Chapter 6 Review

Formulas, definitions, and theorems

Experiment: An experiment can be anything that can have multiple outcomes. Usually in this chapter it is rolling a die, drawing a card, or flipping a coin.

Sample space S: The sample space S is a **set** of all possible *equally-likely* outcomes of the experiment.

For example, the sample space S of rolling a die is $\{1, 2, 3, 4, 5, 6\}$, and the length $n(S)$ is 6.

Event: When performing an experiment, we will ask how likely some *event* is to occur. If the event occurs, we will call this a *success*.

For example, if our event E is rolling a die and getting an even number, then E is $\{2, 4, 6\}$. The length of this, $n(E)$, is 3.

Probability of an event E taking place: The probability that E occurs is written as $Prob(E)$, and can be calculated as $\frac{n(E)}{n(S)}$.

For example, the probability of rolling a die and getting an even number is $\frac{3}{6}$, or $\frac{1}{2}$.

The complement $\bar{E}$ of an event E: For an experiment, an event E can either happen or not happen. The sum of these two outcomes is 1. In other words... $Prob(E) + Prob(\bar{E}) = 1$.

For example, if E is getting a 6 when rolling a die, then $Prob(E)$ is $\frac{1}{6}$. The likelihood of getting anything *besides a 6* when rolling a die is $Prob(\bar{E}) = 1 - Prob(E)$, or $\frac{5}{6}$.

Disjoint events: Two events are disjoint if they cannot occur simultaneously.

For example, you cannot roll one die and get *both* a 1 and a 6.

Independent events: Two events are independent if the occurrence of one does not affect the probability of the other.

For example, if we roll a die twice, the first roll doesn't affect the second roll.

As another example, if we draw a card from a deck without replacement, this affects the second draw, because the size of the deck has been reduced by one.

The General Sum Rule: In an experiment, if we are trying to find the probability that *either* E_1 OR E_2 occurs, we can find this by *adding* together $Prob(E_1)$ and $Prob(E_2)$, as well as subtracting any overlap, $Prob(E_1 \; AND \; E_2)$.

$$Prob(E_1 \; OR \; E_2) = Prob(E_1) + Prob(E_2) - Prob(E_1 \; AND \; E_2)$$

If E_1 and E_2 are **disjoint**, then this will be:

$$Prob(E_1 \; OR \; E_2) = Prob(E_1) + Prob(E_2)$$

The General Product Rule: In an experiment, if we are trying to find the probability that *both* E_1 and E_2 occurs, we can solve this with

$$Prob(E_1 \; AND \; E_2) = Prob(E_2) \cdot Prob(E_1 | E_2)$$

If E_1 and E_2 are **independent**, then this will be:

$$Prob(E_1 \; AND \; E_2) = Prob(E_1) \cdot Prob(E_2)$$

Bernoulli trial: In a Bernoulli trial, we have some experiment where we repeat some experiment n times. The success of the event we're checking in each run of this experiment has a probability p of success, and we are checking for exactly k successes to occur.

We can calculate the probability of k successes occuring over n runs of the experiment with:

$$C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$$

Expected (average) value: Let's say for an experiment, a variable X will receive some value randomly from the set $\{x_1, ..., x_n\}$. We can write the expected value (aka average value) as $E[X]$, and we can calculate this value with the sum...

$$E[X] = (x_1)Prob(X = x_1) + ... + (x_n)Prob(X = x_n)$$

Expected value for a Bernoulli trial: If we're trying to find the expected value for a Bernoulli trial, we can compute it with the simpler formula:

$$E[X] = np$$

Where X is the amount of successful trials in the experiment.

Amount of trials until first value: If we want to run some trial continuously until the first time we receive some value (such as, flip a coin until we get the first HEADS value), we can calculate this.

First, X will be the amount of trials done (rolls, flips, etc.) until the first value is received, and $E[X]$ is the average amount of trials that get run. p is the probability of success (getting the specified value). Then:

$$E[X] = p(1) + (1 - p)(1 + E[X])$$

After plugging in values, you can solve for $E[X]$ algebraically.

Matrix multiplication:

Types of problems

Question 1 (6.1)
Experiment: Drawing a card from a deck of 52 cards.

a. What is the probability that the card is an Ace?

b. What is the probability that the card is an Ace or a Queen?

c. What is the probability that the card is Red or is a Jack?

Question 2 (6.1)
List out the sample space set S for each of the following, and give the size of the sample space $n(S)$.

a. Rolling a die.

b. Flipping two coins.

Question 3 (6.1)
We have a box that contains 12 kittens and 4 puppies. If we're selecting 5 pets, find the probability that...

a. All 5 pets are kittens.

b. At most 1 pet is a puppy.

Question 4 (6.1)
Experiment: Drawing two cards from a deck of 52 cards. What is the probability of getting two cards of the same value (and different suits)?

a. If we have 52 cards and we're selecting 2, what structure type is this?

b. For all items in the sample space S, what is the size $n(S)$ of the sample space?

c. How many possibilities are there for selecting the first card?

d. The second card is more restricted - it has to have the same value as the first card. How many possibilities are there for a second card that has the same value, but a different suit?

e. What is the amount of events $n(E)$ in the experiment?

f. What is the probability of a successful event E?

Answer key

Question 1 (6.1)

a. 4 aces in a deck of 52, so $\frac{4}{52} = \frac{1}{13}$

b. 4 Aces in the deck, and 4 Queens in the deck, so $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

c. Half the deck is red cards, and there are 4 jacks. There is an overlap here of the 2 red jacks, so: $\frac{26}{52} + \frac{4}{52} - \frac{2}{52}$

Question 2 (6.1)

a. $S = \{1, 2, 3, 4, 5, 6\}$; $n(S) = 6$

b. $S = \{HH, HT, TH, TT\}$; $n(S) = 4$

Question 3 (6.1)

a. 12 kittens, select 5 / 16 pets, select 5 $= \frac{C(12,5)}{C(16,5)} = \frac{33}{182}$

b. Can have all kittens, or 4 kittens and 1 puppy.
5 kittens: $C(12, 5)$ 4 kittens, 1 puppy: $C(12, 4) \cdot C(4, 1)$
$= \frac{C(12,5)+C(12,4)\cdot C(4,1)}{C(16,5)}$

Question 4 (6.1)

a. Permutation, P(52,2)

b. $n(S) = P(52, 2) = 52 \cdot 51$

c. 52

d. 3

e. $n(E) = 52 \cdot 3 = 156$

f. $Prob(E) = \frac{n(E)}{n(S)} = \frac{52\cdot 3}{52\cdot 51} = \frac{1}{17}$