FINAL EXAMINATION

Course: **Discrete Structure**	Class: **20CTT2**
Time: **120 minutes**	Term: 1 – Academic year: 2021-2022
Lecturer(s): **Nguyễn Tấn Trung**	
Student name:	Student ID:

(Notes: Books, phones and calculators are NOT allowed)

<Followings are the questions and/or requirements>

Q1. (2.5 points) Draw the Hasse diagram (0.5 pt) for divisibility " | " on the set

$$(\{\mathbf{2}, \mathbf{4}, \mathbf{6}, \mathbf{9}, \mathbf{12}, \mathbf{18}, \mathbf{27}, \mathbf{36}, \mathbf{48}, \mathbf{60}, \mathbf{72}\}, |)$$

a) (0.5 pt) Find the maximal and minimal elements.

b) (0.5 pt) Find the greatest and the least elements if it exists?

c) (0.5 pt) Find all upper bounds and the least upper bound (if it exists) of {2, 9}.

d) (0.5 pt) Find all lower bounds and the greatest lower bound (if it exists) of {60, 72}.

Q2. (2.5 points) Find a minimal expansion as a Boolean sum of Boolean products of the function in the variables of *w*, *x*, *y* and *z*

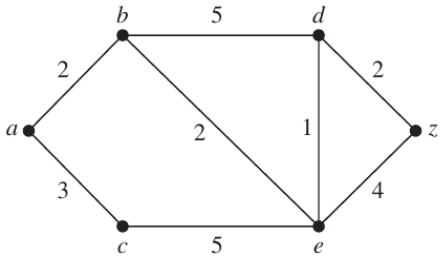
$$\boldsymbol{wxyz + wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\,\bar{y}z}$$

a) (1 pt) Use a K-map.

b) (1 pt) Use the Quine-McCluskey method.

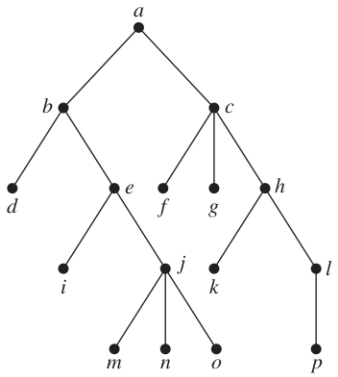
c) (0.5 pt) Draw the circuit of the final result.

Q3. (1 points) Use ***Dijkstra's algorithm*** to find the shortest path between "*a*" and "*z*".
What is the complexity of ***Dijkstra's algorithm***? Explain your solution.



Q4. (2 point)

a) (1.5 pt) Determine the order in which a(n) preorder, inorder and postorder traversal visits the vertices of the given ordered rooted tree



b) (0.5 pt) What is the value of the postfix expressions

$$3\ 2 * \ 2 \uparrow 5\ 3 - 8\ 4 / * \ -$$

Q5. (1 point) Use generating functions to solve the recurrence relation $\boldsymbol{a_k = 3a_{k-1} + 2}$ with the initial condition $\boldsymbol{a_0 = 1}$

Q6. (1 point) Suppose that a connected planar simple graph with *e* edges and *v* vertices contains no simple circuits of length 4 or less. Show that $\boldsymbol{e \leq \frac{5}{3}v - \frac{10}{3}}$ if $\boldsymbol{v \geq 4}$.

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