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A patient assignment algorithm for home care services

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We consider the problem of assigning patients to nurses for home care services. The aim is to balance the workload of the nurses while avoiding long travels to visit the patients. We analyse the case of the CSSS Côte-des-Neiges, Métro and Parc Extension for which a previous analysis has shown that demand fluctuations may create work overload for the nursing staff. We propose a mixed integer programming model with some non-linear constraints and a non-linear objective which we solve using a Tabu Search algorithm. A simplification of the workload measure leads to a linear mixed integer program which we optimize using CPLEX.

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Introduction

The 'Ministère de la Santé et des Services Sociaux' (MSSS) and its network offer health and social services to the entire population of Québec to ensure the welfare of its residents. In 2004–2005, 37% of the overall budget of the government of Québec was allocated to health and social services. Orientations, budgetary resources and results assessment obtained in the entire health care network are established at the central level. At the regional level, the 'Health and Social Services Agencies' are charged with regional planning, resource management and budget allocation to institutions. At the local level, the 95 'Health and Social Services Centres' (CSSS) established in June 2004 and their partners in local services networks share a collective responsibility for the population on their local territories.

The CSSSs were created by merging existing local community health centres (CLSCs), residential and long-term care centres (CHSLDs) and general and specialized hospital centres (CHSGSs). Each CSSS ensures the population on its territory has access to health and social services. The local network of services thus created within a single territory has many objectives such as to promote health and well-being, and offer a cohesive set of services to the public. This enables people to move through the health and social services network and ensure better patient management, particularly of the most vulnerable users (Ministère de la Santé et des Services Sociaux, 2004).

Home care services are a great part of the services managed by the CSSSs. They are provided by health care professionals and are required for acute illness, post-hospitalization and post-operation treatment, long-term health conditions and/or chronic conditions, permanent disability, including physical and mental disability, or terminal illness.

The territorial approach to manage home care services has been used since 1980 in the specific CLSC Côte-des-Neiges site in Montreal (CLSC CDN for short), which catered for 130 000 inhabitants in 2004, among which 5200 are regular home care service users. Given the size of the territory, the management team partitioned the territory into six districts (Blais *et al*, 2003), with each district being assigned to a multidisciplinary team of professionals. This has allowed for increased efficiency in terms of patient assignment (the geographic location of the patient determines which team will be responsible for the care of that patient), reduced transportation time, and therefore allowed for more time for direct patient care (Figure 1).

Since patient assignment to nurses is performed according to a territorial approach (ie the assignment is based on the territorial origin of the demand, and not according to the actual workload of the nurses), partitioning of the territory and assignment of nurses to each district must be carried out carefully so that the nurses can end up with similar workloads. The partitioning in Blais *et al* (2003) was performed on the basis of historical data on number of patients and number of nursing visits. Since the population changes over time, thereby bringing about changes in the demands for services originating in each district, districting exercises must be performed on a regular basis to counterbalance these fluctuations in demands over time. In addition to workload inequities between nurses that such fluctuations tend to create, it has been observed that the availability of nursing services tends to determine the services actually delivered. This in turn leads to inequities on level of service depending on districts. Since reorganizing districts is time and resource consuming and can cause important changes in patients follow-up (by

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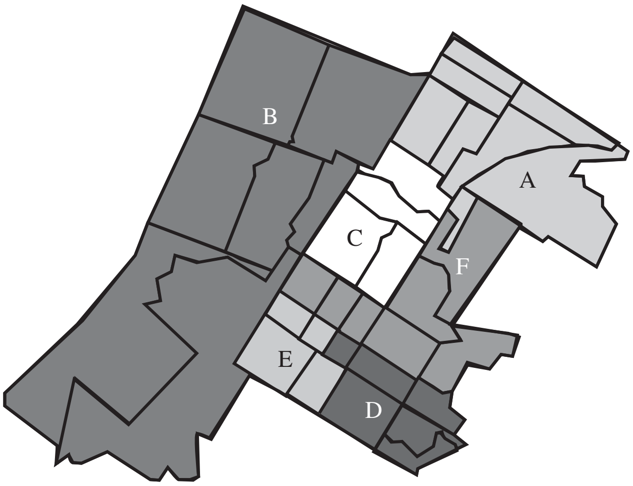

Figure 1 The six districts of the CLSC CDN territory.

changing the case holder), a more dynamic method should be considered to assign patients to nurses (Lahrichi *et al*, 2006). By this, we mean that the patient assignment to nurses should be based on the actual workload of every nurse at the time of the demand for services.

In this paper, we propose a model and an algorithm for assigning patients to nurses that take into account not only the geographic location of the patients but also each nurse's workload. We first give a brief description of the territorial approach used at CLSC CDN, and present measures that help comparing the workload of the nurses. The patient assignment model is then described with the proposed solution method. Experimental results are then reported, and we conclude with final remarks.

The territorial approach at CLSC CDN

The territory of CLSC CDN is currently divided into six *districts*, each one being constituted by several *basic units* which are the census tracts used by Statistics Canada. Requests for home care services arrive at CLSC CDN from a hospital or a physician's office, directly from the patient, a family member or a friend. The intake nurse identifies the district associated with the patient's address and forwards the request to the manager of the team responsible for that particular district. At the same time the nurse performs an analysis of the nature and the urgency of the request for services and then takes a decision to assign the patient to one professional. A patient requiring nursing care as a major component of his care plan will be assigned to a nurse. This decision is confirmed by the manager of the multidisciplinary team who receives every new request for home care services. The professional responsible for the patient will, in most cases, involve other professionals in the care of the patient.

A distinction is made between *case manager nurses* (who typically hold a Bachelor's degree in nursing) and *nurse technicians* (who typically hold a community college degree in nursing). The nurse technicians will be assigned the short-term patients or long-term patients needing punctual nursing care. For instance, a patient requiring short-term and specific nursing care such as a wound dressing or a post-hospitalization home-based antibiotic-therapy treatment, will typically be assigned to a nurse technician.

Conversely, a patient requiring the organization of a more complex service plan such as organizing the activities of daily living, coordinating visits and ensuring links with doctors and specialists as well as with the pharmacist, consulting with and arranging evaluations by other professionals (occupational therapist, physiotherapist, social worker, dietician, etc) will be assigned to a case manager nurse. Such patients typically include frail elderly patients with a great loss of autonomy, palliative patients, patients with cancer, patients suffering from degenerative diseases or chronic illnesses and patients with serious mental health problems.

In addition to these nurses who are part of the six multidisciplinary teams and to whom patients get assigned to, there are three to four nurses who make up the 'surplus' team. These nurses are not assigned any patient nor a particular district. They are asked to deliver specific nursing care treatments by the professionals responsible for the patient and are not responsible for the global care plan of the patient.

Typically case manager nurses and nurse technicians deliver the nursing visits to the patients they are assigned to. The surplus team will handle nursing visits that the team nurses are unable to absorb, given the number of visits they have already scheduled for themselves. Furthermore, since the working hours of the surplus team nurses are extended until late in the evening (11 pm) as well as on week-ends, these nurses are able to absorb visits that are needed outside regular working hours. Some specific cares (such as wound dressing) are required several times a day, seven days a week. Although one nurse will be assigned that particular patient, she cannot be required to perform all the nursing visits needed. However, the surplus team nurses will be able to absorb some, if not most, of the visits required. Another feature of the surplus team is that it can serve as a 'buffer' team to absorb temporary increases in demands for nursing services thereby contributing to the reduction in work overload. It should be noted however that despite the fact that the surplus team can serve as a buffer, it is difficult to absorb increases in demands in all situations due to, on the one hand, the difficulty to predict these peaks in demands and, on the other hand, because the surplus team is often used to compensate for the shortage of nurses during absences of regular nurses. Whether the absence is planned or not (sick day), it is indeed often difficult to find a nurse to replace the one that is absent. For long-term replacement of nurses, the home care department usually resort to outside agencies.

Table 1 shows the number of basic units in each district, as well as the repartition of the case manager nurses, the nurse technicians and the patients over the CLSC CDN territory

Table 1 Number of units, nurses and patients per district for the year 2002–2003

District	*Number of*			
	Units	*Case manager nurses*	*Nurse technicians*	*Patients*
A	8	4	1	712
B	7	2	2	477
C	4	4	1	732
D	6	2	1	550
E	4	3	1	508
F	7	4	1	647

for the year 2002–2003. We observe that the number of case manager nurses in each district varies between 2 and 4 while the number of nurse technicians varies between 1 and 2. Each nurse is associated with a set of basic units in her district and a patient from a basic unit is preferably assigned to a nurse associated with that unit. Each basic unit is assigned to exactly one nurse technician, but can be assigned to more than one case manager nurse. The current division of the CLSC CDN territory results from an analysis performed in 1998–1999, and summarized in Blais *et al* (2003). In 2000, it was considered as optimal in terms of the satisfaction of the professionals, the team managers and the head managers.

Workload measure

Workload balance is known as being a good measure of the performance of systems such as multiprocessor systems (Chu *et al*, 2004), software distributed shared memory systems (Zhuang *et al*, 2004), planning production (Riezebos *et al*, 2003) or health care systems (Hughes, 1999).

In Lahrichi *et al* (2006), we analysed the impact of the demand fluctuations on the workload of the nurses in each district, and thereby concluded that in order to reduce imbalance and inequities, one should consider the possibility of assigning patients from a basic unit to nurses that are not associated with that unit.

Actually at the CLSC CDN there is no fixed measure to evaluate the workload of the nurses. However, in the event of overload, the manager of every team is usually able to designate which nurses are concerned. In case of imbalance, the manager has the latitude to reassign only new requests (since follow-up of the patients already in the system has to be held) to another nurse even if the address of the patient does not correspond to the nurse's set of basic units. Meanwhile, managers do not necessarily have the same information of the workloads of the nurses, since each one works with his own team. The profile of the patients varies a lot from one district to another for demographic and socio-economic regards. By considering the possibility of assigning patients to nurses from a different district, one may better respond to demand fluctuations without creating too much imbalance among the nurses, and this may also result in a closer collaboration between the six team managers. Since CLSC CDN overlaps six districts, there is a need for uniformization of the case load evaluation to promote clarity and efficiency.

Workload measures used in the UK are analysed in Hughes (1999). The author compares activity-based measures (where the workload of each nurse mainly depends on the time spent with the patients) with dependency-based measures (where the patients are grouped into categories, and a workload is associated with each category). He concludes that these two types of measures are neither valid nor reliable. We however believe that a mix of these measures can provide a good picture of the nurses' workload at the CLSC CDN. In cooperation with the CLSC committee, we have defined a workload measure which we now define.

The activities of a nurse can be divided into *direct* work and *indirect* work. Direct work includes every task related to patients, as visits and case management, while indirect work encompass tasks related to the nursing job itself as meetings, syndicate and associations activities and trainings. We consider here only the direct work which we aim to balance.

In the previous section, we highlighted the difference between case manager nurses and nurse technicians. This difference is related to the educational background as much as to the kind of patients they are usually assigned to. In practice, even if nurse technicians are not assigned long-term patients, deterioration of a short-term case or overload of a case manager nurse can lead to this situation. The workload of a nurse depends on the type of patients she is assigned to. For budget purposes, the CLSCs are used to classify the patients according to 16 well-defined categories (Hébert and Dubuc, 2001). With the help of the CLSC CDN committee, we have simplified this classification to obtain only five categories which we use to measure the workload of each nurse. These five categories can be summarized as follows:

- category 1: short-term patients that do not require case management;
- category 2: short-term patients that need post-hospitalization or post-surgery care;
- category 3: long-term patients needing punctual nursing care;
- category 4: patients with loss of autonomy;
- category 5: palliative patients.

In a previous work (Lahrichi *et al*, 2006), we have shown that the duration of a visit at CLSC CDN is independent of the patient category and lasts in average 30 min, with a very small standard deviation. Care procedures have also been used to evaluate nurses workload (Mullinax and Lawley, 2002), but in the context of CLSCs it is considered fastidious by the nurses. Hence, instead of using the duration of a visit or the care procedures to evaluate the workload due to a patient, we prefer to use the *heaviness* of the case which depends on the category of the patient. If the patient is a complex case, we assume that he represents a heavy case for the nurse, while a short-term patient is considered less complex usually. To evaluate the heaviness of a case, we have defined a witness visit. The load of the witness visit includes not only the actual nursing care which has to be provided but also the clerical work and the management associated to the case follow-up. Each visit required by a patient has a weight that indicates how much heavier it is when compared to the witness visit. The CLSC CDN board has fixed these weights to 0.75, 1, 1, 2 and 4 for categories $1, \ldots, 5$, respectively.

The number of patients in each category is also part of the workload. Obviously, a high number of heavy cases is not suitable. It is even more significant for the fifth category of patients which represent palliative cases. These cases usually require very complex nursing cares and heavy case management.

Finally, when a nurse has to travel to a basic unit she is not associated with, inside or outside her district, this creates an additional workload that depends as much on the distance travelled as on the number of visits required by the patients. This consideration is not so important with the actual approach since the nurses typically take care of patients located in the basic units they are associated with, but finds its relevance within the new assignment policy proposed in this paper.

In summary, the workload of every nurse depends on three components:

- the *visit load* which is equal to the weighted sum of the visits that the nurse has to perform, the weight of a visit being defined according to its heaviness when compared to a witness visit;
- the *case load* which depends on the number of patients assigned to the nurse in each category;
- the *travel load* which depends on the distance that the nurse has to travel outside her basic units to visit patients, and the number of visits required by these patients.

To balance the workload of the nurses, we will proceed as follows. We first solve the patient assignment problem with no regards to patients follow-up. This may occur, for example, when a new partition of the territory into districts is defined, or when the total imbalance is so high that a complete reassignment is needed. We then perform a dynamic assignment where new patients have to be assigned to nurses knowing their respective workload.

Note that nurses in most CLSCs practice 'self-scheduling' and the boards are not willing to change this work organization. Thus we do not concentrate on the rostering problem which has been well studied in the literature (see for example Burke *et al*, 2004).

Patient assignment model

We start this section by giving some basic notations which will be used in the proposed mathematical model.

- I is the set of nurses working at the CLSC CDN;
- P is the set of patients;

Figure 2 Graph G associated with the CLSC CDN territory.

- $J = \{1, \ldots, 5\}$ is the set of patient categories (see previous section);
- $K = \{m, t\}$ is the set of nurses types, where m stands for case manager nurses and t for nurse technicians;
- I_k is the set of nurses of type $k \in K$;
- P_{jk} is the set of patients of category $j \in J$ that can be assigned to nurses of type $k \in K$, and $\mathscr{P}_k = \cup_{j \in J} P_{jk}$;
- k_i is the type of nurse i;
- U_i is the set of basic units to which nurse i is associated;
- v_p is the number of visits required by patient p. This value is obtained by considering a period of one month and by multiplying the number of visits needed per week by the number of weeks in the caring plan of the considered month;
- j_p is the category of patient p;
- u_p is the basic unit where patient p is located;
- h_{jk} is the heaviness of a visit to a patient of category j if assigned to a nurse of type k.

To determine the distance between two basic units, we define a graph G in which each vertex is associated with a basic unit, and two vertices are linked by an edge if the corresponding basic units share a common frontier. The graph associated with the CLSC CDN territory is represented in Figure 2. The length of an edge is equal to 1 if it connects two basic units of the same district, and λ otherwise, where λ helps penalizing the move of a nurse from a district to another. The distance ℓ_{ip} that nurse i has to travel to take care of patient p is defined as the length of the shortest chain in G linking the vertex associated with u_p to a vertex associated with a basic unit in U_i. The travel load t_{ip} of patient p for nurse i is defined as $v_p e^{\ell_{ip}}$. It is proportional to v_p to take into account the number of times i will have to move to the basic unit u_p to take care of p. The exponential term is to discourage too long travels.

In the following mathematical formulation, we denote x_{ip} the boolean variable that equals 1 if patient p is assigned to nurse i, and 0 otherwise. The initial idea of the CLSC CDN board was to determine an assignment $\boldsymbol{x}$ with balanced visit loads, case loads, and travel loads. More precisely, the average visit load $\overline{V}_k(\boldsymbol{x})$ of the nurses of type k is equal to

$$\overline{V}_k(\boldsymbol{x}) = \frac{\sum_{p\in\mathscr{P}_k}\sum_{i\in I_k} v_p h_{j_p k} x_{ip}}{|I_k|}$$

and the visit loads are considered as balanced if

$$\sum_{p\in\mathscr{P}_{k_i}} v_p h_{j_p k_i} x_{ip} = \overline{V}_{k_i}(\boldsymbol{x}) \qquad \forall i \in I$$

Similarly, the average number $\overline{N}_{jk}(\boldsymbol{x})$ of patients of category j assigned to nurses of type k is equal to

$$\overline{N}_{jk}(\boldsymbol{x}) = \frac{\sum_{p\in P_{jk}}\sum_{i\in I_k} x_{ip}}{|I_k|}$$

and since the number of patients that every nurse has in every category is an integer, the case loads are considered as balanced if

$$\sum_{p\in P_{jk_i}} x_{ip} \leqslant \lceil \overline{N}_{jk_i}(\boldsymbol{x}) \rceil \qquad \forall i \in I, \forall j \in J$$

Finally, the average travel load $\overline{T}_k(\boldsymbol{x})$ of the nurses of type k is equal to

$$\overline{T}_k(\boldsymbol{x}) = \frac{\sum_{p\in\mathscr{P}_k}\sum_{i\in I_k} t_{ip} x_{ip}}{|I_k|}$$

and the visit loads are considered as balanced if

$$\sum_{p\in\mathscr{P}_{k_i}} t_{ip} x_{ip} = \overline{T}_{k_i}(\boldsymbol{x}) \qquad \forall i \in I$$

Since a perfect balance of the case load, visit load and travel load is typically out of reach, the first objective fixed in collaboration with the board of the CLSC CDN was to determine a patient assignment to the nurses respecting all the following constraints, where ε_v, ε_c and ε_t in constraints (1)–(3) are fixed acceptable deviations from the ideal average visit load, case load and travel load, respectively, and constraints (4) and (5) impose that each patient p is assigned to exactly one nurse of type k with $p \in P_{j_p k}$.

$$\sum_{p\in\mathscr{P}_{k_i}} v_p h_{j_p k_i} x_{ip} \leqslant \overline{V}_{k_i}(\boldsymbol{x}) + \varepsilon_v \qquad \forall i \in I \tag{1}$$

$$\sum_{p\in P_{jk_i}} x_{ip} \leqslant \overline{N}_{jk_i}(\boldsymbol{x}) + \varepsilon_c \qquad \forall i \in I, \quad \forall j \in J \tag{2}$$

$$\sum_{p\in\mathscr{P}_{k_i}} t_{ip} x_{ip} \leqslant \overline{T}_{k_i}(\boldsymbol{x}) + \varepsilon_t \qquad \forall i \in I \tag{3}$$

$$\sum_{k\in K|p\in P_{j_p k}} \sum_{i\in I_k} x_{ip} = 1 \qquad \forall p \in P \tag{4}$$

$$x_{ip} \in \{0, 1\} \qquad \forall i \in I, \quad \forall p \in P \tag{5}$$

If every patient $p \in P$ can only be assigned to one type of nurse (ie there is a unique $k \in K$ with $p \in P_{j_p k}$), then the average visit load $\overline{V}_k(\boldsymbol{x})$ and case load $\overline{N}_{jk}(\boldsymbol{x})$ do not depend on the assignment $\boldsymbol{x}$, and constraints (1), (2), (4) and (5) are then typical of a multi-resource generalized assignment problem (MRGAP for short), where jobs (patients) have to be assigned to agents (nurses) subject to resource constraints (Gavish and Pirkul, 1991). Finding a feasible solution to an MRGAP is an NP-hard problem since, as shown in Martello and Toth (1990), it is already NP-hard with one resource constraint. Heuristic solution methods have been developed to solve the MRGAP with a linear objective function (eg Gavish and Pirkul (1991), Mazzola and Wilcox (2001) and Yagiura *et al* (2004)).

Choosing the most appropriate values for ε_v, ε_c and ε_t is a difficult task. Too high values create workload imbalance while too small values make the problem infeasible. An alternate strategy consists in defining a measure for the imbalance and to minimize this measure rather than imposing an almost perfect balanced workload. A discussion with the CLSC CDN board has resulted in the definition of the three following functions which respectively measure the case load, visit load and travel load imbalance.

The visit load of nurse i is defined as

$$V_i(\boldsymbol{x}) = \sum_{p\in\mathscr{P}_{k_i}} v_p h_{j_p k_i} x_{ip}$$

while the visit load imbalance of nurse i is defined as

$$C_{i1}(\boldsymbol{x}) = \max\{0, V_i(\boldsymbol{x}) - \overline{V}_{k_i}(\boldsymbol{x})\} \tag{6}$$

The first objective is to minimize

$$f_1(\boldsymbol{x}) = \sum_{i\in I} (C_{i1}(\boldsymbol{x}))^2$$

The second objective takes into account the difference between the number of patients of category j assigned to nurse i and the ideal number $\lceil \overline{N}_{jk_i}(\boldsymbol{x}) \rceil$. For every patient in excess, a penalty is added which is equal to the average number $\overline{v}_{ij}(\boldsymbol{x})$ of visits performed by nurse i to patients of category j, multiplied by the weight h_{jk_i} of such patients. More precisely, we have

$$\overline{v}_{ij}(\boldsymbol{x}) \sum_{p\in P_{jk_i}} x_{ip} = \sum_{p\in P_{jk_i}} v_p x_{ip}$$

and

$$C_{i2}(\boldsymbol{x}) = \sum_{j\in J} \max\left\{0, \sum_{p\in P_{jk_i}} x_{ip} - \lceil \overline{N}_{jk_i}(\boldsymbol{x}) \rceil\right\} \overline{v}_{ij}(\boldsymbol{x}) h_{jk_i} \tag{7}$$

The second objective is to minimize

$$f_2(\boldsymbol{x}) = \sum_{i\in I} (C_{i2}(\boldsymbol{x}))^2$$

The travel load of nurse i is defined as

$$T_i(\boldsymbol{x}) = \sum_{p \in \mathscr{P}_{k_i}} t_{ip} x_{ip}$$

and the travel load imbalance of nurse i is defined as

$$C_{i3}(\boldsymbol{x}) = \max\{0, T_i(\boldsymbol{x}) - \overline{T}_{k_i}(\boldsymbol{x})\} \quad (8)$$

The third objective is to minimize

$$f_3(\boldsymbol{x}) = \alpha \sum_{i \in I} (C_{i3}(\boldsymbol{x}))^2 + \sum_{k \in K} (\overline{T}_k(\boldsymbol{x}))^2$$

where the first component of $f_3(\boldsymbol{x})$ aims to reduce imbalance in the travel loads, while the second term minimizes the total distance travelled. Parameter α helps giving more or less importance to one of the two components of $f_3(\boldsymbol{x})$.

The patient assignment problem we solve has for objective to minimize the weighted sum $f(\boldsymbol{x}) = \omega_1 f_1(\boldsymbol{x}) + \omega_2 f_2(\boldsymbol{x}) + \omega_3 f_3(\boldsymbol{x})$, where ω_i $(i = 1, 2, 3)$ are parameters that give more or less importance to each component of $f(\boldsymbol{x})$. The mathematical formulation of the patient assignment problem we solve can now be summarized as follows.

$$\text{Minimize} \quad \sum_{i \in I} (\omega_1 (C_{i1})^2 + \omega_2 (C_{i2})^2 + \omega_3 \alpha (C_{i3})^2) + \omega_3 \sum_{k \in K} (\overline{T}_k)^2$$

$$\text{subject to } \overline{V}_k |I_k| = \sum_{p \in \mathscr{P}_k} \sum_{i \in I_k} v_p h_{j_p k} x_{ip} \qquad \forall k \in K \quad (9)$$

$$\sum_{p \in \mathscr{P}_{k_i}} v_p h_{j_p k_i} x_{ip} - \overline{V}_{k_i} \leqslant C_{i1} \qquad \forall i \in I \quad (10)$$

$$\overline{N}_{jk} |I_k| = \sum_{p \in P_{jk}} \sum_{i \in I_k} x_{ip} \qquad \forall j \in J, \ \forall k \in K \quad (11)$$

$$\sum_{p \in P_{jk_i}} x_{ip} - \overline{N}_{jk_i} \leqslant s_{ij} \qquad \forall i \in I, \ \forall j \in J \quad (12)$$

$$\overline{v}_{ij} \sum_{p \in P_{jk_i}} x_{ip} = \sum_{p \in P_{jk_i}} v_p x_{ip} \qquad \forall i \in I, \ \forall j \in J \quad (13)$$

$$\sum_{j \in J} s_{ij} \overline{v}_{ij} h_{jk_i} \leqslant C_{i2} \qquad \forall i \in I \quad (14)$$

$$\overline{T}_k |I_k| = \sum_{p \in \mathscr{P}_k} \sum_{i \in I_k} t_{ip} x_{ip} \qquad \forall k \in K \quad (15)$$

$$\sum_{p \in P_{k_i}} t_{ip} x_{ip} - \overline{T}_{k_i} \leqslant C_{i3} \qquad \forall i \in I \quad (16)$$

$$\sum_{k \in K | p \in P_{j_p k}} \sum_{i \in I_k} x_{ip} = 1 \qquad \forall p \in P \quad (17)$$

$$x_{ip} \in \{0, 1\} \qquad \forall i \in I, \ \forall p \in P \quad (18)$$

$$C_{i1}, C_{i2}, C_{i3} \geqslant 0 \qquad \forall i \in I \quad (19)$$

$$s_{ij} \geqslant 0 \text{ and integer} \qquad \forall i \in I, \forall j \in J \quad (20)$$

Constraints (9) and (10) coupled with the positivity constraint (19) on C_{i1} imply that $C_{i1}(\boldsymbol{x}) = \max\{0, V_i(\boldsymbol{x}) - \overline{V}_{k_i}(\boldsymbol{x})\}$. Similarly, constraints (11), (12) and the integrality and positivity constraint (20) on s_{ij} imply that $s_{ij} = \max\{0, \sum_{p \in P_{jk_i}} x_{ip} - \lceil \overline{N}_{jk_i}(\boldsymbol{x}) \rceil\}$, which combined with constraints (13), (14) and (19) give definition (7) for C_{i2}. Constraints (15), (16) and (19) define C_{i3} according to (8), and constraints (17) and (18) impose that each patient p is assigned to exactly one nurse of type k with $p \in P_{j_p k}$.

In the next section, we propose a Tabu Search algorithm for solving this problem. Note that all constraints are linear, except constraints (13) and (14), which means that if we remove the constraints on the case load, the considered patient assignment problem has linear constraints and a quadratic objective.

A Tabu Search algorithm

Given a solution space X and a function f that measures the value $f(\boldsymbol{x})$ of every solution $\boldsymbol{x} \in S$, Tabu Search is an algorithm which objective is to determine a solution $\boldsymbol{x}^*$ with minimum value $f(\boldsymbol{x}^*)$ over X. For this purpose, a neighbourhood $N(\boldsymbol{x})$ is defined for every $\boldsymbol{x} \in X$. It corresponds to the set of *neighbour solutions* that can be obtained from s by performing a *local move*. Tabu Search generates a sequence $\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_r$ of solutions such that $\boldsymbol{x}_0$ is an initial solution and $\boldsymbol{x}_i \in N(\boldsymbol{x}_{i-1})$ for $i = 1, \ldots, r$. In order to avoid cycling, a tabu list is created that contains forbidden local moves. Hence, a move m from $\boldsymbol{x}_i$ to $\boldsymbol{x}_{i+1}$ can only be performed if m does not belong to the tabu list, unless $f(\boldsymbol{x}_{i+1}) < f(\boldsymbol{x}^*)$, where $\boldsymbol{x}^*$ is the best solution encountered so far. For more details on Tabu Search, the reader may refer to Glover and Laguna (1997).

For our problem, we define X as the set of assignments such that each patient p is assigned to a nurse i with $p \in P_{j_p k_i}$. Each solution $\boldsymbol{x} \in X$ is measured using the objective function $f(\boldsymbol{x})$ defined in the previous section. The initial solution is generated randomly by assigning a nurse of the right type to each patient. When moving from a solution $\boldsymbol{x}$ to a neighbour one $\boldsymbol{x}'$, we will change the assignment of several patients. If a patient p is transferred from nurse i to nurse i', then we put the pair (i, p) in the tabu list T, with the meaning that it is forbidden for θ iterations to reassign p to i.

Two patients in P are considered as equivalent if they are of the same category and require the same number of visits. By analysing the data set of the CLSC CDN, we have observed that it contains many equivalent patients. Hence, when transferring a patient p from a nurse i to a nurse i', there is a danger that the next move will consist in moving a patient equivalent to p from i' to i, which would create cycles in the algorithm. To avoid such a situation, when moving a patient p from i to i', we also introduce all pairs (i, p') into T, where p' is any patient equivalent to p and assigned to i'.

As mentioned above, when moving from a solution $\boldsymbol{x}$ to a neighbour one $\boldsymbol{x}'$, we possibly change the assignment of several patients. If all these changes belong to the tabu list T, then the move from $\boldsymbol{x}$ to $\boldsymbol{x}'$ is designated *tabu*, while there is no restriction if at least one of the changes is not in T.

For moving from a solution $\boldsymbol{x}$ to a solution $\boldsymbol{x}'$, we first choose a patient p. This is done according to one of the eight following rules, where i denotes the nurse currently assigned to p in $\boldsymbol{x}$, and j is the category of p (ie $j = j_p$):

(a) p is any patient in P;
(b) p is any patient such that $V_i(\boldsymbol{x}) > \overline{V}_{k_i}(\boldsymbol{x})$;
(c) p is any patient such that more than $\lceil \overline{N}_{jk_i}(\boldsymbol{x}) \rceil$ patients of category j are currently assigned to i;
(d) p is any patient such that $\ell_{ip} > 0$;
(e) p is any patient such that $\ell_{ip} > 1$;
(f) p is any patient such that $T_i(\boldsymbol{x}) > \overline{T}_{k_i}(\boldsymbol{x})$ and $\ell_{ip} > 0$;
(g) p is any patient chosen according to (b), (c) or (d);
(h) p is any patient such that $v_p > \sum_{p' \in P_{jk_i}} v_{p'}/|P_{jk_i}|$.

Rule (a) ensures that every patient gets a chance to be moved. Rule (b) helps reducing $f_1(\boldsymbol{x})$, while rule (c) does the same job for $f_2(\boldsymbol{x})$, and rules (d), (e), and (f) for $f_3(\boldsymbol{x})$. Rule (g) is for trying to reduce at least one of the three components $f_i(\boldsymbol{x})$. The transfer of patient p from nurse i to nurse i' may create an important overload for i' when v_p is large. Hence such patients are eventually never moved. However, since the initial solution is randomly generated, it may be necessary to change the assignment of such patients, and rule (h) helps doing it.

Once p is chosen, a move from $\boldsymbol{x}$ to a neighbour $\boldsymbol{x}'$ is performed according to one of the five following procedures (where, as above, i is the nurse assigned to p in $\boldsymbol{x}$)):

(1) A *flip* consists in choosing a nurse $i' \neq i$ with $p \in \mathscr{P}_{k_{i'}}$, and assigning p to i' instead of i.
(2) A 2-*swap* consists in choosing a nurse $i' \neq i$ with $p \in \mathscr{P}_{k_{i'}}$ and a patient p' assigned to i' with $p' \in \mathscr{P}_{k_i}$, and then exchanging patients p and p' between nurses i and i'.
(3) A 3-*swap* consists in choosing two nurses i' and i'' distinct from i and two patients p' and p'' assigned to i' and i'', respectively, such that $p \in \mathscr{P}_{k_{i'}}$, $p' \in \mathscr{P}_{k_{i''}}$ and $p'' \in \mathscr{P}_{k_i}$, and then assigning p to i', p' to i'', and p'' to i.
(4) A 2-*mswap* consists in choosing a nurse $i' \neq i$ with $p \in \mathscr{P}_{k_{i'}}$ and a set P' of patients assigned to i' with $P' \subseteq \mathscr{P}_{k_i}$, and then assigning p to i' and all patients in P' to i.
(5) A 3-*mswap* consists in choosing two nurses i' and i'' distinct from i, and two sets P' and P'' of patients assigned to i' and i'', respectively, such that $p \in \mathscr{P}_{k_{i'}}$, $P' \subseteq \mathscr{P}_{k_{i''}}$ and $P'' \subseteq \mathscr{P}_{k_i}$, and then assigning p to i', all patients in P' to i'', and all patients in P'' to i.

Procedures (1) and (2) are standard moves which are typically used in assignment problems (Ferland *et al*, 1996). The three other procedures are inspired by ejection chain techniques (Laguna *et al*, 1995) where after a flip of patients p from i to i', subsequent moves directly depend on the first one. Moves of type (4) and (5) are especially important in our context where patients may have very different required number of visits. For example, by analyzing the data set of the CLSC CDN, we have noticed that the number of required visits in a given patient category ranges from 1 to 57. If we want to move a patient p with $v_p = 57$ from i to i', without creating a big visit load for i', it may be necessary to remove more than one patient from i'.

We now explain how sets P' and P'' are determined in moves of types (4) and (5). For two nurses i' and i'', let $A_{i'i''}(\boldsymbol{x})$ denote the set of patients currently assigned to i' which can be assigned to i''. More precisely, define $A_{i'i''}(\boldsymbol{x})$ as the set of patients p' with $x_{i'p'} = 1$ and $p' \in \mathscr{P}_{k_{i''}}$. Let $m_{i' \to i''} = \min_{p' \in A_{i'i''}(\boldsymbol{x})} \{t_{i'p'} - t_{i''p'}\} - 1$. Given a patient p assigned to a nurse i and given any two nurses i' and i'' of type k_i with $i' \neq i$, we consider a knapsack problem, denoted $P_{i' \to i''}(p)$, that determines a set of patients in $A_{i'i''}(\boldsymbol{x})$ to be moved from nurse i' to nurse i'':

$$P_{i' \to i''}(p) \begin{cases} \text{Maximize} & \displaystyle\sum_{p' \in A_{i'i''}(\boldsymbol{x})} (t_{i'p'} - t_{i''p'} - m_{i' \to i''}) y_{p'} \\ \text{subject to} & \displaystyle\sum_{p' \in A_{i'i''}(\boldsymbol{x})} v_{p'} h_{j_{p'}k_i} y_{p'} \leqslant v_p h_{j_p k_i} \\ & y_{p'} \in \{0, 1\} \quad \forall p' \in A_{i'i''}(\boldsymbol{x}) \end{cases}$$

The objective of $P_{i' \to i''}(p)$ is to gain as much as possible in the travel loads of nurses i' and i'' when moving patients from i' to i''. The term $m_{i' \to i''}$ is to ensure that each patient in $A_{i'i''}(\boldsymbol{x})$ gets a chance to be moved. The constraint of the knapsack problem ensures that the total visit load of the patients moved from i' to i'' is not larger than the visit load of patient p for nurse i. Let $Q_{i' \to i''}(p)$ be the subset of patients $p' \in A_{i'i''}(\boldsymbol{x})$ such that $y_{p'} = 1$ in the optimal solution of the above problem. For moves of type (4), we define $P' = Q_{i' \to i}(p)$ and for moves of type (5), we define $P' = Q_{i' \to i''}(p)$ and $P'' = Q_{i'' \to i}(p)$.

We solve the knapsack problems using the implementation of Bérubé *et al* (2006) of Martello and Toth's algorithm (1990).

In what follows, we denote $N_{q,r}(\boldsymbol{x})$ the set of solutions that can be obtained from $\boldsymbol{x}$ by choosing patient p according to rule (q) with $q \in \{a, \ldots, h\}$ and then applying procedure (r) with $r \in \{1, \ldots, 5\}$. Also, we denote $N_{q,6}(s) = \bigcup_{r=1}^{5} N_{q,r}(\boldsymbol{x})$. Every neighbourhood $N_{q,r}(\boldsymbol{x})$ is explored until M_I iterations (where M_I is a parameter) have been performed without improvement of $\boldsymbol{x}^*$.

As *intensification* strategy, we check, at each iteration, whether $N_{a,1}(\boldsymbol{x}) \cup N_{a,2}(\boldsymbol{x})$ contains a solution which is better than $\boldsymbol{x}^*$, in which case we determine such a solution and update $\boldsymbol{x}^*$. When all neighbourhoods have been tested, we use a *diversification* strategy which consists in performing M_D moves (where $M_{\rm D}$ is a parameter) using neighbourhoods $N_{d,4}(\boldsymbol{x})$ and $N_{d,5}(\boldsymbol{x})$, but using a different objective

```
Generate an initial solution x ∈ X at random and set x ← x* and T ← ∅;
for loop = 1 to M_L do
  for q = a to g do
    for r = 1 to 6 do
      counter ← 0
      while counter < M_I do
        while N_{a,1}(x) ∪ N_{a,2}(x) contains a solution x' such that f(x') < f(x*)
        do
          Select such a solution x';
          Set x ← x', x* ← x and counter ← 0;
        end while
        Determine the solution x' ∈ N_{q,r}(x) with minimum value f(x') such
        that the move from x to x' is not tabu or f(x') < f(x*);
        Set x ← x' and update T;
        if f(x') < f(x*) then set x* ← x and counter ← 0;
        else set counter ← counter + 1;
      end while
      Set x ← x*;
    end for
  end for
  for diversification =1 to M_D do
    Determine the best solution x' in N_{d,4}(x) ∪ N_{d,5}(x), using the modified
    objective function for the knapsack problem, such that the move from x to
    x' is not tabu or f(x') < f(x*);
    Set x ← x';
  end for
end for
```

Figure 3 Tabu Search for the Patient Assignment Problem.

function in the knapsack problem. More precisely, we maximize $\sum_{p' \in A_{i'i''}(\boldsymbol{x})} (t_{i'p'} - t_{i''p'}) y_{p'}$ (ie the term $-m_{i' \to i''}$ is removed), the consequence being that the patients which are closer to i' than to i'' will not be moved since the increase of the travel load for i'' would be larger than the decrease of the travel load for i'. The output of the knapsack problem is then typically a set of patients with total visit load much smaller than the visit load $v_p h_{j_p k_i}$ of p for i. This means that the visit load of nurse i' will probably increase with such a move in $N_{d,4}(\boldsymbol{x})$ and $N_{d,5}(\boldsymbol{x})$, while the total travel load of the nurses involved in the move will eventually decrease. In summary, the proposed diversification put the emphasis on the decrease of the travel load, even if this induces a large increase in the visit load of some nurses. The process of testing all neighbourhoods $N_{q,r}(\boldsymbol{x})$ followed by a diversification is called a *loop*. We apply M_{L} such loops (where M_{L} is our last parameter) before stopping the algorithm, each new loop starting from the solution produced by the diversification strategy. The proposed algorithm is summarized in Figure 3.

The parameters of our Tabu Search have been fixed on the basis of some preliminary experiments. The following choices have been implemented. For a solution $\boldsymbol{x}$, let $\mu_q(\boldsymbol{x})$ denote the number of patients p which can be chosen according to rule (q) in $\boldsymbol{x}$. There are at most $(|I|-1)$ nurses to which patient p can be reassigned. Hence, $|N_{q,1}(\boldsymbol{x})| \leqslant \mu_q(\boldsymbol{x})(|I|-1)$. When moving from $\boldsymbol{x}$ to $\boldsymbol{x}' \in N_{q,r}(\boldsymbol{x})$, all pairs (i, p) introduced in the tabu list remain in the list for $2 \cdot \sqrt{\mu_q(\boldsymbol{x})(|I|-1)}$ iterations. We have chosen the same value for parameter M_I, which means that every neighbourhood $N_{q,r}(\boldsymbol{x})$ is used until $2\sqrt{\mu_q(\boldsymbol{x})(|I|-1)}$ iterations have been performed without improvement of $\boldsymbol{x}^*$. The diversification strategy is used for $M_{\mathrm{D}} = \sqrt{\mu_q(\boldsymbol{x})(|I|-1)}$, and the number M_{L} of loops is set equal to 10. Parameter α in $f_3(\boldsymbol{x})$ helps giving more or less importance to balanced travel loads in comparison with the total travel load of the nurses. Since the CLSC CDN aims to avoid too much travelling, we set $\alpha = 1/|I|$ which means that journeys have to be minimized in priority before being balanced between nurses.

Experimental results

Since there is no algorithm to which our tabu search can be compared when all constraints (9)–(20) are to be taken into account, we have decided to perform our first experiments on instances without constraints on the case load. As already observed, the resulting patient assignment problem has linear constraints and a quadratic objective and can therefore be solved using CPLEX. In order to evaluate the efficiency of the proposed Tabu Search algorithm, we therefore compare the solutions it provides to those obtained using CPLEX version

Table 2 Comparison between CPLEX and Tabu Search

ω_1–ω_2–ω_3	*Instance*	*CPLEX*									*TABU*			
		O_1	O_{21}	O_{22}	O_{23}	O_{24}	O_{25}	O_{31}	O_{32}	O_{3+}	ΔO_1	ΔO_{31}	ΔO_{32}	ΔO_{3+}
1-0-1	ABCDEF-m	15.49	1.58	0	0	6.25	2.90	2.53	0	0.32	0	0.05	0	0
		5.46	1.58	0	0	7.16	2.52	8.00	0.05	0	0.84	−1.26	0.27	0
	AB,CDEF-m	16.75	1.82	0	0	4.23	2.32	2.21	0	0	0	0	0	0
		10.25	1.58	0	0	6.06	2.31	6.11	0.53	0	1.58	−0.58	−0.06	0
	A,B,C,D,E,F-m	16.75	1.58	0	0	5.75	2.26	2.95	0	0	0	0	0	0
	ABCDEF-t	40.21	5.37	11.35	30.68	0	0	0	0	0	0	0	0	0
		28.08	7.14	10.85	25.93	0	0	11.29	0	0	0	1.00	0	0
	AB,CDEF-t	40.21	5.37	11.35	30.68	0	0	0	0	0	0	0	0	0
		37.17	5.95	11.35	30.68	0	0	1.71	0	0	0.14	2.00	0.29	0
	A,B,C,D,E,F-t	40.21	5.03	11.35	30.68	0	0	0.14	0	0	0	1.29	0	0
100-0-1	ABCDEF-m	4.23	1.58	0	0	6.32	3.30	2.89	0	3.26	0.21	7.06	0.37	−0.10
		0.22	1.50	0	0	7.56	2.66	12.16	0.37	0	−0.04	1.63	1.10	0
	AB,CDEF-m	6.73	1.82	0	0	8.33	3.40	3.42	0	2.68	0.06	2.69	0	−0.05
		3.81	1.82	0	0	6.81	2.88	13.11	1.84	0	−0.15	3.36	0.32	0
	A,B,C,D,E,F-m	16.75	1.82	0	0	6.40	2.58	4.16	0	0	0	1.68	0	0
	ABCDEF-t	33.24	7.19	11.20	30.51	0	0	0	0	5.29	0.04	0	0	0
		2.33	7.63	8.00	9.81	0	0	37.14	0	0	−0.01	0.72	0	0
	AB,CDEF-t	36.88	6.20	11.35	30.68	0	0	0	0	0	0.04	0	0	3.00
		27.54	7.92	8.36	20.99	0	0	39.71	2.00	0	0	2.29	0	0
	A,B,C,D,E,F-t	40.21	4.66	11.35	30.68	0	0	0.86	0	0	0	−0.72	0	0
1-0-100	ABCDEF-m	17.39	1.58	0	0	4.54	2.25	0.68	0	0	0	0	0	0
		16.76	1.82	0	0	4.46	1.77	0.89	0	0	0	0	0	0
	AB,CDEF-m	17.60	1.82	0	0	5.25	2.25	0.37	0	0	0.05	0	0	0
		17.52	1.58	0	0	4.20	1.69	0.53	0	0	0	0	0	0
	A,B,C,D,E,F-m	17.94	1.82	0	0	5.81	2.69	0.05	0	0	0.33	0	0	0
	ABCDEF-t	40.21	5.37	11.35	30.68	0	0	0	0	0	0	0	0	0
		39.92	5.69	11.35	30.68	0	0	0.29	0	0	0	0	0	0
	AB,CDEF-t	40.21	5.37	11.35	30.68	0	0	0	0	0	0	0	0	0
		40.21	5.37	11.35	30.68	0	0	0	0	0	0	0	0	0
	A,B,C,D,E,F-t	40.21	5.37	11.35	30.68	0	0	0	0	0	0	0	0	0

9.1.3 on the proposed mixed integer program, where ω_2 is set equal to 0 and constraints (11)–(14) are removed.

We solve three different problems. The first one, denoted *ABCDEF*, consists in solving the patient assignment problem for the whole territory. We have considered real data from June 2002. The problem contains 19 case manager nurses, seven nurse technicians, 1413 patients and 36 basic units. Since the CLSC CDN board is not convinced that the six team managers in the districts will easily accept to collaborate, we have also considered a problem, denoted AB, $CDEF$, in which we solve two patient assignment problems, one for districts A and B, and the other one for the four other districts. We can then merge the two assignments and compare them with the solution obtained by solving *ABCDEF*. Such a solution requires a collaboration between the two team managers in A and B, and another collaboration between the four other team managers. The patient assignment problem for districts A and B contains nine nurses (six case managers and three technicians), 440 patients and 15 basic units, while there are 17 nurses (13 case managers and four technicians), 992 patients and 21 basic units in districts C, D, E and F. For comparison, we also solve the patient assignment problem in each district separately, and then merge the assignments. The solution thus obtained corresponds to the current situation at CLSC CDN. This last problem will be denoted A, B, C, D, E, F.

Table 2 reports the results obtained with CPLEX and Tabu Search. Instead of reporting the values of each component $f_i(\boldsymbol{x})$ of the objective function which do not clearly indicate the various overloads, we report the average visit overload of the nurses, the average number of visits performed by the nurses in basic units that do not belong to the set of basic units where there are located, and the average number of patients in each category that the nurses have above the ideal average. More precisely, Table 2 can be read as follows:

- The first column indicates the values of the weights ω_i.
- The second column indicates the problem solved. Since the visit, case and travel loads are typically very different when comparing case manager with nurse technicians, we have decided to split the results into two parts, one for the case manager nurses (we add a m at the end of the instance name), and the other one for the nurse

Table 3 Results for the case manager nurses on the whole territory

$\omega_1 - \omega_2 - \omega_3$	O_1	O_{21}	O_{24}	O_{25}	O_{31}	O_{32}	O_{3+}
1-0-1	15.49	1.58	6.25	2.90	2.53	0	0.32
	5.46	1.58	7.16	2.52	8.00	0.05	0
1-1-1	16.33	0.08	2.23	0.43	5.05	0.16	0.11
	6.98	0.08	0.74	0.06	9.32	0.37	0
100-1-100	15.49	1.58	4.93	2.50	2.63	0.05	0.32
	7.11	0.08	2.93	2.04	9.26	0.16	0
1-100-1	15.49	0	0.08	0	4.42	0.16	1.00
	8.52	0	0	0	9.74	0.53	0
1-0-100	17.39	1.58	4.54	2.25	0.68	0	0
	16.76	1.82	4.46	1.77	0.89	0	0
1-1-100	17.84	0.08	3.18	1.62	0.63	0	0
	18.32	1.74	2.02	1.51	2.47	0.05	0
100-1-10 000	17.39	1.58	3.99	1.63	0.68	0	0
	16.76	1.58	4.22	1.79	0.89	0	0
1-100-100	19.15	0.08	1.47	0.08	2.32	0.11	0
	17.85	0.08	0.28	0.05	2.32	0.32	0
1-100-10 000	20.23	0.08	2.95	1.14	0.37	0	0
	19.76	0.08	2.35	0.62	0.63	0	0
1-10 000-100	18.48	0	0.08	0	1.00	0.16	1.00
	18.69	0	0	0	2.53	0.42	0
100-0-1	4.23	1.58	6.32	3.30	2.89	0	3.26
	0.22	1.50	7.56	2.66	12.16	0.37	0
100-1-1	4.23	0.08	3.15	2.43	10.84	0.37	3.26
	0.37	0.08	2.25	0.39	22.32	1.95	0.16
10 000-1-100	4.23	1.58	3.97	2.52	10.68	0.37	3.21
	8.83	1.82	2.23	1.62	24.21	12.05	17.11
100-100-1	4.23	0.08	0.24	0	9.84	0.84	4.16
	3.27	0.47	1.25	0	25.47	12.47	4.79
10 000-100-1	0.13	0	0.94	0.47	9.74	3.42	12.68
	0.05	0	0.51	0	20.21	11.00	0
100-10 000-1	4.86	0	0	0	7.84	0	4.21
	0.61	0	0	0	21.42	5.84	0

technicians (we add a t at the end of the instance name). So, for example, the line with label AB, $CDEF - t$ means that we report results for the nurse technicians after having solved AB, $CDEF$.

- For each instance, except A, B, C, D, E, F, we give two lines of results. The first line was obtained using $\lambda = 3$, which means that travels outside the district are discouraged, while the second line considers $\lambda = 1$ and therefore makes no difference between adjacent basic units of the same district or of neighbour districts. This parameter is not relevant for instance A, B, C, D, E, F since the nurses are not allowed to move to another district, and this explains why we have only one line for this instance.
- The next nine columns contain the results obtained using CPLEX.
  - Column labelled O_1 indicates the average visit overload for the considered type k of nurses. Hence,

$$O_1 = \frac{\sum_{i \in I_k} \max\{0, V_i(\boldsymbol{x}) - \overline{V}_k(\boldsymbol{x})\}}{|I_k|}$$

  - The columns labelled O_{2j} indicate the average number of patients of category j that the nurses have above the ideal average, multiplied by the average number of visits that these nurses have to perform to patients of category j. More precisely, for a type k of nurses, we have

$$O_{2j} = \frac{\sum_{i \in I_k} \max\{0, \sum_{p \in P_{jk_i}} x_{ip} - \lceil \overline{N}_{jk}(\boldsymbol{x}) \rceil\} \overline{v}_{ij}(\boldsymbol{x})}{|I_k|}$$

  Notice that CPLEX does not optimize these values since ω_2=0, but we report them for comparison with solutions obtained using Tabu Search with $\omega_2 > 0$ (see Tables 3 and 4).
  - Column labelled O_{31} reports the average number of visits that the nurses have to do in basic units at distance $\ell_{ip} = 1$ from where they are located. More precisely, for a nurse i, let L_i denote the set of patients p with $x_{ip}\ell_{ip} = 1$. For a type k of nurses, we have

$$O_{31} = \frac{\sum_{i \in I_k} \sum_{p \in L_i} v_p}{|I_k|}$$

Table 4 Results for the technician nurses on the whole territory

$\omega_1 - \omega_2 - \omega_3$	O_1	O_{21}	O_{22}	O_{23}	O_{31}	O_{32}	O_{3+}
1-0-1	40.21	5.37	11.35	30.68	0	0	0
	28.08	7.14	10.85	25.93	11.26	0	0
1-1-1	39.86	5.37	10.24	30.68	0	0	0.43
	27.66	3.42	10.63	7.08	13.43	0	0
100-1-100	40.21	5.37	11.35	30.68	0	0	0
	28.08	5.62	9.62	26.51	11.71	0	0
1-100-1	36.55	1.61	11.35	16.61	0.14	0	7.14
	27.55	0.37	0.69	0.29	21.29	0.29	0
1-0-100	40.21	5.37	11.35	30.68	0	0	0
	39.92	5.69	11.35	30.68	0.29	0	0
1-1-100	40.21	5.37	11.35	30.68	0	0	0
	39.98	5.37	10.35	30.68	0.57	0	0
100-1-10 000	40.21	5.37	11.35	30.68	0	0	0
	39.92	5.69	11.35	30.68	0.29	0	0
1-100-100	40.21	5.03	11.35	30.68	0.14	0	0
	34.61	1.08	10.34	9.46	10.14	0	0
1-100-10 000	40.21	5.37	11.35	30.68	0	0	0
	39.98	5.37	10.35	30.68	0.57	0	0
1-10 000-100	36.55	1.61	11.35	16.61	0.14	0	7.14
	29.21	0.18	0.69	0.29	18.43	0.57	0
100-0-1	33.24	7.63	8	9.81	0	0	5.29
	2.33	7.19	11.2	30.51	37.14	0	0
100-1-1	33.13	6.4	9.81	30.68	0	0	5.43
	2.01	4.29	3.6	8.45	45.57	0	0
10 000-1-100	33.24	7.4	11.35	30.5	0	0	5.29
	2.44	6.02	6.97	14.46	43.29	0	0
100-100-1	33.51	5.24	11.35	9.46	0	0	9.71
	2.02	0.93	0.78	0.35	48	0.14	0
10 000-100-1	3.98	5.51	8.43	22.13	0	0	31
	0.05	0.78	1.26	0.34	70.14	3.43	0
100-10 000-1	27.52	0.21	0.7	0.35	0.29	0	17.14
	2.23	0	0	0	53.43	0	0

Notice that when $\lambda = 3$, O_{31} does not take into account travels to adjacent basic units in different districts. Columns O_{32} and O_{3+} give the same information but for travels to basic units at distance 2 or more.

- The last four columns indicate the values obtained with Tabu Search. The Δ in front of each parameter means that we report differences with the CPLEX solutions. So, for example, the value 0.05 in column ΔO_{31} for the instance $ABCDEF - m$ with $\omega_1 = \omega_3 = 1$ and $\lambda = 3$ means that Tabu Search has produced a solution where the nurses make in average $2.53 + 0.05 = 2.58$ visits in basic units at distance 1 from the basic units where they are located.

We find it important to mention that a positive value in a column for Tabu Search does not necessarily mean that Tabu Search was not able to find the optimal solution. For example, for instance $A, B, C, D, E, F - m$ with $\omega_1 = 1$ and $\omega_3 = 100$, the solutions found by CPLEX and Tabu Search are equal except for district A where the four nurses have a visit load of 192, 260, 172 and 181 in the CPLEX solution, while these values are 192, 260, 160 and 193 for the Tabu Search solution. Since the average visit load in district A is 201.25, the unique nurse with an excess in the visit load is the second one and both solutions have therefore the same value for $f_1(\boldsymbol{x})$. However, by merging the solutions obtained from the six districts, the average visit load of the nurses becomes equal to 186.7, which means that the fourth nurse of district A has no overload in the CPLEX solution, while its overload is equal to $193 - 186.7$ in the Tabu Search solution. This induces an increase of $(193 - 186.7)/19 = 0.33$ of the average visit overload of the nurses, and this is the reported value in Table 2.

Notice also that if three nurses have a visit load of $x - 4$, $x + 2$ and $x + 2$, the average visit overload O_1 is equal to $\frac{4}{3}$ while $f_1(s) = 8$. If the same three nurses have a visit load of $x - 1$, $x - 2$ and $x + 3$, the average visit overload O_1 is equal to $1 < \frac{4}{3}$ while $f_1(s) = 9 > 8$. Hence, a positive value under column ΔO_1 does not mean that the f_1 component of the objective function is larger for Tabu Search. On the opposite, a negative value does not mean that CPLEX has not found the optimal solution.

It clearly appears in Table 2 that the differences between the solutions produced by Tabu Search and CPLEX are very small. The largest difference for O_1 is 1.58, while, as a counterpart, both O_{31} and O_{32} have a lower value in the Tabu Search solution on the same instance. We also observe that the largest gap is 7.06 for O_{31} meaning that nurses perform on average 7.06 more visits to adjacent units. The counterpart for this instance is a decrease of O_{3+} which means that nurses have less visits to perform at patients located very far (ie at distance > 2) from their basic units. We also observe that Tabu Search and CPLEX solutions are very similar when the traveling component is important in the optimization process ($\omega_3 = 100$) while differences are more apparent when $\omega_1 = 100$.

These results lead to important observations. Problem AB, $CDEF$ is a partition of the real problem $ABCDEF$ into two subsets, and it seems that the CLSC CDN board finds it easier to implement since it does not require collaboration of the six team managers. When compared to A, B, C, D, E, F (ie the actual situation) we observe that the visit overload can be drastically reduced. For example, with $\omega_1 = 100$, $\omega_3 = 1$ and $\lambda = 1$, the average visit overload decreases from 16.75 for $A, B, C, D, E, F - m$ to 3.81 for $AB, CDEF - m$. This value can even reduced to 0.22 (ie an almost perfect balanced visit load) if the six team managers are ready to collaborate.

We also observe that the choice for parameter λ makes a big difference. For example, for the instance $ABCDEF - m$ with $\omega_1 = \omega_3 = 1$, the average visit overload decreases from 15.49 with λ=3 to 5.46 with λ=1. Such a decrease is achieved at the expense of an increase of O_{31} from 2.53 to 8.00, and of O_{32} from 0 to 0.05, but with a decrease of O_{3+} from 0.32 to 0. The same phenomenon can be observed for the other instances, and the CLSC CDN board should therefore consider travelling as a good opportunity to reduce imbalance in the visit load. In summary, even though districting simplifies the work of each team managers, authorizing travels to other districts helps obtaining more balanced visit loads.

Notice also that it often happens when $\lambda = 3$ that $O_{32} = 0$ while $O_{3+} > 0$. This is for example the case for $ABCDEF - m$ with weights 100-0-1 where $O_{3+} = 3.26$. This simply means that the nurses make in average 3.26 visits to adjacent basic units of other districts, but do not perform any visit is basic units at distance 2 in their district.

We now report results that include the case load (ie where $\omega_2 > 0$). We have performed tests for the instance $ABCDEF$ with all ω_i equal to 1. with two of them equal to 1 and one equal to 100, with two of them equal to 100 and one to 1, and finally with three different values 1, 100 and 10 000. Each test with $\omega_1 = \omega_3$ is compared with the solution reported in Table 2 for $\omega_1 = \omega_3 = 1$, while the tests with $\omega_1 > \omega_3$ are compared with the solution with $\omega_1 = 100$ and $\omega_3 = 1$ in Table 2, and the tests with $\omega_1 < \omega_3$ are compared with the solution with $\omega_1 = 1$ and $\omega_3 = 100$ in Table 2. The results are given in Tables 3 and 4. The columns are labelled as in Table 2 and we also give two lines of results for each instance, the first one with parameter $\lambda = 3$, and the second one with $\lambda = 1$. Table 3 contains the results for the case manager nurses while Table 4 contains those for the nurse technicians.

We observe that the solutions obtained with $\lambda = 1$ have systematically lower average visit overloads O_1 than those obtained with λ=3, except when $\omega_1 - \omega_2 - \omega_3$ is equal to 1-1-100, 10 000-1-100 and 1-10 000-100, and even in those cases, the increase in the visit overload is only equal to 0.5, 4.6 and 0.2, respectively. The decrease is particularly impressive for nurse technicians where the average visit overload drops from 33.13 to 2.01 for $\omega_1 = 100$ and $\omega_2 = \omega_3 = 1$. For the same instance, O_{3+} is reduced to 0, and the O_{2j} values decrease, for example from 30.68 to 8.45 for O_{23}. However, this is done at the expense of more travels to adjacent basic units from different districts, since O_{31} increase from 0 to 45.57.

We can also observe that Tabu Search is able to balance the case load since all O_{2j} are almost equal to 0 when ω_2 is larger than the two other weights, especially when $\lambda = 1$. If we compare the solution 1-0-1 (obtained with CPLEX) with the solution 1-100-1 (obtained with Tabu Search) for the case manager nurses, we can observe that O_1 does not change and the travel load slightly increases, while the case load is reduced to almost 0. Hence balanced case loads can be obtained without inducing too much increase in the visit and travel overloads.

If we put the emphasis on the travel load (ie ω_3 is the largest weight), we observe that O_{32} and O_{3+} are reduced to almost 0 while the O_{2j} values typically decrease and O_1 slightly increase when we compare the CPLEX solution (obtained with $\omega_2 = 0$) to the Tabu Search solutions (obtained with $\omega_2 > 0$). On the opposite, if we put the emphasis on the visit load (ie ω_1 is the largest weight), Tabu Search is able to reduce the visit overload to very low values while decreasing the O_{2j} values simultaneously. For example weights 10 000-100-1 for the case manager nurses produce a solution with $O_1 = 0.13$ and all O_{2j} smaller than 1, while CPLEX obtains $O_1 = 4.23$, $O_{24} = 1.58$, $O_{24} = 6.32$ and $O_{25} = 3.30$. This is done at the expense of the travel load.

Dynamic assignment

In practice, the list of patients is not known in advance, with the exception of long-term patients who are already assigned to a nurse and cannot be reassigned to a different nurse. When a new request arrives at the CLSC CDN, it is typically immediately assigned to a nurse, although the team managers can consider making the assignment a few days later in the week. By waiting a little, the team managers have the possibility to perform the assignment of several patients at the same time and can thus better control the balance of the workload of the nurses.

In order to analyse the gain that can be obtained by not assigning the new requests on a daily basis, we compare five strategies using historical data from June and July 2002. We have first produced an assignment using Tabu Search for the patients of June, and we have then removed the patients not

Table 5 Dynamic assignment of case management nurses

$\omega_1-\omega_2-\omega_3$		$\lambda=3$					$\lambda=1$				
		$\tau=1$	$\tau=3$	$\tau=7$	$\tau=15$	$\tau=31$	$\tau=1$	$\tau=3$	$\tau=7$	$\tau=15$	$\tau=31$
1-1-1	O_1	11.74	12.05	11.77	11.77	11.43	4.49	5.81	4.23	5.91	5.43
	O_{21}	0	0	0	0	0	0	0	0	0	0
	O_{24}	2.58	2.51	2.06	2.57	2.45	1.29	1.75	1.01	1.74	1.43
	O_{25}	0.49	0.49	0.49	0.49	0.32	0.07	0.09	0	0	0
	O_{31}	5.95	6	5.95	5.84	6.47	9.79	9.63	9.63	9.58	9.74
	O_{32}	0.21	0.32	0.21	0.11	0.05	0.53	0.68	0.37	0.37	0.26
	O_{3+}	0.37	0.32	0.21	0.32	0.26	0	0	0	0	0
1-1-100	O_1	15.2	15.63	16.83	16.98	16.92	17.11	16.13	16.47	16.92	16.99
	O_{21}	0	0	0	0	0	0	0	0	0	0
	O_{24}	3.78	3.95	3.43	4.02	3.89	4.04	3.97	3.47	3.55	3.73
	O_{25}	1.24	1.32	1.82	1.03	1.03	0.69	0.76	0.96	0.75	0.75
	O_{31}	1.42	1.32	1.05	1	1.05	1.16	1.47	1.37	1.32	1.21
	O_{32}	0	0	0	0	0	0	0	0	0	0
	O_{3+}	0.26	0.16	0.32	0.26	0.21	0	0	0	0	0
10 000-1-100	O_1	1.77	1.64	2.06	3.95	3.32	0.38	0.26	0.26	0.52	0.18
	O_{21}	0	0	0	0	0	0.24	0	0	0.24	0
	O_{24}	3.96	5.4	4.99	5.1	5.29	4.88	4.78	3.25	2.79	4.45
	O_{25}	1.74	2.25	1.92	2.25	2.19	1.9	1.56	1.93	2.04	2.29
	O_{31}	12.16	11.95	11.74	12.16	11.58	21.95	22.47	22.84	25.37	23.21
	O_{32}	0.79	0.79	1.16	1.21	1.37	5.37	4.84	5	5.11	4.42
	O_{3+}	4.16	3.89	3.79	3.74	2.84	0	0	0	0.11	0.05
10 000-100-1	O_1	0.3	0.25	0.2	0.15	0.19	0.18	0.19	0.14	0.16	0.16
	O_{21}	0	0	0	0	0	0	0	0	0	0
	O_{24}	2.09	1.06	1.71	1.69	2.1	0.96	1.29	1.23	1.19	0.73
	O_{25}	0.11	0	0	0	0	0.08	0	0	0	0
	O_{31}	10.95	14.47	12.84	12	11.95	24.53	26.47	25	22.63	21.58
	O_{32}	4.11	0.63	3.95	4.74	4.26	17.37	16.11	13.26	11.63	13.16
	O_{3+}	22.26	20.37	18.89	15.63	13.47	3.89	2.68	2.58	2.32	1.58

needing any home care in July. This gives an initial workload for each nurse which can not be modified. We have then considered the new requests in July and these have been assigned to the nurses on a regular basis. We have assigned the new patients to the nurses every τ days, where $\tau=1$ is a daily basis, $\tau=3$ is twice a week, $\tau=7$ is once per week, $\tau=15$ is twice a month, and $\tau=31$ is once per month. When assigning new requests every τ days, we consider that the assignment generated in the previous days can not be modified. Hence, the solutions typically improve when τ increases. In Tables 5 and 6, we report the results obtained with $\lambda=3$ and 1, and with weightings 1-1-1, 1-1-100, 10 000-1-100 and 10 000-100-1 (which are considered as the most realistic by the nurses and the CLSC CDN board). Table 5 contains the results for the case manager nurses while Table 6 contains those for the nurse technicians.

A first observation is that there is no big difference between the solutions obtained with $\tau=1$ and those with $\tau=31$. A decrease of one overload is often obtained at the expense of an increase of another overload. For example, the instance 1-1-100 puts the emphasis on the travel load. For the case manager nurses with $\lambda=3$, we observe that O_{31} and O_{3+} can be decreased from 1.42 and 0.26 to 1.05 and 0.21 when τ increases from 1 to 31. This is obtained at the expense of a slight increase of O_1 and O_{24}, but with a decrease of O_{25} from 1.24 to 1.03.

Parameters 10 000-100-1 produce interesting results for the case manager nurses. Such a weighting gives a higher priority to the visit load, but without neglecting the case load. We can observe that by increasing τ from 1 to 31, one can reduce all overloads since O_1, O_{31}, O_{32}, O_{3+}, O_{21}, O_{24}, and O_{25} decrease from 0.18, 24.53, 17.37, 3.89, 0, 0.96 and 0.08 to 0.16, 21.58, 13.16, 1.58, 0, 0.73 and 0, respectively.

All solutions reported in Tables 5 and 6 show again that fixing λ equal to 1 (ie encouraging travels to adjacent basic units of different districts) helps obtaining much lower visit and case overloads, but at the expense of a higher travel load. For example, for the nurse technicians with parameters 1-1-1, O_1 decreases from 31.6 to 2.33, O_{21}, O_{22} and O_{23} decrease from 8.58, 8.67 and 23.73 to 4.61, 7.53 and 8.58, O_{3+} is reduced from 15 to 0, while O_{31} augments from 0 to 51.

Conclusion

We have considered the problem of assigning patients to nurses for home care services. A previous work has shown

Table 6 Dynamic assignment of nurse technicians

$\omega_1-\omega_2-\omega_3$		$\lambda=3$					$\lambda=1$				
		$\tau=1$	$\tau=3$	$\tau=7$	$\tau=15$	$\tau=31$	$\tau=1$	$\tau=3$	$\tau=7$	$\tau=15$	$\tau=31$
1-1-1	O_1	31.62	31.4	31.11	31.8	31.6	2.01	2.24	2.47	2.36	2.33
	O_{21}	8.37	8.66	5.76	7.88	8.58	5.38	4.72	6.08	6.62	4.61
	O_{22}	5.91	5.13	8.67	7.63	8.67	5.32	6.99	6.27	7.22	7.53
	O_{23}	24.77	24.88	24.36	21.19	23.73	7.61	9.84	6.36	4.85	8.58
	O_{31}	0	0	0	0	0	52.43	50.57	51.43	53.29	51
	O_{32}	0	0	0	0	0	0.14	1	0	0	0
	O_{3+}	16.29	16.43	16.29	16.43	15	0	0	0	0	0
1-1-100	O_1	40.3	40.3	40.3	40.3	40.3	39.79	39.59	39.76	39.82	39.67
	O_{21}	9.67	9.67	9.67	9.67	9.67	8.82	7.78	8.36	9.26	9.03
	O_{22}	8.67	8.67	8.67	8.67	8.67	8.43	7.56	8.36	8.36	8.36
	O_{23}	27.81	27.45	27.81	27.81	27.81	27.01	27.09	27.45	26.99	26.99
	O_{31}	0	0	0	0	0	1	1.57	0.71	0.71	0.86
	O_{32}	0	0	0	0	0	0.14	0	0	0	0
	O_{3+}	0	0.29	0	0	0	0	0	0	0	0
10 000-1-100	O_1	31.8	31	31.17	31.62	31.86	2.38	2.13	2.58	2.41	2.64
	O_{21}	8.17	3.55	5.45	8.92	8.36	5.05	7.09	5.7	4.95	6.45
	O_{22}	5.74	5.11	8.18	7.67	8.67	5.57	2.83	2.09	6.94	2.46
	O_{23}	24.71	24.37	18.87	23.14	21.63	11.36	10.77	9.56	11.28	11.3
	O_{31}	0	0	0	0	0	51.71	51.57	54.71	52.43	55
	O_{32}	0	0	0	0	0	0	1	0	0	0
	O_{3+}	16.14	18.71	18.71	15.43	15.71	0	0	0	0	0
10 000-100-1	O_1	4.93	4.76	4.87	4.67	4.61	0.13	0.18	0.06	0	0.07
	O_{21}	7.04	6.89	5.49	7.02	8.39	1.13	1.07	1.22	0.84	0.82
	O_{22}	7.53	6.06	8.08	8.18	5.77	0.91	1.35	1.16	0.84	1.07
	O_{23}	15.24	14.96	14.04	11.78	16.51	1.37	1.98	2.17	1.12	2.15
	O_{31}	0	0	0	0	0	62.29	59	52.14	58.71	49.86
	O_{32}	0	0	0	0	0	12.29	20	17	10	11.71
	O_{3+}	57.14	58.71	56.29	56.43	56.57	0	0	0	0	3.86

that when the nurses offer their services only in the district where they are located, demand fluctuations may create imbalance and inequities among them, and one should therefore consider the possibility of assigning them patients from basic unit in other districts.

For this purpose, we have developed a measure of the workload of the nurses which takes into account the number of visits performed by the nurses, the heaviness of each patient, the number of patients that the nurses have in each category, and the travels needed to visit the patients. We have then modeled the patient assignment problem as a mixed integer program with some non-linear constraints and a non linear objective. When the case load is not taken into account, while the objective is to minimize the travel and visit overloads, we have shown that the model contains only linear constraints and can therefore be solved using CPLEX. By adding the objective of minimizing the case overload, non-linear constraints must be taken into account and we solve the problem using a Tabu Search algorithm with various neighbourhoods.

The effectiveness of the Tabu Search algorithm has been confirmed by making comparisons with CPLEX on instances where the case load is not considered. The tests performed on real historical data have shown that it is possible to drastically reduce the visit and case loads of the nurses if they accept to move to basic units that are not too far from where they are located, but possibly in another district. Giving the opportunity to nurses to leave their district is comparable to make borders between districts more flexible. This is an interesting alternative when compared to reorganizing districts which is time and resource consuming and can cause important changes in patients follow-up.

It was shown that the proposed Tabu Search algorithm performed well for the addressed problem. However, some more complex and possibly more performing heuristics, such as genetic and memetic algorithms, scatter search and variable neighbourhood search (eg see Ibaraki *et al*, 2005) may be investigated as an extension to this paper.

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