Chapter 6 EXAM PREVIEW

Cheat sheet

Disjoint events Two events are said to be **disjoint** (or *mutually exclusive* if they cannot occur simultaneously.

Independent events Two events are said to be **independent** if the occurrence of one event is not influenced by the occurred (or nonoccurrence) of the other event.

The General Sum Rule If E_1 and E_2 are any events in a given experiment, then the probability that E_1 or E_2 occurs is given by

$$Prob(E_1 or E_2) = Prob(E_1) + Prob(E_2) - Prob(E_1 and E_2)$$

If E_1 and E_2 are disjoint, then $E_1 \cap E_2 = \emptyset$, so $Prob(E_1 and E_2) = 0$.

The General Product Rule If E_1 and E_2 are any events in a given experiment, then the probability that both E_1 and E_2 occur is given by
$Prob(E_1 and E_2) = Prob(E_2) \cdot Prob(E_1|E_2)$
$= Prob(E_1) \cdot Prob(E_2|E_1)$

The probability of E_1 given E_2 Given events E_1 and E_2 for some experiment, we define the probability of E_1 given E_2, denoted by $Prob(E_1|E_2)$, as the probability that E_1 happens given that E_2 occurs. Note that if E_1 and E_2 are independent, then $Prob(E_1|E_2) = Prob(E_1)$.

Cheat sheet

Probability Given an experiment with a sample space S of equally likely outcomes and an event E, the probability of the event occurring, written as $Prob(E)$, is

$$Prob(E) = \frac{n(E)}{n(S)}$$

Complement Given an event E,

$$Prob(E) + Prob(\bar{E}) = 1$$

where $\bar{E}$ is the complement of the event E.

Probability in a Bernoulli Trial For a Bernoulli trial, we run a trial n times. We're looking for some success to happen, and we want it to occur exactly k times. The probability of the success occurring is p. Given these, then you can calculate the probability of k successes occurring with:

$$C(n,k) \cdot p^k \cdot (1-p)^{n-k}$$

Expected (average) value For a given experiment, let X be a random variable whose possible values come from the set $\{ x_1, ..., x_n \}$. The expected value of X, denoted by $E[X]$, is the sum:

$$E[X] = x_1 Prob(X = x_1) + ... + x_n Prob(X = x_n)$$

Expected value in a Bernoulli trial Given a trial performed n times and the probability of succes being p, the expected value $E[X]$ is

$$E[X] = np$$

Questions

Question 1

For an experiment where a single card is drawn from a standard deck of 52 cards, fill out the following table and then answer the questions.

Event	$n(E)$	$n(S)$	$Prob(E)$
The card is a diamond.			
The card is an ace.			
The card is an ace of diamonds.			
The card is a red suit.			
The card is a jack, queen, or king.			
The card has an even number value (2 through 10).			

a. What is the probability that the card is either a diamond or an ace?

b. What is the probability that the card has a suit of diamonds and is a jack, queen, or king?

c. What is the probability that the card has an even number value AND has a red suit?

d. What is the probability that the card has an even number value OR has a red suit?

Question 2

For an experiment where two cards are selected out of a single deck **with no replacement**, find the probability that...

a. The first card has a suit of Diamonds, and the second card has a suit of Hearts.

b. The first card has a value of Jack, and the second card has a value of Queen.

c. Both cards have the same value.

Question 3

In an experiment, you roll one die. What is the probability that the die has a value of at least a 4?

Question 4

In an experiment, you are drawing two cards from a standard deck. Event E_1 is getting an Ace as the first card, and event E_2 is getting an Ace as the second card. What is the probability of E_2 occurring, given that E_1 occurred. In other words, what is $Prob(E_2|E_1)$?

Question 5

In an experiment, we are rolling two dice. Find the probability of each of the following:

a. The probability of getting one 5 and one 6, in any order.

b. The probability of getting at least one 5.

Question 6

For an experiment, we are rolling a die 10 times. We are defining success as getting a 6 exactly half of the time (5 occurrences). Use the formula for k successes in a Bernoulli trial to find the probability of getting 5 successes in 10 rolls of the die.

Question 7

For an experiment, we are going to flip a coin 10 times. Using the Bernoulli formula, find the probability of getting Heads *at least* 8 times.

Question 8

In a game, you are flipping a coin 4 times. Each time you get a Heads, you get \$2. Fill out the following table. Use the Bernoulli formula, $C(n,k) \cdot p^k \cdot (1-p)^{n-k}$, to find the probability of success for each of the rows. Afterward, use the standard formula for expected value, $E[X] = x_1 Prob(X = x_1) + ... + x_n Prob(X = x_n)$, to find the expected amount of money earned.

Outcome x_i	$Prob(E_i)$
$X = \$0$ (No heads)	
$X = \$2$ (One head)	
$X = \$4$ (Two heads)	
$X = \$6$ (Three heads)	
$X = \$8$ (Four heads)	

What is the expected value?

Question 9
In a CS 211 class there are 26 students: 12 CIS majors, 8 IT majors, and 6 undeclared majors. In how many ways can we elect a president, vice president, and secretary for each of the following situations?

a. No restrictions.

b. Three officers are all the same major.

c. None of the officers are undeclared.

d. *At least* one officer must be undeclared.

Question 10
Draw an elephant

Answer key

1.

Event	$n(E)$	$n(S)$	$Prob(E)$
The card is a diamond.	13	52	$13/52$
The card is an ace.	4	52	$4/52 = 1/13$
The card is an ace of diamonds.	1	52	$1/52$
The card is a red suit.	26	52	$26/52 = 1/2$
The card is a jack, queen, or king.	12	52	$12/52 = 3/13$
The card has an even number value.	20	52	$20/52 = 5/13$

For these questions, we're only drawing one card, so the general product rule won't come into play.

1a. What is the probability that the card is either a diamond or an ace?
Probability of getting a diamond: $\frac{13}{52}$
Probability of getting an ace: $\frac{4}{52}$
Probability of getting an ace of diamonds (the overlap): $\frac{1}{52}$
Result (General sum rule): $\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13}$

1b. What is the probability that the card has a suit of diamonds and is a jack, queen, or king?
Probability of getting a diamond: $\frac{13}{52}$
Probability of a face card: $\frac{12}{52}$
Probability of a diamond face card: $\frac{13}{52} \cdot \frac{12}{52} = \frac{3}{52}$

1c. What is the probability that the card has an even number value AND has a red suit?
Probability of an even numbered card: $\frac{20}{52}$
Probability of a red card: $\frac{26}{52}$
Probability of a red even numbered card: $\frac{20}{52} \cdot \frac{26}{52} = \frac{10}{52} = \frac{5}{26}$

1d. What is the probability that the card has an even number value OR has a red suit?
Probability of an even numbered card: $\frac{20}{52}$

Probability of a red card: $\frac{26}{52}$
Probability of a red even numbered card: $\frac{20}{52} \cdot \frac{26}{52} = \frac{10}{52}$
Probability of even numbered card, OR red card: $\frac{20}{52} + \frac{26}{52} - \frac{10}{52} = \frac{36}{52} = \frac{9}{13}$

2. a. The first card has a suit of Diamonds, and the second card has a suit of Hearts.
   E_1 = getting a Diamond card.
   $Prob(E_1) = \frac{13}{52}$
   E_2 = getting a Heart card.
   $Prob(E_2|E_1)$ = The probability of E_2 occurring given than E_1 occurred. In this case, one less item in the deck.
   $Prob(E_2|E_1) = \frac{13}{51}$
   $Prob(E_1 and E_2 = Prob(E_1) \cdot Prob(E_2|E_1) = \frac{13}{52} \cdot \frac{13}{52} = \frac{13}{204}$

   b. The first card has a value of Jack, and the second card has a value of Queen.
   E_1 = getting a Jack card.
   $Prob(E_1) = \frac{4}{52}$
   E_2 = getting a Queen card.
   $Prob(E_2|E_1) = \frac{4}{51}$
   $Prob(E_1 and E_2 = Prob(E_1) \cdot Prob(E_2|E_1) = \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$

   c. Both cards have the same value.
   The first card can be any value, so we don't really have to calculate its probability; it's $\frac{52}{52}$ anyway.
   For event E_2, it has to be the same value. Each card value has 4 in that deck. Given that the first card was one of the values, we have 3 remaining.
   $Prob(E_2|E_1) = \frac{3}{51}$
   $Prob($ both cards have the same value $) = \frac{52}{52} \cdot \frac{3}{51} = \frac{3}{51}$

3. In an experiment, you roll one die. What is the probability that the die has a value of at least a 4?
   For rolling one die, the outcomes would be: Getting a 4, or Getting a 5, or Getting a 6.
   $Prob($ Get a 4 $) = \frac{1}{6}$ $Prob($ Get a 5 $) = \frac{1}{6}$ $Prob($ Get a 6 $) = \frac{1}{6}$
   $Prob($ Getting at least a 4 $) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

4. In an experiment, you are drawing two cards from a standard deck. Event E_1 is getting an Ace as the first card, and event E_2 is getting an Ace as the second card. What is the probability of E_2 occurring, given that E_1 occurred. In other words, what is $Prob(E_2|E_1)$?

E_1 is getting an Ace as the 1st card, E_2 is getting an Ace as the 2nd card.
$Prob(E_1) = \frac{4}{52}$
After we've pulled that first ace from the deck, there are now 3 remaining aces, and 51 remaining cards.
$Prob(E_2|E_1) = \frac{3}{51}$
$Prob(E_1 AND E_2) = Prob(E_1) \cdot Prob(E_2|E_1) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

5. In an experiment, we are rolling two dice. Find the probability of each of the following:

   a. The probability of getting one 5 and one 6, in any order.
   The outcomes where we get one 5 and one 6 are: $(5, 6)$ and $(6, 5)$. So the probability is $\frac{2}{36}$.

   b. The probability of getting at least one 5.
   Outcomes here are (5,1) through (5,6) and (1,5) through (6,5). We only count (5,5) once. So the probability is $\frac{11}{36}$.

6. For an experiment, we are rolling a die 10 times. We are defining success as getting a 6 exactly half of the time (5 occurrences). Use the formula for k successes in a Bernoulli trial to find the probability of getting 5 successes in 10 rolls of the die.
   $n = 10$, $k = 5$, and the probability of getting a 6 is $p = \frac{1}{16}$
   $Prob = C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$
   $Prob = C(10, 5) \cdot (\frac{1}{6})^5 \cdot (\frac{5}{6})^{10-5}$
   $Prob \approx 0.013$

7. For an experiment, we are going to flip a coin 10 times. Using the Bernoulli formula, find the probability of getting Heads <u>at least</u> 8 times.

   Outcome 1: Getting 8 heads; $n = 10$, $k = 8$, $p = \frac{1}{2}$.
   $Prob = C(10, 8) \cdot (\frac{1}{2})^8 \cdot (\frac{1}{2})^2 = 45/1024$

   Outcome 2: Getting 9 heads; $n = 10$, $k = 9$, $p = \frac{1}{2}$.
   $Prob = C(10, 9) \cdot (\frac{1}{2})^9 \cdot (\frac{1}{2})^1 = 5/512$

   Outcome 3: Getting 10 heads; $n = 10$, $k = 10$, $p = \frac{1}{2}$.
   $Prob = C(10, 10) \cdot (\frac{1}{2})^{10} \cdot (\frac{1}{2})^0 = 1/1024$

   Result $= 45/1024 + 5/512 + 1/1024 = 7/128$

8. In a game, you are flipping a coin 4 times. Each time you get a Heads, you get \$2. Fill out the following table. Use the Bernoulli formula,

$C(n,k) \cdot p^k \cdot (1-p)^{n-k}$, to find the probability of success for each of the rows. Afterward, use the standard formula for expected value, $E[X] = x_1 Prob(X = x_1) + ... + x_n Prob(X = x_n)$, to find the expected amount of money earned.

Outcome x_i	$Prob(E_i)$
$X = \$0$	$C(4,0)(1/2)^0(1/2)^4 = \frac{1}{16}$
$X = \$2$	$C(4,0)(1/2)^1(1/2)^3 = \frac{1}{4}$
$X = \$4$	$C(4,0)(1/2)^2(1/2)^2 = \frac{3}{8}$
$X = \$6$	$C(4,0)(1/2)^3(1/2)^1 = \frac{1}{4}$
$X = \$8$	$C(4,0)(1/2)^4(1/2)^0 = \frac{1}{16}$

$E[X] = \$0(\frac{1}{16}) + \$2(\frac{1}{4}) + \$4(\frac{3}{8}) + \$6(\frac{1}{4}) + \$8(\frac{1}{16}) = \$4$