

Bài tập chương 1 Propositional Logic

1 Dẫn nhập

Trong bài tập dưới đây, chúng ta sẽ làm quen với cách diễn đạt trong luận lý mệnh đề và việc diễn dịch chúng trong ngôn ngữ thông thường. Sinh viên cần ôn lại lý thuyết của chương 1 trước khi làm các bài tập bên dưới.

2 Bài tập bắt buộc

Exercise 1.

Construct a truth table for each of these compound poropositions.

Ví dụ. $\neg p \rightarrow (\neg q \vee r)$

p	q	r	$\neg p$	$\neg q$	$\neg q \vee r$	$\neg p \rightarrow (\neg q \vee r)$
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

- a) $(p \wedge q) \rightarrow \neg q$
- b) $(p \vee r) \rightarrow (r \vee \neg p)$
- c) $(p \rightarrow q) \vee (q \rightarrow p)$
- d) $(p \vee \neg q) \wedge (\neg p \vee q)$
- e) $(p \rightarrow \neg q) \vee (q \rightarrow \neg p)$
- f) $\neg(\neg p \wedge \neg q)$
- g) $(p \vee q) \rightarrow (p \oplus q)$
- h) $(p \wedge q) \vee (r \oplus q)$

Exercise 2.

What is the negation of each of these propositions?

- a) Steve has more than 100 GB free disk space on his laptop.
- b) Zach blocks e-mails and texts from Jennifer.
- c) $7 \cdot 11 \cdot 13 = 999$.
- d) Diane rode her bicycle 100 miles on Sunday.

Exercise 3.

What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

- a) if $x + 2 = 3$ then $x := x + 1$
- b) if $(x + 1 = 3)$ OR $(2x + 2 = 3)$ then $x := x + 1$
- c) if $(2x + 3 = 5)$ AND $(3x + 4 = 7)$ then $x := x + 1$
- d) if $(x + 1 = 2)$ XOR $(x + 2 = 3)$ then $x := x + 1$
- e) if $x < 2$ then $x := x + 1$

Exercise 4.

State the converse, contrapositive of each of these conditional statements

- a) If it snows today, I will ski tomorrow.
- b) I come to class whenever there is going to be a quiz.
- c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Exercise 5.

Let p, q , and r be the propositions

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

Express each of these propositions as an English sentence.

- a) $p \rightarrow q$
- b) $\neg q \leftrightarrow r$
- c) $q \rightarrow \neg r$
- d) $p \vee q \vee r$
- e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
- f) $(p \wedge q) \vee (\neg q \wedge r)$

Exercise 6.

Let p, q , and r be the propositions

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using p, q , and r and logical connectives (including negations).

- a) You get an A in this class, but you do not do every exercise in this book.
- b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
- c) To get an A in this class, it is necessary for you to get an A on the final.
- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

- f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Exercise 7.

Evaluate each of these expressions

- a) $11000 \wedge (01011 \vee 11011)$
b) $(01111 \wedge 10101) \vee 01000$

Exercise 8.

Show that each of these conditional statements is a tautology by using truth tables.

- a) $(p \wedge q) \rightarrow p$
b) $p \rightarrow (p \vee q)$
c) $\neg p \rightarrow (p \rightarrow q)$
d) $(p \wedge q) \rightarrow (p \rightarrow q)$
e) $\neg(p \rightarrow q) \rightarrow p$
f) $\neg(p \rightarrow q) \rightarrow \neg q$
g) $[\neg p \wedge (p \vee q)] \rightarrow q$
h) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Exercise 9.

Show that these compound propositionals are logically equivalent.

- a) $\neg(p \leftrightarrow q)$ và $\neg p \leftrightarrow q$
b) $(p \rightarrow q) \wedge (p \rightarrow r)$ và $p \rightarrow (q \wedge r)$
c) $(p \rightarrow r) \wedge (q \rightarrow r)$ và $(p \vee q) \rightarrow r$
d) $(p \rightarrow q) \vee (p \rightarrow r)$ và $p \rightarrow (q \vee r)$
e) $\neg p \rightarrow (q \rightarrow r)$ và $q \rightarrow (p \vee r)$
f) $p \leftrightarrow q$ và $(p \rightarrow q) \wedge (q \rightarrow p)$

Exercise 10.

Show that these compound propositionals are logically equivalent by developing a series of logical equivalences.

- a) $\neg(p \rightarrow (\neg q \wedge r))$ and $p \wedge (q \vee \neg r)$
b) $\neg[p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)]$ and $\neg p \vee \neg r$
c) $\neg[[[(p \wedge q) \wedge r] \vee [(p \wedge r) \wedge \neg r]] \vee \neg q] \rightarrow s]$ and $[(p \wedge r) \vee \neg q] \wedge \neg s$

Exercise 11.

You cannot edit a protected Wikipedia entry unless you are an administrator. Express your answer in terms of e : “You can edit a protected Wikipedia entry” and a : “You are an administrator.”

Exercise 12.

You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of m : “You can see the movie,” e : “You are over 18 years old,” and p : “You have the permission of a parent.”

Exercise 13.

You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of g : “You can graduate,” m : “You owe money to the university,” r : “You have completed the requirements of your major,” and b : “You have an overdue library book.”

Exercise 14.

You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country. Express your answer in terms of

e : “You are eligible to be President of the U.S.A.,”

a : “You are at least 35 years old,”

b : “You were born in the U.S.A.,”

p : “At the time of your birth, both of your parents were citizens,” and

r : “You have lived at least 14 years in the U.S.A.”

Exercise 15.

To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of

w : “You can use the wireless network in the airport,”

d : “You pay the daily fee,” and

s : “You are a subscriber to the service.”

Exercise 16.

Express these system specifications using the propositions p “The message is scanned for viruses” and q “The message was sent from an unknown system” together with logical connectives (including negations).

- a) “The message is scanned for viruses whenever the message was sent from an unknown system.”
- b) “The message was sent from an unknown system but it was not scanned for viruses.”
- c) “It is necessary to scan the message for viruses whenever it was sent from an unknown system.”
- d) “When a message is not sent from an unknown system it is not scanned for viruses.”

Exercise 17.

When three professors are seated in a restaurant, the hostess asks them: “Does everyone want coffee?” The first professor says: “I do not know.” The second professor then says: “I do not know.” Finally, the third professor says: “No, not everyone wants coffee.” The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

3 Tổng kết

Thông qua các bài tập trong phần này, chúng ta đã làm quen với việc sử dụng, khai báo các mệnh đề trong luận lý mệnh đề và việc diễn dịch chúng trong ngôn ngữ thông thường. Và các bài tập này cũng đã giúp chúng ta phần nào hiểu thêm về lý thuyết luận lý mệnh đề (tham khảo chi tiết trong chương 1).