Answer Key

1. a. When tossing a coin four times, no outcome can consist of "exactly two heads" and also "exactly three heads"; hence, these two events are disjoint.

   b. When choosing four cards, if the four cards all have the same value, then they cannot all have the same suit; hence, these two events are disjoint.

   c. When choosing a committee of three people from a club with 8 men and 12 women, there are many ways in which the committee can include a woman and a man, so these events are not disjoint.

2. a.
$$Prob(E_1 or E_2) = Prob(E_1) + Prob(E_2) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

   b. E_1 is the outcomes where you get a diamond, and E_2 is the outcomes where you get a black Jack, King, or Queen. These sets are disjoint, so...
$$Prob(E_1 or E_2) = Prob(E_1) + Prob(E_2) = \frac{1}{4} + \frac{6}{52} = \frac{19}{52}$$

   c. E_1 is the outcomes where the card has an even numbered value, and E_2 is the set of outcomes with a red Jack, King, or Queen. These sets are disjoint, so...
$$Prob(E_1 or E_2) = Prob(E_1 + Prob(E_2) = \frac{5}{13} + \frac{6}{52} = \frac{1}{2}$$

3. a. $n(S) = 36$

   b. $E_1 = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,5),(4,5),(3,5),(2,5),(1,5)\}$
   $n(E_1) = 11$

   c. $E_2 = \{(2,6),(6,2),(3,5),(5,3),(4,4)\}$
   $n(E_2) = 5$

   d. $n(E_1 \quad AND \quad E_2) = 2$

   e. $\frac{11}{36} + \frac{5}{36} - \frac{2}{36} = \frac{7}{18}$

4. $Prob(E_2|E_1) = \frac{4}{51}$

5. a. The events are independent. The first two rolls have no influence on the last two rolls.

b. The events are not independent. The probability of the committee having a corgi (E_2) is different when E_1 occurs than it is when E_1 does not occur. Specifically, if E_1 does not occur, then E_2 happens for sure (i.e., its probability is 1), and if E_1 does occur, then E_2 is not guaranteed to happen (i.e., probability is less than 1).

6. The events E_1, "getting a 5 on the first toss", and E_2, "getting a 6 on the second toss", are independent, so by the product rule, $Prob(E_1 and E_2) = Prob(E_1) \cdot Prob(E_2) = \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{20}$

7.
- $Prob(R_1) = \frac{3}{16}$
- $Prob(R_2) = \frac{2}{15}$
- $Prob(R_1 and R_2) = Prob(R_1) \cdot Prob(R_2|R_1)$
$= \frac{6}{240} = \frac{1}{40}$

8. W_1 will be white as the first, and G_2 will be green as the second.

$Prob(W_1 and G_1 + Prob(G_1 and W_2) =$
$Prob(W_1) \cdot Prob(G_2|W_1) + Prob(G_1) \cdot Prob(W_2|G_1)$
$= \frac{5}{16} \cdot \frac{8}{15} + \frac{8}{16} \cdot \frac{5}{15} = \frac{1}{3}$