Instructions:

Your J-number = 00298436

Keep every 0 in your J-number and replace every non-zero number in your J-number by 1 to get

Your (truth-value) **T-number**:= 00111111

Define $\forall k \in 1..8$,

$T_k :=$ the k^{th} entry in your **truth-value J-number** counting from the left. Therefore,

$T_1 := 0$

$T_2 := 0$

$T_3 := 1$

$T_4 := 1$

$T_5 := 1$

$T_6 := 1$

$T_7 := 1$

$T_8 := 1$

Next construct an expression $G(w, x, y, z)$ as follows:

$$\alpha(T_1, T_2, T_3, T_4)$$

$$= (T_1)(w) + (T_2)(x') + (T_3)(y) + (T_4)(z')$$

$$= (0)(w) + (0)(x') + (1)(y) + (1)(z')$$

$$= y + z'$$

$$\beta(T_3, T_4, T_5, T_6)$$

$$= \quad \left(\left(((T_3)w)'\right)\left(((T_4)x)'\right)\left(((T_5)y)'\right)\left(((T_6)z)'\right)\right)'$$

$$= \left(\left(((1)w)'\right)\left(((1)x)'\right)\left(((1)y)'\right)\left(((1)z)'\right)\right)'$$

$$= (w')' + (x')' + (y')' + (z')'$$

$$= w + x + y + z$$

$$\gamma(T_5, T_6, T_7, T_8)$$

$$= \quad (T_5)(w') + (T_6)(x) + (T_7)(y') + (T_8)(z)$$

$$= (1)(w') + (1)(x) + (1)(y') + (1)(z)$$

$$= (w') + (x) + (y') + (z)$$

$$= w' + x + y' + z$$

$G(w, x, y, z)$

$= \quad (\alpha(T_1, T_2, T_3, T_4))(\beta(T_3, T_4, T_5, T_6))(\gamma(T_5, T_6, T_7, T_8))$

$= \quad (\alpha(0, 0, 1, 1))(\beta(1, 1, 1, 1))(\gamma(1, 1, 1, 1))$

$= \quad (y + z')(w + x + y + z)(w' + x + y' + z)$

(i)

$$\begin{aligned}\varphi(p,q,r) &= (\varphi_1) &&= (p\wedge q\wedge r)\\ &\vee (\varphi_2) &&\vee (p\wedge q\wedge(\neg r))\\ &\vee (\varphi_3) &&\vee (p\wedge(\neg q)\wedge r)\\ &\vee (\varphi_4) &&\vee (p\wedge(\neg q)\wedge(\neg r))\\ &\vee (\varphi_5) &&\vee ((\neg p)\wedge q\wedge r)\\ &\vee (\varphi_6) &&\vee ((\neg p)\wedge q\wedge(\neg r))\end{aligned}$$

(ii)

$$\begin{aligned}F(A,B,C) &= (A\cap B\cap C)\\ &\cup (A\cap B\cap (C^c))\\ &\cup (A\cap (B^c)\cap C)\\ &\cup (A\cap (B^c)\cap (C^c))\\ &\cup ((A^c)\cap B\cap C)\\ &\cup ((A^c)\cap B\cap (C^c))\end{aligned}$$

(iii)

$$\begin{aligned}E(x,y,z) &= (x*y*z) &&= xyz\\ &+ (x*y*(z')) &&+ xyz'\\ &+ (x*(y')*z) &&+ xy'z\\ &+ (x*(y')*(z')) &&+ xy'z'\\ &+ ((x')*y*z) &&+ x'yz\\ &+ ((x')*y*(z')) &&+ x'yz'\end{aligned}$$

6 (1a) Find $\varphi(p,q,r)$ in terms of: p,q and r if $TT(\varphi(p,q,r))$ is given below. Find corresponding expressions $F(A,B,C)$ in <u>Sets</u> and $E(x,y,z)$ in <u>BA</u> on the facing page and record the answers on this page.

p	q	r	$\varphi(p,q,r)$	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	0	$T_1 =$ 0	0	0	0	0	0	0		
0	0	1	$T_2 =$ 0	0	0	0	0	0	0		
0	1	0	$T_3 =$ 1	0	0	0	0	0	1		
0	1	1	$T_4 =$ 1	0	0	0	0	1	0		
1	0	0	$T_5 =$ 1	0	0	0	1	0	0		
1	0	1	$T_6 =$ 1	0	0	1	0	0	0		
1	1	0	$T_7 =$ 1	0	1	0	0	0	0		
1	1	1	$T_8 =$ 1	1	0	0	0	0	0		

4(i) $\varphi(p,q,r)$

$$= (p\wedge q\wedge r)\vee(p\wedge q\wedge(\neg r))\vee(p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q\wedge r)\vee((\neg p)\wedge q\wedge(\neg r))$$

1 (ii) $F(A,B,C)$

$$= (A\cap B\cap C)\cup(A\cap B\cap(C^c))\cup(A\cap(B^c)\cap C)\cup(A\cap(B^c)\cap(C^c))\cup((A^c)\cap B\cap C)\cup((A^c)\cap B\cap(C^c))$$

1 (iii) $E(x,y,z)$

$$= xyz + xyz' + xy'z + xy'z' + x'yz + x'yz'$$

(i) $E(x, y, z)$

$= \underline{xyz} + \underline{xyz'} + xy'z + xy'z' + x'yz + x'yz' \quad (Cns(xyz, xyz') = xy)$

$= \underline{xyz} + \underline{xyz'} + xy'z + xy'z' + x'yz + x'yz' + \underline{xy} \quad (xyz \leq xy \quad xyz' \leq xy)$

$= \underline{xy'z} + \underline{xy'z'} + x'yz + x'yz' + xy \quad (Cns(xy'z, xy'z') = xy')$

$= \underline{xy'z} + \underline{xy'z'} + x'yz + x'yz' + xy + \underline{xy'} \quad (xy'z \leq xy', xy'z' \leq xy')$

$= \underline{x'yz} + \underline{x'yz'} + xy + xy' \quad (Cns(x'yz, x'yz') = x'y)$

$= \underline{x'yz} + \underline{x'yz'} + xy + xy' + \underline{x'y} \quad (x'yz \leq x'y, x'yz' \leq x'y)$

$= \underline{xy} + \underline{xy'} + x'y \quad (Cns(xy, xy') = x)$

$= \underline{xy} + \underline{xy'} + x'y + \underline{x} \quad (xy \leq x, xy' \leq x)$

$= \underline{x'y} + \underline{x} \quad (Cns(x'y, x) = y)$

$= \underline{x'y} + x + \underline{y} \quad (y \leq x'y)$

$= x + y \quad (x \not\leq y, y \not\leq x, \cancel{Cns}(x, y))$

$PI(E) = \{x, y\}$

(ii) ①

$E(x,y,z) =$	x	$x(1)(1)$	$x(y+y')(z+z')$	$\underbrace{xyz}_{1①} + \underbrace{xyz'}_{1②}$	$= x$	$= MSOP(E)$
	$+ y$	$+ (1)y(1)$	$+ (x+x')y(z+z')$	$+ xy'z + xy'z'$	$+ y$	
				$+ \underbrace{xyz}_{2①} + \underbrace{x'yz'}_{2②}$		
				$+ x'yz + x'yz'$		

6 (1b) Use the consensus method to find the following for your answer from (1a):

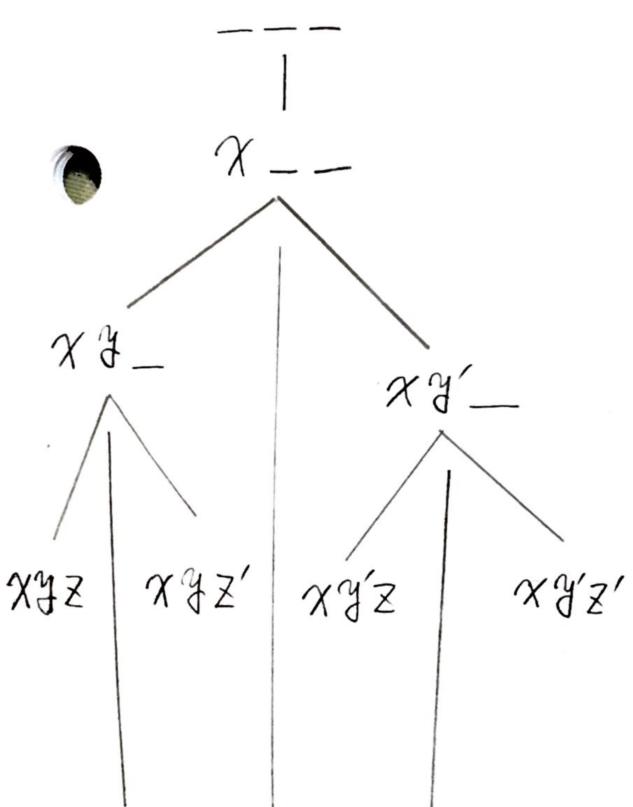
$E(x,y,z) = xyz + xyz' + xy'z + xy'z' + x'yz + x'yz'$

3 (i) the prime implicants of E: $PI(E) = \{x, y\}$

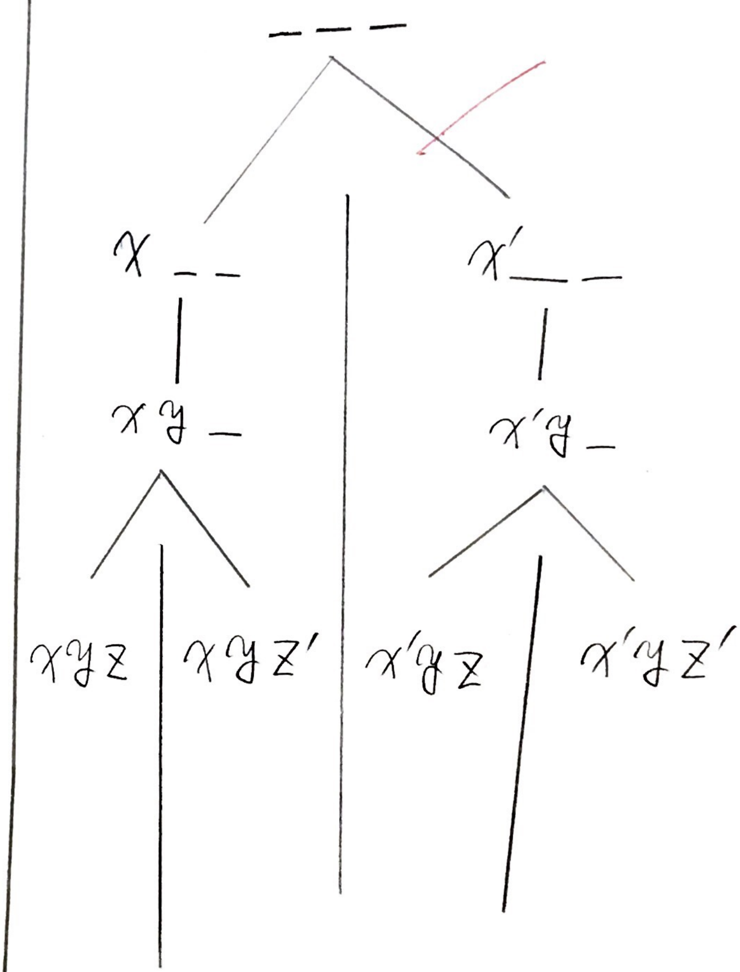
3 (~~i~~) the minimal sum-of-products form: $MSOP(E) = x + y$

(ii)

(ii) ① TREE



(ii) ② TREE



(ii) $\varphi(p, q, r)$

$= (p \wedge q \wedge r) \vee (p \wedge q \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r))$

$= ((p \wedge q) \wedge (r \vee (\neg r))) \vee ((p \wedge (\neg q)) \wedge (r \vee (\neg r))) \vee (((\neg p) \wedge q) \wedge (r \vee (\neg r)))$ $\quad ((x \wedge y) \vee (x \wedge z) = x \wedge (y \vee z))$

$= ((p \wedge q) \wedge T) \vee ((p \wedge (\neg q)) \wedge T) \vee (((\neg p) \wedge q) \wedge T)$ $\quad (x \vee (\neg x) = T)$

$= (p \wedge q) \vee (p \wedge (\neg q)) \vee ((\neg p) \wedge q)$ $\quad (x \wedge T = x)$

$= (p \wedge (q \vee (\neg q))) \vee ((\neg p) \wedge q)$ $\quad ((x \wedge y) \vee (x \wedge z) = x \wedge (y \vee z))$

$= (p \wedge T) \vee ((\neg p) \wedge q)$ $\quad (x \wedge (\neg x) = T)$

$= p \vee ((\neg p) \wedge q)$ $\quad (x \wedge T = x)$

$= (p \vee (\neg p)) \wedge (p \vee q)$ $\quad (x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z))$

$= T \wedge (p \vee q)$ $\quad (x \vee (\neg x) = T)$

$= p \vee q$ $\quad (x \wedge T = x)$

$\varphi(p, q, r) = p \vee q = \psi(p, q, r)$

$\varphi(p, q, r) = \psi(p, q, r)$

5 (1c) From (1b) $MSOP\ (E) =$ $x + y$

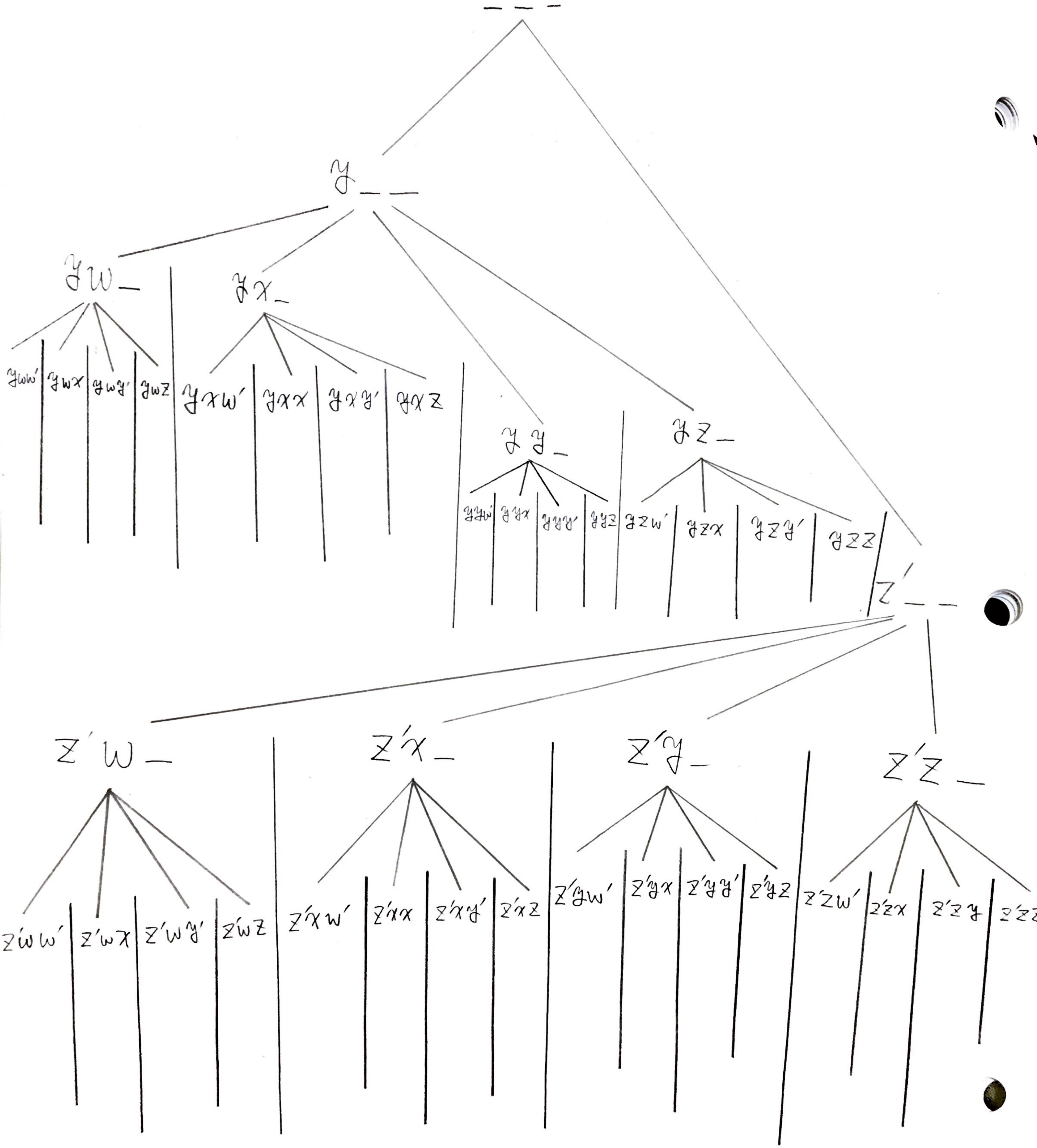
1 (i) Find the version $\psi(p,q,r)$ of $MSOP\ (E)$ in PC

$$\psi(p,q,r) = p \vee q$$

4 (ii) **Prove in PC** that your answer from (1a) $\varphi(p,q,r) = \psi(p,q,r)$

5

(2a) ① TREE


_ _ _
y _ _
yw_
yx_
yy_
yz_
z′ _ _
yww′
ywx
ywy′
ywz
yxw′
yxx
yxy′
yxz
yyw′
yyx
yyy′
yyz
yzw′
yzx
yzy′
yzz
z′w_
z′x_
z′y_
z′z_
z′ww′
z′wx
z′wy′
z′wz
z′xw′
z′xx
z′xy′
z′xz
z′yw′
z′yx
z′yy′
z′yz
z′zw′
z′zx
z′zy
z′zz

6 (2a) For the expression from page 53 below:

$$G(w,x,y,z) = (y + z')(w + x + y + z)(w' + x + y' + z)$$

find a sum-of-products form, $SOP(G)$

$$SOP(G) = wy'z' + w'y + xy + xz' + yz$$

Show the tree on the facing side.

2

(2a) ①

$$G(w,x,y,z)$$

$$= \cancel{yww'} + ywx + \cancel{ywy'} + ywz + yxw' + yxx + \cancel{yxy'} + yxz + yyw' + yyx + \cancel{yyy'} + yyz$$
$$+ yzw' + yzx + \cancel{yzy'} + yzz + \cancel{z'ww'} + z'wx + z'wy' + \cancel{z'wz} + z'xw' + z'xx + z'xy + \cancel{z'xz}$$
$$+ z'yw' + z'yx + \cancel{z'yy'} + \cancel{z'yz} + \cancel{z'zw'} + \cancel{z'zx} + \cancel{z'zy'} + \cancel{z'zz} \qquad (x'x = 0)$$

$$= ywx + ywz + yxw' + yxx + yxz + yyw' + yyx + yyz$$
$$+ yzw' + yzx + yzz + z'wx + z'wy' + z'xw' + z'xx + z'xy$$
$$+ z'yw' + z'yx \qquad (x*x = x)$$

$$= ywx + ywz + yxw' + \underbrace{yx}_{1①} + \underbrace{yxz}_{1②} + yw' + \underbrace{yx}_{2①} + \underbrace{yz}_{1③}$$
$$+ yzw' + \underbrace{yzx}_{2②} + \underbrace{yz}_{2③} + z'wx + z'wy' + z'xw' + z'x + z'xy$$
$$+ z'yw' + z'yx \qquad (x + x = x)$$

$$= \underbrace{wxy} + wyz + \underbrace{w'xy} + \underbrace{xy} + \underbrace{xyz} + w'y + yz + w'yz$$
$$+ wxz' + wy'z' + w'xz' + xz' + xyz' + w'yz' + \underbrace{xyz'} \qquad (x + (x*y) = x)$$

$$= xy + wyz + \underbrace{w'y} + yz + \underbrace{w'yz} + wxz' + wy'z' + w'xz' + xz' + xyz' + \underbrace{w'yz'} \qquad (x + (x*y) = x)$$

$$= xy + wyz + w'y + yz + \underbrace{wxz'} + wy'z' + \underbrace{w'xz'} + \underbrace{xz'} + \underbrace{xyz'} \qquad (x + (x*y) = x)$$

$$= xy + \underbrace{wyz} + w'y + \underbrace{yz} + xz' + wy'z' \qquad (x + (x*y) = x)$$

$$= xy + yz + w'y + xz' + wy'z'$$

$$= wy'z' + w'y + xy + xz' + yz$$

$$= SOP(G)$$

(2b) ⓪

$$
\begin{aligned}
SOP(G) &= wy'z' &&= w(1)y'z' &&= w(x+x')y'z' \\
&\quad + w'y &&\quad + w'(1)y(1) &&\quad + w'(x+x')y(z+z') \\
&\quad + xy &&\quad + (1)xy(1) &&\quad + (w+w')xy(z+z') \\
&\quad + xz' &&\quad + (1)x(1)z' &&\quad + (w+w')x(y+y')z' \\
&\quad + yz &&\quad + (1)(1)yz &&\quad + (w+w')(x+x')yz
\end{aligned}
$$

(2b) ①

$$
\begin{aligned}
SOP(G) &= w(x+x')y'z' \\
&\quad + w'(x+x')y(z+z') \\
&\quad + (w+w')xy(z+z') \\
&\quad + (w+w')x(y+y')z' \\
&\quad + (w+w')(x+x')yz
\end{aligned}
$$

$$
\begin{aligned}
&= \underbrace{wxy'z'}_{1①} + wx'y'z' \\
&\quad + \underbrace{w'xyz}_{1②} + \underbrace{w'xyz'}_{1③} \\
&\quad + \underbrace{w'x'yz}_{1④} + w'x'yz' \\
&\quad + \underbrace{wxyz}_{1⑤} + \underbrace{wxyz'}_{1⑥} \\
&\quad + \underbrace{w'xyz}_{2②} + \underbrace{w'xyz'}_{2③} \\
&\quad + \underbrace{wxyz'}_{2⑥} + \underbrace{wxy'z'}_{2①} \\
&\quad + \underbrace{w'xyz'}_{3③} + w'xy'z' \\
&\quad + \underbrace{wxyz}_{2⑤} + wx'yz \\
&\quad + \underbrace{w'xyz}_{3②} + \underbrace{w'x'yz}_{2④}
\end{aligned}
$$

$$
\begin{aligned}
&= wxyz \\
&\quad + wxyz' \\
&\quad + wxy'z' \\
&\quad + wx'yz \\
&\quad + wx'y'z' \\
&\quad + w'xyz \\
&\quad + w'xyz' \\
&\quad + w'xy'z' \\
&\quad + w'xyz \\
&\quad + w'x'y'z'
\end{aligned}
$$

$$= CSOP(G)$$

6 (2b) For the expression below (**which is the same as in (2a)**):

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

find the complete sum-of-products form, $CSOP(G)$

$$CSOP(G) = wxyz + wxyz' + wxy'z' + wx'yz$$
$$+ wx'y'z' + w'xyz + w'xyz'$$
$$+ w'xy'z' + w'x'yz + w'x'yz'$$

(2b) ② TREE

$w\_\_\_$

$wx\_\_$ $\quad$ $wx'\_\_$

$wxy'\_$ $\quad$ $wx'y'\_$

$wxy'z'$ $\quad$ $wx'y'z'$

(2b) ③ TREE

$w'\_\_\_$

$w'x\_\_$ $\quad$ $w'x'\_\_$

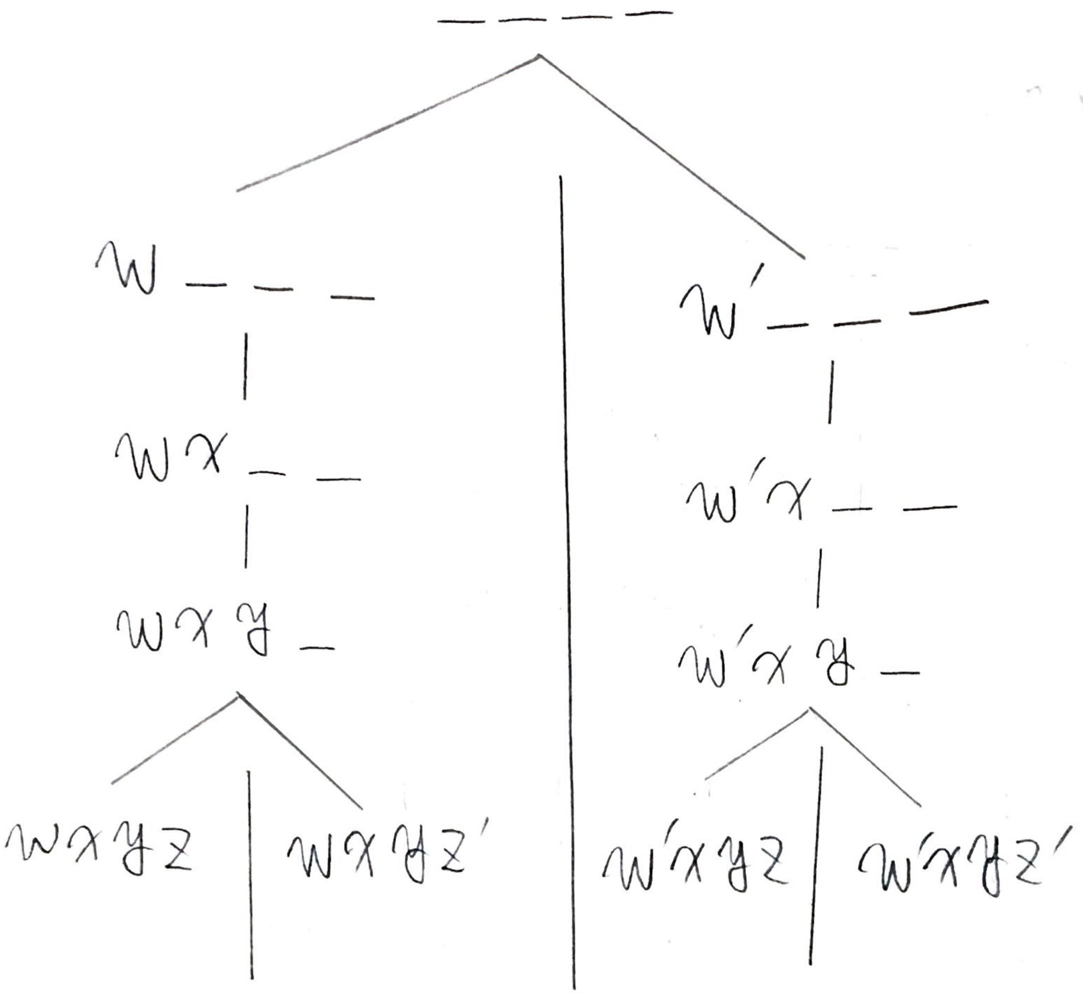
$w'xy\_$ $\quad$ $w'x'y\_$

$w'xyz$ $\quad$ $w'xyz'$ $\quad$ $w'x'yz$ $\quad$ $w'x'yz'$

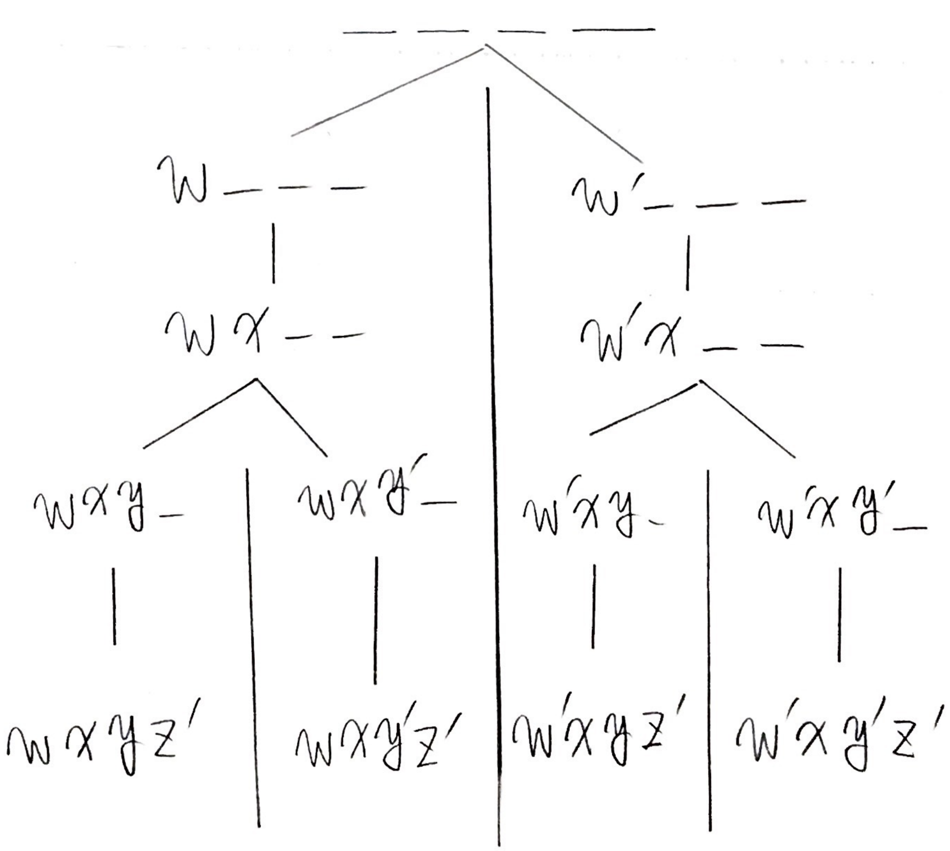
6

You are free to talk to anyone

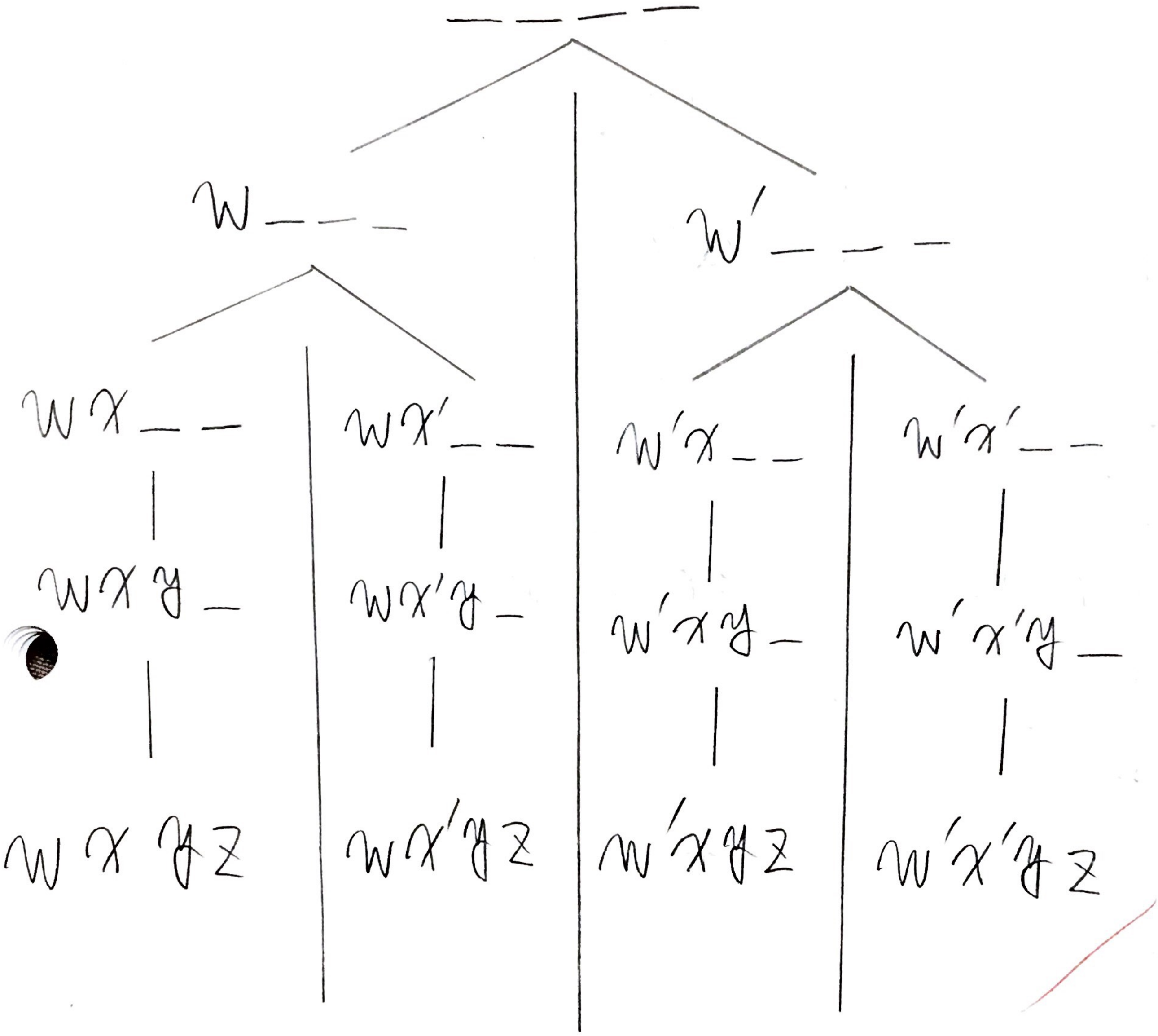
(2b) ④ TREE



(2b) ⑤ TREE



(2b) 6 TREE



(i) $G(w,x,y,z)$

$$= wy'z' + w'y + xy + xz' + yz \qquad ((2a))$$

$$= wy'z' + w'y + xy + xz' + yz$$

$$\Big(wy'z' \nleq w'y,\ wy'z' \nleq xy,\ wy'z' \nleq xz',\ wy'z' \nleq yz,$$
$$w'y \nleq wy'z',\ w'y \nleq xy,\ w'y \nleq xz',\ w'y \nleq yz,$$
$$xy \nleq wy'z',\ xy \nleq w'y,\ xy \nleq xz',\ xy \nleq yz,$$
$$xz' \nleq wy'z',\ xz' \nleq w'y,\ xz' \nleq xy,\ xz' \nleq yz,$$
$$yz \nleq wy'z',\ yz \nleq w'y,\ yz \nleq xy,\ yz \nleq xz',$$
$$\cancel{Cons}(wy'z', w'y),\ \cancel{Cons}(wy'z', xy),\ \cancel{Cons}(wy'z', xz')$$
$$\cancel{Cons}(wy'z', yz),\ \cancel{Cons}(w'y, xy),\ \cancel{Cons}(w'y, xz'),$$
$$\cancel{Cons}(w'y, yz),\ \cancel{Cons}(xy, xz'),\ \cancel{Cons}(xy, yz),$$
$$\cancel{Cons}(xz', yz) \Big)$$

$$PI(G) = \{ wy'z',\ w'y,\ xy,\ xz',\ yz \}$$

(ii) ①

$$SOP(G) = \underbrace{wxy'z'}_{1①} + wx'y'z' + \underbrace{w'xyz}_{1②} + \underbrace{w'xyz'}_{1③}$$
$$+ \underbrace{w'x'yz}_{1④} + w'x'yz' + \underbrace{wxyz}_{1⑤} + \underbrace{wxyz'}_{1⑥}$$
$$+ \underbrace{w'xyz}_{2②} + \underbrace{w'x'yz'}_{2③} + \underbrace{wx'yz'}_{2④} + \underbrace{wxy'z'}_{2①}$$
$$+ \underbrace{w'xyz'}_{3③} + w'xy'z' + \underbrace{wxyz}_{2⑤}$$
$$+ wx'yz + \underbrace{w'xyz}_{3②} + \underbrace{w'x'yz}_{2④}$$

((2b))

$$= wy'z' + w'y + xz' + yz = MSOP(G)$$

5 (2c) For the expression below **which is the same as in (2a) and (2b)**:

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

use your answer from (2a) and (2b) to find the following:

$PI(G)$ and $MSOP\ (G)$

Answer from (2a) $SOP(G) = wy'z' + w'y + xy + xz' + yz$

Answer from (2b) $CSOP\ (G) = wxyz + wxyz' + wxy'z' + wx'yz + wx'y'z' + w'xyz + w'xyz' + w'xy'z' + w'x'yz + w'x'yz'$

2 (i) $PI(G) = \{wy'z',\ w'y,\ xy,\ xz',\ yz\}$

3 (ii) $MSOP\ (G) = wy'z' + w'y + xz' + yz$

(ii) 1

TREE

Note: Please see (2b)

(i) ⓪

$$Smnd(A) = \{wy'z',\ w'y,\ xy,\ xz',\ yz\}$$

(i) ①

$$A_S = \nu(Smnd(A)) = 5$$

(i) ② $MSLtr(A)$

$$= \{w{:}1,\ w'{:}1,\ x{:}2,\ y{:}3,\ y'{:}1,\ z{:}1,\ z'{:}2\}$$

(i) ③ A_L

$$= \nu(MSLtr(A)) = 1+1+2+3+1+1+2 = 11$$

(i) ④ $Smnd(B)$

$$= \{wy'z',\ w'y,\ xz',\ yz\}$$

(i) ⑤ B_S

$$= \nu(Smnd(B)) = 4$$

(i) ⑥ $MSLtr(B)$

$$= \{w{:}1,\ w'{:}1,\ x{:}1,\ y{:}2,\ y'{:}1,\ z{:}1,\ z'{:}2\}$$

(i) ⑦ B_L

$$= \nu(MSLtr(B)) = 1+1+1+2+1+1+2 = 9$$

(ii)

$$\cfrac{\cfrac{(B_L = 9 < 11 = A_L) \wedge (B_S = 4 \leq 11 = A_L)}{(B_L < A_L) \wedge (B_S \leq A_L)}}{Smplr(B, A)}$$

6 (3a) We define:

$$A := SOP(G(w,x,y,z)) = wy'z' + w'y + xy + xz' + yz$$

$$B = MSOP(G(w,x,y,z)) = wy'z' + w'y + xz' + yz$$

Copy your answers from (1a)

Compute:

1 (i) A_S = 5 A_L = 11 B_S = 4 B_L = 9

1 (ii) On the basis of your answers to (i), circle the correct choice below.

$Smplr\ (A, B)$ $Smplr(B, A)$ Neither

Why?

4 (iii) Write down the version of $G(w, x, y, z)$ in PC below and find its **formation-tree on the facing side.**

Version of $G(w, x, y, z)$in PC

$$= (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee (\neg r) \vee s)$$

6

(iii) ⑥

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

$$G(w,x,y,z) = (y+(z)')*(w+x+y+z)*((w)'+x+(y)'+z)$$

$$G(p,q,r,s) = (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee (\neg r) \vee s)$$

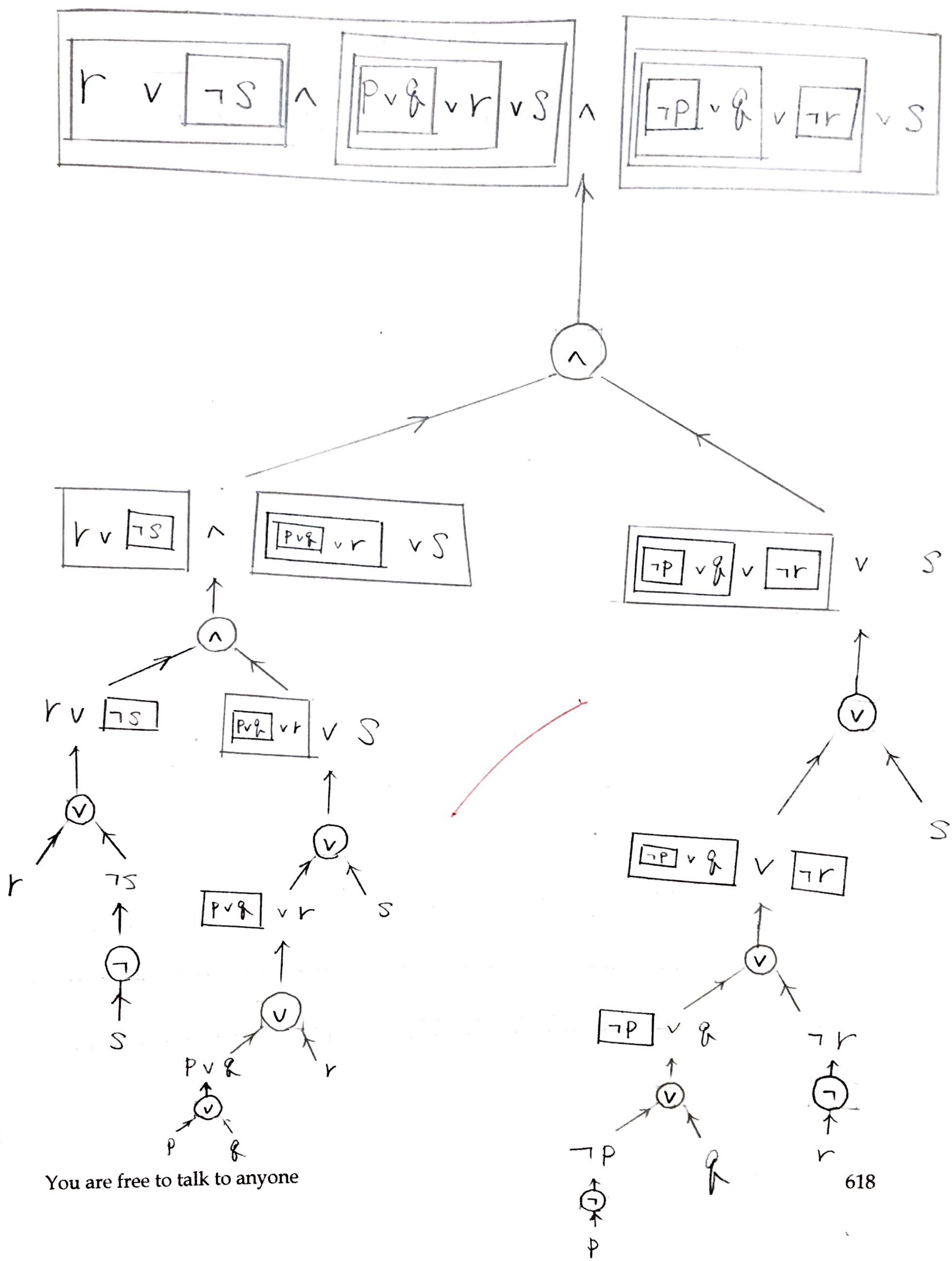
(iii) ①

$$G(w,x,y,z)$$

$$= (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee (\neg r) \vee s)$$

$$\boxed{\boxed{r \vee \boxed{\neg s}} \wedge \boxed{\boxed{\boxed{p \vee q} \vee r} \vee s}} \wedge \boxed{\boxed{\boxed{\boxed{\neg p} \vee q} \vee \boxed{\neg r}} \vee s}$$

(iii) 2



(i) 0

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+y'+z)$$

$$G(w,x,y,z) = (y+(z)')*(w+x+y+z)*((w)'+x+(y)'+z)$$

$$H(A,B,C,D) = (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup (C^c) \cup D)$$

(i) 1

$$MSOP(G(w,x,y,z)) = wy'z' + w'y + xz' + yz$$

$$MSOP(G(w,x,y,z)) = (w*(y')*(z')) + ((w')*y) + (x*(z')) + (y*z)$$

$$K(A,B,C,D) = (A \cap (C^c) \cap (D^c)) \cup ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$$

2 (3b) Find the versions: $H(A,B,C,D)$ of $G(w,x,y,z)$

and $K(A,B,C,D)$ of $MSOP(G(w,x,y,z))$

in Sets.

1 (i) $H(A,B,C,D)$

$$= (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup (C^c) \cup D)$$

1 (i) $K(A,B,C,D)$

$$= (A \cap (C^c) \cap (D^c)) \cup ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$$

2

(3c) ⓪

$H(A,B,C,D)$

$= (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup (C^c) \cup D)$

$= ((C \cap (A \cup B \cup C \cup D)) \cup ((D^c) \cap (A \cup B \cup C \cup D))) \cap ((A^c) \cup B \cup (C^c) \cup D)$ $\quad ((A \cup B) \cap C = (A \cap C) \cup (B \cap C))$

$= ((C \cap A) \cup (C \cap B) \cup (C \cap C) \cup (C \cap D) \cup ((D^c) \cap A) \cup ((D^c) \cap B) \cup ((D^c) \cap C) \cup ((D^c) \cap D)) \cap ((A^c) \cup B \cup (C^c) \cup D)$ $\quad (A \cap (B \cup C) = (A \cap B) \cup (A \cap C))$

$= (\underline{(C \cap A)} \cup \underline{(C \cap B)} \cup \underline{C} \cup \underline{(C \cap D)} \cup ((D^c) \cap A) \cup ((D^c) \cap B) \cup \underline{((D^c) \cap C)} \cup \{\}) \cap ((A^c) \cup B \cup (C^c) \cup D)$ $\quad (A \cap A = A,\ (A^c) \cap A = \{\})$

$= (C \cup (A \cap (D^c)) \cup (B \cap (D^c))) \cap ((A^c) \cup B \cup (C^c) \cup D)$ $\quad (A \cap (B \cup C) = (A \cap B) \cup (A \cap C))$

$= C \cap ((A^c) \cup B \cup (C^c) \cup D) \cup (A \cap (D^c)) \cap ((A^c) \cup B \cup (C^c) \cup D) \cup (B \cap (D^c))$
$\cap ((A^c) \cup B \cup (C^c) \cup D)$

$(A \cap (B \cup C) = (A \cap B) \cup (A \cap C))$

$= (C \cap (A^c)) \cup (C \cap B) \cup (C \cap (C^c)) \cup (C \cap D) \cup (A \cap (D^c) \cap (A^c)) \cup (A \cap (D^c)) \cap B \cup (A \cap (D^c))$
$\cap (C^c) \cup (A \cap (D^c) \cap D) \cup (B \cap (D^c) \cap (A^c))$
$\cup (B \cap (D^c) \cap B) \cup (B \cap (D^c) \cap (C^c)) \cup (B \cap (D^c) \cap D)$

$(A \cap (B \cup C) = (A \cap B) \cup (A \cap C))$

$= (C \cap (A^c)) \cup (C \cap B) \cup (C \cap D) \cup \underline{(A \cap (D^c) \cap B)} \cup (A \cap (D^c) \cap (C^c)) \cup \underline{(B \cap (D^c) \cap (A^c))}$
$\cup \underline{(B \cap (D^c))} \cup \underline{(B \cap (D^c) \cap (C^c))}$ $\quad (A \cap (A^c) = \{\},\ A \cup (A \cap B) = A)$

$= (C \cap (A^c)) \cup (C \cap B) \cup (C \cap D) \cup (A \cap (D^c) \cap (C^c)) \cup (B \cap (D^c))$

9 (3c) Prove in Sets that:

$$H(A,B,C,D) = K(A,B,C,D)$$

(3c) ①

$H(A,B,C,D)$

$= (C \cap (A^c)) \cup (C \cap B) \cup (C \cap D) \cup (A \cap (D^c) \cap (C^c)) \cup (B \cap (D^c))$

$= ((A^c) \cap C) \cup (A \cup (A^c)) \cap B \cap C \cap (D \cup (D^c)) \cup (C \cap D)$

$\cup (A \cap (D^c)) \cap (C^c) \cup (B \cap (D^c)))$ $\quad (A = A \cap \mathcal{U},\ A \cup (A^c) = \mathcal{U})$

$= \underbrace{((A^c) \cap C)}_{1\,\text{①}} \cup \underbrace{(A \cap B \cap C \cap D)}_{1\,\text{②}} \cup \underbrace{(A \cap B \cap C \cap (D^c))}_{1\,\text{③}} \cup \underbrace{((A^c) \cap B \cap C \cap D)}_{2\,\text{①}}$

$\cup \underbrace{((A^c) \cap B \cap C \cap (D^c))}_{3\,\text{①}} \cup \underbrace{(C \cap D)}_{2\,\text{②}} \cup (A \cap (C^c) \cap (D^c)) \cup \underbrace{(B \cap (D^c))}_{2\,\text{③}}$

$(A \cup (A \cap B) = A)$

$= ((A^c) \cap C) \cup (C \cap D) \cup (A \cap (C^c) \cap (D^c)) \cup (B \cap (D^c))$

$H(A,B,C,D) = ((A^c) \cap C) \cup (C \cap D) \cup (A \cap (C^c) \cap (D^c)) \cup (B \cap (D^c)) = K(A,B,C,D)$

$H(A,B,C,D) = K(A,B,C,D)$

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