Use these instructions for the remainder of the exam.

Your J-number $JN := \underline{00266502} := (J_k | k \in 1..8)$

Hence

$J_1 =$ 0 $J_2 =$ 0 $J_3 =$ 2 $J_4 =$ 6

$J_5 =$ 6 $J_6 =$ 5 $J_7 =$ 0 $J_8 =$ 2

R For each of sets defined below **write the elements in increasing order**

D $\mathcal{U} := 0..9 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

D $J := \{J_k \in 0..9 | k \in 1..8\} = \{$ 0, 2, 5, 6 $\}$

D $A := \{J_k \in 0..9 | J_k \text{ is not prime}\} = \{$ 0, 6 $\}$

D $B := \{k \in 1..8 | J_k \neq 0\} = \{$ 3, 4, 5, 6, 8 $\}$

D $C := \{J_k \in 0..9 | J_k < 8\} = \{$ 0, 2, 5, 6 $\}$

D $D := \{J_k \in 0..9 | J_k > 1\} = \{$ 2, 5, 6 $\}$

D Define Y:= Yes N:=No

R **Note that in a set an element may occur exactly once.** Therefore there may not be any repeated elements in any set in the above. **So, if you do not have the correct sets, all your answers will be wrong.**

R If answers to different instances of some question on the exam contradict each other you will get a score of zero for every instance.

R **Do not make any extraneous marks** on the exam.

You are free to talk to anyone

6 (1a) Compute the following, showing work on the facing page, and record ONLY the answers on this page.

1 (i) $C \cup D$

$= \{0, 2, 5, 6\}$

1 (ii) $C \cap D$

$= \{2, 5, 6\}$

1 (iii) $C \times D$ Draw tree on facing page

$= \{(0,2), (0,5), (0,6), (2,2), (2,5), (2,6), (5,2), (5,5), (5,6), (6,2), (6,5), (6,6)\}$

1 (iv) $D \times C$ Draw tree on facing page

$= \{(2,0), (2,2), (2,5), (2,6), (5,0), (5,2), (5,5), (5,6), (6,0), (6,2), (6,5), (6,6)\}$

6

1 (v) $(C \times D) \cap (D \times C)$

$= \{(2,2), (2,5), (2,6), (5,2), (5,5), (5,6), (6,2), (6,5), (6,6)\}$

1 (vi) $(C \cap D) \times (D \cap C)$ Draw tree on facing page

$= \{(2,2), (2,5), (2,6), (5,2), (5,5), (5,6), (6,2), (6,5), (6,6)\}$

You are free to talk to anyone

(i) $C \cup D$

$= \{0,2,5,6\} \cup \{2,5,6\}$

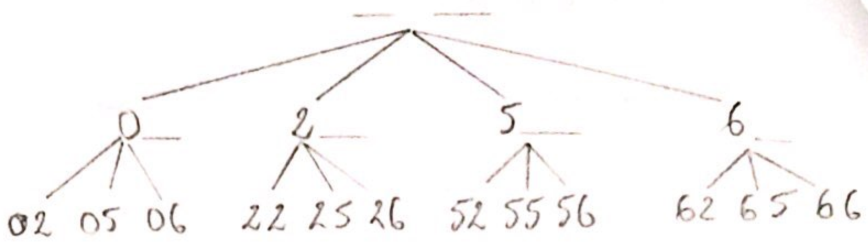
$= \{0,2,5,6\}$

(ii) $C \cap D$

$= \{0,2,5,6\} \cap \{2,5,6\}$

$= \{2,5,6\}$

(iii) Tree



$C \times D$

$= \{0,2,5,6\} \times \{2,5,6\}$

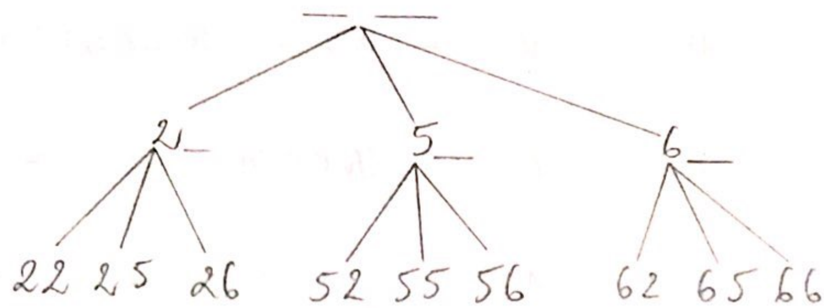
$= \begin{Bmatrix} (0,2),(0,5),(0,6), \\ (2,2),(2,5),(2,6), \\ (5,2),(5,5),(5,6), \\ (6,2),(6,5),(6,6) \end{Bmatrix}$

(vi) $(C \cap D) \times (D \cap C)$

$= \{2,5,6\} \times \{2,5,6\}$

$= \begin{Bmatrix} (2,2)\ (2,5),(2,6) \\ (5,2),(5,5),(5,6) \\ (6,2),(6,5)(6,6) \end{Bmatrix}$

Tree



(v) $(C \times D) \cap (D \times C)$

$= \begin{Bmatrix} (0,2),(0,5),(0,6), \\ (2,2)(2,5),(2,6), \\ (5,2),(5,5),(5,6), \\ (6,2),(6,5),(6,6) \end{Bmatrix} \cap \begin{Bmatrix} (2,0),(2,2),(2,5),(2,6) \\ (5,0),(5,2),(5,5),(5,6), \\ (6,0),(6,2),(6,5),(6,6) \end{Bmatrix}$

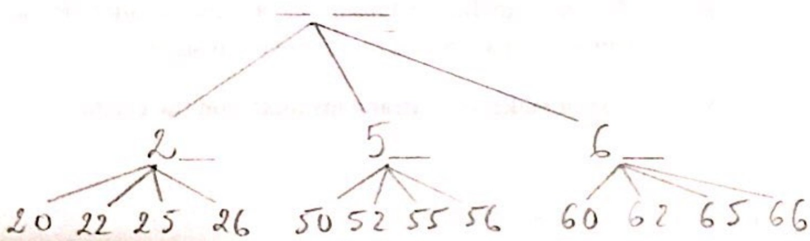
$= \{(2,2),(2,5),(2,6),(5,2),(5,5),(5,6),(6,2),(6,5),(6,6)\}$

(iv) $D \times C$

$= \{2,5,6\} \times \{0,2,5,6\}$

$= \begin{Bmatrix} (2,0),(2,2),(2,5),(2,6), \\ (5,0),(5,2),(5,5),(5,6), \\ (6,0),(6,2),(6,5),(6,6) \end{Bmatrix}$

Tree



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6 (1b) Compute the following, showing work on the facing page, and record only the answers on this page.

2 (i) $(A \cup B)\backslash(A \cap B)$

$= \{0, 3, 4, 5, 8\}$

2 (ii) $(A\backslash B) \cup (B\backslash A)$

$= \{0, 3, 4, 5, 8\}$

6

2 (iii) $\left((A^c) \cap B\right) \cup \left(A \cap (B^c)\right)$

$= \{0, 3, 4, 5, 8\}$

(i) $(A \cup B) \setminus (A \cap B)$

$= (\{0,6\} \cup \{3,4,5,6,8\}) \setminus (\{0,6\} \cap \{3,4,5,6,8\})$

$= \{0,3,4,5,6,8\} \setminus \{6\}$

$= \{0,3,4,5,8\}$

(ii) $(A \setminus B) \cup (B \setminus A)$

$= (\{0,6\} \setminus \{3,4,5,6,8\}) \cup (\{3,4,5,6,8\} \setminus \{0,6\})$

$= \{0\} \cup \{3,4,5,8\}$

$= \{0,3,4,5,8\}$

(iii) $((A^c) \cap B) \cup (A \cap (B^c))$

$= ((\mathcal{U} \setminus A) \cap B) \cup (A \cap (\mathcal{U} \setminus B))$

$= ((\{0,1,2,3,4,5,6,7,8,9\} \setminus \{0,6\}) \cap \{3,4,5,6,8\}) \cup (\{0,6\} \cap (\{0,1,2,3,4,5,6,7,8,9\} \setminus \{3,4,5,6,8\}))$

$= (\{1,2,3,4,5,7,8,9\} \cap \{3,4,5,6,8\}) \cup (\{0,6\} \cap \{0,1,2,7,9\})$

$= \{3,4,5,8\} \cup \{0\}$

$= \{0,3,4,5,8\}$

5 (1c) Prove or disprove: $\forall X, Y \in \mathcal{U}$

2 (i) $(X \cap Y)\times(Y \cap X) = (X\times Y) \cap (Y\times X)$

Pf

$$(a,b) \in (X \cap Y) \times (Y \cap X)$$
$$(a \in (X \cap Y)) \wedge (b \in (Y \cap X))$$
$$(a \in X) \wedge (a \in Y) \wedge (b \in Y) \wedge (b \in X)$$
$$((a \in X) \wedge (b \in Y)) \wedge ((a \in Y) \wedge (b \in X))$$
$$((a,b) \in (X \times Y)) \wedge ((a,b) \in (Y \times X))$$
$$(a,b) \in (X \times Y) \cap (Y \times X)$$

$$(a,b) \in (X \cap Y) \times (Y \cap X)$$
$$(a,b) \in (X \times Y) \cap (Y \times X)$$

$$\forall a,b \in \mathcal{U} \left(\frac{(a,b) \in (X \cap Y) \times (Y \cap X)}{(a,b) \in (X \times X) \cap (Y \times X)} \right)$$

$$(X \cap Y) \times (Y \cap X) = (X \times Y) \cap (Y \times X)$$

$\therefore$

3 (ii) $(X \cup Y)\setminus(X \cap Y) = (X\setminus Y) \cup \cancel{(X\setminus Y)} = ((X^c) \cap Y) \cup (X \cap (Y^c))$ $Y \setminus X$

Pf

$$a \in (X \cup Y) \setminus (X \cap Y)$$
$$(a \in (X \cup Y)) \wedge (a \notin (X \cap Y))$$
$$(a \in X \cup Y) \wedge ((a \notin X) \vee (a \notin Y))$$
$$((a \in X \cup Y) \wedge (a \notin X)) \vee ((a \in X \cup Y) \wedge (a \notin Y))$$
$$(a \in Y \wedge a \notin X) \vee (a \in X \wedge a \notin Y)$$
$$(a \in (Y \setminus X)) \vee (a \in (X \setminus Y))$$
$$a \in (X \setminus Y) \cup (Y \setminus X)$$
$$((a \in X) \wedge (a \in Y^c)) \vee ((a \in Y) \wedge (a \in X^c))$$
$$(a \in X \cap Y^c) \vee (a \in Y \cap X^c)$$
$$a \in ((X^c) \cap Y) \cup (X \cap (Y^c))$$

$$a \in (X \cup Y) \setminus (X \cap Y)$$
$$a \in (X \setminus Y) \cup (Y \setminus X)$$
$$a \in ((X^c) \cap Y) \cup (X \cap (Y^c))$$

$$\forall a \in \mathcal{U} \left(\frac{a \in (X \cup Y) \setminus (X \cap Y)}{\frac{a \in (X \setminus Y) \cup (Y \setminus X)}{a \in ((X^c) \cap Y) \cup (X \cap (Y^c))}} \right)$$

$$(X \cup Y) \setminus (X \cap Y) = (X \setminus Y) \cup (Y \setminus X) = ((X^c) \cap Y) \cup (X \cap (Y^c))$$

$\therefore$

0

6 (2a) My sets are:

$$U := \{x, \{x\}\} \qquad V := \{y, \{y\}\}$$

$$W := \{x, y, \{x, y\}\} \qquad X := \{x, y, \{x\}, \{y\}\}$$

$$Y := \{x, y, \{x\}, \{y\}, \{x, y\}\} \qquad Z := \{x, y, \{x\}, \{y\}, \{x, \{x\}\}, \{y, \{y\}\}\}$$

It is given that: $P, Q \in \{U, V, W, X, Y, Z\}$

Write three true sentences of the type: $P \in Q$ and

three true sentences of the type: $P \subset Q$ below:

You may not use the letters: $U, V, W, X, Y,$ and Z

1 (i) $(P := \{x, \{x\}\})$ $(Q := \{x, y, \{x\}, \{y\}, \{x, \{x\}\}, \{y, \{y\}\}\})$

$\{x, \{x\}\} \in \{x, y, \{x\}, \{y\}, \{x, \{x\}\}, \{y, \{y\}\}\})$

$P \in Q$

1 (ii) $(P := \{y, \{y\}\})$ $(Q := \{x, y, \{x\}, \{y\}, \{x, \{x\}\}, \{y, \{y\}\}\})$

$\{y, \{y\}\} \in \{x, y, \{x\}, \{y\}, \{x, \{x\}\}, \{y, \{y\}\}\})$

$P \in Q$

1 (iii) $(P := \{x, y, \{x, y\}\})$ $(Q := \{x, y, \{x\}, \{y\}, \{x, y\}\})$

$\{x, y, \{x, y\}\} \subset \{x, y, \{x\}, \{y\}, \{x, y\}\}$

$P \subset Q$

1 (iv) $(P := \{y, \{y\}\})$ $(Q := \{x, y, \{x\}, \{y\}, \{x, y\}\})$

$\{y, \{y\}\} \subset \{x, y, \{x\}, \{y\}, \{x, y\}\}$

$P \subset Q$

1 (v) $(P := \{x, y, \{x\}, \{y\}\})$ $(Q := \{x, y, \{x\}, \{y\}, \{x, y\}\})$

$\{x, y, \{x\}, \{y\}\} \subset \{x, y, \{x\}, \{y\}, \{x, y\}\}$

$P \subset Q$

1 (vi)

6

6 (2b) Compute the following, showing work on the facing page, and write only the answers on this page.

1 (i) $\mathcal{P}(A)$

$$= \left\{ \{\}, \{0\}, \{6\}, \{0,6\} \right\}$$

1 (ii) $\mathcal{P}(B)$

$$= \left\{ \begin{array}{l} \{\}, \\ \{3\}, \{4\}, \{5\}, \{6\}, \{8\}, \\ \{3,4\}, \{3,5\}, \{3,6\}, \{3,8\}, \{4,5\}, \{4,6\}, \{4,8\}, \{5,6\}, \{5,8\}, \{6,8\}, \\ \{3,4,5\}, \{3,4,6\}, \{3,4,8\}, \{3,5,6\}, \{3,5,8\}, \{3,6,8\}, \{4,5,6\}, \{4,5,8\}, \{4,6,8\}, \{5,6,8\}, \\ \{3,4,5,6\}, \{3,4,5,8\}, \{3,4,6,8\}, \{3,5,6,8\}, \{4,5,6,8\}, \\ \{3,4,5,6,8\} \end{array} \right\}$$

2 (i) $\mathcal{P}(A \setminus B)$

$$= \left\{ \{\}, \{0\} \right\}$$

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2 (ii) $\mathcal{P}(A) \setminus \mathcal{P}(B)$

$$= \left\{ \{0\}, \{0,6\} \right\}$$

1(i) $\mathcal{P}(A)$

$$= \mathcal{P}(\{0,6\})$$

$$= \left\{ \begin{array}{l} \{\}, \\ \{0\}, \{6\}, \\ \{0,6\} \end{array} \right\}$$

2(i) $\mathcal{P}(A \setminus B)$

$$= \mathcal{P}(\{0,6\} \setminus \{3,4,5,6,8\})$$

$$= \mathcal{P}(\{0\})$$

$$= \left\{ \begin{array}{l} \{\}, \\ \{0\} \end{array} \right\}$$

1(ii) $\mathcal{P}(B)$

$$= \mathcal{P}(\{3,4,5,6,8\})$$

$$= \left\{ \begin{array}{l} \{\}, \\ \{3\}, \{4\}, \{5\}, \{6\}, \{8\}, \\ \{3,4\}, \{3,5\}, \{3,6\}, \{3,8\}, \{4,5\}, \{4,6\}, \{4,8\}, \{5,6\}, \{5,8\}, \{6,8\}, \\ \{3,4,5\}, \{3,4,6\}, \{3,4,8\}, \{3,5,6\}, \{3,5,8\}, \{3,6,8\}, \{4,5,6\}, \{4,5,8\}, \{4,6,8\}, \{5,6,8\}, \\ \{3,4,5,6\}, \{3,4,5,8\}, \{3,4,6,8\}, \{3,5,6,8\}, \{4,5,6,8\}, \\ \{3,4,5,6,8\} \end{array} \right\}$$

2(ii) $\mathcal{P}(A) \setminus \mathcal{P}(B)$

$$= \left\{ \begin{array}{l} \{\}, \\ \{0\}, \{6\}, \\ \{0,6\} \end{array} \right\} \setminus \left\{ \begin{array}{l} \{\}, \\ \{3\}, \{4\}, \{5\}, \{6\}, \{8\}, \\ \{3,4\}, \{3,5\}, \{3,6\}, \{3,8\}, \{4,5\}, \{4,6\}, \{4,8\}, \{5,6\}, \{5,8\}, \{6,8\}, \\ \{3,4,5\}, \{3,4,6\}, \{3,4,8\}, \{3,5,6\}, \{3,5,8\}, \{3,6,8\}, \{4,5,6\}, \{4,5,8\}, \{4,6,8\}, \{5,6,8\}, \\ \{3,4,5,6\}, \{3,4,5,8\}, \{3,4,6,8\}, \{3,5,6,8\}, \{4,5,6,8\}, \\ \{3,4,5,6,8\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \{0\}, \\ \{0,6\} \end{array} \right\}$$

5 (2c) **Show work on facing side and record only the answers on this page.** Define the sequence of sets: $(S_n | n \in \mathbb{N})$ recursively as follows where X and Y are some unknown finite sets with:

$$x := \nu(X) \in \mathbb{N} \qquad y := \nu(Y) \in \mathbb{N} \qquad z := \nu(Z) \in \mathbb{N}$$

Base cases: (B_0) $S_0 := X$

(B_1) $S_1 := Y$

(B_2) $\cancel{S_1}$ $S_2 := Z$

Recursive step: (R) $S_{n+3} := (S_n \times S_{n+1}) \sqcup (S_{n+2})$

1 (i) Compute: S_3

$= (X \times Y) \sqcup Z$

1 (iii) Compute: $\nu(S_3)$

$= xy + z$

1 (ii) Compute: S_4

$= (Y \times Z) \sqcup ((X \times Y) \sqcup Z)$

1 (iv) Compute: $\nu(S_4)$

$= yz + xy + z$

4

1 (v) Find a formula for: $\nu(S_{n+3})$ in terms of n, x, y and z.

= (does not make sense)

(i) S_3

$= S_{0+3}$

$= (S_0 \times S_1) \amalg S_2$

$= (X \times Y) \amalg Z$

(iii) $\nu(S_3)$

$= \nu((X \times Y) \amalg Z)$

$= \nu(X) \times \nu(Y) + \nu(Z)$

$= xy + z$

(ii) S_4

$= S_{1+3}$

$= (S_1 \times S_2) \amalg S_3$

$= (Y \times Z) \amalg ((X \times Y) \amalg Z)$

(iv) $\nu(S_4)$

$= \nu((Y \times Z) \amalg ((X \times Y) \amalg Z))$

$= \nu(Y \times Z) + \nu((X \times Y) \amalg Z)$

$= \nu(Y) \times \nu(Z) + \nu(X) \times \nu(Y) + \nu(Z)$

$= yz + xy + z$

(v) $\nu(S_{n+3})$

$= \nu((S_n \times S_{n+1}) \amalg S_{n+2})$

$= \nu(S_n \times S_{n+1}) + \nu(S_{n+2})$

$= \nu(S_n) \times \nu(S_{n+1}) + \nu(S_{n+2})$

This is a kind of nonlinear recurrence relation, so it does not make sense in finding the formula.

6 (3a) Compute the following, showing work on the facing page, and record the answers below properly.

1 (i) $A \cap B$ $\qquad$ $\nu(A \cap B)$

$= \{6\}$ $\qquad$ $= 1$

1 (ii) $A \cap D$ $\qquad$ $\nu(A \cap D)$

$= \{6\}$ $\qquad$ $= 1$

1 (iii) $B \cap D$ $\qquad$ $\nu(B \cap D)$

$= \{5, 6\}$ $\qquad$ $= 2$

1 (iv) $A \cap B \cap D$ $\qquad$ $\nu(A \cap B \cap D)$

$= \{6\}$ $\qquad$ $= 1$

1 (v) $A \cup B \cup D$ $\qquad$ $\nu(A \cup B \cup D)$

$= \{0, 2, 3, 4, 5, 6, 8\}$ $\qquad$ $= 7$

1 (vi) Verify, by computing LS and RS separately on the facing page that:

$$\nu(A \cup B \cup D) = \nu(A) + \nu(B) + \nu(D) - \nu(A \cap B) - \nu(A \cap D) - \nu(B \cap D) + \nu(A \cap B \cap D)$$

LS

$= \nu(A \cup B \cup D)$

$= 7$

RS

$= \nu(A) + \nu(B) + \nu(D) - \nu(A \cap B) - \nu(A \cap D) - \nu(B \cap D) + \nu(A \cap B \cap D)$

$= 7.$

8

LS = 7 = RS

LS = RS

(i) $A \cap B$

$= \{0,6\} \cap \{3,4,5,6,8\}$

$= \{6\}$

$\nu(A \cap B)$

$= \underline{1}$

(ii) $A \cap D$

$= \{0,6\} \cap \{2,5,6\}$

$= \{6\}$

$\nu(A \cap D)$

$= \underline{1}$

(iii) $B \cap D$

$= \{3,4,5,6,8\} \cap \{2,5,6\}$

$= \{5,6\}$

$\nu(B \cap D)$

$= 2$

(iv) $A \cap B \cap D$

$= \{0,6\} \cap \{3,4,5,6,8\} \cap \{2,5,6\}$

$= \{6\}$

$\nu(A \cap B \cap D)$

$= \underline{1}$

(v) $A \cup B \cup D$

$= \{0,6\} \cup \{3,4,5,6,8\} \cup \{2,5,6\}$

$= \{0,2,3,4,5,6,8\}$

$\nu(A \cup B \cup D)$

$= 7$

(vi) LS

$= \nu(A \cup B \cup D)$

$= 7$

RS

$= \nu(A) + \nu(B) + \nu(D)$
$- \nu(A \cap B) - \nu(A \cap D) - \nu(B \cap D)$
$+ \nu(A \cap B \cap D)$

$= 2 + 5 + 3$
$- 1 - 1 - 2$
$+ 1$

$= 7$

6 (3b) Circle the correct choice on the right and prove your assertion accordingly: (1 : L8)

3 (i) $$Vld\left(\frac{X\subseteq Y}{X\times X \subseteq X\times Y}\right)$$ Y N

Pf

$$\frac{(a,b) \in (X\times X)}{\frac{(a,b \in X) \wedge (X \subseteq Y)}{\frac{(b \in Y) \wedge (a \in X)}{\frac{(a,b) \in (X\times Y)}{}}}}$$

$$\frac{\forall a,b \in U \left(\frac{((a,b) \in (X\times X)) \wedge (X \subseteq Y)}{(a,b) \in X\times Y}\right)}{X\times X \subseteq X\times Y}$$

$\mathcal{J}$9

3 (ii) $$Vld\left(\frac{X\subseteq Y}{X\times X \subseteq Y\times Y}\right)$$ Y N

0

Pf

$$\frac{(a,b) \in (X\times X)}{\frac{(a,b \in X) \wedge (X \subseteq Y)}{\frac{a,b \in Y}{\frac{(a,b) \in (Y\times Y)}{}}}}$$

$$\frac{\forall a,b \in U \left(\frac{((a,b) \in (X\times X)) \wedge (X \subseteq Y)}{(a,b) \in (Y\times Y)}\right)}{X\times X \subseteq Y\times Y}$$

$\mathcal{J}$9

5 (3c) **Circle the correct choice on the right and prove your assertion accordingly:**

$$\dot{V}ld\left(\frac{X\times X\subseteq Y\times Y}{X\subseteq Y}\right) \qquad Y \qquad N$$

Pf

$$\frac{a, b \in X}{((a,b) \in X \times X) \wedge (X \times X \subseteq Y \times Y)}$$

$$\frac{}{(a,b) \in Y \times Y}$$

$$\frac{}{a, b \in Y}$$

$$\frac{\forall a,b \in \mathcal{U}\left(\frac{(a,b \in X) \wedge (X \times X \subseteq Y \times Y)}{a,b \in Y}\right)}{X \subseteq Y}$$

$\sharp$

0