5.3 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

5.3 Combinations

5.3.1 Review

Permutation formula

A permutation is written as $P(n, r)$ where n is the amount of items we have to choose from, and r is the amount of items we are selecting. The formula for this is:

$$P(n, r) = \frac{n!}{(n-r)!}$$

The Rule of Sums

> If we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions. [a]

The Rule of Complements

> If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$. [b]

The Rule of Products

> If there are a ways of doing something and b ways of doing another thing, then there are $a \cdot b$ ways of performing both actions. [c]

[a] From https://en.wikipedia.org/wiki/Rule_of_sum
[b] From Discrete Mathematics, Ensley and Crawley, page 390
[c] From https://en.wikipedia.org/wiki/Rule_of_product

5.3.2 Combinations and Permutations

Combinations

In mathematics, a combination is selection of items from a collection, such that (unlike permutations) the order of selection does not matter. (...) The number of r-combinations from a given set of n elements is often denoted in elementary combinatorics texts by $C(n, r)$. [a]

For a combination of length r from a set of n elements:

$$C(n, r) = \frac{n!}{(n-r)! \cdot r!}$$

(Note that the book uses r and Wikipedia uses k.)

[a] From https://en.wikipedia.org/wiki/Combinations

Question 1

How many ways can you rearrange the letters in the word "DOG"?

Question 2

In how many ways can 10 children line up for lunch? *Hint: Is this a combination problem or a permutation problem? Does order matter?*

Question 3

In a class of 20 students, there are 7 IT majors and 13 CS majors. If 4 board positions had to be filled for the computer club, how many ways would there be to fill the positions with the given constraints?

a. No constraints - any student can be on the board.

b. There must be exactly 2 IT students and 2 CS students on the board.

c. There must be *at least* 2 IT students on the board.

Question 4
Suppose we are going to receive a shipment of 50 games on floppy disk for our vintage game store. Each box of 50 generally has 3 defective floppies. For the shipment, we are going to select 5 games to feature in a display.

a. How many total good floppies are there?

b. How many total bad floppies are there?

c. How many ways could we choose 5 games to feature?

d. How many ways contain *no* defective floppies?

e. Using the Rule of Products, determine how many ways that contain all 3 defective floppies?

f. Using the Rule of Sums and the Rule of Products, determine how many ways contain *at least one* defective floppy.

g. Using the Rule of Complements, determine how many ways contain *at least one* defective floppy. The answer for (f) and (g) should match.

Question 5
There's a bargain bin that has 5 PC games, 3 Playstation games, and 8 Xbox games.

a. How many total games are there?

b. If you're grabbing 4 games to buy, you don't care about the order that you pull the games out. How many ways can 4 games be selected?

c. How many ways can you select 4 games that are all for the same console? *(Hint: This means the Playstation games don't get counted. Also, which Rule are you going to use to solve this?)*

d. How many selections of 4 games are there, where you have 2 for one platform, and 2 for another platform? *(Hint: What Rules apply here?)*