7.4 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

7.4 Connections to Matrices and Relations

7.4.1 Adjacency matrix

Adjacency matrix

Given a graph G with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E, we define the **adjacency matrix** of G as follows. The matrix M is an $n \times n$ array of natural numbers, which we imagine having rows and columns labelled as follows:

$$\begin{array}{rc} & \text{Columns } 1, 2, ..., n \\ \begin{array}{r} \text{Row } 1 \rightarrow \\ \text{Row } 2 \rightarrow \\ \text{Row } 3 \rightarrow \\ \text{Row } ... \\ \text{Row } n \rightarrow \end{array} & \left[\begin{array}{c} \qquad\qquad\qquad\qquad \\ \\ \\ \\ \\ \end{array}\right] \end{array}$$

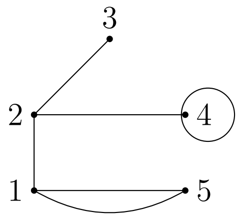
The entry in row i, column j (referred to as the (i, j) - entry of M or, more concisely, M_{ij}) is defined as

$$M_{ij} = \text{the number of edges connecting } v_i \text{ and } v_j \text{ in } G.$$ [a]

[a] Discrete Mathematics, by Ensley and Crawley

Question 1

Fill out the adjacency matrix for the following graph.



$$\begin{array}{cc}
 & \text{Columns} \\
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{Rows} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \quad & \quad & \quad & \quad & \quad \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}
\end{array}$$

7.4.2 Directed graphs

Directed graphs

1. A **directed graph**, like a graph, consists of a set V of vertices and a set E of edges. Each edge is associated with an ordered pair of vertices called its **endpoints**. In other words, a directed graph is the same as a graph, but the edges are described as *ordered pairs* rather than unordered pairs;

2. If the endpoints for edge e are a and b in that order, we say e is an edge **from** a **to** b, and in the diagram we draw the edge as a straight or curved arrow from a to b.

3. For a directed graph, we use (a, b) rather than $[a, b]$ to indicate an edge from a to b. This emphasizes that the edge is an **ordered pair**, by utilizing the usual notation for ordered pairs.

4. A **walk** in a directed graph in a sequence $v_1e_1v_2e_2...v_ne_nv_{n+1}$ of alternating vertices and edges that begins and ends with a vertex, and where each edge in the list between its endpoints in the proper order. (That is, e_1 is an edge from v_1 to v_2, e_2 is an edge from v_2 to v_3, and so on.) If there is no chance of confusion, we omit the edges when we describe a walk.

5. The **adjacency matrix** for a directed graph with vertices $\{v_1, v_2, ..., v_n\}$ is the $n \times n$ matrix where M_{ij} (the entry in row i, column j) is the number of edges from vertex v_i to vertex v_j.

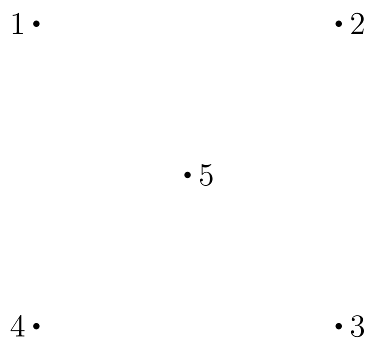
[a]

[a] Discrete Mathematics, Ensley and Crawley

Question 2

Draw a graph that corresponds to the adjacency matrix. This is a directed graph, so the matrix is not symmetric. It should be read as row $i \to$ column j. For example, row 1 shows $1 \to 2$, $1 \to 4$, and $1 \to 5$.

$$\begin{array}{cc} & \text{Columns} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Rows} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



Question 3

Draw a directed graph with vertices $V = \{1, 2, 3, 4, 5\}$ and edges $E = \{(1,4), (1,5), (2,1), (3,4), (4,3), (5,2)\}$.