6 (1a) Answer the following questions:

Your J-number $JN :=$ 00298436 $:= (J_k | k \in 1..8)$

1 (i) $A := \left\{ k \in 1..8 \middle| J_k \text{ is even} \right\} = \{1,2,3,5,6,8\}$

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
J_k is even	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

1 (ii) $B := \left\{ J_k \in 0..9 \middle| k \text{ is prime} \right\} = \{0,2,3,8\}$

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
k is prime	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

2 (iii) Decide if:

$\{1,2,3,5,6,8\} = A \subseteq B = \{0,2,3,8\}$ Y N

Why? By ~~inspection~~ $(\in LS) \wedge (\notin RS)$

2

2 (iii) Decide if:

$\{0,2,3,8\} = B \subseteq A = \{1,2,3,5,6,8\}$ Y N

Why? ~~By inspection.~~

6 (1b) Compute the following, **showing all work including trees on the facing page or pages provided for work,** and record **ONLY** the answers on this page.

Your J-number $JN :=$ 00298436 $:= (J_k | k \in 1..8)$

$C := \left\{k \in 1..8 \middle| J_k \le 2\right\} = \{1, 2, 3\}$

$D := \left\{J_k \in 0..9 \middle| k \in 1..3\right\} = \{0, 2\}$

1 (i) $C \cup D = \{1,2,3\} \cup \{0,2\} = \{0,1,2,3\}$

1 (ii) $C \cap D = \{1,2,3\} \cap \{0,2\} = \{2\}$

1 (iii) $C \backslash D = \{1,2,3\} \backslash \{0,2\} = \{1, 3\}$ −1

1 (iv) $D \backslash C = \{0,2\} \backslash \{1,2,3\} = \{0\}$ −1

1 (v) $C \times D = \{1,2,3\} \times \{0,2\} = \{(1,0),(1,2),(2,0),(2,2),(3,0),(3,2)\}$

1 (vi) $C \times D = \{1,2,3\} \times \{0,2\} = \{(1,0),(1,2),(2,0),(2,2),(3,0),(3,2)\}$

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(C)

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$J_k \leq 2$	Y N	Y N	Y N	Y N	Y N	Y N	Y N	Y N

(D)

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$k \in 1..3$	Y N	Y N	Y N	Y N	Y N	Y N	Y N	Y N

(i) $\{1, 2, 3\} \cup \{0, 2\}$

$= \{0, 1, 2, 3\}$

(ii) $\{1, 2, 3\} \cap \{0, 2\}$

$= \{2\}$

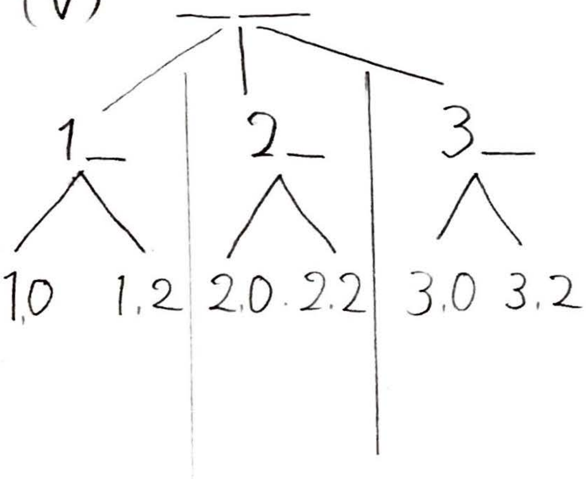
(iii) $\{1, \cancel{2}, 3\} \setminus \{0, 2\}$

$= \{1, 3\}$

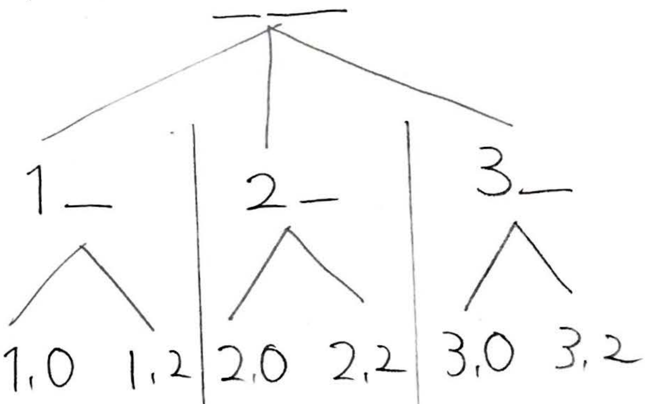
(iv) $\{0, \cancel{2}\} \setminus \{1, 2, 3\}$

$= \{0\}$

(v)



(vi)



5 (1c) Define $\forall X, Y \in \mathcal{U} \quad X \circ Y \quad := \quad (X \cup Y) \setminus (Y \cap X)$

Circle correct choices from among Y, N, Proof, and Witness and provide below a proof or witness as the case may be:

2 (i) $\forall X, Y \in \mathcal{U} \quad \left(X \circ Y = Y \circ X\right)$ Y N

Proof Witness

$$X \circ Y = (X \cup Y) \setminus (Y \cap X)$$
$$= (Y \cup X) \setminus (X \cap Y)$$
$$= Y \circ X$$

Better
LS
=
= RS

-1

3 (ii) $\forall X, Y, Z \in \mathcal{U} \quad \left((X \circ Y) \circ Z = X \circ (Y \circ Z)\right)$ Y N

Proof Witness

$$(X \circ Y) \circ Z = (((X \cup Y) \setminus (Y \cap X) \cup Z) \setminus (Z \cap ((X \cup Y) \setminus (Y \cap X)))$$
$$= (X \cup ((Y \cup Z) \setminus (Z \cap Y)) \setminus (((Y \cup Z) \setminus (Z \cap Y)) \cap X)$$

0

why? ?

$$= X \circ ((Y \cup Z) \setminus (Z \cap Y))$$
$$= X \circ (Y \circ Z)$$

P

6 (2a) Answer the following:

1 (i) $\{a\} = \left\{ a \right\}$ Y N

Why? By inspection.

1 (ii) $\{a\} \neq \left\{ a \right\}$ Y N

Why? By inspection.

1 (iii) $a \in \left\{ a \right\}$ Y N

Why? By inspection.

1 (iv) $\{a\} \notin \left\{ \{a\} \right\}$ Y N

Why? By inspection.

1 (v) $\{a\} \subseteq \left\{ a \right\}$ Y N

Why? By inspection.

1 (vi) $\{a\} \nsubseteq \left\{ \{a\} \right\}$ Y N

Why? By inspection.

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6 (2b) **Show work on facing side and record only the answers on this page.** Define the sequence of sets: $(S_n | n \in \mathbb{N})$ recursively as follows where X is some unknown finite set with:

$$x := \nu(X) \in \mathbb{N}$$

Base case: (B_0) $S_0 := X$

Recursive step: (R) $\forall n \in \mathbb{N}$ $S_{n+1} := \left(S_n\right) \sqcup \left(S_n\right)$

1 (i) Compute: S_1 $= (X \times \{0\}) \cup (X \times \{1\})$

$X \amalg X$

1 (iii) Compute: $\nu(S_1)$ $= 2x$

1 (ii) Compute: S_2 $= (S_1 \times \{0\}) \cup (S_1 \times \{1\})$

1 (iv) Compute: $\nu(S_2)$ $= 4x$

6

2 (v) Find a formula for $\nu(S_n)$ in terms of n and x and prove your result.

$\nu(S_n)$ $= \cancel{2^n x}$

$2n$

labe

Yo

(i) S_1

$= (S_0) \sqcup (S_0)$

$= X \sqcup X$

$= (X \times \{0\}) \cup (X \times \{1\})$

(iii) $\nu(S_1)$

$= \nu(S_0) + \nu(S_0)$

$= 2\nu(X)$

$= 2x$

(ii) S_2

$= (S_1) \sqcup (S_1)$

$= (S_1 \times \{0\}) \cup (S_1 \times \{1\})$

(iv) $\nu(S_2)$

$= \nu(S_1) + \nu(S_1)$

$= 2x + 2x$

$= 4x$

(v) $\nu(S_n)$

$= 2\nu(S_{n-1})$

$= 2 \times 2\nu(S_{n-2})$

$= 2 \times 2 \times 2\nu(S_{n-3})$

$= 2^n \nu(S_0)$

$= 2^n x$

5 (2c) Circle correct choices from among Y, N, Proof, and Witness and provide below a proof or witness as the case may be:

2 (i) $\forall X, Y, Z \in \mathcal{U} \quad \left(X \times Y\right) \cap Z = \left(X \cap Z\right) \times \left(Y \cap Z\right)$

Proof Witness Y N

$X = \{1, 2\}, Y = \{2, 3\}, Z = \{(2, 3)\}$

$(X \times Y) \cap Z = \{(2, 3)\}$,

$(X \cap Z) \times (Y \cap Z) = \{\}$ $\neq$

$\exists X, Y, Z \in \mathcal{U} \quad (X \times Y) \cap Z \neq (X \cap Z) \times (Y \cap Z)$

3 (ii) $\forall X, Y, Z \in \mathcal{U} \quad Z \cup \left(X \times Y\right) = \left(Z \cup X\right) \times \left(Z \cup Y\right)$

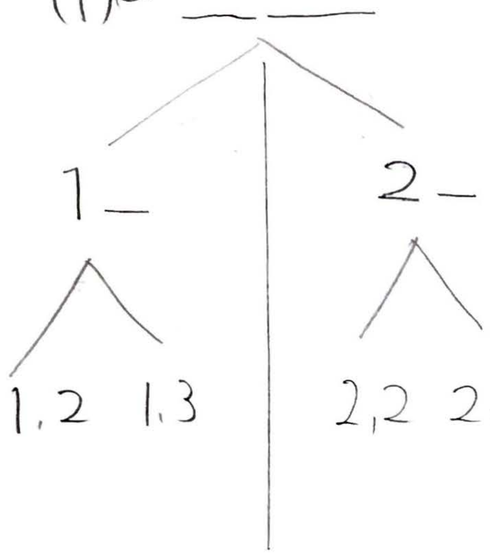
Proof Witness Y N

$\forall X, Y, Z \in \mathcal{U}$

$Z \cup (X \times Y) = (Z \cup X) \times (Z \cup Y)$ (by distribution rule)

2

(i) ⓪



(i) ① $(X \times Y) \cap Z$

$= \{(1,2),(1,3),(2,2),(2,3)\} \cap \{(2,3)\}$

$= \{(2,3)\}$

(i) ② $X \cap Z$

$= \{1,2\} \cap \{(1,2)\}$

$= \{\}$

(i) ③ $Y \cap Z$

$= \{2,3\} \cap \{(1,2)\}$

$= \{\}$

(i) ④ $(X \cap Z) \times (Y \cap Z)$

$= \{\} \times \{\}$

$= \{\}$

6 (3a) Compute the following, **showing all work on the facing page or pages provided for work,** and record **ONLY** the answers on this page.

Your J-number $JN :=$ 00298436 $:= (J_k | k \in 1..8)$

1 (i) $E := \left\{ J_k \in 0..9 \middle| \exists n \in \mathbb{N} \left(J_k = n^3 \right) \right\} = \{ 0, 8 \}$ X

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$\exists n \in \mathbb{N} \left(J_k = n^3 \right)$	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

1 (ii) $F := \left\{ k \in 1..9 \middle| \exists n \in \mathbb{N} \left(k = n^2 \right) \right\} = \{ 1, 4 \}$

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$\exists n \in \mathbb{N} \left(k = n^2 \right)$	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

1 (iii) $E \cap F \quad = \quad \{ \quad \}$ 1

3 (iv) $\mathcal{P}\left(E \cap F \right) \quad = \quad \left\{ \{\} \right\}$

(i) $\{x \mid 1 \le x \le 9 \wedge x = n^3 (n = \mathbb{N})\}$

$= \{1, 8\}$

(ii) $\{y \mid 1 \le y \le 8 \wedge y = n^2 (n = \mathbb{N})\}$

$= \{1, 4\}$

(iii) $E \cap F$

$= \{8\} \cap \{1, 4\}$

$= \{\}$

(iv) $\mathcal{P}(E \cap F)$

$= \mathcal{P}(\{\})$

$= \{\{\}\}$

6 (3b) Circle correct choices from among Y, N, Proof, and Witness and provide below a proof or witness as the case may be:

3 (i) $\exists X, Y, Z \in \mathcal{U} \quad \left(X \cup Y\right) \cap Z = X \cup \left(Y \cap Z\right)$

Proof Witness Y N

$$\frac{\dfrac{X = Y = Z}{(X \cup Y) \cap Z = X, \quad X \cup (Y \cap Z) = X}}{\exists X, Y, Z \in \mathcal{U} \quad (X \cup Y) \cap Z = X \cup (Y \cap Z)}$$

3 (i) $\forall X, Y, Z \in \mathcal{U} \quad \left(X \cap Y\right) \cup Z = X \cap \left(Y \cup Z\right)$

Proof Witness Y N

$$\frac{\dfrac{\forall z \in Z}{\forall z \in ((X \cap Y) \cup Z)), \quad \forall z \notin (X \cap (Y \cup Z))}}{\forall X, Y, Z \in \mathcal{U} \quad (X \cap Y) \cup Z \neq X \cap (Y \cup Z)}$$

0

5 (3c) Circle correct choices from among Y, N, Proof, and Witness and provide below a proof or witness as the case may be:

2 (i) $\forall X, Y, Z \in \mathcal{U}$ $\quad Vld\left(\frac{X \subseteq Y}{X \backslash Z \subseteq Y \backslash Z}\right)$

Proof Witness Y N

$$\frac{\frac{\forall x \in (X \backslash Z), X \subseteq Y}{\forall x \in (Y \backslash Z)}}{\forall X, Y, Z \in \mathcal{U} \left(\frac{X \subseteq Y}{X \backslash Z \subseteq Y \backslash Z}\right)}$$

3 (ii) $\forall X, Y, Z \in \mathcal{U}$ $\quad Vld\left(\frac{X = Y}{X \backslash Z = Y \backslash Z}\right)$

Proof Witness Y N

$$\frac{\frac{\forall x \in (X \backslash Z), X = Y}{\forall x \in (Y \backslash Z)}}{\forall X, Y, Z \in \mathcal{U} \left(\frac{X = Y}{X \backslash Z = Y \backslash Z}\right)}$$

0

5 (3c) **Circle the correct choice** on the right and **prove your assertion** accordingly.

2 (i) $\exists X, Y, Z \in \mathcal{U} \quad \left(X \times Y\right) \times Z = X \times \left(Y \times Z\right)$

Proof Witness Y N

write

3 (i) $\forall X, Y, Z \in \mathcal{U} \quad \left(X \sqcup Y\right) \sqcup Z = X \sqcup \left(Y \sqcup Z\right)$

Proof Witness Y N