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MODELLING APPROACHES FOR THE HOME HEATH CARE DISTRICTING PROBLEM

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ABSTRACT: *The districting problem consists in grouping small geographic areas into larger clusters called "districts" in a way that these latter are "good" according to relevant criteria. Previous districting models that have been proposed in the literature for different contexts such as the design of political districts or the sales territory alignment are first reviewed. Based on these models' analysis, we propose a classification of the criteria considered within the different realms. After that, we treat the problem of designing districts for the home health care structures. The use of the districting approach is motivated by the desire to facilitate the organization of the care delivery, to reduce the transportation costs and to improve the service quality. The relevant characteristics of well-designed districts within this context are: the indivisibility of basic units, the respect of administrative boundaries, the accessibility of the districts, the compactness and the workload equilibrium where the travel time represents an important component of care providers' work time. Finally, we present a set of scenarios that differ according to the importance given to the travel time and "care" workload balance by the decision-makers. An integer programming formulation is then proposed to each one of these scenarios.*

KEYWORDS: *districting problem, home health care, criteria's classification, mathematical programming.*

1. INTRODUCTION

The districting of a territory is a strategic decision which consists in grouping basic units (zip codes areas, streets, etc.) into larger clusters i.e. "districts" so that these districts are "good" according to relevant criteria. These latter can be related to the activity (workload balance), the demography (population equality) or the geographical characteristics (contiguity, compactness, etc.) of the basic units. These basic units which represent an aggregation of people are typically zip codes areas, postal areas, streets, geo-codes addresses, etc.… They are characterized by some measures like the number of inhabitants, the sales potential, the workload, etc. Despites this, the aggregation level of the basic units and the size of the problem significantly influence the computation time necessary to solve the districting problem, the reliability of the solution proposed and consequently its applicability. That is why, it is important to well choose the basic units in order to avoid data aggregation errors which can deteriorate the solution quality. This decision is time and resource consumer and has an important impact on both customers and care providers (their performance and their motivation). In general, managers are motivated by four main factors. First, their desire to better cover a territory with the existing care providers would explain the necessity of the territory adjustment. The second reason is related to the change of human resources or customers numbers that would be alleviated by the districting approach. Moreover, the necessity to balance the activity among the districts also motivates the decision-makers due to the fact that an equitable districting encourages the care providers who would be more productive individually and thus facilitates the increase of the overall performance. Finally, (Kalcsics *et al.*, 2005) present another reason which is related to the introduction of new products within the sales territory context.

This problem has been widely treated in the Operations Research literature, and many models have been proposed for various contexts: political districting, school districting, sales and services territory alignment, salt spreading, etc. In this paper, we are interested in a specific application domain which is the home health care.

The remainder of this paper is organized as follows. In the second section, we begin defining the home health care and an enumerating of the advantages of using the districting approach in this context. Then, the different criteria considered in the districting problem are defined and classified into four main categories. In the fourth section, a review of the districting models proposed in the Operations Research literature is presented. After that, we present the relevant criteria for designing "good" districts within the Home health care context, propose different scenarios which depend on the view points of the decision-makers and develop mathematical formulations for modelling these scenarios. Two possible extensions of these formulations are then presented in the sixth section. The last section provides some concluding remarks.

2. DEFINITION OF HOME HEALTH CARE

The Home Health Care (HHC) represents an alternative to the traditional hospitalisation, such structures have been created fifty years ago in order to solve the problem of hospitals' clogging by providing to the patients, at their home, complex and coordinated medical and paramedical cares for a limited period which can be extended depending on the patients' needs.

These cares are comparable, in terms of nature and intensity, to those who might be delivered to the patients within a traditional hospitalisation.

As the patient's home is one component of the health supply chain, new constraints are to be considered. Consequently, there is a real need for developing innovative approaches in order to improve the organization of the home health care structures.

The main objective of this type of hospitalisation is to contain the health system costs while improving the quality of care delivered to patients in order to help them to reach their best clinical, psychological and social well-being.

It is then clear that the home health care represents an economic and social stake. However despites the interest which it represents, the home health care (HHC) did not know the expected development. Indeed, the HHC structures focused on the therapeutic aspect of the cares delivered to the patients and this explains its need in terms of tools to better organize the conditions of care delivery in order to minimize the operational costs while guaranteeing a satisfactory service quality towards the patients as well as towards the care providers. Minimizing the costs in the home health care consists in minimizing the staffing costs, the medicines' purchase and equipments' acquisition costs and the transportation costs. Guaranteeing a service quality towards the patient involves an improvement of the quality of care, a reduction of the waiting time and a satisfaction of his personal preferences (e.g. affinity with certain care providers, time windows for the visits). Guaranteeing a service quality towards the care providers implies a fair distribution of the workloads between them and the satisfaction of their personal preferences (e.g. periods of vacancies, etc.).

Adopting the districting approach, which consists in partitioning the territory into districts, each district being under the responsibility of a multidisciplinary team composed of care providers responsible of patients' treatments within this district, would help home health care structures to achieve these objectives. Indeed, this approach induces the reduction of the travel time (transportation costs' containment) and consequently the increase of the time dedicated to the direct cares which helps to improve the quality of care (patients' satisfaction) while balancing care providers' workload (care providers' satisfaction). This would be inserted in the strategy of improving the efficiency of the HHC structures by enhancing the service quality and work conditions. Furthermore, the districting approach facilitates the management and the assignment of the patients to their reference care provider (or a group of care providers) since this assignment is subject to a set of parameters among which are the geographical district of both patients and care providers and which must coincide for each patient-reference care provider pair. Finally, the fact that the patients are close to each other would increase the reactivity of the care providers when urgencies occur which would conduct to satisfy more the patients.

This section shows the importance of adopting the districting approach to organize the delivery of cares to patients at home. However, the home health care districting problem shares some similarities with other districting problems previously treated in the literature especially those that has important human impacts such as the police district design or the sales territory alignment. These similarities thus explain the necessity to study the approaches proposed for the other realms.

3. CLASSIFICATION OF THE CRITERIA

The districting problem has been widely treated in the Operations Research literature since the late sixties in a broad range of applications. However, despite the various applications of the districting problem, there are many similarities between the models proposed. As a consequence, the different criteria considered in the districting literature are analogous. We then propose a classification of these criteria into four categories.

To our knowledge, the only two works that propose a classification of the criteria are (Kalcsics *et al.*, 2005) and (Tavares-Pereira, 2007). In the first work, the authors just separate the geographical criteria from the other criteria used in the political districting (demographic criteria) and sales and service territory design (activity-related criteria); while in the second work, (Tavares-Pereira, 2007) propose a classification of the criteria into four categories (the homogeneity criteria, the geographical criteria, the criteria related to the optimisation of the flows between the districts and the similarity criteria). The differences between the classification proposed in (Tavares-Pereira, 2007) and the one that we propose is twofold. First, Tavares-Pereira does not consider the profile balance (see section 4.2.2.); the profitability (see section 4.2.3) and the organizational criteria (see section 4.4). Second, we do not represent the class of the flows' optimisation due to the fact that we do not find any work that considers this type of criteria.

As a consequence, the classification that we propose can be seen as an extension of the two classifications previously mentioned.

The reader is warned that, in the literature, various names can refer to the same criterion. We then nominate the criteria by the most used expressions and put between brackets their synonyms.

3.1. Criteria related to the geographical aspects

The criteria relative to the geographical aspects of the problem are expressed in terms of distances which can be Euclidean distances, straight lines, networks distances, etc. This class contains the following criteria:

- The compactness: there is no rigorous definition of the compactness despite the fact that it has been widely considered in the literature. However many authors have considered that a district is geographically compact if it is somewhat circular or square in shape rather than long and thin. This criterion is very important especially in the political field due to the fact that it prevents from gerrymandering. This latter is defined by (Grilli di Cortona *et al.*, 1999) as a practice that consists in "manipulating the districts in favour of some political party or candidate. The original gerrymander was created in 1812 by Massachusetts governor Elbridge Gerry, who successfully managed to design the district map of the state of Massachusetts in order to guarantee his reelection". The compactness is also very important in the sales and services territory design field as it reduces the total travel times and thus improves a salesman/service provider's efficiency.

- The contiguity (or the connectivity): consists in guaranteeing the connection between the basic units that define one district without having to go through any other district. It is, according to many authors, a desirable property but few of them consider it in an explicit manner. However, according to (Grilli di Cartona *et al.*, 1999), the compactness implies the contiguity. This criterion is generally considered for administrative reasons but also because it protects more against the gerrymandering in the political field.
- The accessibility (or the mobility): this criterion is related to the facility with which the workers would travel within each district; for example by means of public transportation, private cars, etc. Moreover the accessibility of the districts is favoured by the respect of the natural obstacles such as mountains, lakes or rivers. This criterion is very important for sales territories and home health care districts design due to the fact that it reduces the travel time.
- The size: is measured in terms of areas. This criterion, as presented by (D'Amico *et al.*, 2002) is satisfied if the ratio of the areas of the largest and smallest districts does not exceed specified lower and upper bounds.
- The response time to calls for service: is a criterion that characterizes, as mentioned by (D'Amico *et al.*, 2002), the police district design. This response time is defined as the sum of the travel time and the time during which the call is queued while awaiting availability of patrol officers. Note that the travel time is closely related to the distance between the centers of the districts and the scenes of emergencies (places of calls). As police effectiveness is measured by the average response time for calls, this criterion represents a performance objective for the police district design that must respect a predefined service quality level related to threshold values of the customers waiting times necessary to guarantee their satisfaction.

3.2. Criteria relative to the activity measures

This class of criteria can be divided into three subclasses.

3.2.1. Activity balance

These criteria aim to balance the different districts according to specific attributes:

- The population equality (voter equality): this criterion requires the design of districts that have the same number of voters. As the exact equality is very difficult to obtain, deviations from the average population are allowed. Population equality is especially used in the political field as it embodies the respect of the "one-man-one-vote" principle.
- The workload balance (workload equilibrium): is generally used within the context of sales and services territory alignment problems for the design of districts which are balanced relative to the workload that can be expressed as the time spent with the customers or as the sum of the time spent with the customers and the travel time. This criterion is related to the desire of fairly distributing the workload among all the workers (salesmen or service staff).
- The balance of the customers numbers (the balance of product/ service demand): this criterion is analogous to the previous one since the number of customers is an activity measure that could be considered for designing balanced districts within the context of sales and services territory alignment problems. In this case, the main motivation is the fair distribution of potential prospects or profit (see (Kalcsics *et al.*, 2005)).
- The capacity limitation: is generally used for the school district design. It consists in respecting the available space within each school. This limitation is due to the fact that each school has a specified capacity that can not be exceeded.

3.2.2. Profile balance :

This subclass of criteria is used when it is necessary to differ, according to an attribute, between the different profiles of inhabitants, customers or activities done by the workers. Among the criteria related to this differentiation, there are:

- The socio-economic homogeneity: to our knowledge, this criterion has been considered only by (Bozkaya *et al*., 2003). It is based on the personal incomes of the inhabitants and guarantees a better representation of residents who "share common concerns or views". This homogeneity is important in the sense that it ensures that all the socio-economic classes have the same opportunities to be represented.
- The minority representation (racial balance): this criterion is very important in the school districting since it ensures the same educational opportunities to all the races.

3.2.3. Profitability

The profitability of the districts depends essentially on the total time spent by the salesmen within each district. This criterion is used for the sales territory alignment and is especially related to the main objective of the commercial structures: the profit maximization.

3.3. Criteria relative to the comparison between different territory partitions

This class aims to compare different territory partitions that can be of different types or of the same type:

- The conformity of the districts designed to administrative boundaries: in most of the references mentioned in this work, the districts designed must be coherent with the administrative boundaries.
- The similarity with the existing plan: this criterion is essentially used in the redistricting context when the problem consists in constructing new districts as similar as possible to the current ones. The main goal of this criterion is not to change a current partition radically but to improve its efficiency.

3.4. Organizational criteria

Despite the criteria already mentioned, there are two other ones relative to:

- The number of districts to design: in all the references mentioned in this work, the total number of districts must be exactly equal to a number predetermined by the decision makers.
- The indivisibility of basic units (the exclusive assignment of basic units, the integrity of basic units, the non splitting of basic units): it is necessary to assign each basic unit to one and only one district. This indivisibility allows, in the sales or the services territory context, the establishment of long-term relationships between the salesmen/service staff and the customers. Despite this, it avoids the interference of the workers' responsibilities since the integrity constraint guarantees that each basic unit is under the exclusive responsibility of one worker (or a group of workers). Finally, it would be difficult to decide which person (patient, resident, student, customer, etc.) should be assigned to which district since there are no data to indicate how to proceed for dividing the persons living within the same basic unit.

The different criteria presented in this taxonomy have been formulated mathematically in various manners (as objective functions to optimize or hard constraints to satisfy) within the literature related to the districting problem. In the next section, we are going to review a part of this literature.

4. LITERATURE REVIEW

The districting problem has been widely treated in the Operations Research literature since the late sixties in a broad range of applications: the design of political districts; the definition of districts for salesmen; the establishment of districts for schools, salt spreading and police command; the construction of turfs for telecommunications' workers and also the definition of districts for the home health care workers.

Hence, the political and the sales realms are the two most important applications in terms of number of publications.

Several approaches have been proposed in the Operations Research literature to treat this strategic decision. Among these approaches, we can distinguish two categories: the managerial heuristics ((Easingwood, 1973), (Lodish, 1975)) and the exact methods based on mathematical programming techniques. There are two major types of mathematical models related to the districting problems: the set-partitioning models and the location-allocation models.

For the first approach, the districts are designed by grouping basic units so that an objective function is optimised while respecting predefined constraints. However, the second approach consists in defining the centers of the districts and then determining their associated basic units.

In the following section, we present some of the works dealing with the districting problem in the political and the sales territory realms, in the home health care context and also in the police district design field. This latter bears an important similarity with the home health care districting problem which consists in the importance of the human impacts of the partition proposed on both the care providers (police officers) and the patients (population). The works presented in this section are classified according to their realm and sorted by their year of appearance. For each one of them, we precise the different criteria considered as well as the approach used to model the problem.

4.1. Political districting

In this application, a territory has to be partitioned into a predetermined number of districts from which political candidates are elected. The districting procedure must respect the "one-man-one-vote" principle: since each district elects one member, the districts designed must have approximately the same number of voters while satisfying other criteria.

The first mathematical programming approach was proposed by (Hess *et al.*, 1965) who formulate the political problem as a location-allocation problem (capacitated m-median facility location problem) for designing compact and contiguous districts which respect the indivisibility of basic units' constraints and whose populations must lie within a predetermined interval.

(Garfinkel and Nemhauser, 1970) address the problem of political districting as a set partitioning problem. They present a two-stage enumerative procedure which minimizes the maximum deviation from the desired district average size. In the first stage, they generate feasible districts based on criteria related to population equality (total voters within an interval), compactness and contiguity. Then, in the second stage, they determine the set of M optimal districts that minimize the maximum deviation from the average population while respecting the indivisibility of basic units.

More recently, (Hojati, 1996) applies a three-stage location-allocation approach to divide a territory into a given number of districts while respecting criteria related to contiguity, compactness and population equality. By this methodology, district centres are first determined, then basic units are assigned to the district centres and finally the basic units that are assigned to two district centres or more are reassigned to only one of them.

(Mehrotra *et al.*, 1998) build their work on the previous work of (Garfinkel and Nemhauser, 1970) and formulate the problem as a set-partitioning problem. The objective function corresponds to the minimization of the overall compactness of the districts characterized by their population equality, their contiguity, the non-splitting of the basic units and the respect of administrative boundaries as much as possible.

(Bozkaya *et al.*, 2003) propose a weighted multi-criteria approach for the political districting problem by considering five criteria: contiguity, population equality, compactness, socio-economic homogeneity and similarity with the existing plan where the first criterion is considered as a hard constraint while the others are combined in a weighted additive multi-criteria objective function. The advantage of this method is that it allows the users to compare different scenarios.

4.2. Sales territory alignment

The sales territory design consists in grouping sales coverage units into districts of salesmen's responsibility which have approximately similar sizes in terms of number of customers or workloads.

(Hess and Samuels, 1971) first apply a location-allocation model to approach the sales territory alignment problem in order to maximize the total compactness of all districts while balancing the "activity" of the entire salesmen and minimizing the changes in territorial boundaries. The authors propose different "activity" measures such as the number of sales calls or the sales potential and stress on the importance of a good selection of the measure since it influences the solution quality.

After this, (Zoltners and Sinha, 1983)) develop four properties of a "good" sales territory design which are the indivisibility of basic units, the activity balance according to predefined attributes, the contiguity of the districts and the compatibility with geographical obstacles. In order to satisfy these four properties, (Zoltners and Sinha, 1983) propose a location-allocation model whose objective would be the minimization of the travel time or the maximization of the profitability.

(Fleischmann and Paraschis, 1988) also approach the sales territory alignment problem by a location-allocation model which respects the following criteria: the workload balance, the compactness of the districts and the indivisibility of the basic units.

More recently, (Rios-Mercado and Fernandez, 2009) suggest a location-allocation (p-centre) model for the sales territory design problem. The objective is the maximisation of the total compactness while balancing three different activity measures which are the number of customers, the product demand and the workload among the contiguous districts.

4.3. Police district design

In the Operations Research literature, the districting problem within the police patrol context has been treated by (D'Amico *et al.*, 2003). They model the problem as a set-partitioning problem subject to constraints of compactness, contiguity, size and also quality of service considerations related to the response time to calls for service. Note that the "goodness" of a district is related to the disparity between the maximum workload and the minimum workload of the patrol officers and also to the average response time to a call. After the design of the districts, the optimal number of patrol cars is determined for each district.

4.4. Home health care districting problem

To our knowledge, there is a unique Operations Research work that treats the home health care districting problem and which has been conducted by (Blais *et al.*, 2003). Using a model similar to the one proposed by (Bozkaya *et al.*, 2003) for the political districting problem, they propose a multi-criteria approach, where the criteria related to the visiting personnel mobility (via public transportation and respect of the geographical obstacles) and the workload equilibrium (measured by the sum of the travel time and the visit time) are combined into a single objective function whereas the criteria related to the indivisibility of the basic units, the respect of borough boundaries and the contiguity are considered as hard constraints.

As we have mentioned in the second section, adopting the districting approach in the home health care context would be advantageous as it allows the transportation costs containment and the improvement of the service quality and work conditions. Furthermore, the districting approach is performed at the strategic level and is thus related to the managerial organization that should support the tactical and operational decisions i.e. the assignment of the patients to their reference care providers and the scheduling and routing of the care providers' activities within each district.

In the rest of the paper, we are interested in the modelling of the home health care districting problem.

5. MODELING OF THE PROBLEM

The objectives of the districting approach within the home health care context are the containment of the transportation costs and the satisfaction of both the patients and the care providers.

From the patients' view point, a "good" districting should allow the improvement of the quality of care, the reduction of the waiting time and the equilibrium of the quality of the care delivered within each district.

Furthermore, the care providers consider that the main objective of the districting approach is to design districts such that their workloads are balanced and their work time is efficiently used. The analysis of their workload shows that it is essentially composed of the travel time and the time spent with each patient for delivering the necessary cares. While the first depends essentially on the distances between the different patients' home and the accessibility of the districts, the second is related to the number of patients per basic unit and the time necessary to treat each patient.

At this level, it is important to mention that most of the home health care structures classify patients' therapeutic projects by categories named "profiles". This classification depends essentially on the pathology from which the patient suffers, the necessary frequency of the visits, the average duration of the visits and the type(s) of care(s) that he needs. It is then clear that the importance of each profile varies according to its corresponding workload. We then propose to begin by enumerating the different profiles that the HHC structure may use to treat its patients. This step would allow the comparison of these profiles based on their corresponding workload in order to identify the most critical one(s) and after that to realize a certain profile balance since the different basic units may be not homogeneous according to these profiles. Note that (Blais *et al.*, 2003) have not considered the different profiles separately.

In this work, we assume that the HHC structure can treat all the profiles, that each profile is characterized by a predefined number of visits and an average duration of these visits and finally that all the basic units are covered which means that all the patients can be treated by the HHC structure.

5.1. The criteria considered

One of the most important challenges in the home health care system is to provide a satisfactory quality of care and to balance the workload among the medical and paramedical personnel. In order to reach this, we consider the following criteria for the districting problem:

- The number of districts to design is predetermined by the managers.
- The indivisibility of basic units: each basic unit must be assigned to one and only one district. In order to make this criterion manageable, it is important to define the basic units carefully so that the problem would be feasible. This criterion is considered in order to avoid the interference between the care providers' responsibilities and to establish long term relationships with patients.
- The compatibility of basic units: this compatibility is related to the respect of administrative boundaries and the accessibility of districts. The former facilitates the organization of health care delivery procedures and the work with community agencies. The latter is crucial in the home health care field due to the fact that the care providers visit the patients at their home. This accessibility is favoured first, by the respect of geographical obstacles such as mountains, bodies of water, etc. and second by the mean of transport like the public transportation or private cars which means that if the transit between two basic units via the public transport or the routes is too complicated or too long, then these two basic units should not be assigned to the same district.
- The compactness: the most common way to formulate the compactness is to limit the maximum distance between the basic units within the same district. This maximum distance allows the limitation of the travel time necessary to go from one basic unit to another within the same district. As a consequence, the compactness criterion permits the reduction of the unproductive travel time and consequently the increase of the proportion of time devoted to direct cares and the improvement of the cares providers' efficiency. Despite this, the compactness of the districts guarantees their contiguity (according to (Grilli di Cartona *et al.*, 1999)).
- The workload equilibrium: it is essential that the workloads of the different districts are almost the same for the design of "good" districts. As we have seen before, the total workload is defined as the sum of the "care" workload and the travel time. However, a particular attention must be given to the workload's formulations which are intimately related to the managers view point.

Thereby, the proposed criteria allow the modelling of different scenarios depending on the managers' preferences concerning the formulation of the two parts of the workload. Indeed, the direct cares can be considered as an objective function to optimise (or a part of the objective function) or a hard constraint to satisfy. However, the travel time can not be modelled as a hard constraint; the compactness's constraint already exits.

The subsection 5.5 presents the different scenarios and their corresponding mathematical formulations for which we first define the indices, sets, parameters and decision variables.

5.2. Indices and sets

Let:

- N: the number of basic units.
- M: the number of districts to design.
- H: the number of profiles considered.
- i: basic unit index (i=1,…,N).
- j: district index (j=1,…,M).
- h: profile index (h=1,…,H).

5.3. Parameters

To design the districts, we need the following parameters:

- b_h: the frequency of the visits relative to the profile h.
- T_h: the average duration of the visits relative to the profile h.
- Nbr_{ih}: the number of patients having the profile h and living in the basic unit i.
- d_{ik}: the distance between the basic units i and k. The distance metric used could be: the Euclidean distance or the network distance. However, it would be better to consider the network distance since the Euclidean distance does not reflect the real time spent between the basic units while our main objective is to minimize the travel time.

- d_{max}: the maximum distance between two basic units assigned to the same district.
- α: the reverse of the travel speed.
- β: the weight of the "care" workload.
- e_{ik}: the compatibility index where :

$$e_{ik} = \begin{cases} 1 & \textit{if basic units } i \textit{ and } k \textit{ are compatible} \\ 0 & \textit{otherwise} \end{cases} \quad (1)$$

The basic units i and k can be incompatible for three main reasons:

- Existence of geographical obstacles between them.
- Difficulty or impossibility to travel from one basic unit to another by the means of transport used by the HHC structure's care providers (the public transportation, the private cars, etc.).
- They do not belong to the same administrative district.

5.4. Decision variables

To solve this problem, we propose an integer linear programming model for which we define the following decision variables.

- The assignment decision variables :

$$x_{ij} = \begin{cases} 1 & \textit{if basic unit } i \textit{ is assigned to district } j \\ 0 & \textit{otherwise} \end{cases} \quad (2)$$

We also consider three other types of variables:

- the "care" workload of the basic unit i (i=1,…,N)

$$a_i = \sum_{h=1}^{H} Nbr_{ih}.b_h.T_h \qquad \forall \quad i = 1,\ldots, N \qquad (3)$$

- the "care" workload of the district j (j=1,…,M)

$$a\_district_j = \sum_{i=1}^{N} a_i.x_{ij} \qquad \forall \quad j = 1,\ldots, M \qquad (4)$$

- the average "care" workload $\bar{a} = \dfrac{\sum_{j=1}^{M} a\_district_j}{M}$ (5)

5.5. Different scenarios

Figure 1 shows the matrix that combines the "care" workload and the travel time and distinguishes between the different scenarios related to HHC districting problem.

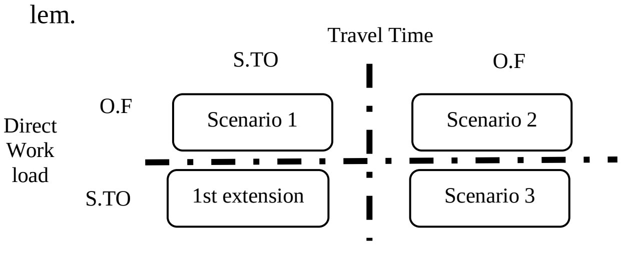

Figure 1: Classification of the scenarios

5.5.1. *First scenario*

Within this scenario, the main objective of the managers would be to design well balanced districts according to the "care" workload. Within this context, the travel time is not explicitly formulated. However, its reduction is guaranteed thanks to the compactness criterion. The mathematical formulation of this scenario is the following. The objective function (6) guarantees the balance of "care" workload between the different districts. This balance would be obtained by minimizing the maximum deviation of the "care" workload of the districts from the average "care" workload. Constraints sets (7), (8) and (9) are used to calculate respectively the "care" workload of each basic unit, the "care" workload of each district and the average "care" workload. Constraints set (10), together with constraints set (13) ensure that each basic unit is assigned to one and only one district. The accessibility and the respect of administrative boundaries criteria are guaranteed by the constraints set (11) related to the compatibility of the basic units assigned to the same district. Finally, constraints set (12) are related to the compactness criterion.

$$Minimize \quad \max_{j=1,\ldots,M} \left| a\_district_j - \bar{a} \right| \qquad (6)$$

$$s.to$$

$$a_i = \sum_{h=1}^{H} Nbr_{ih}.b_h.T_h \qquad \forall \quad i = 1,\ldots, N \qquad (7)$$

$$a\_district_j = \sum_{i=1}^{N} a_i.x_{ij} \qquad \forall \quad j = 1,\ldots, M \qquad (8)$$

$$\bar{a} = \frac{\sum_{j=1}^{M} a\_district_j}{M} \qquad (9)$$

$$\sum_{j=1}^{M} x_{ij} = 1 \qquad \forall \quad i = 1,\ldots, N \qquad (10)$$

$$x_{ij} + x_{kj} \le 1 \qquad \forall \quad (i,k) \in E, \quad j = 1,\ldots,M$$
$$(i,k) \in E \quad iif \quad e_{ik} = 0 \quad \forall \quad i = 1,\ldots N \quad k = 1,\ldots N \qquad (11)$$

$$x_{ij} + x_{kj} \le 1 \qquad \forall \quad (i,k) \in D, \quad j = 1,\ldots,M$$
$$(i,k) \in D \quad iif \quad d_{ik} \ge d_{\max} \quad \forall \quad i = 1,\ldots N \quad k = 1,\ldots N \qquad (12)$$

$$x_{ij} \in \{0,1\} \qquad \forall \quad i = 1,\ldots,N \quad j = 1,\ldots,M \qquad (13)$$

5.5.2. *Second scenario*

It is possible that the decisions-makers choose to balance the districts according to the two parts of the workload i.e. the travel time and the "care" workload. Consequently, it is necessary to assign to each part a weight that reflects its importance compared to the other part. We then formulate the objective function as the weighted sum of these two parts. This scenario corresponds to the following integer linear model:

$$Minimize \quad \beta . \max_{j=1,\dots,M} \left| a\_district_j - \bar{a} \right| + (1-\beta).\alpha . \max_{j=1,\dots,M;\ i=1,\dots,N;\ k=1,\dots,N} (d_{ik}.x_{ij}.x_{kj}) \tag{14}$$

$s.to$

$$a_i = \sum_{h=1}^{H} Nbr_{ih}.b_h.T_h \quad \forall \quad i = 1,\dots, N \tag{15}$$

$$a\_district_j = \sum_{i=1}^{N} a_i.x_{ij} \quad \forall \quad j = 1,\dots, M \tag{16}$$

$$\bar{a} = \frac{\sum_{j=1}^{M} a\_district_j}{M} \tag{17}$$

$$\sum_{j=1}^{M} x_{ij} = 1 \quad \forall \quad i = 1,\dots, N \tag{18}$$

$$x_{ij} + x_{kj} \leq 1 \quad \forall \quad (i,k) \in E, \quad j = 1,\dots, M$$
$$(i,k) \in E \quad iif \quad e_{ik} = 0 \quad \forall \quad i = 1,\dots,N \quad k = 1,\dots,N \tag{19}$$

$$x_{ij} \in \{0,1\} \quad \forall \quad i = 1,\dots,N \quad j = 1,\dots,M \tag{20}$$

The analysis of the different constraints sets of the second model is the same as for the first one (constraint sets (15), (16), (17), (18), (19) and (20)) expect for the objective function defined as the weighted sum of the two indicators that correspond respectively to the maximum deviation of the district "care" workload from the average "care" workload and the maximum travel time between two basic units within the same district.

5.5.3. Third scenario

In the two previous scenarios, we model the cases where the main objective is to minimize the maximum deviation of the "care" workload from the average "care" workload. However, the managers could prefer defining a tolerance interval that insures that no district "care" workload may deviate from the average "care" workload by more than a pre-specified percentage ±τ. The choice of this type of constraints can be explained by the fact that the indivisibility of basic units makes the perfect balance of "care" workload very difficult. The quality of the results depends on the tolerance interval chosen by the HHC managers. In this scenario, the travel time is considered, as in the second model, in the objective function (21) which consists in minimizing only the maximum travel time between two basic units within the same district, the "care" workload equilibrium being considered in the constraints sets (27) and (28). These constraints represent the conditions related to the minimum and maximum allowable "care" workload within each district as a percentage of $\bar{a}$. The analysis of the different constraints sets (22), (23), (24), (25), (26) and (29) of the third model is the same as for the second one.

$$Minimize \quad \alpha . \max_{j=1,\dots,M;\ i=1,\dots,N;\ k=1,\dots,N} (d_{ik}.x_{ij}.x_{kj}) \tag{21}$$

$s.to$

$$a_i = \sum_{h=1}^{H} Nbr_{ih}.b_h.T_h \quad \forall \quad i = 1,\dots, N \tag{22}$$

$$a\_district_j = \sum_{i=1}^{N} a_i.x_{ij} \quad \forall \quad j = 1,\dots, M \tag{23}$$

$$\bar{a} = \frac{\sum_{j=1}^{M} a\_district_j}{M} \tag{24}$$

$$\sum_{j=1}^{M} x_{ij} = 1 \quad \forall \quad i = 1,\dots, N \tag{25}$$

$$x_{ij} + x_{kj} \leq 1 \quad \forall \quad (i,k) \in E, \quad j = 1,\dots, M$$
$$(i,k) \in E \quad iif \quad e_{ik} = 0 \quad \forall \quad i = 1,\dots,N \quad k = 1,\dots,N \tag{26}$$

$$(1-\tau)\bar{a} \leq a\_district_j \quad \forall \quad j = 1,\dots, M \tag{27}$$

$$a\_district_j \leq (1+\tau)\bar{a} \quad \forall \quad j = 1,\dots, M \tag{28}$$

$$x_{ij} \in \{0,1\} \quad \forall \quad i = 1,\dots, N \quad j = 1,\dots, M \tag{29}$$

6. MODEL'S EXTENSION

6.1. First extension

In the three scenarios, we have not separated the different profiles when we have calculated the "care" workload. However, important gaps between the workload related to these different profiles may exist. Consequently, they can influence the productivity of the districts differently. This difference explains the necessity of considering them separately. To do this, it is important to enumerate all the profiles treated by the home health care structure and after that sort them according to the importance of their relative "care" workload. By doing this, we give the decisions-makers the opportunity to treat the profiles differently: for the first part of the profiles, we minimize the maximum deviation of the "care" workload while for the second one, we define the tolerance interval depending on the decisions-makers' preferences.

In the following model, we apply the method of profiles' distinction to the first scenario. Note that this distinction could be applied to the two others scenarios.

In this model, h* represents the profile whose "care" workload must be within a predefined interval, that is why the balance of the h*'s "care" workload is considered as a hard constraint in this model. However, the "care" workload related to the other profiles (//h*) is balanced in the objective function due to the fact that this imbalance could conduct to a large inequity between the district workload.

All the other constraints sets of this model are similar to the ones used in the first one.

Apart from the profiles, the "care" workload varies according to the care providers' specialities. Similarly to the profiles' distinction, it would be also possible to separate the various specialities and to model them differently (objective function versus hard constraint) in order to respond to the priorities relative to the different care providers' specialities.

$$Minimize \quad \max_{j=1,\dots,M} \left| a\_district_{j//h^*} - \overline{a} \right| \tag{30}$$

$$s.to$$

$$a_i // h^* = \sum_{h=1, h \neq h^*}^{H} Nbr_{ih}.b_h.T_h \quad \forall \quad i = 1,\dots, N \tag{31}$$

$$a_{ih^*} = Nbr_{ih^*}.b_{ih^*}.T_{h^*} \quad \forall \quad i = 1,\dots, n \tag{32}$$

$$a\_district_{j//h^*} = \sum_{i=1}^{N} a_{i//h^*}.x_{ij} \quad \forall \quad j = 1,\dots, M \tag{33}$$

$$a\_district_{jh^*} = \sum_{i=1}^{N} a_{ih^*}.x_{ij} \quad \forall \quad j = 1,\dots, M \tag{34}$$

$$\overline{a_{h^*}} = \frac{\sum_{j=1}^{M} a\_district_{jh^*}}{M} \tag{35}$$

$$\overline{a_{//h^*}} = \frac{\sum_{j=1}^{M} a\_district_{j//h^*}}{M} \tag{36}$$

$$\sum_{j=1}^{M} x_{ij} = 1 \quad \forall \quad i = 1,\dots, N \tag{37}$$

$$x_{ij} + x_{kj} \le 1 \quad \forall \quad (i,k) \in E, \quad j = 1,\dots, M$$
$$(i,k) \in E \quad iif \quad e_{ik} = 0 \quad \forall \quad i = 1,\dots,N \quad k = 1,\dots,N \tag{38}$$

$$x_{ij} + x_{kj} \le 1 \quad \forall \quad (i,k) \in D, \quad j = 1,\dots, M$$
$$(i,k) \in D \quad iif \quad d_{ik} \ge d_{\max} \quad \forall \quad i = 1,\dots,N \quad k = 1,\dots,N \tag{39}$$

$$(1-\tau)\overline{a}_{h^*} \le a\_district_{jh^*} \quad \forall \quad j = 1,\dots, M \tag{40}$$

$$a\_district_{jh^*} \le (1+\tau)\overline{a_{h^*}} \quad \forall \quad j = 1,\dots, M \tag{41}$$

$$x_{ij} \in \{0,1\} \quad \forall \quad i = 1,\dots,N \quad j = 1,\dots,M \tag{42}$$

6.2. Second Extension

One of the most important challenges of the home health care is related to the continuity of care. Indeed, it is defined by (Woodward *et al.*, 2004) as the degree with which the formal (medical and paramedical) and informal (delivered by the caregivers) cares are delivered by a sequence of coordinated and uninterrupted activities that convince to the patients needs. Thus, the patients should be treated by the same care provider (or a set of care providers) during their stay within the HHC structure.
Consequently, when a home health care's partition is revised, it is necessary to limit the changes of the assignments of each basic unit to different districts for a period of few years. To do this, we need to use the dynamic programming by replacing the decision variables set x_{ij} by another one that integrates the time dimension:

$$x_{ijt} = \begin{cases} 1 & if \;\; basic\ unit \;\; i \;\; is \;\; assigned \;\; to \;\; district\ j \;\; in \;\; year\ t \\ 0 & otherwise \end{cases} \tag{43}$$

Two other parameters must then be considered:

- L: the maximum number of changes permitted.
- S: the time horizon for the districting decision.

We then add to the previous models the following constraints set:

$$\sum_{j=1}^{M} \sum_{i=1}^{S} x_{ijt} \le L \quad \forall \; i = 1,\dots, N \quad \forall \; s = 1,\dots, S \tag{44}$$

Thus, the main purpose of the section five and six was to propose a general approach for designing a predetermined number of compact, accessible and balanced districts according to the "care" workload and the travel time. In this approach, we formulate the districting problem in different ways. This helps the managers to design balanced districts according to their priorities but also give them the possibility to separate the different profiles and/or the different care providers' specialities based on their corresponding workload. Finally, we propose the possibility of adopting the dynamic programming in order to respect the continuity of care by limiting the changes in the partitions proposed for a horizon of few years.

7. CONCLUSION

In this paper, we present an analysis of the literature related to the districting problem and propose a classification of the different criteria considered. Furthermore, we focus on the districting problem applied to the home health care structures by modelling it in different ways in order to allow the managers to design the districts according to their priorities.
This partitioning permits the reduction of the travel time and the increase of the proportion of time allocated to the direct care. Despite this, the fact that each care provider is responsible of one specific district allows him to be more familiar with his patients and also improve the work conditions which results on a greater performance and a better motivation. However, according to (Lahrichi *et al.*, 2006), the shortcoming of this approach is that it can create a workload imbalance between care providers due to the difficulty to predict the fluctuations of the demand. These fluctuations can be related to the change of the patients' home, the evolution of their clinical conditions, the arrival of new patients or the exit of some patients (death or recovery), etc.
This inequity would lead to the non uniformity of the services delivered since each care provider adjusts his work according to the number of patients he must visit. In order to alleviate the shortcoming of this strategic decision, it is important, at the tactical level, to adjust

periodically the number of the human resources to the variation of the demand within each district. Furthermore, the assignment of patients to the human resources at the short level must take into account their workload related to the cares needed by the patients already assigned to them.
To strengthen more this alleviation, we can permit to the care providers, when there are urgencies, to visit patients in the neighbouring districts when neither the reference care providers nor the other ones that are working in the district where the patient lives are available. Furthermore, this also encourages the collaboration between the multidisciplinary teams working in different districts. The home heath care structure can also, as proposed by (Lahrichi *et al.*, 2006), opt for creating a surplus team working in all the districts.
All these propositions would conduct to solve the problem of workload imbalance between care providers and also the non uniformity of the quality of care which must not depends on the particular district to which the patients belong but only on their needs in such a way that all the patients could receive services of the same quality level.
We also remember that the main purpose of this work is to offer to the decision-makers different possibilities to model the districting problem. The next step of this work would be to propose an approach to solve real instances as these mathematical formulations are known as NP-hard which explains the necessity of developing a heuristic approach to solve real instances.
Finally, we must keep in mind that the use of a total automatic procedure is not possible due to the fact that some criteria are very difficult to quantify. Despite this, the importance of the human factor in the home health care context necessitates a continuous interaction with both the decision makers and the care providers.

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