

Question 1.(1.5 marks) How many solutions are there to the equation

$$x_1 + x_2 + x_3 = 22,$$

where x_1, x_2 and x_3 are nonnegative integers such that

- a) $x_1 > 2$ and $5 \leq x_2 \leq 10$? b) $x_1 \geq 3$ or $x_2 > 10$?

Question 2.(1.5 marks) What is the solution of the recurrence relation $a_{n+1} - 6a_n + 8a_{n-1} = 2^n(2 - 4n)$ with $a_0 = 5$ and $a_1 = 20$?

Question 3.(2 marks) A number n is called a square number if there exists an integer number m such that $n = m^2$. On the set $A = \{1, 2, 3, 4, 6, 8, 9, 10, 12, 16, 18\}$, we define a relation \mathcal{R} as follows

$$x\mathcal{R}y \Leftrightarrow xy \text{ is a square number.}$$

- a) Prove that \mathcal{R} is an equivalence relation.
- b) Write down the equivalence class [2].
- c) Find all equivalence classes.

Question 4.(1.5 marks)

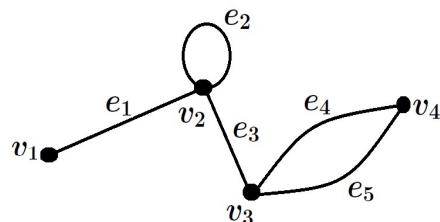
- a) Draw the Hasse diagram of the divisibility relation on the set

$$\{1, 2, 4, 5, 7, 10, 12, 15, 30, 60\}.$$

- b) Find the maximal and minimal elements.
- c) Find the least and greatest elements, if they exist.

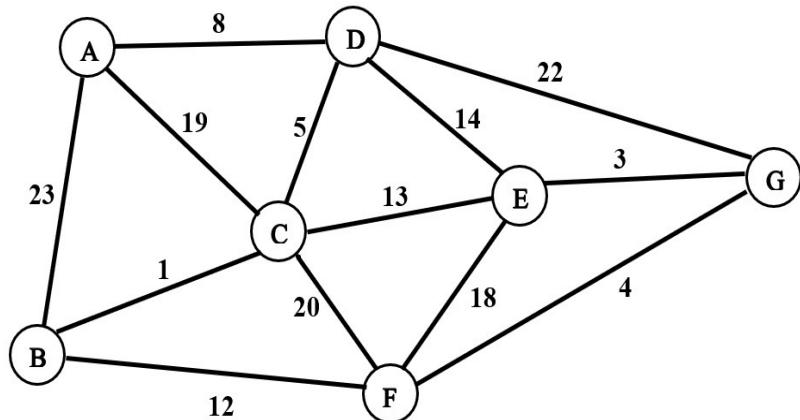
Question 4.(0.5 marks) How many edges does a graph have if its degree sequence is 5, 2, 2, 2, 2, 1? Draw such a graph.

Question 5.(1 mark) Given the graph with vertices v_1, v_2, v_3 and v_4 .



- a) Find the number of walks of length 3 from v_2 to v_3 .
- b) Find the number of walks of length 5 from v_1 to v_4 .

Question 7.(2 marks) Let \mathcal{X} be the following weighted graph



- Is it an Eulerian graph or a Hamiltonian graph? Why?
- Find a minimum spanning tree of G by using Kruskal's algorithm.
- Find a maximum spanning tree containing edge BC of \mathcal{X} by using Prim's algorithm.