6 (1a) Find the truth-tables for:

2 (i) 
$$((\neg p) \rightarrow (\neg q)) \rightarrow p$$

p	q	¬р	$\neg q$	$(\neg p) \rightarrow (\neg q)$	$((\neg p) \to (\neg q)) \to p$
0	0	1	1	1	0
0	1	1	0	0	1 1
1	0	0	1	1	1
1	1	0	0	1	1

4 (ii) 
$$(p \rightarrow (\neg r)) \land (q \rightarrow r)$$

p	q	r	$\neg r$	$p \rightarrow (\neg r)$	$q \rightarrow r$	$(p \to (\neg r)) \land (q \to r)$
0	0	0	1	1	1	1
0	0	1	0	. 1	1	. 1
0	1	0	1	Λ	0	0
0	1	1	0	1	1	7 × 1
1	0	0	1	1	1	4
1	0	1	Q	0	4	0
What						
l	1	0	1	1	0	0
	1	1	O	0	1	0

6 (1b) Find truth-tables for the following expressions:

$$2 \text{ (i)} \quad \textit{LS} \coloneqq (p \lor q) \longrightarrow r,$$

2 (ii) 
$$RS \coloneqq (p \rightarrow r) \lor (q \rightarrow r)$$
, and

2 (iii) decide, if 
$$(p \lor q) \rightarrow r \Rightarrow (p \rightarrow r) \lor (q \rightarrow r)$$

[	p	q	r	$p \lor q$	$p \rightarrow r$	$q \rightarrow r$	$(p \lor q) \longrightarrow r$	$(p \to r) \lor (q \to r)$	$LS \rightarrow$	RS
L										1
0	0	)	0	0	1	A	1	1	1	
0	0	1	1	0	1	1	1	1	1	
0	1	10		4	1	0	0	1	1	
0	1	1	$\perp$	1	1	4	1	. 1	1	
		$\perp$	$\perp$							
	0	0		1	0	Λ	0	1	1	
	0	1	$\vdash$	1	Λ	1	1	1	1	
1	1	0	$\vdash$	1	0	0	0	0	1	
7	1	1		1	1	4	1	1	1	

$$(p \lor q) \to r \Rrightarrow (p \to r) \lor (q \to r)$$

 $\widehat{\mathbf{Y}}$  N

Why?

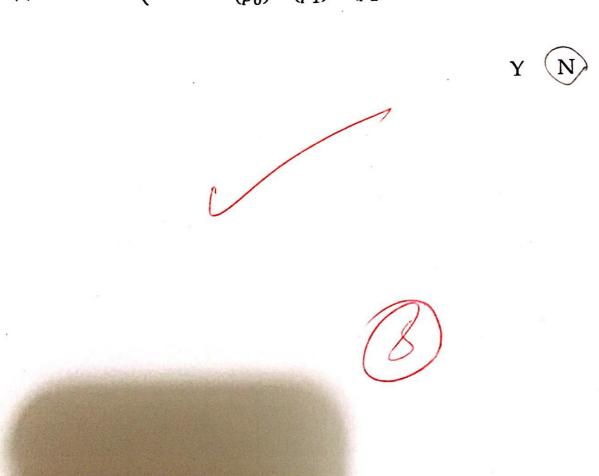






$$2 \text{ (i)} \qquad Vld\left(\frac{(p_0) \rightarrow (p_1) \quad (p_1) \rightarrow (p_0)}{(p_0) \leftrightarrow (p_1)}\right) \qquad \text{Y} \qquad \text{N}$$

$$3 \text{ (ii)} \qquad Vld\left(\frac{(p_0) \rightarrow (p_1) \quad (p_1) \rightarrow (p_2) \quad (p_1) \rightarrow (p_0)}{(p_0) \leftarrow (p_1) \leftarrow (p_2)}\right)$$



(i)	(f,)	(Po) -> (P.)	$(P_{\bullet}) \rightarrow (P_{\bullet})$	$(P_{\bullet})\rightarrow (P_{\bullet})$ $\rangle \wedge ((P_{\bullet})\rightarrow (P_{\bullet}))$	(P)~(P)	(P) (P)	((e) -10)) ((a)
0	0	,	1	1	1	1	(1) -> (10) -> ( (Seal))
0	1	1	0	0	0	1	
1	0	σ	1	0	0		
1	1	1	1	1	1	1	

$$VId\left(\frac{(P_0) \rightarrow (P_1)}{(P_0) \leftrightarrow (P_1)} \rightarrow (P_0)\right)$$

$$\begin{array}{ll} (Ci) & LS := ((P_0) \rightarrow (P_1)) \land ((P_1) \rightarrow (P_1)) \land ((P_1) \rightarrow (P_0))) \\ RS := ((P_0) \leftrightarrow (P_1) \leftrightarrow (P_1)) \end{array}$$

25										
1000000	P.	(P,)	(P2)	(Po)-1P.)	$(P_{\bullet}) \rightarrow (P_{\epsilon})$	(P1) -> (Ps)	LS	RS	LS - RS	
	0	0	0	1	1	1	1	1	(1) -> K>	
	0	0	1	1	1	1	1	10	1	
The second	ç	1	0	1	0	0	0	0	N	
	0	1	1	1	1	0	0	0	1	1
Philogopy No.	1	O	0	0	1	1	0	0	1	
	1	0	1	0	1	1	0	0	4	
	1	1	0	1	0		0	0	,	
	1	1	1	1	1	1	1	1	,	

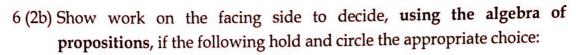
$$TYG\left(\begin{pmatrix} (P_0) \rightarrow (P_1) \end{pmatrix} \land \begin{pmatrix} (P_1) \rightarrow (P_2) \end{pmatrix} \land \begin{pmatrix} (P_1) \rightarrow (P_2) \end{pmatrix} \right)$$

$$YG\left(\frac{(P_0) \rightarrow (P_1)}{(P_0) \rightarrow (P_1)} \begin{pmatrix} (P_1) \rightarrow (P_2) \end{pmatrix} \begin{pmatrix} (P_0) \rightarrow (P_1) \end{pmatrix} \begin{pmatrix} (P_0) \rightarrow (P_1$$

6 (2a) Find  $\varphi(p,q,r)$  in terms of p,q,r, and connectives, if  $\varphi(p,q,r)$  is to have the following truth table:

p		r	$\varphi(p,q,r)$	$\varphi_1$	$\varphi_2$	$\varphi_3$	$\varphi_4$	$\varphi_5$	$\varphi_6$	$\varphi_7$	$\varphi_8$
Р	q	+	Ψφιαιί	71	1						
0	0	0	1	0	0	0	1				
0	0	1	0	0	. 0	0	0				-
				0	0	0	0				-
0	1	0	0		-	0					
0	1	1	1	0	0	1	0				
1	0	0	0	0	0	0	0				
1	0	1	1	0	1	0	0				-
											-
1	1	0	1	1	0	0	G				
1	1	1	0	0	0	0	0				

 $\varphi(p,q,r)$ 



- 1 (i)  $((\bot \rightarrow p) \rightarrow q) = \top$
- Y
- (Circle the correct choice)

1 (i) 
$$((p \rightarrow \bot) \rightarrow q) = \top$$

(N)

(Circle the correct choice)

1 (i) 
$$((p \rightarrow q) \rightarrow \bot) = T$$

Y

(Circle the correct choice) (N)

1 (i) 
$$(1 \rightarrow (p \rightarrow q)) = T$$

(Y)

(Circle the correct choice) N

1 (i) 
$$(p \rightarrow (\bot \rightarrow q)) = T$$

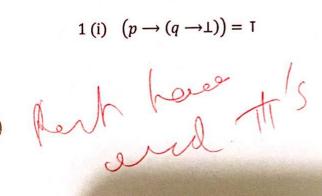
(Y)

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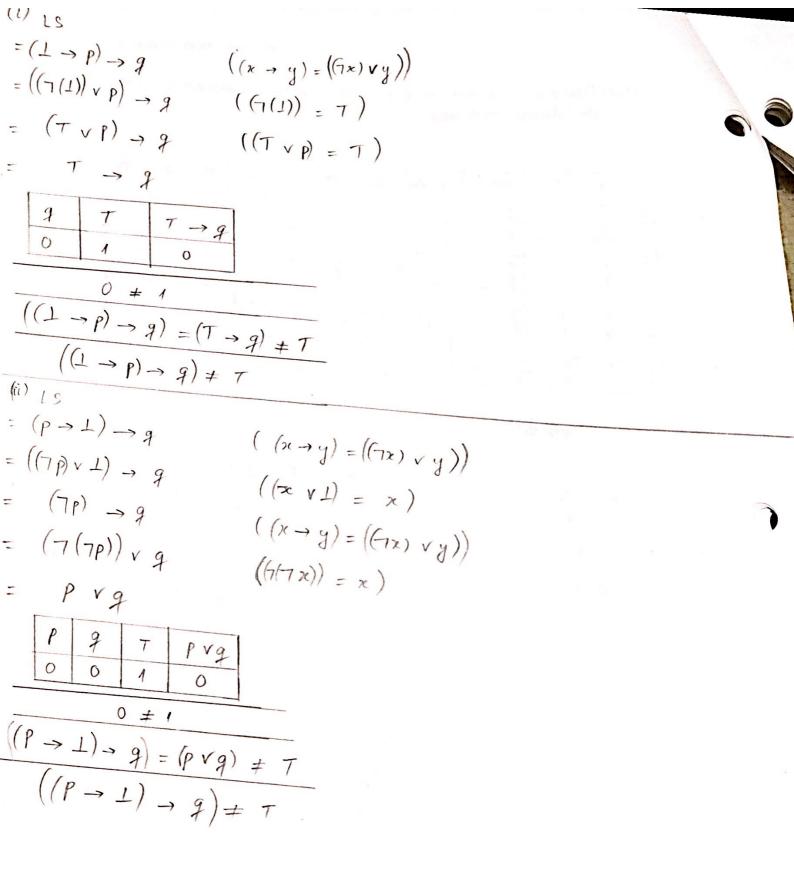
(Circle the correct choice)

1 (i) 
$$(p \rightarrow (q \rightarrow \bot)) =$$

(Circle the correct choice)







 $\frac{(P \rightarrow (1 \rightarrow g)) = 7}{(11 \downarrow 5)}$ 

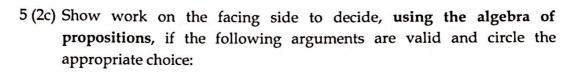
$$= p \rightarrow (q \rightarrow 1)$$

$$= (7P) \vee (g \rightarrow 1)$$

((x	-y)	= (7)	x) v	y	>)
		1		0	

$$\frac{0 \neq 1}{(P \rightarrow (q \rightarrow 1)) = (7p) \vee (7q)) + T}$$

$$\begin{array}{lll}
(\partial D) & \leq \\
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& = (T \circ P) \vee A \to \bot \\$$



$$Vld\left(\frac{(p \lor q) \to r}{(p \to r) \lor (q \to r)}\right) \qquad \qquad (Y) \qquad \qquad N$$

$$= RS$$

$$((P \vee g) \rightarrow r) \rightarrow ((P \rightarrow r) \vee (q \rightarrow r)) = T$$

$$Tig(((p \vee q) \rightarrow r) \rightarrow ((p \rightarrow r) \vee (q \rightarrow r)))$$

$$Vld\left(\frac{(\rho \vee g) \to r}{(\rho \to r) \vee (g \to r)}\right)$$



$$\frac{(2i) \vee 10!}{(p \to r) \vee (q \to r)} (e)! (p \vee q) \to r) \to ((p \to r) \vee (q \to r)) = T$$

$$= ((p \vee q) \to r) \to ((p \to r) \vee (q \to r)) \qquad (x \to y = f \to x) \vee y)$$

$$= ((p \vee q) \to r) \to ((f \to p) \vee r) \vee (f \to q)) \qquad (x \vee y = y \vee x)$$

$$= ((p \vee q) \to r) \to ((f \to p) \vee r) \vee (r \vee r) \vee (f \to q)) \qquad (x \vee y = y \vee x)$$

$$= ((p \vee q) \to r) \to ((f \to p) \vee (r \vee r) \vee f \to q)) \qquad (x \vee y = x \vee y = y \vee x)$$

$$= ((p \vee q) \to r) \to ((f \to p) \vee (f \to q)) \vee r) \qquad (x \to y = (f \to x) \vee y)$$

$$= (f((p \vee q)) \vee r) \to (((f \to p) \vee (f \to q)) \vee r) \qquad (x \to y = (f \to x) \vee y)$$

$$= (f((p \vee q)) \vee r) \vee ((f \to p) \vee (f \to q)) \vee r) \qquad (x \to y = (f \to x) \vee y)$$

$$= ((f((p \vee q)) \vee r) \vee ((f \to p) \vee (f \to q)) \vee r) \qquad (x \to y = y \vee x)$$

$$= ((f((p \vee q)) \vee r) \vee ((f \to p) \vee (f \to q)) \vee r) \qquad (x \to y = y \vee x)$$

$$= ((f((p \vee q)) \vee (f \to y)) \vee ((f \to p) \vee (f \to q)) \vee r) \qquad (x \vee y = y \vee x)$$

$$= ((f((p \vee q)) \vee (f \to x)) \vee ((f \to p) \vee (f \to y)) \vee r) \qquad (x \vee y = y \vee x)$$

$$= ((f((p \vee r)) \vee (f \to x)) \vee ((f \to p) \vee (f \to y)) \vee r) \qquad (x \vee y = y \vee x)$$

$$= ((f((p \vee r)) \vee (f \to x)) \vee ((f \to p) \vee r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f \to x) \wedge r)) \vee ((f((f \to y) \vee r)) \vee (f((f \to x) \wedge r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f \to x) \wedge r)) \vee ((f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f \to x) \wedge r)) \vee ((f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f \to x) \wedge r)) \vee ((f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f \to x) \wedge r)) \vee ((f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \qquad (x \vee y = y \vee x)$$

$$= (f((f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \vee (f((f \to x) \wedge r)) \qquad (f((f \to x) \wedge$$

6 (3a) Show work on the facing side to decide if the following proposition is a tautology, contradiction, or contingency, and circle an appropriate answer.

1 (i) 
$$((\bot \rightarrow p) \rightarrow q)$$

$$Tlg(\psi)$$

$$Cdn(\psi)$$



1 (ii) 
$$((p \rightarrow \bot) \rightarrow q)$$

$$Tlg(\psi)$$

$$Cdn(\psi)$$



1 (iii) 
$$((p \rightarrow q) \rightarrow \bot)$$

$$Cdn(\psi)$$

$$Cng(\psi)$$

1 (iv) 
$$(1 \rightarrow (p \rightarrow q))$$

$$Tlg(\psi)$$

 $Cdn(\psi)$ 

 $Cng(\psi)$ 

1 (v) 
$$(p \rightarrow (\bot \rightarrow q))$$

$$Tlg(\psi)$$

 $Cdn(\psi)$ 

 $Cng(\psi)$ 



1 (vi) 
$$(p \rightarrow (q \rightarrow \bot))$$

$$Tlg(\psi)$$

$$Cdn(\psi)$$



(1)					(4)						
P	9	1	$\perp \rightarrow \rho$	$(1 \rightarrow p) \rightarrow g$	P	9	1	1	79	$\rho \rightarrow (\perp \rightarrow g)$	
0	0	0	1	OT	0	0	0		1	1	
0	1	0	1	1	0	0	0		1	1.	
1	0	0	1			1	0		1	1/4	•
1	1	0	1		-		1				
(ii)					(vi`	) .					
P	9	1	P -> 1	$(P \rightarrow \bot) \rightarrow g$	P	9		4	9 -> 1	$P \rightarrow (g \rightarrow$	1)
0	O	0	1	0	10		(	5	4	1	
0	1	0	1	1	110	1	1	C	0	1	
1	0	0	0		111	0	1	0	4	1	
1	1	0	0		. [[		1	0	0		-
(i ii)					7						

6 (29) Show work on the facing side to decide, using axioms and rules of inference, if the following arguments are valid and circle the appropriate choice:

$$Vld\left(\frac{(p \vee q) \to r}{(p \to r) \vee (q \to r)}\right) \qquad \qquad Y \qquad \qquad N$$

5 (3c) Prove or disprove:  $\forall n \in \mathbb{N}$   $Vld\left(\frac{(p_0) \rightarrow (p_1) \quad (p_1) \rightarrow (p_2) \quad \dots \quad (p_{n-1}) \rightarrow (p_n) \quad (p_n) \rightarrow (p_0)}{\forall k, l \in 0 \dots n \quad \left((p_k) \leftrightarrow (p_l)\right)}\right)$ 

Y N

