6 (1a) Find the truth-tables for:

2 (i) $\left(p \vee (\neg q)\right) \rightarrow (\neg p)$

p	q	$\neg p$	$\neg q$	$p \vee (\neg q)$	$(p \vee (\neg q)) \rightarrow (\neg p)$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	1	0
1	1	0	0	1	0

4 (ii) $\left(q \rightarrow p\right) \rightarrow \left(r \rightarrow p\right)$

p	q	r	$q \rightarrow p$	$r \rightarrow p$	$(q \rightarrow p) \rightarrow (r \rightarrow p)$
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

6

6 (1b) Find truth-tables for the following expressions:

2 (i) $LS := \Big((p \longrightarrow q) \longrightarrow r\Big)$,

2 (ii) $RS := \Big(q \longrightarrow (r \longrightarrow p)\Big)$, and

2 (iii) decide, if $\Big((p \longrightarrow q) \longrightarrow r\Big) \Rrightarrow \Big(q \longrightarrow (r \longrightarrow p)\Big)$

p	q	r	$p \longrightarrow q$	$r \longrightarrow p$	$(p \longrightarrow q) \longrightarrow r$	$q \longrightarrow (r \longrightarrow p)$	$LS \Rrightarrow RS$
0	0	0	1	1	0	1	1
0	0	1	1	0	1	1	1
0	1	0	1	1	0	1	1
0	1	1	1	0	1	0	$\boxed{0}$
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

$\Big((p \longrightarrow q) \longrightarrow r\Big) \Rrightarrow \Big(q \longrightarrow (r \longrightarrow p)\Big)$ Y N

Why? See the box 0.

boxed

6

5 (1c) Decide if:

2 (i) $$Vld\left(\frac{p_0}{(p_0)\vee(p_1)}\right)$$ Y N

3 (ii) $$Vld\left(\frac{(p_0)\wedge(p_1)}{(p_1)\vee(p_2)}\right)$$ Y N

5

1c.i

P_0	P_1	$P_0 \vee P_1$	$P_0 \to (P_0 \vee P_1)$
0	0	0	1
0	1	1	1
1	0	1	$\boxed{0}$
1	1	1	1

$\cancel{\mathrm{Tlg}}(P_0 \to (P_0 \vee P_1))$

$\cancel{\mathrm{Vld}}\left(\frac{P_0}{(P_0)\vee(P_1)}\right)$

1c.ii

P_0	P_1	P_2	$(P_0)\wedge(P_1)$	$(P_1)\vee(P_2)$	$((P_0)\wedge(P_1)) \to ((P_1)\vee(P_2))$
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	1	1	1
1	1	1	1	1	1

$\mathrm{Tlg}(((P_0)\wedge(P_1)) \to ((P_1)\vee(P_2)))$

$\mathrm{Vld}\left(\frac{(P_0)\wedge(P_1)}{(P_1)\vee(P_2)}\right)$

6 (2a) Find $\varphi(p,q,r)$ in terms of $p,q,r,$ and connectives, if $\varphi(p,q,r)$ is to have the following truth table:

p	q	r	$\varphi(p,q,r)$	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	0	1	0	0	0	1				
0	0	1	1	0	0	1	0				
0	1	0	0	0	0	0	0				
0	1	1	0	0	0	0	0				
1	0	0	1	0	1	0	0				
1	0	1	0	0	0	0	0				
1	1	0	0	0	0	0	0				
1	1	1	1	1	0	0	0				

$\varphi(p,q,r)$

$$= \varphi_1 \vee \varphi_2 \vee \varphi_3 \vee \varphi_4$$

$$= (p \wedge q \wedge r) \vee (p \wedge (\neg p) \wedge (\neg r)) \vee ((\neg p) \wedge (\neg q) \wedge r) \vee (\neg p) \wedge (\neg q) \wedge (\neg r)$$

6

6 (2b) Show work on the facing side to decide, **using the algebra of propositions,** if the following hold and circle the appropriate choice:

1 (i) $\Big((p \longrightarrow p) \longrightarrow q\Big) = \mathrm{T}$ Y N (Circle the correct choice)

1 (ii) $\Big((p \longrightarrow q) \longrightarrow q\Big) = \mathrm{T}$ Y N (Circle the correct choice)

1 (iii) $\Big((p \longrightarrow q) \longrightarrow p\Big) = \mathrm{T}$ Y N (Circle the correct choice)

1 (iv) $\Big(p \longrightarrow (p \longrightarrow q)\Big) = \mathrm{T}$ Y N (Circle the correct choice)

1 (v) $\Big(p \longrightarrow (q \longrightarrow q)\Big) = \mathrm{T}$ Y N (Circle the correct choice)

1 (vi) $\Big(p \longrightarrow (q \longrightarrow p)\Big) = \mathrm{T}$ Y N (Circle the correct choice)

6

2b.i $((P\to P)\to q)=T$

$(P\to P)\to q$	$x\to y=(\neg x)\vee y$
$=\neg(P\to P)\vee q$	$x\to y=(\neg x)\vee y$
$=\neg((\neg P)\vee P)\vee q$	$\neg(x\vee y)=(\neg x)\wedge(\neg y)$
$=\neg(\neg P)\wedge(\neg P)\vee q$	$\neg(\neg x)=x$
$=P\wedge(\neg P)\vee q$	$x\wedge(\neg x)=\bot$
$=\bot\vee q$	$\bot\vee x=x$
$=q$	
$=DNF((P\to P)\to q)$	

$DNF((P\to P)\to q)=q\neq T=DNF(T)$

$((P\to P)\to q)\neq T$

2b.ii $((P\to q)\to q)=T$

$(P\to q)\to q$	$x\to y=(\neg x)\vee y$
$=\neg(P\to q)\vee q$	$x\to y=(\neg x)\vee y$
$=\neg((\neg P)\vee q)\vee q$	$\neg(x\vee y)=(\neg x)\wedge(\neg y)$
$=\neg(\neg P)\wedge(\neg q)\vee q$	$\neg(\neg x)=x$
$=P\wedge(\neg q)\vee q$	$x\vee(\neg x)=T$
$=P\wedge T$	$x\wedge T=x$
$=P$	
$=DNF((P\to q)\to q)$	

$DNF((P\to q)\to q)=P\neq T=DNF(T)$

$((P\to q)\to q)\neq T$

2b.iii $((P\to q)\to P)=T$

$(P\to q)\to P$	$x\to y=(\neg x)\vee y$
$=\neg(P\to q)\vee P$	$x\to y=(\neg x)\,y$
$=\neg((\neg P)\vee q)\vee P$	$\neg(x\vee y)=(\neg x)\wedge(\neg y)$
$=(\neg(\neg P)\wedge(\neg q)\vee P$	$\neg(\neg x)=x$
$=P\wedge(\neg q)\vee P$	
$=DNF((P\to q)\to P)$	

$DNF((P\to q)\to P)=P\wedge(\neg q)\vee P\neq T=DNF(T)$

$(P\to q)\to P\neq T$

3b.iv $P\to(P\to q))=T$

$P\to(P\to q)$	$x\to y=(\neg x)\vee y$
$=\neg P\vee(P\to q)$	$x\to y=(\neg x)\vee y$
$=(\neg P)\vee(\neg P)\vee q$	$^{2}\ (\neg p)\cup q$
$=DNF(P\to(P\to q))$	

$DNF(P\to(P\to q))=((\neg P)\vee(\neg P)\vee q)\neq T=DNF(T)$

$(P\to(P\to q))\neq T$

3b.v $(P\to(q\to q))=T$

LS	
$=P\to(q\to q)$	$x\to y=(\neg x)\vee y$
$=(\neg P)\vee(q\to q)$	$x\to y=(\neg x)\vee y$
$=(\neg P)\vee((\neg q)\vee q)$	$(\neg x)\vee x=T$
$=(\neg P)\vee T$	$x\vee T=T$
$=T$	
$=RS$	

$LS=(P\to(q\to q))=T=RS$

$LS=RS$

3b.vi $(P\to(q\to P))=T$

LS	
$=P\to(q\to P)$	$x\to y=(\neg x)\vee y$
$=(\neg P)\vee(q\to P)$	$x\to y=(\neg x)\vee y$
$=(\neg P)\vee(\neg q)\vee P$	$x\vee y=y\vee x$
$=(\neg P)\vee P\vee(\neg q)$	$(\neg x)\vee x=T$
$=T\vee(\neg q)$	$T\vee x=T$
$=T$	
$=RS$	

$LS=(P\to(q\to P))=T=RS$

$LS\equiv RS$

6 (3a) Show work on the facing side to decide if the following proposition is a tautology, contradiction, or contingency, and circle an appropriate answer.

1 (i) $\left((p \longrightarrow p) \longrightarrow q\right)$ $\quad Tlg(\psi) \quad Cdn(\psi) \quad Cng(\psi)$

1 (ii) $\left((p \longrightarrow q) \longrightarrow q\right)$ $\quad Tlg(\psi) \quad Cdn(\psi) \quad Cng(\psi)$

1 (iii) $\left((p \longrightarrow q) \longrightarrow p\right)$ $\quad Tlg(\psi) \quad Cdn(\psi) \quad Cng(\psi)$

1 (iv) $\left(p \longrightarrow (p \longrightarrow q)\right)$ $\quad Tlg(\psi) \quad Cdn(\psi) \quad Cng(\psi)$

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1 (v) $\left(p \longrightarrow (q \longrightarrow q)\right)$ $\quad Tlg(\psi) \quad Cdn(\psi) \quad Cng(\psi)$

1 (vi) $\left(p \longrightarrow (q \longrightarrow p)\right)$ $\quad Tlg(\psi) \quad Cdn(\psi) \quad Cng(\psi)$

3a.i $(p \to p) \to q$

p	q	$p \to p$	$(p \to p) \to q$
0	0	1	0
0	1	1	1
1	0	1	$\boxed{0}$
1	1	1	$\boxed{1}$

3a.ii $(p \to q) \to q$

p	q	$p \to q$	$(p \to q) \to q$
0	0	1	$\boxed{0}$
0	1	1	$\boxed{1}$
1	0	0	1
1	1	1	1

3a.iii $(p \to q) \to p$

p	q	$p \to q$	$(p \to q) \to p$
0	0	1	0
0	1	1	$\boxed{0}$
1	0	0	$\boxed{1}$
1	1	1	1

3a.iv $p \to (p \to q)$

p	q	$p \to q$	$p \to (p \to q)$
0	0	1	1
0	1	1	1
1	0	0	$\boxed{0}$
1	1	1	$\boxed{1}$

3a.v $p \to (q \to q)$

p	q	$q \to q$	$p \to (q \to q)$
0	0	1	$\boxed{1}$
0	1	1	$\boxed{1}$
1	0	1	$\boxed{1}$
1	1	1	$\boxed{1}$

3a.vi $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
0	0	1	$\boxed{1}$
0	1	0	$\boxed{1}$
1	0	1	$\boxed{1}$
1	1	1	$\boxed{1}$