6.4 Expected value in games of chance

Question 1 ____ / 3

Suppose you pay \$2 each time to play the following game: Two dice are rolled, and you win \$5 for each 6 that comes up. Do you expect to win more than you pay if you play many, many times?

Let X represent the amount of money you win in one play of the game. So, you can win either \$0, \$5, or \$10, so the values are $\{0, 5, 10\}$.

What is $Prob(X = 0)$? (The probability of getting no 6's?) $\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$

What is $Prob(X = 5)$? (The probability of getting one 6?) $C(2,1)(\frac{1}{6}) \cdot \frac{5}{6} = \frac{10}{36}$

What is $Prob(X = 10)$? (The probability of getting two 6's?) $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Then, $E[X] = 0 \cdot Prob(X = 0) + 5 \cdot Prob(X = 5) + 10 \cdot (X = 10)$
$= 0 \cdot \frac{25}{36} + 5 \cdot \frac{10}{36} + 10 \cdot \frac{1}{36} \quad = \frac{60}{36} \quad \approx 1.67$

Question 2 ____ / 3

Suppose that the payoff for the game (in question 1) is \$5 for rolling one 6, and \$25 for rolling two 6's. Now is it worth \$2 to play? (Recalculate the expected value)

$E[X] = 0 \cdot Prob(X = 0) + 5 \cdot Prob(X = 5) + 25 \cdot Prob(X = 25)$

$= 0 \cdot \frac{25}{36} + 5 \cdot \frac{10}{36} + 25 \cdot \frac{1}{36}$

$= \frac{75}{36} \approx \$2.08$

You will win about 8 cents each time you play, over the long term.

6.4.1 Expectation in Bernoulli trials

Question 3 ____ / 3

If two teams, team Anteater and team Badger, play a best-of-three series, and if team Anteater has a (2/3) probability of winning any given game, then what is the average number of games in the series?

Since it is a best-of-three match, there can be either two games (if one team wins the first and second match), or three games (if one team wins the first, the other the second, and either win the third).

a. What are the two possible values of X? (The amount of games played in the series?) 2 or 3

b. Using "A" and "B" to symbolize which team won, draw all the possible outcomes for only 2 games. AA or BB

c. So what is the probability $Prob(X = 2)$? Use the sum rule to combine each outcome's probability. $= (2/3)^2 + (1/3)^2 = 5/9$

d. For the other case with 3 games. If the series doesn't have 2 games, then the only other option is $Prob(X = 3) = 1 - Prob(X = 2)$. What is $Prob(X = 3)$? $\frac{4}{9}$

e. Now calculate the value of $E[X]$ with the equation in Theorem 1. $2 \cdot Prob(X = 2) + 3 \cdot Prob(X = 3)$
$= 2(5/9) + 3(4/9) \quad = (22/9) \approx 2.44$ games