

Please write down all people in your team.

1.

2.

3.

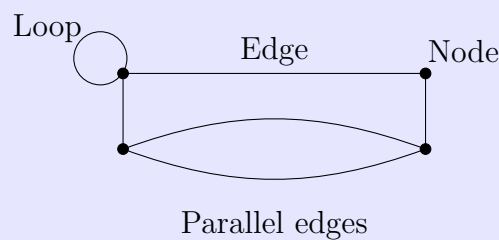
4.

7.1 Graph Theory

7.1.1 Terminology

Since we're introducing a new concept, Graph Theory, we need to go over the various terms so that we can communicate about these graphs properly.

- **Graph:** A graph is a type of diagram that contains *vertices* (aka nodes) and *edges*.

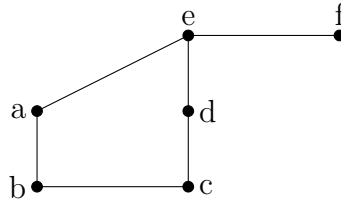


- **Node:** A vertex of the graph, drawn as a dot.
 - **Adjacent nodes:** Two nodes that are connected by an edge.
 - **Node degree:** The amount of edges that are connected to a node. Loops are counted twice.
- **Edge:** A line that connects two nodes together.
 - **Parallel edges:** Two edges that have the same two endpoints.
 - **Loop:** An edge that begins and ends at the same node, creating a loop.
 - $[a, b]$ is used to indicate an edge with a and b as endpoints, though direction can be either way.

Question 1

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Identify each item for the graph G given.



- a. How many nodes (vertices) are there?
- b. How many edges are there?

- c. Write down the degree of each node:

| Vertex v | $\deg(v)$ |
|------------|-----------|
| a | |
| b | |
| c | |
| d | |
| e | |
| f | |

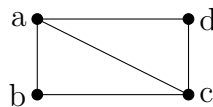
- d. The **maximum degree** of a graph is the highest $\deg(v)$ value. What is this graph's maximum degree?
- e. The **minimum degree** of a graph is the lowest $\deg(v)$ value. What is this graph's minimum degree?

- **Walk:** A series of alternating nodes and edges, traversing between adjacent nodes.
 - **Closed walk:** When the beginning and ending node of a walk are the same.
 - **Length of a walk:** The amount of edges in the walk.
 - **Trivial walk:** A walk of length 0.
 - **Path:** A walk with no repeated vertices.
 - **Trail:** A walk with no repeated edges.
 - * **Circuit:** A closed trail.
 - **Trivial circuit:** A circuit with one vertex and no edges.
 - **Cycle:** A nontrivial circuit where the only repeated node is the first/last one.
 - * **Eulerian:** A trail or circuit where every edge is traversed.

Question 2

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Answer the following questions, using the graph H given.

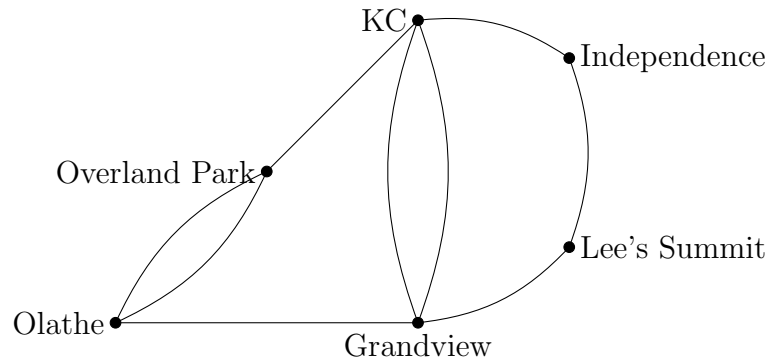


- a. Come up with several walks from a to c . Write all steps (each node visited). Also label the **length** of each walk.
- b. Come up with a **closed walk**, beginning and ending at a . You can choose to visit all nodes or not.
- c. Come up with a **path**, where no vertices are repeated.

Question 3

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Answer the following questions, using the graph I given.



- Come up with a **trail**, a walk with no repeated edges.
- Come up with a **circuit**, a closed trail.
- Come up with a **cycle**, a circuit where the only repeated node is the first/last one..
- Identify: Did you come up with any **Eulerian Trails**?
If not, create one.
- Identify: Are there any **parallel edges**?

- **Simple graph:** A graph that has no loops or parallel edges.
- **Directed graph:** The edges in the graph are given a direction, which can only be traversed in that way.
 - Edges are denoted with parentheses (a, b) , showing that it goes from a to b .

Question 4

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Draw a **Directed Graph** using the following list of edges:

$$(1, 2), (2, 1), (3, 3), (4, 2)$$

(Don't confuse these for points on an x, y plane that are interconnected, each ordered pair is its own set of information - beginning and end nodes.)

- A graph is **connected** if there is a walk between any pair of distinct nodes.
- A graph H is a **subgraph** of a graph G if all nodes and edges in H are also nodes and edges in G .
- A **connected component** of a graph G is a connected subgraph H of G such that no other connected subgraph of G containing H exists.

$$a$$

^aDiscrete Mathematics, Ensley and Crawley**Question 5**

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Draw a graph that is **not connected**, and draw a **subgraph** of your graph.