Ngô Đình Thiên Lộc (EA125093)

HW2

~~Problem 16~~:

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Homework 2

5) a) $\{1,3,3,5,5,5,5,5,5\}$

$= \{6, 3, 1\}$

b) $\{\{1\}\} \neq \{1; \{1\}\}$

c) $\emptyset \neq \{\emptyset\}$

10) a) T c) F e) T

b) T d) T f) T

g) T

33) $A = \{0; 1; 3\}$

$\Rightarrow A^2 = \{\{0;1\}, \{0;3\}, \{1;3\}; \{1;0\}$

$\{3;0\}, \{3;1\}; \{0;0\}; \{1;1\}; \{3;3\}\}$

43) a) $x^2 < 3$

$\Leftrightarrow x\sqrt{3} < x < \sqrt{3}$

$\Leftrightarrow x \in \{-1; 0; 1\}$

Domain is $\{-1; 0; 1\}$

b) $x^2 > x \Leftrightarrow x(x-1) > 0$

$\Leftrightarrow \begin{cases} x > 0 \ \& \ x-1 > 0 \\ x < 0 \ \& \ x-1 < 0 \end{cases}$

$\Leftrightarrow \begin{cases} x > 1 \\ x < 0 \end{cases}$

Truth set is:

~~$(0;0) \cup ($~~

$(-\infty, 0) \cup (1; +\infty)$

c) $2x + 1 = 0$

$\Leftrightarrow x = -\frac{1}{2} \notin \mathbb{Z}$

Truth set is $\emptyset$

Nguyễn Đình Thiên Lợi
Homework 2 (cont)
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3) $A = \{1; 2; 3; 4; 5\}$

$B = \{0; 3; 6\}$

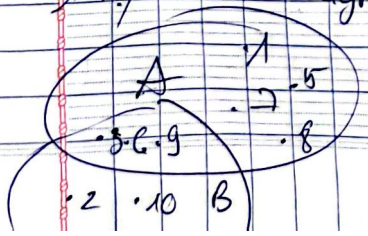
a) $A \cup B = \{0; 1; 2; 3; 4; 5; 6\}$

b) $A \cap B = \{1; 2; 3; 4; 5\} \cap \{0; 3; 6\}$
$= \{3\}$

c) $A - B = \{1; 2; 3; 4; 5\} - \{0; 3; 6\}$
$= \{1; 2; 4; 5\}$

d) $B - A = \{0; 6\}$

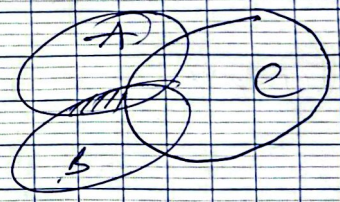
14) Venn Diagram



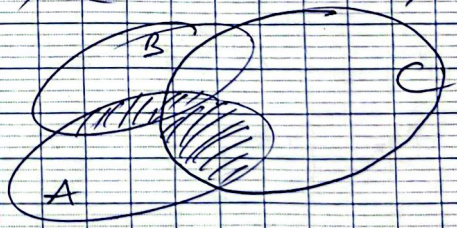
$A = \{1; 3; 5; 6; 7; 8; 9\}$

$B = \{2; 3; 6; 9; 10\}$

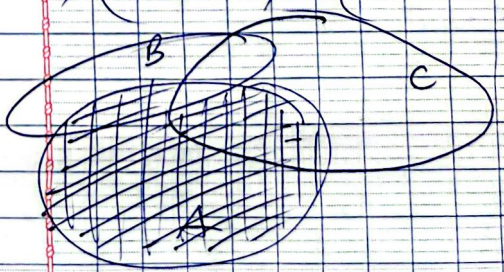
27) a) $A \cap (B - C)$



b) $(A \cap B) \cup (A \cap C)$



c) $(A \cap \overline{B}) \cup (A \cap \overline{C})$



52) a)

52 b) 1010010001

53 b) 0111001110

54 a) $D = \mathbb{N}^2$; $E = \mathbb{N} - \{0\}$

b) $D = \mathbb{N}$; $E = \{0; 1; \dots 8\}$

c) $D = \{0; 1\}$; $E = \mathbb{N}$

d) $D = \{0; 1\}$; $E = \mathbb{N}$

23) a) $f(x) = 2x + 1$, $D = \mathbb{R}$

+) $\forall x, y \in \mathbb{R}$:

$f(x) = f(y)$

$\Leftrightarrow x = y$

Thus f is one-to-one func ①

+) Random y

If $x = \frac{y-1}{2}$ then $f(x) = y$

Thus f is onto ②

From ① and ② $\Rightarrow$ Bijection ✓

b) $f(x) = x^2 + 1$

$x^2 \geq 0 \quad \forall x \in \mathbb{R}$

$\Leftrightarrow x^2 + 1 \geq 1$

$\Leftrightarrow f(x) \geq 1 \quad | \forall x \in \mathbb{R}$

So if $y < 1$, x would not exist

f is not bijection (not onto)

c) $f(x) = x^3$

This is bijection

Cause +) $f(x) = f(y)$

$\Leftrightarrow x^3 = y^3 \Leftrightarrow x = y$

+) Onto:

$\sqrt[3]{y} = x$ for all $y \in \mathbb{R}$

a) $f(x) = \frac{x^2+1}{x^2+2}$

~~Bài tập~~

$f(x) > 0 \quad \forall x \in \mathbb{R}$

Therefore $y \leq 0$ would not have corresponding x-values

$\Rightarrow y$ ∉ is not onto

$\Rightarrow f$ is NOT bijection

31) a) $S = \{-2; -1; 0; 1; 2; 3\}$

$f\{S\} = \{0; 1; 3\}$

$\left(f\{S\} = \{x^2 / 3\}\right)$

b) $S = \{0; 1; 2; 3; 4; 5\}$

$\Rightarrow f(S) = \{0; 1; 3; 5; 8\}$

c) $f(S) = \{0; 1; 16; 40\}$

d) $f(P) = \{1; 12; 33; 65\}$

39) Showing they are invertible

$\Leftrightarrow$ Proving they are ~~one-to-one~~ bijections

functions (exists f^{-1})

Consider:

$x_1 = x_2 \in \mathbb{R}$

$f(x_1) = f(x_2)$

$\Leftrightarrow (x_1 - x_2)a = 0$

$\Leftrightarrow x_1 = x_2 \quad (a \neq 0)$

f is one-to-one func

f is also ~~invertible~~ onto:

$\forall y \in \mathbb{R}: x = \frac{y-b}{a} \quad (a \neq 0)$

Therefore: f is bijection

$\rightarrow$ invertible

$f^{-1}(x) = \frac{x-b}{a}$