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An effective greedy method for the Meals-On-Wheels service districting problem

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ABSTRACT

This paper focuses on a specific districting problem related to home health care (HHC) services, that is, the Meals-On-Wheels service districting (MOWSD) problem which is formulated as an integrated mixed-integer programming (MIP) model. The MOWSD problem aims at finding the minimum number of districts to cover all basic units while satisfying the constraints, including capacity and time window limitation, accessibility, compactness, and the indivisibility of locations. Inspired from the thought of the planner who has to solve the MOWSD problem in practice, an effective greedy heuristic method is proposed to quickly construct good districts. The results indicate that the greedy heuristic method can achieve as many good solutions as the Gurobi Optimizer, which is applied to solve the MIP, but with an extremely shorter computation time. We firstly conduct the sensitivity analysis on the key parameters of the MOWSD problem, which reveals that the performance is significantly affected by the available time period for the service delivery, the capacity of a meal cart, and the maximum travel duration between any two basic units in a district. We further compare the resulting districts determined using these two methods with the existing districts identified using manual planning. This comparison claims that the proposed greedy heuristic method is capable of not only improving the design of districts but also achieving better compactness than Gurobi Optimizer.

1. Introduction

The number of elderly citizens in Hong Kong has grown at an average rate of 4.8% over the past 50 years (i.e., from 1961 to 2011). Specifically, the proportion of the elderly in the total population rose from 2.8% in 1961 to 13.3% in 2011, and the elderly dependency ratio increased from 50 in 1961 to 177 in 2011 (CENSUS2011.GovHK., 2013). The life expectancy of the elderly also changed from 67.8 years for males and 75.3 years for females in 1971 to 81.2 years for males and 86.9 years for females in 2014 (CHP.GovHK., 2016). The increasing life expectancy corresponds to an increase in the demand for elderly care. Furthermore, the elderly prefer to grow old in the privacy of their homes rather than in a nursing home, whereas their relatives show a decreasing willingness to perform informal care because of work commitments. Accordingly, organizations providing home health care (HHC) are inclined to optimize their activities to meet the constantly increasing demand for HHC. The Hong Kong Government, in particular, has increased the amount of the annual Elderly Health Care Voucher to HK$2000 per elderly (aged 70 years or above); such increase is aimed at encouraging the elderly to choose a private health care service that suits their needs, seek consultation, and establish a close relationship with private care providers who are familiar with their health conditions (GovHK., 2015). Thus, exploring ways to enhance productivity by reducing cost and improving service quality has become necessary for HHC structures. The Salvation Army-Tai Po Integrated Home Care Service Center (SA-TPIHCS) in Hong Kong is a "social profit" organization providing HHC services, such as personal care, household care, nursing care, and Meals-On-Wheels (MOW) service. In this paper, we focus on the MOW service districting (MOWSD) problem faced by the SA-TPIHCS.

MOW service providers deliver meals to individuals who are at home and unable to purchase or prepare their own meals (Wikipedia., 2016). In the MOW service delivery procedure, vehicles typically start from the same central kitchen to deliver meals to customers; they then pick up the packing and return it to the kitchen. The procedure consists of two steps: first, the vehicles

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Nomenclature

Abbreviations

ANOVA	Analysis of Variance
CCP	Capacitated Clustering Problem
GA	Genetic Algorithm
g-CCCP	Generic Capacitated Centered Clustering Problem
GRASP	Greedy Randomized Adaptive Search Procedure
HHC	Home Health Care
HHCD	Home Health Care Districting
MIP	Mixed-Integer Programming
MOW	Meals-On-Wheels
MOWSD	MOW Service Districting
P&D	Pickup and Delivery
SA	Simulated Annealing
SA-TPIHCS	Salvation Army-Tai Po Integrated Home Care Service center
VNS	Variable Neighborhood Search
VRPTWMD	Vehicle Routing Problem with Time Window and Multiple Deliverymen
TS	Tabu Search

Notations

OBJVAL	the value of the objective function of the proposed MIP model
GAP	the gap between the current optimal objective value and the best lower bound
RUNTIME	the computation time of the proposed MIP model
WM	the maximum walking duration among basic units in the same district
Q	the capacity of a meal cart
T	the total available duration of MOW service

travel from the kitchen to the parking sites of districts; second, the deliverymen (care workers) deliver the meals to the customers' locations on foot. From the daily operational perspective, the process can be modeled as a vehicle routing problem with time window and multiple deliverymen (VRPTWMD) (Pureza, Morabito, & Reimann, 2012), the inputs of which consist of the known parking sites of customer districts and the known service times of each district. The former depends on customer locations, and the latter depends on crew size, deliveryman service strategy, and the demand and geographical dispersion of customers in districts. As a long-term planning strategy, the districting decision of customers can be the base of the VRPTWMD problem, the clustered capacitated vehicle routing problem (Expósito-Izquierdo, Rossi, & Sevaux, 2016), and the capacitated location routing problem (Yu, Lin, Lee, & Ting, 2010). Generally, the districting problem in the service context is to partition the customers in the service region into a number of districts such that each district satisfies a collection of capacity, temporal, and geometric constraints (Jarrah & Bard, 2012). Once the districts are determined, each of them is assigned with one care worker. Planners generally employ the average monthly data of customer demands to design the work areas without considering the dynamic and uncertain nature of customer demands (Jarrah & Bard, 2011).

In this paper, we focus our attention on MOWSD problem because of their critical role as the base of daily operations and their importance in improving the MOW delivery efficiency. As a result of the sustainability of using care workers, the capacity of a meal cart is the primary determinant of district size. Furthermore, the expected time duration of serving all customers in a district and the desire to achieve higher workload utilization must be considered. The challenge faced by the SA-TPIHCS lies in determining a feasible partition of the service region resulting in the minimum number of districts, which leads to the minimum number of care workers, each of whom is responsible for one district, to satisfy all customer demands. The objective of the MOWSD problem is therefore achieve the minimum number of districts to cover all basic units while considering different constraints, such as accessibility, compactness, indivisibility of locations, capacity of districts, and time limitation of service delivery. As a minimum capacitated districting problem, it mainly focuses on district design without regard to tour construction, while VRPTWMD considered by Pureza et al. (2012) focused on tour construction without considering district design. The difference of the MOWSD problem from related districting problems in literature lies in its exclusive focus on the minimum number of districts, whereas minimizing either the workload imbalance or total travel distances was considered by Benzarti, Sahin, and Dallery (2013), or minimizing the dissimilarity and the minimum number of districts simultaneously was studied by Bard and Jarrah (2009). The main contributions of this study are presented as follows:

(1) An integrated mixed-integer programming (MIP) model is proposed and solved by the Gurobi Optimizer for the MOWSD problem. To the best of our knowledge, it is the first study that the operational aspects, such as compactness, the selection of the parking site of each district, and packages and delivery duration limitation, are integrated into an MIP model in the literature of the districting problems in HHC context.
(2) An effective greedy heuristic method, inspired from the thought of the planner of SA-TPIHCS, is proposed for the MOWSD problem. Given that the Gurobi Optimizer requires a long computation time to generate good feasible solutions, the greedy heuristic method is applied to find a good feasible solution with fewer districts in a shorter computation time.
(3) A case study based on a real instance from the SA-TPIHCS is investigated. In the case study, we conduct a sensitivity analysis on some key parameters of the MOWSD problem related to the minimum number of districts and computation time. Furthermore, the existing districts obtained with manual planning are compared with the resulting districts to illustrate the improvement of the district design, which was achieved with the application of the proposed MIP model and greedy heuristic method.

This paper is organized as follows. In Section 2, we present a survey of the literature related to our work, which seeks to explore the districting problems from different aspects. In Section 3, we describe a formal problem statement and formulate the MOWSD problem as an MIP model with objective functions and constraints, followed by the model data preparation and model analysis. In Section 4, we explain the design and analysis of the proposed greedy heuristic method. In Section 5, we detail the results of computational experiments conducted on real data provided by the SA-TPIHCS in Hong Kong. We also verify the proposed model and greedy heuristic method by comparing the resulting and existing districts. Finally, in Section 6, we discuss our conclusions and perspectives for future research.

2. Literature review

The MOWSD problem falls into the category of the districting problem intended for grouping small geographic areas (also called basic units) into large geographic clusters (also called districts) that are balanced, contiguous, and compact according to relevant criteria (Benzarti, Sahin, & Dallery, 2010; Kalcsics, 2015). Especially, it falls into two mKalcsics, 2015ajor areas of districting problems, namely, service districting and distribution districting according to the classification work (Kalcsics, 2015), given that it bears several similarities to the applications in the service and distribution contexts. The first type of application treats the problem of designing districts for HHC structures, called as HHC districting (HHCD) problem (Benzarti et al., 2010, 2013; Blais, Lapierre, & Laporte, 2003). The other important application is the design of P&D districts in logistics (Jarrah & Bard, 2011, 2012; Lei, Laporte, & Bo, 2012). Workload in these applications always includes the service time and the average travel time within a district and to a centralized depot (Jarrah & Bard, 2012; Lei et al., 2012).

A districting problem mainly consists of two components, according to the definition (Kalcsics, 2015), basic unit and district. One main attribute associated to basic unit is the distance to other basic unit, which are typically Euclidean (Jarrah & Bard, 2012) or road distance (Ríos-Mercado & Salazar-Acosta, 2011) which can properly reflect obstacles, such as rivers or mountain ranges (Kalcsics, 2015). By setting the maximum distance between any two basic units in a district, the neighborhood or contiguity graph denoted with binary parameters was used (Benzarti et al., 2010, 2013). Moreover, activity measures are often associated with basic unit such as workload consisting of service time and estimated travel time for visiting a basic unit (Benzarti et al., 2010; Jarrah & Bard, 2012). The number of districts can either be fixed in advance, e.g., Benzarti et al. (2013) assumed that it is predetermined by HHC managers, or be subject to planning, e.g., the minimal number of vehicles to make all pickups and deliveries within the legal limit (Jarrah & Bard, 2012). Similarities and differences exist in terms of the components of the MOWSD problem. First, the distance between two basic units is measured on the basis of road distance specified in Google Maps. Second, the activity measures similar to those in (Bard & Jarrah, 2009) are associated with each basic unit. Third, the number of districts is the target to be minimized, and the parking site of the district is supposed to be the closest basic unit to the depot, which is an outcome of the districting model as well (Jarrah & Bard, 2012).

The districting problems always employ a list of typical criteria, a classification of which was proposed by Benzarti et al. (2010), presented in Table 1. These categories of criteria, related to different aspects, such as geographical aspects, activity measures, comparison between different territory partitions, and organizational criteria, are formulated into objective functions or constraints in the existing districting problems. For example, Benzarti et al. (2013) formulated the HHC districting problem as two MIP models by considering the indivisibility of basic units, compactness, compatibility as constraints in both models and workload balance either as constraints in Model 2 or as objective functions in Model 1. Their models are aimed at either balancing the personnel care workload (Model 1) or minimizing the travel distance to reach patients (Model 2). A study of the design of pickup and delivery (P&D) work areas for drivers was conducted with the goal of partitioning a set of customers into a minimum number of contiguous clusters that satisfy criteria such as capacity limitation, contiguity, and working time limitation of drivers (Jarrah & Bard, 2011, 2012). They assumed that the parking site of the cluster was the closest customer to the depot in the cluster (Jarrah & Bard, 2012). The criteria considered in the MOWSD problem are contiguous and geographically compact districts (Butsch, Kalcsics, & Laporte, 2014), meal time restriction of all customers (D'Amico, Wang, Batta, & Rump, 2002; Jarrah & Bard, 2012), capacity limitation and maximal or average walking duration for care workers (Ferland & Guénette, 1990), and minimum number of districts and indivisibility of basic units (Jarrah & Bard, 2012). One characteristic is that care workers must deliver meals directly to the customers' homes on foot (Pureza et al., 2012).

Districting problems in practice are mainly modeled as a capacitated clustering problem (CCP) introduced by Mulvey and Beck (1984) and further studied (Ahmadi & Osman, 2004, 2005; Lorena & Senne, 2004; Scheuerer & Wendolsky, 2006; Yang, Chen, & Chu, 2011). A variant of the CCP known as the

Table 1
Comparison between different studies.

Study	Criteria				Obj.	Method		
	Geographical aspects	Activity measure	Compare partition	Organization criteria		Exact	Heuristic	meta-heuristic
Benzarti et al. (2010, 2013)	Compatibility; Compactness; Conformity of districts	Workload (care load and travel load)	No	Pre-determined number of districts; Indivisibility of basic units	Balance care and travel load	MIP models	No	No
Blais et al. (2003)	Respect of borough boundaries; Connectivity; Mobility	Workload (care load and travel load)	Districts by manual and by TS	Known number of districts; Indivisibility of basic units	Mobility level and workload balance	No	No	Tabu search
Jarrah and Bard (2011, 2012) and Bard and Jarrah (2009)	Contiguity; Compactness	Workload (service load, travel load); Capacity limitation of districts	No	Indivisibility of customers; Selection of center of district; Delivery duration limitation	Squared distance of each customer to center of assigned district; Total number of districts created	Mixed-integer goal; Set covering	Heuristic column generation	Grid search procedure
Negreiros and Palhano (2006)	Similarity	Capacity of clusters	No	Indivisibility	Number of clusters; Sum of dissimilarity	Mathematical programming	Constructive phase	VNS
Our study	Compactness; Compatibility	Workload (service load, travel load); Capacity limitation of districts	Districts by manual and by greedy heuristic	Indivisibility of customers; Selection of center of district; Delivery duration limitation	Number of districts created	MIP model	Greedy heuristic method	No

p-centered capacitated clustering problem arises when the median is replaced by the centroid (Chaves & Lorena, 2010; Chaves & Nogueira Lorena, 2011; Negreiros & Palhano, 2006). The MOWSD problem is similar to the minimum capacitated clustering problem (MCCP) (Jarrah & Bard, 2011, 2012) applied to the P&D work area and the generic capacitated centered clustering problem (*g*-CCCP) applied to sales force territorial design (Negreiros & Palhano, 2006). Both studies simultaneously minimize dissimilarity and the number of clusters, while the MOWSD problem only minimizes the number of districts. Many different approaches for the districting problems have been proposed in the literature. These approaches can roughly be divided into exact methods utilizing a mathematical programming model (Benzarti et al., 2010, 2013), such as location-allocation and set partitioning methods (D'Amico et al., 2002; Hess, Weaver, Siegfeldt, Whelan, & Zitlau, 1965; Jarrah & Bard, 2011, 2012; Ríos-Mercado & Fernández, 2009), and those depending merely upon heuristic methods, such as geometric algorithms (Fortune, 1997; Kalcsics, Nickel, & Schröder, 2005; Ricca, Scozzari, & Simeone, 2008), simple construction methods (Bozkaya, Erkut, & Laporte, 2003; Negreiros & Palhano, 2006), and meta-heuristic methods, examples of which include the column generation approach (Jarrah & Bard, 2011; Lorena & Senne, 2004), genetic algorithm (GA) (Agustín-Blas, Salcedo-Sanz, Ortiz-García, Portilla-Figueras, & Pérez-Bellido, 2009; Forman & Yue, 2003), simulated annealing (SA) (D'Amico et al., 2002) and Tabu Search (TS) (Bozkaya et al., 2003; Osman & Christofides, 1994), scatter search (Scheuerer & Wendolsky, 2006), variable neighborhood search (VNS) (Negreiros & Palhano, 2006), and greedy randomized adaptive search procedure (GRASP) (Ahmadi & Osman, 2005; Roger Z. Ríos-Mercado & Fernández, 2009). These methods maximize their flexibility to include nearly any practical criterion and measure for the design of districts.

In the present study, the MOWSD problem can be considered as an MCCP, with the indivisibility of basic units, compactness and compatibility constraints in the HHCD problem (Benzarti et al., 2013), and with the constraints for the selection of parking sites of each district, capacity limitation constraints, and working time limitation constraints in the P&D work area (Jarrah & Bard, 2012), but without the minimization of the dissimilarity in the objective function (Bard & Jarrah, 2009). As shown in Table 1, in which comparison is only conducted among the most relevant studies, the MOWSD problem integrates the geographical criteria of districting problem in HHC context and the activity measures and organizational criteria of districting problem in P&D area. The compactness criterion leads the long computation time of exact methods adopted in optimizer solvers, such as Cplex (Benzarti et al., 2013). Our study also compares the results of greedy heuristic method with the existing districts. In terms of objective, most studies in literature focus on multi-criteria objectives, such as dissimilarity and the number of districts at the same time (Jarrah & Bard, 2012; Negreiros & Palhano, 2006), or the care load balance and minimization of travel load at the same time (Benzarti et al., 2013). However, our study focuses on minimizing the number of districts created to cover all basic units. Note that the MOWSD problem is formulated on the basis of the operation in SA-TPIHCS, the good districts should be reasonable and obtained in short computation time. Therefore, inspired from the thought of planner, an effective greedy heuristic method is developed to construct good districts, because it is simple and easy to implement when considering the characteristics of the MOWSD problem.

3. Modeling the MOWSD problem

The SA-TPIHCS extensively provides various services to patients, one of which is the MOW service. Fig. 1 presents the work flowchart of a care worker, which reflects that the number of care workers for the MOW delivery is dynamic and affects the daily schedule.

Therefore, having a good knowledge of the minimum number of care workers required for MOW delivery is necessary for the scheduler to improve the practicability of the daily schedule of care workers and vehicles in use. The first step is to design good districts. Such design would allow the improvement of service quality

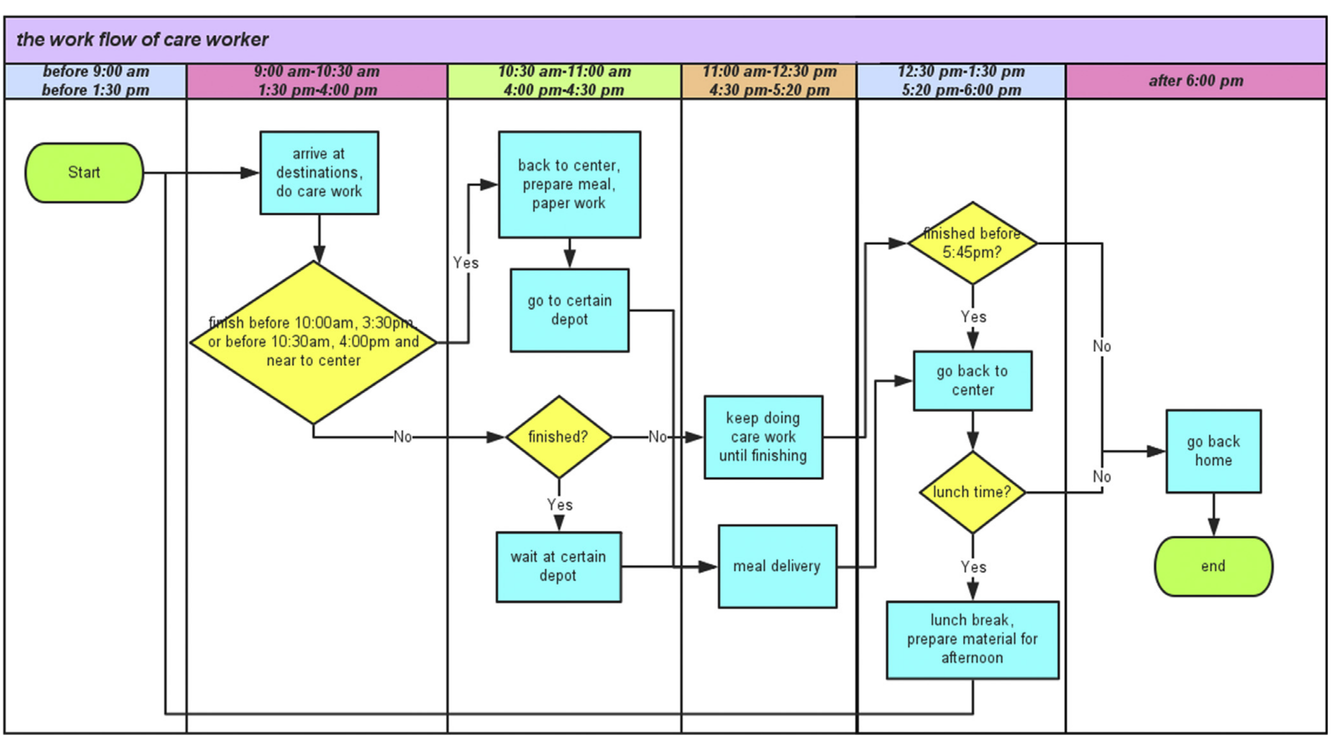


Fig. 1. Work flowchart of a care worker.

for the benefit of customers and care workers. The grouping of customers in a district indeed increases the reactivity of care workers (e.g., in case of emergencies), which in turn leads to customer satisfaction. Furthermore, the design would induce the reduction of travel time and consequently improve the efficiency of the MOW delivery process.

3.1. Problem definition

Given a service region, i.e., Tai Po, Hong Kong, and a set of customers who may be co-located at the same basic unit as shown in Google Maps, the MOWSD problem is aimed at grouping customers' locations into as few good districts as possible according to relevant criteria. Delivery is followed by the immediate retrieval of meal boxes over the planning horizon, a typical month with a six-day working week.

3.1.1. Assumptions

By counting as many practice operational constraints as possible and without losing generality, we make the following assumptions:

(1) The number of customers requesting for meal services is known in advance. The average of meals per delivery for each customer on any working day is also known.
(2) Customers living in the same location may be aggregated into different basic units depending on the total number of customers living in the same location and the capacity of a meal cart. After aggregation, the package and workload of each basic unit is calculated on the basis of the data provided by customers (Bard & Jarrah, 2009).
(3) All the basic units are covered. That is, all customers admitted to the HHC structure and request for MOW service are assigned to districts. The basic units with different transit modes cannot be assigned in the same district.
(4) Care workers delivering meals to customers are homogeneous in terms of skills and workload capacity.
(5) The duration of travel between the parking site of a district and a depot (the location of the HHC structure) is measured by the driving duration between the depot and its closest basic unit in the district.

3.1.2. Criteria considered

To effectively capture the districting problem, we summarize the operational requirements from the following criteria:

(a) The compactness that can be formulated as a hard constraint by limiting the maximum walking/driving duration between any two basic units in the same district.
(b) The indivisibility of basic units, each of which must be assigned to one and only one district. The criterion is considered to avoid an overlap between care workers' responsibilities and to establish long-term relationships with customers.
(c) The number of districts to design is subject to minimization according to operational factors.
(d) The capacity limitation should be satisfied because maintaining the sustainability of care workers by limiting the maximum number of meal packages is important.
(e) The delivery time period limitation should be obeyed to ensure the satisfaction of customers, who expectedly prefer hot meals.

3.2. Modeling the MOWSD problem

We model the MOWSD problem as an MIP model according to the concerns from SA-TPIHCS. The model can be used in cases in which a manager wants to ensure that the number of care workers available for MOW delivery at certain time periods satisfies the customer demand.

3.2.1. Notations

We present the parameters and decision variables for modeling the MOWSD problem in Table 2.

3.2.2. Proposed MIP model

$$\text{Minimize} \quad f = \sum_{k=1}^{M+V} y_k \tag{1}$$

$$\text{Subjectto}: \quad \sum_{k=1}^{M+V} x_{ik} = 1, \quad \forall i = 1,2,\ldots,N \tag{2}$$

$$\sum_{i=1}^{N} q_i x_{ik} \leqslant Q_k, \quad \forall k = 1,2,\ldots,M+V \tag{3}$$

$$x_{ik} + x_{jk} \leqslant y_k, \quad \forall (i,j) \in E, \quad \forall k = 1,2,\ldots,M+V \tag{4}$$

$$\sum_{i=1}^{N} t_i x_{ik} + \zeta_k \leqslant T, \quad \forall k = 1,2,\ldots,M+V \tag{5}$$

$$\zeta_k \leqslant d_{0i} x_{ik} + h_{max}(1 - x_{ik}), \quad \forall i = 1,2,\ldots,N; \quad \forall k = 1,2,\ldots,M+V \tag{6}$$

$$\zeta_k = \sum_{i=1}^{N} d_{0i} z_{ik}, \quad \forall k = 1,2,\ldots,M+V \tag{7}$$

$$\sum_{i=1}^{N} z_{ik} = y_k, \quad \forall k = 1,2,\ldots,M+V \tag{8}$$

$$z_{ik} \leqslant x_{ik}; \quad \forall i = 1,2,\ldots,N; \quad \forall k = 1,2,\ldots,M+V \tag{9}$$

$$x_{ik} \leqslant \lambda_{ik}, \quad \forall i = 1,2,\ldots,N; \quad \forall k = 1,2,\ldots,M+V \tag{10}$$

$$x_{ik} \leqslant y_k, \quad \forall i = 1,2,\ldots,N; \quad \forall k = 1,2,\ldots,M+V \tag{11}$$

$$x_{ik} \in \{0,1\}; z_{ik} \in \{0,1\}; \quad \forall i = 1,2,\ldots,N; \quad \forall k = 1,2,\ldots,M+V \tag{12}$$

$$y_k \in \{0,1\}, \zeta_k \in \{0, h_{max}\} \quad \forall k = 1,2,\ldots,M+V \tag{13}$$

The objective function is designed to minimize the total number of districts created, as mentioned in Criterion (c). According to Assumption (4), which states that care workers are homogeneous, except for their transit modes, the cost of district k is constant and assumed to be 1. The objective function is then formulated in Eq. (1). Constraint (2) ensures the individuality of basic units, as described in Criterion (b), that is, each basic unit is assigned to one and only one district. Constraint (3), according to Criterion (d), limits the capacity of district k on the basis of Assumption (1). Constraint (4) is related to the compactness criterion, as shown in Criterion (a), in which the duration between two basic units assigned to the same district ($m_i = m_j$) is bounded by the walking duration w_{max} or driving duration d_{max}. Constraint (5) limits the total available time for MOW delivery according to Criterion (e). It consists of three parts, namely, the service time of basic units, the travel time within districts, and the driving time from the depot to the parking site of districts ζ_k. The former two parts are estimated as t_i for basic unit i. The calculation of t_i is on the basis of Assumption (2) and similar to the calculation in (Bard & Jarrah,

Table 2
Notations for the MOWSD problem.

Parameters	
$\boldsymbol{T}$	Total time available for MOW delivery at lunch or dinner
N	Total number of basic units in the service area
M	Upper bound on the number of care workers on foot
V	Upper bound on the number of care workers on board vehicles
$\boldsymbol{B_l}$	The set of basic units visited by walking $l = 0$, or by driving $l = 1$
$\boldsymbol{Q_k}$	Capacity of district k (meal cart capacity), equal to a constant Q for all districts in the case study
$\boldsymbol{q_i}$	Average number of meal packages to be delivered to the basic unit i at each meal period
$\boldsymbol{t_i}$	Estimated workload of serving at basic unit i (service time plus travel time to the next basic unit)
$\boldsymbol{m_i}$	Available travel mode to visit basic unit i. $m_i = 1$ if basic unit i is mainly visited by a care worker in driving mode, and $m_i = 0$ if the visiting care worker is in walking mode
$\boldsymbol{\lambda_{ik}}$	Compatibility of basic unit i to be assigned to district k. $\lambda_{ik} = 1, if m_i = 0, k \in [1, M] or m_i = 1, k \in [M+1, M+V]$ (basic unit i can be assigned to district k); otherwise, $\lambda_{ik} = 0$
$\boldsymbol{w_{ij}}$	Walking duration between basic units i and j
$\boldsymbol{w_{max}}$	Maximum walking duration allowed between two basic units in the same district
$\boldsymbol{d_{max}}$	Maximum driving duration allowed between two basic units in the same district
$\boldsymbol{d_{ij}}$	Driving duration between basic units i and j
$\boldsymbol{d_{0i}}$	Driving duration between depot and basic unit i
$\boldsymbol{h_{max}}$	Maximum driving duration from depot to the farthest basic unit
$\boldsymbol{a_{ij}}$	Non-compactness value of basic units' pair (i, j). $a_{ij} = a_{ji} = 1$ if $w_{ij} > w_{max}$ or $d_{ij} > d_{max}$ when $m_i = m_j$; otherwise, $a_{ij} = a_{ji} = 0$
$\boldsymbol{c_{ij}}$	Duration between basic unit i and j. If mode = 0, c_{ij} is equal to w_{ij}; otherwise, c_{ij} is equal to d_{ij}
E	Set of basic units' pair (i, j), where $(i, j) \in E$ if and only if $a_{ij} = a_{ji} = 1$
Decision variables	
$\boldsymbol{x_{ik}} = \begin{cases} 1, & \textit{if the basic unit } i\ (i = 1, 2 \ldots, N) \textit{ is assigned to district } k\ (k = 1, 2, \ldots, M+V), \\ 0, & \textit{otherwise}. \end{cases}$	
$\boldsymbol{y_k} = \begin{cases} 1, & \textit{if district } k \textit{ is selected}, \\ 0, & \textit{otherwise}. \end{cases}$	
$\boldsymbol{z_{ik}} = \begin{cases} 1, & \textit{if } d_{0i} \textit{ is minimum for all } i \textit{ that } x_{ik} = 1 \textit{ in the solution}, \\ 0, & \textit{otherwise}. \end{cases}$	
$\boldsymbol{\zeta_k}$	Integer decision variables (estimated driving duration from depot to district k).

2009). Constraints (6) to (9) guarantee that exactly one basic unit in district k is chosen as the basis for ζ_k and that the basic unit in the district is the closest one to the depot according to Assumption (5). Constraints (10) and (11) define the availability of basic units to districts. Constraints (12) and (13) define the binary and integer decision variables.

3.3. Model analysis

We assume that the number of care workers is always adequate under the rules that one care worker is in charge of one district and that basic units in walking and driving mode can only be visited by care workers on foot and by car, respectively. On this assumption, the upper bound of care workers on foot M and the upper bound of care workers by car V are large enough. The capacity of a district (care worker) Q_k is assumed to be a common constant Q for $k = 1, 2, \ldots, M+V$, which means the capacity of all districts even in different modes is the same. Note that the sets of basic units in walking and driving mode are B_0 and B_1, respectively (see Table 2). The computation time according to solvers may be shortened by providing the lower bound (noted as L) of the proposed MIP model. To obtain the lower bound, most constraints could be relaxed except those related to resource limitation, such as Constraint (3) and Constraint (5). For instance, if all constraints except Constraint (5) are relaxed, the lower bound can be calculated on the condition that Constraint (5) is satisfied, which is noted as L_c and indicated in Eq. (15), where $\sum_{i \in B_0} q_i$, $\sum_{i \in B_1} q_i$ are the total meal demand of basic units in walking and driving mode, respectively. Eq. (15) means that the number of care workers needed to deliver on foot and by car are at least $\left\lceil \frac{\sum_{i \in B_0} q_i}{Q} \right\rceil$ and $\left\lceil \frac{\sum_{i \in B_1} q_i}{Q} \right\rceil$, respectively. In a similar way, when only Constraint (5) is satisfied, the lower bound can be calculated, which is noted as L_s and indicated in Eq. (16), where $\sum_{i \in B_0} t_i$, $\sum_{i \in B_1} t_i$ are the total service time of basic units in walking and driving mode, respectively. Eq. (16) means that the number of care workers needed to deliver on foot and by car are at least $\left\lceil \frac{\sum_{i \in B_0} t_i}{T} \right\rceil$ and $\left\lceil \frac{\sum_{i \in B_1} t_i}{T} \right\rceil$, respectively, without considering the travel duration from depot to each parking site. As a whole, the lower bound of the proposed MIP model should be the maximum one of the lower bounds determined by Eqs. (15) and (16), as shown in Eq. (14).

$$L = \max\{L_c, L_s\} \tag{14}$$

$$L_c = \left\lceil \frac{\sum_{i \in B_0} q_i}{Q} + \frac{\sum_{i \in B_1} q_i}{Q} \right\rceil \tag{15}$$

$$L_s = \left\lceil \frac{\sum_{i \in B_0} t_i}{T} + \frac{\sum_{i \in B_1} t_i}{T} \right\rceil \tag{16}$$

Expectedly, the computation time for solving the MIP model above increases with the increasing scale of instance even if lower bounds are provided. First, the model has $(M+V) \times N \times N$ compactness constraints and $M+V$ or N other constraints. The computation time for the optimal solution increases when N and $M + V$ increase. Second, if the optimal objective value is $m + v$, where m and v are the minimum number of care worker on foot and on board vehicles correspondingly, $C_M^m \times C_V^v$ alternative optima exist for the decision variable of district selection y_k. Furthermore, symmetry causes havoc during branch and bound, thus implying the existence of at least $(m-1)! \times (v-1)!$ alternative optima for the decision variable of basic unit assignment x_{ik}. As a result of the characteristics of the proposed MIP model in terms of the objective functions and specific constraints, finding the optimal solutions for the model is computationally intensive with the use of the exact method, such as the branch-and-bound algorithm and branch-and-cut techniques.

4. Proposed greedy heuristic method for solving the MOWSD problem

The Gurobi Optimizer may be a feasible choice for solving the above model because it is an outstanding solver for all major mathematical programming problems (Gurobi., 2016a). However, finding the optima within an acceptable computation time tends to be difficult when using the Gurobi Optimizer; this difficulty stems from the slow convergence rate of exact methods for partial enumeration that use the branch-and-bound algorithm. With the Gurobi Optimizer, finding the optimal solution for the proposed MIP model is time consuming (see Section 5.2) because branch-and-bound and branch-and-cut techniques are adopted (Gurobi., 2016b). Given these challenges, considerable effort should be focused on discovering heuristic and meta-heuristic methods that can be used to find optimal or near optimal solutions to the MOWSD problem within a reasonable amount of time.

As discussed in Section 2, heuristic and meta-heuristic methods are applied to solve the different applications of districting problems; these methods include Grouping-GA (Agustín-Blas et al., 2009), TS (Bozkaya et al., 2003), SA (D'Amico et al., 2002), and GRASP (Roger Z. Ríos-Mercado & Fernández, 2009). The application of these heuristic and meta-heuristic methods requires considerable modification or design to solve the MOWSD problem caused by the characteristics of the model. These methods start from a population of initial solutions (consisting of at least one initial solution) obtained through a random generation method or construction method. The random generation of initial solutions ignores the characteristics of the specific problem. The construction method should deal with constraints when constructing solutions. The crossover and mutation operators of Grouping-GA may lead to the violation of compactness constraints and capacity constraints; given this limitation, the construction method should adopt the correct repair steps to ensure the feasibility of the solutions (Agustín-Blas et al., 2009). It is difficult to reduce the minimum number of districts by exploring neighborhood structures, such as insertions, λ -interchange, and add/drop and swap, which are applied in TS (Bozkaya et al., 2003) and SA (D'Amico et al., 2002); GRASP consists of a constructive phase and local search, which also require defined moves (Ríos-Mercado & Fernández, 2009). Neighborhood structures inevitably differ to a considerable degree when these heuristic methods are applied in solving the MOWSD problem. Such difference is complex because of the specific constraints and objective functions of the proposed MIP model. Greedy heuristic methods are fast and simple to implement, but they are usually used only in the construction phase of complex meta-heuristic methods; for example, a greedy search algorithm is used to form a cluster of customers in a capacitated location-routing problem (Zare Mehrjerdi & Nadizadeh, 2013). In this study, we propose a greedy heuristic method to provide a comprehensive solution to the MOWSD problem.

4.1. Proposed greedy heuristic method

The proposed greedy heuristic method follows the assumptions mentioned in Section 3.3. According to Assumption (3), the basic units in different modes cannot be assigned in the same district. Therefore the proposed greedy heuristic method runs twice, one for the basic units in walking mode and the other for the basic units in driving mode. We take the districting of basic units in walking mode as the sample explanation.

The inputs of the greedy heuristic method for districting basic units in walking mode are prepared before running. The inputs are as follows: the set of basic units in walking mode B_0 and the non-compactness matrix A. Each basic unit $i \in B_0$ comprises three attributes, namely, the average number of meal packages q_i, the total load t_i, and the driving duration from depot to basic unit i, d_{0i}. Before the implementation of the greedy heuristic method, each basic unit $i \in B_0$ comprises an additional attribute obtained on the basis of A, the compact set A_i, which consists of basic units j where $a_{ij} = a_{ji} = 0\ \forall j \in B_0, j \neq i$. The output for the MOWSD problem is a list of districts each of which consists of a list of compact basic units and satisfies Constraints (3) and (5). Let U be the list of unassigned basic units, where $U \subset B_0$, let D_k be the set of basic units that have been assigned in district k, let P_k be the set of all available basic units that can be assigned to district k, let q be the current packages of a district and t be the current total load of a district, and let *flag* be the flag that stops or prompts the filling of districts. ζ_k is the duration between the depot and the parking site of district k. Q is the capacity of a district and T is the total available duration for delivery. Fig. 2 illustrates the greedy heuristic process in detail.

The greedy heuristic method for the MOWSD problem examines the relative size of compact set A_i, the number of meal packages q_i, and the total load t_i of each basic unit to select those to assign to districts. The priority of basic unit i to be assigned is determined according to the size of A_i (presented as $|A_i|$). Initially, all the districts are empty and all the basic units are unassigned. The basic units are sorted increasingly in terms of $|A_i|$. The smaller $|A_i|$ is, the higher priority of basic unit i to be assigned is. The unassigned list consists of all basic units in decreasing priority order. For each district, the proposed greedy heuristic method starts by finding the first basic unit with the smallest compact set in the list of unassigned basic units. The basic unit with the highest priority is then selected from the set of available basic units to fill the district until either Constraint (3) or Constraint (5) is violated. Each assignment of a basic unit to the district k mandates the revision of the smallest driving duration ζ_k from the depot to the district k, the set of available basic units P_k of the district k, the current packages q of the district k, and the current total load t of district k.

The following sketch summarizes the greedy heuristic method. This sketch is based on the notations presented in Section 3.2 and the flowchart of the proposed greedy heuristic method in Fig. 2. All $q_i < Q$ and $t_i < T$ are set according to the data preparation (Bard & Jarrah, 2009).

Step 1. Let $k = 0$, $U = B_0$, and sort U by $|A_i|, i \in U$.
Step 2. Let $flag = 0, k = k + 1, D_k = \varnothing, q = 0, t = 0$, and $\zeta_k = h_{max}$.
Step 3. Get basic unit i with highest priority from U (positioned at the top of list U), and let $P_k = A_i$, $q = q_i$, $t = t_i$, and $\zeta_k = d_{0i}$.
Step 4. Assign basic unit i to district k, delete basic unit i from U, and update the available set of district k, P_k. $D_k = D_k \cup \{i\}, U = U - \{i\}, P_k = P_k \cap U$.
Step 5. Find the basic unit j with highest priority from P_k. The basic unit j to be assigned to the district should ensure that Constraints (3) and (5) are not violated. If basic unit j is not found, then stop the filling of district k, and set $flag = 1$; otherwise, let $i = j$, and $P_k = P_k \cap A_i$, $q = q + q_i$, $t = t + t_i$, and $\zeta_k = d_{0i}$ *if* $d_{0i} < \zeta_k$.
Step 6. Repeat Steps 4 to 5 until $flag = 1$.
Step 7. Repeat Steps 2 to 6 until $U = \varnothing$.

When $|A_i| = |A_j|, \forall i, j \in U, i \neq j$, the greedy heuristic method finds the basic unit with a small index because of the implementation of the algorithm. The elements in P_k are stored in the sequence according to the sequence of sorted U. During each step, U keeps the order sorted by Step 1. There exists a basic unit i (the first assigned basic unit) in district k that satisfies two criteria: (1) $D_k \subset A_i \cup \{i\}$ and (2) $|A_i| \leqslant |A_j|, \forall j \neq i, j \in D_k$. District k also satisfies two criteria: (1) $\sum_{j \in D_k} q_j \leqslant Q$ and (2) $\sum_{j \in D_k} t_j + \min(d_{0j} | j \in D_k) \leqslant T$.

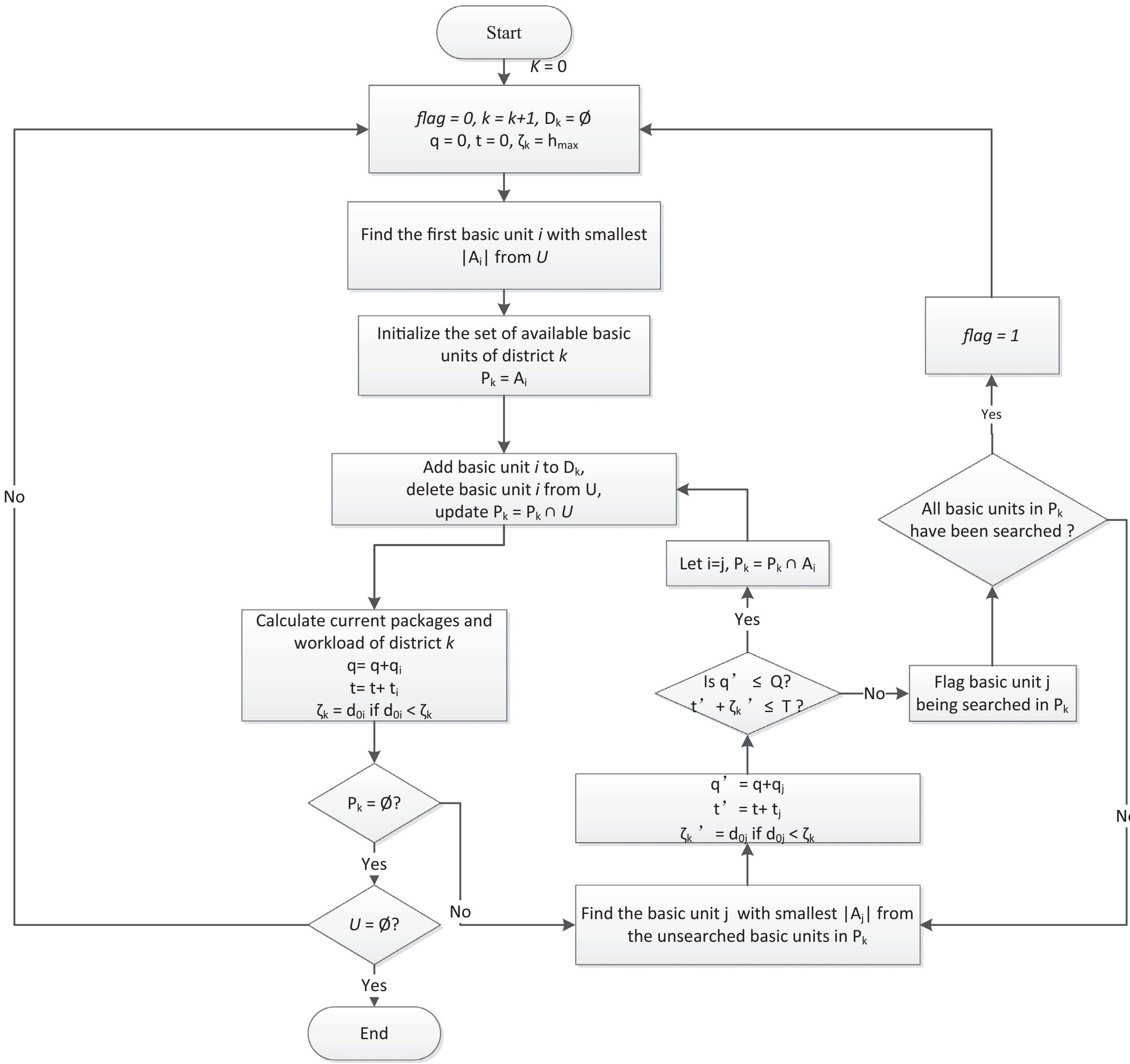


Fig. 2. Proposed greedy heuristic method.

4.2. Analysis of the greedy heuristic method

The justification for the greedy characterization of the proposed method stems from the fact that basic units with small $|A_i|$ have few chances to be assigned to the same districts. Given this justification, the basic unit with low compactness is given higher priority for assignment, considering that the basic units comprise high numbers of compact basic units at a later period. The greedy heuristic method is applied in Step 7 until a complete district design for all basic units in B_0 is obtained. Sections 4.2.1 and 4.2.2 discuss the correctness and efficiency of the greedy heuristic method.

4.2.1. Analysis of correctness

Let F_k be the compact set of the first basic unit of district k and remain unchangeable during the analysis. For example, if the basic unit j is the first basic unit assigned to district k, then $F_k = A_j$. D_k and U keep updated according to the steps of the greedy heuristic method.

Invariant of the greedy heuristic method: Each basic unit $j \in D_k$ falls into one of the three different cases:

(1) $|A_j| \leqslant |A_r|,\ \forall r \neq j, r \in D_k \cup U$
(2) $|A_j| \leqslant |A_r|,\ \forall r \in F_k \cap U$
(3)
$|A_j| \geqslant |A_r|, \sum_{s\in D_k-\{j\}} q_s + q_j \leqslant Q, \sum_{s\in D_k-\{j\}} t_s + \min(d_{0s}|s \in D_k) \leqslant T,$

$$\sum_{s\in D_k-\{j\}} q_s + q_r > Q or \sum_{s\in D_k-\{j\}} t_s + \min(d_{0s}|s \in (D_k - \{j\}) \cup \{r\}) > T; \quad \forall r \in F_k \cap U$$

Analysis (by induction on D_k):

- *Base case*: k = 0, $D_k = \varnothing$.
- *Inductive hypothesis*: We assume that the invariant is true for $D_{k-1},\ k > 1$.We consider two different situations. First, let basic unit j be the first basic unit added to D_k, which falls under Case

(1). If $\exists r \in U$ that satisfies $|A_r| < |A_j|$, then the greedy heuristic method chooses basic unit r as the first basic unit in D_k. The result contradicts the definition of the first basic unit j in D_k. Therefore, basic unit $r \in U$ cannot be selected as the first basic unit in D_k. Second, let basic unit j be the next basic unit added to D_k, which falls under Case (2) or Case (3). If $\exists r \in F_k \cap U$ satisfies all criteria $|A_r| < |A_j|, \sum_{s \in D_k - \{j\}} q_s + q_r \leqslant Q, \sum_{s \in D_k - \{j\}} t_s + \min(d_{0s} | s \in D_{D_k - \{j\}} \cup \{r\}) \leqslant T$, then the greedy heuristic method assigns basic unit r to D_k. The result contradicts the definition of basic unit j in D_k. Therefore, basic unit $r \in F_k \cap U$ cannot be added in D_k. In summary, the invariant of the greedy heuristic method is true for the basic units in each district.

4.2.2. Order of growth

The greedy heuristic method can rapidly find a solution. As described in Section 4.1, the greedy heuristic method initially sorts the n unassigned basic units (n equals $|B_0|$ in walking mode or $|B_1|$ in driving mode) according to a given criterion (the size of compact sets of basic units). Sorting may be conducted at rate $O(n \log n)$ (Fill & Janson, 2002), followed by subsequent decisions that are based on the sorted unassigned basic units. The running time of Step 2 to Step 4 of the greedy heuristic method is $O(1)$. To finish the construction of the solution, the iteration of these steps is conducted at a maximum of n times. The running time for these steps is $O(n)$. Step 5 of the greedy heuristic method depends on the size of P_k. The best case is that the greedy heuristic method executes Step 5 once for each basic unit. Then executing Step 5 takes $O(n)$. The worst case is that all basic units are compact and the size of P_k of each unassigned basic unit is $n, n-1, n-2, \ldots 1$. The worst case of Step 5 can be implemented in $O(n \log n)$ time by storing the unassigned basic units in decreasing order according to the packages and workload of each unassigned basic unit. The sorting of unassigned basic units takes $O(n \log n)$. Therefore, we can easily determine the convergence rate of the greedy heuristic method at $O(n \log n)$. The logarithmic order of convergence clearly demonstrates that the proposed greedy heuristic method can be successfully applied as the solution for the large MOWSD problem. Further details concerning the computation time required for running the greedy heuristic method in comparison with the running time required by the Gurobi Optimizer are presented in Section 5.2.

5. Computational tests

The purpose of this section is to analyze the behavior of the proposed model and the proposed greedy heuristic method in solving the MOWSD problem. These two methods are verified with the data set, a monthly record of the lunch MOW service delivery in the Tai Po area in August 2014, which is provided by the SA-TPIHCS. Two experiments were conducted to evaluate the performance of the proposed MIP model and greedy heuristic method.

The first experiment analyzed the effects of the key parameters of the model for the MOWSD problem. As mentioned in Section 3.3, finding an optimal solution within a limited computation time is difficult. Given this difficulty, the computation time of the proposed MIP model solved with the Gurobi Optimizer is limited to 3600 s, and the gap between the current objective function value and the best lower bound is set to 15% to guarantee acceptable solutions to the MOWSD problem. Once the model achieves a gap lower than 15% or its runs out of time (3600 s), the experiment was interrupted. To obtain the mean acceptable solutions and to observe the stability of the model, each experiment was run three times under the same setting of parameters in the first experiment because of the long computation time.

The second experiment compared the performance of the Gurobi Optimizer and the proposed greedy heuristic method when solving the MOWSD problem. The resulting districts obtained by these two methods were compared with the existing districts obtained via manual planning to verify the fit of the proposed model and greedy heuristic method to the MOWSD problem in Section 5.3. The experiments were conducted and coded with Python on a personal computer with Intel 2.2 GHz, 8 GB usable memory, and OS X operating system.

5.1. Sensitivity analysis on the key parameters of the MOWSD problem

To analyze the effects of key parameters on the performance of the model, we adopted different levels of key parameters (Q, T, w_{max}) in the problem. The capacity of each district Q ranges from 8 to 12, that is, $Q \in \{8, 9, 10, 11, 12\}$. Parameter Q relates to the physical burden of care workers, which is neither extremely heavy that it hurts their health nor too easy that it reduces the workload utilization of care workers. The total available duration for MOW delivery T ranges from 4500 s to 7200 s, with an increment of 900 s (15 min), that is, $T \in \{4500, 5400, 6300, 7200\}$ as a result of the limitation of meal periods. According to practice, the maximum walking duration between two basic units w_{max} (noted as WM in Table 3) in a district was set from 420 s to 840 s, with an increment of 60 s or 120 s (1 min or 2 min), that is, $w_{max} \in \{420, 540, 600, 720, 840\}$.

The results of the analysis of variance (ANOVA) are presented in Table 3. The results show that Q, T, w_{max} and their interactions $Q \times T, Q \times WM, T \times WM$ and $Q \times T \times WM$ have significant effects (P-value = 0) on the value of the objective function (the number of districts needed, noted as *OBJVAL* in Table 3), the gap between the current optimal objective value and the best lower bound (noted as *GAP* in Table 3), and computation time (noted as *RUNTIME* in Table 3). Given that $Q \times T \times WM$ has a significant effect on these three performance factors, the levels of Q, T and WM with significant effects should be analyzed. The details of the effects of these three parameters on each performance factor are discussed in the following section.

5.1.1. Effects of key parameters on OBJVAL

As shown in Table 3, the interaction of Q, T, and w_{max} (WM) has a significant effect on *OBJVAL*. Fig. 3, which is composed of Fig. 3 (a)–(i), indicates the simple effects of each parameter on the value of the objective function.

The effect of Q on *OBJVAL* is significant, as indicated in Fig. 3(a)–(d). Except when $T = 4500, WM = 600$, as shown by a line in Fig. 3 (a), which indicates the lack of difference for Q among the means of *OBJVAL*. As shown in Fig. 3(a)–(d), *OBJVAL* tends to decrease with increasing Q, especially when $T \in \{5400, 6300, 7200\}$ (see Fig. 3 (b)–(d)). Given this finding, the minimum number of districts that cover the total demand of basic units decreases when the capacity of each district increases.

w_{max} (WM) exerts a significant effect on *OBJVAL*, as indicated in Fig. 3(e)–(i), except when $Q = 12, T = 7200$, which is depicted as a stable line in Fig. 3(i). Fig. 3(e)–(i) shows that *OBJVAL* reaches the bottom either at $WM = 540, 600$, with one exception, that is, the bottom occurs at point $WM = 840$ when $Q = 8, T = 7200$. This finding means that the maximum walking duration between two basic units should have a proper threshold.

Fig. 3(e)–(i) indicates that T exerts a significant effect on *OBJVAL* because most of the values differ from one another when Q and WM are settled, except when $Q = 8, WM = 540$. This exception is shown as a point in Fig. 3(e). The values of *OBJVAL* at $T = 4500$ are not smaller than those at $T = 5400$, followed by $T = 6300$,

Table 3
Test of between-subject effects.

Source	Performance Factors	Sum of squares	Df	Mean square	F	P-value
Q	OBJVAL	407.267	4	101.817	3818.125	**0.000**
	GAP	0.021	4	0.005	84.247	**0.000**
	RUNTIME	64429542.318	4	16107385.579	706.452	**0.000**
T	OBJVAL	802.170	3	267.390	10027.125	**0.000**
	GAP	0.008	3	0.003	44.147	**0.000**
	RUNTIME	35149109.117	3	11716369.706	513.867	**0.000**
WM	OBJVAL	38.700	4	9.675	362.813	**0.000**
	GAP	0.179	4	0.045	715.554	**0.000**
	RUNTIME	1.052E8	4	26311543.214	1153.996	**0.000**
$Q \times T$	OBJVAL	155.747	12	12.979	486.708	**0.000**
	GAP	0.026	12	0.002	34.378	**0.000**
	RUNTIME	25981738.477	12	2165144.873	94.961	**0.000**
$Q \times WM$	OBJVAL	33.033	16	2.065	77.422	**0.000**
	GAP	0.063	16	0.004	63.031	**0.000**
	RUNTIME	44327862.883	16	2770491.430	121.511	**0.000**
$T \times WM$	OBJVAL	66.580	12	5.548	208.062	**0.000**
	GAP	0.029	12	0.002	39.172	**0.000**
	RUNTIME	14911880.403	12	1242656.700	54.502	**0.000**
$Q \times T \times WM$	OBJVAL	30.087	48	0.627	23.505	**0.000**
	GAP	0.162	48	0.003	54.206	**0.000**
	RUNTIME	62765912.507	48	1307623.177	57.351	**0.000**
Error	OBJVAL	5.333	200	0.027		
	GAP	0.012	200	6.238E−5		
	RUNTIME	4560076.678	200	22800.383		
Corrected total	OBJVAL	1538.917	299			
	GAP	0.501	299			
	RUNTIME	3.574E8	299			

The bold values highlights that the column p-value less than 0.05, which indicates the significant effects of corresponding elements.

which is not smaller than those at $T = 7200$ (see Fig. 3(e)–(i)). A conclusion can be drawn, that is, the longer the available duration for MOW delivery, the fewer districts are needed. *OBJVAL* is smaller at $T = 6300, 7200$ than at $T = 4500, 5400$. When $WM = 540, Q = 8, 9, 10, 11, 12$ and $WM = 600, Q = 9, 10$, *OBJVAL* is the same for $T = 6300$ and $T = 7200$. When $WM = 600, Q = 8$, *OBJVAL* is smaller at $T = 6300$ than at $T = 7200$. When $WM = 600, Q = 11, 12$, *OBJVAL* is smaller at $T = 7200$ than at $T = 6300$ (see Fig. 3(e)–(i)).

The conclusion for these three parameters affecting *OBJVAL* in an actual situation can be drawn as follows. The larger the Q is, the smaller the *OBJVAL* is. *OBJVAL* is small when $WM = 540, 600$. The longer T is, the smaller the *OBJVAL* is, especially when $T = 6300, 7200$. In other words, suppose one care worker is responsible for one district, the capacity of which is also the capacity of cart, the HHC structure should buy carts of proper size. The larger the cart is, the less the care workers are need. The proper walk duration between any two customers for a care worker makes less care workers needed. When there are limited care workers for great MOW meal demand, extending the total available time for MOW service delivery is one way to cover the demand.

5.1.2. Effects of key parameters on GAP

GAP depends on *OBJVAL* and the lower bound of the model. Fig. 4, which comprises Fig. 4(a)–(i), shows the effects of Q, T, and WM on *GAP*.

Q exerts a significant effect on *GAP* in most cases, as indicated in Fig. 4(a)–(d). However, when $T = 4500, WM = 600, 840$ and $T = 5400, WM = 600$, a slight difference exists among the means of *GAP*. This slightness is shown as stable lines in Fig. 4 (a) and (b). Fig. 4(a)–(d) also indicates that the values of *GAP* are always lower than 0.15 at $Q = 8$. This result means that an acceptable solution can be found within the limitation of computation time with the proper capacity of each district. Fig. 4(c) shows that *GAP* increases with increasing Q from 9 to 12 when $WM = 540, 600$.

Fig. 4(e)–(i) shows that w_{max} (WM) exerts a significant effect on *GAP*, except when $Q = 12, WM = 720$. This exception is shown as a stable line in Fig. 4(i). Except when $WM = 420$, *GAP* is not higher than 15% for Q, T, and the remaining WM levels (see Fig. 4). This result means that an acceptable solution can be found within the given threshold of *GAP* and the limitation of computation time given a proper WM. When $WM = 540, 600$, *GAP* is lower than 15%.

T also exerts a significant effect on *GAP*, except when $Q = 8, WM = 420, 540, 720$ and $Q = 9, WM = 540$ because the points according to the setting are almost at the same position (see Fig. 4(e)–(i)). As summarized in Section 5.1.1, $T = 6300, 7200$ is a good choice in terms of *OBJVAL*. Therefore, the discussion on the effect of T on *GAP* is narrowed down to two levels, namely, $T = 6300, 7200$. A conclusion can be derived from Fig. 4(e)–(i), wherein *GAP* is higher at $T = 7200$ than at $T = 6300$. Therefore, $T = 6300$ is a good choice for MOW delivery in terms of *OBJVAL* and *GAP*.

In summary, lower *GAP* with lower *OBJVAL* means *OBJVAL* is not only more close to the lower bound but also to the optimal value. With proper walking duration, say $WM = 540, 600$, where the number of care workers needed (*OBJVAL*) is smaller, *GAP* increases when the capacity of cart increases. However, *OBJVAL* decreases when the capacity of cart increases. Therefore, the HHC structure should determine the capacity of carts with the aim of balancing the trade-off between *OBJVAL* and *GAP*. With proper available time for meal delivery, say $T = 6300, 7200$, where *OBJVAL* is smaller, the HHC structure should select the proper T where GAP is lower.

5.1.3. Effects of key parameters on RUNTI $Q = 8, T = 7200$ ME

The model stops in one of two situations: (1) the model finds the solution that satisfies the gap between the current optimal solution and the best lower bound of the model lower than 0.15;

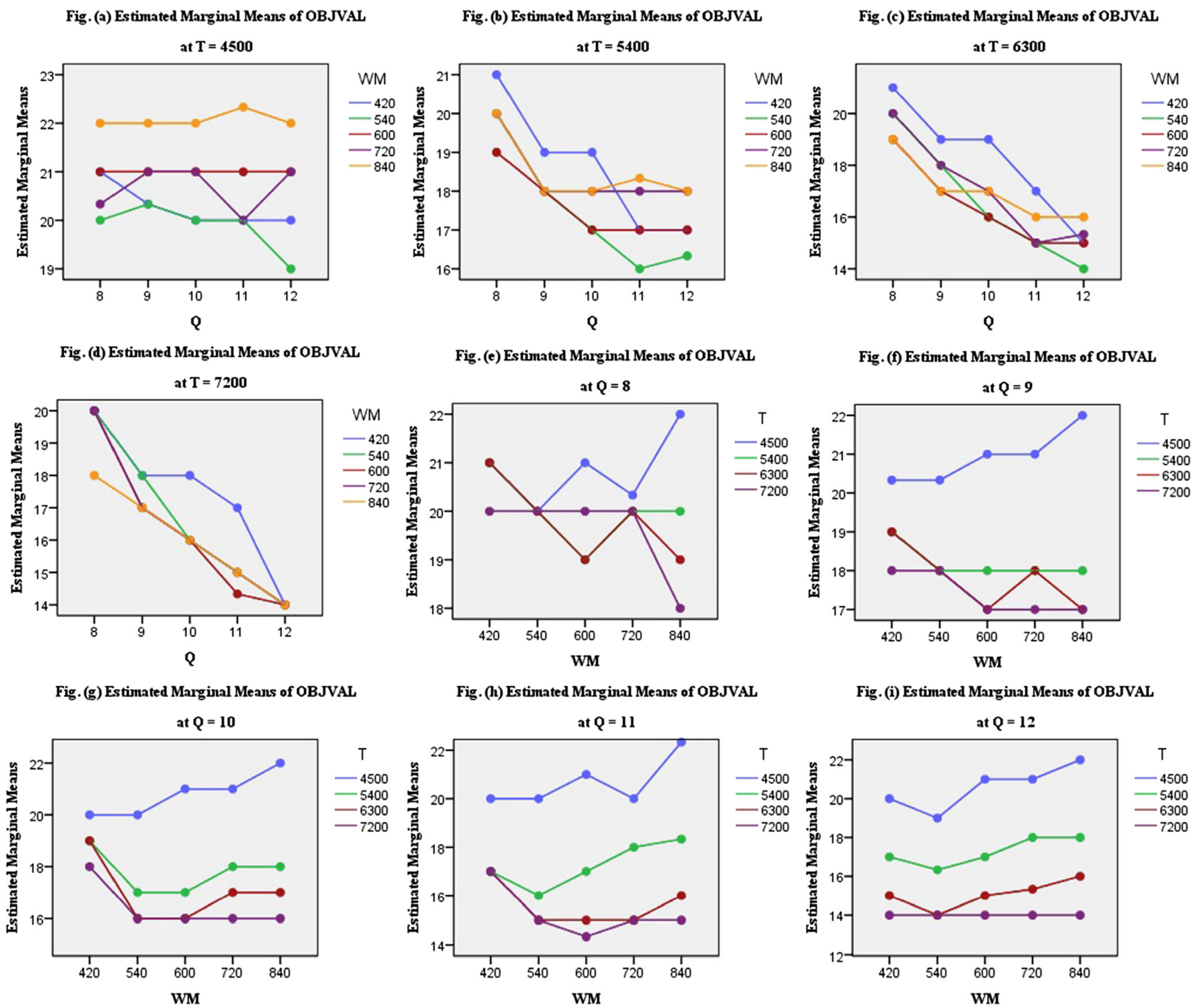


Fig. 3. Estimated marginal means of *OBJVAL*.

(2) the computation time (e.g., 3600 s) for finding improvement runs out. Fig. 5 shows the effect of key parameters on *RUNTIME*. This effect is also shown in Fig. 5(a)–(i).

Fig. 5(a)–(d) indicates that Q exerts a significant effect on *RUNTIME*, except when $T = 7200$, $WM = 840$. This exception is shown as a stable line in Fig. 5(d). Both w_{max} (WM) and T significantly affect *RUNTIME*, as shown in Fig. 5(e)–(i). Only three cases are excluded, namely, and $Q = 8, T = 7200$, and $Q = 8, WM = 540, 720$ as shown in Fig. 5(e). This exclusion is due to deviation, which is smaller than that in other cases. $T = 7200$ is more stable at $Q = 8$ than in other cases, as shown in Fig. 5(e)–(i). Points at $Q = 8, WM = 540, 720$ are closer than those at different levels of W (see Fig. 5(e)). The trend of *RUNTIME* increases with increasing Q and decreasing T. Fig. 5 (e)–(i) indicates that when $WM = 540$, *RUNTIME* reaches the bottom in most cases. Hence, when $WM = 540$, based on the selection $T = 6300$, the model can achieve improved results at a short time in terms of *GAP* and *RUNTIME*.

Lower *RUNTIME* with lower *OBJVAL* and lower *GAP* means higher efficiency and effectiveness for the HHC structure. Therefore, with proper walking duration and total available time, say $T = 6300, WM = 540$, where the model runs quicker, the HHC structure should determine the proper capacity of cart to obtain higher quality of solution with lower *OBJVAL* and lower *GAP*. According to Sections 5.1.1 and 5.1.2, a large Q leads to a small *OBJVAL* and a large *GAP* when $T = 6300, WM = 540$; while hen the computational time is nearly 2 min when $Q = 9, 10, 12$. Therefore, $Q = 10$ should be set to balance the quality of solution (*OBJVAL* and *GAP*) and computation time (*RUNTIME*).

5.1.4. Effect summary

Q, T, and WM exert significant effects on *OBJVAL*, *GAP*, and *RUNTIME*. In general, low *OBJVAL*, low *GAP*, and low *RUNTIME* facilitate the trade-off among these three performance factors. Such trade-off results in improved choices of Q, T, and WM. On the basis of the above analysis, Q is determined as 10, T as 6300 s, and WM as 540 s. The decision on proper value of key factors could be made by the HHC structure with their specific concerns. For instance, if they prefer to obtain as good quality of solutions as possible within acceptable computational time, the selection of key factors should lead to the lowest *OBJVAL* without considering *GAP* and *RUNTIME*.

In a word, the sensitivity analysis helps understanding the relationships between inputs and outputs of an optimization model. When we construct an optimization model for a practical problem, the inputs of the model covering key factors of the problem and the

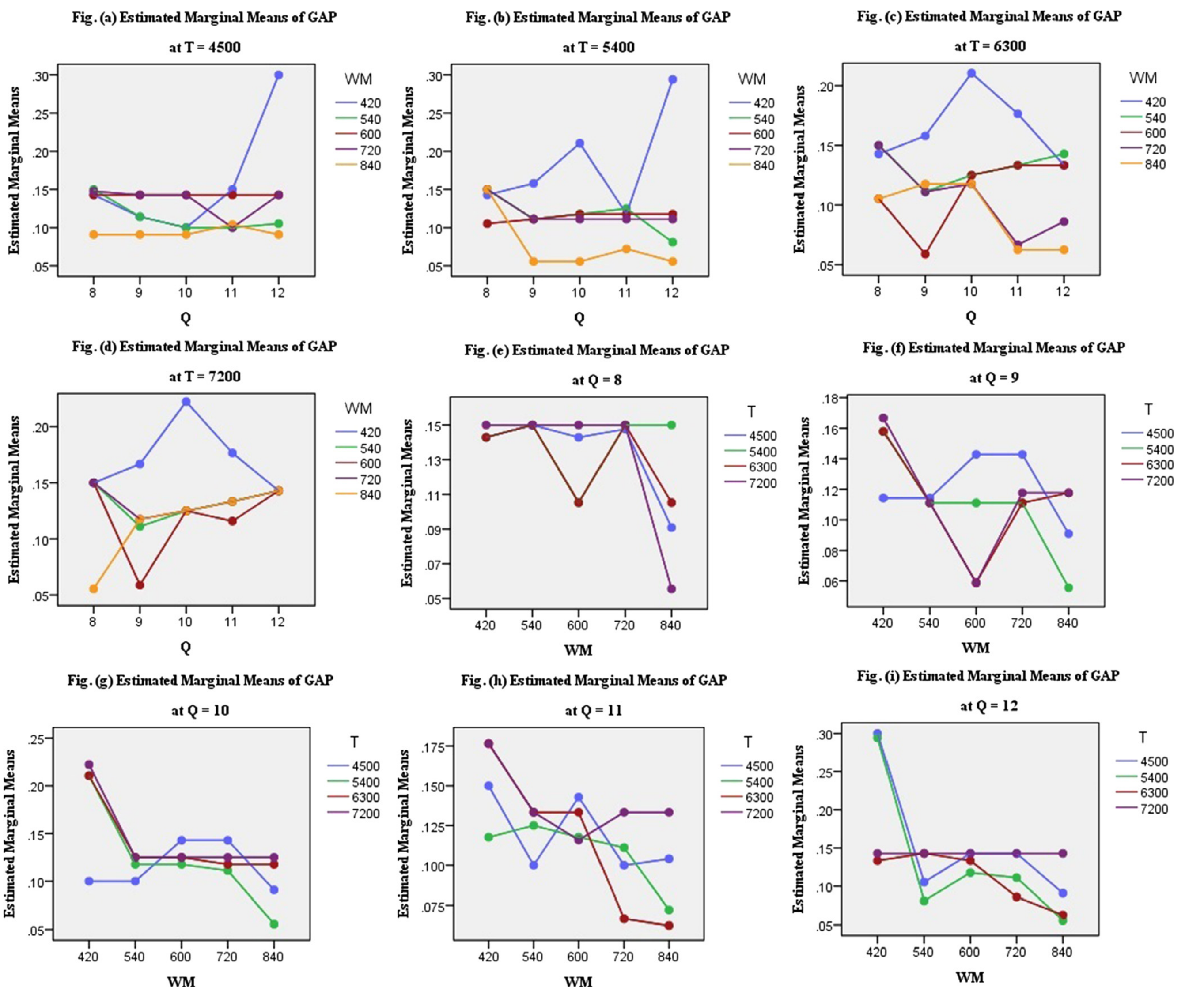


Fig. 4. Estimated marginal means of *GAP*.

outputs including solution quality and computation time can be identified from the perspective of the structures (organizations). Appropriate methods are then developed to find effective or acceptable outputs for the constructed model as quickly as possible. This reveals that sensitivity analysis significantly contributes to understanding and solving a practical problem in a satisfactory or at least acceptable way.

5.2. Comparison between the proposed greedy heuristic method and the Gurobi Optimizer

The proposed greedy heuristic method and the Gurobi Optimizer are compared in this section. The horizontal axis in Figs. 6–8 is the parameter (pairs of Q, T, and WM) index. Given 5 levels of Q, 4 levels of T, and 5 levels of WM, the parameter index ranges from 1 to 100 with an increment of 1. Suppose i, j, and k denote the corresponding levels of Q, T, and WM. Moreover, suppose *indx* denotes the corresponding index of parameters. Then, $indx = (i-1) \times 5 \times 4 + (j-1) \times 5 + k$, where $i \in [1,5], j \in [1,4], k \in [1,5]$. For instance, *indx* that is equal to 80 presents the following parameter settings: i equals 4 ($Q = 11$), j equals 4 ($T = 7200$), and k equals 5 ($WM = 840$). When i is equal to 3 ($Q = 10$), j is equal to 3 ($T = 6300$), and k is equal to 2 ($WM = 540$), *indx* is equal to 52.

Fig. 6 indicates the value of the objective function in Eq. (1) obtained by Gurobi Optimizer and the proposed greedy heuristic method. The value shows the number of districts that cover all basic units. The current objective function value depends on the gap and runtime when obtained through the Gurobi Optimizer, while it depends on the application of the constructive greedy heuristic method. As indicated in Fig. 6, the 54 solutions obtained with the proposed greedy heuristic method can achieve objective function values that are same to those obtained with the Gurobi Optimizer. The 14 solutions obtained with the greedy heuristic method are better than those obtained with the Gurobi Optimizer. The 32 solutions obtained with the Gurobi Optimizer are better than those obtained with the greedy heuristic method; most of these solutions occur when $WM = 420$ (17 cases) and $WM = 540$ (7 cases), followed by $WM = 600$, 720 (6 cases) and $WM = 840$ (1 case) (see Table A1 of Appendix A). In terms of the objective function value, the greedy heuristic method can achieve feasible solutions, 68% of which are as good as those obtained by the Gurobi Optimizer. Moreover, the proposed greedy heuristic method can

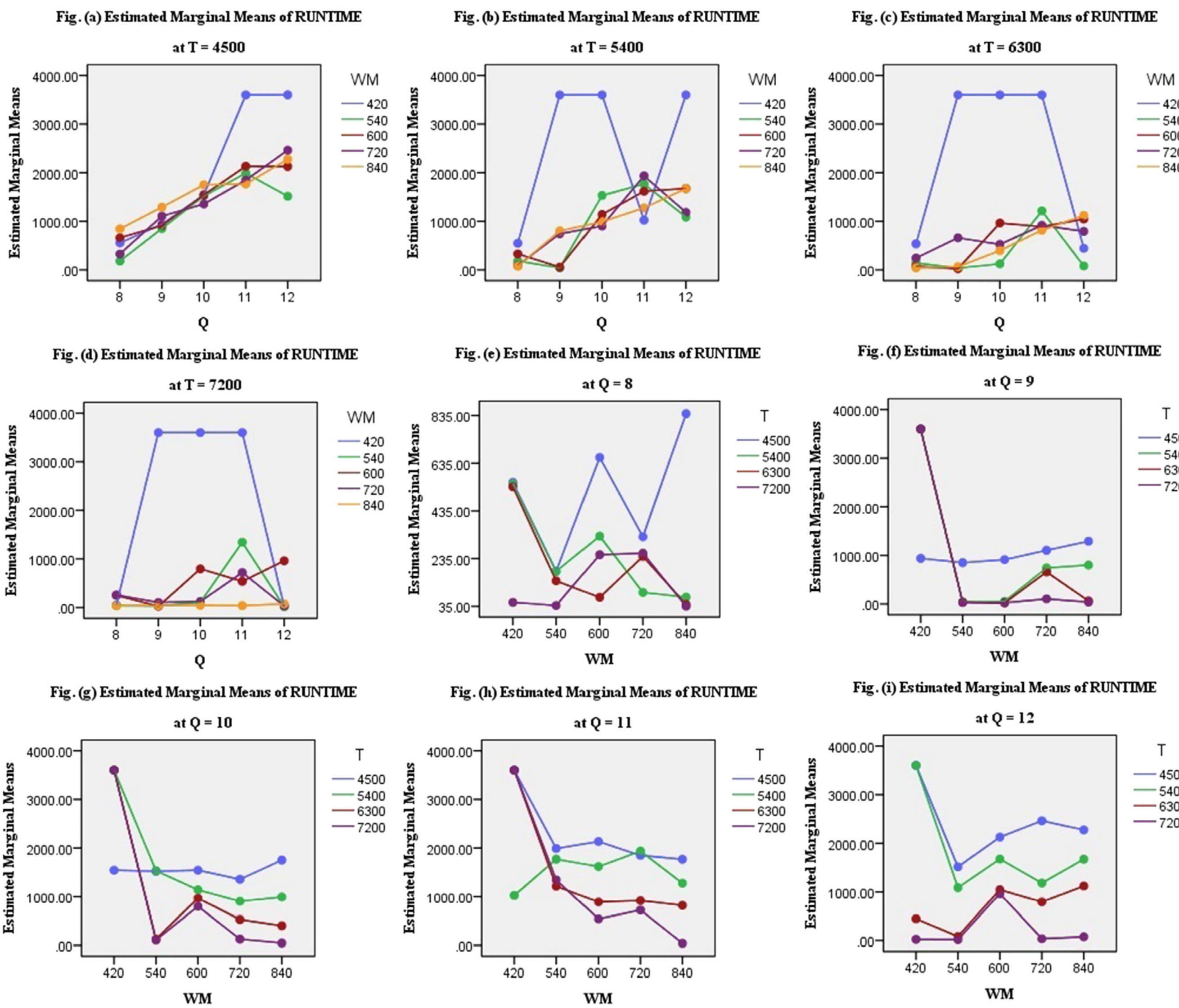


Fig. 5. Estimated marginal means of runtime.

achieve 81% good solutions as those obtained by the Gurobi Optimizer without the 20 cases when $WM = 420$. Therefore, this case study derives the following conclusion: the greedy heuristic method can achieve as many good solutions as the Gurobi Optimizer.

Figs. 7 and 8 present the computation time for solving the district model using the constructive heuristic method and Gurobi Optimizer. Fig. 7 shows that the computation time for the MOWSD problem under different parameter settings varies in the range of [0.5 s, 1.1 s]. Fig. 8 shows some cases wherein the

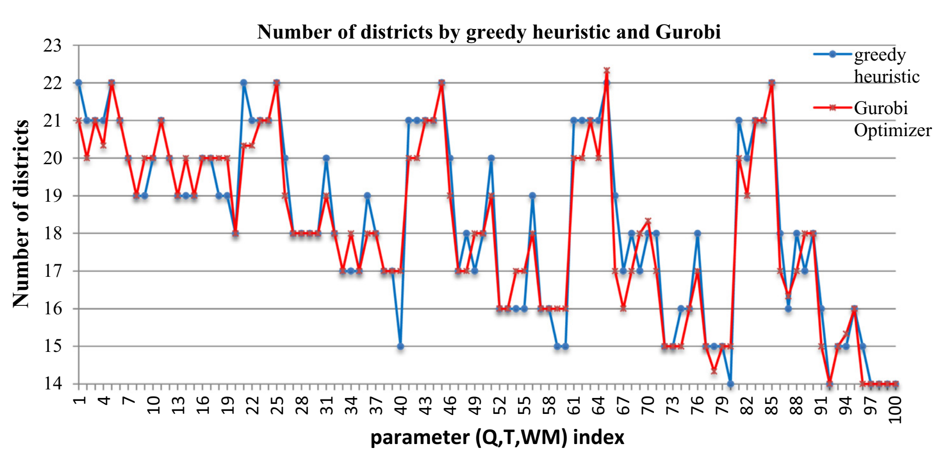


Fig. 6. Objective function values obtained by the greedy heuristic method and Gurobi Optimizer.

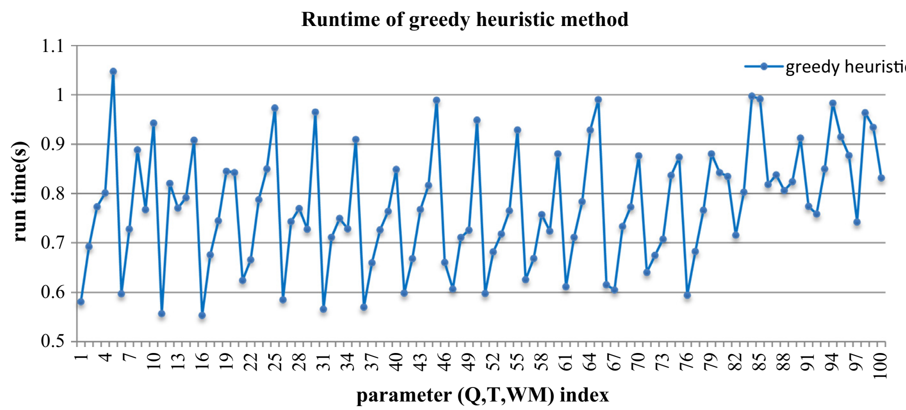


Fig. 7. Computation time of the greedy heuristic method.

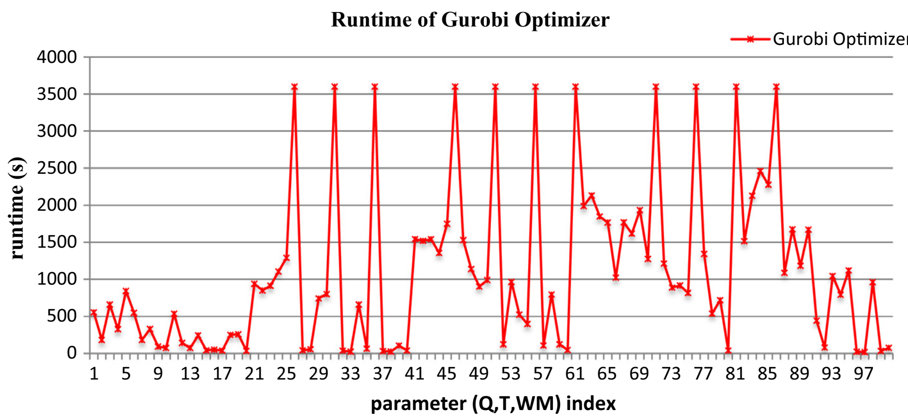


Fig. 8. Computation time of the Gurobi Optimizer.

computation time reaches the maximum time limitation (3600 s) without achieving an acceptable gap, specifically, almost all cases when $WM = 420, Q = 9, 10, 11, 12$, where $indx \in \{26, 31, 36, 46, 51, 56, 61, 71, 76, 81, 86\}$ (also see Table A1 of Appendix A). The computation time for achieving the solutions with a gap of less than 15% falls in the range of [18 s, 2500 s]. Under the best choice of key parameters ($Q = 10, T = 6300, WM = 540$), the greedy heuristic method can achieve a solution that is as good as that obtained with the Gurobi Optimizer but within a shorter computation time. As shown in Figs. 6–8, when $indx = 52$ ($Q = 10, T = 6300$ s, $WM = 540$ s), the average computation time for achieving 16 districts with 12.5% *GAP* is 124 s when using the Gurobi Optimizer and 0.68 s when using the proposed greedy heuristic method (also see Table A1 of Appendix A).

Figs. 6–8 show the performance trade-off in terms of the value of the objective function and computation time. In summary, finding an improved solution indicates increased computation time. When the balance between the number of districts and computation time is considered, the greedy heuristic method achieves good performance.

5.3. Comparison between the resulting districts and existing districts

As indicated in Sections 5.1 and 5.2, when Q is set as 10, T as 6300 s, and WM as 540 s, the solutions obtained with the Gurobi Optimizer and greedy heuristic method are good in terms of the number of districts, gap, and computation time. The resulting districts under this key parameter setting are applied to verify the MOWSD model and the proposed greedy heuristic method. The existing districts obtained via manual planning, which have been applied in practice for many years, are obtained randomly from one working day delivery record of historical data. The comparison between the resulting districts and existing districts is conducted on the basis of the compactness of districts and the number of districts needed for covering basic units. The compactness of districts is measured by the maximum duration between any two basic units in a district. Figs. 9 and 10 present the main service districts planned manually, obtained with the Gurobi Optimizer, and obtained with the greedy heuristic method. Figs. 11 and 12 show the districts near the SA-TPIHCS and those far from the center (visited by driving).

Figs. 9–12 show that the existing districts are more compact than the resulting districts because the maximum duration between any two basic units in an existing district is smaller than that in a resulting district. From the perspective of practice, the shorter the walking duration each care worker needs, the higher the utilization of the available duration for MOW delivery. However, high compactness, where the walking duration between two locations decreases, increases the number of districts (22 districts) needed to serve daily demands. The four districts in Figs. 9 and 11 only cover the basic units in one location, as shown by an arrowhead without lines. This result means that the demand (packages) required from this location has reached the capacity of a meal cart. Given the compactness constraint claimed in the district model in Section 3.2.2, the resulting districts obtained with the greedy heuristic method and Gurobi Optimizer show a certain level of compactness for districts in two different transit modes.

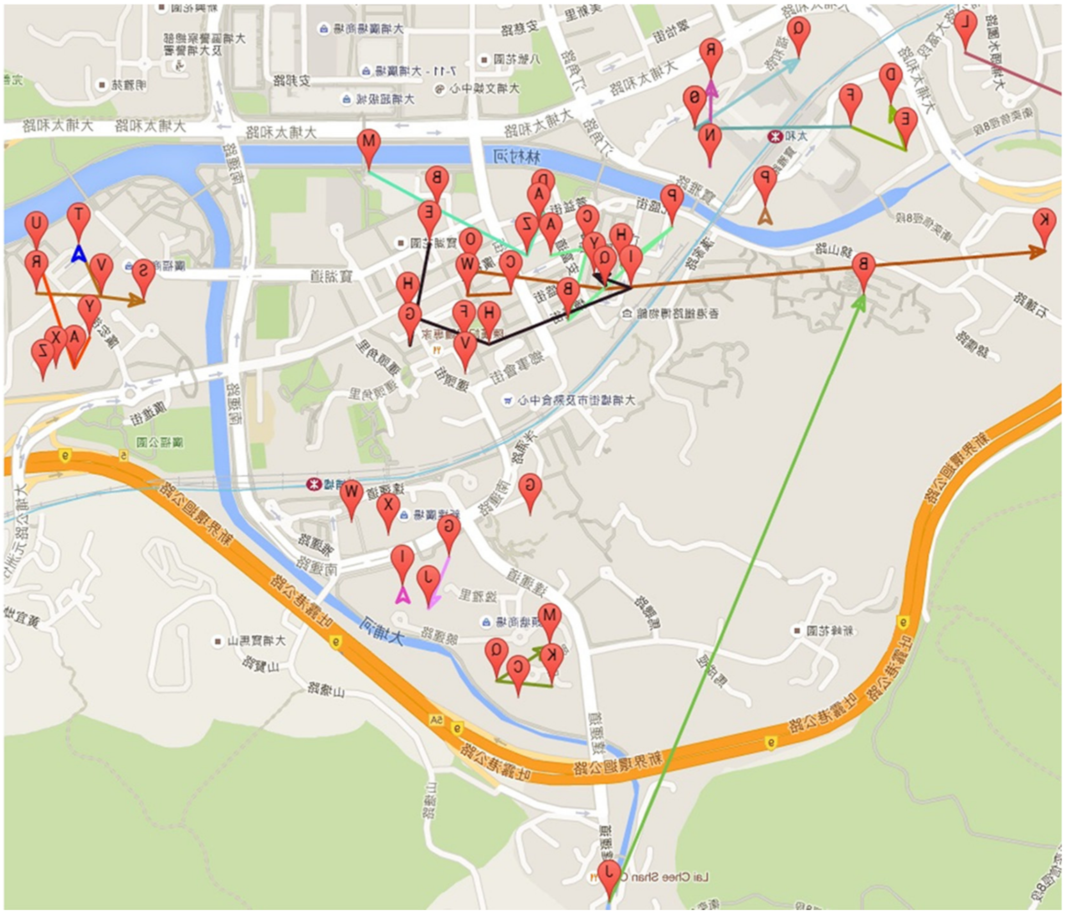
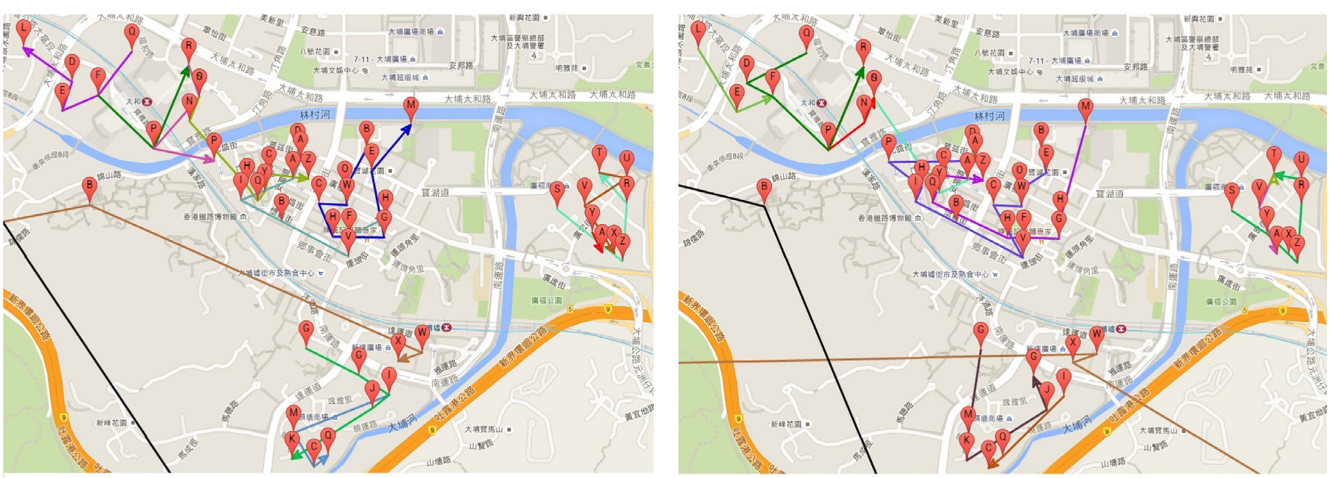


Fig. 9. Existing main service districts via manual planning.



(a). The resulting main services districts by Greedy heuristic **(b). The resulting main services districts by Gurobi**

Fig. 10. Resulting main service districts via greedy heuristic and Gurobi.

Moreover, less care workers (e.g., 14 care workers) are needed to deliver meals by foot (see Figs. 10 and 11), and less care workers (e.g., 2 care workers) are needed to deliver meals by driving (see Fig. 12). The comparison between the existing districts and the resulting districts indicates that not all basic units require meals at the same time. For instance, some basic units in existing districts (see Figs. 9, 11 and 12) are not aligned with any district, whereas all basic units in resulting districts are covered in districts (see Figs. 10–12) because the monthly demand is statistically estimated. Therefore, we can conclude that the MIP model works well in describing the MOWSD problem and that the greedy heuristic method and Gurobi Optimizer can derive good solutions in terms

Fig. 11. Districts near SA-TPIHCS via greedy heuristic (left), Gurobi (middle), and manual planning (right).

of the low number of districts needed. The resulting districts indicate the availability of the district model to be applied in districting problems by considering different transit modes, compactness criterion, and workload/package constraints.

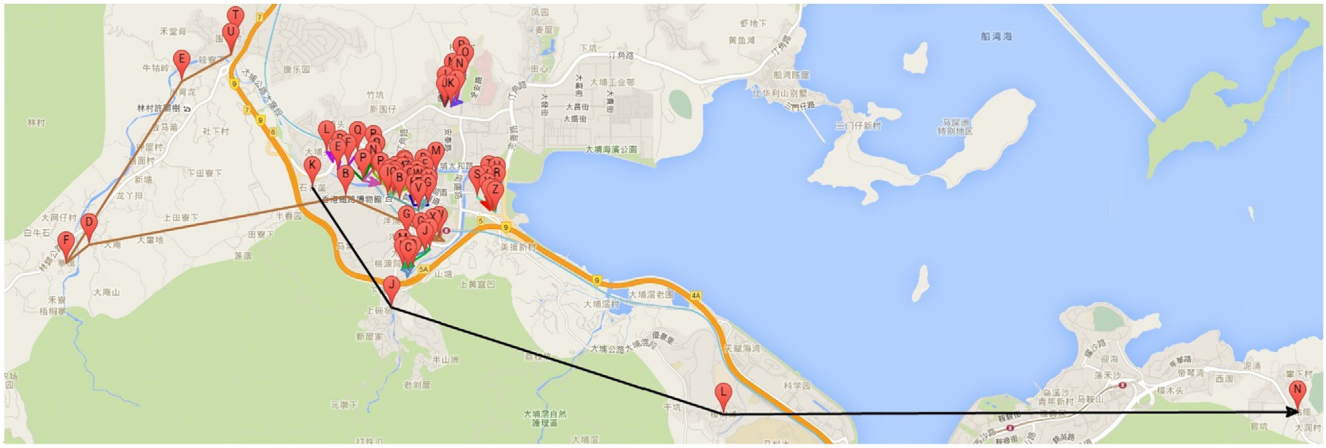
A discussion on the resulting districts obtained with the greedy heuristic method and Gurobi Optimizer was conducted to obtain a comprehensive knowledge of the quality of solutions in terms of the compactness of districts. As shown in Fig. 10, most of the main service districts obtained with the greedy heuristic method (shown in Fig. 10(a)) are more compact than those obtained with the Gurobi Optimizer (shown in Fig. 10(b)), because the maximum duration between any two basic units in a district in Fig. 10(a) is shorter than that in Fig. 10(b). For the districts near the center, the greedy heuristic method finds three districts with one comprising only one location and each of the other two consisting of four locations near the center (see the left side of Fig. 11). The Gurobi Optimizer also finds three different districts, however each of which consists of three or four basic units (see the middle part of Fig. 11). This finding shows that the districts near the center obtained with the greedy heuristic method are more compact than those obtained with the Gurobi Optimizer. The districts visited by driving obtained with the greedy heuristic method and Gurobi Optimizer are shown as black and brown lines in Fig. 12, respectively. This result indicates that the former can obtain more compact districts than the latter. In summary, the greedy heuristic method can achieve better districts than the Gurobi Optimizer in terms of the compactness criterion when Q is set as 10, T as 6300 s, and WM as 540 s in an actual situation.

6. Conclusion

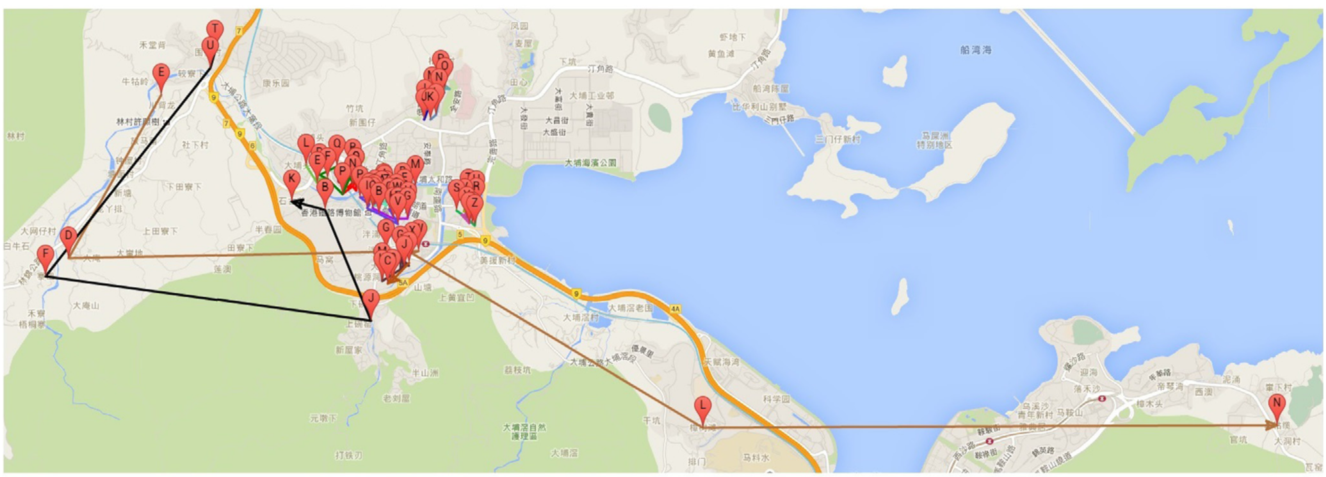
In this study, we developed an integrated MIP model for the MOWSD problem faced by HHC structures. The model considers several criteria, including the compactness of districts, the individuality of each basic unit, workload and package limitation, and the available delivery duration. The model was validated by a real instance collected from the SA-TPIHCS. A numerical analysis was conducted by varying the key parameters of the problem. The analysis indicates that Q, T, and WM exert significant effects on the value of the objective function and computation time. A good choice of key parameters can lead to improved resulting districts, as indicated by the results of the simple effect analysis of these parameters. For instance, when Q is equal to 10, T is equal to 6300 s, and WM is equal to 540 s, the Gurobi Optimizer can find a good feasible solution within a short computation time.

We also proposed a greedy heuristic method to shorten computation because, in general, the Gurobi Optimizer requires a long computation time to find a good solution to the MOWSD problem. The results obtained with the greedy heuristic method and Gurobi Optimizer were then analyzed and compared. The greedy heuristic method can achieve as many good solutions as the Gurobi Optimizer. Moreover, the greedy heuristic method requires a shorter computation time (less than 1.1 s) than the Gurobi Optimizer which takes at least 18 s and even more than 3600 s. Besides, the proposed greedy heuristic method did better than the Gurobi Optimizer in finding good districts that are more compact. We compared the resulting districts obtained with the Gurobi Optimizer and greedy heuristic method with the existing districts recorded in historical data. The proposed MIP model and the greedy heuristic method work well to satisfy the criteria and to improve the design of districts from the perspective of practice when solving the MOWSD problem.

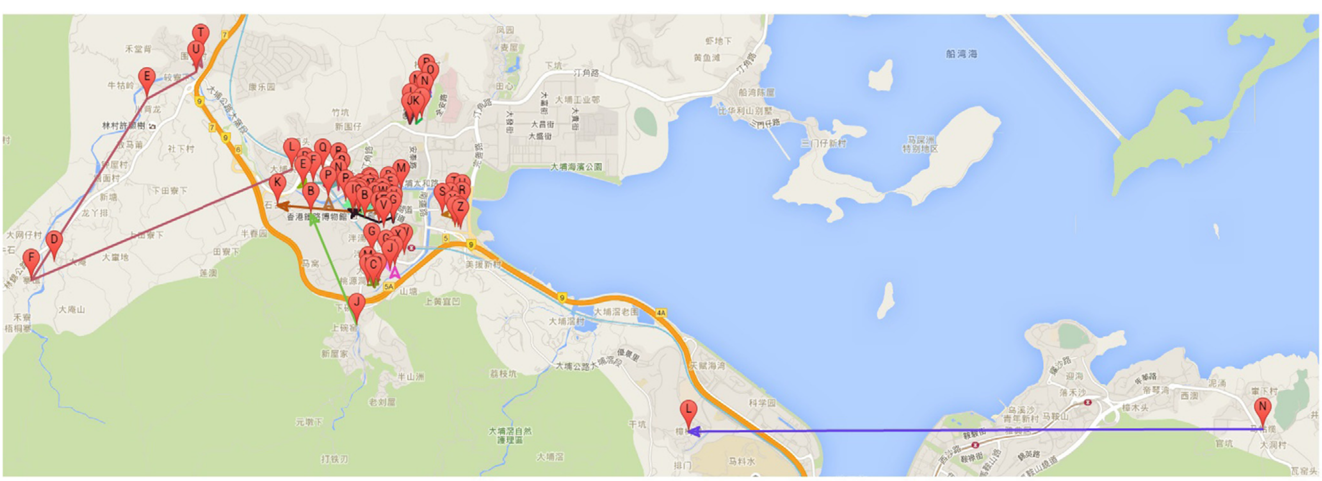
In our ongoing research, we present an interesting research priority of developing an improved heuristic approach to improve the quality of solutions obtained with the greedy heuristic method. The greedy heuristic method finds only good solutions with a satisfied gap threshold. We also intend to consider several extensions of the proposed model. The first extension concerns the possibility of simultaneously optimizing travel duration and the minimum number of districts. This optimization can be achieved by using a multi-criteria approach, which includes formulating the problem as a weighted sum of these two objectives and solved by the Gurobi Optimizer. Optimization can also be achieved by modeling it a bi-objective optimization problem solved by heuristic and meta-heuristic methods, such as the ε -constraint method, elitist non-dominated sorting GA (NSGA-II), and Pareto envelope-based selection algorithm (PESA).



(a). The resulting districts using driving mode by Greedy heuristic



(b). The resulting districts using driving mode by Gurobi



(c). The existing districts using driving mode

Fig. 12. Districts using driving mode.

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Appendix A. Comparison results between two methods

See Table A1.

Table A1
Results obtained by the greedy heuristic method and the Gurobi Optimizer.

Parameters			Greedy heuristic		Gurobi Optimizer		OBJVAL Greedy > Gurobi
Q	T	WM	RUNTIME	OBJVAL	OBJVAL	RUNTIME	
8	**4500**	**420**	**0.58**	**22**	**21**	**555.32**	**1**
8	**4500**	**540**	**0.69**	**21**	**20**	**181.91**	**1**
8	4500	600	0.77	21	21	659.52	0
8	**4500**	**720**	**0.8**	**21**	**20**	**326.67**	**1**
8	4500	840	1.05	22	22	842.69	0
8	5400	420	0.6	21	21	548.55	0
8	5400	540	0.73	20	20	181.91	0
8	5400	600	0.89	19	19	329.17	0
8	5400	720	0.77	19	20	93.47	−1
8	5400	840	0.94	20	20	73.83	0
8	6300	420	0.56	21	21	536.32	0
8	6300	540	0.82	20	20	141.92	0
8	6300	600	0.77	19	19	72.91	0
8	6300	720	0.79	19	20	244.1	−1
8	6300	840	0.91	19	19	43.26	0
8	7200	420	0.55	20	20	51.76	0
8	7200	540	0.68	20	20	38.61	0
8	7200	600	0.74	19	20	251.44	−1
8	7200	720	0.85	19	20	257.94	−1
8	7200	840	0.84	18	18	35.26	0
9	**4500**	**420**	**0.62**	**22**	**20**	**938.69**	**1**
9	**4500**	**540**	**0.67**	**21**	**20**	**850.84**	**1**
9	4500	600	0.79	21	21	912.2	0
9	4500	720	0.85	21	21	1104.77	0
9	4500	840	0.97	22	22	1290.51	0
9	**5400**	**420**	**0.58**	**20**	**19**	**3600.08**	**1**
9	5400	540	0.74	18	18	42.17	0
9	5400	600	0.77	18	18	55.47	0
9	5400	720	0.73	18	18	742.64	0
9	5400	840	0.96	18	18	800.66	0
9	**6300**	**420**	**0.57**	**20**	**19**	**3600.03**	**1**
9	6300	540	0.71	18	18	37.48	0
9	6300	600	0.75	17	17	23.18	0
9	6300	720	0.73	17	18	659.78	−1
9	6300	840	0.91	17	17	64.1	0
9	**7200**	**420**	**0.57**	**19**	**18**	**3600.07**	**1**
9	7200	540	0.66	18	18	31.44	0
9	7200	600	0.73	17	17	25.68	0
9	7200	720	0.76	17	17	105.18	0
9	7200	840	0.85	15	17	39.98	−1
10	**4500**	**420**	**0.6**	**21**	**20**	**1542.31**	**1**
10	**4500**	**540**	**0.67**	**21**	**20**	**1519.11**	**1**
10	4500	600	0.77	21	21	1541.68	0
10	4500	720	0.82	21	21	1354.43	0
10	4500	840	0.99	22	22	1749.21	0
10	**5400**	**420**	**0.66**	**20**	**19**	**3600.12**	**1**
10	5400	540	0.61	17	17	1530.56	0
10	5400	600	0.71	18	17	1138.93	1
10	5400	720	0.73	17	18	902.24	−1
10	5400	840	0.95	18	18	989.59	0
10	**6300**	**420**	**0.6**	**20**	**19**	**3600.03**	**1**
10	6300	540	0.68	16	16	124.03	0
10	6300	600	0.72	16	16	964.55	0
10	6300	720	0.77	16	17	523.88	−1
10	6300	840	0.93	16	17	396.63	−1
10	**7200**	**420**	**0.63**	**19**	**18**	**3600.14**	**1**
10	7200	540	0.67	16	16	106.97	0
10	7200	600	0.76	16	16	795.29	0
10	7200	720	0.72	15	16	124.26	−1
10	7200	840	0.88	15	16	46.26	−1
11	**4500**	**420**	**0.61**	**21**	**20**	**3600.03**	**1**
11	**4500**	**540**	**0.71**	**21**	**20**	**1989.28**	**1**
11	4500	600	0.78	21	21	2131.87	0
11	**4500**	**720**	**0.93**	**21**	**20**	**1847.06**	**1**
11	**4500**	**840**	**0.99**	**22**	**22**	**1766.51**	**1**
11	**5400**	**420**	**0.62**	**19**	**17**	**1022.93**	**1**
11	**5400**	**540**	**0.6**	**17**	**16**	**1768.1**	**1**
11	**5400**	**600**	**0.73**	**18**	**17**	**1617.46**	**1**
11	5400	720	0.77	17	18	1934.1	−1
11	5400	840	0.88	18	18	1275.55	0
11	**6300**	**420**	**0.64**	**18**	**17**	**3600.07**	**1**
11	6300	540	0.67	15	15	1212.78	0
11	6300	600	0.71	15	15	888.78	0
11	**6300**	**720**	**0.84**	**16**	**15**	**915.84**	**1**
11	6300	840	0.87	16	16	815.57	0
11	**7200**	**420**	**0.59**	**18**	**17**	**3600.05**	**1**
11	7200	540	0.68	15	15	1342.91	0
11	**7200**	**600**	**0.77**	**15**	**14**	**537.62**	**1**
11	7200	720	0.88	15	15	720.04	0
11	7200	840	0.84	14	15	39.58	−1
12	**4500**	**420**	**0.83**	**21**	**20**	**3600.05**	**1**
12	**4500**	**540**	**0.72**	**20**	**19**	**1514.14**	**1**
12	4500	600	0.8	21	21	2126.83	0
12	4500	720	1	21	21	2462.46	0
12	4500	840	0.99	22	22	2275.31	0
12	**5400**	**420**	**0.82**	**18**	**17**	**3600.02**	**1**
12	**5400**	**540**	**0.84**	**16**	**16**	**1086.96**	**1**
12	**5400**	**600**	**0.81**	**18**	**17**	**1675.07**	**1**
12	5400	720	0.82	17	18	1182.61	−1
12	5400	840	0.91	18	18	1671.36	0
12	**6300**	**420**	**0.77**	**16**	**15**	**443.03**	**1**
12	6300	540	0.76	14	14	80.52	0
12	6300	600	0.85	15	15	1045.65	0
12	6300	720	0.98	15	15	792.22	0
12	6300	840	0.91	16	16	1120.68	0
12	**7200**	**420**	**0.88**	**15**	**14**	**23.47**	**1**
12	7200	540	0.74	14	14	18.84	0
12	7200	600	0.96	14	14	961.97	0
12	7200	720	0.93	14	14	34.42	0
12	7200	840	0.83	14	14	74.8	0
Greedy heuristic as good as Gurobi Optimizer							54
Gurobi Optimizer works better than greedy heuristic							32
Greedy heuristic works better than Gurobi Optimizer							14

The bold values in table highlight that these results obtained by greedy are better than by Gurobi.

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