Please write down all people in your team.

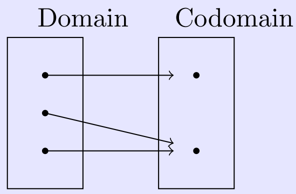
1. 2.

3. 4.

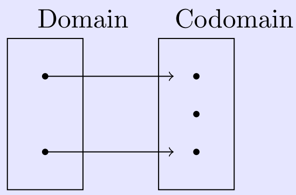
Review: Functions

Properties of Functions

- **Onto:** A function is **onto** if every element of the codomain has at least one element in the domain pointing to it. (Every output is attainable via at least one input.)
- **One-to-one:** A function is **one-to-one** if none of the elements in the codomain is the output from two *different* inputs from the domain.



Onto but not one-to-one

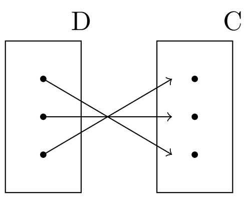


One-to-one but not onto

Question 1

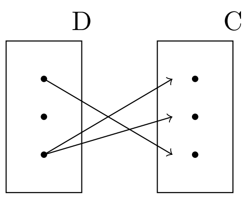
____ / 3

Identify the properties for the following graphs.



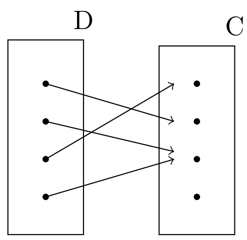
Onto?

One-to-one?



Onto?

One-to-one?



Onto?

One-to-one?

7.3 Isomorphism and Planarity

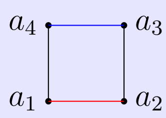
7.3.1 Isomorphism

Isomorphism

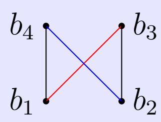
Simple graphs G and H are called **isomorphic** if there is a one-to-one and onto function f from the nodes of G to the nodes of H such that $\{v, w\}$ is an edge of G if and only if $\{f(v), f(w)\}$ is an edge of H. The function f is called an isomorphism. Hence, an isomorphism is simply a **rule** associating nodes that preserves the edges joining the nodes. [a]

In other words, two graphs are isomorphic if they're essentially the same graph, even if the vertices are in different positions.

Example:



G



H

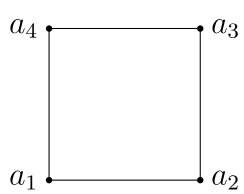
These are isomorphic... imagine taking a_2 and a_3 from graph G and physically flipping them with the edges still connected. In this case, our mapping is... $a_1 \to b_1$ $a_2 \to b_3$ $a_3 \to b_2$ $a_4 \to b_4$.

[a] Discrete Mathematics, Ensley and Crawley

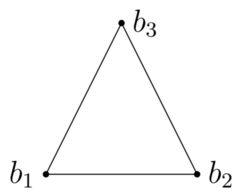
Question 2 ______ / 5

Redraw the following graphs by moving the vertices around, but keeping the edges connected.

a.



b.



Properties of isomorphic graphs

Two graphs that are isomorphic to one another must have...:[a]

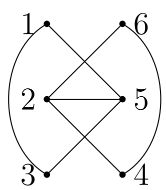
- The same number of nodes
- The same number of edges
- The same number of nodes of any given degree.
- The same number of cycles.
- The same number of cycles of any given size.

[a] Discrete Mathematics, Ensley and Crawley

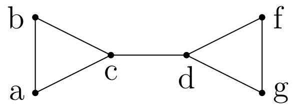
Question 3 ____ / 4

Given the following two graphs...

G



H



a. Write out all edges for both graphs.

   G: {2, 5}

   H: {d, c}

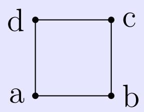
b. For each edge from G, write out what edge in H corresponds to it.

   Example: {2,5} $\rightarrow$ {d, c}

7.3.2 Adjacency matrix

We can also use a matrix to list out which vertices are adjacent to which other vertices in order to help us figure out if two graphs are isomorphic.

Example:

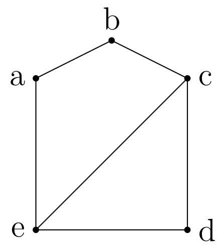


	a	b	c	d
a	0	**1**	0	**1**
b	**1**	0	**1**	0
c	0	**1**	0	**1**
d	**1**	0	**1**	0

a is adjacent to b and d, so in the a row we have 1's under the b and d columns.

Question 4 _____ / 4

For the following graph...



a. Finish the adjacency matrix:

	a	b	c	d	e
a					
b					
c					
d					
e					

b. Fill out the degrees of each:

a	b	c	d	e

7.3.3 Planarity

1. A simple, connected graph is called **planar** if there is a way to draw it (on a plane) so that no edges cross (i.e., they can only meet at a node). We will call "drawing" of a graph on a plane surface with no edge-crossings an **embedding** of the graph in the plane.

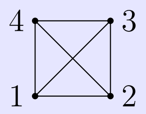
2. A graph is called **bipartite** if its set of nodes can be partitioned into two disjoint sets S_1 and S_2 so that every edge in the graph has one endpoint in S_1 and one endpoint in S_2.

3. The **complete graph** on n nodes, denoted by K_n, is the simple graph with nodes $\{1, ..., n\}$ and an edge between every pair of distinct nodes.

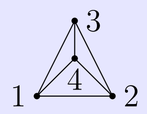
4. The **complete bipartite graph** on n, m nodes, denoted by $K_{n,m}$, is the simple bipartite graph with nodes $S_1 = \{a_1, a_2, ..., a_n\}$ and $S_2 = \{b_1, b_2, ..., b_m\}$ and with edges connecting each node in S_1 to every node in S_2.

[a]

Example: Let's redraw the graph K_4 so it has no overlapping edges.



K_3

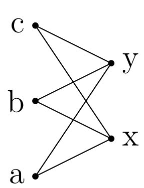


K_3 redrawn

[a]Discrete Mathematics, Ensley and Crawley

Question 5 ____ / 2

Redraw the following graph, $K_{3,2}$, so that no edges are overlapping.



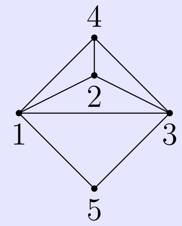
Faces

For a planar graph G embedded in the plane, a **face** of the graph is a region of the plane created by the drawing. Since the plane is an unbounded surface, every embedding of a finite planar graph will have exactly one **unbound** face. [a]

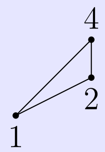
Unbound (external) face: Think of the external face as the "canvas" that all other faces are painted on to. Or, if you were viewing a silhouette of the drawing, you would only see the unbounded face - the sum of all the faces.

Example:

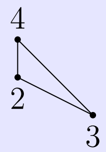
For the drawing, identify the faces by giving the cycle that creates each face, and highlight the unbounded face.



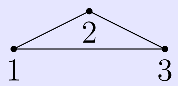
Faces:



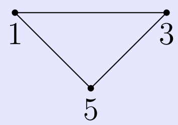
1, 2, 4, 1



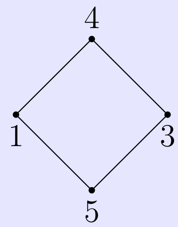
2, 3, 4, 2



1, 2, 3, 1



1, 3, 5, 1



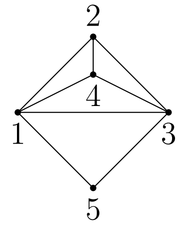
1, 4, 3, 5, 1
(unbounded)

[a] Discrete Mathematics, Ensley and Crawley

Question 6 ____ / 2

For both graphs, draw out each of its **faces**, then write out all the cycles bordering faces and identify the unbounded cycle.

a.



b.