3a.i $(p \to p) \to q$

p	q	$p \to p$	$(p \to p) \to q$
0	0	1	0
0	1	1	1
1	0	1	0
1	1	1	1

3a.ii $(p \to q) \to q$

p	q	$p \to q$	$(p \to q) \to q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	1	1

3a.iii $(p \to q) \to p$

p	q	$p \to q$	$(p \to q) \to p$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

3a.iv $p \to (p \to q)$

p	q	$p \to q$	$p \to (p \to q)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

3a.v $p \to (q \to q)$

p	q	$q \to q$	$p \to (q \to q)$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	1	1

3a.vi $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	1	1

6 (1a) Find $\varphi(p,q,r)$ in terms of: p, q and r if $\varphi(p,q,r)$ has the following truth table:

p	q	r	$\varphi(p,q,r)$	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	0	$T_1 = 0$	0	0	0	0	0	0		
0	0	1	$T_2 = 0$	0	0	0	0	0	0		
0	1	0	$T_3 = 1$	0	0	0	0	0	1		
0	1	1	$T_4 = 1$	0	0	0	0	1	0		
1	0	0	$T_5 = 1$	0	0	0	1	0	0		
1	0	1	$T_6 = 1$	0	0	1	0	0	0		
1	1	0	$T_7 = 1$	0	1	0	0	0	0		
1	1	1	$T_8 = 1$	1	0	0	0	0	0		

4(i) $\varphi(p,q,r) = (\varphi_1)\vee(\varphi_2)\vee(\varphi_3)\vee(\varphi_4)\vee(\varphi_5)\vee(\varphi_6)$

$= (p\wedge q\wedge r)$
$\vee (p\wedge q\wedge(\neg r))$
$\vee (p\wedge(\neg q)\wedge r)$
$\vee (p\wedge(\neg q)\wedge(\neg r))$
$\vee ((\neg p)\wedge q\wedge r)$
$\vee ((\neg p)\wedge q\wedge(\neg r))$

6

1(ii) Find the version of $F(A,B,C)$ of $\varphi(p,q,r)$ in Sets:

$F(A,B,C) = (A\cap B\cap C)\cup(A\cap B\cap(C)^c)\cup(A\cap(B)^c\cap C)$
$\cup(A\cap(B)^c\cap(C)^c)\cup((A)^c\cap B\cap C)\cup((A)^c\cap B\cap(C)^c)$

1(~~ii~~ iii) Find the version of $E(x,y,z)$ of $\varphi(p,q,r)$ in BA:

$E(x,y,z) = xyz + xyz' + xy'z$
$+ xy'z' + x'yz + x'yz'$

1a.i

$$\varphi(p,q,r)$$

$$
\begin{aligned}
= \varphi_1 &= (p \wedge q \wedge r) \\
+ \varphi_2 &\quad \vee (p \wedge q \wedge (\neg r)) \\
+ \varphi_3 &\quad \vee (p \wedge (\neg q) \wedge r) \\
+ \varphi_4 &\quad \vee (p \wedge (\neg q) \wedge (\neg r)) \\
+ \varphi_5 &\quad \vee ((\neg p) \wedge q \wedge r) \\
+ \varphi_6 &\quad \vee ((\neg p) \wedge q \wedge (\neg r))
\end{aligned}
$$

1a.ii

$$F(A,B,C)$$

$$
\begin{aligned}
= &(A \cap B \cap C) \\
&\cup (A \cap B \cap (C)^c) \\
&\cup (A \cap (B)^c \cap C) \\
&\cup (A \cap (B)^c \cap (C)^c) \\
&\cup ((A)^c \cap B \cap C) \\
&\cup ((A)^c \cap B (C)^c)
\end{aligned}
$$

1a.iii

$$E(x,y,z)$$

$$
\begin{aligned}
= &(x * y * z) &= &\; xyz \\
&+ (x * y * (z)') & &+ xyz' \\
&+ (x * (y)' * z) & &+ xy'z \\
&+ (x * (y)' * (z)') & &+ xy'z' \\
&+ ((x)' * y * z) & &+ x'yz \\
&+ ((x)' * y * (z)') & &+ x'yz'
\end{aligned}
$$

6 (1b) Use the consensus method to find the following for your answer from (1a):

$$E(x, y, z) = xyz + xyz' + xy'z' + x'yz + \cancel{x'yz} + x'yz'$$

3 (i) the prime implicants of E: $PI(E) = \{x, y\}$

3 (i) the minimal sum-of-products form: $MSOP(E) = x + y$

6

You are free to talk to anyone

(1b.i.a) $E(x,y,z)$

$= xyz + xyz' + xy'z + xy'z' + x'yz + x'yz'$

$= xyz + xyz' + xy'z + xy'z' + x'yz + x'yz' + xy$

$= xy'z + xy'z' + x'yz + x'yz' + xy$

$= xy'z + xy'z' + x'yz + x'yz' + xy + xy'$

$= x'yz + x'yz' + xy + xy'$

$= x'yz + x'yz' + xy + xy' + x'y$

$= xy + xy' + x'y$

$= xy + xy' + x'y + x$

$= x'y + x$

$= x'y + x + y$

$= x + y$

$(x \not\subseteq y,\ y \not\subseteq x)$

~~Cns~~(x,y)

$PI(E) = \{x, y\}$

$(cns(xyz, xyz') = xy)$

$(xy \subseteq xyz, xy \subseteq xyz')$

$(cns(xy'z, xy'z') = xy')$

$(xy' \subseteq xy'z,\ xy' \subseteq xy'z')$

$(cns(x'yz, x'yz') = x'y)$

$(x'y \subseteq x'yz,\ x'y \subseteq x'yz')$

$(cns(xy, xy') = x)$

$(x \subseteq xy,\ x \subseteq xy')$

$(cns(x'y, x) = y)$

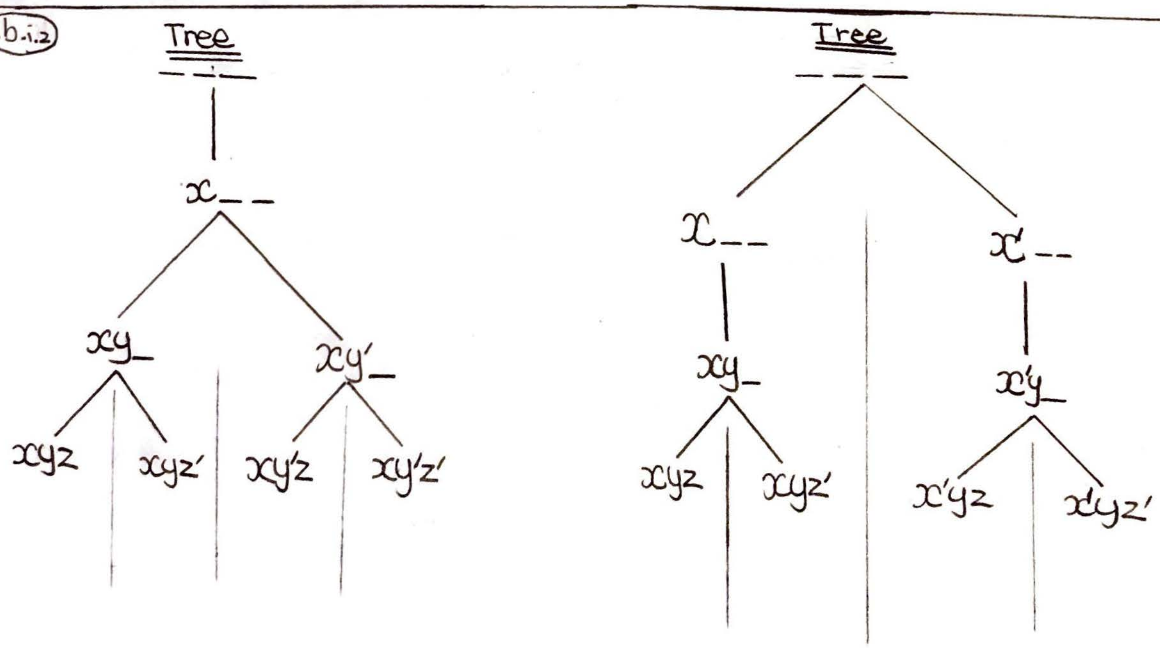
$(y \subseteq x'y)$

(1b.ii) E

$= x = x(1)(1) = x(y+y')(z+z') = xyz + xyz' + xy'z + xy'z' = x = MSOP(E)$

$+y \mid +(1)y(1) \mid +(x+x')y(z+z') \mid + xyz + xyz' + x'yz + x'yz' \mid + y$

(1b.i.2)



5 (1c) From (1b) $MSOP\ (E) = x + y$

1 (i) Find the version $\psi(p,q,r)$ of $MSOP\ (E)$ in PC

$$\psi(p,q,r) = p \vee q$$

4 (ii) **Prove in PC** that your answer from (1a) $\varphi(p,q,r) = \psi(p,q,r)$

8

c.ii

$$
\begin{aligned}
&\varphi(p,q,r)\\
&=(p\wedge q\wedge r)\vee(p\wedge q\wedge(\neg r))\vee(p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q\wedge r)\vee((\neg p)\wedge q\wedge(\neg r)) && \mid (x\wedge y)\vee(x\wedge z)=x\wedge(y\vee z)\\
&=((p\wedge q)\wedge(r\vee(\neg r)))\vee((p\wedge(\neg q))\wedge(r\vee(\neg r)))\vee(((\neg p)\wedge q)\wedge(r\vee(\neg r))) && \mid (x\vee(\neg x)=T)\\
&=((p\wedge q)\wedge T)\vee((p\wedge(\neg q))\wedge T)\vee(((\neg p)\wedge q)\wedge T) && \mid (x\wedge T=x)\\
&=(p\wedge q)\vee(p\wedge(\neg q))\vee((\neg p)\wedge q) && \mid ((x\wedge y)\vee(x\wedge z)=x\wedge(y\vee z))\\
&=(p\wedge(q\vee(\neg q)))\vee((\neg p)\wedge q) && \mid (x\vee(\neg x)=T)\\
&=(p\wedge T)\vee((\neg p)\wedge q) && \mid (x\wedge T=x)\\
&=p\vee((\neg p)\wedge q) && \mid (x\vee(y\wedge z)=(x\vee y)\wedge(x\vee z))\\
&=(p\vee(\neg p))\wedge(p\vee q) && \mid (x\vee(\neg x)=T)\\
&=T\wedge(p\vee q) && \mid T\wedge x=x\\
&=p\vee q
\end{aligned}
$$

$$\varphi(p,q,r)=p\vee q=\psi(p,q,r)$$

$$\varphi(p,q,r)=\psi(p,q,r)$$

6 (2a) For the expression from page 53 below:

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

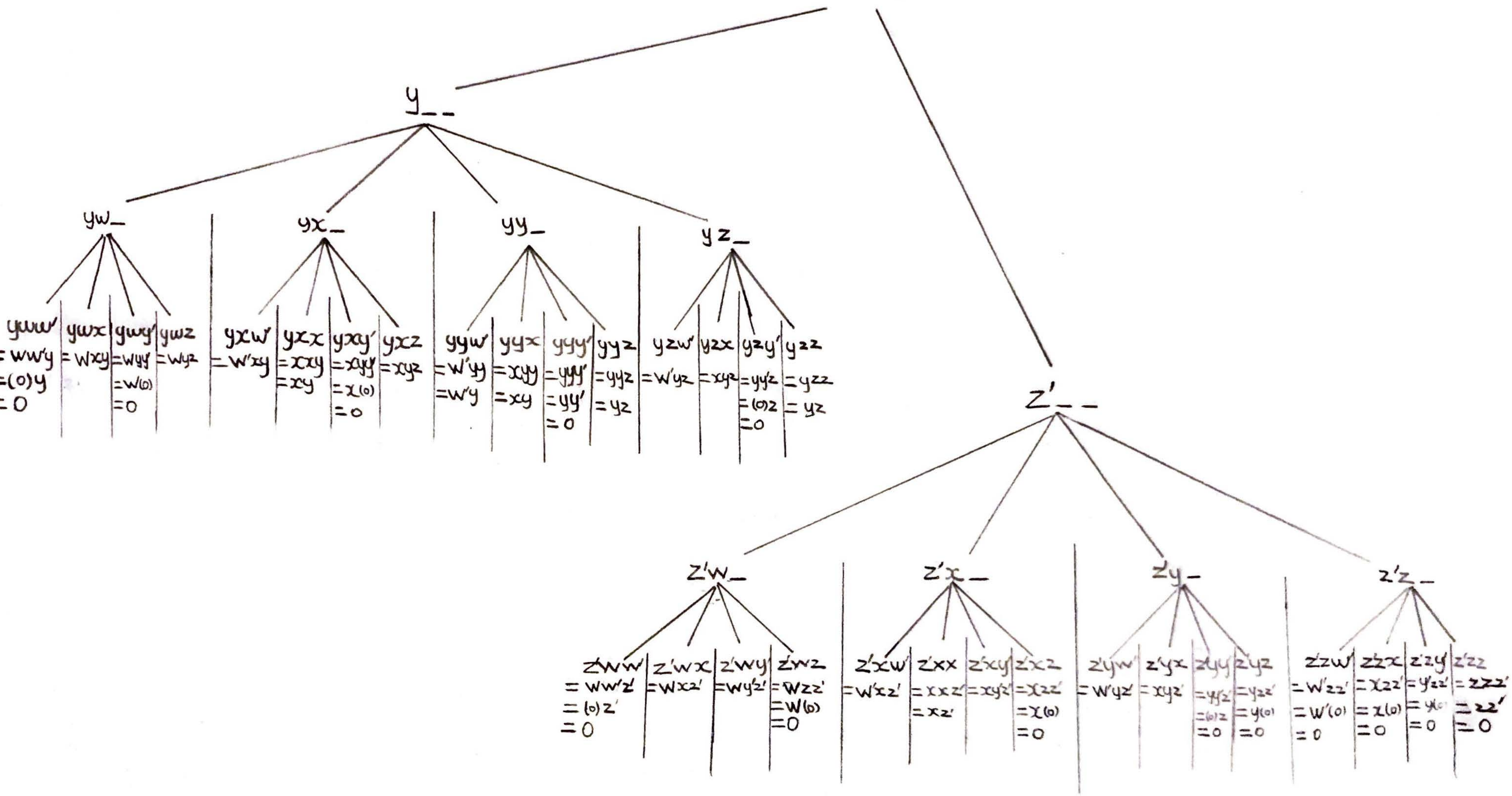
find a sum-of-products form, $SOP(G)$

$$SOP(G) \quad = wy'z'+w'y+xy+xz'+yz$$

Show the tree on the facing side.

8

Tree



y_ _
yw_
yx_
yy_
yz_
ywwʹ = wwʹy = (0)y = 0
ywx = wxy
ywyʹ = wyyʹ = w(0) = 0
ywz = wyz
yxwʹ = wʹxy
yxx = xxy = xy
yxyʹ = xyyʹ = x(0) = 0
yxz = xyz
yywʹ = wʹyy = wʹy
yyx = xyy = xy
yyyʹ = yyyʹ = yyʹ = 0
yyz = yyz = yz
yzwʹ = wʹyz
yzx = xyz
yzyʹ = yyʹz = (0)z = 0
yzz = yzz = yz
zʹ_ _
zʹw_
zʹx_
zʹy_
zʹz_
zʹwwʹ = wwʹzʹ = (0)zʹ = 0
zʹwx = wxzʹ
zʹwy = wyzʹ
zʹwz = wzzʹ = w(0) = 0
zʹxwʹ = wʹxzʹ
zʹxx = xxzʹ = xzʹ
zʹxy = xyzʹ
zʹxz = xzzʹ = x(0) = 0
zʹywʹ = wʹyzʹ
zʹyx = xyzʹ
zʹyy = yyzʹ = (0)z = 0
zʹyz = yzzʹ = y(0) = 0
zʹzwʹ = wʹzzʹ = wʹ(0) = 0
zʹzx = xzzʹ = x(0) = 0
zʹzy = yzzʹ = y(0) = 0
zʹzz = zzzʹ = zzʹ = 0

2a(ii)

G

$$= wxy + wyz + w'xy + \underbrace{xy}_{1} + \underbrace{xyz}_{2} + w'y + \underbrace{xy}_{1} + \underbrace{yz}_{3} + w'yz + \underbrace{xyz}_{2} + \underbrace{yz}_{3} + wxz' + wy'z' + w'xz' + xz' + xy'z' + w'yz' + xyz' \qquad (s+s=s)$$

$$= wxy + wyz + w'xy + xy + xyz + w'y + yz + w'yz + wxz' + wy'z' + w'xz' + xz' + xy'z' + w'yz' + xyz' \qquad (s+(s*t)=s)$$

$$= xy + wyz + w'y + yz + w'yz + wxz' + wy'z' + w'xz' + xz' + xy'z' + w'yz' \qquad (s+(s*t)=s)$$

$$= xy + wyz + w'y + yz + wxz' + wy'z' + w'xz' + xz' + xy'z' \qquad (s+(s*t)=s)$$

$$= xy + yz + w'y + wxz' + wy'z' + w'xz' + xz' + xy'z' \qquad (s+(s*t)=s)$$

$$= xy + yz + w'y + wy'z' + xz'$$

$$= wy'z' + w'y + xy + xz' + yz$$

$$= SOP(G)$$

6 (2b) For the expression below (**which is the same as in (2a)**):

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

find the complete sum-of-products form, $CSOP(G)$

$$CSOP(G) = wxyz + wxyz' + wxy'z' + wx'yz + wx'y'z' + w'xyz + w'xyz' + w'xy'z' + w'x'yz + w'x'yz'$$

0

(2b.0) SOP(G)

$$= wy'z' + w'y + xy + xz' + yz$$

$$= w(1)y'z' + w'(1)y(1) + (1)xy(1) + (1)x(1)z' + (1)(1)yz$$

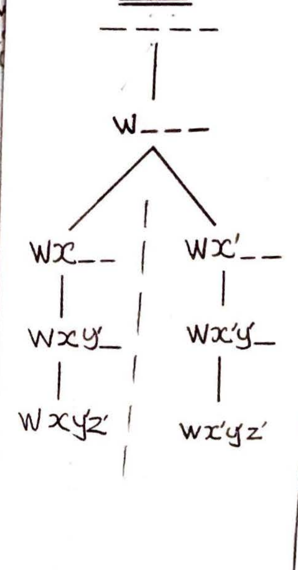
$$= w(x+x')y'z' + w'(x+x')y(z+z') + (w+w')xy(z+z') + (w+w')x(y+y')z' + (w+w')(x+x')yz$$

$$= wxy'z' + wx'y'z' + w'xyz + w'xyz' + w'x'yz + w'x'yz' + wxyz + wxyz' + w'xyz + w'xyz' + wxyz' + wxy'z' + w'xyz' + w'xy'z' + wxyz + wx'yz + w'xyz + w'x'yz$$

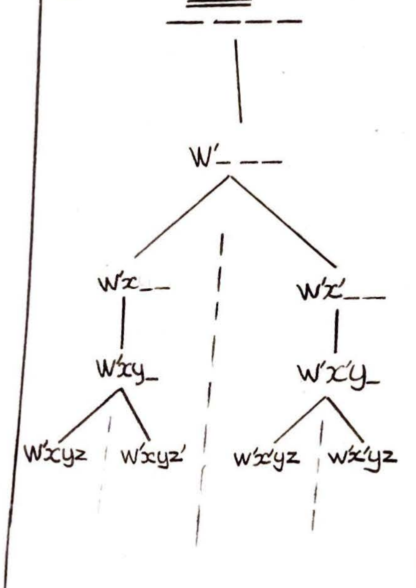
$$= wxy'z' + wx'y'z' + w'xyz + w'xyz' + w'x'yz + w'x'yz' + wxyz + wxyz' + w'xy'z' + wx'yz$$

$$= wxyz + wxyz' + wxy'z' + wx'yz + wx'y'z' + w'xyz + w'xyz' + w'xy'z' + w'xyz + w'x'yz' = CSOP(E)$$

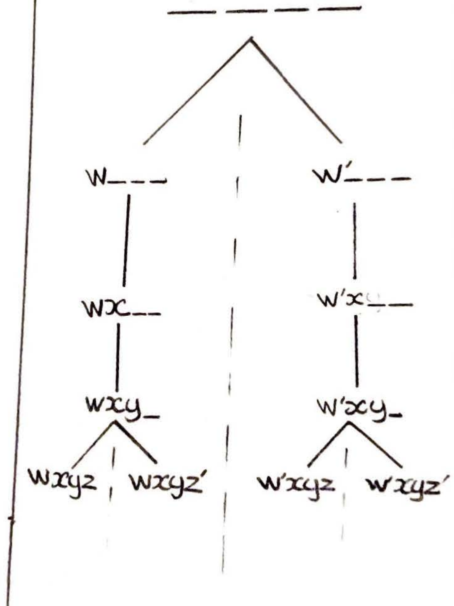
(2b.1) **Tree**



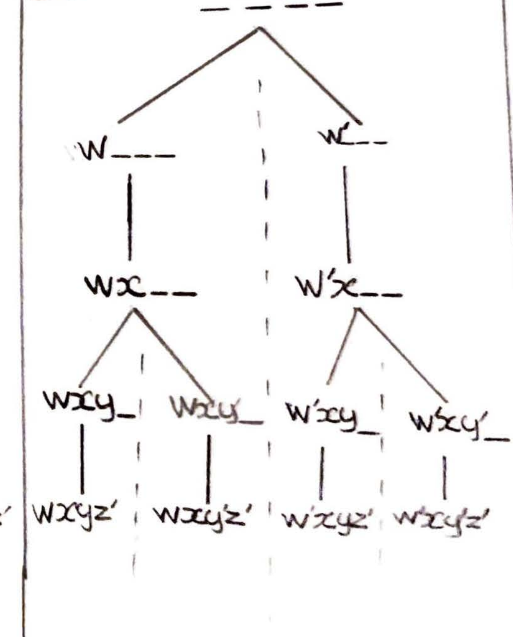
(2b.2) **Tree**



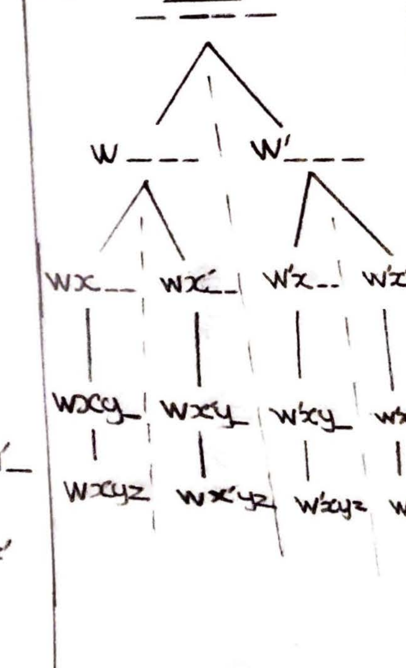
(2b.3) **Tree**



(2b.4) **Tree**



(2b.5) **Tree**



5 (2c) For the expression below **which is the same as in (2a) and (2b)**:

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

use your answer from (2a) and (2b) to find the following:

$PI(G)$ and $MSOP\ (G)$

Answer from (2a) $SOP(G) = wy'z' + w'y + xy + xz' + yz$

Answer from (2b) $CSOP\ (G) = wxyz + wxyz' + wxy'z' + wx'yz + wx'y'z' + w'xyz + w'xyz' + w'xy'z' + w'x'yz + w'x'yz'$

2 (i) $PI(G) = \{wy'z', w'y, xy, xz', yz\}$

3 (ii) $MSOP\ (G) = wy'z' + w'y + xz' + yz$

8

2c.ii

SOP(G)

$= wy'z' + w'y + xy + xz' + yz$

$PI(G) = \{wy'z', w'y, xy, xz', yz\}$

2c.i.i

$wy'z' \$ w'y$, $wy'z' \$ xy$, $wy'z' \$ xz'$, $wy'z' \$ yz$, $w'y \$ wy'z'$, $w'y \$ xy$, $w'y \$ xz'$, $w'y \$ yz$, $xy \$ wy'z'$, $xy \$ w'y$, $xy \$ xz'$, $xy \$ yz$, $xz' \$ wy'z'$, $xz' \$ w'y$, $xz' \$ xy$, $xz' \$ yz$, $yz \$ wy'z'$, $yz \$ w'y$, $yz \$ xy$, $yz \$ xz'$

$cns(xz', yz) = xy$

2c.ii

SOP(G)

$$
\begin{array}{l|l|l|l|l}
= wy'z' & = w(1)y'z' & = w(x+x')y'z' & = \underline{wxy'z'} + wx'y'z' & = wy'z' \quad = MSOP(G) \\
+ w'y & + w'(1)y(1) & + w'(x+x')y(z+z') & + \underline{w'xyz} + \underline{w'xyz'} + \underline{w'x'yz} + w'x'yz' & + w'y \\
+ xy & + (1)xy(1) & + (w+w')xy(z+z') & + \underline{wxyz} + \underline{wxyz'} + \underline{w'xyz} + \underline{w'xyz'} & + \\
+ xz' & + (1)x(1)z' & + (w+w')x(y+y')z' & + \underline{wxyz'} + \underline{wxy'z'} + \underline{w'xyz} + w'xy'z' & + xz' \\
+ yz & + (1)(1)yz & + (w+w')(x+x')yz & + \underline{wxyz} + wx'yz + \underline{w'xyz} + \underline{w'x'yz} & + yz
\end{array}
$$

3c.ii

Tree

Note: Please see 2b

6 (3a) We define:

$$A := SOP(G(w,x,y,z)) = wy'z' + w'y + xy + xz' + yz$$

$$B = MSOP(G(w,x,y,z)) = wy'z' + w'y + xz' + yz$$

Copy your answers from (1a)

Compute:

1 (i) $A_S = 5$ $A_L = 11$ $B_S = 4$ $B_L = 9$

1 (ii) On the basis of your answers to (i), circle the correct choice below.

Smplr (A, B) *Smplr*(B, A) Neither

Why?

4 (iii) Write down the version of $G(w,x,y,z)$ in PC below and find its **formation-tree on the facing side.**

Version of $G(w,x,y,z)$in PC

$= (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee (\neg r) \vee s)$

0

3a.i.0 Smnd (A)

$= \{wy'z', w'y, xy, xz', yz\}$

3a.i.1 As

$= \nu(\text{Smnd}(A))$

$= 5$

3a.i.2 MSLtr(A)

$= \{w{:}1, w'{:}1, x{:}2, y{:}3, y'{:}1, z{:}1, z'{:}2\}$

3a.i.3 A_L

$= \nu(\text{MSLtr}(A))$

$= 1+1+2+3+1+1+2$

$= 11$

3a.i.4 Smnd (B)

$= \{wy'z', w'y, xz', yz\}$

3a.i.5 Bs

$= \nu(\text{Smnd}(B))$

$= 4$

34.i.6 MSLtr (B)

$= \{w{:}1, w'{:}1, x{:}1, y{:}2, y'{:}1, z{:}1, z'{:}2\}$

3a.i.7 B_L

$= \nu(\text{MSLtr}(B))$

$= 1+1+1+2+1+1+2$

$= 9$

3a.ii

$$\frac{\dfrac{(B_L = 9 < 11 = A_L) \wedge (B_s = 4 \leqslant 11 = A_L)}{(B_L < A_L) \wedge (B_s \leqslant A_L)}}{\text{Smplr}(B, A)}$$

3a.iii

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z$$

$$G(w,xy,z) = (y+(z'))*(w+x+y+z)*((w')+x+(y')+z)$$

$$G(p,q,r,s) = (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee (\neg r) \vee s)$$

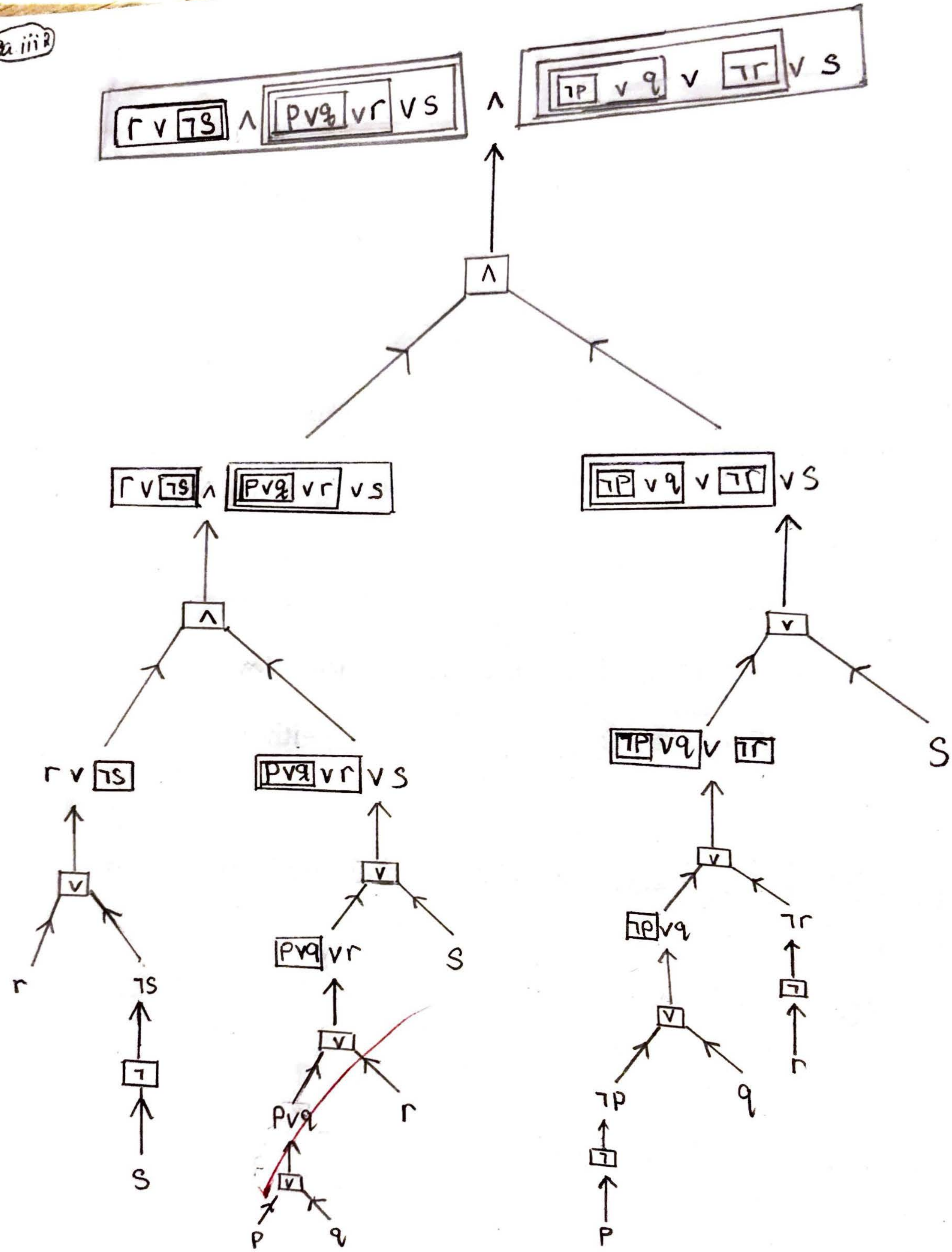
3a iii.1

$G(w,x,y,z)$

$= (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee (\neg r) \vee s)$

$$\boxed{\boxed{r \vee \boxed{\neg s}}} \wedge \boxed{\boxed{\boxed{p \vee q} \vee r} \vee s} \wedge \boxed{\boxed{\boxed{\neg p} \vee q} \vee \boxed{\neg r} \vee s}$$

3a iii 2



2 (3b) Find the versions: $H(A,B,C,D)$ of $G(w,x,y,z)$

and $K(A,B,C,D)$ of $MSOP(G(w,x,y,z))$

in Sets.

1 (i) $H(A,B,C,D)$

$$= (C\cup(D^c))\cap(A\cup B\cup C\cup D)\cap((A^c)\cup B\cup(C^c)\cup D)$$

1 (ii) $K(A,B,C,D)$

$$= (A\cap(C^c)\cap(D^c))\cup((A^c)\cap C)\cup(B\cap(D^c))\cup(C\cap D)$$

3bi)

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

$$G(w,x,y,z) = (y+(z'))*(w+x+y+z)*((w')+x+(y')+z)$$

$$G_{sets}(A,B,C,D) = (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup (C^c) \cup D)$$

3b.ii)

$$MSOP(G(w,x,y,z) = wy'z' + w'y + xz' + yz$$

$$MSOP(G(w,x,y,z) = (w*(y')*(z')) + ((w')*y) + (x*(z')) + (y*z)$$

$$MSOP_{sets}(G(A,B,C,D) = (A \cap (C^c) \cap (D^c)) \cup ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$$

9 (3c) Prove in Sets that:

$$H(A,B,C,D) = K(A,B,C,D)$$

9

(3c)

$$H(A,B,C,D)$$

$$=(C\cup(D^c))\cap(A\cup B\cup C\cup D)\cap((A^c)\cup B\cup(C^c)\cup D)$$

$$=((C\cup(D^c))\cap(A\cup B\cup C\cup D))\cap((A^c)\cup B\cup(C^c)\cup D) \qquad (A\cap B=B\cap A)$$

$$=((A\cup B\cup C\cup D)\cap(C\cup(D^c)))\cap((A^c)\cup B\cup(C^c)\cup D) \qquad (A\cap(B\cup C)=(A\cap B)\cup(A\cap C))$$

$$=(((A\cup B\cup C\cup D)\cap C)\cup((A\cup B\cup C\cup D)\cap(D^c)))\cap((A^c)\cup B\cup(C^c)\cup D) \qquad (A\cap B=B\cap A)$$

$$=((C\cap(A\cup B\cup C\cup D))\cup((D^c)\cap(A\cup B\cup C\cup D)))\cap((A^c)\cup B\cup(C^c)\cup D) \qquad (A\cap(B\cup C)=(A\cap B)\cup(A\cap C))$$

$$=((C\cap A)\cup(C\cap B)\cup(C\cap C)\cup(C\cap D)\cup((D^c)\cap A)\cup((D^c)\cap B)\cup((D^c)\cap C)\cup((D^c)\cap D))\cap((A^c)\cup B\cup(C^c)\cup D) \qquad (A\cap A=A,\ A\cap(A^c)=\{\})$$

$$=(\underline{(A\cap C)}\cup\underline{(B\cap C)}\cup\underline{C}\cup\underline{(C\cap D)}\cup(A\cap(D^c))\cup(B\cap(D^c))\cup\underline{(C\cap(D^c))}\cup(\{\}))\cap((A^c\cup B\cup(C^c)\cup D) \qquad (A\cup\{\}=A,\ A\cup(A\cap B)=A)$$

$$=(C\cup(A\cap(D^c))\cup(B\cap(D^c)))\cap((A^c)\cup B\cup(C^c)\cup D) \qquad (A\cap(B\cup C)=(A\cap B)\cup(A\cap C))$$

$$=C\cap((A^c)\cup B\cup(C^c)\cup D)\cup(A\cap(D^c))\cap((A^c)\cup B\cup(C^c)\cup D)\cup(B\cap(D^c))\cap((A^c\cup B\cup(C^c)\cup D) \qquad (A\cap(B\cup C)=(A\cap B)\cup(A\cap C))$$

$$=(C\cap A^c)\cup(C\cap B)\cup\underline{(C\cap(C^c))}\cup(C\cap D)\cup\underline{(A\cap(D^c)\cap(A^c))}\cup(A\cap(D^c)\cap B\cup(A\cap(D^c)\cap(C^c))\cup\underline{(A\cap(D^c)\cap D)}$$
$$\cup(B\cap(D^c)\cap(A^c))\cup\underline{(B\cap(D^c)\cap B)}\cup(B\cap(D^c)\cap(C^c))\cup\underline{(B\cap(D^c)\cap D)} \qquad (A\cap(A^c)=\{\},\ A\cap(A^c)=\{\})$$

$$=(C\cap(A^c))\cup(C\cap B)\cup(C\cap D)\cup\underline{(A\cap(D^c)\cap B)}\cup(A\cap(D^c)\cap(C^c))\cup\underline{(B\cap(D^c)\cap(A^c))}\cup\underline{(B\cap D^c)}\cup\underline{(B\cap(D^c)\cap(C^c))} \qquad (A\cup(A\cap B)=A)$$

$$=((C\cap(A^c))\cup(C\cap B)\cup(C\cap D)\cup(A\cap(D^c)\cap(C^c)\cup(B\cap(D^c))$$

$$=((A^c)\cap C)\cup(A\cup(A^c))\cap B\cap C\cap(D\cup(D^c)))\cup(C\cap D)\cup(A\cap(D^c)\cap(C^c))\cup(B\cap(D^c)) \qquad (A=A\cap\mathcal{U})$$

$$=\underline{((A^c)\cap C)}\cup\underline{(A\cap B\cap C\cap D)}\cup\underline{(A\cap B\cap C\cap(D^c))}\cup\underline{((A^c)\cap B\cap C\cap D)}\cup\underline{((A^c)\cap B\cap C\cap(D^c))} \qquad (A\cap(B\cup C)=(A\cap B)\cup(A\cap C))$$
$$\cup\underline{(C\cap D)}\cup(A\cap(C^c)\cap(D^c))\cup\underline{(B\cap(D^c))} \qquad A\cup(A\cap B)=A$$

$$=((A^c)\cap C)\cup((C\cap D)\cup(A\cap(C^c))\cap(D^c))\cup(B\cap(D^c))$$

$$H(A,B,C,D)=((A^c)\cap C)\cup((C\cap D)\cup(A\cap(C^c))\cap(D^c))\cup(B\cap(D^c))=K(A,B,C,D)$$

$$H(A,B,C,D)=K(A,B,C,D)$$