6 (1a) Compute the following, **showing work on the facing page,** and record **ONLY** the answers on this page.

1 (i) $C \cup D$

$= \{1, 2, 3, 6\}$

1 (ii) $C \cap D$

$= \{1, 2\}$

1 (iii) $C \times D$ Draw tree on facing page

$= \{(1,1), (1,2), (1,3), (1,6), (2,1), (2,2), (2,3), (2,6)\}$

1 (iv) $D \times C$ Draw tree on facing page

$= \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (6,1), (6,2)\}$

1 (v) $(C \times D) \cap (D \times C)$

$= \{(1,1), (1,2), (2,1), (2,2)\}$

Q

1 (vi) $(C \cap D) \times (D \cap C)$ Draw tree on facing page

$= \{(1,1), (1,2), (2,1), (2,2)\}$

1a.i.0

$C \cup D$

$= \{1,2\} \cup \{1,2,3,6\}$

$= \{1,2,3,6\}$

$C \cup D$

$=$

1a.ii.0

$C \cap D$

$= \{1,2\} \cap \{1,2,3,6\}$

$= \{1,2\}$

1a.iii.0

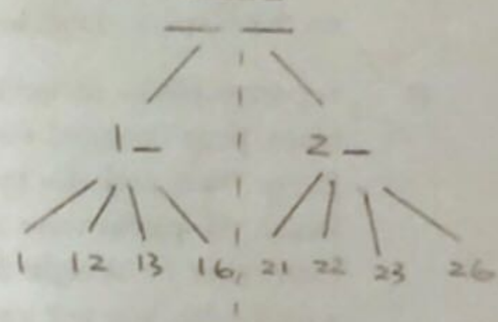
$C \times D$

$= \{1,2\} \times \{1,2,3,6\}$

$= \left\{ \begin{matrix} (1,1),(1,2),(1,3),(1,6), \\ (2,1),(2,2),(2,3),(2,6) \end{matrix} \right\}$

1a.iii.1

Tree



1a.iv.0

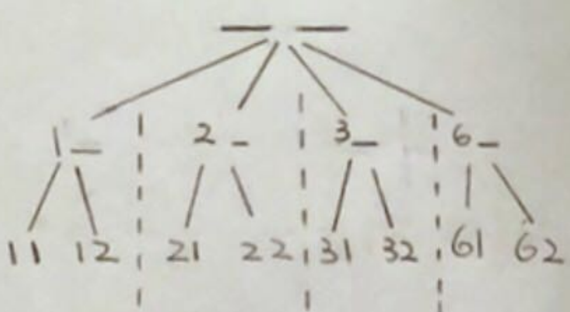
$D \times C$

$= \{1,2,3,6\} \times \{1,2\}$

$= \left\{ \begin{matrix} (1,1),(1,2), \\ (2,1),(2,2), \\ (3,1),(3,2), \\ (6,1),(6,2) \end{matrix} \right\}$

1a.iv.1

Tree



Ia.v.0

$$(C \times D) \cap (D \times C)$$

$$= \left\{ \begin{matrix} (1,1),(1,2),(1,3),(1,6), \\ (2,1),(2,2),(2,3),(2,6) \end{matrix} \right\} \cap \left\{ \begin{matrix} (1,1),(1,2), \\ (2,1),(2,2), \\ (3,1),(3,2), \\ (6,1),(6,2) \end{matrix} \right\}$$

$$= \{(1,1),(1,2),(2,1),(2,2)\}$$

Ia.vi.0

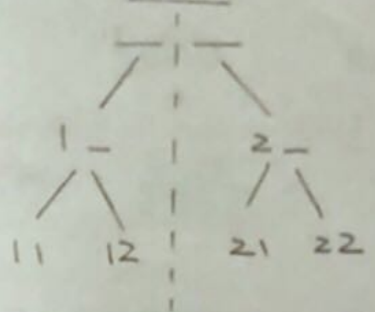
Note: See Ia.ii.0

$$D \cap C$$
$$= \{1,2,3,6\} \cap \{1,2\}$$
$$= \{1,2\}$$

$$(C \cap D) \times (D \cap C)$$
$$= \{1,2\} \times \{1,2\}$$
$$= \left\{ \begin{matrix} (1,1),(1,2), \\ (2,1),(2,2) \end{matrix} \right\}$$

Ia.vi.1

<u>Tree</u>



6 (1b) Compute the following, **showing work on the facing page,** and record **only the answers** on this page. You will get **0 points on this page unless all three answers are correctly calculated and are equal.**

2 (i) $(A \cup B) \backslash (A \cap B)$

$= \{0, 1, 4, 6, 7\}$

2 (ii) $(A \backslash B) \cup (B \backslash A)$

$= \{0, 1, 4, 6, 7\}$

2 (iii) $\left((A^c) \cap B\right) \cup \left(A \cap (B^c)\right)$

6

$= \{0, 1, 4, 6, 7\}$

1b.ii.0 $(A \setminus B) \cup (B \setminus A)$

$= (\{1, \cancel{2}, 4, 7\} \setminus \{0, 2, 6\}) \cup (\{0, \cancel{2}, 6\} \setminus \{1, 2, 4, 7\})$

$= \{1, 4, 7\} \cup \{0, 6\}$

$= \{0, 1, 4, 6, 7\}$

1b.iii.1 A^c

$= \mathcal{U} \setminus A$

$= \{0, \cancel{1}, \cancel{2}, 3, \cancel{4}, 5, 6, \cancel{7}, 8, 9\} \setminus \{1, 2, 4, 7\}$

$= \{0, 3, 5, 6, 8, 9\}$

1b.iii B^c

$= \mathcal{U} \setminus B$

$= \{\cancel{0}, 1, \cancel{2}, 3, 4, 5, \cancel{6}, 7, 8, 9\} \setminus \{0, 2, 6\}$

$= \{1, 3, 4, 5, 7, 8, 9\}$

1b.iii.2 $((A^c) \cap B) \cup (A \cap (B^c))$

$= (\{0, 3, 5, 6, 8, 9\} \cap \{0, 2, 6\}) \cup (\{1, 2, 4, 7\} \cap \{1, 3, 4, 5, 7, 8, 9\})$

$= \{0, 6\} \cup \{1, 4, 7\}$

$= \{0, 1, 4, 6, 7\}$

5 (1c) **Circle the correct choice** on the right and **prove your assertion** accordingly

2 (i) $\forall X, Y \in \mathcal{U} \quad (X \backslash Y) \times (X \backslash Y) = (X \times X) \backslash (Y \times Y)$ Y N

3 (ii) $\forall X, Y \in \mathcal{U} \quad (X \cup Y) \backslash (Y \cap X) = (X \cap Y) \backslash (X \cup Y)$ Y N

8

(1c.i.0) $X := \{1,2\}$

$Y := \{1\}$

LS:

$= (X \setminus Y) \times (X \setminus Y)$

$= (\{\cancel{1},2\} \setminus \{1\}) \times (\{\cancel{1},2\} \setminus \{1\})$

$= \{2\} \times \{2\}$

$= \{(2,2)\}$

(1c.i.1) RS

$= (X \times X) \setminus (Y \times Y)$

$= (\{1,2\} \times \{1,2\}) \setminus (\{1\} \times \{1\})$

$= \left\{ \begin{matrix} \cancel{(1,1)}, (1,2), \\ (2,1), (2,2) \end{matrix} \right\} \setminus \{(1,1)\}$

$= \left\{ \begin{matrix} (1,2), \\ (2,1), (2,2) \end{matrix} \right\}$

(1c.i.2) $LS = \{(2,2)\} \neq \left\{ \begin{matrix} (1,2), \\ (2,1), (2,2) \end{matrix} \right\} = RS$

$$LS \neq RS$$

1c.ii0

$X := \{1, 2\}$

$Y := \{1\}$

LS:

$= (X \cup Y) \setminus (Y \cap X)$

$= (\{1,2\} \cup \{1\}) \setminus (\{1\} \cap \{1,2\})$

$= \{\cancel{1}, 2\} \setminus \{1\}$

$= \{2\}$

1c.ii1

RS:

$= (X \cap Y) \setminus (X \cup Y)$

$= (\{1,2\} \cap \{1\}) \setminus (\{1,2\} \cup \{1\})$

$= \cancel{\{1\}} \setminus \{1, 2\}$

$= \{\ \}$

1c.ii2

$$\frac{LS := \{2\} \neq \{\ \} = RS}{LS \neq RS}$$

6 (2a) My sets are:

$U \coloneqq \{s, b\}$ $= \{2, 7\}$

$V \coloneqq \{\{s\}, \{b\}\}$ $= \{\{2\}, \{7\}\}$

$W \coloneqq \{s, b, \{s, b\}\}$ $= \{2, 7, \{2, 7\}\}$

$X \coloneqq \{\{s\}, \{b\}, \{s, b\}\}$ $= \{\{2\}, \{7\}, \{2, 7\}\}$

$Y \coloneqq \{s, b, \{s\}, \{b\}\}$ $= \{2, 7, \{2\}, \{7\}\}$

$Z \coloneqq \{s, b, \{s\}, \{b\}, \{s, b\}\}$ $= \{2, 7, \{2\}, \{7\}, \{2, 7\}\}$

Given that $P, Q \in \{U, V, W, X, Y, Z\}$, write, if possible, exactly one valid assertion of each of the following types **without using the names:** U, V, W, X, Y, Z. If not possible, write not possible, and prove on the facing side that this is the case.

1 (i) $P \neq Q$: $\{2, 7\} \neq \{\{2\}, \{7\}\}$

1 (ii) $P \in Q$: $\{2, 7\} \in \{2, 7, \{2, 7\}\}$

1 (iii) $P \notin Q$: $\{\{2\}, \{7\}\} \notin \{2, 7, \{2, 7\}\}$

8

1 (iv) $P \subseteq Q$: $\{\{2\}, \{7\}\} \subseteq \{\{2\}, \{7\}, \{2, 7\}\}$

1 (v) $P \nsubseteq Q$: $\{\{2\}, \{7\}\} \nsubseteq \{2, 7\}$

1 (vi) Do the answers to the above depend on s and b? Y N

-1

Why?

2a.vi.o

$$\dfrac{\dfrac{\dfrac{\{s, b\} \notin \{\{s\}, \{b\}\}}{\{b, b\} \notin \{\{b\}, \{b\}\}}\left\langle s \quad b \right\rangle}{\{b\} \notin \{\{b\}\}}}{\bot}$$

6 (2b) Compute the following, **showing work on the facing page,** and write **only the answers** on this page.

1 (i) $\mathcal{P}\left(A \cap B\right)$

$= \left\{ \{\ \}, \{2\} \right\}$

1 (ii) $\mathcal{P}\left(C \cap D\right)$

$= \left\{ \{\ \}, \{1\}, \{2\}, \{1, 2\} \right\}$

2 (~~i~~ iii) $\mathcal{P}\left((A \cap B) \setminus (C \cap D)\right)$

$= \left\{ \{\ \} \right\}$

2 (~~ii~~ iv) $\mathcal{P}\left(C \cap D\right) \setminus \mathcal{P}\left(A \cap B\right)$

$= \left\{ \{1\}, \{1, 2\} \right\}$

0

2b.i.0

$$\mathcal{P}(A \cap B)$$
$$= \mathcal{P}(\{1,2,4,7\} \cap \{0,2,6\})$$
$$= \mathcal{P}(\{2\})$$
$$= \left\{ \begin{array}{l} \{\ \}, \\ \{2\} \end{array} \right\}$$

2b.ii.0

$$\mathcal{P}(C \cap D)$$
$$= \mathcal{P}(\{1,2\} \cap \{1,2,3,6\})$$
$$= \mathcal{P}(\{1,2\})$$
$$= \left\{ \begin{array}{l} \{\ \}, \\ \{1\}, \{2\}, \\ \{1,2\} \end{array} \right\}$$

2b.iii.0

Note: See 2b.ii.0 $\wedge$ 2b.iii.0

$$\mathcal{P}((A \cap B) \setminus (C \cap D))$$
$$= \mathcal{P}(\cancel{\{2\}} \setminus \{1,2\})$$
$$= \mathcal{P}(\{\ \})$$
$$= \{\{\ \}\}$$

2b.iv.0

Note: See 2b.iii.0 $\wedge$ 2b.ii.0

$$\mathcal{P}(C \cap D) \setminus \mathcal{P}(A \cap B)$$
$$= \left\{ \begin{array}{l} \cancel{\{\ \}}, \\ \{1\}, \cancel{\{2\}}, \\ \{1,2\} \end{array} \right\} \setminus \left\{ \begin{array}{l} \{\ \}, \\ \{2\} \end{array} \right\}$$
$$= \left\{ \begin{array}{l} \{1\}, \\ \{1,2\} \end{array} \right\}$$

5 (2c) **Show work on facing side and record only the answers on this page.** Define the sequence of sets: $(S_n | n \in \mathbb{N})$ recursively as follows where X, Y and Z are some unknown finite sets with:

$$x := \nu(X) \in \mathbb{N} \qquad y := \nu(Y) \in \mathbb{N} \qquad z := \nu(Z) \in \mathbb{N}$$

Base cases: (B_0) $S_0 := X$

(B_1) $S_1 := Y$

(B_2) $S_2 := Z$

Recursive step: (R) $S_{n+3} := \left(S_n \times S_{n+1}\right) \sqcup \left(S_{n+2}\right)$

1 (i) Compute: S_3 $= (X \times Y) \sqcup Z$

1 (iii) Compute: $\nu(S_3)$ $= xy + z$

1 (ii) Compute: S_4 $= (Y \times Z) \sqcup ((X \times Y) \sqcup (Z))$

1 (iv) Compute: $\nu(S_4)$ $= yz + (xy + z)$

1 (v) Find a formula for $\nu(S_{n+3})$ in terms of n, x, y and z and prove your result.

$\nu(S_{n+3})$ $=$

4

2c.i.o $S_3 = S_{0+3} := (S_0 \times S_1) \sqcup S_2 = (X \times Y) \sqcup Z$

2c.iii.o $\nu(S_3) = \nu(X \times Y) \sqcup \nu(Z)$

$= (\nu(X))(\nu(Y)) + \nu(Z)$

$= xy + z$

2c.ii.o $S_4 = S_{1+3} := (S_1 \times S_2) \sqcup S_3 = (Y \times Z) \sqcup ((X \times Y) \sqcup (Z))$

2c.iv.o $\nu(S_4) = \nu(Y) \cdot \nu(Z) + ((\nu(X))(\nu(Y)) + \nu(Z))$

$= yz + (xy + z)$

6 (3a) Compute the following, **showing work on the facing page,** and record the answers below **properly**.

1 (i) $A \cap B$ $\qquad$ $\nu(A \cap B)$

$= \{2\}$ $\qquad$ $= 1$

1 (ii) $A \cap D$ $\qquad$ $\nu(A \cap D)$

$= \{1, 2\}$ $\qquad$ $= 2$

1 (iii) $B \cap D$ $\qquad$ $\nu(B \cap D)$

$= \{2, 6\}$ $\qquad$ $= 2$

1 (iv) $A \cap B \cap D$ $\qquad$ $\nu(A \cap B \cap D)$

$= \{2\}$ $\qquad$ $= 1$

1 (v) $A \cup B \cup D$ $\qquad$ $\nu(A \cup B \cup D)$

$= \{0, 1, 2, 3, 4, 6, 7\}$ $\qquad$ $= 7$

1 (vi) **Verify**, by computing LS and RS **separately** on the facing page that:

$$\nu(A \cup B \cup D) = \nu(A) + \nu(B) + \nu(D) - \nu(A \cap B) - \nu(A \cap D) - \nu(B \cap D) + \nu(A \cap B \cap D)$$

LS

$= \nu(A \cup B \cup D)$

$= 7$

LS = 7 = RS

LS = RS.

RS

$= \nu(A) + \nu(B) + \nu(D) - \nu(A \cap B) - \nu(A \cap D) - \nu(B \cap D) + \nu(A \cap B \cap D)$

$= 7$

-1

6

3a.i.0 $A \cap B$

$= (\{1,2,4,7\} \cap \{0,2,6\})$

$= \{2\}$

3a.i.1 $\nu(A \cap B)$

$= \nu(\{2\})$

$= 1$

3a.ii.0 $A \cap D$

$= (\{1,2,4,7\} \cap \{1,2,3,6\})$

$= \{1,2\}$

3a.ii.1 $\nu(A \cap D)$

$= \nu(\{1,2\})$

$= 2$

3a.iii.0 $B \cap D$

$= (\{0,2,6\} \cap \{1,2,3,6\})$

$= \{2,6\}$

3a.iii.1 $\nu(B \cap D)$

$= \nu(\{2,6\})$

$= 2$

3a.iv.0 $A \cap B \cap D$

$= (\{1,2,4,7\} \cap \{0,2,6\} \cap \{1,2,3,6\})$

$= (\{2\} \cap \{1,2,3,6\})$

$= \{2\}$

3a.iv.1 $\nu(A \cap B \cap D)$

$= \nu(\{3\})$

$= 1$

3a.v.0 $A \cup B \cup D$

$= (\{1,2,4,7\} \cup \{0,2,6\} \cup \{1,2,3,6\})$

$= (\{0,1,2,4,6,7\} \cup \{1,2,3,6\})$

$= \{0,1,2,3,4,6,7\}$

3a.v.1 $\nu(A \cup B \cup D)$

$= \nu(\{0,1,2,3,4,6,7\})$

$= 7$

3a.vi.0 LS

Note: See 3a.iv.1

$n(A \cup B \cup D)$
$= 7$

1

3a.vi.1 RS

$n(A)+n(B)+n(D)-n(A\cap B)$
$-n(A\cap D)-n(B\cap D)+n(A\cap B\cap D)$
$= 4+3+4-1-2-2+1$
$= 7$

Note ⓪ $n(A)$
$= n(\{1,2,4,7\})$
$= 4$

① $n(B)$
$= n(\{0,2,6\})$
$= 3$

② $n(D)$
$= n(\{1,2,3,6\})$
$= 4$

③ Please See:
3a.ii.1
3a.iii.1
3a.iii.1
3a.iv.1

3a.vi.2 $\dfrac{LS = 7 = RS}{LS = RS}$

6 (3b) **Circle the correct choice** on the right and **prove your assertion** accordingly:

3 (i) $$Vld\left(\frac{X = X \cap Y}{X \subseteq Y}\right)$$ Y N

3 (ii) $$Vld\left(\frac{X \cup Y = Y}{X \subseteq Y}\right)$$ Y N

$$\frac{X \subseteq X \cup Y \quad X \cup Y = Y}{X \subseteq Y}$$

3

3b.i.o

$$X = X \cap Y$$

$$X \subseteq (X \cap Y) \quad (X \cap Y) \subseteq X$$

$$\forall x \in X \quad x \in (X \cap Y)$$

$$\forall x \in X \quad x \in Y$$

$$X \subseteq Y$$

$$\mathrm{VId}\left(\frac{X = X \cap Y}{X \subseteq Y}\right)$$

3b.ii.o

$$a \in X \cup Y \quad X \cup Y = Y$$

$$a \in Y$$

$$\forall a \in \mathcal{U} \quad \frac{a \in X}{a \in Y}$$

$$X \subseteq Y$$

$$\mathrm{VId}\left(\frac{X \cup Y = Y}{X \subseteq Y}\right)$$