6.4 Expected value in games of chance

Definition
For a given probability experiment, let X be a random variable whose possible values come from the set of numbers $x_1, ..., x_n$. Then the **expected value of** X, denoted by $E[X]$, is the sum

$$(x_1) \cdot Prob(X = x_1) + (x_2) \cdot Prob(X = x_2) + ... + (x_n) \cdot Prob(X = x_n)$$

This is sometimes called the *average value* of the random variable, thinking of the average of the values X takes on over many repetitions of the experiment. [a]

Example 2 Suppose I have a "loaded" die for which the probability of a 6 appearing is $\frac{1}{2}$, while the probability of each of the other faces appearing is $\frac{1}{10}$. What is the expected value on one roll? Compare to the expected value of a fair die.

For the loaded die:

$$E[X] = (1)(\frac{1}{10}) + (2)(\frac{1}{10}) + (3)(\frac{1}{10}) + (4)(\frac{1}{10}) + (5)(\frac{1}{10}) + (6)(\frac{1}{2})$$

$$= \frac{1}{10}(15) + \frac{1}{2}(6) = 4.5$$

For the fair die:

$$E[X] = (1)(\frac{1}{6}) + (2)(\frac{1}{6}) + (3)(\frac{1}{6}) + (4)(\frac{1}{6}) + (5)(\frac{1}{6}) + (6)(\frac{1}{6})$$

$$= \frac{1}{6}(21) = 3.5$$

[a] From Discrete Math by Ensley and Crawley, page 460

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Suppose you pay \$2 each time to play the following game: Two dice are rolled, and you win \$5 for each 6 that comes up. Do you expect to win more than you pay if you play many, many times?

Let X represent the amount of money you win in one play of the game. So, you can win either \$0, \$5, or \$10, so the values are $\{0, 5, 10\}$.

What is $Prob(X = 0)$?

What is $Prob(X = 5)$?

What is $Prob(X = 10)$?

Then, $E[X] = 0 \cdot Prob(X = 0) + 5 \cdot Prob(X = 5) + 10 \cdot (X = 10)$