7.1 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

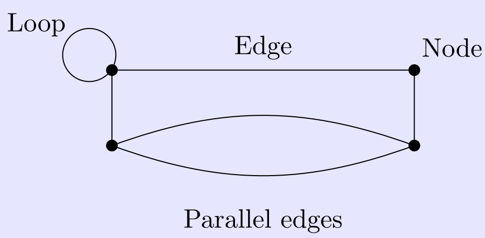
7.1 Graph Theory

7.1.1 Terminology

Since we're introducing a new concept, Graph Theory, we need to go over the various terms so that we can communicate about these graphs properly.

Graphs, nodes, and edges

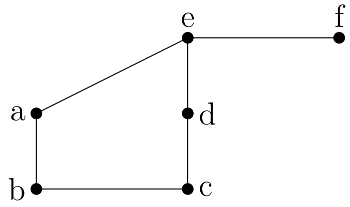
- **Graph:** A graph is a type of diagram that contains *vertices* (aka nodes) and *edges*.



- **Node:** A vertex of the graph, drawn as a dot.
  - **Adjacent nodes:** Two nodes that are connected by an edge.
  - **Node degree:** The amount of edges that are connected to a node. Loops are counted twice.
- **Edge:** A line that connects two nodes together.
  - **Parallel edges:** Two edges that have the same two endpoints.
  - **Loop:** An edge that begins and ends at the same node, creating a loop.
  - $[a, b]$ is used to indicate an edge with a and b as endpoints, though direction can be either way.

Question 1

Identify each item for the graph G given.



a. How many nodes (vertices) are there?

b. How many edges are there?

c. Write down the degree of each node:

Vertex v	$deg(v)$
a	
b	
c	
d	
e	
f	

d. The **maximum degree** of a graph is the highest $deg(v)$ value. What is this graph's maximum degree?

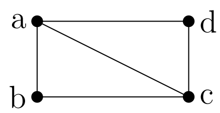
e. The **minimum degree** of a graph is the lowest $deg(v)$ value. What is this graph's minimum degree?

Types of walks

- **Walk:** A series of alternating nodes and edges, traversing between adjacent nodes.
  - **Closed walk:** When the beginning and ending node of a walk are the same.
  - **Length of a walk:** The amount of edges in the walk.
  - **Trivial walk:** A walk of length 0.
  - **Path:** A walk with no repeated vertices.
  - **Trail:** A walk with no repeated edges.
    - **Circuit:** A closed trail.
      - **Trivial circuit:** A circuit with one vertex and no edges.
      - **Cycle:** A nontrivial circuit where the only repeated node is the first/last one.
    - **Eulerian:** A trail or circuit where every edge is traversed.

Question 2

Answer the following questions, using the graph H given.



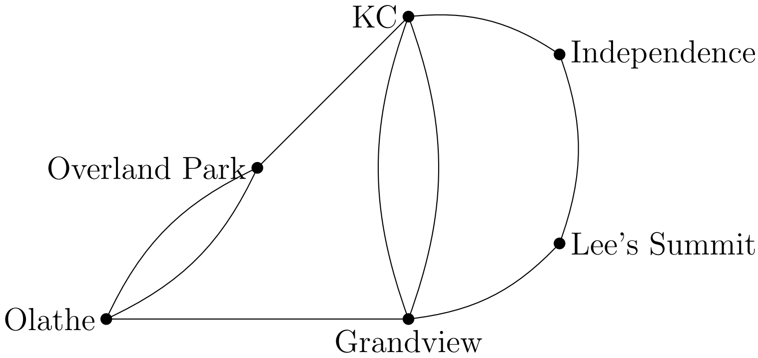
a. Come up with several walks from a to c. Write all steps (each node visited). Also label the **length** of each walk.

b. Come up with a **closed walk**, beginning and ending at a. You can choose to visit all nodes or not.

c. Come up with a **path**, where no vertices are repeated.

Question 3

Answer the following questions, using the graph I given.



a. Come up with a **trail**, a walk with no repeated edges.

b. Come up with a **circuit**, a closed trail.

c. Come up with a **cycle**, a circuit where the only repeated node is the first/last one..

d. Identify: Did you come up with any **Eulerian Trails**?
If not, create one.

e. Identify: Are there any **parallel edges**?

Simple vs. directed graphs

- **Simple graph:** A graph that has no loops or parallel edges.
- **Directed graph:** The edges in the graph are given a direction, which can only be traversed in that way.
  - Edges are denoted with parentheses (a, b), showing that it goes from a to b.

Question 4
Draw a **Directed Graph** using the following list of edges:

$$(1, 2), (2, 1), (3, 3), (4, 2)$$

(Don't confuse these for points on an x, y plane that are interconnected, each ordered pair is its own set of information - beginning and end nodes.)

Connected, subgraphs

- A graph is **connected** if there is a walk between any pair of distinct nodes.
- A graph H is a **subgraph** of a graph G if all nodes and edges in H are also nodes and edges in G.
- A connected component of a graph G is a connected subgraph H of G such that no other connected subgraph of G containing H exists.

[a]

[a] Discrete Mathematics, Ensley and Crawley

Question 5

Draw a graph that is **not connected**, and draw a **subgraph** of your graph.