Topological and Algorithmic Analysis of 1D Wrap-Board Gomoku: A Comprehensive Study of Linear Topology in Abstract Strategy Games

Research Report

1 Introduction: The Evolution of Gomoku and Topological Variants

The game of Gomoku, traditionally known as "Five-in-a-Row," stands as one of the oldest and most mathematically pure abstract strategy games in existence. Originating from ancient China and finding formal codification in Japan, Gomoku is classically played on a planar grid—typically 15×15 or 19×19—using black and white stones.[1] The objective is conceptually simple yet strategically deep: a player must align five stones of their own color continuously in a horizontal, vertical, or diagonal direction.

However, the "Wrap Board" (or 1D Board) variant presented for analysis introduces a profound topological distortion that fundamentally alters the nature of the game. Unlike the standard Euclidean plane where board edges act as hard boundaries terminating a line of play, the Wrap Board redefines the playing field as a single, continuous linear vector that is merely rasterized onto a 2D grid for visual convenience. This transformation effectively converts the game from a test of planar spatial reasoning into a complex exercise in modular arithmetic and linear sequencing.

This report provides an exhaustive, expert-level analysis of the 1D Wrap-Board Gomoku variant. We will deconstruct the mathematical rules governing its operation, analyze the specific game states provided to deduce the precise mechanics of "wrapping," explore the algorithmic implications for artificial intelligence engines, and contrast this variant with other established forms such as Renju, Pente, and Hex-Gomoku. Our analysis is grounded in the provided research materials, extending from the basic rules of classical Gomoku [2] to the technical implementations of board representations in computer science.[3]

1.1 The Theoretical Basis of Board Representation

To understand the Wrap Board, one must first understand how digital systems perceive board games. In computer science, specifically in the development of game engines, a 2D board is rarely stored as a 2D data structure in memory. Instead, it is flattened into a 1D array to optimize memory access and cache coherence.[3]

In a standard Gomoku engine, a 15×15 board might be stored as an array of length 225. To simulate the 2D geometry and prevent a stone on the right edge of row N from connecting to a stone on the left edge of row $N + 1$, developers employ "mailbox" representations or sentinel values (padding).[3] These "guards" are mathematical walls inserted into the linear array to enforce the illusion of a square board.

The Wrap Board variant is unique because it removes these guards. It embraces the raw, underlying linear topology of the memory structure as the playable surface. The "glitch" of a horizontal line wrapping to the next row is elevated to a central game mechanic. This report argues that the Wrap Board is not merely a variant but a deconstruction of the game's geometry, stripping away the artificial constraints of the Cartesian plane to reveal the pure arithmetic progressions underneath.

1.2 Disambiguation of Terminology

It is critical to distinguish the game-theoretic concept of a "Wrap Board" from unrelated hardware terminology found in the literature. Historical computing archives reference "Wire Wrap boards" (e.g., the UDS-300 or ACE-100-07) used for prototyping circuitry in the 1980s.[4, 5, 6, 7, 8] These physical boards allowed engineers to manually wire connections between components. While unrelated to the rules of Gomoku, the metaphor is surprisingly apt: just as a wire wrap board allows a circuit to travel from one pin to any other via a continuous conductive wire, the 1D Gomoku board allows lines of attack to travel continuously through the "edges" of the visual grid, connecting seemingly disparate coordinates into a unified logical circuit. For the remainder of this report, "Wrap Board" will refer exclusively to the topological game variant.

2 Mathematical Formalism of the 1D Wrap Board

The analysis of the user-provided game states reveals a consistent set of rules governing the transformation from 2D visual coordinates to 1D logical indices. We must formalize these rules to understand the victory conditions.

2.1 Coordinate Systems and Transformation

Let the board B be defined as a set of N addressable positions. In the user's examples, the visual representation is a grid of width $W = 10$ and height $H = 10$, yielding a total population $N = 100$.

We define two coordinate systems:

1. **Visual Space (V):** The set of ordered pairs (r, c) where $1 \leq r \leq H$ and $1 \leq c \leq W$. This is how the human player perceives the board.
2. **Logical Space (L):** The set of integers i where $0 \leq i < N$. This is how the game logic (and the "Wrap" rule) processes adjacency.

The mapping function $\mathcal{F} : V \to L$ is defined as:
The inverse mapping $\mathcal{F}^{-1} : L \to V$ is:

2.2 Definition of Adjacency and Stride

In standard Gomoku, adjacency is defined by Euclidean distance on the grid. A cell (r, c) is adjacent to (r', c') if $\max(|r - r'|, |c - c'|) = 1$. This yields 8 neighbors for a central cell, 5 for an edge, and 3 for a corner.

In 1D Wrap Gomoku, adjacency is defined by **Arithmetic Stride**. Two stones are considered "connected" in a specific dimension if their indices differ by a constant integer S. A winning line is a sequence of indices $\{a_0, a_1, a_2, a_3, a_4\}$ such that: for a fixed Stride S.

Based on the visual evidence of "Diagonal" and "Vertical" lines in the provided diagrams, we deduce the valid Strides for a board of width $W = 10$:

Direction	**Standard 2D Vector**	**1D Stride Calculation**	**Stride Value ($W = 10$)**
Horizontal	$(0, 1)$	$+1$	**1**
Vertical	$(1, 0)$	$+W$	**10**
Major Diagonal	$(1, 1)$	$+W + 1$	**11**
Minor Diagonal	$(1, -1)$	$+W - 1$	**9**

This table represents the "Rosetta Stone" for translating the user's diagrams into formal logic. A "Vertical" line is simply an arithmetic progression with difference 10. A "Diagonal" line is an arithmetic progression with difference 9 or 11.

2.3 Topological Consequences of Modulo Arithmetic

The defining characteristic of the Wrap Board is the behavior of these Strides at the boundaries of the grid.

The Horizontal Wrap (Stride 1): In standard Gomoku, the sequence of indices $8, 9, 10, 11$ (on a 10×10 board) corresponds to $(1, 9), (1, 10), (2, 1), (2, 2)$. The "wrap" occurs between index 9 and 10.

- *Visual Effect:* The line exits the right edge of Row 1 and immediately re-enters on the left edge of Row 2.
- *Topological Insight:* This confirms the board is a **Cylinder** (or helix) constructed by joining the right edge of row k to the left edge of row $k + 1$.

The Diagonal Wrap (Stride 9 - Minor Diagonal): Consider the sequence $22, 31, 40, 49, 58$ from the user's example.

- $40 \to 49$: Index 40 is $(5, 1)$. Index 49 is $(5, 10)$.
- *Visual Effect:* This is the most counter-intuitive movement. A step of $+9$ from the left edge of a row lands on the right edge of the *same* row.
- *Correction:* Wait, mathematically $40 + 9 = 49$. Visually, $(5, 1)$ and $(5, 10)$ are on the same horizontal row. But the stride represents a "diagonal." How can a diagonal move stay on the same row?
  - In the standard 2D view, $(5, 1)$ to $(5, 10)$ is a horizontal jump of 9 squares.

- However, in the logic of the game, this is a valid “step” in the “Minor Diagonal” sequence.
- The sequence continues: $49 + 9 = 58$. Index 58 is $(6, 9)$.
- So the full line is $(4, 2) \to (5, 1) \to (5, 10) \to (6, 9)$.
- The “Wrap” connects the left wall of Row N to the right wall of Row N.

The Diagonal Wrap (Stride 11 - Major Diagonal): Consider a stone at $(1, 10)$ [Index 9].

- Next in sequence: $9 + 11 = 20$.
- Index 20 is $(3, 1)$.
- *Visual Effect:* The line exits the right edge of Row 1 and re-enters on the left edge of **Row 3**. It effectively “skips” Row 2.
- This creates a “Knight’s Move” shift at the boundary. A diagonal line that hits the edge does not continue on the immediate next row (like a horizontal line) but jumps an extra row down.

3 Analysis of Specific Game Scenarios

The user provided four distinct board states. We will analyze these as case studies to confirm the rules derived above.

3.1 Case Study: The Horizontal Split (Board #4)

Visual Observation: The board shows a line of stones at the end of Row 5 and the beginning of Row 6.

- Row 5: `... X X X X` (Cols 7, 8, 9, 10)
- Row 6: `X...` (Col 1)

Indices:

- Row 5, Col 7 $\to$ Index 46
- Row 5, Col 8 $\to$ Index 47
- Row 5, Col 9 $\to$ Index 48
- Row 5, Col 10 $\to$ Index 49
- Row 6, Col 1 $\to$ Index 50

Analysis: The difference between consecutive indices is exactly 1 ($47 - 46 = 1$, etc.). This confirms that the “Wrap” is linear. The end of one row is mathematically contiguous with the start of the next. In traditional topology, this is equivalent to writing the text of a book: when a line ends, the sentence continues on the next line. The “board” is essentially a paragraph of text 100 characters long.

Strategic Implication: This fundamentally changes the defense of the “Edge.” In standard Gomoku, a row of 4 stones ending at the board’s edge is harmless because it cannot be extended.[1] In Wrap Gomoku, that line is fully alive and threatens to win on the next row. A defender cannot use the wall as an anvil; they must block on the other side of the board.

3.2 Case Study: The Broken Diagonal (Board #3)

Visual Observation: A diagonal line descends from left to right, hits the left wall, and reappears on the right wall.

- Stones at: $(3, 3), (4, 2), (5, 1), (5, 10), (6, 9)$.
- Indices: $22, 31, 40, 49, 58$.

Analysis:

- $31 - 22 = 9$
- $40 - 31 = 9$

- $49 - 40 = 9$
- $58 - 49 = 9$

This confirms the "Stride 9" rule. The transition $40 \to 49$ is the critical anomaly.

- Index 40 is $(5, 1)$ — Left edge.
- Index 49 is $(5, 10)$ — Right edge.
- The "Diagonal" step of +9 acts as a "Teleporter" between the extreme columns of the same row.

Interpretation: This suggests the board topology for Stride 9 is akin to a Möbius strip behavior where the "left" and "right" are connected, but with a vertical shear. The fact that a "diagonal" move results in two stones on the same row $(5, 1)$ and $(5, 10)$ is a unique feature of the $W = 10$ grid width combined with the arithmetic of the stride.

3.3 Case Study: Vertical Line Stability

Although not explicitly highlighted as a "wrap" anomaly in the snippets, the Vertical Stride ($S = 10$) is the only one that preserves visual intuition.

- Sequence: $5, 15, 25, 35, 45$.
- Visual: $(1, 6), (2, 6), (3, 6), (4, 6), (5, 6)$.
- Since $W = 10$, adding 10 simply increments the row number r while keeping column c constant ($c = i \pmod{10}$).
- Therefore, vertical lines do not wrap horizontally. They only wrap if the board is Toroidal (connecting Row 10 to Row 1). The user prompt does not show a vertical wrap, so we assume the board is a **Linear Strip** (finite cylinder), not a torus. Vertical lines terminate at the top and bottom of the grid.

4 Algorithmic Implementation and Complexity

Implementing a Gomoku engine for a 1D Wrap Board requires significant deviations from standard bitboard or 2D array techniques. This section outlines the computational architecture required.

4.1 Data Structures

Standard Gomoku engines often use a 1D array with "padding" to detect edges.[3] For a 15-wide board, the array might be width 16, with the 16th column storing a "guard" value. For the Wrap Board, we deliberately use a **compact array** of size $N = W \times H$.

```
Board: Array[0..99] of Integers (0=Empty, 1=Black, 2=White)
```

4.2 Victory Check Algorithm

The victory check is simplified in code complexity but conceptually distinct. Instead of checking `(x+1, y)`, `(x, y+1)`, etc., we iterate through the 4 fundamental strides.

Pseudocode for Win Detection:

```python
def check_win(board, last_move_index):
    player = board[last_move_index]
    # Horizontal, Vertical, Minor Diag, Major Diag
    strides =

    for stride in strides:
        count = 1

        # Check Positive Direction
        next_idx = last_move_index + stride
        while 0 <= next_idx < 100 and board[next_idx] == player:
            count += 1
            next_idx += stride
```

```
        # Check Negative Direction
        prev_idx = last_move_index - stride
        while 0 <= prev_idx < 100 and board[prev_idx] == player:
            count += 1
            prev_idx -= stride

        if count >= 5:
            return True
    return False
```

4.3 Computational Complexity and Search Space

- **State Space Complexity:** The 10×10 board has 100 spots. This is significantly smaller than the standard 15×15 (225 spots). The number of possible games is roughly bounded by 3^{100}, compared to 3^{225} for standard Gomoku.
- **Branching Factor:** The branching factor b begins at 100 and decreases. In standard Gomoku, it begins at 225.
- **Implication:** The 1D Wrap Gomoku is computationally much "lighter" than the standard game. It is highly likely that this variant is **Strongly Solved** (meaning a perfect strategy for the first player has been or can be easily computed). The smaller board combined with the "Wrap" rules (which increase the connectivity of the graph) drastically favors the attacker (Black).

In standard Gomoku, the first-player advantage is so large that professional rules (Renju) enforce strict handicaps on Black.[1] On a 10x10 Wrap Board, where edges no longer defend against attacks, the first-player advantage is likely decisive. Black can open in the center (index 45) and project power in 8 directions (4 strides $\times$ 2 directions) that all wrap around to form continuous threats.

5 Strategic Theory and Gameplay

The transition from 2D to 1D topology necessitates a complete overhaul of strategic intuition. Players must unlearn the safety of edges and the geometry of diagonals.

5.1 The "Death of the Edge"

In standard Gomoku, the edge of the board is a defensive resource. A player pushed against the edge has fewer degrees of freedom, but the attacker also cannot extend lines past the edge. In Wrap Gomoku, the left and right edges are illusions.

- **Horizontal Attack:** An attacker building a line on Row 3, Col 9 is simultaneously building a line on Row 4, Col 1.
- **Defensive Pivot:** A defender cannot simply "block the right side." They must calculate where that line emerges on the left.

The "Seam" Control: The columns 1 and 10 (Indices $x0$ and $x9$) constitute the "Seam" of the cylinder. Control of the seam is critical because it allows a player to transition attacks between rows. A stone at $(5, 10)$ is a pivot point that threatens $(5, 9)$ [Left], $(5, 1)$, $(4, 9)$, and $(6, 1)$.

5.2 The "Shear" Effect and Spatial Aliasing

One of the most difficult aspects for human players is the "Shear." Because the grid width ($W = 10$) is even, and the diagonal strides $(9, 11)$ are odd, the wrapping behavior is asymmetric.

- **Major Diagonal (11):** Shifts the row by +1 visually (standard diagonal) plus an *extra* row when it wraps. This double-jump makes tracking long-range diagonal attacks extremely prone to error.
- **Cognitive Load:** This phenomenon is known in cognitive science as **Spatial Aliasing**. The visual signal (2D grid) suggests one geometry, but the logical signal (1D array) enforces another. The mismatch creates a "Blind Spot" where players consistently fail to see threats that cross the wrap boundary.

5.3 Opening Theory

Given the high connectivity, the center of the 1D array (Indices 40-60) is the optimal opening zone.

- **Center Opening:** A stone at Index 45 (Row 5, Col 6) has maximum extension potential in both positive and negative directions for all strides.
- **Corner Opening:** A stone at Index 0 (Row 1, Col 1) is still weak compared to the center, but stronger than in standard Gomoku. It connects to Index 1 (Right), Index 10 (Down), Index 11 (Diag), and Index 9 (Horizontal Wrap from Row 1, Col 10?? No. Index -1 is invalid).
  - *Note:* Assuming a linear strip, Index 0 has no "negative" neighbors. The "Top" and "Bottom" of the strip (Indices 0-9 and 90-99) act as true boundaries. Thus, the board is a cylinder with open ends.

6 Comparative Analysis with Other Variants

To contextualize the Wrap Board, we compare it with other Gomoku variants mentioned in the research material.

6.1 vs. Renju

Renju [1] is the professional standard for Gomoku. It mitigates the first-player advantage by forbidding Black from making "double threes" (two simultaneous open lines of 3).

- *Comparison:* The Wrap Board *exacerbates* the first-player advantage. It adds connectivity without adding restrictions. A Renju-style ruleset (e.g., banning "Wrapped Double Threes") would be essential to make the Wrap Board competitively viable.

6.2 vs. Hex-Gomoku

Hex-Gomoku [9] uses a hexagonal grid.

- *Comparison:* Hexagonal grids naturally provide 6 directions of movement (3 axes). A 1D Wrap Board provides 4 axes (Horizontal, Vertical, 2 Diagonals). However, the "Wrap" creates a quasi-toroidal topology that makes the board feel "infinite" in the horizontal dimension, whereas Hex-Gomoku usually retains hard boundaries.
- *Connectivity:* A Hex board has uniform connectivity (6 neighbors) everywhere except edges. A Wrap board has uniform connectivity (2 neighbors per stride) everywhere except the very start and end of the array.

6.3 vs. Pente

Pente [10] introduces captures (capturing a pair of opponent stones).

- *Comparison:* Captures on a Wrap Board would be chaotic. A pair of stones at $(1, 10)$ and $(2, 1)$ could be captured by bracketing them at $(1, 9)$ and $(2, 2)$. This would require players to visualize captures that snake across rows, significantly increasing the difficulty of defensive play.

7 Strategic Synthesis and Recommendations

The "Wrap Board" puzzle presented by the user is likely designed to test a specific type of lateral thinking: the ability to ignore visual boundaries.

Key Takeaways for the Player:

1. **Count, Don't Look:** Do not rely on visual lines. Calculate the indices. If you have stones at 34 and 45, you have a Stride 11 connection. The next stone is 56.
2. **Respect the Seam:** The wrap occurs between Column 10 and Column 1. Always double-check "broken" lines at this interface.
3. **Dominate the Center:** The middle rows (4, 5, 6, 7) are the most powerful because they are furthest from the true (Top/Bottom) boundaries of the linear strip.

Conclusion: The 1D Wrap Board is a fascinating deconstruction of Gomoku. By stripping away the 2D padding, it exposes the game's underlying arithmetic nature. While it likely suffers from severe first-player imbalance due to increased connectivity and smaller board size, it serves as an excellent pedagogical tool for understanding board representations in computer science and the topology of games. The "Rule of the Wrap Board" is simply that the board is a single line of 100 squares, and the grid is a lie.

Table 1: Summary of Rules and Mechanics

Feature	**Standard Gomoku**	**1D Wrap Gomoku**
Topology	Euclidean Plane (15×15)	Linear Strip (1×100)
Grid Visualization	Faithful Representation	Rasterized Artifact
Horizontal Wrap	Impossible (Edge blocks)	**Active** (Right edge $\rightarrow$ Left edge)
Diagonal Wrap	Impossible (Edge blocks)	**Active** (Calculated via Stride)
Vertical Wrap	Impossible	Impossible (unless Toroidal)
Winning Condition	5 contiguous (2D adjacency)	5 arithmetic sequence (Stride 1, 9, 10, 11)
Complexity	High (3^{225} states)	Moderate (3^{100} states)
Cognitive Load	Spatial Pattern Matching	Arithmetic Calculation