Please write down all people in your team.

1. 2.

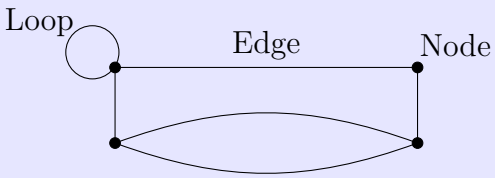
3. 4.

7.1 Graph Theory

7.1.1 Terminology

Since we're introducing a new concept, Graph Theory, we need to go over the various terms so that we can communicate about these graphs properly.

- **Graph:** A graph is a type of diagram that contains *vertices* (aka nodes) and *edges*.

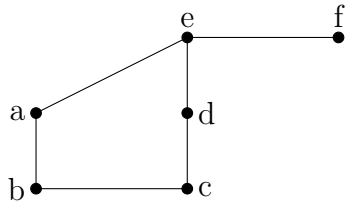


Parallel edges

- **Node:** A vertex of the graph, drawn as a dot.
  - **Adjacent nodes:** Two nodes that are connected by an edge.
  - **Node degree:** The amount of edges that are connected to a node. Loops are counted twice.
- **Edge:** A line that connects two nodes together.
  - **Parallel edges:** Two edges that have the same two endpoints.
  - **Loop:** An edge that begins and ends at the same node, creating a loop.
  - $[a, b]$ is used to indicate an edge with a and b as endpoints, though direction can be either way.

Question 1 _____ / 6

Identify each item for the graph G given.



a. How many nodes (vertices) are there?

b. How many edges are there?

c. Write down the degree of each node:

Vertex v	$deg(v)$
a	
b	
c	
d	
e	
f	

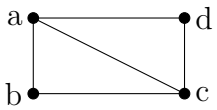
d. The **maximum degree** of a graph is the highest $deg(v)$ value. What is this graph's maximum degree?

e. The **minimum degree** of a graph is the lowest $deg(v)$ value. What is this graph's minimum degree?

- **Walk:** A series of alternating nodes and edges, traversing between adjacent nodes.
  - **Closed walk:** When the beginning and ending node of a walk are the same.
  - **Length of a walk:** The amount of edges in the walk.
  - **Trivial walk:** A walk of length 0.
  - **Path:** A walk with no repeated vertices.
  - **Trail:** A walk with no repeated edges.
    - **Circuit:** A closed trail.
      - **Trivial circuit:** A circuit with one vertex and no edges.
      - **Cycle:** A nontrivial circuit where the only repeated node is the first/last one.
    - **Eulerian:** A trail or circuit where every edge is traversed.

Question 2 ____ / 3

Answer the following questions, using the graph H given.



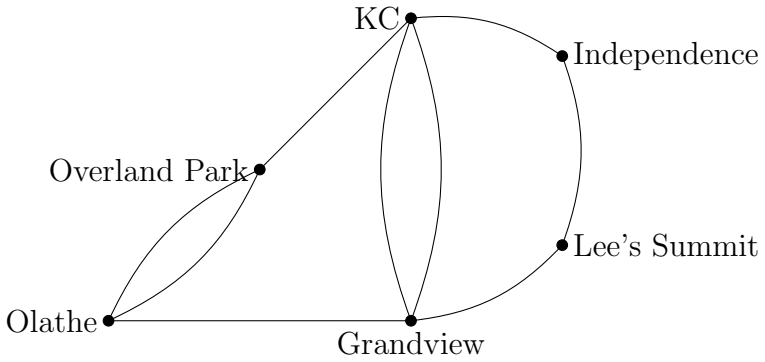
a. Come up with several walks from a to c. Write all steps (each node visited). Also label the **length** of each walk.

b. Come up with a **closed walk**, beginning and ending at a. You can choose to visit all nodes or not.

c. Come up with a **path**, where no vertices are repeated.

Question 3 _____ / 5

Answer the following questions, using the graph I given.



a. Come up with a **trail**, a walk with no repeated edges.

b. Come up with a **circuit**, a closed trail.

c. Come up with a **cycle**, a circuit where the only repeated node is the first/last one..

d. Identify: Did you come up with any **Eulerian Trails**?
If not, create one.

e. Identify: Are there any **parallel edges**?

- **Simple graph:** A graph that has no loops or parallel edges.
- **Directed graph:** The edges in the graph are given a direction, which can only be traversed in that way.
  - Edges are denoted with parentheses (a, b), showing that it goes from a to b.

Question 4 _____ / 1

Draw a **Directed Graph** using the following list of edges:

$$(1, 2), (2, 1), (3, 3), (4, 2)$$

(Don't confuse these for points on an x, y plane that are interconnected, each ordered pair is its own set of information - beginning and end nodes.)

- A graph is **connected** if there is a walk between any pair of distinct nodes.
- A graph H is a **subgraph** of a graph G if all nodes and edges in H are also nodes and edges in G.
- A **connected component** of a graph G is a connected subgraph H of G such that no other connected subgraph of G containing H exists.

[a]

[a] Discrete Mathematics, Ensley and Crawley

Question 5 _____ / 2

Draw a graph that is **not connected**, and draw a **subgraph** of your graph.