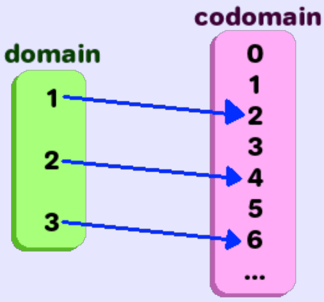
1. Review

CS 210 - Functions

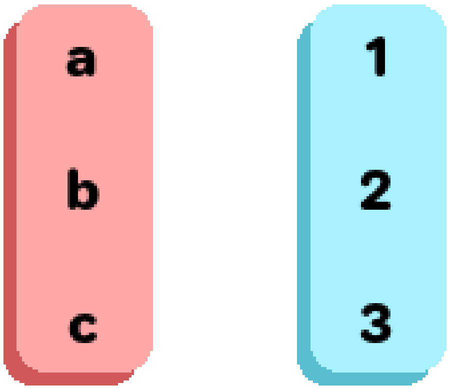
$f : A \to B$ specifies a function named f, with some set of inputs (A, the domain), and a set of outputs (B, the codomain). The function f associates each input A with one and only one output from B.



A function f with domain $\{1, 2, 3\}$, a codomain $\mathbb{N}$, and the rule, $2k$.

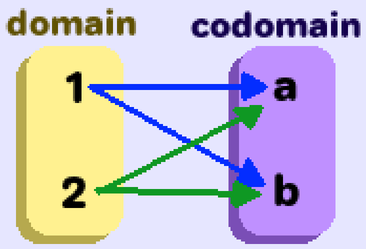
Question 1 _____ / 5%

Let f be the function with the domain $\{a, b, c\}$ and codomain $\{1, 2, 3\}$ defined by the set of ordered pairs: $\{(a, 2), (b, 3), (c, 1)\}$. Complete the arrow diagram.



CS 210 - Binary Relations

A binary relation R has a domain A, a codomain B, and its rule is the subset of $A \times B$ (the Cartesian* product of A and B).



Domain: $A = \{1, 2\}$, Codomain: $B = \{2, 3\}$,
Rule: $L = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$.

Cartesian product: a Cartesian product is a mathematical operation that returns a set (...) from multiple sets. That is, for sets A and B, the Cartesian product $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. [a]

So if we took the Cartesian product from A and B above, the result would be L.

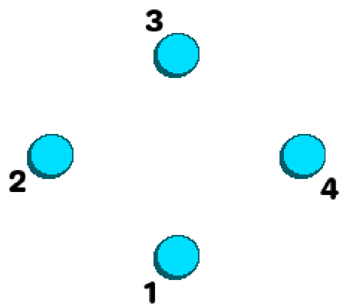
[a] From https://en.wikipedia.org/wiki/Cartesian_product

Question 2 ______ / 6%

Finish the diagram for the relation R_1 on the set $\{1, 2, 3, 4\}$ with the rule $(x, y) \in R_1$ if $x \leq y$

Hints:

- If a relationship exists between some point (x, y), it means that the value of x is $\leq y$. This will be represented graphically as an arrow starting at x and pointing to y.
- For node (1), it is *less than* (2), (3), and (4), and it is equal to (1), so it will have arrows pointing to all of these.
- For node (4), it is only $\leq$ itself, so it will only have an arrow looping back to itself.



2. Subsets

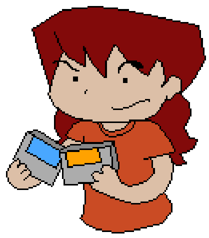
Permutations: A **permutation** is an ordered list where elements cannot repeat. For example, if we have a set of numbers {1, 2, 3}, then we could have permutations (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), and so on.

Permutations of length r, pulling elements from $\{1, 2, ..., n\}$, can be written as $P(n, r)$.

$P(n, n)$ can be calculated as $n!$ (n-factorial). If we are not selecting all items from $\{1, 2, ..., n\}$, then we can compute the amount of results as $P(n, r) = \frac{n!}{(n-k)!}$.

Question 3 ______ / 12%

Rose wants to buy some game cartridges at a garage sale but only brought enough cash for two of the four games they have: LM: Legacy of Magic, TC: Temporal Catalyst, CC: Concluding Chronicle, and DT: Dream of Terra



a. What would n be? b. What would r be?
c. What is the result of $P(n, r)$?

Subsets: A **subset** is a permutation, except that we *do not* care about order. So, if we have (2, 3) and (3, 2) as different permutations, as two subsets, {2, 3} and {3, 2} are considered the same.

Question 4 ______ / 15%

Going back to Question 3, answer the following questions:

a. List all the choices that are equivalent to each other (example, choosing "Legacy of Magic" and "Temporal Catalyst", vs. choosing "Temporal Catalyst" and "Legacy of Magic".

b. How many equivalent choices are there?

c. So if Rose still selects two games from the four possible ones, but the order she selects the games doesn't matter, how many different outcomes are there?

3. Equivalence Relations

Equivalence Relations & Classes: [a]
An **Equivalence Relation**
$(X, Y) \in R$ if permutations X and Y use the same objects. R is the equivalence relation, "use the same objects."

An **Equivalence Class** is the set of objects (in this case, permutations) that are equivalent to one another. For example, $(Apple, Orange)$ and $(Orange, Apple) \in R$. So, these two permutation smake up an equivalence class.

Example 1: Given a set of 7 items, {a, s, d, f, j, k, l}, if we were to choose 3 items from this set, how many permutations would there be?

$P(n, r) => P(7, 3) => \frac{7!}{(7-3)!} = \frac{5\times4\times3\times2\times1}{4\times3\times2\times1} = \frac{5040}{24}$
$= 210$ total permutations

If we were thinking about it in terms of "buckets" visually, it would look like this:

______	______	______
7 options	6 options	5 options

$$= 7 \times 6 \times 5 = \mathbf{210}$$

Example 2: For the permutation, **asd**, what are all the elements of the equivalence relation?

These are all the subsets using **asd** that would be equivalent. There are a total of **6** items in the equivalence class, if we are looking at a permutation of length 3.
{asd, ads, sad, sda, dsa, das}

Example 3: How many subsets of the {a, s, d, f, j, k, l} set have 3 elements?

For this, we divide the amount of permutations of length 3 with the amount of items in an equivalence class of length 3. So: $\frac{210}{6} = 35$.

[a] From Jim Van Horn's notes

Question 5 _____ / 25%

Given a set {2000, 2004, 2008, 2012, 2016}...

a. How many permutations of length 3 are there?

b. For the length-3 permutation of **2000, 2004, 2008**, list out all the permutations in the equivalence class that are related (i.e., "permutationA and permutationB would be the same if they were subsets".

c. How many permutations are there in any given equivalence class of length 3?

d. How many subsets of length 3 are there for the set {2000, 2004, 2008, 2012, 2016}?

e. If $n = 5$, the length of our set, and $r = 3$, the length of the permutations, what is the result of the following?

$$C(n,r) = \frac{n!}{(n-r)! \cdot r!}$$

4. Combinations

Combinations In mathematics, a combination is selection of items from a collection, such that (unlike permutations) the order of selection does not matter. (...) The number of r-combinations from a given set of n elements is often denoted in elementary combinatorics texts by $C(n,r)$. [a]

For a combination of length r from a set of n elements is equal to the binomial coefficient,

$$\binom{n}{r} = \frac{n(n-1)...(n-r+1)}{r(r-1)...1}$$

which can be simplified to:

$$C(n,r) = \frac{n!}{(n-r)! \cdot r!}$$

(Note that the book uses r and Wikipedia uses k.)

[a] From https://en.wikipedia.org/wiki/Combinations

Question 6 ____ / 25%

There are 4! possible ways for four students to be seated around a table. But, what if we only care about relative position and not the specific chair. That is, who you are seated next to - the students on your left and right. Assuming there is Alice, Bob, Chen, and David (Going clockwise around the table starting with Alice).

	A	
B		D
	C	

If they were to each move one seat clockwise the arrangement would be: David, Alice, Bob, and Chen. This is equivalent to the original arrangement for the relation of "same person on my left and right". [1]

a. How many possible seating arrangements are there?

b. How many times can they move one seat clockwise before they are back to their original positions?

c. We know that groups of 4 permutations are equivalent when only the relative position is considered and not the specific seat. So, if the equivalence relation is "same person on my left and right", how many permutations are in each equivalence class? List them all out.

d. How many seating arrangements are possible if we are only concerned about how is on our left and who is on our right?

e. Instead of individual students, assume that at each table of 4 people, there are two pairs working together against the other team at their table. For a single team-pair, how many pair arrangements could there be? List them out.

5. Wind-down

Question 6 ____ / 12%

Solve the following.

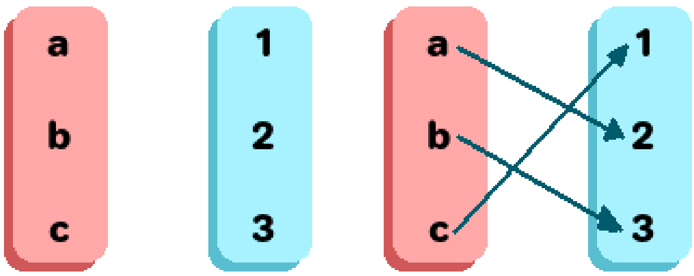
a. $P(6,2)$	b. $C(6,2)$	c. $C(6,4)$
d. $P(5,5)$	e. $C(5,5)$	f. $C(7,0)$

[1]From Jim Van Horn's POGIL exercises

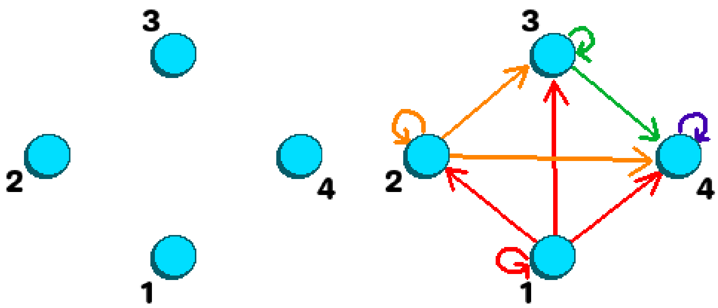
Chapter 5.3 In-class Exercise Worksheet

ANSWER KEY

5% Question 1: Function diagram □ 0 □ 1 □ 2 □ 3 □ 4



6% Question 2: Binary relation diagram □ 0 □ 1 □ 2 □ 3 □ 4



12% Question 3: Garage sale games (part 1) □ 0 □ 1 □ 2 □ 3 □ 4

a. $n =$ 4

b. $r =$ 2

c. $P(n,r) =$ $P(4,2) = \frac{4!}{(4-2)!} = \frac{(4 \cdot 3 \cdot 2 \cdot 1)}{(2 \cdot 1)} = 12$

15% Question 4: Garage sale games (part 2) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

a. ({ LM, TC } { TC, LM }), ({ LM, CC }, { CC, LM }), ({ LM, DT }, { DT, LM })
({ TC, CC } { CC, TC }), ({ TC, DT }, { DT, TC }),
({ CC, DT }, { DT, CC })

b. 6

c. 6

25% Question 5: 2000 - 2016 ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

a. $\frac{5!}{(5-3)!} = \frac{120}{2} = 60$

b. 2000-2004-2008 2000-2008-2004 2004-2000-2008
2004-2008-2000 2008-2000-2004 2008-2004-2000

c. 6

d. $\frac{60}{6} = 10$

e. $C(5,3) = \frac{5!}{(2!)(3!)} = 10$

25% Question 6: ABCD Seating ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

a. $4! = 24$

b. 4

c. 4: ABCD, BCDA, CDAB, DABC

d. $\frac{24}{4} = 6$

e. 6: AB, AC, AD, BC, BD, CD

12% Question 7: Wind-down ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

a. $P(6,2) = \frac{6!}{4!} = \frac{720}{24} = 30$

b. $C(6,2) = \frac{6!}{4! \cdot 2!} = \frac{720}{(24 \cdot 2)} = 15$

c. $C(6,4) = \frac{6!}{2! \cdot 4!} = \frac{720}{(2 \cdot 24)} = 15$

d. $P(5,5) = \frac{5!}{5!} = \frac{120}{120} = 1$

e. $C(5,5) = \frac{5!}{(0! \cdot 5!)} = \frac{5!}{5!} = 1$

f. $C(7,0) = \frac{7!}{(7! \cdot 0!)} = \frac{7!}{7!} = 1$

Team: Please write down all people in your team.

1.
2.
3.
4.

Grading

Question	Weight	0-4	Adjusted score
1	5%		
2	6%		
3	12%		
4	15%		
5	25%		
6	25%		
7	12%		