Homework 1

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1 Airplane seat overselling

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a

Looking at this question, the binomial distribution represents the theoretical distribution since we want to count the amount of sucesses within a certain amount of samples. However, if we count a passenger missing a flight being a success with probability $p = .02$, with a large sample of $n = 100$ and an expected value of $np = 2$. We can approximate it with a poisson distribution where $\lambda = np = 2$

The probability of not having enough seats occurs when the event of only 0 or 1 passengers misses their flight.

$$P(X \leq 1) = P(X = 0) + P(X = 1) = \frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} = .4060 \tag{1}$$

There is a .4169 probability that there will not be enough seats on the plane.

b

Binomial distribution pdf is $\binom{n}{k}(1-p)^{n-k}p^k$ where $n = 100$ and $p = .02$

$$P(X = 0) = \binom{100}{0}(1-p)^{100}p^0 = 1 * .98^{100} = .1326 \tag{2}$$

$$P(X = 1) = \binom{100}{1}(1-p)^{99}p^1 = 100 * .98^{99} * .02 = .2707 \tag{3}$$

$$P(X \leq 1) = P(X = 0) + P(X = 1) = .4033 \tag{4}$$

$$\tag{5}$$

The approximation using the poisson distribution is slightly larger then the binomial distribution but is overall pretty close.

c

This is a conditional probabiliy where given the plane has all its passenger, there are exactly two no shows. The probability of 2 no shows and the plane being full can be modeled with a binomial distribution in equation (6). The probability that every seat is filled occurs when there are two or less no shows which is modeled in equation (7). The conditional probability is calculated in equation (8). The results indicate a probability of .4040 that an airline will not need to reimburse passengers given the plan is full.

$$P(X \geq 2 \cap X \leq 2) = P(X = 2) = \binom{100}{2}(1-p)^{98}p^2 = \frac{100!}{2!98!}.98^{98}.2^2 = .2734 \tag{6}$$

$$P(X \leq 2) = P(X = 2) + P(X \leq 1) = .2734 + .4033 = .6767 \tag{7}$$

$$P(X \geq 2 \cap x \leq 2 | X \leq 2) = \frac{P(X = 2)}{P(X \leq 2)} = \frac{.2734}{.6767} = .4040 \tag{8}$$

2 Joint distributions

$$f(x,y) = \begin{cases} Cxy & 0 \le y \le x, 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

a

We need to find some c where the joint cummulative density equals 1 when the entire sample space is integrated.

$$\begin{aligned} 1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} CxydR = 0 + \int_0^1 \int_0^x Cxydydx && (10) \\ = \frac{1}{2}\int_0^1 Cx^3dx && (11) \\ = \frac{1}{2}\frac{1}{4}cx^4|_0^1 = C\frac{1}{8} && (12) \\ \implies c = 8 && (13) \end{aligned}$$

Hence $C = 8$ for the joint distribution

b

The joint distribution is not independent. By looking at it, when x, is 0, the probability of get 0 for y is 100%, but clearly when x is some other value, the probability of y being 0 is not 1. This can be proven more rigorously by showing the product of the marginal distribution does not equal the joint distribution.

$$f_x = \int_0^x 8xydy = 4x^3 \tag{14}$$

$$f_y = \int_y^1 8xydx = 4(y - y^3) \tag{15}$$

$$f_x * f_y = 8(x^3y - x^3y^3) \neq 8xy \tag{16}$$

c

3 Numerical evaluation of integrals via Monte Carlo simulation

Consider the following equation

$$I = \int_1^{\infty} \frac{1}{1+x^6} dx \tag{17}$$

a

I will be using $u = \frac{1}{x}$ and $du = \frac{-1}{x^2} dx$ for the substitution yielding:

$$\int_1^0 \frac{1}{1+\frac{1}{u^6}} * -x^2 du \tag{18}$$

$$\int_0^1 \frac{1}{u^2+\frac{1}{u^4}} du \tag{19}$$

$$\int_0^1 \frac{u^4}{u^6+1} du \tag{20}$$

$$\tag{21}$$

Note that