7.4 Connections to Matrices and Relations

7.4.1 Adjacency matrix

Question 1

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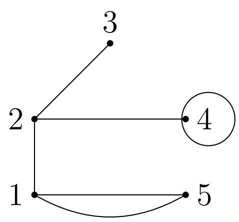
Fill out the adjacency matrix for the following graph.



$$\begin{array}{ccc} & & \text{Columns} \\ & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Rows} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Question 2

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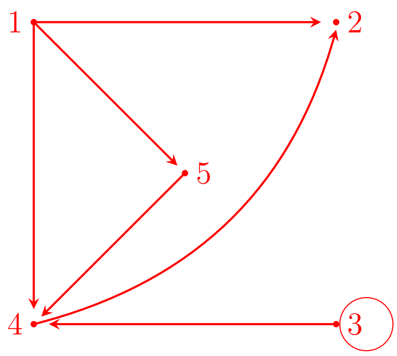
Draw a graph that corresponds to the adjacency matrix

$$\begin{array}{ccc} & & \text{Columns} \\ & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Rows} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



7.4.2 Directed graphs

1. A **directed graph**, like a graph, consists of a set V of vertices and a set E of edges. Each edge is associated with an ordered pair of vertices called its **endpoints**. In other words, a directed graph is the same as a graph, but the edges are described as *ordered pairs* rather than unordered pairs;

2. If the endpoints for edge e are a and b in that order, we say e is an edge **from** a **to** b, and in the diagram we draw the edge as a straight or curved arrow from a to b.

3. For a directed graph, we use (a, b) rather than $[a, b]$ to indicate an edge from a to b. This emphasizes that the edge is an **ordered pair**, by utilizing the usual notation for ordered pairs.

4. A **walk** in a directed graph in a sequence $v_1 e_1 v_2 e_2 ... v_n e_n v_{n+1}$ of alternating vertices and edges that begins and ends with a vertex, and where each edge in the list between its endpoints in the proper order. (That is, e_1 is an edge from v_1 to v_2, e_2 is an edge from v_2 to v_3, and so on.) If there is no chance of confusion, we omit the edges when we describe a walk.

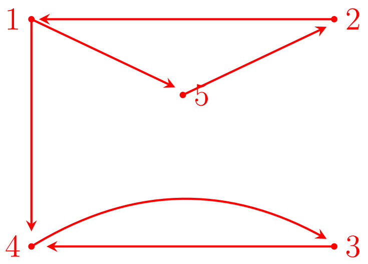
5. The **adjacency matrix** for a directed graph with vertices $\{v_1, v_2, ..., v_n\}$ is the $n \times n$ matrix where M_{ij} (the entry in row i, column j) is the number of edges from vertex v_i to vertex v_j.

[a]

[a]Discrete Mathematics, Ensley and Crawley

Question 3 _____ / 2

Draw a directed graph with vertices $V = \{1, 2, 3, 4, 5\}$ and edges $E = \{(1, 4), (1, 5), (2, 1), (3, 4), (4, 3), (5, 2)\}$.


1
2
5
4
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