6.3 Probability in games of chance

Theorem 1
Given a simple experiment, called a **Bernoulli trial**, and an event that occurs with a probability p, if the trial is repeated independently n times, then the probability of having exactly k successes is

$$C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$$

[a]

Example 1 What is the probability that in 10 successive rolls of a fair, six-sided die, we get exactly five results of 6?

Here, we have $n = 10$, $k = 5$, and $p = \frac{1}{6}$, so:

$$C(10, 5) \cdot (\frac{1}{6})^5 \cdot (1 - \frac{1}{6})^{10-5}$$

$$\frac{10!}{5!(10-5)!} \cdot (\frac{1}{6})^5 \cdot (\frac{5}{6})^5$$

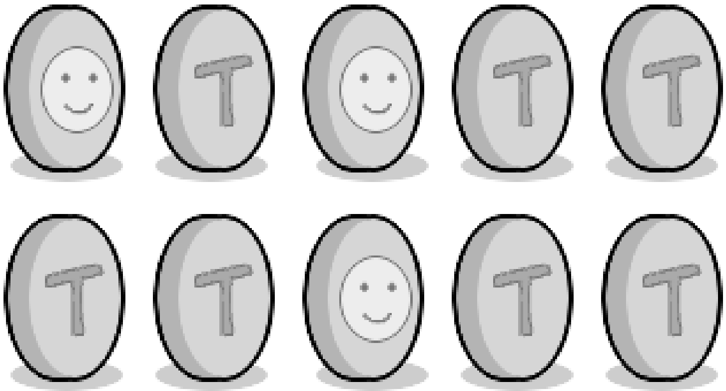
$$\frac{3628800}{14400} \cdot \frac{1}{7776} \cdot \frac{3125}{7776}$$

$$\approx 0.013$$

[a] From Discrete Math by Ensley and Crawley, page 460

Question 1 ____ / 3

What is the probability of getting exactly 3 heads on 10 tosses of a fair coin?



n, the amount of trial repeats: 10

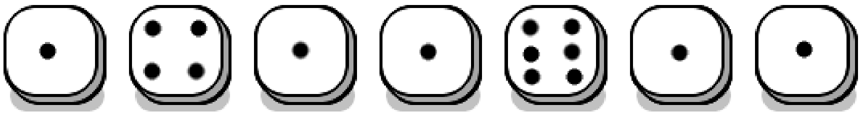
k, the amount of successes (heads): 3

p, the probability of success: (1/2)

Use the formula of $C(n,k) \cdot p^k \cdot (1-p)^{n-k}$ to find the probability.
$C(10,3) \cdot (1/2)^3 \cdot (1/2)^7 = \frac{15}{128}$

Question 2 ____ / 3

What is the probability that in seven rolls of a six-sided die, the result of 1 appears *at least* five times?



Hint

For this one, we will need to use the **rule of sums** to combine several outcomes: Getting 5 1's, 6 1's, OR 7 1's.

		repeats n	successes k	probability p
A	Getting five 1's	7	5	1/6
B	Getting six 1's	7	6	1/6
C	Getting seven 1's	7	7	1/6

Now, using the formula $C(n,k) \cdot p^k \cdot (1-p)^{n-k}$ three different times for case (A), (B), and (C).

(A) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} = C(7,5) \cdot (1/6)^5 \cdot (5/6)^2$

(B) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} = C(7,6) \cdot (1/6)^6 \cdot (5/6)^1$

(C) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} = C(7,7) \cdot (1/6)^7 \cdot (5/6)^0$

To find the probability of getting at least five 1's in seven rolls, add (A), (B), and (C) together. (Just write out the formula; don't solve.)

$Prob($ at least five 1's $) = C(7,5) \cdot (1/6)^5 \cdot (5/6)^2 + C(7,6) \cdot (1/6)^6 \cdot (5/6)^1 + C(7,7) \cdot (1/6)^7 \cdot (5/6)^0$

Question 3 ____ / 3

What is the probability of getting exactly one 6 on 10 tosses of a fair six-sided die?

$n = 10$, $k = 1$, $p = (1/6)$

$C(10, 1) \cdot (1/6)^1 \cdot (5/6)^9 \approx 0.323$