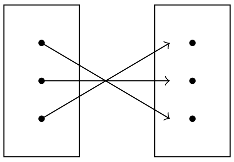
# Review: Functions

### Question 1

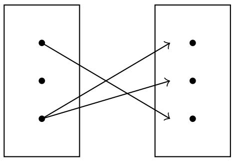
_____ / 3

Identify the properties for the following graphs.



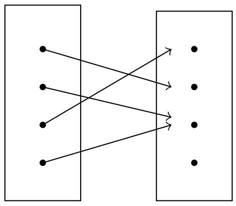
Onto? yes, every domain element has an input

One-to-one? yes, every domain element has max 1 input



Onto? yes, every domain element has an input

One-to-one? yes, every domain element has max 1 input



Onto? no, the bottom domain element doesn't have an input

One-to-one? no, one element has 2 inputs

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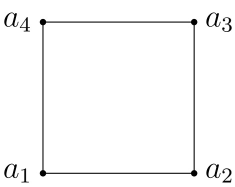
# 7.3 Isomorphism and Planarity

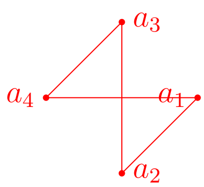
## 7.3.1 Isomorphism

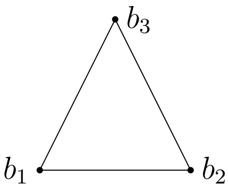
### Question 2

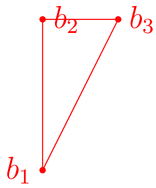
_____ / 5

Redraw the following graphs by moving the vertices around, but keeping the edges connected.

a. 


Example: 


b. 

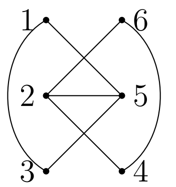

Example: 


Multiple solutions...

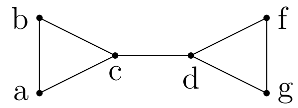
**Question 3** ____ / 4

Given the following two graphs...

G




H




a. Write out all edges for both graphs.

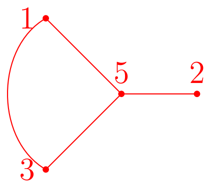
$G$: {2, 5} { 2, 3 } { 3, 4 } { 4, 5 } { 2, 5 } { 5, 6 }

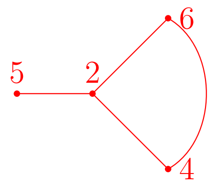
$H$: {d, c} { y, x } { x, w } { w, v } { y, v } { v, u }

b. For each edge from $G$, write out what edge in $H$ corresponds to it.

Example: {2,5} $\rightarrow$ {d, c}

Let's split up $G$ into two subgraphs to see it more clearly...




$\{1,3\} \rightarrow \{b,a\}$
$\{2,5\} \rightarrow \{d,c\}$
$\{4,6\} \rightarrow \{e,f\}$

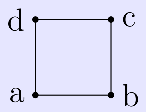
$\{1,5\} \rightarrow \{b,c\}$
$\{2,6\} \rightarrow \{d,f\}$

$\{2,4\} \rightarrow \{d,e\}$
$\{3,5\} \rightarrow \{a,c\}$

## 7.3.2 Adjacency matrix

We can also use a matrix to list out which vertices are adjacent to which other vertices in order to help us figure out if two graphs are isomorphic.

**Example:**

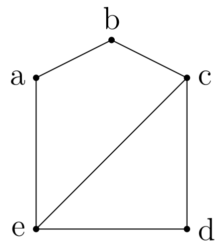



| | a | b | c | d |
|---|---|---|---|---|
| a | 0 | **1** | 0 | **1** |
| b | **1** | 0 | **1** | 0 |
| c | 0 | **1** | 0 | **1** |
| d | **1** | 0 | **1** | 0 |

$a$ is adjacent to $b$ and $d$, so in the $a$ row we have 1's under the $b$ and $d$ columns.

### Question 4 ____ / 4

For the following graph...




a. Finish the adjacency matrix:

| | a | b | c | d | e |
|---|---|---|---|---|---|
| a | 0 | 1 | 0 | 0 | 1 |
| b | 1 | 0 | 1 | 0 | 0 |
| c | 0 | 1 | 0 | 1 | 1 |
| d | 0 | 0 | 1 | 0 | 1 |
| e | 1 | 0 | 1 | 1 | 0 |

b. Fill out the degrees of each:

| a | b | c | d | e |
|---|---|---|---|---|
| 2 | 2 | 3 | 2 | 3 |

### 7.3.3 Planarity

1. A simple, connected graph is called **planar** if there is a way to draw it (on a plane) so that no edges cross (i.e., they can only meet at a node). We will call "drawing" of a graph on a plane surface with no edge-crossings an **embedding** of the graph in the plane.

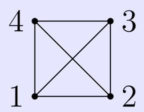
2. A graph is called **bipartite** if its set of nodes can be partitioned into two disjoint sets $S_1$ and $S_2$ so that every edge in the graph has one endpoint in $S_1$ and one endpoint in $S_2$.

3. The **complete graph** on $n$ nodes, denoted by $K_n$, is the simple graph with nodes $\{1, ..., n\}$ and an edge between every pair of distinct nodes.

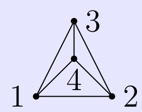
4. The **complete bipartite graph** on $n, m$ nodes, denoted by $K_{n,m}$, is the simple bipartite graph with nodes $S_1 = \{a_1, a_2, ..., a_n\}$ and $S_2 = \{b_1, b_2, ..., b_m\}$ and with edges connecting each node in $S_1$ to every node in $S_2$.

[a]

**Example:** Let's redraw the graph $K_4$ so it has no overlapping edges.




$K_3$

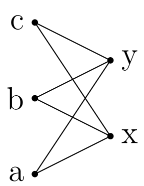



$K_3$ redrawn

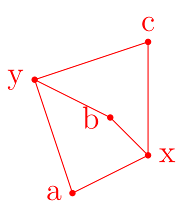
[a]Discrete Mathematics, Ensley and Crawley

### Question 5 _____ / 2

Redraw the following graph, $K_{3,2}$, so that no edges are overlapping.




Example:



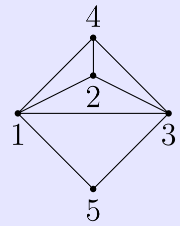
**Faces**

For a planar graph $G$ embedded in the plane, a **face** of the graph is a region of the plane created by the drawing. Since the plane is an unbounded surface, every embedding of a finite planar graph will have exactly one **unbound** face. [a]

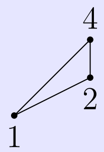
**Unbound (external) face:** Think of the external face as the "canvas" that all other faces are painted on to. Or, if you were viewing a silhouette of the drawing, you would only see the unbounded face - the sum of all the faces.

**Example:**

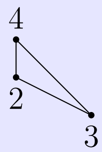
For the drawing, identify the faces by giving the cycle that creates each face, and highlight the unbounded face.



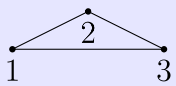

Faces:



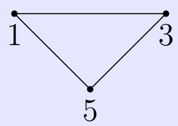

1, 2, 4, 1



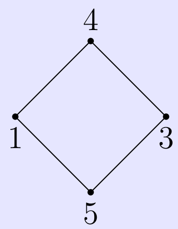

2, 3, 4, 2




1, 2, 3, 1




1, 3, 5, 1




1, 4, 3, 5, 1
(unbounded)

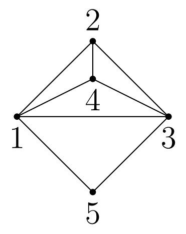
[a] Discrete Mathematics, Ensley and Crawley

## Question 6

_____ / 2

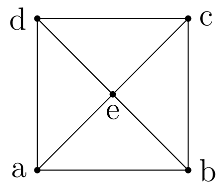
For both graphs, draw out each of its **faces**, then write out all the cycles bordering faces and identify the unbounded cycle.

a.




1, 2, 4, 1    1, 3, 5, 1    2, 3, 4, 2
1, 3, 4, 1    1, 2, 3, 5, 1 (unbounded).

b.




a, b, e, a    a, e, d, a    d, e, c, d    b, c, e, b
a, b, c, d, a (unbounded)