, T: 1 3. H: 2, H: 0

b. Draw the transition matrix for this game.

	State 1	State 2	State 3
State 1 →	1	0	0
State 2 →	1/2	0	1/2
State 3 →	0	0	1

Question 3 _____ / 2

Calculate the product $M \cdot M$ (aka M^2) for our original game with 3 markers.

	Col 1	Col 2	Col 3	Col 4
Row 1	1	0	0	0
Row 2	1/2	0	1/2	0
Row 3	0	1/2	0	1/2
Row 4	0	0	0	1

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/4 & 0 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

The result ends up being the probability that the game processes from state i to state j in **two** moves.

$$M^2_{1,1} = \begin{matrix} M_{1,1} \cdot M_{1,1} + & M_{1,2} \cdot M_{2,1} + & M_{1,3} \cdot M_{3,1} + & M_{1,4} \cdot M_{4,1} \\ 1 \cdot 1 & 0 \cdot 1/2 & 0 \cdot 0 & 0 \cdot 0 \end{matrix} = 1$$

$M^2_{1,2}$

$M^2_{1,3}$

$M^2_{1,4}$

$M^2_{2,1}$

$M^2_{2,2}$

$M^2_{2,3}$

$M^2_{2,4}$

$$M^2_{3,1} = \begin{matrix} M_{3,1} \cdot M_{1,1} + & M_{3,2} \cdot M_{2,1} + & M_{3,3} \cdot M_{3,1} + & M_{3,4} \cdot M_{4,1} \\ 0 \cdot 1 & 1/2 \cdot 1/2 & 0 \cdot 0 & 1/2 \cdot 0 \end{matrix} = 1/4$$

$M^2_{3,2}$

$M^2_{3,3}$

$M^2_{3,4}$

$M^2_{4,1}$

$M^2_{4,2}$

$M^2_{4,3}$

$M^2_{4,4}$

Draw the matrix M^2: