

Homework 1

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See <https://github.com/nguyencquan/Markov2026> for code used in simulations.

1 Power Law Sampling

Assume we have some distribution

$$\begin{cases} Cx^{-\gamma} & x \geq x_0, \\ 0, & x < x_0 \end{cases} \quad (1)$$

a)

Where $x_0 > 0$ and $\gamma > 3$

A valid pdf must have a total probability of 1, which is calculated below:

$$1 = \int_{-\infty}^{\infty} Cx^{-\gamma} dx = C \int_{x_0}^{\infty} x^{-\gamma} dx = \frac{C}{x^{\gamma-1}(1-\gamma)} \Big|_{x_0}^{\infty} = \frac{C}{x_0^{\gamma-1}(\gamma-1)} \quad (2)$$

$$C = x_0^{\gamma-1}(\gamma-1) \quad (3)$$

b) First calculating the cumulative distribution we take a step from equation 2 except take the integral from x_0 to x instead.

$$1 = \int_{-\infty}^{\infty} Cx^{-\gamma} dx = C \int_{x_0}^x x^{-\gamma} dx = \frac{C}{x^{\gamma-1}(1-\gamma)} \Big|_{x_0}^x = \frac{C}{x_0^{\gamma-1}(\gamma-1)} + \frac{C}{x^{\gamma-1}(1-\gamma)} \quad (4)$$

Substituting in C

$$F(x) = \begin{cases} 1 - \frac{x_0^{\gamma-1}}{x^{\gamma-1}}, & x \geq x_0 \\ 0, & x < x_0 \end{cases} \quad (5)$$

To calculate the inverse, note we can do root math since $x_0 > 0$

$$p = 1 - \frac{x_0^{\gamma-1}}{x^{\gamma-1}} \quad (6)$$

$$\frac{x_0^{\gamma-1}}{x^{\gamma-1}} = 1 - p \quad (7)$$

$$\frac{x_0}{x} = (1 - p)^{\frac{1}{\gamma-1}} \quad (8)$$

$$x = \frac{x_0}{(1 - p)^{\frac{1}{\gamma-1}}} \quad (9)$$

c) Also we can note that we can use $U(0, 1)$ without $1 - p$ to speed it up since their distribution is exactly the same.

1. Sample from a normal distribution $U(0, 1)$

2. substitute value into p for $\frac{x_0}{p^{\frac{1}{\gamma-1}}}$

d)

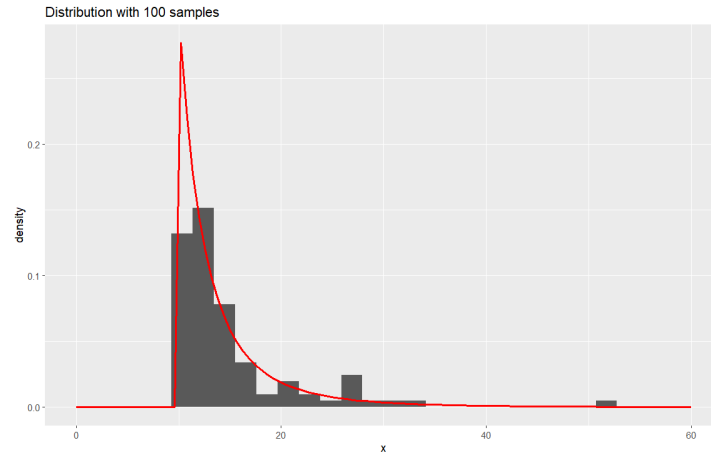


Figure 1: Graph of pdf and random sampling of 100 points, given $\gamma = 4$ and $x_0 = 10$

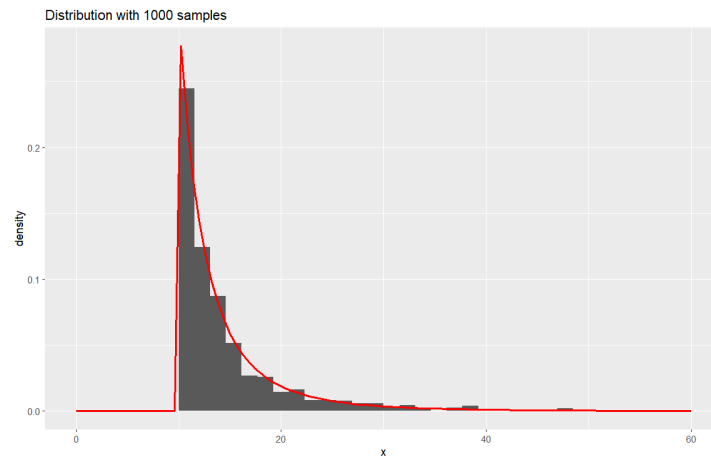


Figure 2: Graph of pdf and random sampling of 1000 points, given $\gamma = 4$ and $x_0 = 10$

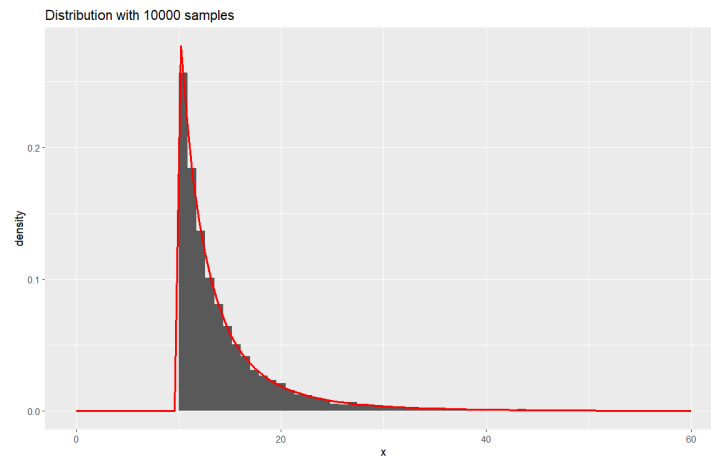


Figure 3: Graph of pdf and random sampling of 10000 points, given $\gamma = 4$ and $x_0 = 10$