

Please write down all people in your team.

- 1.
 - 2.
 - 3.
 - 4.
-

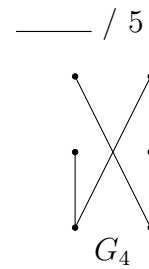
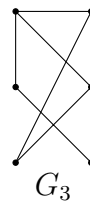
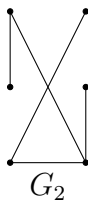
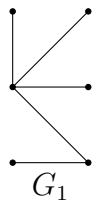
7.2 Proofs about Graphs and Trees

Although this section is named “Proofs”, we are actually going to focus on Trees for this section.

7.2.1 Introduction to Trees

Question 1

¹



- a. How many vertices does each graph have?

G_1 _____ G_2 _____ G_3 _____ G_4 _____

- b. How many edges does each graph have?

G_1 _____ G_2 _____ G_3 _____ G_4 _____

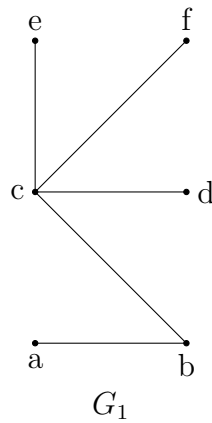
- c. Which graph is NOT a connected graph?
- d. Which of the graphs has at least one cycle?
- e. Which of the graphs is a tree?

A simple connected graph with no cycles is a **tree**.

¹From Jim Van Horn's POGIL Activity 16

Question 2

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²

- a. What is the degree of each of the vertices in G_1 ?

$\deg(a)$ ____ $\deg(b)$ ____ $\deg(c)$ ____

$\deg(d)$ ____ $\deg(e)$ ____ $\deg(f)$ ____

- b. List the leaves for G_1 .

Vertices of degree 1 in a tree are called **leaves** of the tree.

²From Jim Van Horn's POGIL Activity 16

A **Tree** is a connected simple graph that has no cycles. Vertices of degree 1 in a tree are called **Leaves** of the tree. ^a

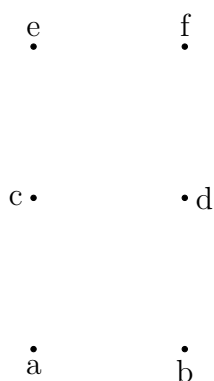
^aDiscrete Mathematics, Ensley and Crawley

Question 3

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3

Given these 6 vertices, draw a tree other than G_1 or G_2 .



- How many edges are in your new tree?
- How many leaves on your new tree?
- If you removed one edge, would the graph still be connected?

A tree with n vertices will have $n - 1$ edges. In other words, it is a connected graph and if you remove an edge then it will become a disconnected graph.

³From Jim Van Horn's POGIL Activity 16

7.2.2 Subgraphs and Trees

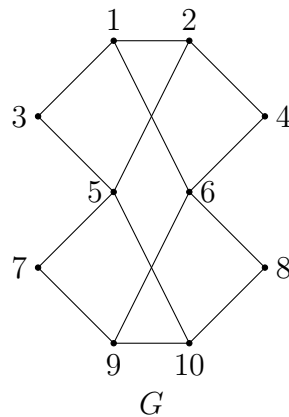
A graph H is a **subgraph** of a graph G if all nodes and edges in H are also nodes and edges in G .^a

^aDiscrete Mathematics, Ensley and Crawley

Question 4

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4



G

G_1

G_2

- a. Draw a graph G_1 above using vertices and edges from G ...
Vertices: 1, 2, 5, Edges: $\{1, 2\}$ and $\{2, 5\}$.

Is this a **subgraph**?

Are all the vertices of G_1 also nodes of G ?

Are all the edges of G_1 also edges of G ?

- b. Draw a graph G_2 above using vertices and edges from G ...
Vertices: 1, 3, 4, Edges: $\{1, 3\}$ and $\{3, 4\}$

Is this a **subgraph**?

Are all the vertices of G_2 also nodes of G ?

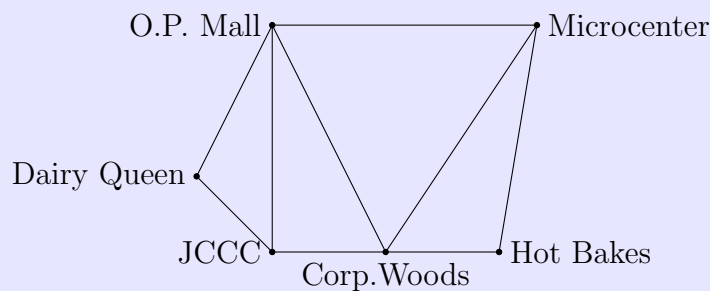
Are all the edges of G_2 also edges of G ?

⁴From Jim Van Horn's POGIL Activity 16

7.2.3 Spanning Trees

Let G be a simple connected graph. The subgraph T is a **spanning tree** of G if T is a tree and every node in G is a node in T .^a

Example:^b

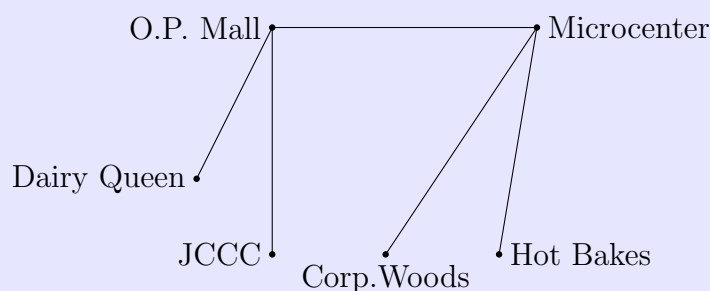


To get from Corporate Woods to JCCC, there are three paths leading into JCCC:

- (1) Directly from CW \rightarrow JCCC,
- (2) CW \rightarrow OP Mall \rightarrow JCCC, and
- (3) CW \rightarrow OP Mall \rightarrow Dairy Queen \rightarrow JCCC

We want to make a graph that connects all locations with the fewest paths. One way to do this is to remove edges of a cycle until no additional edges can be removed without getting a disconnected graph.

One example result is this:



^aDiscrete Mathematics, Ensley and Crawley

^bFrom Jim Van Horn's POGIL Activity 16

Spanning Tree algorithm ^a

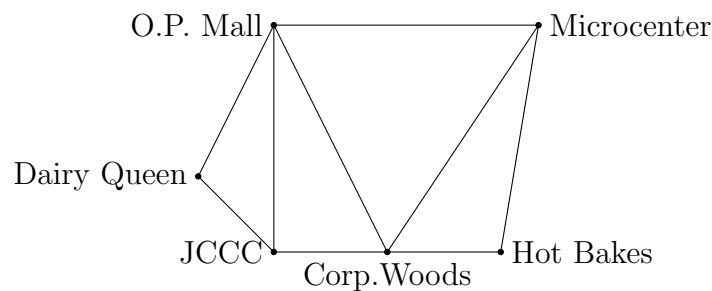
1. Begin with a simple connected graph G_0 .
2. For each $i \geq 1$, as long as there is a cycle in G_{i-1} ...
 - (a) Choose an edge e in any cycle of G_{i-1} , and form the subgraph G_i of G_{i-1} by deleting e from G_{i-1}
3. The final result G_k will be a spanning tree of G_0 . This is a spanning tree.

^aDiscrete Mathematics, Ensley and Crawley

Question 5

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Follow the algorithm to create a Spanning Tree from this map. “x” out edges that you choose to delete as you go. Draw your spanning tree below.



7.2.4 Minimal Spanning Trees

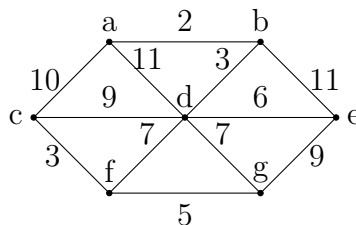
Prim's Minimal Spanning Tree algorithm ^a

1. Given a connected simple graph G with $n + 1$ nodes.
2. Let v_0 be any node in G , and let $T_0 = \{v_0\}$ be a tree with one node and no edges.
3. For each k from $\{1, 2, \dots, n\}$...
 - (a) Let $E_k = \{e \text{ an edge in } G : e \text{ has one endpoint in } T_{k-1} \text{ and the other endpoint not in } T_{k-1}\}$.
 - (b) Let e_k be the edge in E_k with the smallest weight. (In case of a tie, choose any edge of the smallest weight.)
 - (c) Let T_k be the tree obtained by adding edge e_k (along with its node not already in T_{k-1} to T_{k-1} .
4. T_n is the tree returned by the algorithm.

^aDiscrete Mathematics, Ensley and Crawley

Question 6

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Use Prim's algorithm to find a minimal spanning tree for the graph.