6 (1a) Find the truth-tables for:

2 (i) $(p \rightarrow q) \vee (q \rightarrow p)$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
0	0	1	1	1	1	1
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	1	1	1

4 (ii) $\Big(p \rightarrow q\Big) \rightarrow \Big(q \rightarrow r\Big)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \rightarrow (q \rightarrow r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

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6 (1b) Find truth-tables for the following expressions:

2 (i) $LS := \Big(q \to r\Big)$,

2 (ii) $RS := \Bigg(\Big(p \to q\Big) \to \Big(q \to r\Big)\Bigg)$, and

2 (iii) decide, if $\Big(q \to r\Big) \Rrightarrow \Bigg(\Big(p \to q\Big) \to \Big(q \to r\Big)\Bigg)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \to (q \to r)$	$LS \Rrightarrow RS$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

$\Big(q \to r\Big) = LS \Rrightarrow RS = \Bigg(\Big(p \to q\Big) \to \Big(q \to r\Big)\Bigg)$ Y (N)

Why? By inspection.

(1c)

P	q	r	$q \to r$	$p \to q$	$(p \to q) \to (q \to r)$	$(q \to r) \to ((p \to q) \to (q \to r))$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	0	1	0	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	1	1	1	1	1

$$\frac{\mathrm{Tlg}\left((q \to r) \to ((p \to q) \to (q \to r))\right)}{\mathrm{Vld}\left(\dfrac{q \to r}{(p \to q) \to (q \to r)}\right)}$$

5 (1c) Decide if:

$$Vld\left(\frac{q \rightarrow r}{(p \rightarrow q) \rightarrow (q \rightarrow r)}\right)$$

Y N

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6 (2a) Find $\varphi(p,q,r)$ in terms of p, q, r, and connectives, if $\varphi(p,q,r)$ is to have the following truth table:

p	q	r	$\varphi(p,q,r)$	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	0	0	0	0	0	0				
0	0	1	1	0	0	0	1				
0	1	0	1	0	0	1	0				
0	1	1	0	0	0	0	0				
1	0	0	1	0	1	0	0				
1	0	1	1	1	0	0	0				
1	1	0	0	0	0	0	0				
1	1	1	0	0	0	0	0				

$\varphi(p,q,r)$

$= \varphi_1$

$\vee \varphi_2$

$\vee \varphi_3$

$\vee \varphi_4$

$= (p \wedge (\neg q) \wedge r)$

$\vee (p \wedge (\neg q) \wedge (\neg r))$

$\vee ((\neg p) \wedge q \wedge (\neg r))$

$\vee ((\neg p) \wedge (\neg q) \wedge r)$

(i) LS

$= (p \to p) \to q$

$= \neg(p \to p) \vee q \quad (x \to y = (\neg x) \vee y)$

$= \neg((\neg p) \vee p) \vee q) \quad (x \to y = (\neg x) \vee y)$

$= (p \wedge (\neg p)) \vee q \quad (\neg(x \vee y) = (\neg x) \wedge (\neg y))$

$= \bot \vee q \quad (x \wedge (\neg x) = \bot)$

$= q \quad (\bot \vee x = x)$

$LS = q \neq T = RS$

$LS \neq RS$

(ii) LS

$= (p \to q) \to p$

$= \neg(p \to q) \vee p \quad (x \to y = (\neg x) \vee y)$

$= \neg((\neg p) \vee q) \wedge p) \quad (x \to y = (\neg x \vee y)$

$= (p \wedge (\neg q)) \vee p \quad (\neg(x \vee y) = (\neg x) \wedge (\neg y))$

$= (p \vee p) \wedge (p \wedge (\neg q)) \quad ((x \wedge y) \vee z = z \wedge (x \vee z))$

$= p \wedge p \wedge (\neg q) \quad (x \vee x = x)$

$= p \wedge (\neg q) \quad (x \wedge x = x)$

$LS = p \wedge (\neg q) \neq T = RS$

$LS \neq RS$

6 (2b) Show work on the facing side to decide, **using the algebra of propositions**, if the following hold and circle the appropriate choice:

1 (i) $\Big((p \to p) \to q\Big) = \top$ Y (N) (Circle the correct choice)

1 (ii) $\Big((p \to q) \to p\Big) = \top$ Y (N) (Circle the correct choice)

1 (iii) $\Big((q \to p) \to p\Big) = \top$ Y (N) (Circle the correct choice)

1 (iv) $\Big(p \to (p \to q)\Big) = \top$ Y (N) (Circle the correct choice)

1 (v) $\Big(p \to (q \to p)\Big) = \top$ (Y) N (Circle the correct choice)

1 (vi) $\Big(q \to (p \to p)\Big) = \top$ (Y) N (Circle the correct choice)

(iii) LS

$= (q \to p) \to p$

$= \neg(q \to p) \vee p \quad (x \to y = (\neg x) \vee y)$

$= \neg(\neg q \vee p) \vee p \quad (x \to y = (\neg x) \vee y)$

$= (q \wedge (\neg p)) \vee p \quad (\neg(x \vee y) = (\neg x) \wedge (\neg y))$

$= (p \vee q) \wedge (p \vee (\neg p)) \quad ((x \wedge y) \vee z = (x \vee z) \wedge (y \vee z))$

$= (p \wedge q) \wedge T \quad (x \vee (\neg x) = T)$

$= p \wedge q \quad (x \wedge T = x)$

$LS = p \wedge q \neq T = RS$

$LS \neq RS$

(iv) LS

$= p \to (p \to q)$

$= (\neg p) \vee (p \to q) \quad (x \to y = (\neg x) \vee y)$

$= (\neg p) \vee ((\neg p) \vee q) \quad (x \to y = (\neg x \vee y))$

$= (\neg p) \vee q \quad (x \vee (x \vee y) = x \vee y)$

$LS = (\neg p) \vee q \neq T = RS$

$LS \neq RS$

(v) LS

$= p \to (q \to p)$

$= (\neg p) \vee (q \to p) \quad (x \to y = (\neg x) \vee y)$

$= (\neg p) \vee ((\neg q) \vee p) \quad (x \to y = (\neg x) \vee y)$

$= \big((\neg p) \vee (\neg q)\big) \vee (p \vee (\neg p)) \quad \big(x \vee (y \vee z) = (x \vee y) \vee (x \vee z)\big)$

$= \big((\neg p) \vee (\neg q)\big) \vee T \quad (x \vee (\neg x) = T)$

$= T \quad (x \vee T = T)$

$LS = T = T = RS$

$LS = RS$

(vi) LS

$= q \to (p \to p)$

$= (\neg q) \vee (p \to p) \quad (x \to y = (\neg x) \vee y)$

$= (\neg q) \vee ((\neg p) \vee p) \quad (x \to y = (\neg x) \vee y)$

$= (\neg q) \vee T \quad (x \vee (\neg x) = T)$

$= T \quad (x \vee T = T)$

$LS = T = T = RS$

$LS = RS$

2c) ①

$$\text{Vld}\left(\frac{p\to\bot,\ \bot\to q}{p\to q}\right)=\left(\left(\left((p\to\bot)\wedge(\bot\to q)\right)\to(p\to q)\right)=\top\right)$$

---

① LS

$= \left((p\to\bot)\wedge(\bot\to q)\right)\to(p\to q)$

$= \neg\left((p\to\bot)\wedge(\bot\to q)\right)\vee(p\to q)$ $\quad (x\to y = (\neg x)\vee y)$

$= \neg\left((\neg p)\vee\bot)\wedge((\neg\bot)\vee q)\right)\vee((\neg p)\vee q)$ $\quad (x\to y=(\neg x)\vee y)$

$= \neg\left((\neg p)\wedge((\neg\bot)\vee q)\right)\vee((\neg p)\vee q)$ $\quad (x\vee\bot = x)$

$= \neg\left((\neg p)\wedge(\top\vee q)\right)\vee((\neg p)\vee q)$ $\quad (\neg\bot=\top)$

$= \neg((\neg p)\wedge\top)\vee((\neg p)\vee q)$ $\quad (\top\vee x=\top)$

$= (p\vee\bot)\vee((\neg p)\vee q)$ $\quad (\neg(x\wedge\top)=(\neg x)\vee\bot)$

$= p\vee((\neg p)\vee q)$ $\quad (x\vee\bot=x)$

$= (p\vee(\neg p))\vee(p\vee q)$ $\quad (x\vee(y\vee z)=(x\vee y)\vee(x\vee z))$

$= \top\vee(p\vee q)$ $\quad (x\vee(\neg x)=\top)$

$= \top$ $\quad (\top\vee x=\top)$

$LS = \top = \top = RS$

$LS = RS$

5 (2c) Show work on the facing side to decide, **using the algebra of propositions**, if the following arguments are valid and circle the appropriate choice:

$$Vld\left(\frac{p\to\bot \quad \bot\to q}{p\to q}\right)$$

(Y) N

(2c) (2) $\Big(\big((p\to\bot)\wedge(\bot\to q)\big)\to(p\to q)\Big) = \top$

$$Vld\left(\frac{p\to\bot,\ \bot\to q}{p\to q}\right)$$

6

(i)

p	q	$p \to p$	$(p \to p) \to q$
0	0	1	0
0	1	1	1
1	0	1	0
1	1	1	1

Cng $((p \to p) \to q)$

(ii)

p	q	$p \to q$	$(p \to q) \to p$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

Cng $((p \to q) \to p)$

6 (3a) Show work on the facing side to decide if the following proposition is a tautology, contradiction, or contingency, and circle an appropriate answer.

1 (i) $\Big((p \to p) \to q\Big)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (ii) $\Big((p \to q) \to p\Big)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (iii) $\Big((q \to p) \to p\Big)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (iv) $\Big(p \to (p \to q)\Big)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (v) $\Big(p \to (q \to p)\Big)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$ 6

1 (vi) $\Big(q \to (p \to p)\Big)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

(iii)

P	q	$q \to P$	$(q \to p) \to P$
0	0	1	0
0	1	0	1
1	0	1	1
1	1	1	1

$Cng\left((q \to p) \to p\right)$

(iv)

P	q	$p \to q$	$p \to (p \to q)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

$Cng\left(p \to (p \to q)\right)$

(v)

P	q	$q \to P$	$P \to (q \to P)$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	1	1

Tlg $(P \to (q \to P))$

(vi)

P	q	$(P \to P)$	$q \to (P \to P)$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	1	1

Tlg $(q \to (P \to P))$

(3b) ②

$$\mathrm{Vld}\left(\frac{p\to\bot,\ \bot\to q}{p\to q}\right) = \mathrm{Vld}\Big(\big((p\to\bot)\wedge(\bot\to q)\big)\to(p\to q)\Big)$$

(3b) ①

$$\big((p\to\bot)\wedge(\bot\to q)\big)$$

$$\to \Big(\big((r\vee p)\to(r\vee\bot)\big)\wedge(\bot\to q)\Big) \quad \big((p\to q)\to(r\vee p)\to(r\vee q)\big)$$

$$\to \Big(\big((r\vee p)\to(r\vee\bot)\big)\wedge\big((r\vee\bot)\to(r\vee q)\big)\Big) \quad \big((p\to q)\to(r\vee p)\to(r\vee q)\big)$$

$$= \Big(\big((r\vee p)\to r\big)\wedge\big(r\to(r\vee q)\big)\Big) \quad (p\vee\bot = p)$$

$$\to \big((r\vee p)\to(r\vee q)\big) \quad \big(\big((p\to q)\wedge(q\to r)\big)\to(p\to r)\big)$$

$$\to \big((p\vee r)\to(r\vee q)\big) \quad \big((p\vee q)\to(q\vee p)\big)$$

$$\to (p\to q) \quad \big(\big((p\vee q)\to(q\vee r)\big)\to(p\to r)\big)$$

$$\frac{\big((p\to\bot)\wedge(\bot\to q)\big)\to(p\to q)}{\mathrm{Vld}\Big(\big((p\to\bot)\wedge(\bot\to q)\big)\to(p\to q)\Big)}$$

$$\mathrm{Vld}\left(\frac{p\to\bot,\ \bot\to q}{p\to q}\right)$$

6 (3b) Show work on the facing side to decide, **using axioms and rules of inference**, if the following arguments are valid and circle the appropriate choice:

$$Vld\left(\frac{p \rightarrow \bot \quad \bot \rightarrow q}{p \rightarrow q}\right)$$

Y N

(3c)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$((p \to q) \wedge (q \to r)) \to (p \to q)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

$$\frac{Tlg\left((p \to q) \wedge (q \to r)\right) \to (p \to q)}{Vld\left(\dfrac{p \to q,\ q \to r}{p \to q}\right)}$$

5 (3c) Decide if:

$$Vld\left(\frac{p \rightarrow q \qquad q \rightarrow r}{p \rightarrow q}\right)$$

(Y) N

Same method as allowed.

6