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Multi-objective optimization in partitioning the healthcare system of Parana State in Brazil ☆

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A B S T R A C T

Motivated by a proposal of the local authority for improving the existing healthcare system in the Parana State in Brazil, this article presents an optimization-based model for developing a better system for patients by aggregating various health services offered in the municipalities of Parana into some microregions. The problem is formulated as a multi-objective partitioning of the nodes of an undirected graph (or network) with the municipalities as the nodes and the roads connecting them as the edges of the graph. Maximizing the population homogeneity in the microregions, maximizing the variety of medical procedures offered in the microregions, and minimizing the inter-microregion distances to be traveled by patients are considered as three objective functions of the problem. An integer-coded multi-objective genetic algorithm is adopted as the optimization tool, which yields a significant improvement to the existing healthcare system map of the Parana State. The results obtained may have a strong impact on the healthcare system management in Parana. The model proposed here could be a useful tool to aid the decision-making in health management, as well as for better organization of any healthcare system, including those of other Brazilian States.

1. Introduction

One of the challenges in logistics is to provide need-basis products or services. A bottom line task in logistic handling is to structure systems or distribution configurations so that markets away from production sources can be served in an optimized way or services can be provided in possible shorter time [2]. The healthcare system of Parana State in Southern Brazil is studied here in that direction.

The Parana State, which is divided into 399 municipalities, has a population close to 11 million inhabitants. In the case of health services in Parana, funds are transferred from the Federal and State authorities to local units based upon the population size and variety of health procedures expected to be performed. There is a proposal for decentralization of the healthcare services in Parana by aggregating its municipalities into smaller groups, known as microregions, where the majority of health procedures can be provided so as to reduce the number of inter-microregion trips and the distances to be traveled by patients [7]. Motivated by the proposal, the present optimization-based computational work is carried out. For this purpose, Parana State is first transformed into an undirected graph/network with its municipalities as the elementary units (nodes) and connecting roads of the municipalities as the edges of the network. The network is then partitioned into some non-empty zones, each zone representing a microregion, so as to optimize three objectives subject to five constraints. The first objective is to maximize the homogeneity of population in the microregions, which would ease the distribution of facilities and funds. The second objective is to maximize the variety of medical procedures offered within a microregion so that precious time and money can be saved by reducing the need to send patients from one microregion to another. The third objective is to minimize the distances to be traveled by patients on going from one microregion to another. In regard of constraints satisfaction during the partitioning process, the integrity of the municipalities is maintained by avoiding the inclusion of the same municipality in multiple

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microregions, and the contiguity of the microregions is maintained by forming a microregion with inter-connected municipalities only. Care is also taken to avoid the complete embedding (surrounding) of a microregion by another. Finally, the municipalities are aggregated (partitioned) into a number of microregions within a predefined range, and the number of municipalities to be included in each microregion is also maintained within another given range. A customized version of the integer-coded multi-objective genetic algorithm, proposed by Datta et al. [12,15], is employed as the optimization tool for the problem. In the experimental part, the best compromised scenario is presented by comparing the results of the genetic algorithm with the existing healthcare system map partitioning the 399 municipalities of Parana into 83 microregions.

It is to be emphasized that the main scientific contribution in this article is related to the modeling aspect of the problem, in particular the design of the three objective functions taking into account the real need in Parana and considering the proposal of the local authority for improving the existing healthcare system in the State. It seems to be very significant in the context of the graph partitioning problem in general, and the territory districting problem in particular. To the best knowledge of the authors after surveying the specialized literature, it is the very first attempt for developing such a model for this problem. Additionally, the application of the multi-objective optimization approach to the healthcare system partitioning, in which way it is done here, is also new. Surveying the specialized literature (as reported in Section 2), only two works, conducted by Datta et al. [13] and Benzarti et al. [4], could be obtained where healthcare systems were partitioned as a multi-objective optimization problem, but with very different approaches.

The rest of the article is organized as follows. The relevant works dealing with territory partitioning are reviewed in Section 2. The present healthcare system in Parana State is highlighted in Section 3, followed by Section 4 giving the present problem formulation and Section 5 describing the methodology used in this work. Section 6 presents the results and managerial implications. Finally, the article is concluded in Section 7.

2. Literature review

Under the broad name of the territorial districting or network partitioning problem, numerous mathematical programming models have been proposed for grouping smaller territorial units into larger ones with applications to different fields. Although the deterministic mixed-integer linear programming (MILP) is the most appropriate approach for handling such models, its inadequacy for working with large-size real-world situations has led to the development of different heuristics and metaheuristics for the territorial partitioning problem [3]. However, such metaheuristics are generally real-valued and their direct application to the integer variables of a problem is likely to generate infeasible solutions, whose steering to the feasible region becomes a challenging task (*see*, e.g., [14]). Hence, many problem-specific versions of different metaheuristics are being proposed for generating feasible solutions directly. Such approaches reported in the specialized literature in the last decade are reviewed here in detail in chronological order (and also summarized in Table 1).

D'Amico et al. [9] applied a single-objective simulated annealing (SA) to the police management problem in Buffalo City, USA, in which the aim was to obtain an optimum police district map that minimizes the disparity between the extreme workloads of the police officers in the process of determining the number of patrol cars on duty at various time periods of a day. As a major merit, their proposed map helped to alleviate an annoying dispatching quirk that appeared in the practicing configuration. Muyldermans et al. [25] studied salt spreading in the province of Antwerp, Belgium, through a multi-objective heuristic search procedure, in which the road network was partitioned in order to minimize both the traveling distance and number of trucks required in the operation. Applying a multi-objective genetic algorithm (GA) to partition the power grid of the Republic of Ghana into economically viable districts (distribution companies), Bergey et al. [5] identified some fundamental characteristics required to correctly model and solve an electrical power districting problem. A tabu search (TS) approach coupled with an adaptive memory heuristic was proposed by Bozkaya et al. [6] with the application to the electoral districting problem of the city of Edmonton, Canada. Galvão et al. [19] presented a multiplicatively-weighted Voronoi diagram for treating the parcel delivery in the city of São Paulo, Brazil, as a logistics districting problem, which resulted in more balanced time/capacity utilization compared with other approaches. A benchmark transportation delivery planning system is studied by Haugland et al. [20] in two stages using TS and a multi-start metaheuristic, in which the districting decision is made in the first stage and then the vehicle routing problem is solved for each district in the second stage. Combining a local search with a multi-objective GA, Tavares-Pereira et al. [32] analyzed the public transportation system of the Paris region in order to suggest a reform to its pricing system. A spatial-GIS based multi-objective GA is investigated by Datta et al. [14] for the problem of land-use management in Baixo Alentejo, Portugal, with the objectives of both natural balance of the environment and financial profit. Targeting population equality, compactness and administrative conformity, Ricca and Simeone [28] studied descent, TS, SA and old bachelor acceptance algorithm in a comparative way on the multi-objective electoral districting problem in the Italian regions. Ricca et al. [27] investigated the potentiality of weighted Voronoi diagrams (WVR) in locating the Italian electoral centers with the objectives of population equality and compactness. With transportation and logistics applications in an urban distribution service in a part of the city of São Paulo, Brazil, Novaes et al. [26] also investigated Voronoi diagrams, but in association with continuous approximation models, for solving the location-districting problem by minimizing total daily delivery costs and balancing the distribution effort among vehicles. Salazar-Aguilar et al. [30] applied the ϵ-constraint method for handling the beverage distribution in the city of Monterrey, Mexico, as a multi-objective problem with respect to the number of customers and sales volume. Shirabe [31] applied a single-objective map algebra based heuristic for solving MILP models of several illustrative instances of the well-known school bus problem. Contreras et al. [8] presented two MILP formulations, as the generalization of the classical p-center problem, for the location of facilities and the design of its underlying network so as to minimize the maximum customer-facility travel time. A model for locating emergency medical services by incorporating survival functions for capturing multiple-classes of heterogeneous patients was proposed by Knight et al. [21] for maximizing the overall expected survival probability of multiple-classes of patients, which was solved for the ambulance service in Wales using an approximation approach. An integer-coded multi-objective GA was proposed by Datta et al. [15] to deal with the census tracts in the Census Metropolitan area of London, Ontario, Canada, by aggregating the census units so as to obtain a higher level of compactness and population/area uniformity. A customized version of this integer-coded multi-objective GA was proposed by Datta et al. [13] for presenting an optimal administrative healthcare geography for East England with five objectives of geographical compactness, co-extensiveness with current local authorities, size homogeneity, age homogeneity and economic homogeneity. Lin et al. [23] proposed a framework for multi-objective simulation optimization that combines GA with data envelopment analysis, which was applied to determine the optimal resource levels in surgical services. Benzarti et al. [4] studied the home healthcare as a mixed-integer programming districting problem under four objectives of indivisibility of basic units, compactness, workload balance between

Table 1
Selected publications on the territorial or graph partitioning problem.

Publication	Problem	Location	Model	Approach	Method
D'Amico et al. [9]	Police management	Buffalo city, USA	SO	MH	SA
Muyldermans et al. [25]	Salt spreading operations	Antwerp, Belgium	MO	H	MHP
Bergey et al. [5]	Electrical power districting	Republic of Ghana	MO	MH	GA
Bozkaya et al. [6]	Electoral problem	Edmonton, Canada	SO	MH	TS
Galvão et al. [19]	Delivery problem	São Paulo, Brazil	MO	H	WVD
Tavares-Pereira et al. [32]	Public transportation	Paris, France	MO	MH	GA
Haugland et al. [20]	Routing problem	Benchmark instances	SO	MH	TS
Datta et al. [14]	Land-use management	Southern Portugal	MO	MH	GA
Ricca and Simeone [28]	Electoral problem	Italian regions	MO	MH	LS
Ricca et al. [27]	Electoral problem	Italian regions	MO	H	WVD
Novaes et al. [26]	Delivery problem	São Paulo, Brazil	MO	H	VD
Salazar-Aguilar et al. [30]	Distribution problem	Monterrey, Mexico	MO	EX	ϵ-constraint
Shirabe [31]	School bus transportation	Sweden	SO	H	Map algebra
Contreras et al. [8]	Combined facility location and network design	Benchmark instances	SO	EX	MILP
Knight et al. [21]	Locating emergency medical services	Wales	SO	MH	GA
Datta et al. [15]	Census tracts	London, Ontario, Canada	MO	MH	GA
Datta et al. [13]	Healthcare	East England	MO	MH	GA
Lin et al. [23]	Determination of resource levels in surgical services	Benchmark instances	MO	MH	GA
Benzarti et al. [4]	Home healthcare	France	MO	EX	MILP
Ríos-Mercado and López-Pérez [29]	Distribution problem	Monterrey, Mexico	SO	EX	MILP
Assis et al. [1]	Power meter reading	São Paulo, Brazil	MO	MH	GRASP
Li et al. [22]	Urban land use	Southern California, USA	SO	EX	MILP
Liu et al. [24]	Periodic vehicle routing problem	Benchmark instances	SO	MH	TS

Legend – EX, exact; GA, genetic algorithm; H, heuristic; LS, local search; MH, metaheuristic; MHP, multicriteria heuristic procedures; MILP, mixed-integer linear programming; MO, multi-objective optimization; SA, simulated annealing; SO, single-objective optimization; TS, tabu search; VD, Voronoi diagram; and WVD, weighted Voronoi diagram.

human resources, and compatibility. A single-objective branch-and-bound based approach was proposed by Ríos-Mercado and López-Pérez [29] for solving several instances of a bottled beverage distribution company in Monterrey, Mexico, where the entire set of city blocks was partitioned into a given number of territories subject to several planning constraints. A framework involving a GRASP procedure was addressed by Assis et al. [1] for solving the multicriteria capacitated redistricting problem with the application to the power meter reading in São Paulo, Brazil, considering two objectives of compactness and homogeneity. Li et al. [22] proposed a single-objective metaheuristic approach combining TS and SA for maximizing compactness in the MILP model of districting the urban land-use and transportation in the field of metropolitan transportation economy in Southern California, USA. Liu et al. [24] proposed a TS combined with different local search schemes for solving the periodic vehicle routing problem encountered in home health care (HHC) logistics as minimizing the maximal routing costs among all routes over the horizon.

As mentioned in Section 1, this article deals with the healthcare system of Parana State by developing a customized version of the integer-coded multi-objective GA proposed by Datta et al. [12,15]. The proposed model should serve as a useful tool to aid the management decision-making ensuring a "smart" partitioning of the Parana State, taking into account three important objectives in the context of the public health system as presented in Section 4.

3. The Brazilian healthcare system

The public health services provided in Brazil are divided into three categories according to the complexity levels of low, medium and high. The complexity is measured based upon the kind of equipment, facilities and skills required for treatment. The low level of complexity basically consists of ambulatory or minor invasive procedures. The procedures demanding more sophisticated equipment, facilities or personnel are considered to be of medium level, while the high level procedures require larger teams, longer duration, and very specific equipment and facilities. Because of a huge complaints about the effectiveness of the public health system, it is generally accessed by the low income population only. A private health system of a similar size has also been developed, which is usually availed by wealthy people or offered by companies to their employees. Accordingly, there exists a two-tier health system.

Funding to the public health system is released by the federal government through the SUS (*Sistema Único de Saúde* or Unified Health System), which is distributed among the 28 Brazilian States based on population size and age range. The amount received by a State authority is either spent for the services provided by the State owned facilities or forwarded to the municipalities. The fund received by the municipalities is processed by partitioning the State into microregions, and it is shared taking into account the population characteristics (age, size, etc.) as well as the amount and variety of medical procedures offered by the municipalities belonging to each microregion. Each health service unit of every municipality receives an amount based on the number of procedures, of each available type of services, expected to be conducted every month. However, it is a very difficult/challenging job to distribute the funding effectively among the units, so as to provide the best services to the population.

The municipalities with simpler facilities basically offer low complexity services. The high complexity services are offered by few larger hospitals only, which are normally located in cities with bigger populations, and owned and operated by the State level authorities. The total number of different services or medical procedures offered in the State is higher than 6000. The basic healthcare units provide only out-patient ambulatory services. The patients in need of more complex services have to be sent to the nearest facilities, offering the required procedures, either on their own transport cost or providing ambulance by the local authorities. Hence, although complex, the partitioning is necessary in the drive to reduce expenditure and at the same time to offer quality services.

The partitioning of a State into microregions not only defines the type of services offered in the primary health area (the microregion), but also the budget for services (people, equipment, suppliers, drugs, etc.), thus establishes a quality level of services provided to the population of the microregion. A wrong partitioning will certainly result in an unbalanced health service and might cause extensive flow of patients from one microregion to another for the sole purpose of a required procedure not available in the patient's home microregion, and thus it may increase cost and reduce quality.

Table 2
Health services offered in the Parana State (Source: DATASUS [10]).

Type of facility	Public	Non-profit	Private	Unions	Total
Hospitals	160	91	307	–	558
Community health centers	1488	12	13	6	1519
Specialized units	147	139	1743	1	2030
Doctor's offices	126	3	10134	58	10321
Basic health units	962	1	5	2	970
Others	455	29	1772	6	2262
Total	3338	275	13974	73	17660

The Parana State has a total area of 199,314.85 km^2 and its population (close to 11 million inhabitants) is around 5% of the total Brazilian population. The health services in Parana are offered through a public system managed by the government and also complemented by private companies, but the latter are paid services affordable only by a section of the population. The public health services are offered by SUS and include a high variety of procedures. Table 2, which contains data obtained directly from a database published by the SUS [10], provides a notion of all the health services available in the Paraná State at the time of this study. For the purpose of healthcare services, each of the 399 municipalities of Parana is assigned to one of its 83 microregions and each microregion has an annual budget for health services provided by the local authorities. The existing map of Parana partitioning its 399 municipalities into 83 microregions, as shown in Fig. 1, involves some undesirable situations. Many microregions are composed of a single municipality while some others are formed by as many as 19 municipalities, irrespective of the number of inhabitants or variety of medical procedures offered. Moreover, according to SESA, the number of medical procedures varies from 26 (in the municipality of Entre Rios do Oeste) to 4499 (in Londrina) with the State capital located in Curitiba offering 3514 medical procedures. The number of inhabitants also varies from 1390 (in Nova Aliança do Ivaí) to 1,697,703 (in Curitiba).

For standardizing the existing healthcare system map of Parana, SESA (*Secretaria de Saúde do Estado do Paraná* or Parana State Health Secretary) considered that the microregions should meet the following criteria – geographical accessibility among their municipalities, apart from other aspects such as cultural and historical ties; territory based on the requirement of health services and services provided; cooperation between municipalities in solving common problems; establishing inter-sectorial culture within each region; and integration and organization of assistance networks with the assurance of connectivity. In order to reduce paperwork and bureaucracy as well as to ensure cooperation between different administrative units, the maximum number of municipalities to be covered by each microregion was set to eight and the minimum number of that was fixed at two. Moreover, each microregion should have at least one key city with a large number of medical procedures, so that patients would not have to travel long distances to get procedures which are not offered in low-service cities.

4. Problem formulation

The health service problem of the Parana State, discussed in Section 3, can be modeled as an undirected graph $G=(V,E)$; where $V=\{v_i;\ i=1,2,\ldots,|V|\}$ is the set of $|V|$ vertices (nodes) of the graph with the node v_i located in a two-dimensional space by its centroidal coordinates (x_i,y_i) with respect to a given coordinate system. Similarly, $E=\{e_{ij};\ i,j=1,2,\ldots,|V|;\ i\neq j; e_{ij}=e_{ji}\}$ is the set of $|E|$ edges with e_{ij} as the edge (connection) between the nodes v_i and v_j. Each of the node v_i and edge e_{ij} is usually involved with a set of "weights" (or parameters), $\{\sigma_1^{(i)},\sigma_2^{(i)},\ldots,\sigma_p^{(i)}\}$ and $\{w_1^{(ij)},w_2^{(ij)},\ldots,w_q^{(ij)}\}$, respectively. Partitioning G is the task of grouping its nodes into a set of K non-empty and disjoint partitions, $\{P_1,\ldots,P_l,\ldots,P_K\}$, where $P_l=\{v_1^{(l)},v_2^{(l)},\ldots\}$ is the set of some nodes of

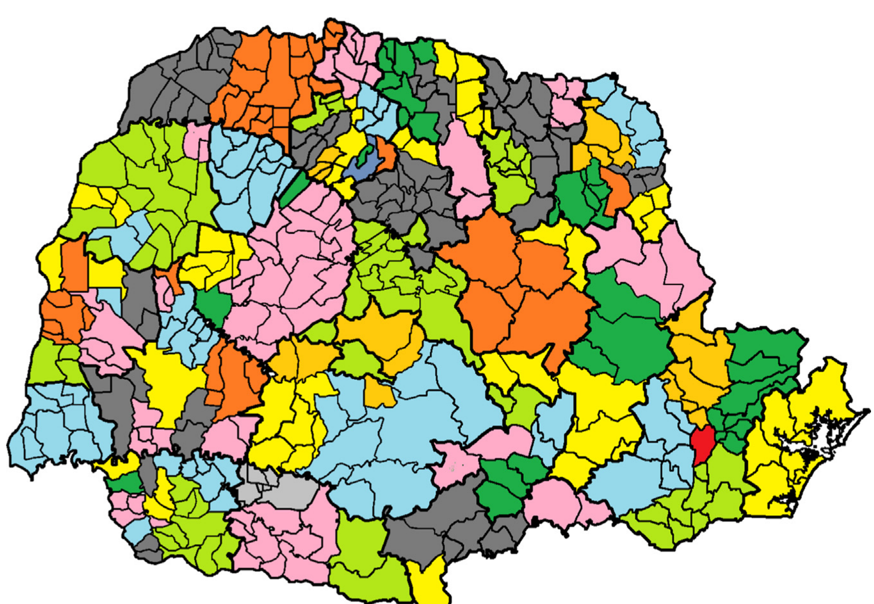

Fig. 1. Existing map of Parana partitioning its 399 municipalities into 83 microregions (Source: *Secretaria de Saúde do Estado do Paraná* (SESA) or Parana State Health Secretary).

G and represents the lth partition. The partitioning of G is to be performed in such a way that some objectives are optimized by respecting a given set of constraints, both are usually functions of node and edge weights [11]. Apart from the problem-specific objectives and constraints, the partitioning of G has also to satisfy some general conditions given by the following equation:

$$\left.\begin{array}{ll} \text{Number of partitions}: 2 \le K \le |V| & \\ \text{Non-empty partition}: P_l \ne \varnothing; & l = 1, 2, \ldots, K \\ \text{Size of a partition}: 1 \le |P_l| \le |V|; & l = 1, 2, \ldots, K \\ \text{Disjoint partitions}: P_l \cap P_m = \varnothing; & l, m = 1, 2, \ldots, K; l \ne m \\ \text{Inclusion of all nodes}: P_1 \cup \ldots \cup P_l \cup \ldots \cup P_K = V & \end{array}\right\} \tag{1}$$

In the healthcare problem of Parana at hand, the node v_i will represent the ith municipality with the edge e_{ij} as the traveling route between v_i and v_j, P_l as the lth microregion, and K as the total number of microregions. Apart from the traveling distance between v_i and v_j as the "weight" of the edge e_{ij}, the node v_i also has two "weights" representing the numbers of inhabitants and medical procedures offered in the ith municipality. The problem aims at partitioning the $|V|$ municipalities of Parana into K microregions by optimizing three objective functions in terms of these three "weights" subject to five constraints. These are addressed, respectively, in Sections 4.1 and 4.2 using the following notations:

Parameters :

$|V| =$ Number of municipalities

$K =$ Number of microregions

$h_i =$ Number of inhabitants in $v_i; i = 1, 2, \ldots, |V|$

$a_i =$ Number of medical procedures in $v_i; i = 1, 2, \ldots, |V|$

$d_{ij} =$ Distance between v_i and v_j; $i, j = 1, 2, \ldots, |V|$

$$X_{ij} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are connected; } i, j = 1, 2, \ldots, |V|; \; i \ne j \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Variable}: \quad Y_{il} = \begin{cases} 1 & \text{if } v_i \in P_l; \; i = 1, 2, \ldots, |V| \text{ and } l = 1, 2, \ldots, K \\ 0 & \text{otherwise.} \end{cases}$$

4.1. Objective functions

1. Maximize the homogeneity of inhabitants in the microregions, which can be achieved by minimizing the total deviations of the inhabitants in the microregions from their average value as expressed by Eq. (2a).

$$\text{Minimize} \quad f_1 \equiv \frac{1}{K} \sum_{l=1}^{K} \left| \sum_{i=1}^{|V|} h_i Y_{il} - \overline{H} \right| \tag{2a}$$

where $\overline{H} = (1/K) \sum_{i=1}^{|V|} h_i$ is the average number of inhabitants per microregion. The total deviation in Eq. (2a) is divided by K in order to minimize the per microregion average deviation without loosing any generality.

2. Maximize the variety of medical procedures offered in the microregions, which can be achieved as follows:

$$\text{Maximize} \quad f_2 \equiv \frac{1}{K} \sum_{l=1}^{K} \max_{i=1,\ldots,|V|} \{a_i Y_{il}\} \tag{2b}$$

Eq. (2b) takes care that each microregion should have at least one municipality with a large number of variety in the offered medical procedures, so that patients would not have to travel to other microregions for many procedures. It is to be noted that Eq. (2b) emphasizes on the variety of medical procedures offered in a microregion, not on the total amount of procedures obtained by counting multiple times the same procedure if offered in different municipalities of the microregion. Since the detail information about different medical procedures (names, types, etc.) offered in a municipality could not be obtained, just the highest number of variety of procedures offered in a single municipality of a microregion is considered in the evaluation of f_2 in Eq. (2b).

3. Minimize the inter-microregion traveling distance, which can be achieved, as follows, by evaluating the distance of each municipality of a microregion to the nearest municipality of every other microregion:

$$\text{Minimize} \quad f_3 \equiv \sum_{l=1}^{K} \left[\sum_{i=1}^{|V|} \min_{j=1,\ldots,|V|} \{h_i d_{ij} (1 - Y_{il}) Y_{jl}\} \right] \tag{2c}$$

Eq. (2c) estimates the minimum distance from v_i to v_j, where $Y_{il} = 0$ (i.e., $v_i \notin P_l$) and $Y_{jl} = 1$ (i.e., $v_j \in P_l$), and $l = 1, 2, \ldots, K$ and $i, j = 1, 2, \ldots, |V|$. The formulation of f_3 is adapted from Teitz and Bart [33], where it was used for the p-median problem.

4.2. Constraints

1. Integrity of a municipality – a municipality should belong to one and only one microregion at a time, which can be expressed mathematically as follows:

$$\sum_{l=1}^{k} Y_{il} = 1; \quad i = 1, 2, \ldots, |V| \tag{3a}$$

Fig. 2(a) shows a situation, where the integrity of municipalities is violated by putting municipality 4 in two microregions at the same time.

2. Contiguity of a microregion – the municipalities of a microregion should be inter-connected with one another, i.e., a microregion cannot be formed by several disconnected municipalities. In order to check this constraint, consider that T is a set initially formed with an arbitrary municipality of P_l (T and P_l are mutually exclusive for every l), and then it is gradually augmented with other municipalities of P_l one by one, which is directly connected to at least one element (municipality) of T. At the end of the process, T

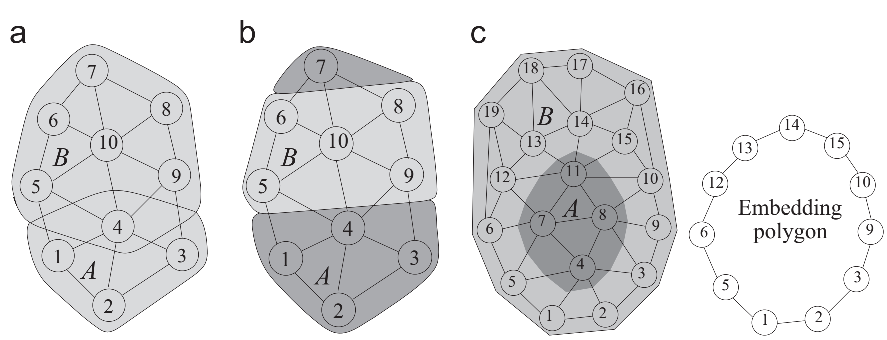

Fig. 2. Constraints violation: (a) violation of integrity of municipality, (b) violation of contiguity of microregions, and (c) embedded microregion.

will be equal to P_l if all the municipalities of P_l are interconnected. The process can be expressed mathematically as follows:

$$\left.\begin{array}{ll} T=\{v_r\}; & \text{where, } Y_{rl}=1 \\ & \quad r\in\{1,2,\ldots,|V|\};\ l\in\{1,2,\ldots,K\} \\ T\leftarrow T\cup\{v_j\}; & \text{if, } X_{ij}=1; Y_{jl}=1;\ v_i\in T;\ v_j\notin T \\ & \quad i,j=1,2,\ldots,|V|;\ i\neq j \end{array}\right\} \tag{3b}$$

The violation of contiguity of a microregion is shown in Fig. 2(b), where the microregion A is not contiguous as municipality 7 is not directly connected with any other municipality of A.

3. Absence of embedded microregions – a microregion should not be embedded (surrounded) by another microregion. The microregion P_l is said to be embedded by the microregion P_m if and only if the following necessary and sufficient conditions are met:

$$\left.\begin{array}{lll} \text{Necessary condition: } & v_j\in T; & \text{if, } X_{ij}=1;\ Y_{il}=1;\ Y_{jl}=0 \\ & & T\subseteq P_m \\ & & i,j=1,2,\ldots,|V|;\ i\neq j \\ & & l,m\in\{1,2,\ldots,K\};\ l\neq m \\ \text{Sufficient condition: } & \exists r: X_{ir}=1; & \text{where,} \\ & & v_i,v_r\in T \\ & & i=1,2,\ldots,|V| \\ & & r\in\{1,2,\ldots,|V|\};\ r\neq i \end{array}\right\} \tag{3c}$$

The necessary condition in Eq. (3c) says that all the neighboring municipalities of P_l (i.e., the municipalities belonging to other microregions, but connected with some municipalities of P_l) belong to T only (T is the set of the neighboring municipalities of P_l and $T\subseteq P_m$), while the sufficient condition finds that the municipalities belonging to T are inter-connected in the form of a closed polyhedron (a polygon in a two-dimensional space), thus ensuring that P_l is embedded (surrounded completely) by P_m. It is to be noted that the necessary condition in Eq. (3c) may be satisfied in the case of both interior and boundary microregions of the territory, while the sufficient condition may be satisfied in the case of an interior microregion only. Since only an interior microregion can be an embedded one, it (embedding) can be ensured if and only if both the conditions of Eq. (3c) are satisfied. Illustrating the above situation, Fig. 2(c) shows the microregion A embedded by the microregion B, as well as the embedding closed polygon formed by the neighboring municipalities of A (municipalities 16–19 of B are not included in the embedding polygon as they are not neighboring municipalities of A).

4. Range of microregions – the number of microregions (K) into which the healthcare map of Parana is to be partitioned should be within a given range or at a fixed value, i.e.,

$$K_{\min}\leq K\leq K_{\max};\quad \text{or } K=K_{\text{fix}} \tag{3d}$$

where $[K_{\min},K_{\max}]$ is the range and K_{fix} is a fixed value of K.

5. Size of a microregion – the number of municipalities within each microregion (or, size of a microregion) should also be within a given range, i.e.,

$$\mathrm{P}_{\min}\leq|P_l|\leq\mathrm{P}_{\max};\quad l=1,2,\ldots,K \tag{3e}$$

where, $[P_{\min},P_{\max}]$ is the range of each microregion, and $|P_l|$ is the size of P_l.

5. Genetic algorithm for partitioning the healthcare map

The genetic algorithm (GA) is a stochastic search technique that mimics the mechanisms of the Darwinian evolution based on the concept of the *survival of the fittest*. The basic component of a GA is an individual (solution representation), which is usually an array representing the design variables of a problem as a complete solution. A GA begins with a set of random solutions, known as a population, which is evolved over generations (iterations) by repeated applications of some genetic operators analogous to ones from natural evolution, namely selection, crossover, and mutation. A selection operator emphasizes good solutions and eliminates weak solutions by forming a temporary population, known as a mating pool, with multiple copies of good solutions from the original population. A crossover operator generates offspring (children solutions) by applying some transition rules to the parent solutions of the mating pool. A mutation operator explores the neighborhood of a child solution generated by the crossover operator. In this way, the population is gradually improved towards the optima of some given objective functions, also called fitness functions, which are the measures of qualities of a solution. The process of evolution of the population is continued until some termination criteria are met, usually until a predefined maximum number of generations are performed or the desired objective values are obtained.

The integer-coded multi-objective GA, proposed by Datta et al. [12,15] based on NSGA-II [18], is customized here for partitioning the healthcare system map of the Parana State formulated in Section 4. The main steps of the customized GA are addressed in Sections 5.1–5.6.

5.1. Individual representation and initialization

The GA solution, chosen for the present healthcare system partitioning problem of the Parana State, is an array of $|V|$ elements representing the municipalities of Parana. The value of an element of the solution is the microregion in which the representing municipality belongs.

Since it is very difficult to obtain a feasible solution for the graph partitioning problem through random assignment, a greedy algorithm is used for initializing the GA solution by satisfying the constraint on the upper size ($P_{\max}$) of a microregion, given by Eq. (3e), as much as possible. The algorithm works as follows – a microregion is first formed with a randomly chosen unassigned municipality (i.e., a municipality which is not yet included in any microregion), and then it is gradually expanded to the neighboring unassigned municipalities until the permitted maximum size ($P_{\max}$) of the microregion is obtained or all municipalities are already assigned.

5.2. Selection operation

After initializing solutions as in Section 5.1, the crowded binary tournament selection operator [18] is applied to the GA population, which picks up two random solutions at a time and a copy of the best one, based on the convergence and diversity of the solutions as stated in the next three paragraphs, is stored in the mating pool. The process is continued until the mating pool reaches a predefined size, usually equal to that of the GA population.

In single-objective optimization, solutions are compared based on their scalar objective values. However, two solutions in multi-objective optimization are compared through the concept of domination, where solution z_i is said to be dominated by solution z_j if and only if (i) z_j is not worse than z_i in any objective value and (ii) z_j is strictly better than z_i in at least one objective value, on violation of which they are called non-dominated solutions. As an example of a population of 6 solutions shown in Fig. 3(a), z_4 dominates z_5 as z_4 is not worse than z_5 in any objective value (better in f_1 and equal in f_2) and strictly better than z_5 in f_1. On the same ground, z_4 dominates z_2 and z_3 also. On the other hand, z_4 and z_6 are non-dominated solutions as z_4 is worse than z_6 in f_2 and z_6 is worse than z_4 in f_1,

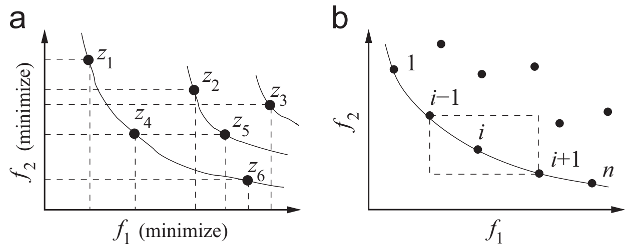

Fig. 3. Procedures for comparison of solutions in multi-objective optimization. (a) Concept of domination and (b) crowding distance estimation.

thus both violate the first condition of the domination concept. Extending the analysis, it is seen that none of z_1, z_4 and z_6 is dominated by any other solution of the population. After obtaining the domination relations between each pair of solutions, the population is sorted according to the convergence-based non-domination levels of the solutions. The set of the best non-dominated solutions is called the first non-dominated front, which is known as the Pareto front. The subsequent non-dominated fronts are identified from the rest of the population by excluding the solutions of the preceding fronts (the characteristic of the solutions of a non-dominated front is that the value of an objective function cannot be improved without degrading the value of at least another objective function). The solutions are finally ranked according to their levels of non-dominance (convergence). For example, the rank of each of the solutions of the first front (Pareto front) is 1, that of the second front is 2, and so on. Accordingly, z_1, z_4 and z_6 in Fig. 3(a) form the first non-dominated front (shown connected by a line for easy recognition), and hence each scores the rank of 1. Similarly, z_2 and z_5 form the second non-dominated front with rank 2, and z_3 comes under the third non-dominated front with rank 3.

One of the primary targets of a multi-objective optimizer is to maintain diversity among the solutions of a non-dominated front. The diversity of such a solution can be expressed in terms of its crowding distance, representing the density of other solutions around it [18], which can be estimated applying Eq. (4) to the perimeters of the cuboid formed by the nearest neighbors of the solution (Fig. 3(b) shows the two-dimensional cuboid for solution i):

$$d_i^{(\mathcal{I})} = \sum_{j=1}^{q} \frac{f_j^{(\mathcal{I}_{i+1}^{(j)})} - f_j^{(\mathcal{I}_{i-1}^{(j)})}}{f_j^{\max} - f_j^{\min}} \tag{4}$$

where $\mathcal{I}$ is the index of the non-dominated front, $\mathcal{I}_i^{(j)}$ is the ith solution of $\mathcal{I}$ in the direction of the jth objective function, $f_j^{(\mathcal{I}_i^{(j)})}$ is the jth objective value of $\mathcal{I}_i^{(j)}$, $d_i^{(\mathcal{I})}$ is the crowding distance (diversity) of the ith solution of $\mathcal{I}$, $(f_j^{\min}, f_j^{\max})$ is the range of the jth objective function in the entire population, and q is the number of objective functions in the problem.

After determining the non-dominated rank and diversity of each solution as above, solution z_i can be said to be better than solution z_j if and only if (i) z_i has a better (smaller) convergence rank than that of z_j, or (ii) both have the same rank but z_i has a better (higher) diversity than that of z_j.

5.3. Crossover operation

The crossover operator, adopted for the healthcare system partitioning, draws two random solutions from the mating pool, and generates a new solution (offspring) by inserting some random microregions into one parent solution from another. It also takes care of any overlapping, during this insertion, by relabeling the partially overlapped microregions as well as other microregions. For illustrating the working procedure of the crossover operator, consider two random solutions of a map composed of 29 municipalities. As shown in Fig. 4, the solutions are named as Parent 1 and Parent 2, each of which is partitioned into 4 microregions marked by A_1 to A_4 and B_1 to B_4, respectively. A new solution, Child 1, is now generated by inserting B_4 of Parent 2 into Parent 1. In doing so, A_2 to A_4 in the child have been partially overlapped by B_4, which leads to a total of 5 microregions in the child. Finally, the microregions in the child are labeled as A_1, B_4, C_1, C_2 and C_3.

5.4. Mutation operation

Since the objectives of the healthcare partitioning problem of Parana, expressed by Eq. (2), can be achieved only by balancing the sizes of its microregions, a special mutation operator is applied to the solutions, generated by the crossover operator, for altering the sizes of the microregions by shifting some randomly chosen outer municipalities of a microregion to one of its boundary microregions. Fig. 5 shows an example of the mutation operation on Child 1 of Fig. 4 generated by the crossover operator. Two boundary municipalities of the microregion B_4 are shifted to the bordering microregion C_2, thus the size of B_4 is reduced and that of C_2 is increased. These two altered microregions are finally relabeled as D_1 and D_2, respectively.

5.5. Repairing mechanism

Since an element of the considered solution array represents a municipality of the healthcare map of the Parana State and it is assigned only one value at a time as the microregion of the representing municipality, the municipality integrity constraint, given by Eq. (3a), is automatically satisfied. If the microregion contiguity constraint, given by Eq. (3b), is violated under the proposed crossover operator, it is taken care by a labeling mechanism, which relabels a disconnected portion of a microregion as a new microregion. However, the other constraints, given by Eqs. (3c)–(3e), may get violated at any stage of initialization/

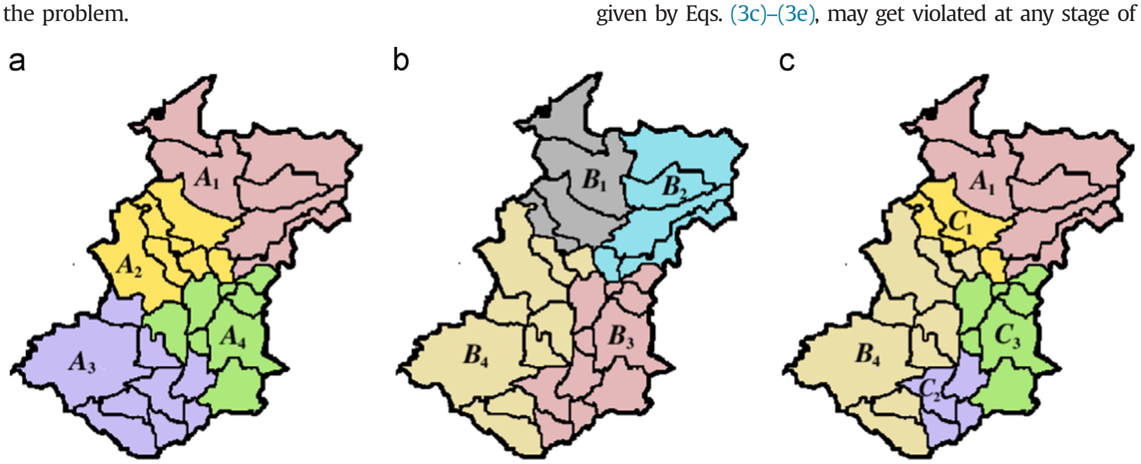

Fig. 4. Crossover operation where Child 1 is generated by inserting a microregion into Parent 1 from Parent 2. (a) Parent 1, (b) Parent 2 and (c) Child 1.

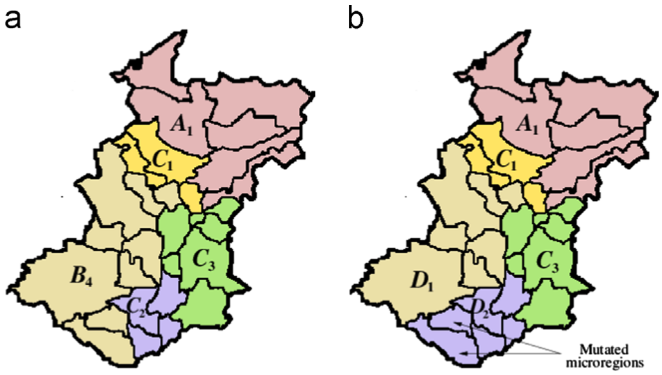

Fig. 5. Mutation operation where some municipalities are shifted from one microregion to another microregion. (a) Before mutation and (b) after mutation.

generation of a solution. Therefore, a repairing mechanism is applied here to forcibly satisfy those constraints, as much as possible. The mechanism works in four steps, as explained below, by attempting to satisfy some constraints in each step:

1. Avoiding embedded microregions – if a microregion is found fully surrounded by another microregion, as stated under the constraint given by Eq. (3c), the former is merged in the latter.
2. Maintaining the minimum size of a microregion – it attempts to have microregions at least of minimum size as specified by the lower limit in Eq. (3e), for which an under-sized microregion is eliminated by merging its municipalities in one or more adjacent microregion(s). The process is continued until all such microregions are eliminated, or the number of microregions comes down to its predefined minimum value as given by the lower limit in Eq. (3d).
3. Maintaining the maximum number of microregions – during the crossover and mutation operations, the maximum number of microregions may go beyond the limit as specified in Eq. (3d). Some extra microregions are eliminated above in Step (2). Another attempt is taken for further elimination, if required, in which a smaller microregion is merged in adjacent microregions, if they have the capacity to take more municipalities.
4. Maintaining the maximum size of a microregion – the size of an embedding microregion is increased above in Step (1) by merging the embedded microregion in it. Similarly, when eliminating a smaller microregion above in Steps (2) and (3), the sizes of some other microregions are increased. In these processes, the size of a microregion may go beyond its upper limit as specified in Eq. (3e). In that case, an attempt is made to reduce the size of an over-sized microregion by merging some of its outer municipalities in adjacent microregions, if the latter still have the capacity to take more municipalities.

If the above repairing mechanism fails to satisfy all the constraints, the penalty-parameter-less constraint handling approach, proposed by Deb [16], is applied to take care of an infeasible solution. In this approach, an infeasible solution is first made inferior to any feasible solution by assigning it a fitness value, which in a minimization problem is the sum of the fitness of the worst feasible solution and the total amount of constraint violation by the infeasible solution. After assigning such fitness values to the infeasible solutions, all the feasible and infeasible solutions are handled as feasible solutions only.

5.6. Elite preservation

Finally, the elite preserving mechanism, proposed by Deb et al. [18], is applied in order to carry out good solutions over generations. In this mechanism, both the old and new populations of a generation are first combined, and then those are ordered, from better to worse, according to their quality measured in terms of convergence and diversity as stated in Section 5.2. After that, the first 50% of the sorted combined solutions is taken as the population for the next generation. This mechanism guarantees that, even if no good solution is generated at a generation, the GA never moves opposite to the optima from the current position.

6. Numerical experimentation

The GA for partitioning the healthcare map of the Parana State, as described in Sections 4 and 5, is coded in C programming language and executed in Linux platform. In this section, the numerical computation and obtained results are analyzed in detail, emphasizing the possibility for providing with a compromised solution as per managerial implications. It is to be mentioned at this juncture that there is a long-pending desire of the concerned managers to develop a more balanced solution by re-partitioning the healthcare system in the Parana State. Hence, their own inputs have been very important and often used by researchers in order to find a compromised solution that would be very close to what they think is the best one.

6.1. Experimental setup

It is stated in Section 3 that the Parana State houses around 11 million inhabitants over its total area of 199,314.85 km^2, which is divided into 399 municipalities. The healthcare system of the State seeks to aggregate the municipalities into some bigger segments, known as microregions, based on number of inhabitants and variety of medical procedures offered in the municipalities, as well as their (municipalities) inter-connectivity. As per the formulation of the problem and its solution algorithm, presented in Sections 4 and 5 respectively, the corresponding information is gathered in the formats shown in Tables 3 and 4. The municipality-wise centroidal coordinates, number of inhabitants, number of variety of medical

Table 3
Inhabitants and medical procedures ("weights" of the municipalities), and municipality adjacency.

Municipality ID	Centroidal coordinates (x,y)	Number of inhabitants	Variety of medical procedures offered	IDs of adjacent municipalities
1	(23,304, 50,318)	7457	611	29, 85, 180, 305, 320, 335
2	(24,657, 48,990)	6224	631	40, 73, 383
3	(25,986, 49,323)	7685	424	200, 265, 295, 379
⋮	⋮	⋮	⋮	⋮
397	(26,259, 52,804)	6206	417	211, 258, 301
398	(23,866, 49,812)	19,840	629	19, 334, 349, 367, 381
399	(23,727, 53,499)	5579	521	49, 106, 263, 388, 395

Table 4
Traveling distance between a pair of municipalities (distance matrix).

Municipality ID	Distance						
	1	2	3	…	397	398	399
1	0	311	377	…	525	122	355
2	311	0	164	…	524	206	602
3	377	164	0	…	412	283	561
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
397	525	524	412	…	0	489	358
398	122	206	283	…	489	0	417
399	355	602	561	…	358	417	0

procedures offered and list of adjacent municipalities are given in Table 3, while inter-municipality traveling distances (distance matrix) is given in Table 4. Since the inter-municipality distances are provided separately, the centroidal coordinates of the municipalities are not required in the algorithm.

Since the existing healthcare map of Parana is divided into 83 microregions, for the purpose of comparison of the computational result with the existing scenario, the number of microregions into which the map is to be partitioned here is also fixed at 83, i.e., $K=83$ is used in Eq. (3d). The range of municipalities in a microregion is taken as [2, 10], i.e., $[P_{\min}, P_{\max}] = [2, 10]$ is used in Eq. (3e).

In regard of the computation, since it is well known that the performance of a stochastic optimizer (like GA) may be influenced by its various user-defined parameter values (mainly random seed defining initial solutions, crossover probability and mutation probability in the case of a GA), 90 trial runs are performed with different parameter values in order to verify the stability of the integer-coded multi-objective GA used here and to identify the best set of parameter values for the final run. The 90 trial runs are performed in three parts with 30 runs in each part. The crossover probabilities in these three parts are fixed at 90%, 80% and 75%, respectively. In the 30 runs of each part, the random seed is varied from 0.01 to 0.30 with an incremental step size of 0.01. In all the 90 trial runs, a population of 40 solutions is evolved over 500 generations. Since it is already learned that the influence of parameter values on an optimizer can be reduced at least to some extent by using self-adaption of some parameter values [11,13,15], the mutation probability in all the 90 trial runs is randomly chosen within the range of $(0, 5\%]$. Some statistical analyses of the 90 trial runs are summarized in Table 5 with better values marked in boldface. It is observed in Table 5 that the mean value of the best f_1 are almost the same in all the three parts of the trial runs, with a relatively better result for the crossover probability of 90%. The same happens to f_2 and f_3, with marginally better values for the crossover probabilities of 80% and 90%, respectively. Considering the very small values of the coefficient of variation for f_1, f_2 and f_3, calculated using Eq. (5), it can be concluded that the chosen integer-coded multi-objective GA is stable in the case of the healthcare problem of Parana.

$$\text{Variation coefficient for } f_i = \frac{\text{Standard deviation of the best values of } f_i}{\text{Mean of the best values of } f_i}; \quad i = 1, 2, 3. \qquad (5)$$

After scrutinizing the results of the trial runs as above, the following set of parameter values is considered for the final run:

Random seed = 0.03
Crossover probability = 90%
Mutation probability = (0, 5%]
Population size = 1000
Number of generations to be performed = 10,000

6.2. Results and discussion

Executing the GA with the parameter values stated above, it was observed that the number of non-dominated solutions on each generation, after the first 20 runs, varied in the range of [88,100], and the final generation came out with a Pareto front consisting of 92 non-dominated solutions. The total execution time for the population of 1000 solutions over 10,000 generations was 75 h 15 min in the Ubuntu 12.04 LTS Linux environment in a 64-bit LG Notebook with the specification of 3.8GiB, Intel Core i3-2310 M CPU @2.10 GHz × 4, 448GB HDD.

In the obtained three-objective Pareto front consisting of 92 solutions, $(f_1, f_2, f_3) = (\mathbf{74284}, 1764, 2.23183 \times 10^{11})$, $(82401, \mathbf{1817}, 2.25414 \times 10^{11})$ and $(91225, 1688, \mathbf{2.14945 \times 10^{11}})$ are the three extreme (corner) solutions, each containing the best value (shown in boldface) of one of the three objective functions, where f_1 to f_3 are the objective functions as defined in Eq. (2). The ranges of f_1, f_2 and f_3 in the entire Pareto front are [74284, 92950], [1674, 1817] and $[2.14945 \times 10^{11}, 2.26706 \times 10^{11}]$, respectively. In contrast, the set of the objective values of the existing healthcare map of Parana is $(f_1, f_2, f_3) = (99537, 1644, 2.38082 \times 10^{11})$, which is clearly much inferior to any solution of the computed Pareto front. The three extreme solutions of the Pareto front and the existing solution of Parana are presented in Table 6, along with a compromised solution and its improvement over the existing solution. Since the size of the Pareto front is large and it is difficult (also not the aim of the present study) to device an interactive-based technique for identifying a particular solution for implementation, any solution out of the many solutions of the Pareto front can be chosen as a compromised solution as per the preference information received from the managers of the healthcare authorities of Parana. Accordingly, without going through a systematic interactive procedure, $(76100.84, 1773.14, 2.21134 \times 10^{11})$ is taken here from the Pareto front as the compromised solution, which has improvement of 23.54%, 7.83% and 7.12% on f_1, f_2 and f_3, respectively, over the existing solution of Parana. It is to be noted that improvements in the ranges of [6.62%, 25.37%], [1.80%, 10.50%] and [4.78%, 9.72%] on f_1, f_2 and f_3, respectively, can be obtained over the existing solution by selecting different solutions of the Pareto front as compromised solutions.

The plot of the computed Pareto front, along with the existing and compromised solutions, of the three-objective healthcare problem of Parana is shown in Fig. 6. Since it is difficult to study a three-dimensional scenario from a two-dimensional plot, the

Table 5
Statistical results for 90 trial runs.

Runs	1–30	31–60	61–90
Population size	40	40	40
Number of generations	500	500	500
Random seed	[0.01, 0.30]	[0.01, 0.30]	[0.01, 0.30]
Crossover probability	90%	80%	75%
Mutation probability	(0, 5%]	(0, 5%]	(0, 5%]
Mean of the number of non-dominated solutions	37.2	**37.6**	36.8
Mean of the best values of f_1 (minimization)	**81,335.47**	81,596.27	82,202.84
Mean of the best values of f_2 (maximization)	1788.16	**1788.67**	1787.11
Mean of the best values of f_3 (minimization)	$\mathbf{2.20134 \times 10^{11}}$	2.20314×10^{11}	2.20474×10^{11}
Standard deviation of the best values of f_1	1,283.75	1,068.58	1,108.97
Standard deviation of the best values of f_2	8.36	7.43	7.25
Standard deviation of the best values of f_3	8.5999×10^{10}	7.6317×10^{10}	5.8364×10^{10}
As in Eq. (5), variation coefficient for the best values of f_1	0.0158	0.0131	0.0135
As in Eq. (5), variation coefficient for the best values of f_2	0.0047	0.0042	0.0041
As in Eq. (5), variation coefficient for the best values of f_3	0.39	0.35	0.26

Table 6
Extreme solutions of the Pareto front, existing solution and a compromised solution of the healthcare map of Parana.

Objective functions	f_1	f_2	f_3 ($\times 10^8$)
Solution with the best (minimum) value of f_1	**74,284.14**	1,764.22	2,231.83
Solution with the best (maximum) value of f_2	82,401.13	**1,816.66**	2,254.14
Solution with the best (minimum) value of f_3	91,224.58	1,688.01	**2,149.45**
Existing solution	99,536.76	1,644.34	2,380,82
Compromised solution	76,100.84	1,773.14	2,211.34
Improvement in the compromised solution $\left(=\frac{\lvert\text{Existing solution}-\text{compromised solution}\rvert}{\text{Existing solution}}\times 100\%\right)$	23.54%	7.83%	7.12%

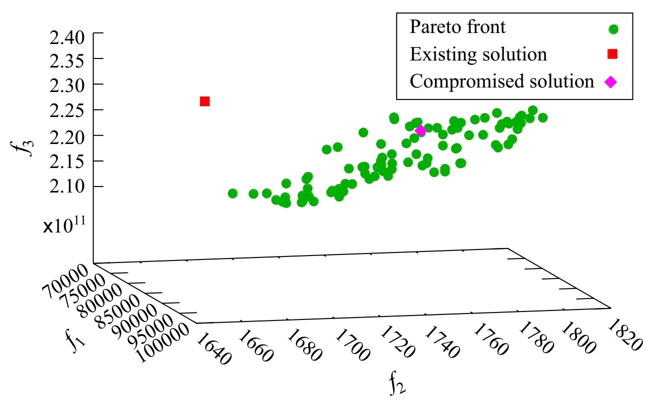

Fig. 6. Computed Pareto front along with the existing and compromised solutions of the healthcare map of Parana.

value path plots of the Pareto front and the existing and compromised solutions are shown in Fig. 7 by normalizing the objective values in the range of [0, 1], where the horizontal axis represents the objective functions and the vertical axis shows their values for different solutions (Deb [17] may be referred for detail of the value path plot). The value path plots in Fig. 7 show the diversity among different solutions of the healthcare map of Parana, as well as the crossing lines depict clearly the trade-off (conflicting) nature of the considered objective functions, i.e., no single solution can be the best one in terms of all the objective functions, and one solution can be improved in an objective value only if it is degraded at least in another objective value. Fig. 7 also shows that, in comparison to the solutions of the computed Pareto front, the existing solution in practice is the worst in all the three objective functions, while the chosen compromised solution has some balanced values of the objective functions.

Finally, the distributions of the municipalities among different microregions of the existing and compromised solutions are given in Table 7, and also shown through histogram plots in Fig. 8. It is observed that the number of municipalities in different microregions of the compromised solution varies in the range of [2, 8] in contrast to the huge range of [1, 19] that in the existing solution. Municipalities in different microregions of the compromised solution are obtained in the range of [2, 8] against the range of [2, 10] set for the same in Eq. (3e). It is to be noted that the municipalities can easily be distributed in any other desirable range by setting suitable values in Eq. (3e).

6.3. Managerial implications

All the solutions contained by the computed final Pareto front are feasible and non-dominated with respect to each other, i.e., none is better than another solution in all the objective functions. Hence, any solution of the Pareto front can be selected for implementation. It is important to note that, however, many efficient solutions might be lesser desirable by the management due to the preferences of the managers and several other factors not taken into consideration when defining the problem. Such factors could include historical reasons, past partnerships, political reasons, or other administrative situations which would require some municipalities to belong to the same or different microregion(s). Therefore, it is a managerial duty to select the most appropriate solution. Fig. 9 shows the map of the compromised solution taken here, out of the 92 efficient solutions of the computed Pareto front, based on the preference information received from the concerned managers. As seen in Fig. 9, each of

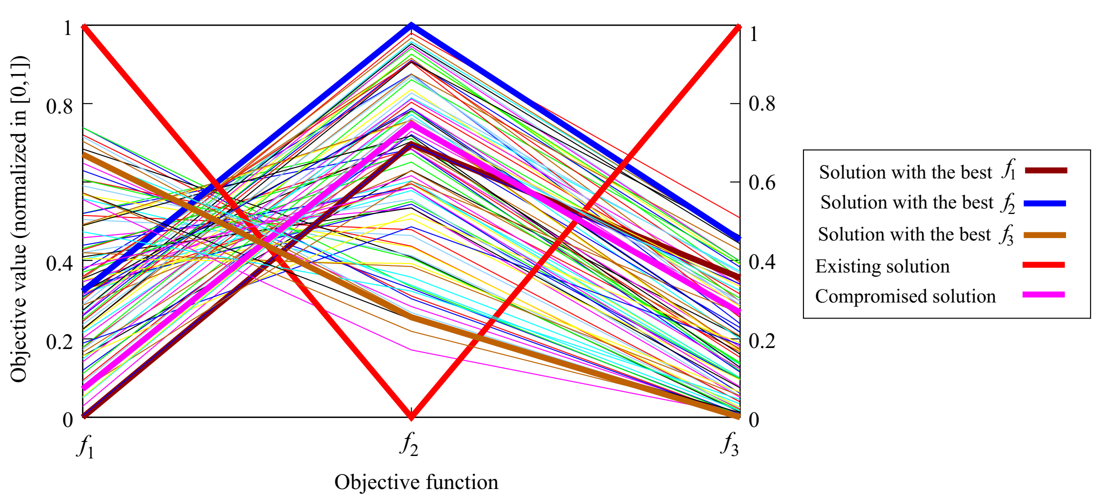

Fig. 7. Value path plots of the Pareto front, and the existing and compromised solutions of the healthcare map of Parana.

Table 7
Distribution of the municipalities among the microregions.

Existing solution			Compromised solution		
Number of microregions	Number of municipalities in a microregion	Total number of municipalities	Number of microregions	Number of municipalities in a microregion	Total number of municipalities
13	1	13	8	2	16
16	2	32	14	3	42
11	3	33	12	4	48
8	4	32	15	5	75
9	5	45	22	6	132
9	6	54	10	7	70
2	7	14	2	8	16
2	8	16	–	–	–
5	9	45	–	–	–
2	11	22	–	–	–
1	12	12	–	–	–
1	14	14	–	–	–
1	15	15	–	–	–
1	16	16	–	–	–
1	17	17	–	–	–
1	19	19	–	–	–
Total 83	–	399	83	–	399

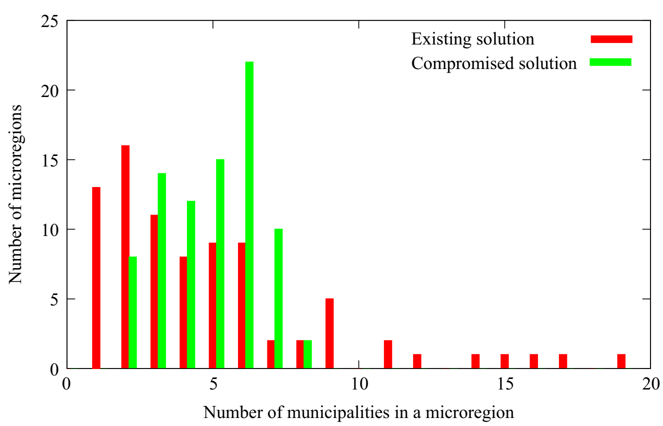

Fig. 8. Histogram plots of the number of microregions versus number of municipalities in a microregion.

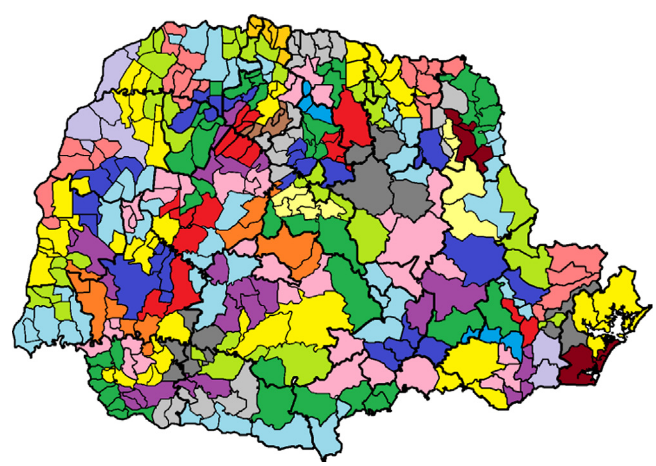

Fig. 9. Compromised optimum map of the healthcare system of Parana, partitioning 399 municipalities into 83 microregions based upon the preferences of the concerned authorities.

the five constraints (integrity, contiguity, embedding, number of microregions, and size of a microregion as defined in Eq. (3) is clearly obeyed in the compromised map. At a first glance, one may ask why the compromised map has some peculiar microregions, as they are not round in shape and some are apparently crossing paths. It is to be mentioned that desired shapes of the microregions could be taken into account by imposing different compactness constraints on the microregions as done by Datta et al. [13,15] in their studies with the healthcare system in East England and the census metropolitan area of Ontario in Canada, respectively. However, the only primary objectives in this study were the homogeneity of inhabitants in the microregions, maximization of the variety of medical procedures in different microregions, and minimization of the inter-microregion traveling distances. Meetings with the healthcare authority SESA also reinforced the need to seek partitioning considering the three objectives studied in this work. Although SESA has attempted for many years to develop a good partitioning, due to many technical, administrative and political complexities, the best map that SESA could obtain is the one shown in Fig. 1, which is found very inferior to any solution of the Pareto front computed in the present study.

It is important to mention that, if implemented, the compromised map presented in Fig. 9 should be re-evaluated regularly. The partitions are to be updated if there is substantial change in the number of inhabitants or variety of medical procedures offered in any municipality. Since the change in the population size is not so radical, and the changes in the variety of offering medical procedures are also not so frequent as funding is provided only on year-basis, re-evaluation of the partitioning can be carried out at the earliest once in a year. Since it is a decision strategy, the computational time to be required in obtaining an optimized solution may not be an issue in that case.

In the managerial level, the variety of medical procedures offered in different municipalities should be dealt with priority as the maximization of the number of types of offering medical procedures is a very important objective. With more procedures offered, faster medical assistance can be provided in each microregion by reducing inter-microregion transport of patients for procedures, thus resulting to reduced suffering, time and travel expenses. By analyzing the data of the proposed compromised solution, it is observed that the number of types of offered medical procedures in the microregions varies from 498 to 4,499. This variation shows that many microregions do not offer even the required minimum number of varieties. There are 10 microregions with less than 1000 different procedures being offered, and increasing that number in those microregions would be a wise move. The management can launch a program for improving the health assistance to the inhabitants by attempting to offer services in different microregions in a uniform manner. Such a program may aim to deal with three factors: facilities, equipment and people.

7. Conclusions

In this paper, an application of a multi-objective optimization approach is presented for dealing with a real-world spatial problem of aggregating the municipalities, of the Parana State in Brazil, into some microregions. The partitioning is needed for better management of the public health services in the State, especially since most of the SUS patients cannot pay to alternative service providers. The 399 municipalities of 11 million inhabitants of Parana are currently divided into 83 microregions. Accordingly, in the numerical experimentation of this work, the municipalities are partitioned into 83 microregions so as to compare the proposed solutions with the existing map.

The proposed approach involves the formulation of three conflicting objective functions in relation to the microregions (homogenizing the population size in the microregions, maximizing the variety of offered medical procedures in the microregions, and minimizing the inter-microregion traveling distances). The objectives are also subject to five basic constraints (integrity of a municipality, contiguity of a microregion, non-embedding of a microregion, number of microregions, and size of a microregion). The problem is solved by a customized version of an existing integer-coded multi-objective genetic algorithm. Different algorithmic parameter values are considered based on a set of trials. The computed final Pareto front contains 92 efficient solutions, all of which are feasible and non-dominated with respect to each other (i.e., none is better than another). It is also found that all the 92 solutions of the Pareto front are much superior to the existing healthcare system map of Parana. In accordance with the preference of the local healthcare authority, a solution taken from the Pareto front is also presented as a compromised solution for the problem, which has objective values very close to that what the authority feels to be the best one. The presented solution reflects the *status quo* at the time of the data collection and the beliefs of the health system management.

If there is any social, technical, administrative or political change, a new solution may be obtained by solving the proposed model of the problem. According to the health specialists, the obtained results could be very helpful in providing better and more homogeneous health services to the population as a whole acting as a powerful managerial tool that could be extended to other States in Brazil, which like the Parana State are in need of improvements to their healthcare systems. The complexity of the task itself explains the importance of the use of an optimization tool for obtaining a better solution for the problem. Additional objectives can also be considered for the problem, such as the age and birth rate of inhabitants (older people and babies require more services), and employment status and income level of inhabitants, among others.

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