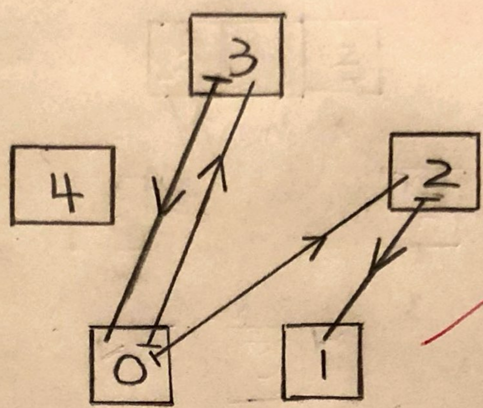
17 (1abc) Carry out the following instructions, **doing all computations on the facing side** and **recording only the answers on this side.**

1 (i) Draw directed graph for the relation R in the above with 0, 1, 2, 3, 4 as nodes **arranged in the shape of a regular pentagon with a horizontal base,** with the left vertex of the horizontal side labelled 0, and labelling all the vertices in order traversing them in the counter-clockwise sense.



DG(R)

1 (ii) Find the 5× 5 incidence matrix $M(R)$ for the relation R.

$$M(R) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3

1 (iii) Find the 5× 5 incidence matrix $M(R^{op})$ for the relation R^{op}.

$$M(R^{op}) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

labc.ii.0

	0	1	2	3	4
0	0	0	1	1	0
1	0	0	0	0	0
2	0	1	0	0	0
3	1	0	0	0	0
4	0	0	0	0	0

$$M(R) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

labc.iii.0 $M(R^{op}) = \{(0,3),(1,2),(2,0),(3,0)\}$

labc.iii.1

	0	1	2	3	4
0	0	0	0	1	0
1	0	0	1	0	0
2	1	0	0	0	0
3	1	0	0	0	0
4	0	0	0	0	0

$$M(R^{op}) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

5 (iv) Compute: on the **facing side**:

$$R \circ R^{op}, \qquad R^{op} \circ R,$$

$$M(R \circ R^{op}), \qquad M(R^{op} \circ R),$$

$$(M(R))(M(R^{op})), \qquad (M(R^{op}))(M(R))$$

Prove on **this side** that:

(1) $$\left(M(R \circ R^{op})\right)(0,1) = \left(\left(M(R^{op})\right)\left(M(R)\right)\right)(0,1)$$

$$(M(R \circ R^{op}))(0,1) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = ((M(R^{op}))(M(R)))(0,1)$$

$$M(R \circ R^{op}) = (M(R^{op}))(M(R))$$

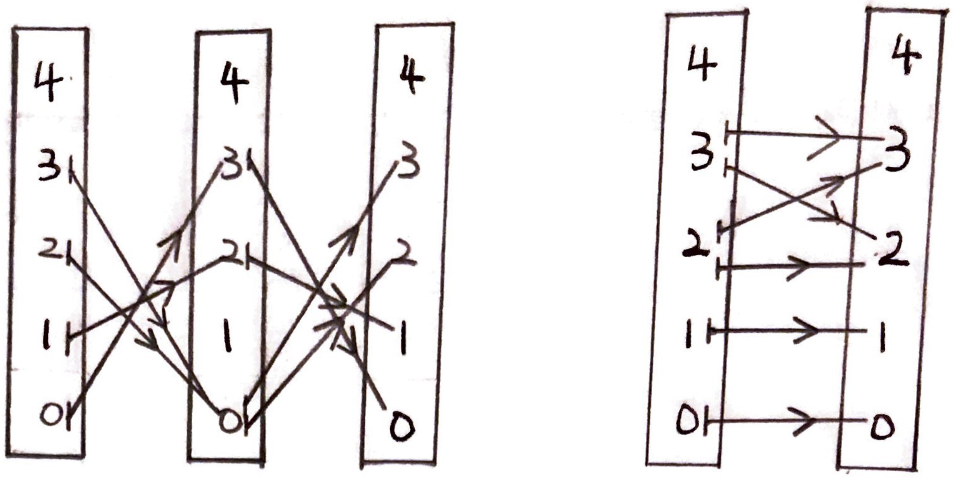
(2) $$\left(M(R^{op} \circ R)\right)(0,1) = \left(\left(M(R)\right)\left(M(R^{op})\right)\right)(0,1)$$

8

$$(M(R^{op} \circ R))(0,1) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = ((M(R))(M(R^{op})))(0,1)$$

$$M(R^{op} \circ R) = (M(R))(M(R^{op}))$$

labc.iv.1.0



AD(R^{op})   AD(R)   AD($R \circ R^{op}$)

labc.iv.1.1

Find $M(R \circ R^{op})$

$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$M(R \circ R^{op}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

labc.iv.1.2

$((M(R^{op}))(M(R)))$

NOTE: See labc.iii.1 $\wedge$ labc.ii.0

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

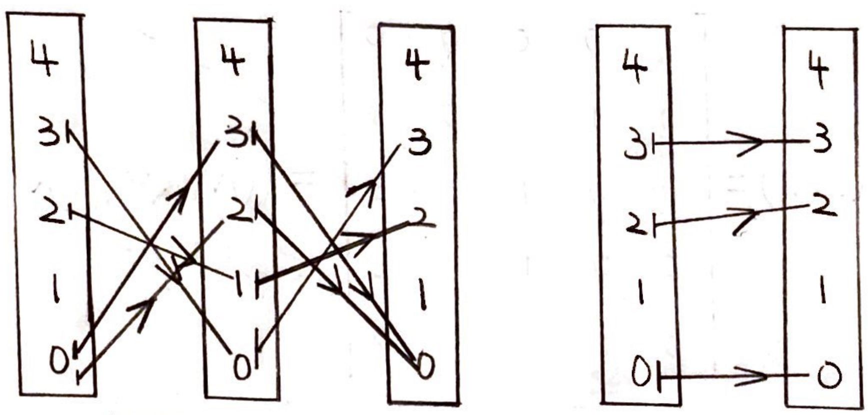
labc.iv.1.3

$$= \begin{bmatrix} 0(0)+0(0)+0(0)+1(1)+0(0) & 0(0)+0(0)+0(1)+1(0)+0(0) & 0(1)+0(0)+0(0)+1(0)+0(0) & 0(1)+0(0)+0(0)+1(0)+0(0) & 0(0)+0(0)+0(0)+1(0)+0(0) \\ 0(0)+0(0)+1(0)+0(1)+0(0) & 0(0)+0(0)+1(1)+0(0)+0(0) & 0(1)+0(0)+1(0)+0(0)+0(0) & 0(1)+0(0)+1(0)+0(0)+0(0) & 0(0)+0(0)+1(0)+0(0)+0(0) \\ 1(0)+0(0)+0(0)+0(1)+0(0) & 1(0)+0(0)+0(1)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) \\ 1(0)+0(0)+0(0)+1(1)+0(0) & 1(0)+0(0)+0(1)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) \\ 0(0)+0(0)+0(0)+0(1)+0(0) & 0(0)+0(0)+0(1)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$(M(R^{op}))(M(R))$

labc.iv.2.0



AD(R) AD(R^{op}) AD($R^{op} \circ R$)

labc.iv.2.1 Find $M(R^{op} \circ R)$

	0	1	2	3	4
0	1	0	0	0	0
1	0	0	0	0	0
2	0	0	1	0	0
3	0	0	0	1	0
4	0	0	0	0	0

$M(R^{op} \circ R)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

labc.iv.2.2 $(M(R))(M(R^{op}))$

Note: See labc.ii.0 $\wedge$ labc.iii.1

$$= \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0(0)+0(0)+1(1)+1(1)+0(0) & 0(0)+0(0)+1(0)+1(0)+0(0) & 0(0)+0(1)+1(0)+1(0)+0(0) & 0(1)+0(0)+1(0)+1(0)+0(0) & 0(0)+0(0)+1(0)+1(0)+0(0) \\ 0(0)+0(0)+0(1)+0(1)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(1)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \\ 0(0)+1(0)+0(1)+0(1)+0(0) & 0(0)+1(0)+0(0)+0(0)+0(0) & 0(0)+1(1)+0(0)+0(0)+0(0) & 0(1)+1(0)+0(0)+0(0)+0(0) & 0(0)+1(0)+0(0)+0(0)+0(0) \\ 1(0)+0(0)+0(1)+0(1)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(1)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) \\ 0(0)+0(0)+0(1)+0(1)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(1)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (v) On the **facing side**

draw: $DG(R)$, $DG(RflxClsr(R))$ and

find: $M\left(RflxClsr(R)\right)$, $M\left(R \cup (\Delta_{0..4})\right)$

Prove on **this side** that:

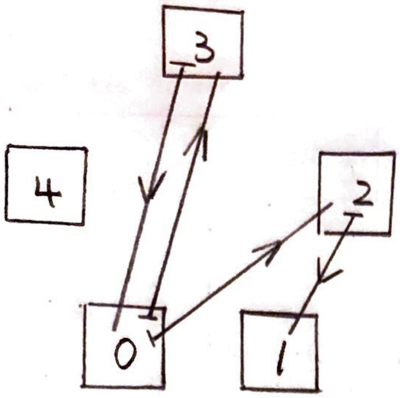
$$\left(M\left(RflxClsr(R)\right)\right)(0,1) = \left(M\left(R \cup (\Delta_{0..4})\right)\right)(0,1)$$

$$(M(RflxClsr(R)))(0,1) = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = (M(R \cup (\Delta_{0..4})))(0,1)$$

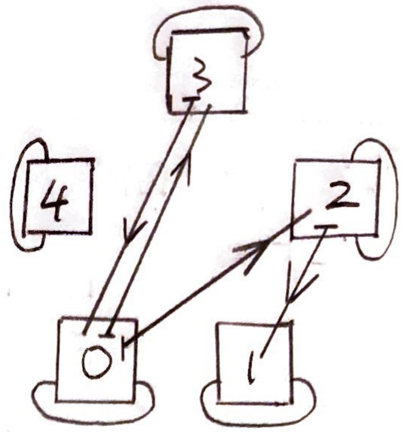
$$M(RflxClsr(R)) = M(R \cup (\Delta_{0..4}))$$

1

labc.v.0



DG(R)



DG(RflxClsr(R))

labc.v.1

Find M(RflxClsr(R))

	0	1	2	3	4
0	1	0	1	1	0
1	0	1	0	0	0
2	0	1	1	0	0
3	1	0	0	1	0
4	0	0	0	0	1

M(RflxClsr(R))

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

labc.v.2

$M(R \cup (\Delta_{0..4}))$

$= \{(0,2),(0,3),(2,1),(3,0)\} \cup \{(0,0),(1,1),(2,2),(3,3),(4,4)\}$

$= \{(0,0),(0,2),(0,3),(1,1),(2,1),(2,2),(3,0),(3,3),(4,4)\}$

Find $M(R \cup (\Delta_{0..4}))$

	0	1	2	3	4
0	1	0	1	1	0
1	0	1	0	0	0
2	0	1	1	0	0
3	1	0	0	1	0
4	0	0	0	0	1

$M(R \cup (\Delta_{0..4}))$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (vi) On the **facing side**

draw: $DG(R)$, $DG(SymClsr(R))$ and

find: $M\left(SymClsr(R)\right), M\left(R \cup (R^{op})\right)$

Prove on **this side** that:

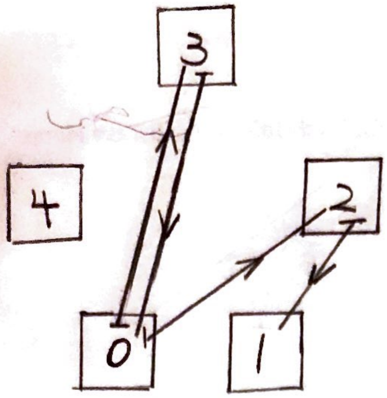
$$\left(M\left(SymClsr(R)\right)\right)(0,1) = \left(M\left(R \cup (R^{op})\right)\right)(0,1)$$

$$(M(SymClsr(R))(0,1) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = (M(R \cup (R^{op})))(0,1)$$

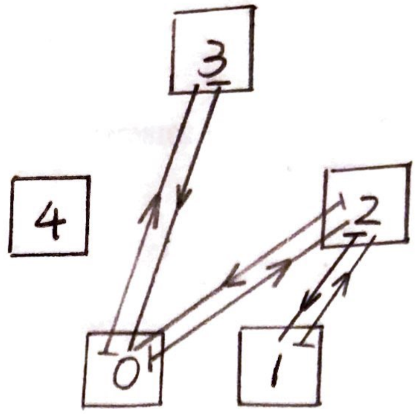
$$M(SymClsr(R)) = M(R \cup (R^{op}))$$

1

(labc.vi.0)



DG(R)



DG(SymClsr(R))

(labc.vi.1) Find M(SymClsr(R))

	0	1	2	3	4
0	0	0	1	1	0
1	0	0	1	0	0
2	1	1	0	0	0
3	1	0	0	0	0
4	0	0	0	0	0

M(SymClsr(R))

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(labc.vi.2) $M(R \cup (R^{op}))$

$= \{(0,2),(0,3),(2,1),(3,0)\} \cup \{(0,3),(1,2),(2,0),(3,0)\}$

$= \{(0,2),(0,3),(1,2),(2,0),(2,1),(3,0)\}$

Find $M(R \cup (R^{op}))$

	0	1	2	3	4
0	0	0	1	1	0
1	0	0	1	0	0
2	1	1	0	0	0
3	1	0	0	0	0
4	0	0	0	0	0

$M(R \cup (R^{op}))$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (vii) On the **facing side**

draw: $DG(R)$, $DG(TrnsClsr(R))$ and

find: $M\left(TrnsCls(R)\right)$, $M\left(\cup_{k=1}^{5}(R^{\circ k})\right)$

Prove on **this side** that:

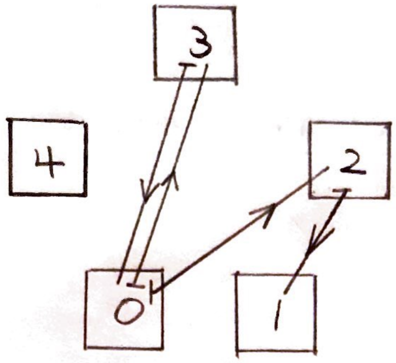
$$\left(M\left(TrnsCls(R)\right)\right)(0,1) = \left(M\left(\cup_{k=1}^{5}(R^{\circ k})\right)\right)(0,1)$$

$$(M(TrnsCls(R))(0,1) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = (M(\cup_{k=1}^{5}(R^{\circ k}))) (0,1)$$

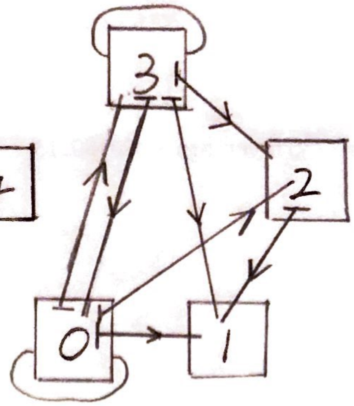
$$M(TrnsCls(R)) = M(\cup_{k=1}^{5}(R^{\circ k}))$$

1

labc.vii.0



DG(R)



DG(TrnsClsr(R))

labc.vii.1

Find M(TrnsClsr(R))

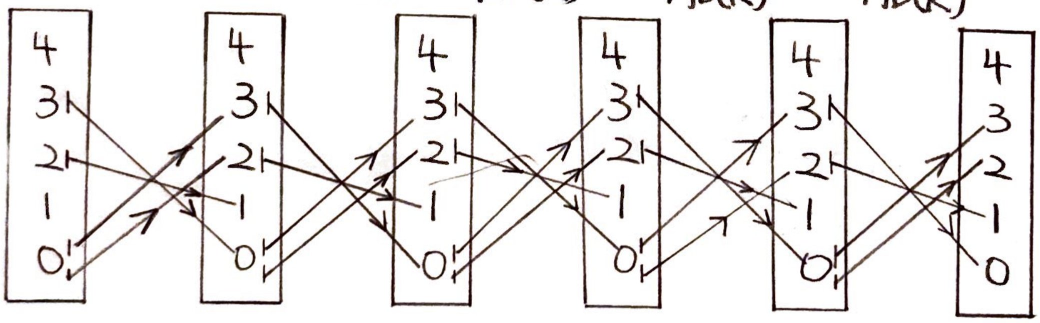
	0	1	2	3	4
0	1	1	1	1	0
1	0	0	0	0	0
2	0	1	0	0	0
3	1	1	1	1	0
4	0	0	0	0	0

M(TrnsCls(R))

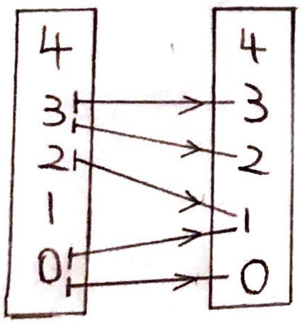
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

labc.vii.2

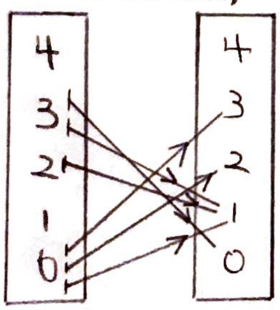
AD(R) AD(R) AD(R) AD(R) AD(R)



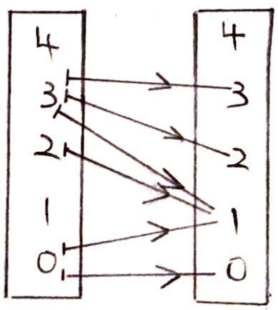
AD(RoR)



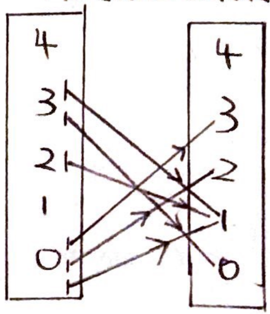
AD(RoRoR)



AD(RoRoRoR)



AD(RoRoRoRoR)



(1abc.vii.3) $\bigcup_{k=1}^{5}(R^{\circ k})$

$= (R) \cup (R \circ R) \cup (R \circ R \circ R) \cup (R \circ R \circ R \circ R) \cup (R \circ R \circ R \circ R \circ R)$

$= \{(0,2),(0,3),(2,1),(3,0)\} \cup \{(0,0),(0,1),(2,1),(3,2),(3,3)\}$
$\cup \{(0,1),(0,2),(0,3),(2,1),(3,0),(3,1)\} \cup \{(0,0),(0,1),(2,1),$
$(3,1),(3,2),(3,3)\} \cup \{(0,1),(0,2),(0,3),(2,1),(3,0),(3,1)\}$

$= \{(0,0),(0,1),(0,2),(0,3),(2,1),(3,0),(3,1),(3,2),(3,3)\}$

(1abc.vii.4) Find $M(\bigcup_{k=1}^{5}(R^{\circ k}))$

	0	1	2	3	4
0	1	1	1	1	0
1	0	0	0	0	0
2	0	1	0	0	0
3	1	1	1	1	0
4	0	0	0	0	0

$$M(\bigcup_{k=1}^{5}(R^{\circ k})) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (viii) On the **facing side**

draw: $DG(R)$, $DG\left(EqvClsr(R)\right)$ and

find: $EqvClsr(R)$, $M\left(EqvClsr(R)\right)$

Record on **this side:**

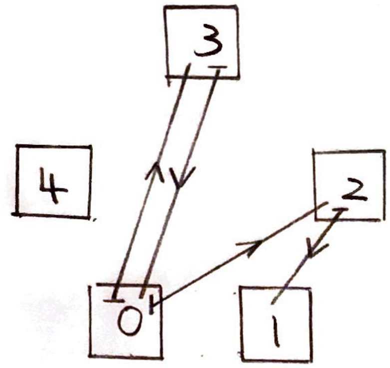
$$EqvCls(R)$$

$$= \frac{\{0,1,2,3,4\}}{EqvClsr(R)}$$

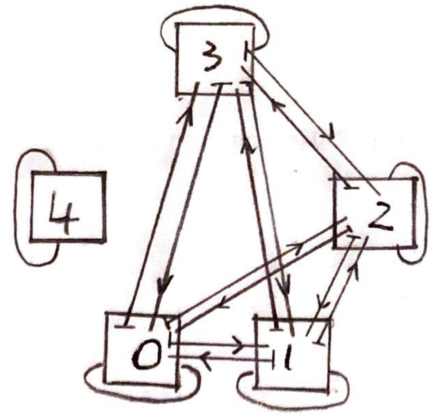
$$= \{\{0,1,2,3\},\{4\}\}$$

1

labc.viii.0



DG (R)



DG(EquClsr(R))

labc.viii.1

$EquClsr(R) = \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,2),(2,3),(3,0),(3,1),(3,2),(3,3),(4,4)\}$

labc.viii.2

	0	1	2	3	4
0	1	1	1	1	0
1	1	1	1	1	0
2	1	1	1	1	0
3	1	1	1	1	0
4	0	0	0	0	1

M(EquClsr(R))

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

labc.viii.3

$EquClsr(0, EquClsr(R)) = \{0,1,2,3\}$

$EquClsr(1, EquClsr(R)) = \{0,1,2,3\}$

$EquClsr(2, EquClsr(R) = \{0,1,2,3\}$

$EquClsr(3, EquClsr(R) = \{0,1,2,3\}$

$EquClsr(4, EquClsr(R) = \{4\}$

17 (1abc) Carry out the following instructions

5 (viii) Find a formula for: $EqvClsr(R)$ and prove your assertion.

Use your formula to find $M\left(EqvClsr(R)\right)$ and check that it agrees with your answer on (viii)

$(EqvClsr(R))(0,1) = ((\bigcup_{k=1}^{3}((R \cup R^{op})^{\circ k})) \cup (\Delta_{0..4}))(0,1)$

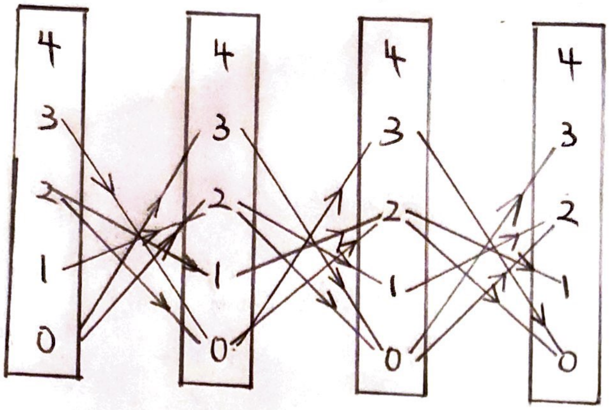
$$M(EqvClsr(R)) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = M(\bigcup_{k=1}^{3}((R \cup R^{op})^{\circ k})) \cup (\Delta_{0..4})$$

$M(EqvClsr(R)) = M(\bigcup_{k=1}^{3}((R \cup R^{op})^{\circ k})) \cup (\Delta_{0..4})$

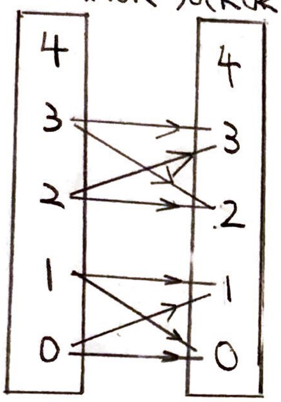
3

labc.viii.0 $\quad \bigcup_{k=1}^{3}((R \cup R^{op})^{\circ k})$

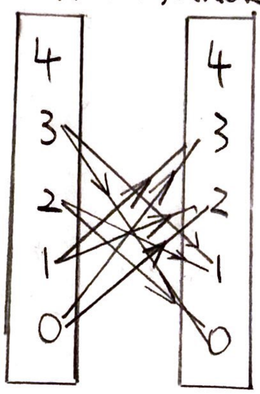
AD$(R \cup R^{op})$ $\quad$ AD$(R \cup R^{op})$ $\quad$ AD$(R \cup R^{op})$



AD$((R \cup R^{op}) \circ (R \cup R^{op}))$



AD$((R \cup R^{op}) \circ (R \cup R^{op}) \circ (R \cup R^{op}))$



$\bigcup_{k=1}^{3}((R \cup R^{op})^{\circ k})$

$= (R \cup R^{op}) \cup ((R \cup R^{op}) \circ (R \cup R^{op})) \cup ((R \cup R^{op}) \circ (R \cup R^{op}) \circ (R \cup R^{op}))$

$= \{(0,2),(0,3),(1,2),(2,0),(2,1),(3,0)\} \cup \{(0,0),(0,1),(1,0),(1,1),(2,2),(2,3),$
$(3,2),(3,3)\} \cup \{(0,2),(0,3),(1,2),(1,3),(2,0),(2,1),(3,0),(3,1)\}$

$= \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,2),$
$(2,3),(3,0),(3,1),(3,2),(3,3)$

Note: Please see labc.vi.2

labc.viii.1 $\quad (\bigcup_{k=1}^{3}((R \cup R^{op})^{\circ k})) \cup (\Delta_{0..4})$

$= \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1)$
$(2,2),(2,3),(3,0),(3,1),(3,2),(3,3)\} \cup \{(0,0),(1,1),(2,2),(3,3),(4,4)\}$

$= \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,2),(2,3),(3,0),(3,1),(3,2),(3,3),(4,4)\}$

abc.viii.2

1a

	0	1	2	3	4
0	1	1	1	1	0
1	1	1	1	1	0
2	1	1	1	1	0
3	1	1	1	1	0
4	0	0	0	0	1

$$M\left(\bigcup_{k=1}^{3}(R\cup R^{op})^{\circ k}\right)\cup(\Delta_{0..4})$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

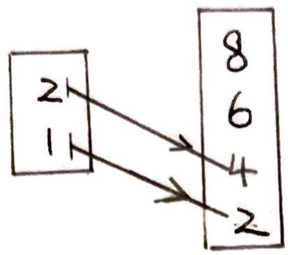
2a.v.o

$\gamma_1 : \boxed{1} \longrightarrow \boxed{4} : \{1\} \longrightarrow \{4\}$

$\gamma_2 : \boxed{2} \longrightarrow \boxed{4} : \{2\} \longrightarrow \{4\}$

6 (2a) **A, B are as defined on page 202. Define, if possible, functions: $\alpha, \beta . \gamma : A \longrightarrow B$ that have the indicated properties. If such a function does not exist, you have to prove why this is so. Express your answers as arrow-diagrams and include the source and target as sets with elements.**

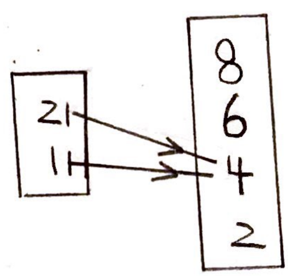
1(i) **α is injective but NOT surjective.**

$\alpha ::=$  $: \{1,2\} \rightarrow \{2,4,6,8\}$

1(ii) **β is surjective but NOT injective.**

$$\beta : \quad \frac{\nu(A) = 2 \not\geq \nu(B) = 4}{Srj(A,B) = \{\ \}}$$

1(iii) **γ is NEITHER injective NOR surjective.**

$\gamma :=$  $: \{1,2\} \longrightarrow \{2,4,6,8\}$

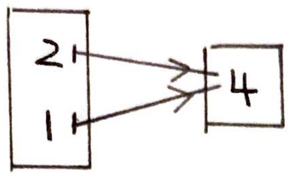
1 (iv) Find: $Im(\gamma) = \{4\}$

2 (v) **On the facing page,** draw arrow-diagrams for **every** adjusted maximal bijective restriction of $\gamma : A \longrightarrow B$ and count the total number of **such restrictions:** 2

You may talk to anyone

6 (2b) Carry out the following instructions:

1 (i) Draw: $AD\left(\gamma\Big|^{Im} : A \longrightarrow Im(\gamma)\right)$ below.



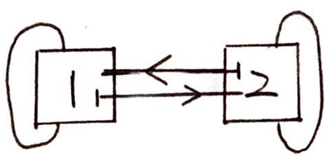
$: \{1,2\} \longrightarrow \{4\}$

$AD\left(\gamma\Big|^{Im} : A \rightarrow Im(\gamma)\right)$

4 (ii) Define the relation $S \subseteq A \times A$ as follows:

$$\forall a, b \in A \quad \left(aSb\right) :\Leftrightarrow \left(\gamma(a) = \gamma(b)\right)$$

Draw $DG(S)$ below.



$DG(S)$

6

1 (iii) $Eqv(S)$, that is, is S an equivalence relation? Y N

Why? By inspection.

5 (2c) Prove or disprove:

$$\forall \varphi \in Fnc(S,T) \quad Eqv(R(\varphi))$$

where the relation

$$R(\varphi) := \left\{ (u,v) \in S \times S \,\middle|\, \varphi(u) = \varphi(v) \right\} \subseteq S \times S$$

$$\frac{(Rflx(R(\varphi)) \,((Sym(R(\varphi)) \,((Trns(R(\varphi))\,)}{Eqv(R(\varphi))}$$

6

2c.0

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{x\in S}{((x,x)\in S\times S)\quad(\varphi(x)=\varphi(x))}}{(x,x)\in R(\varphi)}}{x\;R(\varphi)\;x}}{\forall x\in S\quad (x\;R(\varphi)\;x)}}{\mathrm{Rflx}\,(R(\varphi))}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(a\in S)\;(b\in S)\;(a\;R(\varphi)\;b)}{\varphi(a)=\varphi(b)}}{\varphi(b)=\varphi(a)}}{b\,R(\varphi)\,a}}{(\forall a\in S)\;(\forall b\in S)\;\left(\frac{a\,R(\varphi)\,b}{b\,R(\varphi)\,a}\right)}}{\mathrm{Sym}\,(R(\varphi))}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(c\in S)\;(d\in S)\;(e\in S)\;(c\,R(\varphi)\,d)\;(d\,R(\varphi)\,e)}{(\varphi(c)=\varphi(d))\;(\varphi(d)=\varphi(e))}}{\varphi(c)=\varphi(d)=\varphi(e)}}{\varphi(c)=\varphi(e)}}{c\;R(\varphi)\;e}}{(\forall c\in S)\;(\forall d\in S)\;(\forall e\in S)\;\left(\frac{(c\,R(\varphi)\,d)\;(d\,R(\varphi)\,e)}{c\,R(\varphi)\,e}\right)}}{\mathrm{Trns}\,(R(\varphi))}$$

$$\mathrm{Eqv}\,(R(\varphi))$$

6 (3a) Define the relations $P \subseteq J \times J$ and $Q \subseteq E \times E$ as follows:

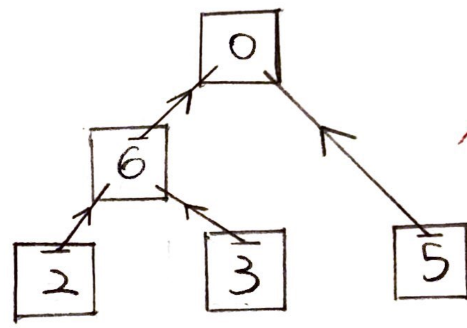
$$P := \left\{ (j, k) \in J \times J \,\middle|\, j|k \right\} \subseteq J \times J$$

$$Q := \left\{ (e, f) \in E \times E \,\middle|\, e|f \right\} \subseteq E \times E$$

and answer the following questions.

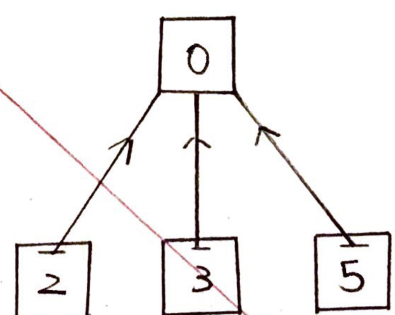
3 (i) Prove that $PO(P)$ on the facing side and draw $HsDgrm(P)$ on this side.

$HsDgrm(P)$:



3 ~~(i)~~ (ii) Prove that $PO(Q)$ on the facing side and draw $HsDgrm(Q)$ on this side.

$HsDgrm(Q)$:



3

$0 \notin E$

3a.i.o $Po(P)$

$$\cfrac{\cfrac{\cfrac{\cfrac{(a \in J)}{((a,a) \in J \times J) \quad (a|a)}}{a\,P\,a}}{\forall a \in J\ (a\,P\,a)}}{Rflx(P)}
\qquad
\cfrac{\cfrac{\cfrac{\cfrac{(b \in J)\ (c \in J)\ (b\,P\,c)\ (c\,P\,b)}{(b|c) \quad (c|b)}}{b = c}}{(\forall b \in J)\ (\forall c \in J)\ \left(\frac{(b\,P\,c)(c\,P\,b)}{b=c}\right)}}{AntSym(P)}
\qquad
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(d \in J)\ (e \in J)(f \in J)\ (d\,P\,e)(e\,P\,f)}{(d|e)\ (e|f)}}{d|f}}{d\,P\,f}}{(\forall d \in J)\ (\forall e \in J)\ (\forall f \in J)\left(\frac{(d\,P\,e)\ (e\,P\,f)}{d\,P\,f}\right)}}{Trns(P)}$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$Po(P)$$

3a.ii.o $Po(Q)$

$$\cfrac{\cfrac{\cfrac{\cfrac{a \in E}{((a,a) \in E \times E) \quad (a|a)}}{a\,Q\,a}}{\forall a \in E\ (a\,Q\,a)}}{Rflx(Q)}
\qquad
\cfrac{\cfrac{\cfrac{\cfrac{(b \in E)\ (c \in E)\ (b\,Q\,c)\ (c\,Q\,b)}{(b|c) \quad (c|b)}}{b = c}}{(\forall b \in E)\ (\forall c \in E)\ \left(\frac{(b\,Q\,c)(c\,Q\,b)}{b=c}\right)}}{AntSym(Q)}
\qquad
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(d \in E)(e \in E)(f \in E)\ (d\,Q\,e)(e\,Q\,f)}{d|e \quad e|f}}{d|f}}{d\,Q\,f}}{(\forall d \in E)\ (\forall e \in E)(\forall f \in E)\left(\frac{(d\,Q\,e)(e\,Q\,f)}{d\,Q\,f}\right)}}{Trns(Q)}$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$Po(Q)$$

5 (3c) Define: $Prtn(S) :=$ the set of partitions of S and

$EqvRln(S) :=$ the set of equivalence relations on S

Draw the digraph of equivalence relations on the facing side and give them names: E_n where $n \in \mathbb{N}$ using as many n's as you need and list these names on this side.

Determine the following:

2 (i) $\nu(Prtn(B)) = 15$

$Prtn(B)$

$$= \left\{ \begin{array}{l} \{\{2\},\{4\},\{6\},\{8\}\}, \{\{2,4\},\{6\},\{8\}\}, \{\{2,6\},\{4\},\{8\}\}, \\ \{\{2,8\},\{4\},\{6\}\}, \{\{4,6\},\{2\},\{8\}\}, \{\{4,8\},\{2\},\{6\}\}, \\ \{\{6,8\},\{2\},\{4\}\}, \{\{2,4\},\{6,8\}\}, \{\{2,8\},\{4,6\}\}, \\ \{\{2,6\},\{4,8\}\}, \{\{2,4,6\},\{8\}\}, \{\{2,4,8\},\{6\}\}, \\ \{\{2,6,8\},\{4\}\}, \{\{4,6,8\},\{2\}\}, \{\{2,4,6,8\}\} \end{array} \right\}$$

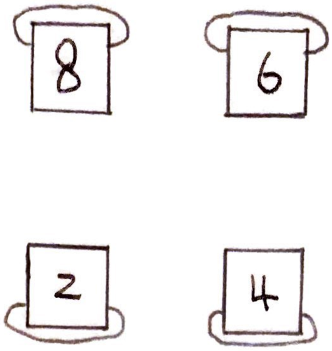
3 (ii) $\nu(EqnRln(B)) = 15$

8

$EqnRln(B)$

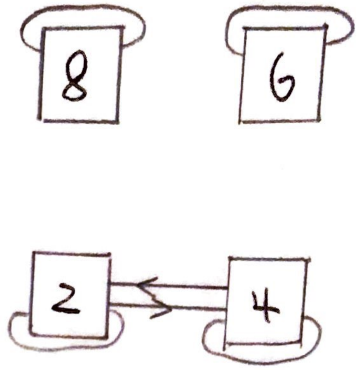
$$= \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10}, E_{11}, E_{12}, E_{13}, E_{14}, E_{15}\}$$

3c.ii.0



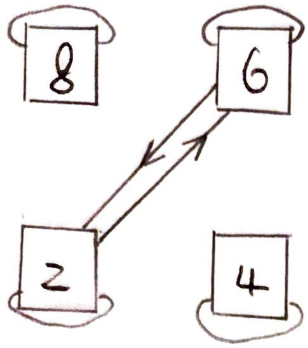
DG(E_1)

3c.ii.1



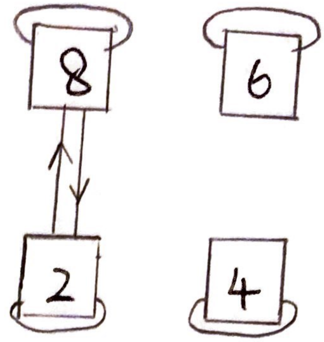
DG(E_2)

3c.ii.2



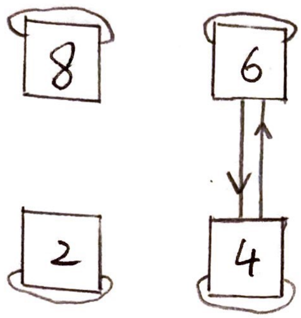
DG(E_3)

3c.ii.3



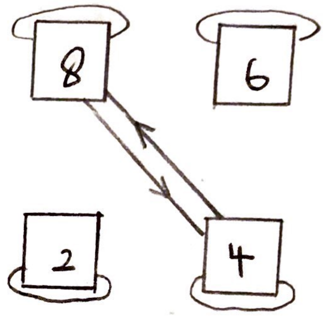
DG(E_4)

3c.ii.4



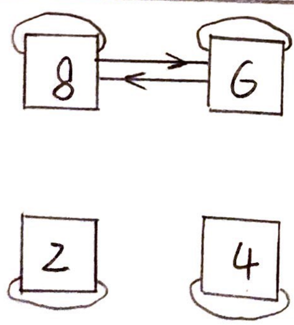
DG(E_5)

3c.ii.5



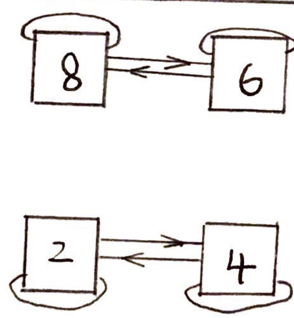
DG(E_6)

3c.ii.6



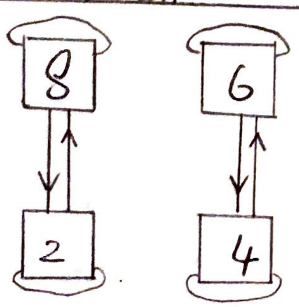
DG(E_7)

3c.ii.7



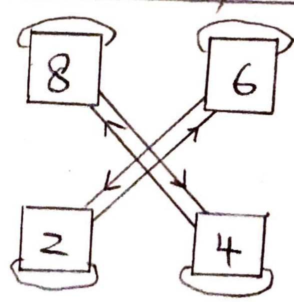
DG(E_8)

3c.ii.8



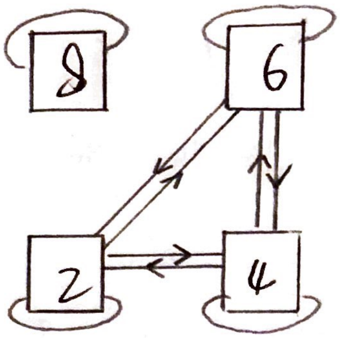
DG(E_9)

3c.ii.9



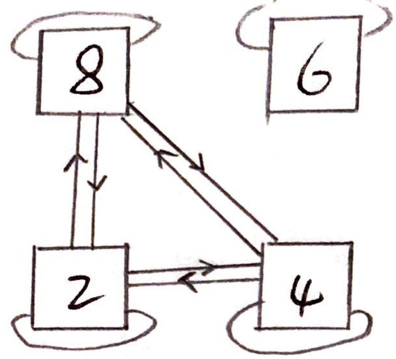
DG(E_{10})

3c.ii.10



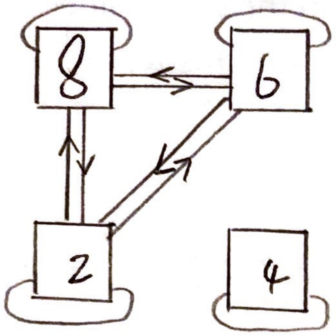
DG(E_{11})

3c.ii.11



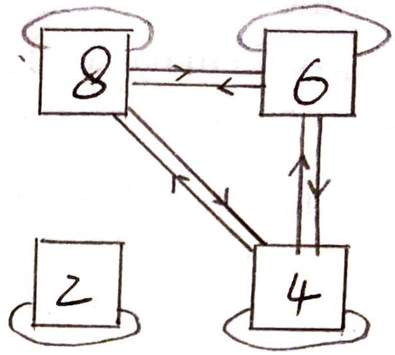
DG(E_{12})

3c.ii.12



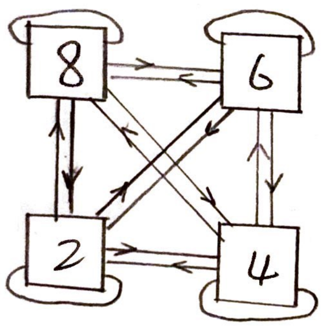
DG(E_{13})

3c.ii.13



DG(E_{14})

3c.ii.14



DG(E_{15})