Intro to Combinatorics

Written by Rachel J. Morris, last updated Jan 8, 2018

About

In this chapter we are getting introduced to the idea of “counting problems” and what kind of problems we will be looking at.

Topics

1. What is “Combinatorics”?
2. Counting structures
3. Practice problems

What is "Combinatorics"?

1. What is "Combinatorics"?

"Combinatorics is an area of mathematics primarily concerned with counting, both as a means and an end in obtaining results, and certain properties of finite structures."

From https://en.wikipedia.org/wiki/Combinatorics

Notes

Combinatorics = counting problems

1. What is "Combinatorics"?

In this chapter we are concerned with finding out "how many ways can we do [thing]?"

In chapter 6, we will be asking about probability, which will build on top of combinations.

Notes

Combinatorics = counting problems

1. What is "Combinatorics"?

For example: If we have a deck of cards, how many ways are there to select one card?

Notes

Combinatorics = counting problems

1. What is "Combinatorics"?

For example: If we have a deck of cards, how many ways are there to select one card? **52 ways**

Notes

Combinatorics = counting problems

1. What is "Combinatorics"?

For example: If we have a deck of cards, how many ways are there to select one card? **52 ways**

How many ways are there to select two cards out of a desk (without replacement)?

Notes

Combinatorics = counting problems

1. What is "Combinatorics"?

For example: If we have a deck of cards, how many ways are there to select one card? **52 ways**

How many ways are there to select two cards out of a desk (without replacement)?
52 x 51 ways = 2,652

Notes

Combinatorics = counting problems

Counting Structures

2. Counting Structures

There are different types of structures for different types of counting problems. We differentiate these structures based on whether **repetition is allowed** and whether **order matters**.

Type of structure	Repetition allowed?	Order matters?
Ordered list	**yes**	**yes**
Unordered list	**yes**	**no**
Permutation	**no**	**yes**
Set / Combination	**no**	**no**

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

2. Counting Structures – Ordered List

Type of structure	Repetition allowed?	Order matters?
Ordered list	**yes**	**yes**

An example of an Ordered List counting problem:

If you're filling out your initials in a high score table in an arcade, you can use the letters A-Z. How many ways are there to fill in a 3-letter name?

- **Can repeat letters:**
  MOO, ZZZ, POP are valid outcomes.
- **Order of letters matters:**
  RAT, TAR, and ART are all considered different outcomes.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

2. Counting Structures – Unordered List

Type of structure	Repetition allowed?	Order matters?
Unordered list	**yes**	**no**

An example of an Unordered List counting problem:

You're flipping a coin 5 times. In how many ways can you get more HEADS than TAILS?

- **Can repeat outcomes:**
  H-H-H-H-H, H-H-T-H-T, T-T-H-T-H, etc.
- **Order of outcome doesn't matter:**
  We would count H-H-H-H-T and H-H-T-H-H as being the same outcome.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

2. Counting Structures – Permutation

Type of structure	Repetition allowed?	Order matters?
Permutation	**no**	**yes**

An example of a Permutation counting problem:

Five people are running a race, how many different ways are there to award Gold, Silver, and Bronze?

- **Cannot repeat:**
  A person cannot win multiple positions.
- **Order does matter:**
  Let's say we have GOLD-SILVER-BRONZE, if a person is at position 1, 2, or 3, it would represent different things.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

2. Counting Structures – Sets / Combination

Type of structure	Repetition allowed?	Order matters?
Sets / Combination	**no**	**no**

An example of a Set counting problem:

Out of 10 people, 3 will be chosen to be on a steering committee. How many different ways are there to choose 3 volunteers?

- **Cannot repeat:**
  A person cannot win multiple positions.
- **Order doesn't matter:**
  It doesn't matter what order people are chosen in, they're still part of the committee.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

2. Counting Structures

Each of these have formulas associated with them, which you can use to figure out the amount of outcomes.

This is what we will be covering during Chapter 5.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

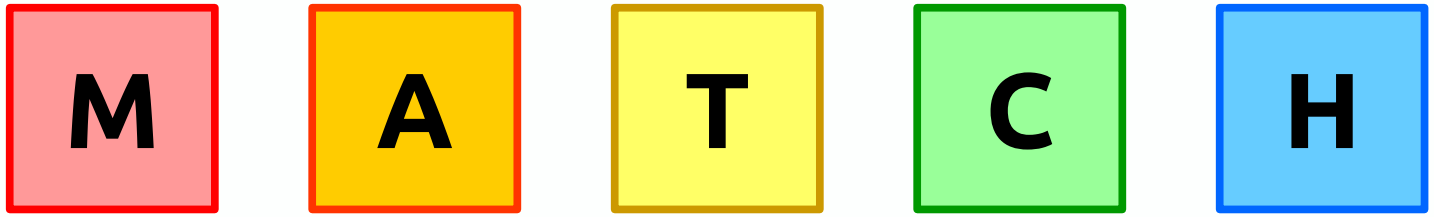
Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

Practice Problems

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



- Are repetitions allowed?
- Does order matter?

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
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3. Practice Problems

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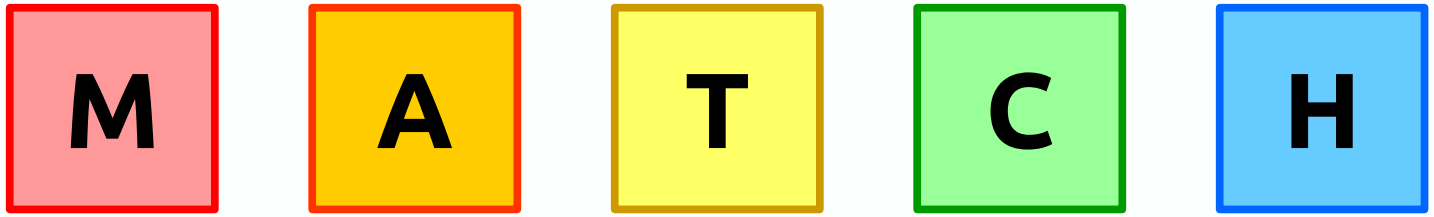

- Are repetitions allowed? **No**
- Does order matter? **Yes**

Notes

Combinatorics = counting problems

Ordered List
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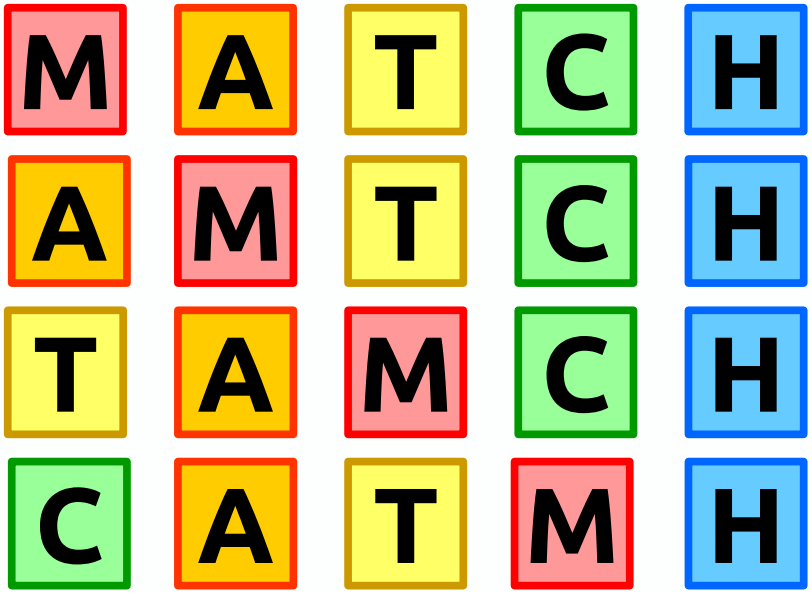
3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



And so on…

If there were few enough combinations, we could just try to write out all possibilities…
… but not here.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?

Instead, think of each position, and the options we have for each…

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
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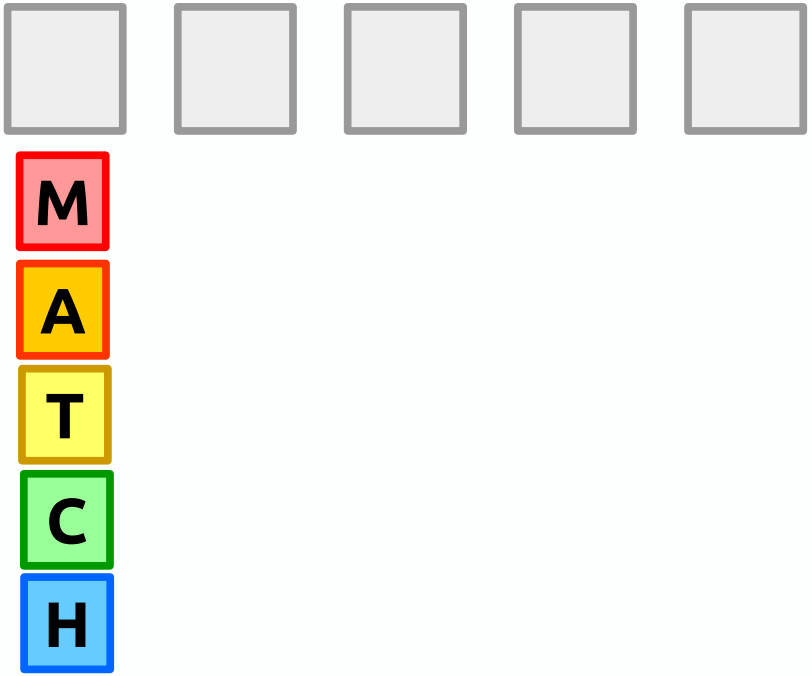
3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



For the first position, we have any of the 5 letters to choose from.

Notes

Combinatorics = counting problems

Ordered List
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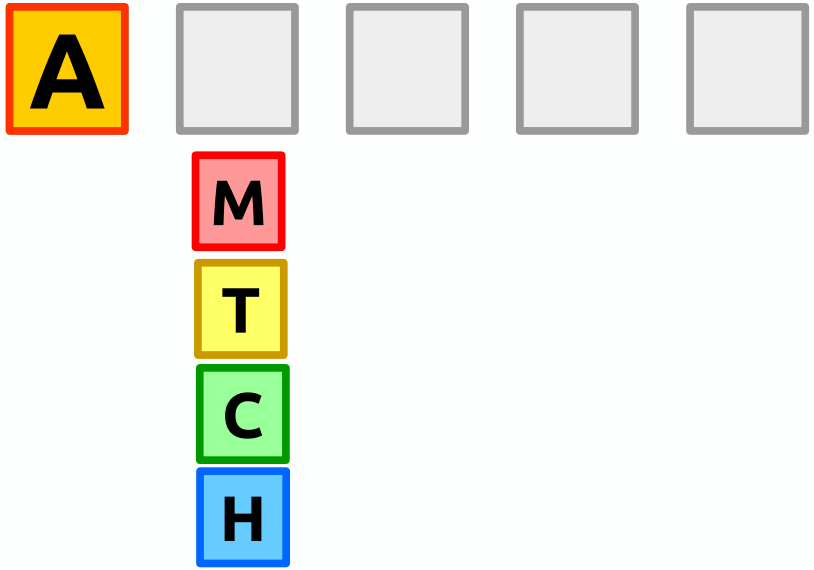
3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



Once we choose the first letter, we have 4 options to choose from for the 2nd position (because no repeats.)

Notes

Combinatorics = counting problems

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Unordered List
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Sets
- Repeats? no
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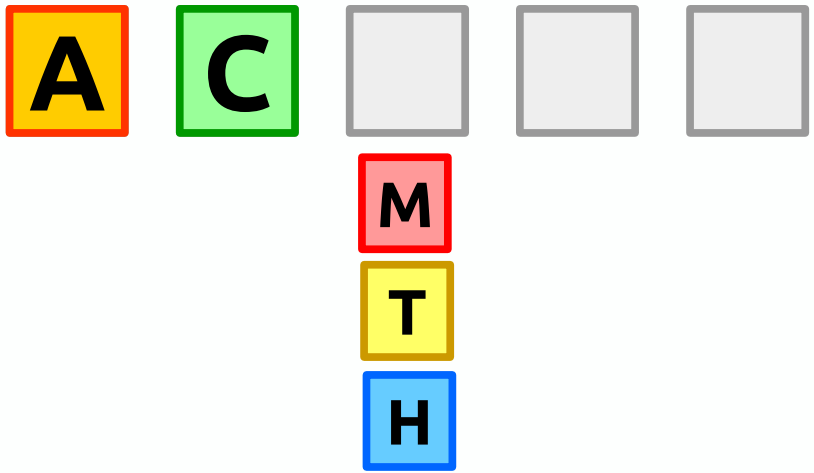
3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



Then 3 options for the 3rd position…

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
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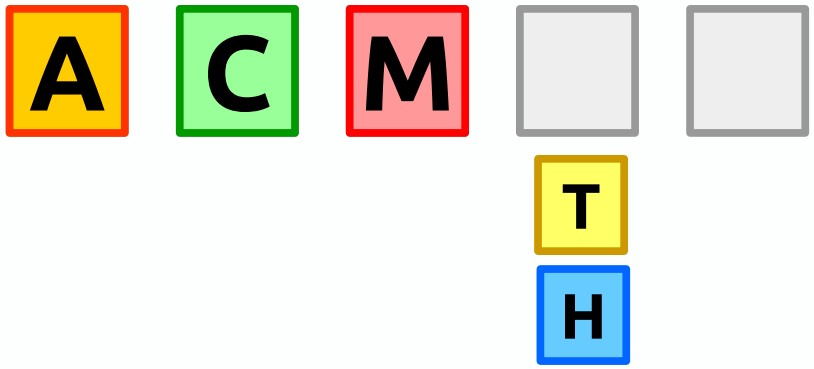
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



Then 3 options for the 3rd position…

2 options for the 4th position…

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

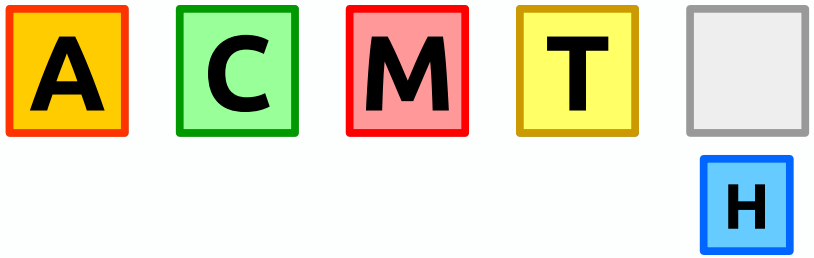
Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?



And only 1 option remaining for the 5^{th} position.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?

The letters I chose here are just arbitrary to illustrate a point.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
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Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?

5	4	3	2	1

We don’t really care *what* is selected, just how many options we have for each selection.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?

So the result here will be

5 x 4 x 3 x 2 x 1

… which can be written as **5!**
and is equal to **120**.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?

5! = 120 ways to arrange the letters.

What kind of structure is this?

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

1. How many arrangements are there of the letters in the word “MATCH”?

5! = 120 ways to arrange the letters.

What kind of structure is this?

Permutation

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?

- Are repetitions allowed?

- Does order matter?

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?

- Are repetitions allowed? **Yes**
  It wasn't specified that there's only one of each type.

- Does order matter? **Yes**
  One option is for you, one option is for your friend.

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
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Permutation
- Repeats? no
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Sets
- Repeats? no
- Order? no

3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?


1 2 **Mine**


3 4 **Friend's**

Repetitions are allowed, so how many options do you have

- for you?
- for your friend?

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?

4
Mine

4
Friend's

Repetitions are allowed, so how many options do you have

- for you? 4
- for your friend? 4

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
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3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?


1


2

4

Mine

So the amount of selections would be **4x4**, or **4^2**, **= 16**.


3


4

4

Friend's

Notes

Combinatorics = counting problems

Ordered List
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- Order? yes

Unordered List
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Sets
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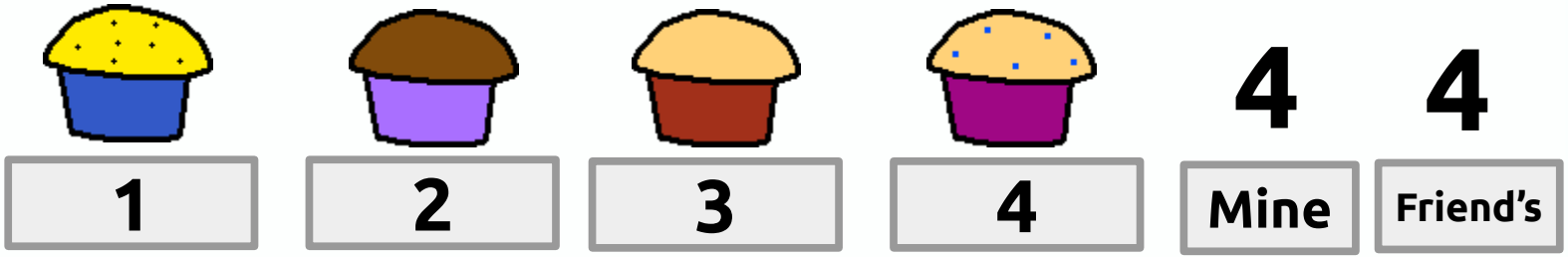
3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?



$4^2 = 16$ ways to choose.

What kind of structure is this?

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

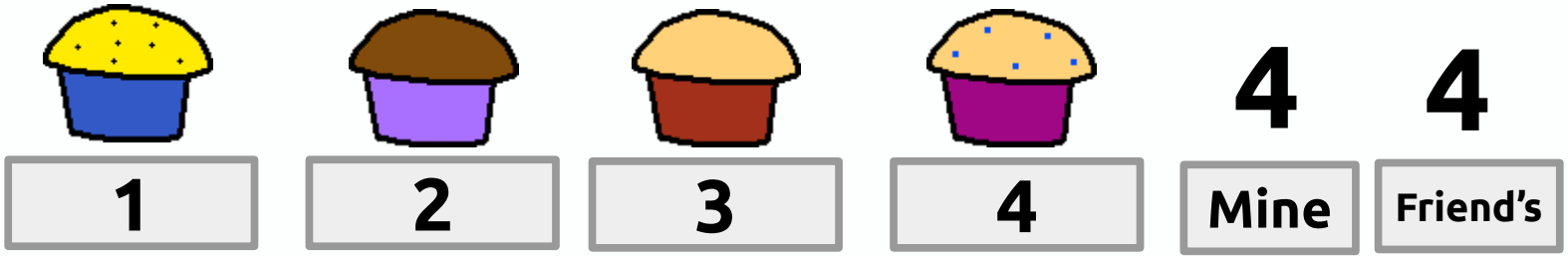
Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

3. Practice Problems

2. You're going to get a muffin for you and a muffin for your friend. You can choose from: Lemon, Chocolate, Cream Cheese, and Blueberry.
How many ways are there to choose?



$4^2 = 16$ ways to choose.

What kind of structure is this?

Ordered list

Notes

Combinatorics = counting problems

Ordered List
- Repeats? yes
- Order? yes

Unordered List
- Repeats? yes
- Order? no

Permutation
- Repeats? no
- Order? yes

Sets
- Repeats? no
- Order? no

Conclusion

This section of Chapter 5 is just meant to introduce you to these concepts. In the next chapters we will learn about the formulas associated with each structure and how to solve them.