K. H. Rosen, Discrete Math and Applications (7th edition), McGraw Hill

Page 53: 8, 10, 12, 28, 32

8/53:

a) For every animal, if it is a rabbit then it hops

b) All animals are rabbits and they hop

c) There exists one animal that, if it is a rabbit then it hops

d) There exists one animal that is a rabbit and hops

10/53:

a) $\exists x\,(C(x) \wedge F(x) \wedge D(x))$

b) $\forall x\,(C(x) \vee D(x) \vee F(x))$

c) $\exists x\,(C(x) \wedge F(x) \wedge \neg D(x))$

d) $\neg\exists x\,(C(x) \vee D(x) \vee F(x))$

e) $(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$

12/53:

$Q(n)$: $n+1 > 2n$ and $D = \mathbb{Z}$

a) $Q(0)$: $0+1 > 0 \rightarrow$ True

b) $Q(-1)$: $-1+1 > -2 \rightarrow$ True

c) $Q(1)$: $1+1 > 2 \rightarrow$ False

d) $\exists x Q(x) \to$ True ($x = 0$ satisfies)

e) $\forall x Q(x) \to$ False ($x = 1$ does not satisfy)

f) $\exists x \neg Q(x) \to$ True ($x = 1$ satisfies)

g) $\forall x \neg Q(x) \to$ False ($\exists x Q(x)$ is proven)

28/54

Let $P(x)$ mean "x is in correct place"

$E(x)$ " "x is in exellent condition"

$T(x)$ " "x is a tool"

and the domain is all our things

a) $\exists x \neg P(x)$

b) $\forall x (T(x) \to E(x) \wedge P(x)))$

c) $\forall x (E(x) \wedge P(x))$

d) $\forall x (\neg P(x) \wedge E(x))$

e) $\exists x (T(x) \wedge \neg P(x) \wedge E(x))$

32/55 Let the domain be all animals

a) $\forall x (D(x) \to F(x))$

$\exists x (D(x) \wedge \neg F(x))$

$\longrightarrow$ Some dogs do not have fleas

b) $\exists x (H(x) \wedge A(x))$

$\forall x (H(x) \to \neg A(x))$

→ No horses can add

d) $\forall x(K(x) \to S(x))$

$\exists x\,(K(x) \wedge \neg S(x))$

→ Some monkeys can speak French

e) $\exists x\,(P(x) \wedge W(x) \wedge G(x))$

$\forall x\,(P(x) \to (\neg W(x) \vee \neg G(x)))$

Every pig either cannot swim or cannot catch fish

6 / P64

a) Randy Goldberg is enrolled in CS252

b) There is at least 1 student who is enrolled in Math 695

c) Carol Sitea is enrolled in at least one class

d) There exists one student that is both enrolled in Math 222 and CS252

e) There exists 2 students such that whenever the former is enrolled in a class, the latter is also enrolled is also enrolled in the same class

f) There exists 2 students such that their classes are exactly the same

10/P65

a) $\forall x\, F(x, \text{Fred})$

b) $\forall y\, F(\text{Evelyn}, y)$

c) $\forall x \exists y\, (x, y)$

d) $\neg \exists x \forall y\, F(x, y)$

e) $\exists x \forall y\, (x, y)$

f) $\neg \exists x\, [(F(x, \text{Fred}) \wedge F(x, \text{Jerry})]$

g) $\exists y \exists z\, (y \neq z \wedge F(\text{Nancy}, y) \wedge F(\text{Nancy}, z) \wedge \forall w\, (F(\text{Nancy}, w) \to (w = y \vee w = z)))$

h) $\exists y (\forall x\, F(x, y) \wedge \forall z ((\forall x F(x, z)) \to z = y))$

i) $\neg \exists x F(x, x)$

j) $\exists x (\exists y\, (y = x \wedge F(x, y)) \wedge \forall z\, ((z \neq x \wedge F(x, z)) \to z = y))$

12/P65

a) $\neg I(\text{Jerry})$

b) $\neg C(\text{Rachel}, \text{Chelsea})$

c) $\neg C(\text{Jan}, \text{Sharon})$

d) $\forall x\, C(x, \text{Bob})$

e) $\forall x (C(\text{Sanjay}, x) \land \neg C(\text{Sanjay}, \text{Joseph}))$

f) $\exists x\, \neg I(x)$

g) $\neg \forall x\, I(x)$

h) $\exists x (I(x) \land (\forall y (I(y) \to x = y)))$

i) $\exists x (\neg I x \land \forall y (y \neq x \to I(y)))$

j) $\forall x (I(x) \to \exists y (y \neq x \land C(x,y)))$

k) $\exists x (I(x) \land \forall y (y \neq x \to \neg C(x,y)))$

l) $\exists x \exists y (x \neq y \land \neg C(x,y))$

m) $\exists x \forall y\, C(x,y)$

n) $\exists x \exists y \exists z (x \neq y \land \neg C(x,z) \land \neg C(y,z))$

o) $\exists x \exists y (x \neq y \land \forall z ((z \neq x \land z \neq y) \to (C(x,z) \lor C(y,z))))$

28/P67:

a) $\forall x \exists y (x^2 = y) \to$ True

b) $\forall x \exists y (x = y^2) \to$ False $(x = -1)$

c) $\exists x \forall y (xy = 0) \to$ True $(x = 0)$

d) $\exists x \exists y (x + y \neq y + x) \to$ False

e) $\forall x (x \neq 0 \to \exists y (xy = 1)) \to$ True because

$y = \frac{1}{x} \wedge xy = 1$ is true)

f) $\exists x \forall y (y \neq 0 \to xy = 1) \to$ False

g) $\forall x \exists y (x + y = 1) \to$ True

h) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

$\to$ False cause no solutions

i) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$

$\to$ False cause only $x = 1$ works

j) $\forall x \forall y \exists z (z = (x + y)/2)$

$\to$ True cause $z = \frac{x+y}{2}$ exists $\forall x, y \in \mathbb{R}$

37/68

Let x be everyone in the world

a) Let $S(x)$ be "student x is in this class"

Let $M(x, y)$ be "x has taken mathematics class called y" and domain for y is all math classes in this school

$\forall x \{ S(x) \to \exists y_1 \exists y_2 [y_1 \neq y_2 \wedge M(x, y_1) \wedge M(x, y_2) \wedge \forall y (M(x, y) \to (y = y_1 \vee y = y_2))] \}$

Negation:

$\exists x \{ S(x) \wedge \forall u_1 \forall u_2 [u_1 \neq u \ \ldots \}$

b) $V(x,y)$: "x has visited country y"

Domain: all countries

$\exists x \forall y \, (y \neq Libya \to V(x,y))$

$\forall x \exists y \, (y \neq Libya \wedge \neg V(x,y))$

c) $\neg \exists x \forall y \, C(x,y)$

$\exists x \forall y \, C(x,y)$

d) $\forall x \left[(A(x) \to M(x,k) \vee \exists y (M(x,y) \wedge M(y,k))) \right]$

$\exists x \left\{ A(x) \wedge \neg \left[M(x,k) \vee \exists y (M(x,y) \wedge M(y,k)) \right] \right\}$

4/78

a) Simplification

b Disjunctive syllogism

c) Modus pollens

d) Addition

9/78: a)

"Tues or Thurs" $\vee$ "not Tues" $\to$ Thurs

) "Day off" → "rain or snow" and "no snow on Thurs"
→ "rain on Thurs"

Rules: DS + MP

b) Conclusions:

- I did not eat spicy foods
- There was no thunder

Rules: MT (twice)

c) Conclusion: I am clever

Rule: Disjunctive syllogism

"Clever or lucky" & "¬lucky"

→ clever

d) Conclusion: Ralph is not a CS major

Rule: Modus tollens

If CS major ⇒ has PC

Ralph no PC ⇒ not CS major

e) Conclusion: Buying lots of stuff is good for you

Rule: Hypothetical syllogism

$p \to q \wedge q \to r \to p \to r$

f) Conclusion: Rabbits are not rodents

Mice gnaw their food

16/79:

a) Correct by modus tollens

b) Incorrect. Fallacy of denying the antecedent

c) Incorrect. Fallacy of affirming the consequent (converse error)

d) Correct by universal instantiation and modus ponens

37/91 We have a cycle

$$p_1 \to p_4 \to p_2 \to p_5 \to p_3 \to p_1 \to p_4 \to \dots$$

Therefore

$$p_1 \leftrightarrow p_2 \leftrightarrow p_3 \leftrightarrow p_4 \leftrightarrow p_5$$