# MID-TERM EXAMINATION

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| Course: Discrete Mathematics | |
| Time: **75** minutes | Term: 1 – Academic year: 2021-2022 |
| Lecturer(s): **Dr Nguyen Tan Trung** | |
| Student name: | Student ID: |

***(Notes: Books, laptops, phones and calculators are NOT allowed)***

***<u>Question 1</u>: (2p) a)*** Use rules of inference to show that:

$$\begin{array}{l} (p \land q) \to \neg r \\ s \\ t \land p \\ p \to (u \to q) \\ s \to (r \lor \neg t) \\ \hline \therefore \ \neg u \end{array}$$

***<u>b)</u>)*** *Show that* $\boldsymbol{p} \lor \big((\boldsymbol{p} \land \boldsymbol{q}) \lor (\boldsymbol{p} \land \neg \boldsymbol{r})\big)$ *and* $\boldsymbol{p} \land \big[(\neg \boldsymbol{q} \to \boldsymbol{r}) \lor \neg\big(\boldsymbol{q} \lor (\boldsymbol{r} \land \boldsymbol{s}) \lor (\boldsymbol{r} \land \neg \boldsymbol{s})\big)\big]$ *are logically equivalent by developing a series of logical equivalences.*

***<u>Question 2</u>: (1.5p)*** *Find the solution to each of these recurrence relations and initial conditions.*

***a)*** $a_n = -a_{n-1} + n - 1\ ,\ a_0 = 7$

***b)*** $a_n = 2n a_{n-1}\ ,\ a_0 = 3$

***<u>Question 3</u>: (2p)*** *Show that if $n$ is a positive integer, then* $\binom{2n}{2} = 2\binom{n}{2} + n^2$

*a) Using a combinatorial argument.*

*b) By algebraic manipulation.*

***<u>Question 4:</u> (2p)*** *How many solution are there to the equation*

$$\boldsymbol{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29}$$

*where $x_i, i = 1,2,3,4,5,6$, is a nonnegative integer such that*

a) $x_i > 1 \quad for \quad i = 1,2,3,4,5,6$ ?

b) $x_1 < 8$ và $x_2 > 8$

***Question 5*: (1p)** *Determine whether each of these functions is one-to-one, onto or a bijection and find the inverse function of that function.*

a) ***(0.5p)*** $f: \mathbb{R}\backslash\{2\} \to \mathbb{R}$

$$f(x) = \frac{3x-1}{x-2}$$

b) **(0.5p)** $f: [1; +\infty) \to [-5; +\infty)$

$$f(x) = x^2 - 2x - 4$$

***Question 6*: (1p)** *Find a recurrence relation and initial conditions for the number of bit strings of length n that contain three consecutive 0s.*

***Question 7*: (0,5p)** *Find a closed form for the generating function for each of these sequences. (Assume a general form for the terms of the sequence, using the most obvious choice of such a sequence.)*

$$0, 1, -2, 4, -8, 16, -32, 64, \ldots$$

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