Answer Key

1. Sequence: 7, 12, 17, 22, 27, ...

   Closed formula:

n	0	1	2	3	4
s_n	7	12	17	22	27
$\triangle_n$	5	5	5	5	...

   $$s_n = s_0 + \sum_{k=0}^{n-1}(\triangle_k)$$

   $$s_n = 7 + 5n$$

   Recursive formula:
   $s_1 = 7$, $s_n = s_{n-1} + 5$

2. Sequence: 3, 10, 17, 24, 31, ...

   Closed formula:

n	0	1	2	3	4
s_n	3	10	17	24	31
$\triangle_n$	7	7	7	7	...

   $$s_n = 3 + 7n$$

   Recursive formula:
   $s_1 = 3$, $s_n = s_{n-1} + 7$

3. Sequence: 1, 3, 8, 16, 27, 41, ...

Closed formula:

n	0	1	2	3	4	5
s_n	1	3	8	16	27	41
$\triangle_n^1$	2	5	8	11	14	...
$\triangle_n^2$	3	3	3	3	...	...

Finding the 1st level difference equation:

$\triangle_n^1 = \triangle_0^1 + \sum_{k=0}^{n-1}(\triangle_k^2)$

$\triangle_n^1 = 2 + 3n$

Finding s_n:

$$s_n = s_0 + \sum_{k=0}^{n-1}(\triangle_k^1)$$

$$s_n = 1 + \sum_{k=0}^{n-1}(2 + 3k)$$

$$s_n = 1 + \sum_{k=0}^{n-1}(2) + 3\sum_{k=0}^{n-1}(k)$$

$$s_n = 1 + 2n + 3\frac{n(n-1)}{2}$$

$$s_n = 1 + \frac{4n + 3n(n-1)}{2}$$

$$s_n = 1 + \frac{4n + 3n^2 - 3n}{2}$$

$$s_n = \frac{3}{2}n^2 + \frac{1}{2}n + 1$$