Instructions:

Use these instructions for the remainder of the exam.

Your J-number $JN :=$ 00266502 $:= (J_k | k \in 1..8)$

Hence

$J_1 =$ 0 $J_2 =$ 0 $J_3 =$ 2 $J_4 =$ 6

$J_5 =$ 6 $J_6 =$ 5 $J_7 =$ 0 $J_8 =$ 2

R For each of sets defined below write the elements in increasing order

D Define set $\mathcal{U} := 0..9$

$= \{0,1,2,3,4,5,6,7,8,9\}$

D Define set $J := \{J_k \in 0..9 | k \in 1..8\}$

$= \{ 0, 2, 5, 6 \}$

D Define set $E := \{1\} \cup \{J_k \in 0..9 | Evn(J_k)\}$

$= \{ 0, 1, 2, 6 \}$

D Define set $A := \{k \in 1..8 | (2|J_k) \vee (3|J_k)\}$

$= \{ 1, 2, 3, 4, 5, 7, 8 \}$

D Define set $B := \{k \in 1..8 | J_k = 0\}$

$= \{ 1, 2, 7 \}$

Your J-number $JN :=$ 00 266 502

We define $\forall k \in 1..8$,

D $J_k :=$ the k^{th} entry in your J-number counting from the left.

D $r_k :=$ the remainder obtained upon dividing J_k by 5.

Therefore,

$J_1 :=$ 0

$J_2 :=$ 0

$J_3 :=$ 2

$J_4 :=$ 6

$J_5 :=$ 6

$J_6 :=$ 5

$J_7 :=$ 0

$J_8 :=$ 2

Show all long divisions on the facing page

$r_1 :=$ 0

$r_2 :=$ 0

$r_3 :=$ 2

$r_4 :=$ 1

$r_5 :=$ 1

$r_6 :=$ 0

$r_7 :=$ 0

$r_8 :=$ 2

$$r_1 := 5\begin{array}{l}0\\ \overline{)0}\\ \underline{0}\\ 0\end{array} = 0$$

$$r_2 := 5\begin{array}{l}0\\ \overline{)0}\\ \underline{0}\\ 0\end{array} = 0$$

$$r_3 := 5\begin{array}{l}0\\ \overline{)2}\\ \underline{0}\\ 2\end{array} = 2$$

$$r_4 := 5\begin{array}{l}1\\ \overline{)6}\\ \underline{5}\\ 1\end{array} = 1$$

$$r_5 := 5\begin{array}{l}1\\ \overline{)6}\\ \underline{5}\\ 1\end{array} = 1$$

$$r_6 := 5\begin{array}{l}1\\ \overline{)5}\\ \underline{5}\\ 0\end{array} = 0$$

$$r_7 := 5\begin{array}{l}0\\ \overline{)0}\\ \underline{0}\\ 0\end{array} = 0$$

$$r_8 := 5\begin{array}{l}0\\ \overline{)2}\\ \underline{0}\\ 2\end{array} = 2$$

[illegible]

D Define a relation $R \subseteq \{0,1,2,3,4\} \times \{0,1,2,3,4\} =: \mathcal{U}$ as follows:

Compute *R* on the facing side, and write the answer below:

$$R \quad := \bigcup_{k \in 1..4}\{(r_{9-2k}, r_{2k})\}$$

$$= \{(0,0),\ (0,2),\ (1,1),\ (2,0)\}$$

R In order to finish this exam you need to learn to produce the incidence matrix or a relation and need to learn how to multiply matrices. The book and the internet contains information on both items but does not use the word incidence.

You will need the following definitions.

D Given and incidence matrix for a relation $M(R)$ we define:

$$(M(R))(0,1)$$

$$:= \quad \textit{Matrix obtained by replacing all non-zero entries in } M(R) \textit{ by } 1$$

R **Note that in a set an element may occur exactly once.** Therefore there may not be any repeated elements in either A or B in the above. **The sets *A* and *B* will occur in several problems on this test. So, if you do not have the correct sets, all your answers will be wrong.**

R **For arrow-diagrams put the numbers in rectangles rather than ovals and draw the rectangles using rulers.**

R **Any sloppiness, untidiness, and any departure from proper format (as indicated in class) will lead to a score of 0.**

R **Do not write anything down on any page if you cannot completely solve the problem or some part thereof that is indexed by a roman numeral.**

$$R := \bigcup_{k \in 1..4} \{(r_{9-2k}, r_{2k})\}$$

$$= \{(r_{9-2(1)}, r_{2(1)})\} \cup \{(r_{9-2(2)}, r_{2(2)})\} \cup \{(r_{9-2(3)}, r_{2(3)})\} \cup \{(r_{9-2(4)}, r_{2(4)})\}$$

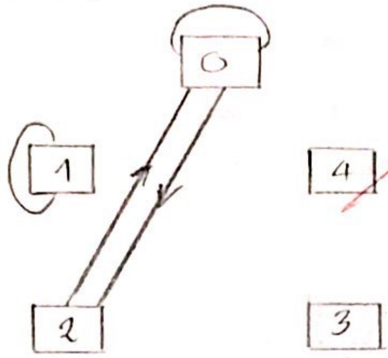
$$= \{(r_7, r_2)\} \cup \{(r_5, r_4)\} \cup \{(r_3, r_6)\} \cup \{(r_1, r_8)\}$$

$$= \{(0, 0)\} \cup \{(1, 1)\} \cup \{(2, 0)\} \cup \{(0, 2)\}$$

$$= \{(0,0), (0,2), (1,1), (2,0)\}$$

17 (1abc) Carry out the following instructions

1 (i) Draw directed graph for the relation R in the above with 0, 1, 2, 3, 4 as nodes **arranged in the shape of a regular pentagon**, with the top vertex labelled 0, and labelling all the vertices counter-clockwise.



DG(R)

1 (ii) Find the 5× 5 incidence matrix $M(R)$ for the relation R.

$$M(R) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1 (iii) Find the 5× 5 incidence matrix $M(R^{op})$ for the relation R^{op}.

$$M(R^{op}) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3

(ii)

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0

$$M(R) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii) $R^{OP} := \{(0,0), (0,2), (1,1), (2,0)\}$

(ii)

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0

$$M(R^{op}) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

5 (iv) Compute: on the **facing side**:

$$R \circ R^{op}, M(R \circ R^{op}), (M(R))(M(R^{op}))$$

Prove on **this side** that:

$$\cancel{(M(R \circ R^{op}))(0,1) = ((M(R))(M(R^{op})))(0,1)}$$

$$(M(R \circ R^{op}))(0,1) = ((M(R^{op}))(M(R)))(0,1)$$

$$(M(R^{op} \circ R))(0,1) = ((M(R))(M(R^{op})))(0,1)$$

$(M(R \circ R^{op}))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$= ((M(R^{op}))(M(R)))(0,1)$

$(M(R \circ R^{op}))(0,1) = ((M(R^{op}))(M(R)))(0,1)$

$(M(R^{op} \circ R))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

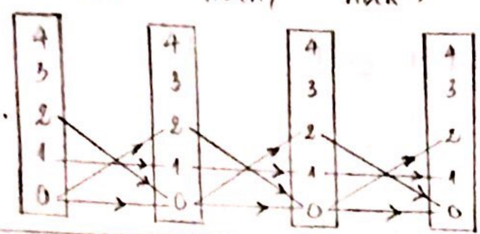
$= ((M(R))(M(R^{op})))(0,1)$

$(M(R^{op} \circ R))(0,1) = ((M(R))(M(R^{op})))(0,1)$

8

(v)

$AD(R^{op})$ $AD(R)$ $AD(R^{op})$



(iv) find $M(R \circ R^{op})$

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	0	0
4	0	0	0	0	0

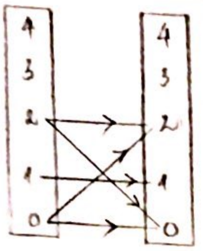
$$M(R \circ R^{op}) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iv) find $M(R^{op} \circ R)$

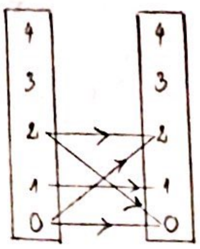
	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	0	0
4	0	0	0	0	0

$$M(R^{op} \circ R) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iv)



$AD(R \circ R^{op})$



$AD(R^{op} \circ R)$

(iv) $R \circ R^{op} = \{(0,0), (0,2), (1,1), (2,0) (2,2)\}$

(iv) $R^{op} \circ R = \{(0,0), (0,2), (1,1), (2,0), (2,2)\}$

(iv) $(M(R))(M(R^{op}))$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0(0)+1(1)+0(0)+0(0) & 1(0)+0(1)+1(0)+0(0)+0(0) & 1(1)+0(0)+1(0)+0(0)+0(0) & 1(0)+0(0)+1(0)+0(0)+0(0) & 1(0)+0(0)+1(0)+0(0)+0(0) \\ 0(1)+1(0)+0(1)+0(0)+0(0) & 0(0)+1(1)+0(0)+0(0)+0(0) & 0(1)+1(0)+0(0)+0(0)+0(0) & 0(0)+1(0)+0(0)+0(0)+0(0) & 0(0)+1(0)+0(0)+0(0)+0(0) \\ 1(1)+0(0)+0(1)+0(0)+0(0) & 1(0)+0(1)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) \\ 0(1)+0(0)+0(1)+0(0)+0(0) & 0(0)+0(1)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \\ 0(1)+0(0)+0(1)+0(0)+0(0) & 0(0)+0(1)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iv) $(M(R^{op}))(M(R))$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0(0)+1(1)+0(0)+0(0) & 1(0)+0(1)+1(0)+0(0)+0(0) & 1(1)+0(0)+1(0)+0(0)+0(0) & 1(0)+0(0)+1(0)+0(0)+0(0) & 1(0)+0(0)+1(0)+0(0)+0(0) \\ 0(1)+1(0)+0(1)+0(0)+0(0) & 0(0)+1(1)+0(0)+0(0)+0(0) & 0(1)+1(0)+0(0)+0(0)+0(0) & 0(0)+1(0)+0(0)+0(0)+0(0) & 0(0)+1(0)+0(0)+0(0)+0(0) \\ 1(1)+0(0)+0(1)+0(0)+0(0) & 1(0)+0(1)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) \\ 0(1)+0(0)+0(1)+0(0)+0(0) & 0(0)+0(1)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \\ 0(1)+0(0)+0(1)+0(0)+0(0) & 0(0)+0(1)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (v) On the **facing side**

draw: $DG(R)$, $DG(RflxClsr(R))$ and

find: $M(RflxClsr(R))$, $M\big(R \cup (\Delta_{0..4})\big)$

Prove on **this side** that:

$$\big(M(RflxClsr(R))\big)(0,1) = \Big(M\big(R \cup (\Delta_{0..4})\big)\Big)(0,1)$$

RS

$= \big(M(R \cup (\Delta_{0..4}))\big)(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

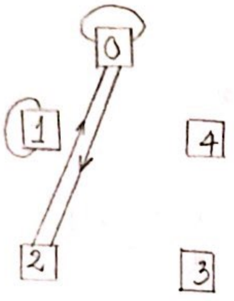
$= \big(M(RflxClsr(R))\big)(0,1)$

$= LS$

$LS = RS$

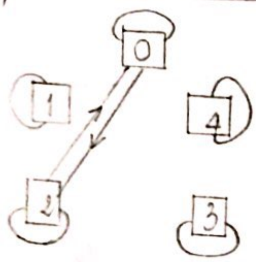
1

(iv)



$DG(R)$

(v)



$DG(Rflx\,Clsr(R))$

(v) Find $M(Rflx\,Clsr(R))$

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	1

$$M(Rflx\,Clsr(R)) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(v) $R \cup (\Delta_{0..4})$

$= \{(0,0),(0,2),(1,1),(2,0)\} \cup \{(0,0),(1,1),(2,2),(3,3),(4,4)\}$

$= \{(0,0),(0,2),(1,1),(2,0),(2,2),(3,3),(4,4)\}$

(v) Find $M(R \cup (\Delta_{0..4}))$

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	1

$$M(R \cup (\Delta_{0..4})) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(v) $(M(Rflx\,Clsr(R)))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(v) $(M(R \cup (\Delta_{0..4})))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (vi) On the **facing side**

draw: $DG(R)$, $DG(SymClsr(R))$ and

find: $M(SymClsr(R))$, $M\big(R \cup (R^{op})\big)$

Prove on **this side** that:

$$\cancel{\big(M(SymCls(rR))\big)}(0,1) = \Big(M\big(R \cup (R^{op})\big)\Big)(0,1)$$

$(M(SymClsr(R)))$

RS

$= \big(M(R \cup (R^{op}))\big)(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$= \big(M(SymClsr(R))\big)(0,1)$

$= LS$

$LS = RS$

1

$R \cup (R^{op})$

$= \{(0,0), (1,1), (0,2), (2,0)\} \cup \{(0,0), (1,1), (0,2), (2,0)\}$

$= \{(0,0), (1,1), (0,2), (2,0)\}$

(vii) $M(SymClsr(R))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(vi) $(M(R \cup (R^{op})))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (vii) On the **facing side**

draw: $DG(R)$, $DG(TrnsClsr(R))$ and

find: $M(TrnsClsr(R))$, $M(\cup_{k=1}^{5}(R^{\circ k}))$

Prove on **this side** that:

$$(M(TrnsCls(R)))(0,1) = \left(M(\cup_{k=1}^{5}(R^{\circ k}))\right)(0,1)$$

$(M(TrnsClsr(R)))$

RS

$$= \left(M\left(\bigcup_{k=1}^{5}(R^{\circ k})\right)\right)(0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

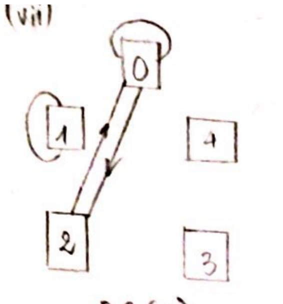
$$= (M(TrnsClsr(R)))(0,1)$$

$= LS$

$LS = RS$

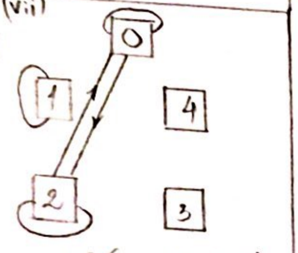
1

(vii)



DG(R)

(vii)



DG(Trns(lsr(R)))

(vii) Find M(Trns(lsr(R))

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	0	0
4	0	0	0	0	0

$$M(Trns(lsr(R))) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

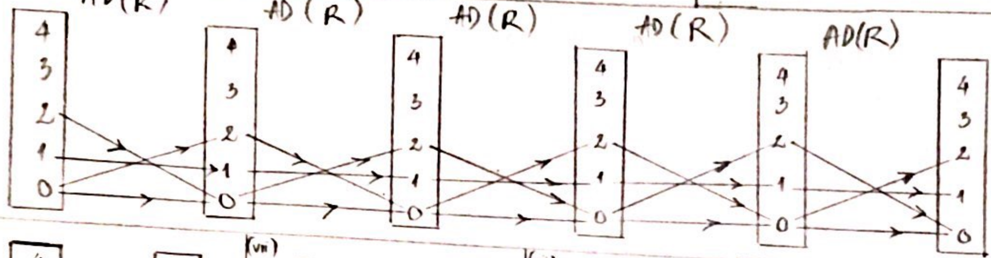
(vii) Find $M(\bigcup_{k=1}^{5}(R^{ok}))$

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	0	0
4	0	0	0	0	0

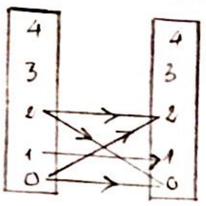
$$M\left(\bigcup_{k=1}^{5}(R^{ok})\right) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(vii)

AD(R) AD(R) AD(R) AD(R) AD(R)

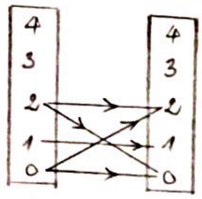


(vii)



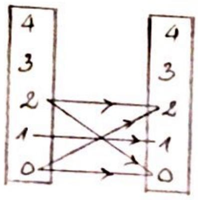
AD(R∘R)

(vii)



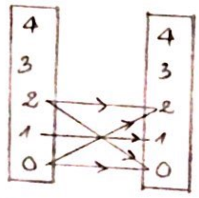
AD(R∘R∘R∘R)

(vii)



AD(R∘R∘R∘R)

(vii)



AD(R∘R∘R∘R∘R)

(vii) $\bigcup_{k=1}^{5}(R^{ok})$

$= (R) \cup (R \circ R) \cup (R \circ R \circ R) \cup (R \circ R \circ R \circ R) \cup (R \circ R \circ R \circ R \circ R)$

$= \{(0,0),(0,2),(1,1),(2,0)\} \cup \{(0,0),(0,2),(1,1),(2,0),(2,2)\} \cup \{(0,0),(0,2),(1,1),(2,0),(2,2)\} \cup \{(0,0),(0,2),(1,1),(2,0),(2,2)\} \cup \{(0,0),(0,2),(1,1),(2,0),(2,2)\}$

$= \{(0,0),(0,2),(1,1),(2,0),(2,2)\}$

(vii) $(M(Trns(lsr(R))))(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(vii) $\left(M\left(\bigcup_{k=1}^{5}(R^{ok})\right)\right)(0,1)$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (viii) On the **facing side**

draw: $DG(R)$, $DG(EqvClsr(R))$ and

find: $\cancel{EqvCls(R)}$, $M(EqvClsr(R))$, $EqvCls(EqvClsr(R))$

$EqvClsr(R)$

Record on **this side** that:

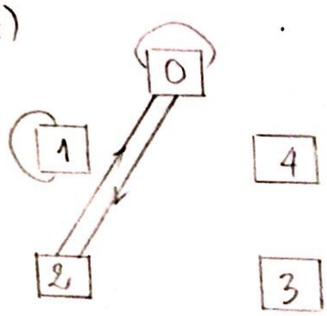
$$\frac{\{0,1,2,3,4\}}{\cancel{EqvCls(R)}}$$

$EqvClsr(R)$

$= \{\{1\},\{0,2\},\{3\},\{4\}\}$

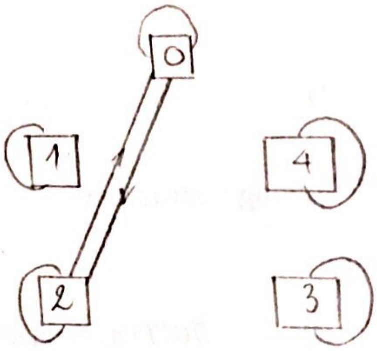
1

(viii)



DG(R)

(viii)



DG(EqvClsr(R))

(viii) $EqvClsr(R) = \{(0,0), (0,2), (2,0), (1,1), (2,2), (3,3), (4,4)\}$

(viii)

$EqvCls(0, EqvClsr(R)) = \{0, 2\}$

$EqvCls(1, EqvClsr(R)) = \{1\}$

$EqvCls(2, EqvClsr(R)) = \{0, 2\}$

$EqvCls(3, EqvClsr(R)) = \{3\}$.

$EqvCls(4, EqvClsr(R)) = \{4\}$

(viii)

	0	1	2	3	4
0	1	0	1	0	0
1	0	1	0	0	0
2	1	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	1

$$M(EqvClsr(R)) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

5 (viii) Find a formula for: $EqvClsr(R)$ and prove your assertion.

Use your formula to find $M(EqvClsr(R))$ and check that it agrees with your answer on (viii)

0

6 (2a) **A, B are as defined on page 202.** Define, **if possible,** functions: $\alpha, \beta. \gamma: A \longrightarrow B$ that have the indicated properties. **If such a function does not exist, you have to prove why this is so.** Express your answers as arrow-diagrams and include the source and target as sets with elements.

1(i) α is injective but NOT surjective.

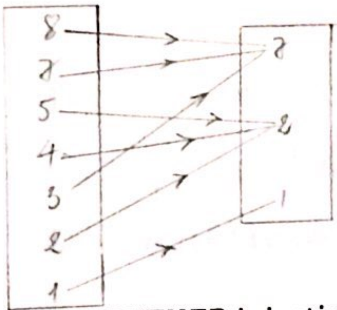
$$\frac{(\alpha \in InjFnc(A,B)) \wedge (InjFnc(A,B) \neq \{\}) \wedge (\neg(Srj(\alpha)))}{\nu(A) < \nu(B)}$$

$$\frac{7 < 3}{\perp}$$

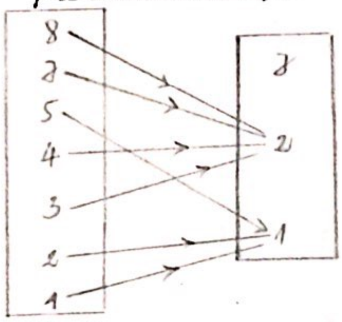
$$\frac{\neg((\alpha \in InjFnc(A,B)) \wedge (InjFnc(A,B) \neq \{\}) \wedge (\neg(Srj(\alpha))))}{(\alpha \notin InjFnc(A,B)) \vee (InjFnc(A,B) = \{\}) \vee (Srj(\alpha))}$$

$$InjFnc(A,B) = \{\}$$

1(ii) β is surjective but NOT injective.

$\beta :=$  $: \{1,2,3,4,5,7,8\} \longrightarrow \{1,2,7\}$

1(iii) γ is NEITHER injective NOR surjective.

$\gamma :=$  $: \{1,2,3,4,5,7,8\} \longrightarrow \{1,2,7\}$

1 (iv) Find: $Im(\gamma) = \{1, 2\}$

5

2 (v) On the facing page, draw arrow-diagrams for every ~~adjusted-restriction~~ maximal adjusted restriction of $\gamma: A \longrightarrow B$ that are bijective and count the total number of such ~~adjusted-restrictions~~: maximal adjusted restrictions 12

You may talk to anyone

(v)

$$\gamma_1 := \begin{array}{ccc} 3 & \longrightarrow & 2 \\ 1 & \longrightarrow & 1 \end{array} : \{1,3\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_2 := \begin{array}{ccc} 4 & \longrightarrow & 2 \\ 1 & \longrightarrow & 1 \end{array} : \{1,4\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_3 := \begin{array}{ccc} 7 & \longrightarrow & 2 \\ 1 & \longrightarrow & 1 \end{array} : \{1,7\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_4 := \begin{array}{ccc} 8 & \longrightarrow & 2 \\ 1 & \longrightarrow & 1 \end{array} : \{1,8\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_5 := \begin{array}{ccc} 3 & \longrightarrow & 2 \\ 2 & \longrightarrow & 1 \end{array} : \{2,3\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_6 := \begin{array}{ccc} 4 & \longrightarrow & 2 \\ 2 & \longrightarrow & 1 \end{array} : \{2,4\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_7 := \begin{array}{ccc} 7 & \longrightarrow & 2 \\ 2 & \longrightarrow & 1 \end{array} : \{2,7\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_8 := \begin{array}{ccc} 8 & \longrightarrow & 2 \\ 2 & \longrightarrow & 1 \end{array} : \{2,8\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_9 := \begin{array}{ccc} 5 & \longrightarrow & 1 \\ 3 & \longrightarrow & 2 \end{array} : \{3,5\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_{10} := \begin{array}{ccc} 5 & \longrightarrow & 1 \\ 4 & \longrightarrow & 2 \end{array} : \{4,5\} \longrightarrow \{1,2\}$$

(v)

$$\gamma_{11} := \begin{array}{ccc} 7 & \longrightarrow & 2 \\ 5 & \longrightarrow & 1 \end{array} : \{5,7\} \longrightarrow \{1,2\}$$

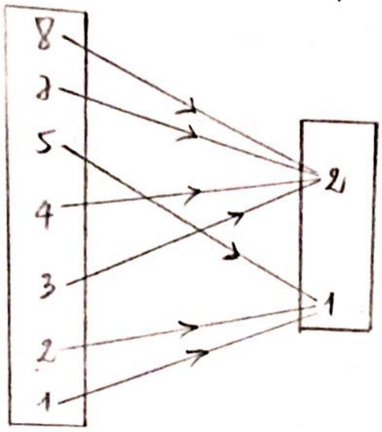
(v)

$$\gamma_{12} := \begin{array}{ccc} 8 & \longrightarrow & 2 \\ 5 & \longrightarrow & 1 \end{array} : \{5,8\} \longrightarrow \{1,2\}$$

Scanned with CamScanner

6 (2b) Carry out the following instructions:

1 (i) Draw: $AD\left(\gamma\Big|^{Im} : A \longrightarrow Im(\gamma)\right)$ below.

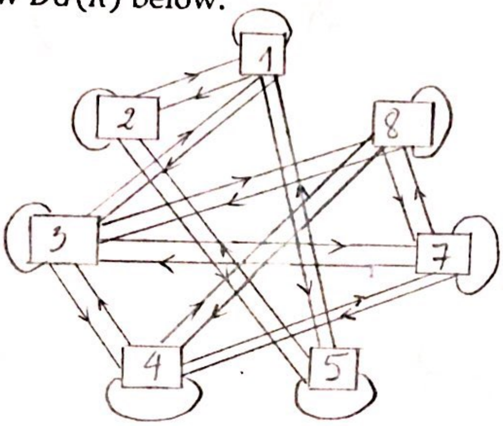


$: \{ \} \rightarrow \{ \}$

4 (ii) Define the relation $R \subseteq A \times A$ as follows:

$$\forall a, b \in A \quad aRb :\Leftrightarrow \gamma(a) = \gamma(b)$$

Draw $DG(R)$ below.



1 (iii) $Eqv(R)$, that is, is R an equivalence relation? Y N

Why?

By inspection

5

5 (2c) Prove or disprove:

$$\forall \varphi \in Fnc(S,T) \quad Eqv(R(\varphi))$$

where the relation

$$R(\varphi) := \{\cancel{(u,v)}^{(s,t)} \in S \times S | \varphi(s) = \varphi(t)\} \subseteq S \times S$$

Pf

$x \in S$

$((x,x) \in S \times S) \wedge (\varphi(x) = \varphi(x))$

$(x,x) \in R(\varphi)$

$x\ R(\varphi)\ x$

$\forall x \in S \quad (x\ R(\varphi)\ x)$

$Rflx(R(\varphi))$

0 ?

$(a \in S) \wedge (b \in S) \wedge (a\ R(\varphi) b)$

$\varphi(a) = \varphi(b)$

$\varphi(b) = \varphi(a)$

6

$b\ R(\varphi)\ a$

$(\forall a \in S) \wedge (\forall b \in S) \left(\dfrac{a\ R(\varphi) b}{b\ R(\varphi) a}\right)$

$Sym(R(\varphi))$

0 ?

$(c \in S) \wedge (d \in S) \wedge (e \in S) \wedge (c\ R(\varphi) d) \wedge (d\ R(\varphi) e)$

$(\varphi(c) = \varphi(d)) \wedge (\varphi(d) = \varphi(e))$

$\varphi(c) = \varphi(e)$

$c\ R(\varphi) e$

$(\forall c \in S) \wedge (\forall d \in S) \wedge (\forall e \in S) \left(\dfrac{(c\ R(\varphi) d) \wedge (d\ R(\varphi) e)}{c\ R(\varphi) e}\right)$

$Trns(R(\varphi))$

$(Rflx(R(\varphi))) \wedge (Sym(R(\varphi))) \wedge (Trns(R(\varphi)))$

$Eqv(R(\varphi))$

fq

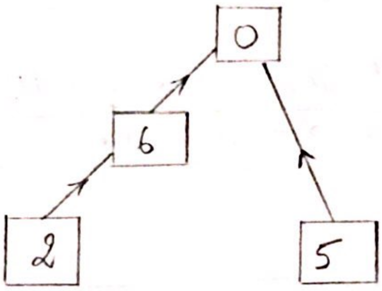
6 (3a) Define the relations $R \subseteq J \times J$ and $S \subseteq E \times E$ as follows:

$$R := \{(j,k) \in J \times J \mid j|k\} \subseteq J \times J$$

$$S := \{(e,f) \in E \times E \mid e|f\} \subseteq \cancel{J \times J}\ E \times E$$

and answer the following questions.

3 (i) Prove that $PO(R)$ on the facing side and draw $HsDgrm(R)$ on this side.



3 (i) Prove that $PO(S)$ on the facing side and draw $HsDgrm(S)$ on this side.

6

(i) Prove $PO(R)$

Pf

$$\frac{\dfrac{\dfrac{(a \in J)}{((a,a) \in J \times J) \wedge (a \mid a)}}{a R a}}{\forall a \in J\ (a R a)}$$

Irrelevant!

$$Rflx(R)$$

$$\frac{\dfrac{(b \in J) \wedge (c \in J) \wedge (bRc) \wedge (cRb)}{(b \mid c) \wedge (c \mid b)}}{b = c}$$

$$\frac{(\forall b \in J) \wedge (\forall c \in J)\left(\dfrac{(bRc) \wedge (cRb)}{b = c}\right)}{AntSym(R)}$$

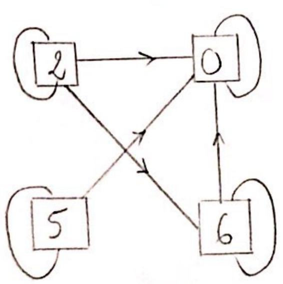
$$\frac{\dfrac{\dfrac{(d \in J) \wedge (e \in J) \wedge (f \in J) \wedge (d R e) \wedge (e R f)}{(d \mid e) \wedge (e \mid f)}}{d \mid f}}{d R f}$$

$$\frac{(\forall d \in J) \wedge (\forall e \in J) \wedge (\forall f \in J)\left(\dfrac{(dRe) \wedge (eRf)}{dRf}\right)}{Trns(R)}$$

$$\frac{(Rflx(R)) \wedge (AntSym(R)) \wedge (Trns(R))}{PO(R)}$$

(ii) Pf

(i) $DG(R)$



(i) Prove $PO(S)$

Pf

$$\frac{\dfrac{\dfrac{a \in E}{((a,a) \in E \times E) \wedge (a \mid a)}}{a S a}}{\forall a \in E\ (a S a)}$$

$$Rflx(S)$$

$$\frac{\dfrac{(b \in E) \wedge (c \in E) \wedge (bSc) \wedge (cSb)}{(b \mid c) \wedge (c \mid b)}}{b = c}$$

$$\frac{(\forall b \in E)(\forall c \in E)\left(\dfrac{(bSc) \wedge (cSb)}{b = c}\right)}{AntSym(S)}$$

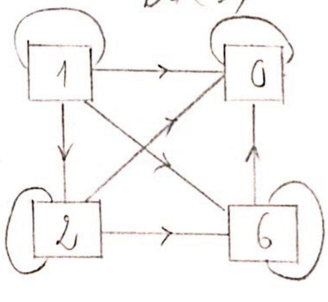
$$\frac{\dfrac{\dfrac{(d \in E) \wedge (e \in E) \wedge (f \in E) \wedge (dSe) \wedge (eSf)}{(d \mid e) \wedge (e \mid f)}}{d \mid f}}{dSf}$$

$$\frac{(\forall d \in E) \wedge (\forall e \in E) \wedge (\forall f \in E)\left(\dfrac{(dSe) \wedge (eSf)}{dSf}\right)}{Trns(S)}$$

$$\frac{(Rflx(S)) \wedge (AntSym(S)) \wedge (Trns(S))}{PO(S)}$$

Pf

(i) $DG(S)$



6 (3b) Define the relations $R \vee S,\ R \wedge S \subseteq (J \times E) \times (J \times E)$ as follows:

$\forall (j, e) \in (J \times E), \forall (k, f) \in (J \times E)$

$$(j, e)(R \vee S)(k, f) :\Leftrightarrow (jRk) \vee (eSf)$$

$$(j, e)(R \wedge S)(k, f) :\Leftrightarrow (jRk) \wedge (eSf)$$

Prove or disprove on the facing side:

3 (i) $PO(R \vee S)$ Y N

and if yes draw $HsDgrm(R \vee S)$ on this side.

aRb bRa

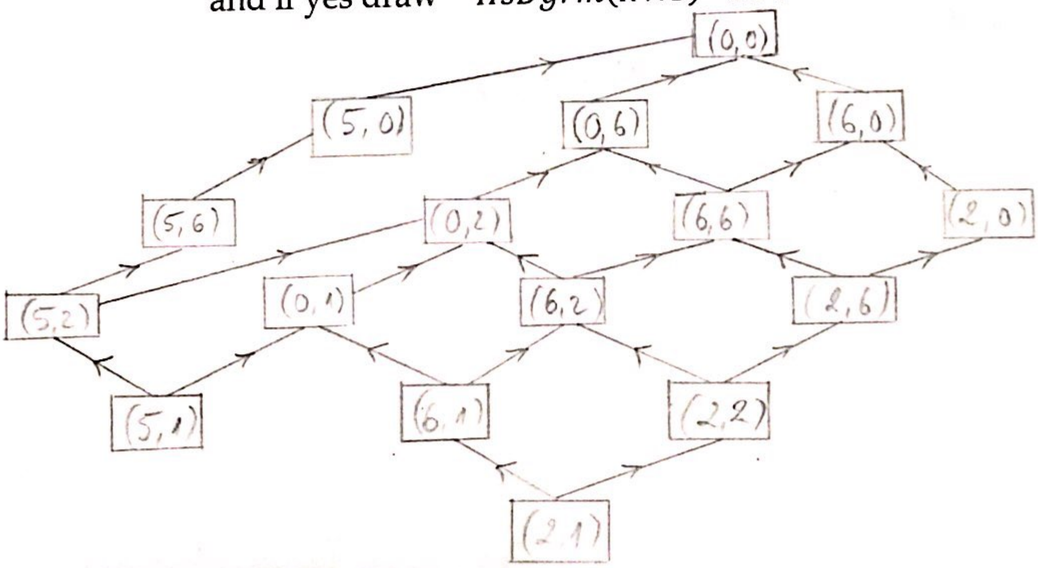
$a = b$ R

$(a,b)(R \cup S)(c,d)$ $(a,b)R \cup S(a,b)$

$\wedge\ (a,b) \neq (c,d)$

3 (i) $PO(R \wedge S)$ Y N

and if yes draw $HsDgrm(R \wedge S)$ on this side.



1

$((2,0)(R\vee S)(6,1)) \wedge ((6,1)(R\vee S)(2,0)) \wedge ((2,0) \neq (6,1))$

$\cancel{\text{AntSym}}(R\vee S)$

$(2R6) \vee' (\quad)$

$\cancel{PO}(R\vee S)$

fg

(i) Prove $PO(R\wedge S)$

Pf

$((a,b) \in (J\times E)) \quad (Rflx(R)) \quad (Rflx(S))$

$(((a,b),(a,b)) \in ((J\times E)\times(J\times E))) \quad (a R a) \quad (b S b)$

$(a,b)(R\wedge S)(a,b)$

$\forall (a,b) \in (J\times E) \quad ((a,b)(R\wedge S)(a,b))$

$Rflx(R\wedge S)$

$((c,d) \in (J\times E)) \wedge ((e,f) \in (J\times E)) \wedge ((c,d)(R\wedge S)(e,f)) \wedge ((e,f)(R\wedge S)(c,d))$

$(cRe) \quad (dSf) \quad (eRc) \quad (fSd) \quad AntSym(R) \quad AntSym(S)$

$(c=e) \quad (d=f)$

$(c,d) = (e,f)$

$$(\forall (c,d) \in (J\times E))(\forall (e,f) \in (J\times E)) \left(\frac{((c,d)(R\wedge S)(e,f)) \wedge ((e,f)(R\wedge S)(c,d))}{(c,d) = (e,f)}\right)$$

$AntSym(R\wedge S)$

$((g,h) \in (J\times E)) \wedge ((i,j) \in (J\times E)) \wedge ((k,l) \in (J\times E)) \wedge ((g,h)(R\wedge S)(i,j)) \wedge ((i,j)(R\wedge S)(k,l))$

$(gRi) \quad (hSj) \quad (iRk) \quad (jSl) \quad Trns(R) \quad Trns(S)$

$(gRk) \quad (hSl)$

$(g,h)(R\wedge S)(k,l)$

$$(\forall (g,h) \in (J\times E)) \wedge (\forall (i,j) \in (J\times E)) \wedge (\forall (k,l) \in (J\times E)) \left(\frac{((g,h)(R\wedge S)(i,j)) \wedge ((i,j)(R\wedge S)(k,l))}{(g,h)(R\wedge S)(k,l)}\right)$$

$Trns(R\wedge S)$

$(Rflx(R\wedge S)) \wedge (AntSym(R\wedge S)) \wedge (Trns(R\wedge S))$

$PO(R\wedge S)$

fg

5 (3c) Define: $Prtn(S) :=$ the set of partitions of S and

$EqvRln(S) :=$ the set of equivalence relations on S

Draw the digraph of equivalence relation on the facing side and give them names: E_n where $n \in \mathbb{N}$ using as many n's as you need and list these names on this side.

Determine the following:

2 (i) $\nu(Prtn(B)) =$ 5

$Prtn(B)$

$$= \left\{ \begin{array}{l} \{\{1\},\{2\},\{7\}\}, \\ \{\{1,2\},\{7\}\}, \{\{1,7\},\{2\}\}, \{\{1\},\{2,7\}\}, \\ \{\{1,2,7\}\} \end{array} \right\}$$

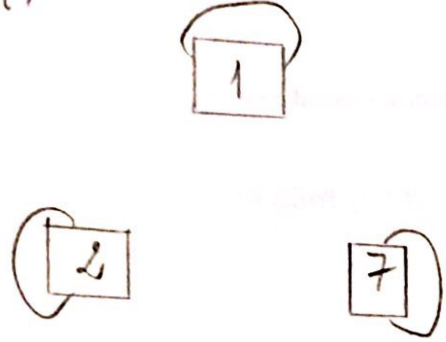
3 (ii) $\nu(EqnRln(B)) =$ 5

$EqnRln(B)$

5

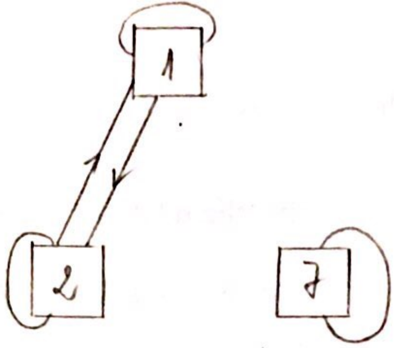
$$= \{E_1, E_2, E_3, E_4, E_5\}$$

(ii)



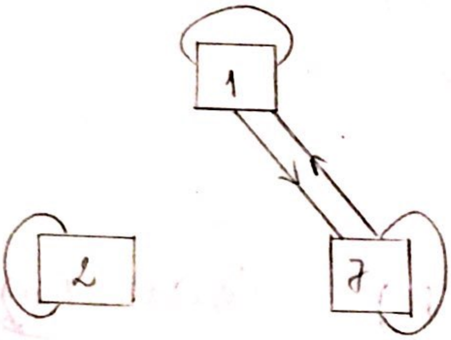
$DG(E_1)$

(ii)



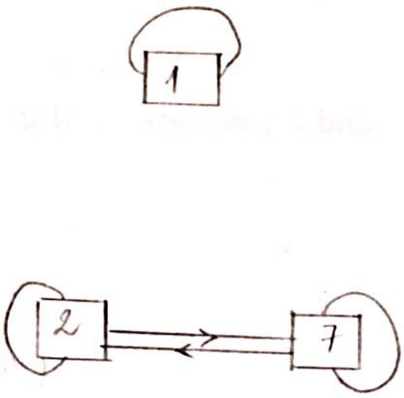
$DG(E_2)$

(ii)



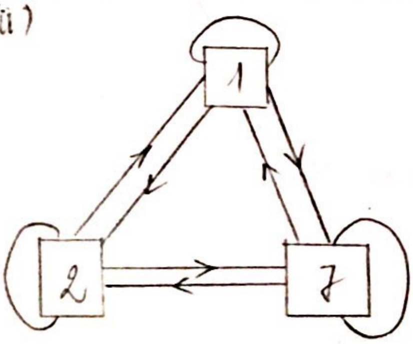
$DG(E_3)$

(ii)



$DG(E_4)$

(ii)



$DG(E_5)$

(1abc)
(v)

$$\left(M\left(R \circ R^{op}\right)\right)(0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(1abc)
(iv)

$$\left(M\left(R^{op} \circ R\right)\right)(0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{p \wedge q}{p \vee q}$$

(1abc)
(iv)

$$\left(\left(M(R)\right)\left(M\left(R^{op}\right)\right)\right)(0,1)$$

$$= \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(1abc)
(iv)

$$\left(\left(M\left(R^{op}\right)\right)\left(M(R)\right)\right)(0,1)$$

$$= \begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} (0,1)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$