5.2 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

5.2 Basic Rules for Counting

5.2.1 Permutations

Permutation formula

A permutation is a type of structure that describes counting when **order matters** and **repetitions are not allowed**.

A permutation is written as $P(n, r)$ where n is the amount of items we have to choose from, and r is the amount of items we are selecting. The formula for this is:

$$P(n, r) = \frac{n!}{(n-r)!}$$

Where $n!$ is n-factorial. Also note that $0! = 1$.

Question 1

Calculate the following by hand using the formula:

a. $P(5, 2)$

b. $P(5, 5)$

c. $P(5, 1)$

Question 2

Use `www.wolframalpha.com` or a graphing calculator to compute the following:

a. $P(26, 3)$

b. $P(52, 6)$

c. $P(26, 2) + P(10, 2)$

5.2.2 The Rule of Sums

The Rule of Sums

In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, **it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.** [a]

[a] From https://en.wikipedia.org/wiki/Rule_of_sum

Question 3
Uttam wants to read a new book series. He can either pick up the whole series of "Native Tongue", which is *3 books*, or the whole series of "Seed to Harvest", which is *4 books*. How many ways can Uttam select books, if he is choosing 2 Native Tongue books, OR 2 Seed to Harvest books, but not both?

Question 4
Jennifer is trying to set up their class schedule so it won't interfere with work. They can fit in the following:

- Morning classes: 3 per week
- Night classes: 3 per week
- Weekend classes: 4 per week

Jennifer can only take all morning, all night, or all weekend classes - they cannot mix them. If there are 12 classes to choose from, how many combinations of classes can Jennifer have?

When questions are phrased as "can choose **either ... OR ...**", this usually points to **adding** the size of both sets.

The rule of sums with overlap

If the list to count can be split into two pieces of size z and y, and the pieces have z objects in common, then the original list has $x+y-z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B. [a]

[a]From Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

5.2.3 The Rule of Complements

The rule of complements

> If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$. [a]

[a]From Discrete Mathematics, Ensley and Crawley, page 390

Question 5

Assume you are rolling two dice.

a. List out all possible results, assuming order does matter.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

b. How many total outcomes are there?

c. How many results have 6 show up at least once?

d. Use the rule of complements to solve how many outcomes there are where 6 does not show up.

Question 6
Ryan has 107 games in his Steam library. Of those, 32 have the category "action", 14 have the category "RPG", and 8 are categorized as both "action" AND "RPG", so they end up getting double-counted.

a. How many games are action or RPG?
(Use the rule of sums with overlap)

b. How many games are NOT action or RPG?
(Use the rule of complements)

5.2.4 The Rule of Products

The rule of products

> In combinatorics, the rule of product or multiplication principle is a basic counting principle (a.k.a. the fundamental principle of counting). Stated simply, **it is the idea that if there are a ways of doing something and b ways of doing another thing, then there are $a \cdot b$ ways of performing both actions.** [a]

[a] From https://en.wikipedia.org/wiki/Rule_of_product

Question 7
On an old arcade machine, the highscore entries allow for 3 letters for a player to sign their name. Many only offer capital letters, so only 26 options, and each letter can be used more than once.

a. How many options are there for the first letter?

b. How many options are there for the second letter?

c. How many options are there for the third letter?

d. If you're choosing a first letter **and** a second letter **and** a third letter, how many possible combinations are there?