6 (1a) Find the truth-tables for:

2 (i) $((\neg p) \longrightarrow (\neg q)) \longrightarrow p$

p	q	$\neg p$	$\neg q$	$(\neg p) \longrightarrow (\neg q)$	$((\neg p) \longrightarrow (\neg q)) \longrightarrow p$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	1	1
1	1	0	0	1	1

4 (ii) $(p \longrightarrow (\neg r)) \wedge (q \longrightarrow r)$

p	q	r	$\neg r$	$p \longrightarrow (\neg r)$	$q \longrightarrow r$	$(p \longrightarrow (\neg r)) \wedge (q \longrightarrow r)$
0	0	0	1	1	1	1
0	0	1	0	1	1	1
0	1	0	1	1	0	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	0	1	0
1	1	0	1	1	0	0
1	1	1	0	0	1	0

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6 (1b) Find truth-tables for the following expressions:

2 (i) $LS := (p \vee q) \longrightarrow r$,

2 (ii) $RS := (p \longrightarrow r) \vee (q \longrightarrow r)$, and

2 (iii) decide, if $(p \vee q) \longrightarrow r \Rrightarrow (p \longrightarrow r) \vee (q \longrightarrow r)$

p	q	r	$p \vee q$	$p \longrightarrow r$	$q \longrightarrow r$	$(p \vee q) \longrightarrow r$	$(p \longrightarrow r) \vee (q \longrightarrow r)$	$LS \longrightarrow RS$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	1	1	0	0	1	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

$(p \vee q) \longrightarrow r \Rrightarrow (p \longrightarrow r) \vee (q \longrightarrow r)$ Y N

Why?

By inspection

6

5 (1c) Decide if:

2 (i) $$Vld\left(\frac{(p_0)\rightarrow(p_1) \quad (p_1)\rightarrow(p_0)}{(p_0)\leftrightarrow(p_1)}\right)$$

Y N

3 (ii) $$Vld\left(\frac{(p_0)\rightarrow(p_1) \quad (p_1)\rightarrow(p_2) \quad (p_1)\rightarrow(p_0)}{(p_0)\leftrightarrow(p_1)\leftrightarrow(p_2)}\right)$$

Y N

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(i)

(p_0)	(p_1)	$(p_0)\to(p_1)$	$(p_1)\to(p_0)$	$((p_0)\to(p_1))\wedge((p_1)\to(p_0))$	$(p_0)\leftrightarrow(p_1)$	$(((p_0)\to(p_1))\wedge((p_1)\to(p_0)))\to((p_0)\leftrightarrow(p_1))$
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	1	1	1	1	1

$$Tlg\left(\left(((p_0)\to(p_1))\wedge((p_1)\to(p_0))\right)\to((p_0)\leftrightarrow(p_1))\right)$$

$$Vld\left(\frac{(p_0)\to(p_1)\quad (p_1)\to(p_0)}{(p_0)\leftrightarrow(p_1)}\right)$$

(ii) $LS := (((p_0)\to(p_1))\wedge((p_1)\to(p_2))\wedge((p_1)\to(p_0)))$

$RS := ((p_0)\leftrightarrow(p_1)\leftrightarrow(p_2))$

(p_0)	(p_1)	(p_2)	$(p_0)\to(p_1)$	$(p_1)\to(p_2)$	$(p_1)\to(p_0)$	LS	RS	$LS\to RS$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	0	$\boxed{0}$
0	1	0	1	0	0	0	0	1
0	1	1	1	1	0	0	0	1
1	0	0	0	1	1	0	0	1
1	0	1	0	1	1	0	0	1
1	1	0	1	0	1	0	0	1
1	1	1	1	1	1	1	1	1

$$\cancel{Tlg}\left(((p_0)\to(p_1))\wedge((p_1)\to(p_2))\wedge((p_1)\to(p_0))\right)$$

$$\cancel{Vld}\left(\frac{(p_0)\to(p_1)\quad (p_1)\to(p_2)\quad (p_1)\to(p_0)}{(p_0)\leftrightarrow(p_1)\leftrightarrow(p_2)}\right)$$

6 (2a) Find $\varphi(p,q,r)$ in terms of p, q, r, and connectives, if $\varphi(p,q,r)$ is to have the following truth table:

p	q	r	$\varphi(p,q,r)$	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	0	1	0	0	0	1				
0	0	1	0	0	0	0	0				
0	1	0	0	0	0	0	0				
0	1	1	1	0	0	1	0				
1	0	0	0	0	0	0	0				
1	0	1	1	0	1	0	0				
1	1	0	1	1	0	0	0				
1	1	1	0	0	0	0	0				

$\varphi(p,q,r)$

$$= \varphi_1 \vee \varphi_2 \vee \varphi_3 \vee \varphi_4$$

$$= (p \wedge q \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge r) \vee ((\neg p) \wedge (\neg q) \wedge (\neg r))$$

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6 (2b) Show work on the facing side to decide, **using the algebra of propositions**, if the following hold and circle the appropriate choice:

1 (i) $\big((\bot \longrightarrow p) \longrightarrow q\big) = \top$ Y N (Circle the correct choice)

1 (i) $\big((p \longrightarrow \bot) \longrightarrow q\big) = \top$ Y N (Circle the correct choice)

1 (i) $\big((p \longrightarrow q) \longrightarrow \bot\big) = \top$ Y N (Circle the correct choice)

1 (i) $\big(\bot \longrightarrow (p \longrightarrow q)\big) = \top$ Y N (Circle the correct choice)

1 (i) $\big(p \longrightarrow (\bot \longrightarrow q)\big) = \top$ Y N (Circle the correct choice)

1 (i) $\big(p \longrightarrow (q \longrightarrow \bot)\big) = \top$ Y N (Circle the correct choice)

Both here and TT's

2

(i) LS

$= (\bot \to p) \to q \qquad ((x \to y) = ((\neg x) \vee y))$

$= ((\neg(\bot)) \vee p) \to q \qquad ((\neg(\bot)) = T)$

$= (T \vee p) \to q \qquad ((T \vee p) = T)$

$= T \to q$

q	T	$T \to q$
0	1	0

$0 \neq 1$

$((\bot \to p) \to q) = (T \to q) \neq T$

$((\bot \to p) \to q) \neq T$

(ii) LS

$= (p \to \bot) \to q \qquad ((x \to y) = ((\neg x) \vee y))$

$= ((\neg p) \vee \bot) \to q \qquad ((x \vee \bot) = x)$

$= (\neg p) \to q \qquad ((x \to y) = ((\neg x) \vee y))$

$= (\neg(\neg p)) \vee q \qquad ((\neg(\neg x)) = x)$

$= p \vee q$

p	q	T	$p \vee q$
0	0	1	0

$0 \neq 1$

$((p \to \bot) \to q) = (p \vee q) \neq T$

$((p \to \bot) \to q) \neq T$

Scanned with CamScanner

(v) LS

$= p \to (\bot \to q)$ $\quad ((x \to y) = ((\neg x) \vee y))$

$= (\neg p) \vee (\bot \to q)$ $\quad ((x \to y) = ((\neg x) \vee y))$

$= (\neg p) \vee ((\neg(\bot)) \vee q)$ $\quad ((\neg(\bot)) = T)$

$= (\neg p) \vee (T \vee q)$ $\quad ((x \vee y = (y \vee x))$

$= (T \vee q) \vee (\neg p)$ $\quad (((x \vee y) \vee z) = (x \vee (y \vee z)))$

$= T \vee (q \vee (\neg p))$ $\quad ((x \vee y) = (y \vee x))$

$= (q \vee (\neg p)) \vee T$ $\quad ((x \vee T) = T)$

$= T$

$= RS$

$LS = RS$

$(p \to (\bot \to q)) = T$

(vi) LS

$= p \to (q \to \bot)$ $\quad ((x \to y) = ((\neg x) \vee y))$

$= (\neg p) \vee (q \to \bot)$ $\quad ((x \to y) = ((\neg x) \vee y))$

$= (\neg p) \vee ((\neg q) \vee \bot)$ $\quad ((x \vee \bot) = x)$

$= (\neg p) \vee (\neg q)$

p	q	$(\neg p)$	$(\neg q)$	T	$(\neg p) \vee (\neg q)$
1	1	0	0	1	0

$0 \neq 1$

$(p \to (q \to \bot)) = (\neg p) \vee (\neg q) \neq T$

$(p \to (q \to \bot)) \neq T$

(2b)

(iii) LS

$= (p \to q) \to \bot \qquad ((x \to y) = ((\neg x) \vee y))$

$= ((\neg p) \vee q) \to \bot \qquad ((x \to y) = ((\neg x) \vee y))$

$= (\neg((\neg p) \vee q)) \vee \bot \qquad ((x \vee \bot) = x)$

$= (\neg((\neg p) \vee q)) \qquad ((\neg(x \vee y)) = (\neg x) \wedge (\neg y))$

$= (\neg(\neg p)) \wedge (\neg q) \qquad ((\neg(\neg x)) = x)$

$= p \wedge (\neg q)$

p	q	$(\neg q)$	T	$p \wedge (\neg q)$
0	0	1	1	0

$0 \neq 1$

$((p \to q) \to \bot) = (p \wedge (\neg q)) \neq T$

$((p \to q) \to \bot) \neq T$

(iv) LS

$= \bot \to (p \to q) \qquad ((x \to y) = ((\neg x) \vee y))$

$= (\neg(\bot)) \vee ((\neg p) \vee q) \qquad ((\neg(\bot)) = T)$

$= T \vee ((\neg p) \vee q) \qquad ((x \vee y) = (y \vee x))$

$= ((\neg p) \vee q) \vee T \qquad ((x \vee T) = T)$

$= T$

$= RS$

$LS = RS$

$(\bot \to (p \to q)) = T$

Scanned with CamScanner

5 (2c) Show work on the facing side to decide, **using the algebra of propositions,** if the following arguments are valid and circle the appropriate choice:

$$Vld\left(\frac{(p \vee q) \longrightarrow r}{(p \longrightarrow r) \vee (q \longrightarrow r)}\right) \qquad \text{Y} \qquad \text{N}$$

(2c)

$= (\neg q) \vee T \vee ((\neg r) \wedge q)$

$= T$

$= RS$

$LS = RS$

$((p \vee q) \to r) \to ((p \to r) \vee (q \to r)) = T$

$Tlg(((p \vee q) \to r) \to ((p \to r) \vee (q \to r)))$

$Vld\left(\frac{(p \vee q) \to r}{(p \to r) \vee (q \to r)}\right)$

8

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(2c) $\text{vld}\left(\dfrac{(p\vee q)\to r}{(p\to r)\vee(q\to r)}\right) \Leftrightarrow \Big(\big((p\vee q)\to r\big)\to\big((p\to r)\vee(q\to r)\big)\Big) = T$

LS

$= ((p\vee q)\to r)\to((p\to r)\vee(q\to r))$ $\quad (x\to y = (\neg x)\vee y)$

$= ((p\vee q)\to r)\to(((\neg p)\vee r)\vee((\neg q)\vee r))$ $\quad (x\vee y = y\vee x)$

$= ((p\vee q)\to r)\to(((\neg p)\vee r)\vee(r\vee(\neg q)))$ $\quad ((x\vee y)\vee z = x\vee(y\vee z))$

$= ((p\vee q)\to r)\to((\neg p)\vee(r\vee r)\vee(\neg q))$ $\quad (x\vee x = x\ ,\ x\vee y = y\vee x)$

$= ((p\vee q)\to r)\to(((\neg p)\vee(\neg q))\vee r)$ $\quad (x\to y = (\neg x)\vee y)$

$= ((\neg(p\vee q))\vee r)\to(((\neg p)\vee(\neg q))\vee r)$ $\quad (x\to y = (\neg x)\vee y)$

$= (\neg((\neg(p\vee q))\vee r))\vee(((\neg p)\vee(\neg q))\vee r)$ $\quad ((\neg(\neg x)) = x, (\neg(x\vee y)) = (\neg x)\wedge(\neg y))$

$= ((p\vee q)\wedge(\neg r))\vee(((\neg p)\vee(\neg q))\vee r)$ $\quad (x\wedge y = y\wedge x)$

$= ((\neg r)\wedge(p\vee q))\vee(((\neg p)\vee(\neg q))\vee r)$ $\quad ((x\wedge(y\vee z)) = (x\wedge y)\vee(x\wedge z))$

$= (((\neg r)\wedge p)\vee((\neg r)\wedge q))\vee(((\neg p)\vee(\neg q))\vee r)$ $\quad (x = (\neg(\neg x)))$

$= (((\neg r)\wedge(\neg(\neg p)))\vee((\neg r)\wedge q))\vee(((\neg p)\vee(\neg q))\vee r)$ $\quad ((\neg x)\wedge(\neg y) = (\neg(x\vee y)))$

$= ((\neg(r\vee(\neg p)))\vee((\neg r)\wedge q))\vee(((\neg p)\vee(\neg q))\vee r)$ $\quad (x\vee y = y\vee x)$

$= ((\neg((\neg p)\vee r))\vee((\neg r)\wedge q))\vee(((\neg q)\vee(\neg p))\vee r)$ $\quad ((x\vee y)\vee z = x\vee(y\vee z))$

$= ((\neg((\neg p)\vee r))\vee((\neg r)\wedge q))\vee((\neg q)\vee((\neg p)\vee r))$ $\quad (x\vee y = y\vee x)$

$= ((\neg q)\vee((\neg p)\vee r))\vee((\neg((\neg p)\vee r))\vee((\neg r)\wedge q))$ $\quad ((x\vee y)\vee z = x\vee(y\vee z))$

$= (\neg q)\vee(((\neg p)\vee r)\vee(\neg((\neg p)\vee r)))\vee((\neg r)\wedge q)$ $\quad (x\vee(\neg x) = T)$

6 (3a) Show work on the facing side to decide if the following proposition is a tautology, contradiction, or contingency, and circle an appropriate answer.

1 (i) $((\perp \rightarrow p) \rightarrow q)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (ii) $((p \rightarrow \perp) \rightarrow q)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (iii) $((p \rightarrow q) \rightarrow \perp)$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (iv) $(\perp \rightarrow (p \rightarrow q))$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

1 (v) $(p \rightarrow (\perp \rightarrow q))$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

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1 (vi) $(p \rightarrow (q \rightarrow \perp))$ $Tlg(\psi)$ $Cdn(\psi)$ $Cng(\psi)$

(i)

p	q	$\bot$	$\bot \to p$	$(\bot \to p) \to q$
0	0	0	1	0
0	1	0	1	1
1	0	0	1	
1	1	0	1	

(ii)

p	q	$\bot$	$p \to \bot$	$(p \to \bot) \to q$
0	0	0	1	0
0	1	0	1	1
1	0	0	0	
1	1	0	0	

(iii)

p	q	$\bot$	$p \to q$	$(p \to q) \to \bot$
0	0	0	1	0
0	1	0	1	0
1	0	0	0	1
1	1	0	1	

(v)

p	q	$\bot$	$\bot \to q$	$p \to (\bot \to q)$
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	0	1	1

(vi)

p	q	$\bot$	$q \to \bot$	$p \to (q \to \bot)$
0	0	0	1	1
0	1	0	0	1
1	0	0	1	1
1	1	0	0	0

6 (3b) Show work on the facing side to decide, **using axioms and rules of inference**, if the following arguments are valid and circle the appropriate choice:

$$Vld\left(\frac{(p \vee q) \rightarrow r}{(p \rightarrow r) \vee (q \rightarrow r)}\right) \qquad Y \qquad N$$

0

5 (3c) Prove or disprove: $\forall n \in \mathbb{N}$

$$Vld\left(\frac{(p_0)\longrightarrow(p_1) \quad (p_1)\longrightarrow(p_2) \quad \dots \quad (p_{n-1})\longrightarrow(p_n) \quad (p_n)\longrightarrow(p_0)}{\forall k,l\in 0..n \quad ((p_k)\longleftrightarrow(p_l))}\right)$$

Y N

0