HUST

TRƯỜNG ĐẠI HỌC BÁCH KHOA HÀ NỘI

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

TRƯỜNG ĐẠI HỌC
BÁCH KHOA HÀ NỘI
HANOI UNIVERSITY
OF SCIENCE AND TECHNOLOGY

LÝ THUYẾT MẬT MÃ

Cryptography Theory

ET3310

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Symmetric Ciphers

Symmetric encryption, also referred to as conventional encryption or single-key encryption, was the only type of encryption in use prior to the development of public key encryption in the 1970s. It remains by far the most widely used of the two types of encryption.

Terms:

An original message is known as the **plaintext**, while the coded message is called the **ciphertext**. The process of converting from plaintext to ciphertext is known as **enciphering** or **encryption**; restoring the plaintext from the ciphertext is **deciphering** or **decryption**. The many schemes used for encryption constitute the area of study known as **cryptography**. Such a scheme is known as a **cryptographic system** or a **cipher**.

Symmetric Ciphers

Classical Encryption Techniques

Symmetric Cipher Model

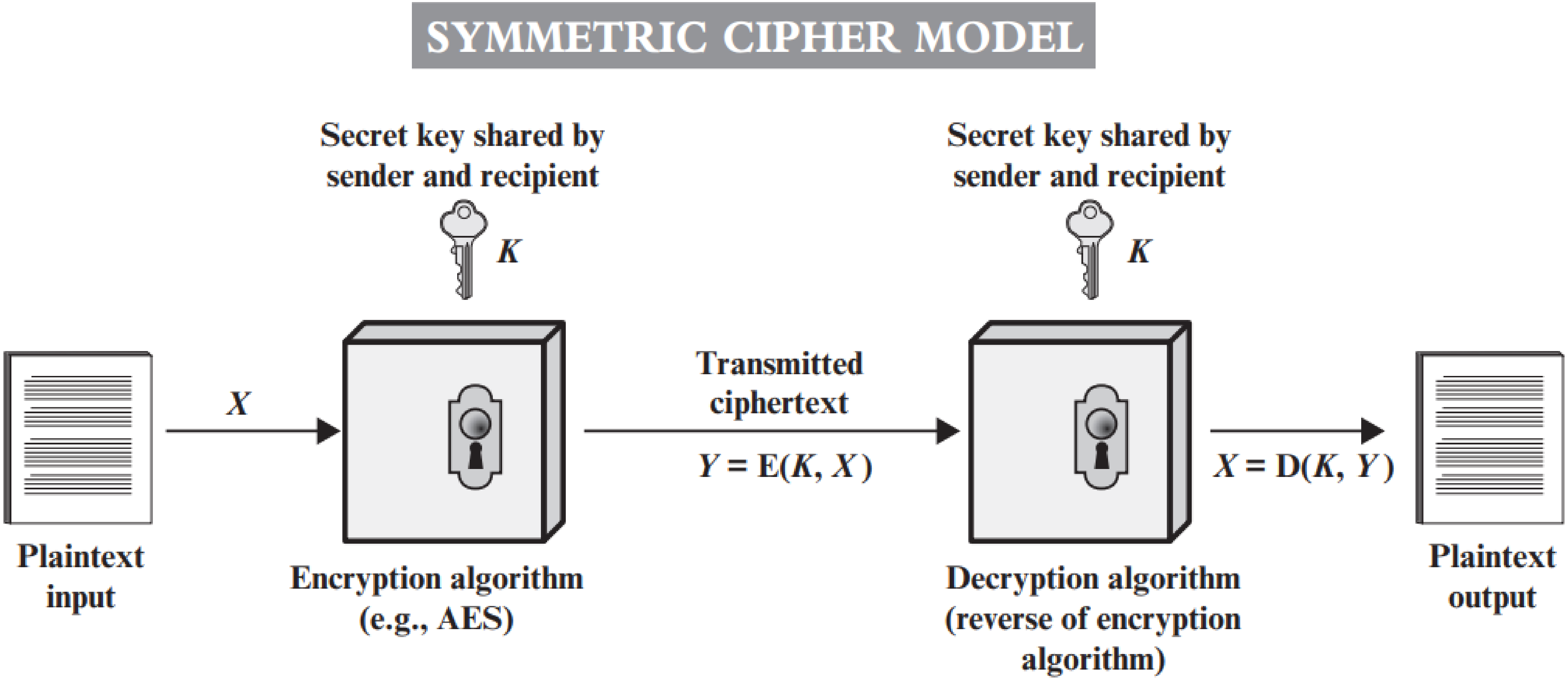
- Cryptography
- Cryptanalysis and Brute-Force Attack

Substitution Techniques

- Caesar Cipher
- Monoalphabetic Ciphers
- Playfair Cipher
- Hill Cipher
- Polyalphabetic Ciphers
- One-Time Pad

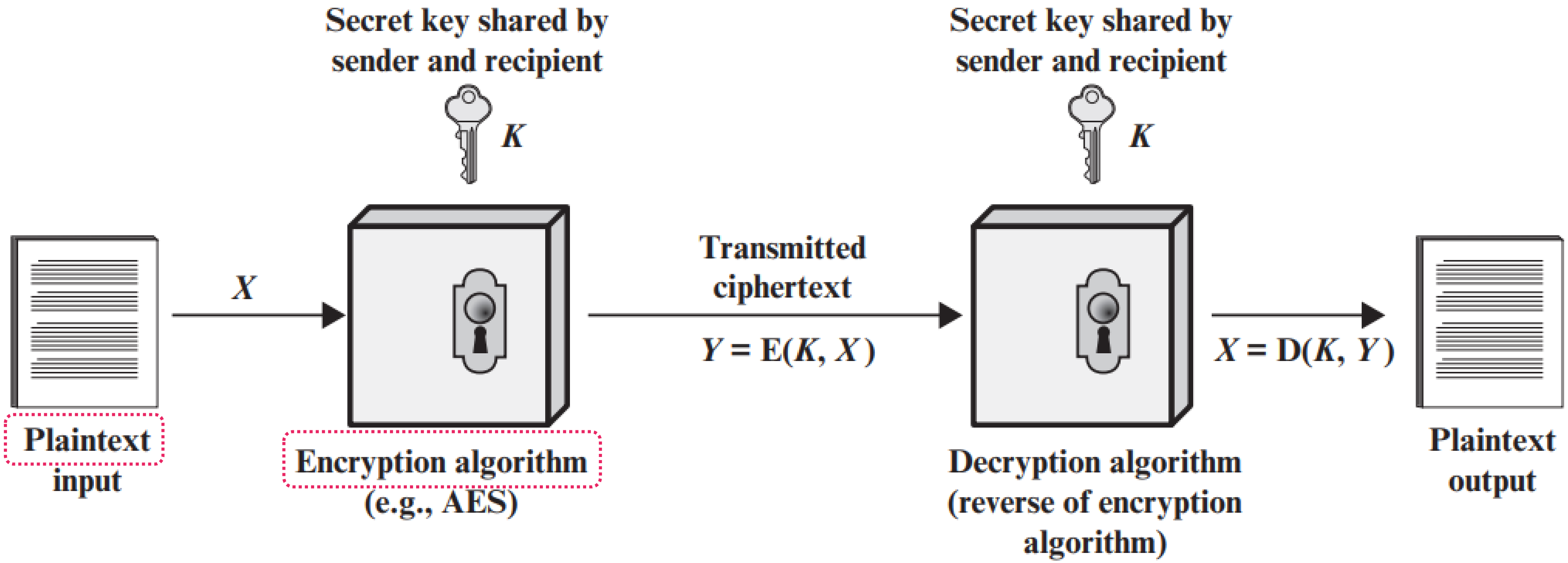
Transposition Techniques

Mật mã khóa đối xứng



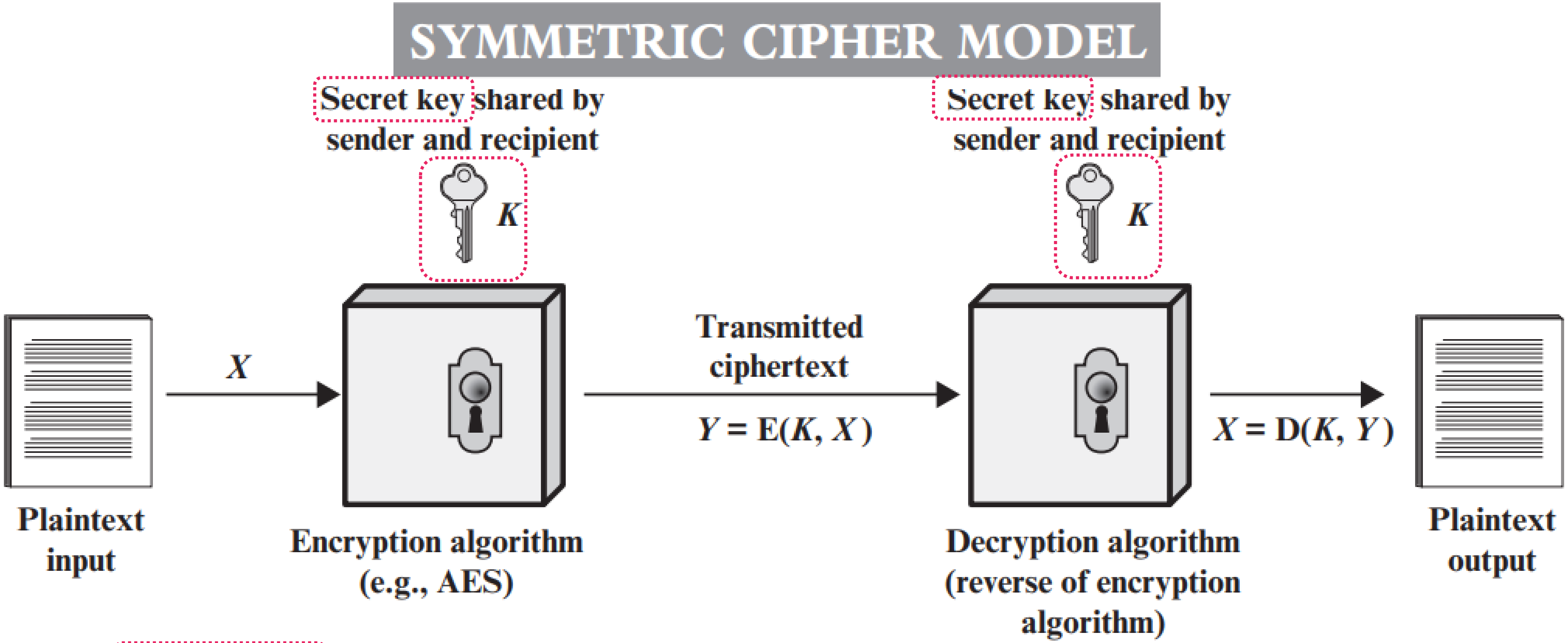
Mật mã khóa đối xứng

SYMMETRIC CIPHER MODEL



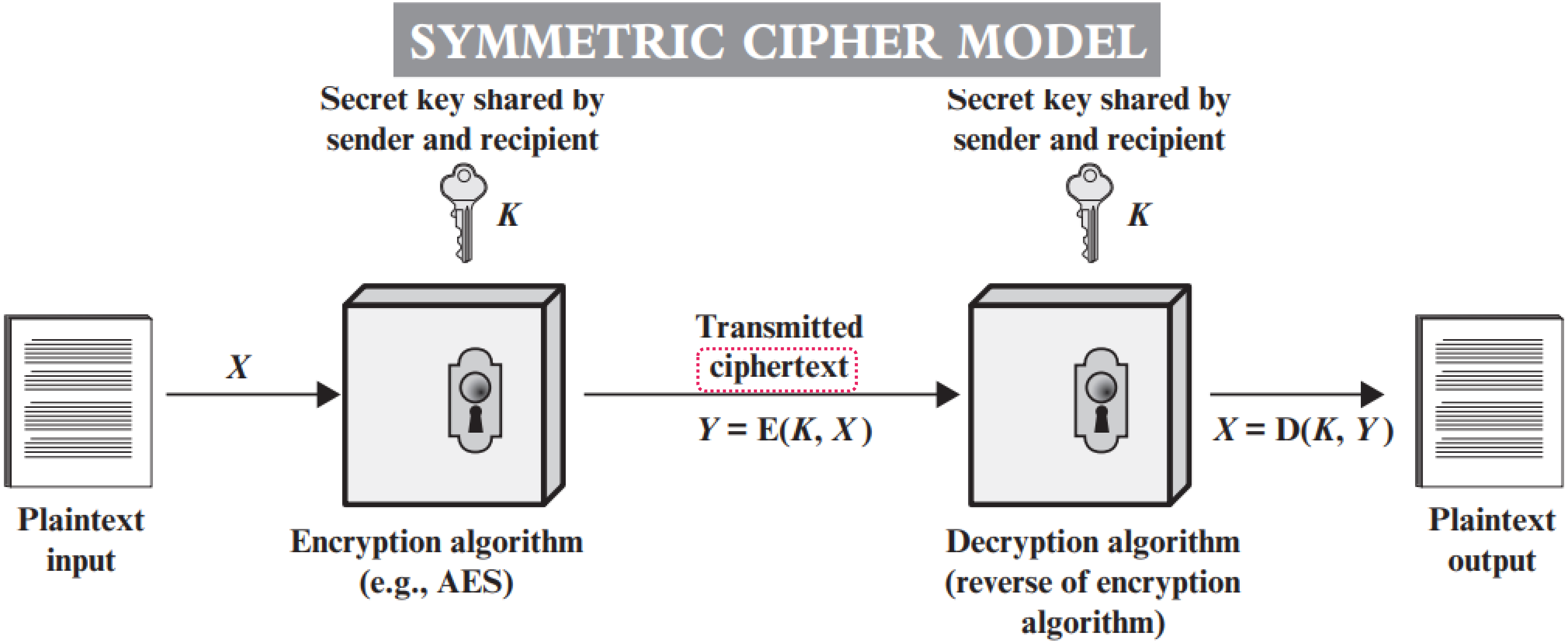
- **Plaintext:** This is the original intelligible message or data that is fed into the algorithm as input.
- **Encryption algorithm:** The encryption algorithm performs various substitutions and transformations on the plaintext.

Mật mã khóa đối xứng



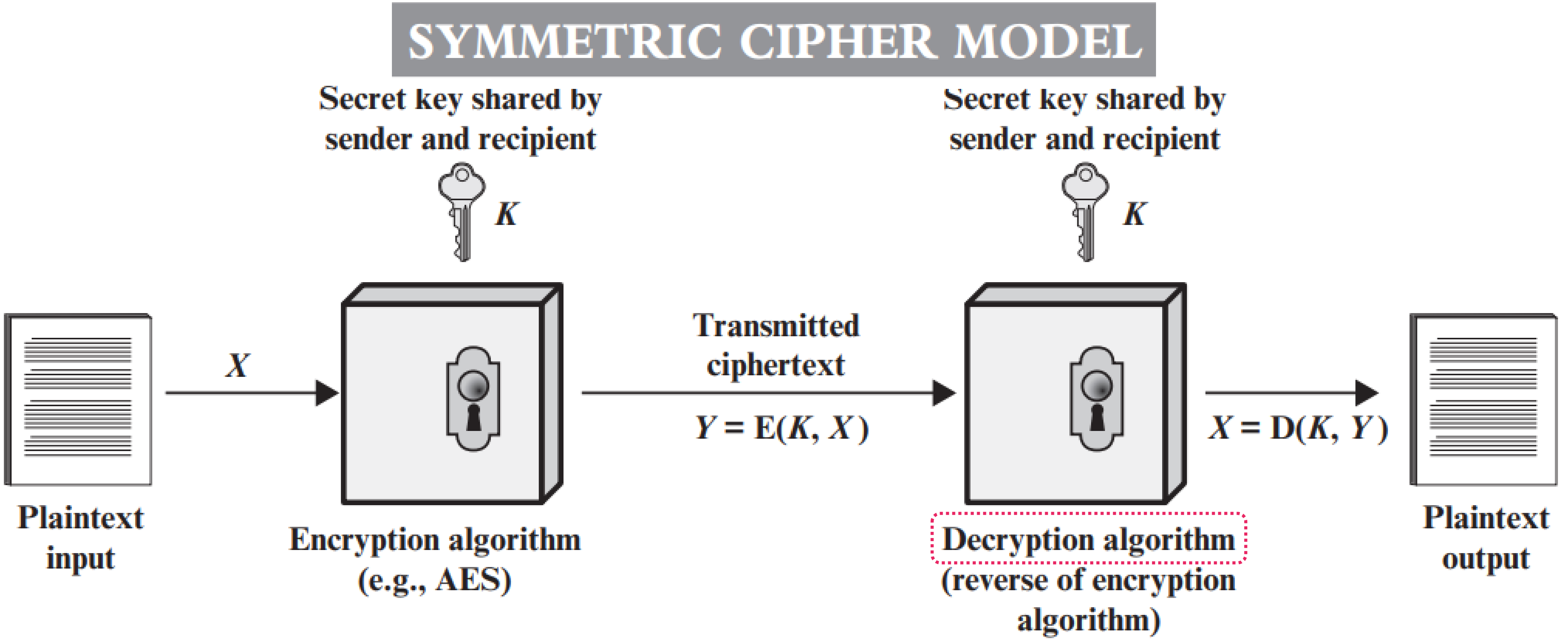
- **Secret key:** The secret key is also input to the encryption algorithm. The key is a value independent of the plaintext and of the algorithm. The algorithm will produce a different output depending on the specific key being used at the time. The exact substitutions and transformations performed by the algorithm depend on the key.

Mật mã khóa đối xứng



- **Ciphertext:** This is the scrambled message produced as output. It depends on the plaintext and the secret key. For a given message, two different keys will produce two different ciphertexts. The ciphertext is an apparently random stream of data and, as it stands, is unintelligible.

Mật mã khóa đối xứng



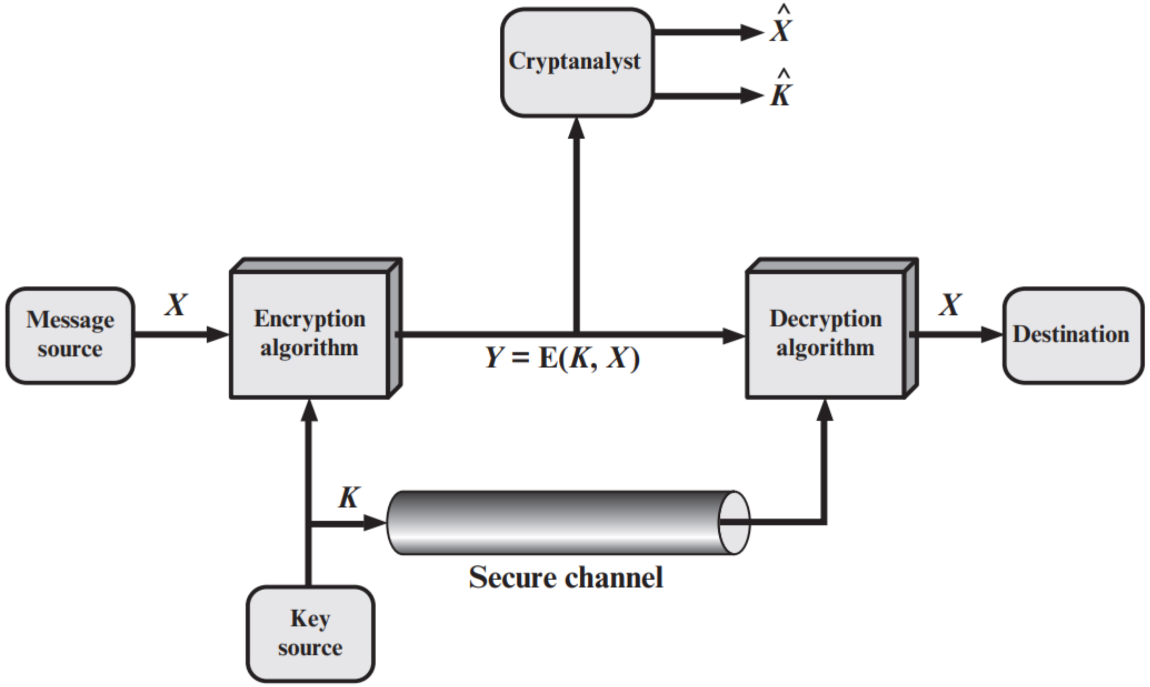
- **Decryption algorithm:** This is essentially the encryption algorithm run in reverse. It takes the ciphertext and the secret key and produces the original plaintext.

Mật mã khóa đối xứng

SYMMETRIC CIPHER MODEL

There are two requirements for secure use of conventional encryption:

1. We need a strong encryption algorithm. At a minimum, we would like the algorithm to be such that an opponent who knows the algorithm and has access to one or more ciphertexts would be unable to decipher the ciphertext or figure out the key. This requirement is usually stated in a stronger form: The opponent should be unable to decrypt ciphertext or discover the key even if he or she is in possession of a number of ciphertexts together with the plaintext that produced each ciphertext.



2. Sender and receiver must have obtained copies of the secret key in a secure fashion and must keep the key secure. If someone can discover the key and knows the algorithm, all communication using this key is readable.

Cryptography

Cryptographic systems are characterized along three independent dimensions:

1. **The type of operations used for transforming plaintext to ciphertext**. All encryption algorithms are based on two general principles: substitution, in which each element in the plaintext (bit, letter, group of bits or letters) is mapped into another element, and transposition, in which elements in the plaintext are rearranged. The fundamental requirement is that no information be lost (i.e., that all operations are reversible). Most systems, referred to as product systems, involve multiple stages of substitutions and transpositions.

2. **The number of keys used**. If both sender and receiver use the same key, the system is referred to as symmetric, single-key, secret-key, or conventional encryption. If the sender and receiver use different keys, the system is referred to as asymmetric, two-key, or public-key encryption.

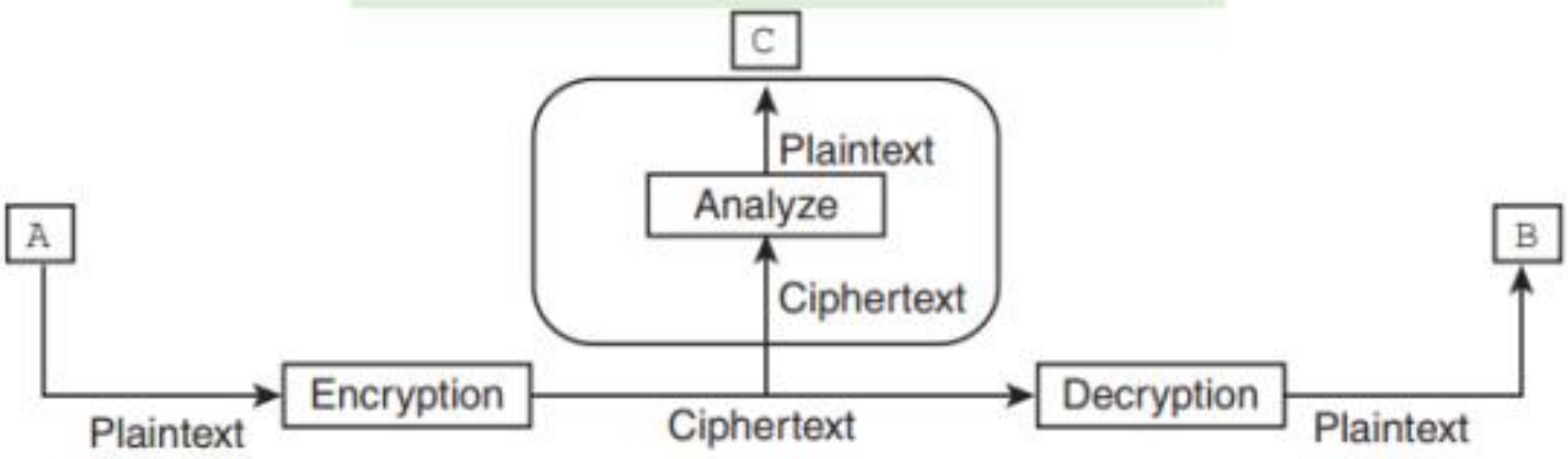
3. **The way in which the plaintext is processed**. A block cipher processes the input one block of elements at a time, producing an output block for each input block. A stream cipher processes the input elements continuously, producing output one element at a time, as it goes along

Cryptanalysis

Type of Attack	Known to Cryptanalyst
Ciphertext Only	■ Encryption algorithm ■ Ciphertext
Known Plaintext	■ Encryption algorithm ■ Ciphertext ■ One or more plaintext–ciphertext pairs formed with the secret key
Chosen Plaintext	■ Encryption algorithm ■ Ciphertext ■ Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key
Chosen Ciphertext	■ Encryption algorithm ■ Ciphertext ■ Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key
Chosen Text	■ Encryption algorithm ■ Ciphertext ■ Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key ■ Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

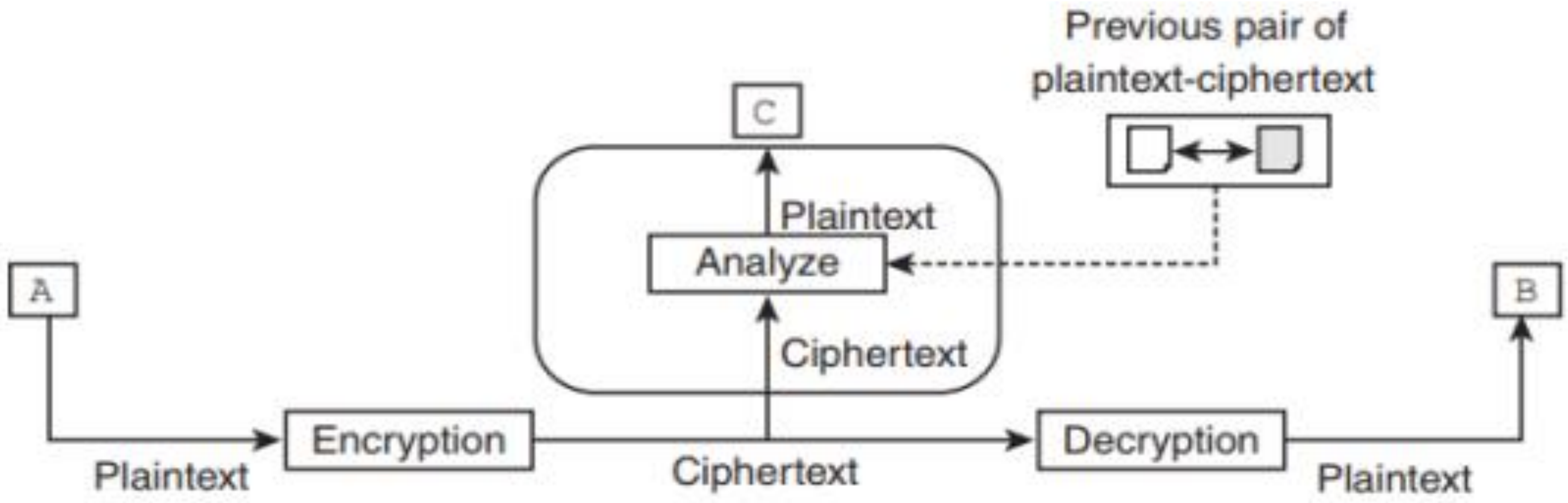
Cryptanalysis

Type of Attack	Known to Cryptanalyst
Ciphertext Only	▪ Encryption algorithm ▪ Ciphertext



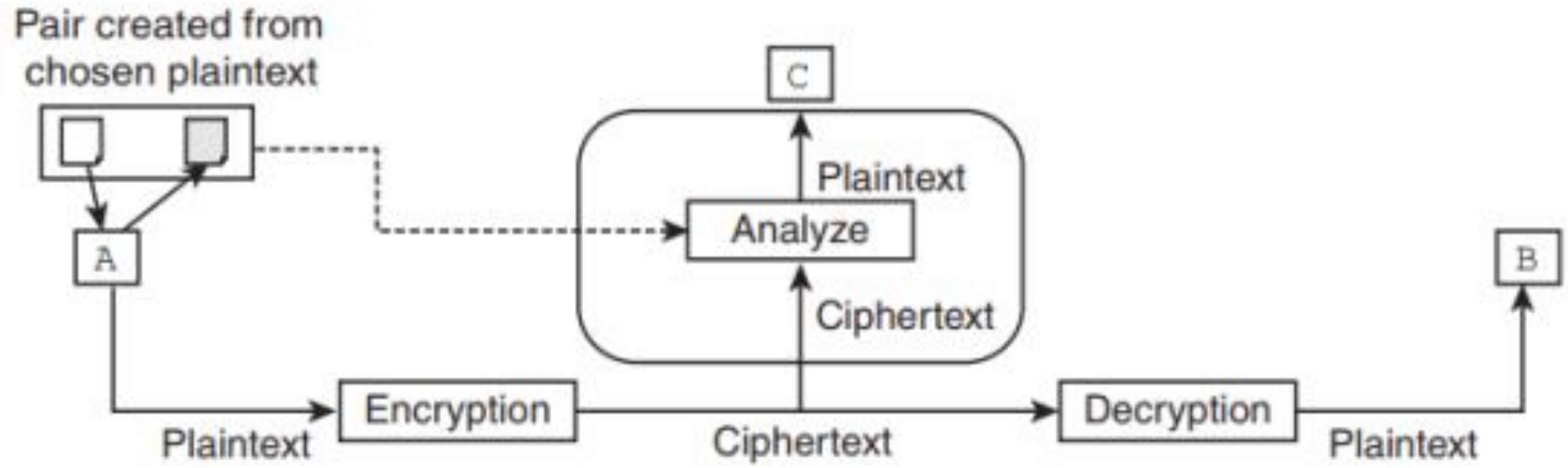
Cryptanalysis

Type of Attack	Known to Cryptanalyst
Known Plaintext	■ Encryption algorithm ■ Ciphertext ■ One or more plaintext–ciphertext pairs formed with the secret key



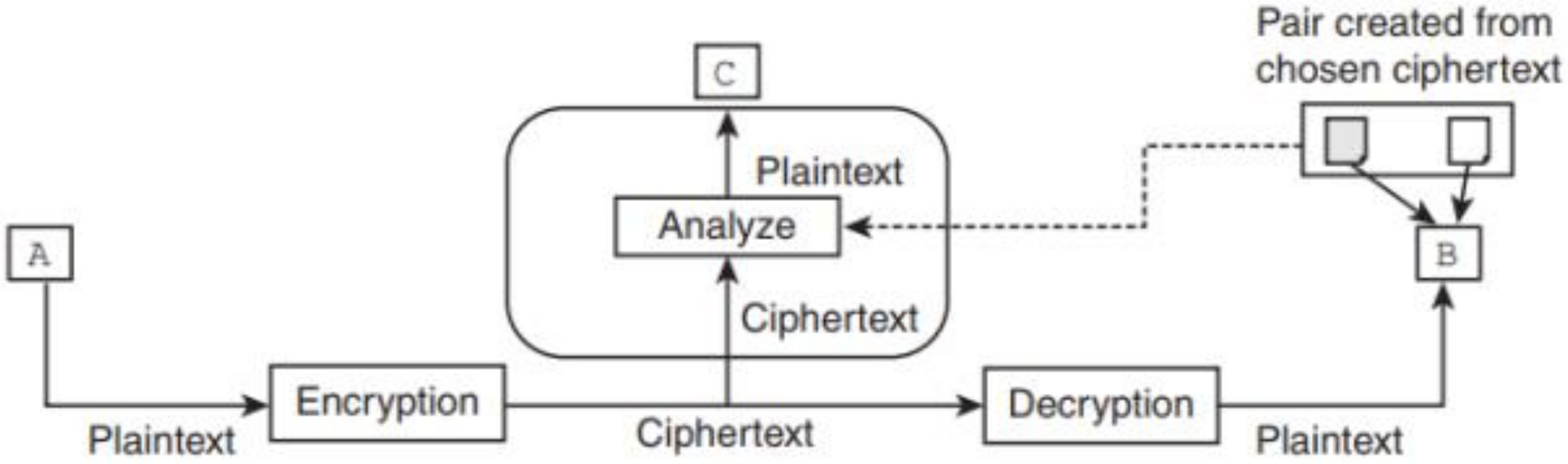
Cryptanalysis

Type of Attack	Known to Cryptanalyst
Chosen Plaintext	■ Encryption algorithm ■ Ciphertext ■ Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key



Cryptanalysis

Type of Attack	Known to Cryptanalyst
Chosen Ciphertext	■ Encryption algorithm ■ Ciphertext ■ Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key



Cryptanalysis

Type of Attack	Known to Cryptanalyst
Chosen Text	■ Encryption algorithm ■ Ciphertext ■ Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key ■ Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

Mật mã khóa đối xứng cổ điển

Caesar Cipher

The earliest known, and the simplest, use of a substitution cipher was by Julius Caesar. The Caesar cipher involves replacing each letter of the alphabet with the letter standing three places further down the alphabet. For example,

```
plain:  meet me after the toga party
cipher: PHHW PH DIWHU WKH WRJD SDUWB
```

Note that the alphabet is wrapped around, so that the letter following Z is A. We can define the transformation by listing all possibilities, as follows:

```
plain:  a b c d e f g h i j k l m n o p q r s t u v w x y z
cipher: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
```

a	b	c	d	e	f	g	h	i	j	k	l	m
0	1	2	3	4	5	6	7	8	9	10	11	12

n	o	p	q	r	s	t	u	v	w	x	y	z
13	14	15	16	17	18	19	20	21	22	23	24	25

Mật mã khóa đối xứng cổ điển

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```

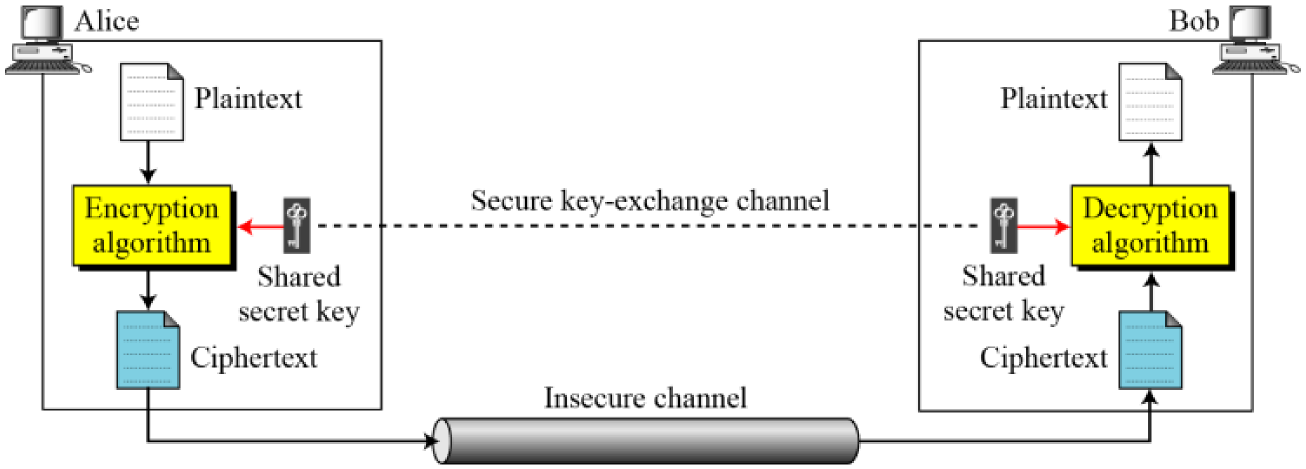
$$C = \mathrm{E}(k, p) = (p + k) \bmod 26$$

$$p = \mathrm{D}(k, C) = (C - k) \bmod 26$$

Three important characteristics of this problem enabled us to use a brute-force cryptanalysis:

1. The encryption and decryption algorithms are known.
2. There are only 25 keys to try.
3. The language of the plaintext is known and easily recognizable.

Mật mã khóa đối xứng cổ điển



Plaintext and Ciphertext in Z_{26}

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

A Substitution Cipher replaces one symbol with another

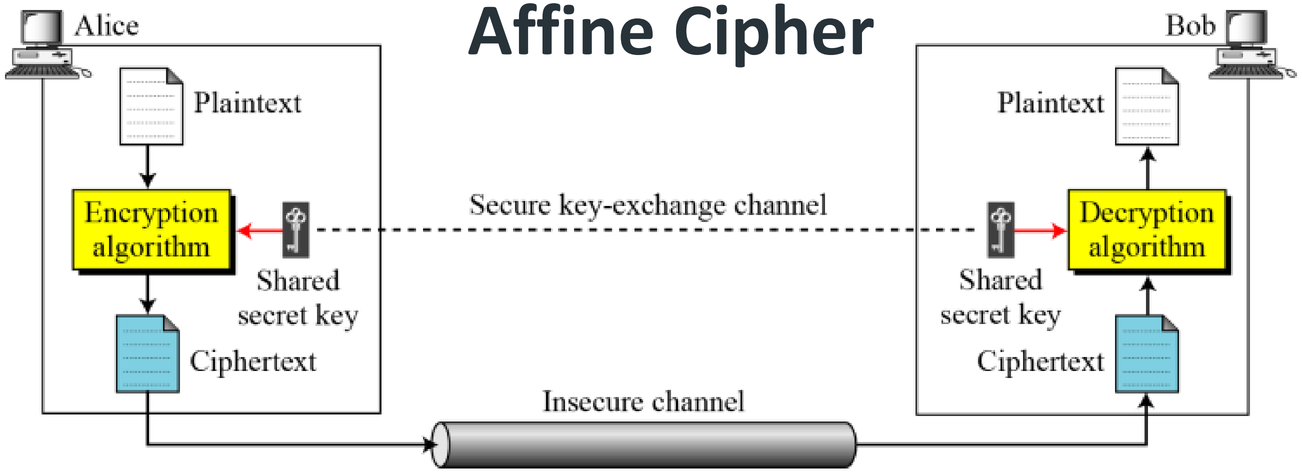
Ceasar, Affine, Vigenere Cipher, Hill Cipher…

A Transposition Cipher reorders symbols.

Rail Fence Cipher, AutoKey …

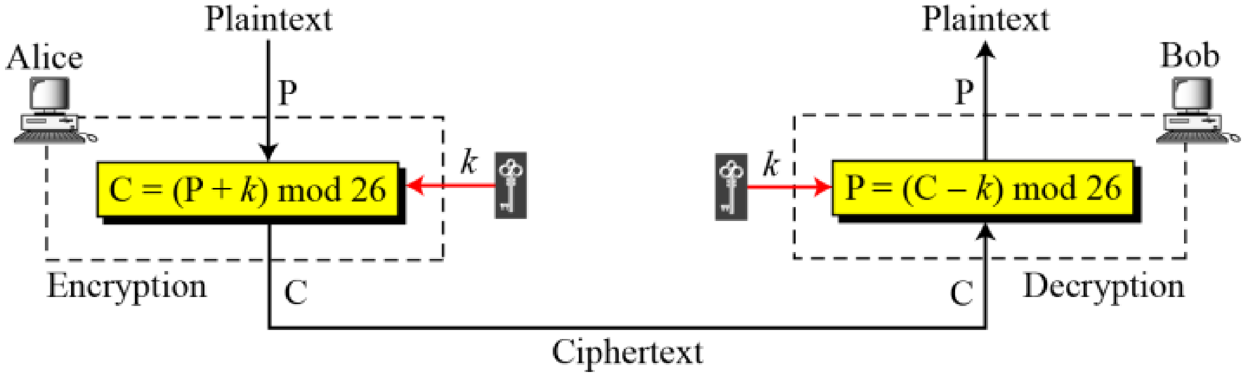
Mật mã khóa đối xứng cổ điển

Affine Cipher



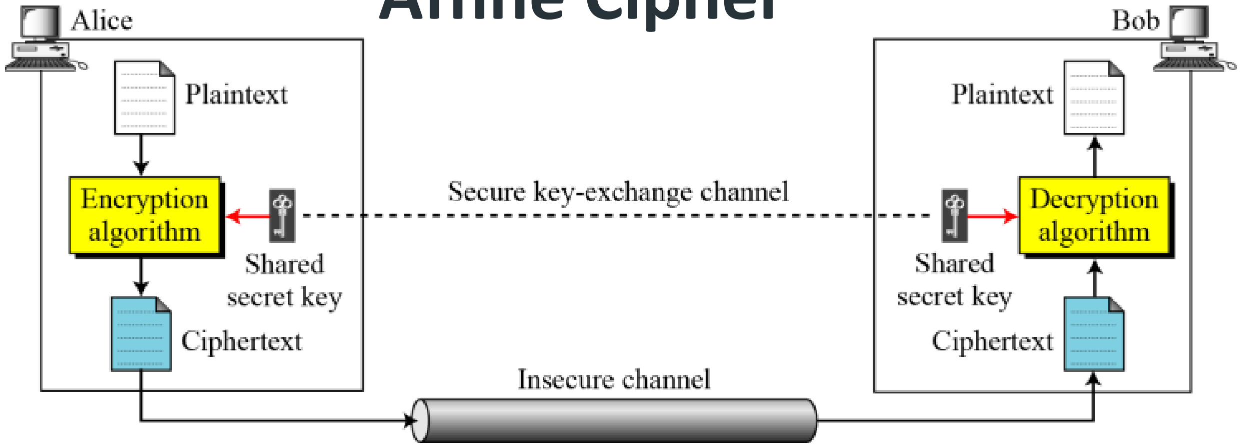
Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Plaintext and Ciphertext in Z_{26}



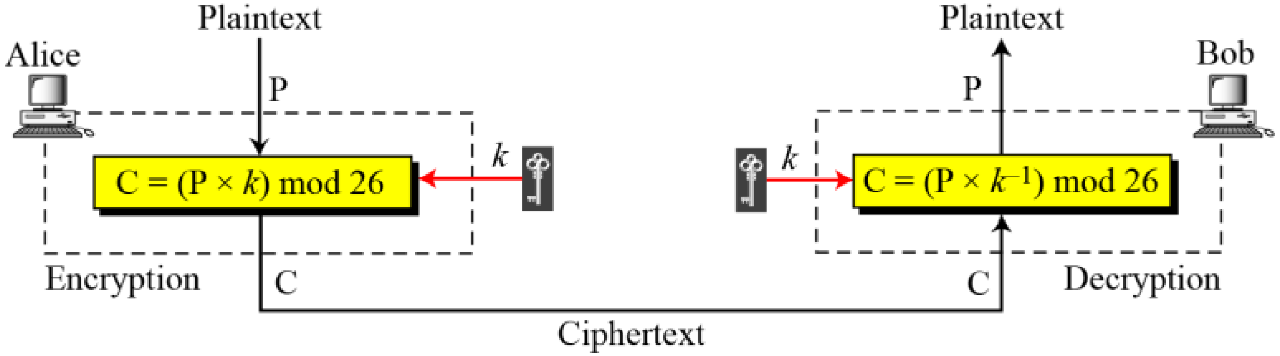
Additive ciphers are sometimes referred to as shift ciphers or Caesar cipher.

Affine Cipher



Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Plaintext and Ciphertext in Z_{26}

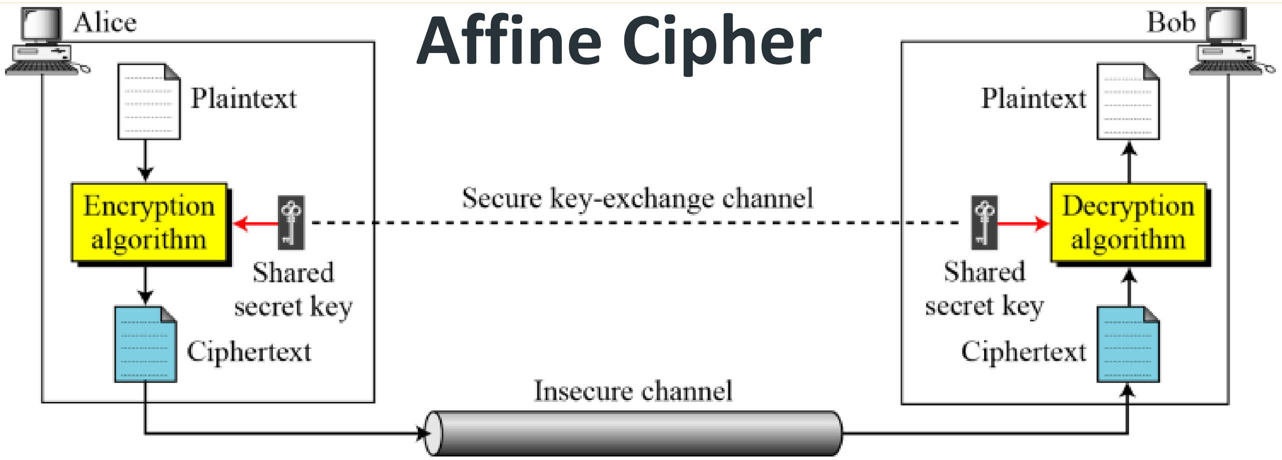


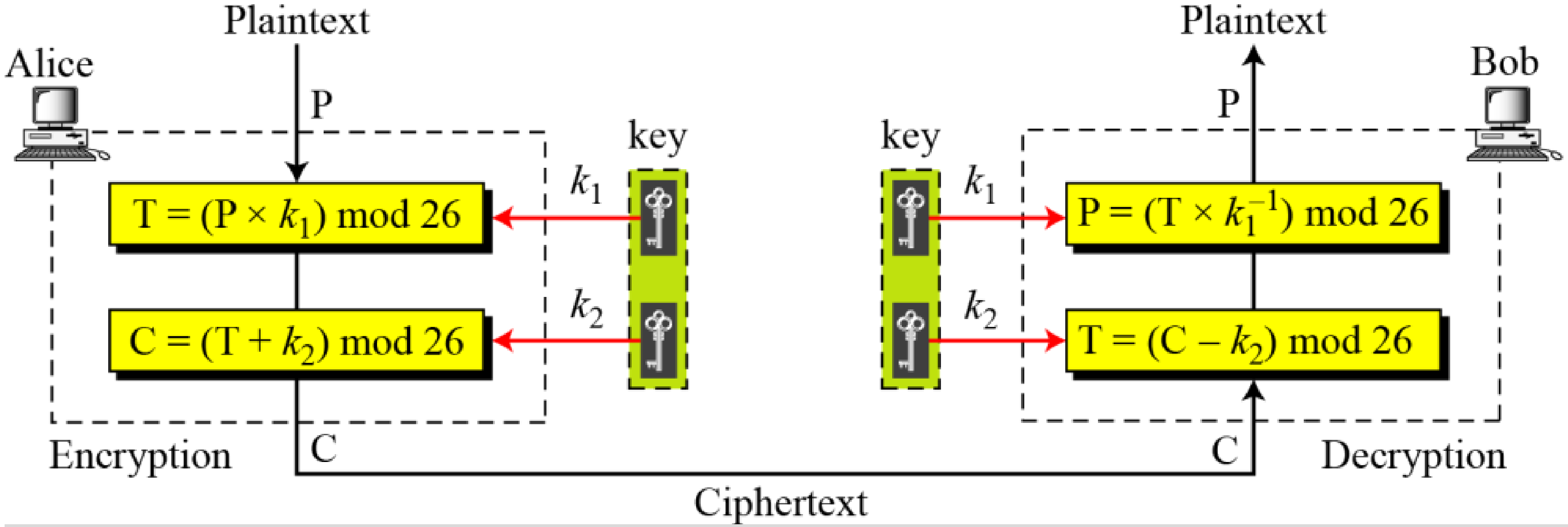
In a multiplicative cipher, the plaintext and ciphertext are integers in Z26; the key is an integer in Z26*.

The affine cipher uses a pair of keys in which the first key is from Z_{26}* and the second is from Z_{26}.

The size of the key domain is $26 \times 12 = 312$.

Affine Cipher





$C = (P \times k_1 + k_2) \bmod 26$ $\qquad$ $P = ((C - k_2) \times k_1^{-1}) \bmod 26$

where k_1^{-1} is the multiplicative inverse of k_1 and $-k_2$ is the additive inverse of k_2

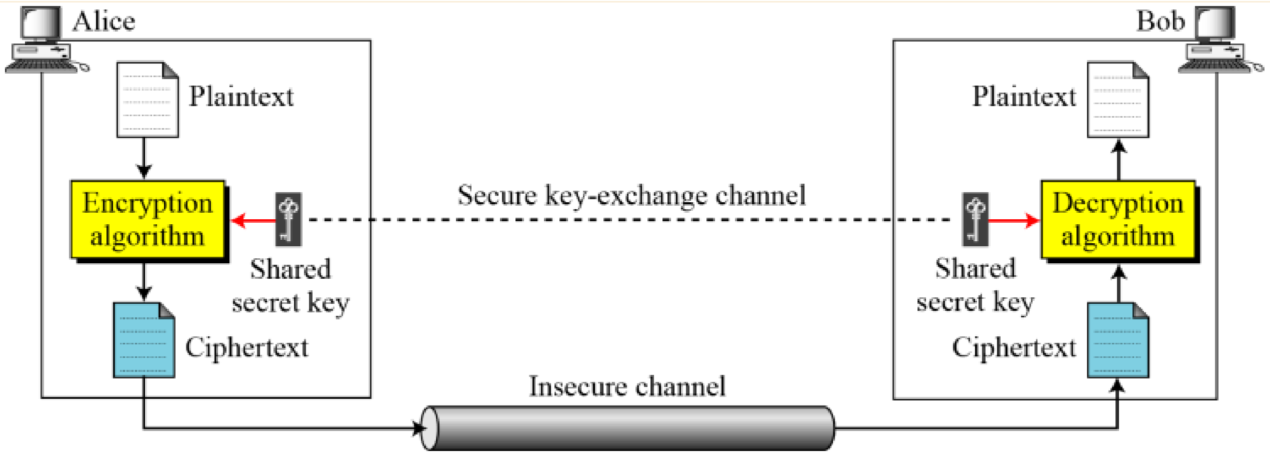
Vigenère Cipher

- To encrypt, pick a letter in the plaintext and its corresponding letter in the keyword, use the keyword letter and the plaintext letter as the row index and column index, respectively, and the entry at the row-column intersection is the letter in the ciphertext.
- To decrypt, pick a letter in the ciphertext and its corresponding letter in the keyword, use the keyword letter to find the corresponding row, and the letter heading of the column that contains the ciphertext letter is the needed plaintext letter.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Vigenère Tableau, *Vigenère Table* or *Vigenère Square*

Mật mã khóa đối xứng cổ điển



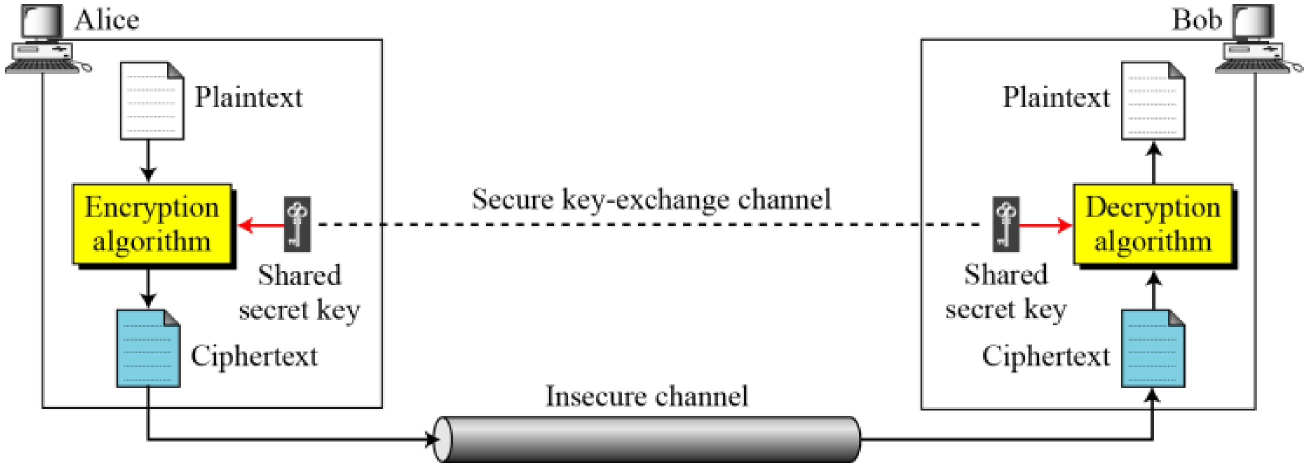
Rail Fence Cipher

Original Message: Hello World

with **3** "rails"

H				o				r		
	e		l				o		l	
		l				W				d

Encrypted Message: Horel ollWd




```
Plaintext = "HELLO"
Autokey = N
Ciphertext = "ULPWZ"
```

```
Given plain text is : H E L L O
Key is              : N H E L L
```

AutoKey

```
Plain Text(P)        : H    E    L    L    O
Corresponding Number: 7    4    11   11   14
Key(K)               : N    H    E    L    L
Corresponding Number: 13   7    4    11   11
                      ---------------------
Applying the formula: 20   11   15   22   25

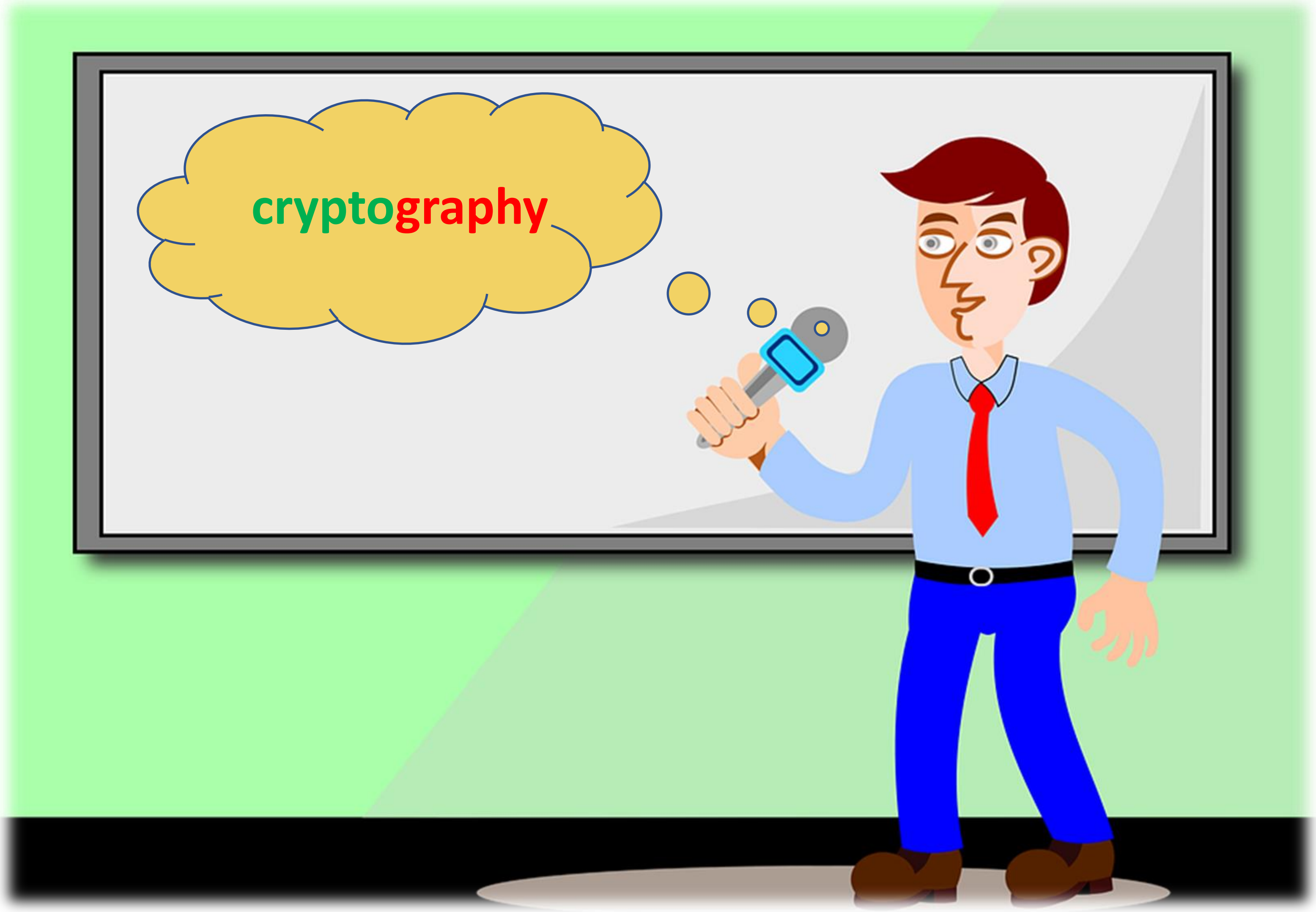
Corresponding
Letters are          : U    L    P    W    Z
```

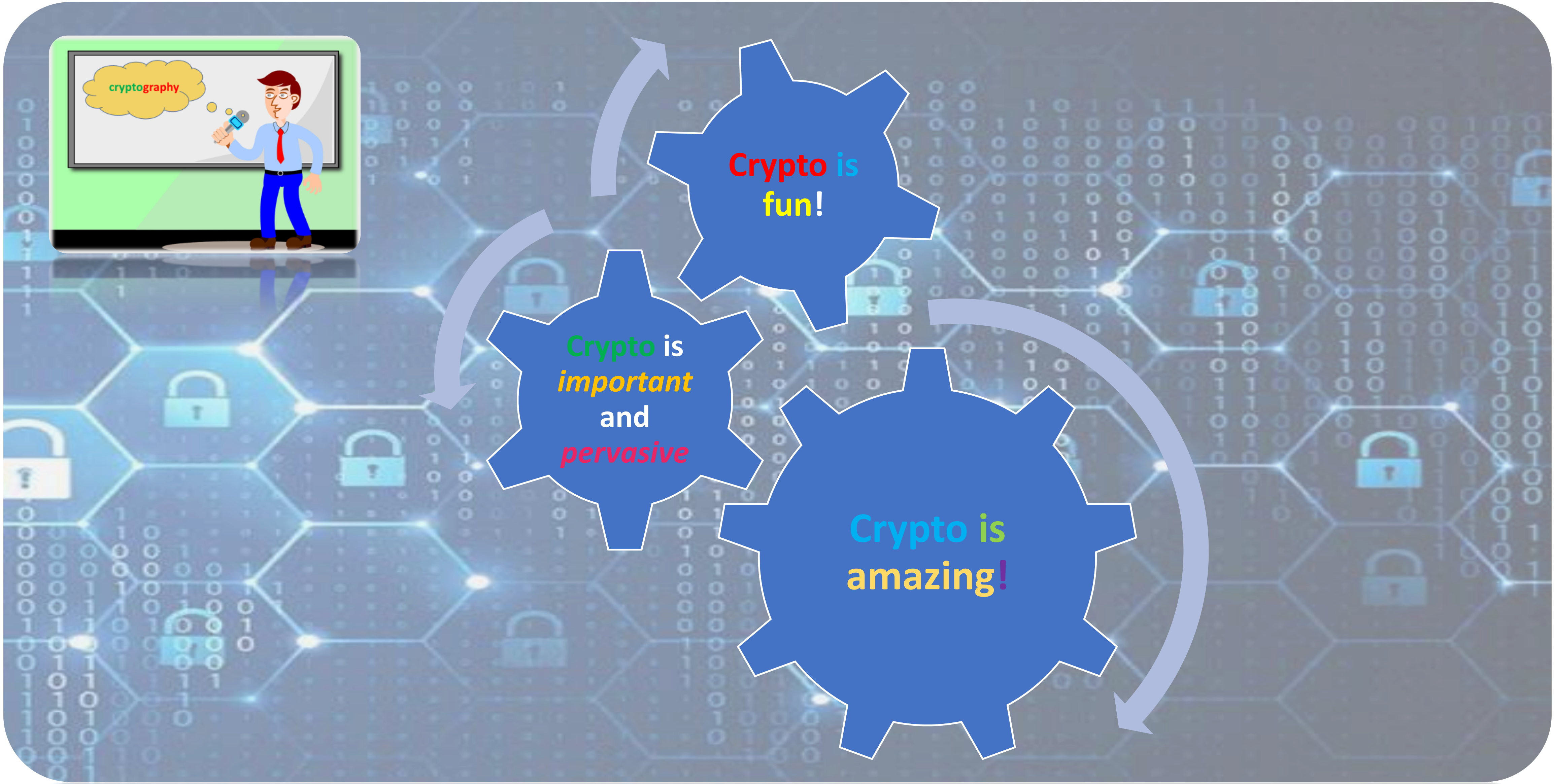
Hence Ciphertext is: **ULPWZ**

```
Let's decrypt:

Cipher Text(C)       : U    L    P    W    Z
Key(K)               : N    H    E    L    L
                      ---------------------
Applying the formula: H    E    L    L    O
```

Hence Plaintext is: **HELLO**


cryptography


cryptography
Crypto is fun!
Crypto is *important* and *pervasive*
Crypto is amazing!

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THANK YOU !

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