Instructions:

Use these instructions for the remainder of the exam.

Your J-number $JN :=$ 00298436 $:= (J_k | k \in 1..8)$

Hence

$J_1 =$ 0 $J_2 =$ 0 $J_3 =$ 2 $J_4 =$ 9

$J_5 =$ 8 $J_6 =$ 4 $J_7 =$ 3 $J_8 =$ 6

R For each of sets defined below **write the elements in increasing order**

D Define $\mathcal{U} := 0..9 = \{0,1,2,3,4,5,6,7,8,9\}$

D Define $J := \{J_k \in 0..9 | k \in 1..8\} =$ {0,2,3,4,6,8,9}

D Define $\forall n \in \mathbb{N}$ $Prm(n) :\Leftrightarrow n \text{ is a prime number}$

D Define $E := \left\{J_k \in 0..9 \middle| Prm(k)\right\} =$ { 0, 2, 3, 8 }

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$Prm(k)$	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

D Define

$$A := \left\{J_k \in 0..9 \,\middle|\, \exists n \in \mathbb{N} \quad \left(J_k = n^3\right)\right\} = \left\{ \quad 0,8 \quad \right\}$$

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$\exists n \in \mathbb{N} \quad \left(J_k = n^3\right)$	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

D Define

$$B := \left\{k \in 1..8 \,\middle|\, \exists n \in \mathbb{N} \quad \left(k = n^2\right)\right\} = \left\{ \quad 1,4 \quad \right\}$$

k	1	2	3	4	5	6	7	8
J_k	0	0	2	9	8	4	3	6
$\exists n \in \mathbb{N} \quad \left(k = n^2\right)$	Y	Y	Y	Y	Y	Y	Y	Y
	N	N	N	N	N	N	N	N

Your J-number $JN :=$ 00298436

We define $\forall k \in 1..8$,

D $J_k :=$ the k[th] entry in your J-number counting from the left.

D $r_k :=$ the remainder obtained upon dividing J_k by 5.

Therefore,

$J_1 :=$ 0

$J_2 :=$ 0

$J_3 :=$ 2

$J_4 :=$ 9

$J_5 :=$ 8

$J_6 :=$ 4

$J_7 :=$ 3

$J_8 :=$ 6

Show all long divisions on the facing page

$r_1 :=$ 0

$r_2 :=$ 0

$r_3 :=$ 2

$r_4 :=$ 4

$r_5 :=$ 3

$r_6 :=$ 4

$r_7 :=$ 3

$r_8 :=$ 1

$J_1 = 0$	$J_2 = 0$	$J_3 = 2$	$J_4 = 9$	$J_5 = 8$
$\begin{array}{r} 0 \\ 5 \overline{)\,0} \\ 0 - \\ \hline 0 \end{array}$	$\begin{array}{r} 0 \\ 5 \overline{)\,0} \\ 0 - \\ \hline 0 \end{array}$	$\begin{array}{r} 0 \\ 5 \overline{)\,2} \\ 0 - \\ \hline 2 \end{array}$	$\begin{array}{r} 1 \\ 5 \overline{)\,9} \\ 5 - \\ \hline 4 \end{array}$	$\begin{array}{r} 1 \\ 5 \overline{)\,8} \\ 5 - \\ \hline 3 \end{array}$
$r_1 = 0$	$r_2 = 0$	$r_3 = 2$	$r_4 = 4$	$r_5 = 3$

$J_6 = 4$	$J_7 = 3$	$J_8 = 6$
$\begin{array}{r} 0 \\ 5 \overline{)\,4} \\ 0 - \\ \hline 4 \end{array}$	$\begin{array}{r} 0 \\ 5 \overline{)\,3} \\ 0 - \\ \hline 3 \end{array}$	$\begin{array}{r} 1 \\ 5 \overline{)\,6} \\ 5 - \\ \hline 1 \end{array}$
$r_6 = 4$	$r_7 = 3$	$r_8 = 1$

D Define a relation $R \subseteq \{0,1,2,3,4\} \times \{0,1,2,3,4\} =: \mathcal{U}$ as follows:

Compute R on the facing side, and write the answer below:

R

$$:= \{(0,0)\} \cup \left(\bigcup_{k \in 1..4} \{(r_k, r_{9-k})\} \right)$$

$$= \{(0,0), (0,1), (0,3), (2,4), (4,3)\}$$

R In order to finish this exam you need to learn to produce the incidence matrix or a relation and need to learn how to multiply matrices. The book and the internet contains information on both items but does not use the word incidence.

You will need the following definitions.

D Given and incidence matrix for a relation $M(R)$ we define:

$$(M(R))(0,1)$$

$$:= \quad Matrix\ obtained\ by\ replacing\ all\ non-zero\ entries\ in\ M(R)\ by\ 1$$

R **Note that in a set an element may occur exactly once.** Therefore there may not be any repeated elements in either A or B in the above. **The sets A and B will occur in several problems on this test. So, if you do not have the correct sets, all your answers will be wrong.**

R **In arrow-diagrams, put the numbers in rectangles rather than ovals and draw the rectangles using rulers.**

R **Any sloppiness, untidiness, and any departure from proper format (as indicated in class) will lead to a score of 0.**

$$R$$

$$:= \{(0,0)\} \cup \left(\bigcup_{k=1..4} \{(r_k, r_{9-k})\} \right)$$

$$= \{(0,0)\} \cup \{(r_1, r_{9-1})\} \cup \{(r_2, r_{9-2})\} \cup \{(r_3, r_{9-3})\} \cup \{(r_4, r_{9-4})\}$$

$$= \{(0,0)\} \cup \{(r_1, r_8)\} \cup \{(r_2, r_7)\} \cup \{(r_3, r_6)\} \cup \{(r_4, r_5)\}$$

$$= \{(0,0)\} \cup \{(0,1)\} \cup \{(0,3)\} \cup \{(2,4)\} \cup \{(4,3)\}$$

$$= \{(0,0), (0,1), (0,3), (2,4), (4,3)\}$$

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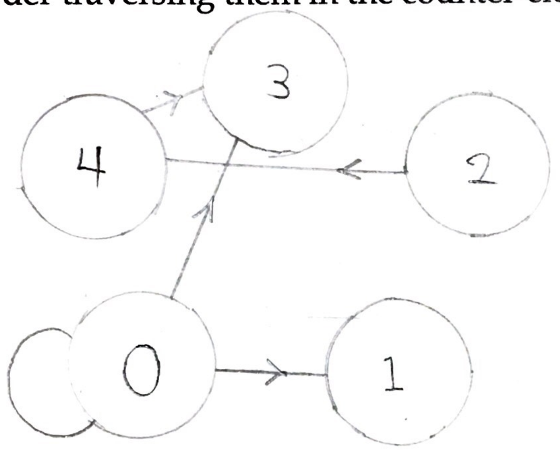
17 (1abc) Carry out the following instructions, **doing all computations on the facing side** and **recording only the answers on this side.**

1 (i) **Draw directed graph for the relation *R*** earlier **in the space below** with 0, 1, 2, 3, 4 as nodes **arranged in the shape of a regular pentagon with a horizontal base,** with the left vertex of the horizontal side labelled 0, and labelling all the vertices in order traversing them in the counter-clockwise sense.



1 (ii) Find the 5× 5 incidence matrix $M(R)$ for the relation R.

$$M(R) = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

3

1 (iii) Find the 5× 5 incidence matrix $M(R^{op})$ for the relation R^{op}.

$$M(R^{op}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(ii)

	0	1	2	3	4
0	1	1	0	1	0
1	0	0	0	0	0
2	0	0	0	0	1
3	0	0	0	0	0
4	0	0	0	1	0

$$M(R) = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(iii) ⓪

$$R^{op} = \left\{ (0,0), (1,0), (3,0), (3,4), (4,2) \right\}$$

(iii) ①

	0	1	2	3	4
0	1	0	0	0	0
1	1	0	0	0	0
2	0	0	0	0	0
3	1	0	0	0	1
4	0	0	1	0	0

$$M(R^{op}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

5 (iv) Compute: on the **facing side**:

$$R \circ R^{op}, \qquad R^{op} \circ R,$$

$$M(R \circ R^{op}), \qquad M(R^{op} \circ R),$$

$$(M(R))(M(R^{op})), \qquad (M(R^{op}))(M(R))$$

Prove on **this side** that:

(1) $\left(M(R \circ R^{op})\right)(0,1) = \left(\left(M(R^{op})\right)\left(M(R)\right)\right)(0,1)$

LS | RS

$$= \begin{bmatrix} 1&1&0&1&0 \\ 1&1&0&1&0 \\ 0&0&0&0&0 \\ 1&1&0&1&0 \\ 0&0&0&0&1 \end{bmatrix} \qquad = \begin{bmatrix} 1&1&0&1&0 \\ 1&1&0&1&0 \\ 0&0&0&0&0 \\ 1&1&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$$

$$LS = \begin{bmatrix} 1&1&0&1&0 \\ 1&1&0&1&0 \\ 0&0&0&0&0 \\ 1&1&0&1&0 \\ 0&0&0&0&1 \end{bmatrix} = RS$$

$$\left(M(R \circ R^{op})\right)(0,1) = \left(\left(M(R^{op})\right)\left(M(R)\right)\right)(0,1)$$

(2) $\left(M(R^{op} \circ R)\right)(0,1) = \left(\left(M(R)\right)\left(M(R^{op})\right)\right)(0,1)$

LS | RS

$$= \begin{bmatrix} 1&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \end{bmatrix} \qquad = \begin{bmatrix} 1&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \end{bmatrix}$$

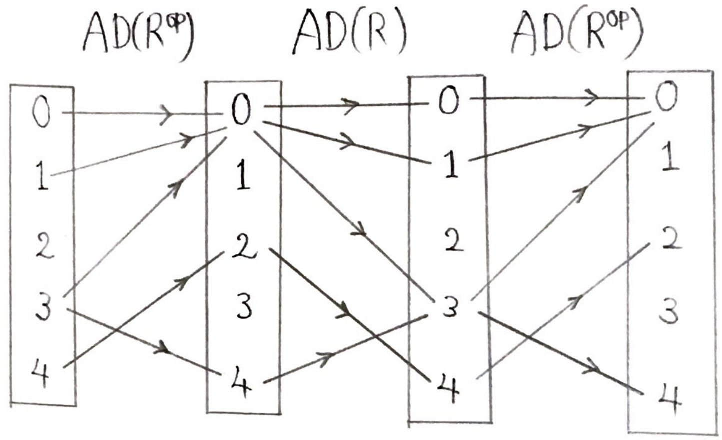
$$LS = \begin{bmatrix} 1&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \end{bmatrix} = RS$$

$$\left(M(R^{op} \circ R)\right)(0,1) = \left(\left(M(R)\right)\left(M(R^{op})\right)\right)(0,1)$$

8

(iv)

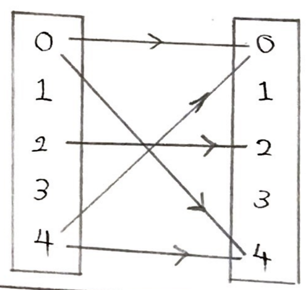
0



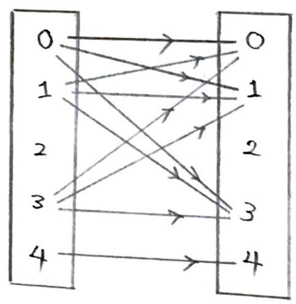
(iv)

1

$AD(R^{op} \circ R)$



$AD(R \circ R^{op})$



(iv) 2

$$R^{op} \circ R = \{(0,0),(0,4),(2,2),(4,0),(4,4)\}$$

(iv) 3

$$R \circ R^{op} = \{(0,0),(0,1),(0,3),(1,0),(1,1),(1,3),(3,0),(3,1),(3,3),(4,4)\}$$

(iv)

4

	0	1	2	3	4
0	1	0	0	0	1
1	0	0	0	0	0
2	0	0	1	0	0
3	0	0	0	0	0
4	1	0	0	0	1

$$M(R^{op} \circ R) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(iv)

5

	0	1	2	3	4
0	1	1	0	1	0
1	1	1	0	1	0
2	0	0	0	0	0
3	1	1	0	1	0
4	0	0	0	0	1

$$M(R \circ R^{op}) = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(iv)

(2) ⓪ $(M(R))(M(R^{op}))$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix} \begin{bmatrix} 1&0&0&0&0 \\ 1&0&0&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \\ 0&0&1&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+1(1)+0(0)+1(1)+0(0) & 1(0)+1(0)+0(0)+1(0)+0(0) & 1(0)\ 1(0)+0(0)+1(0)+0(1) & 1(0)+1(0)+0(0)+1(0)+0(0) & 1(0)+1(0)+0(0)+1(1)+0(0) \\ 0(1)+0(1)+0(0)+0(1)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(1) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(1)+0(0) \\ 0(1)+0(1)+0(0)+0(1)+1(0) & 0(0)+0(0)+0(0)+0(0)+1(0) & 0(1)+0(0)+0(0)+0(0)+1(1) & 0(0)+0(0)+0(0)+0(0)+1(0) & 0(0)+0(0)+0(0)+0(1)+1(0) \\ 0(1)+0(1)+0(0)+0(1)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(1) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(1)+0(0) \\ 0(1)+0(1)+0(0)+1(1)+0(0) & 0(0)+0(0)+0(0)+1(0)+0(0) & 0(1)+0(0)+0(0)+1(0)+0(1) & 0(0)+0(0)+0(0)+1(0)+0(0) & 0(0)+0(0)+0(0)+1(1)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 3&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \end{bmatrix}$$

(iv)

(2) ① $((M(R))(M(R^{op})))(0,1)$

$$= \begin{bmatrix} 1&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \end{bmatrix}$$

(iv)

(1) ① $(M(R^{op}))(M(R))$

$$= \begin{bmatrix} 1&0&0&0&0 \\ 1&0&0&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&1 \\ 0&0&1&0&0 \end{bmatrix} \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0(0)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(1) & 1(0)+0(1)+0(0)+0(1)+0(0) \\ 1(1)+0(0)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(0) & 1(0)+0(0)+0(0)+0(0)+0(0) & 1(1)+0(0)+0(0)+0(0)+0(1) & 1(0)+0(1)+0(0)+0(1)+0(0) \\ 0(1)+0(0)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(0) & 0(0)+0(0)+0(0)+0(0)+0(0) & 0(1)+0(0)+0(0)+0(0)+0(1) & 0(0)+0(0)+0(0)+0(1)+0(0) \\ 1(1)+0(0)+0(0)+0(0)+1(0) & 1(1)+0(0)+0(0)+0(0)+1(0) & 1(0)+0(0)+0(0)+0(0)+1(0) & 1(1)+0(0)+0(0)+0(0)+1(1) & 1(0)+0(0)+0(0)+0(1)+1(0) \\ 0(1)+0(0)+1(0)+0(0)+0(0) & 0(1)+0(0)+1(0)+0(0)+0(0) & 0(0)+0(0)+1(0)+0(0)+0(0) & 0(1)+0(0)+1(0)+0(0)+0(1) & 0(0)+0(0)+1(0)+0(1)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 1&1&0&1&0 \\ 0&0&0&0&0 \\ 1&1&0&2&0 \\ 0&0&0&0&1 \end{bmatrix}$$

R

(iv)

(1) ① $((M(R^{op}))(M(R)))(0,1)$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 1&1&0&1&0 \\ 0&0&0&0&0 \\ 1&1&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (v) On the **facing side**

draw: $DG(R)$, $DG(RflxClsr(R))$ and

find: $M\left(RflxClsr(R)\right)$, $M\left(R \cup (\Delta_{0..4})\right)$

Prove on **this side** that:

$$\left(M\left(RflxClsr(R)\right)\right)(0,1) = \left(M\left(R \cup (\Delta_{0..4})\right)\right)(0,1)$$

LS

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix}(0,1)$$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix}$$

RS

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix}(0,1)$$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix}$$

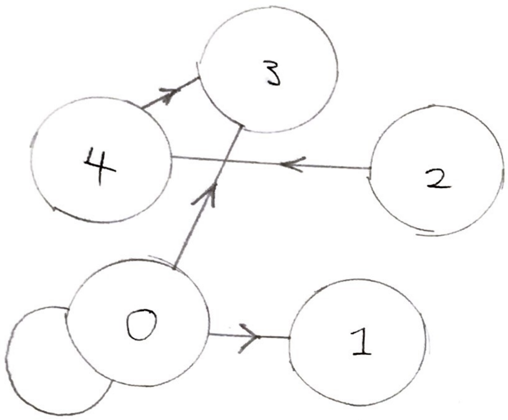
$$LS = \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix} = RS$$

1

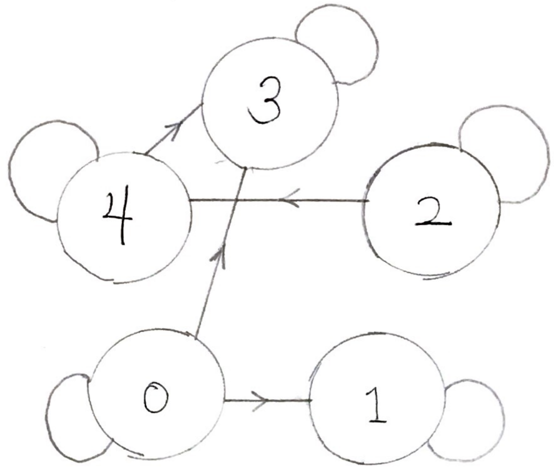
$$\left(M\left(RflxClsr(R)\right)\right)(0,1) = \left(M\left(R \cup (\Delta_{0..4})\right)\right)(0,1)$$

You may talk to anyone

(v) ⓪ DG(R)



(v) ① DG(RflxClsr(R))



(v) ②

	0	1	2	3	4
0	1	1	0	1	0
1	0	1	0	0	0
2	0	0	1	0	1
3	0	0	0	1	0
4	0	0	0	1	1

$M(RflxClsr(R))$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix}$$

(v) ③ $R \cup (\Delta_{0..4})$

$$= \{(0,0),(0,1),(0,3),(2,4),(4,3)\}$$
$$\cup \{(0,0),(1,1),(2,2),(3,3),(4,4)\}$$

$$= \left\{ \begin{array}{l} (0,0),(0,1),(0,3), \\ (1,1),(2,2),(2,4), \\ (3,3),(4,3),(4,4) \end{array} \right\}$$

(v) ④

	0	1	2	3	4
0	1	1	0	1	0
1	0	1	0	0	0
2	0	0	1	0	1
3	0	0	0	1	0
4	0	0	0	1	1

$M(R \cup (\Delta_{0..4}))$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&1&1 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (vi) On the **facing side**

draw: $DG(R)$, $DG(SymClsr(R))$ and

find: $M\left(SymClsr(R)\right), M\left(R \cup (R^{op})\right)$

Prove on **this side** that:

$$\left(M\left(SymClsr(R)\right)\right)(0,1) = \left(M\left(R \cup (R^{op})\right)\right)(0,1)$$

LS | RS

$$= \begin{bmatrix} 1&1&0&1&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \end{bmatrix}(0,1) \qquad = \begin{bmatrix} 1&1&0&1&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \end{bmatrix}(0,1)$$

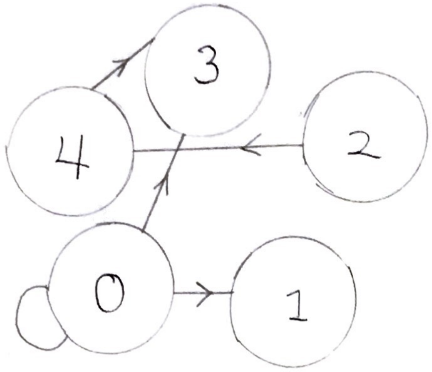
$$= \begin{bmatrix} 1&1&0&1&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \end{bmatrix} \qquad = \begin{bmatrix} 1&1&0&1&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \end{bmatrix}$$

$$LS = \begin{bmatrix} 1&1&0&1&0 \\ 1&0&0&0&0 \\ 0&0&0&0&1 \\ 1&0&0&0&1 \\ 0&0&1&1&0 \end{bmatrix} = RS$$

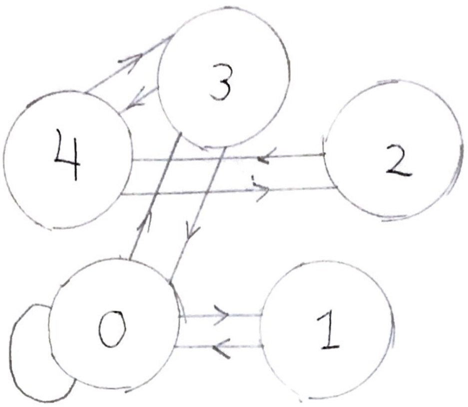
1

$$\left(M\left(SymClsr(R)\right)\right)(0,1) = \left(M\left(R \cup (R^{op})\right)\right)(0,1)$$

(vi) ⓪ DG(R)



(vi) ① DG(SymClsr(R))



(vi) ②

	0	1	2	3	4
0	1	1	0	1	0
1	1	0	0	0	0
2	0	0	0	0	1
3	1	0	0	0	1
4	0	0	1	1	0

M(SymClsr(R))

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(vi) ③

$R \cup (R^{op})$

$= \{(0,0),(0,1),(0,3),(2,4),(4,3)\}$

$\cup \{(0,0),(1,0),(3,0),(3,4),(4,2)\}$

$$= \left\{ \begin{matrix} (0,0), (0,1), (0,3), \\ (1,0), (2,4), (3,0), \\ (3,4), (4,2), (4,3) \end{matrix} \right\}$$

(vi) ④

	0	1	2	3	4
0	1	1	0	1	0
1	1	0	0	0	0
2	0	0	0	0	1
3	1	0	0	0	1
4	0	0	1	1	0

$M(R \cup (R^{op}))$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

17 (1abc) Carry out the following instructions

1 (vii) On the **facing side**

draw: $DG(R)$, $DG(TrnsClsr(R))$ and

find: $M\left(TrnsClsr(R)\right), M\left(\cup_{k=1}^{5}(R^{\circ k})\right)$

Prove on **this side** that:

$$\left(M\left(TrnsClsr(R)\right)\right)(0,1) = \left(M\left(\cup_{k=1}^{5}(R^{\circ k})\right)\right)(0,1)$$

LS

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}(0,1)$$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

RS

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}(0,1)$$

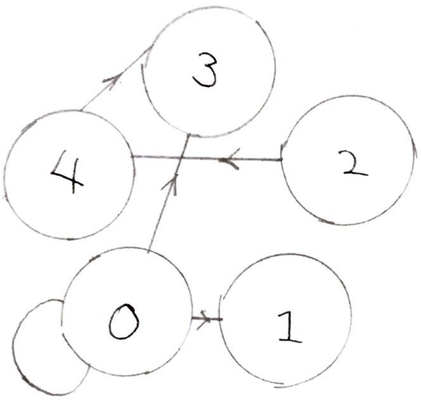
$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

$$LS = \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix} = RS$$

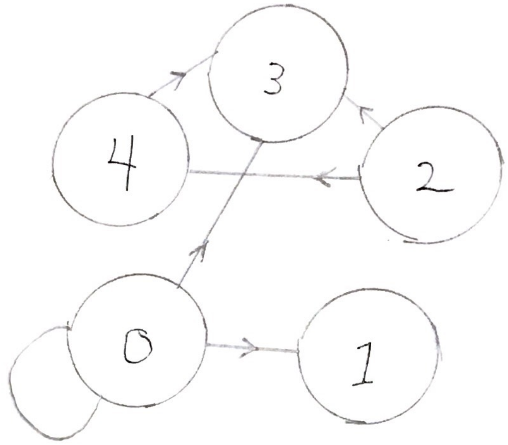
1

$$\left(M\left(TrnsClsr(R)\right)\right)(0,1) = \left(M\left(U_{k=1}^{5}(R^{\circ K})\right)\right)(0,1)$$

(vii) ⓪ DG(R)



(vii) ① DG(TrnsClsr(R))



(vii) ②

	0	1	2	3	4
0	1	1	0	1	0
1	0	0	0	0	0
2	0	0	0	1	1
3	0	0	0	0	0
4	0	0	0	1	0

$M(\mathrm{TrnsClsr}(R))$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

(vii) ③ ⓐ

$$\bigcup_{k=1}^{5} (R^{\circ k})$$

$$= R^{\circ(1)} \cup R^{\circ(2)} \cup R^{\circ(3)} \cup R^{\circ(4)} \cup R^{\circ(5)}$$

$$= \{(0,0),(0,1),(0,3),(2,4),(4,3)\} \cup \{(0,0),(0,1),(0,3),(2,3)\}$$
$$\cup \{(0,0),(0,1),(0,3)\} \cup \{(0,0),(0,1),(0,3)\}$$
$$\cup \{(0,0),(0,1),(0,3)\}$$

$$= \{(0,0),(0,1),(0,3),(2,3),(2,4),(4,3)\}$$

(vii) ④

	0	1	2	3	4
0	1	1	0	1	0
1	0	0	0	0	0
2	0	0	0	1	1
3	0	0	0	0	0
4	0	0	0	1	0

$M\left(\bigcup_{k=1}^{5} (R^{\circ k})\right)$

$$= \begin{bmatrix} 1&1&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

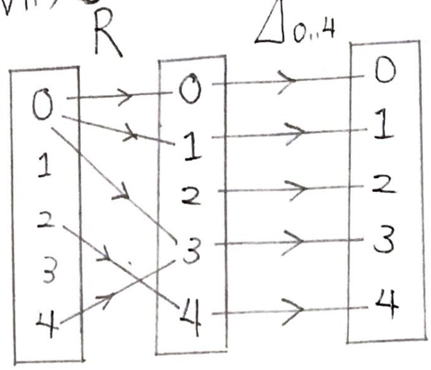
(vii) ③ ⓑ

$R^{\circ(1)}$

$= R^{\circ(0)} \circ R$

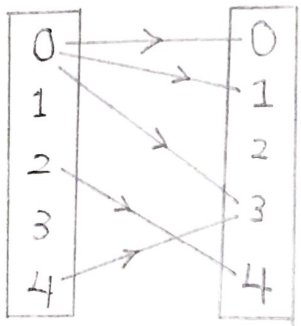
$= \Delta_{0..4} \circ R$

(vii) ③ ⓒ



$AD(\Delta_{0..4} \circ R)$

(vii) ③ ⓓ



$AD(R^{\circ(1)})$

(vii) ③ ⓕ

$R^{\circ(2)}$

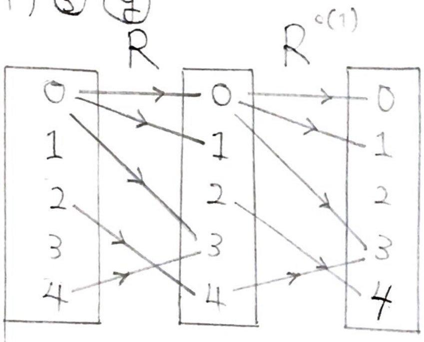
$= R^{\circ(1+1)}$

$= R^{\circ(1)} \circ R$

(vii) ③ ⓔ

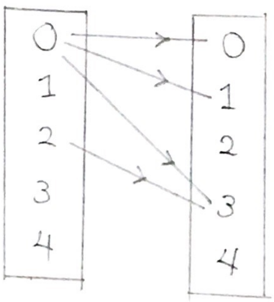
$R^{\circ(1)} = \{(0,0),(0,1),(0,3),(2,4),(4,3)\}$

(viii) ③ ⓖ



$AD(R^{\circ(1)} \circ R)$

(viii) ③ ⓗ



$AD(R^{\circ(2)})$

(viii) ③ ⓘ

$R^{\circ(2)}$

$= \{(0,0),(0,1),(0,3),(2,3)\}$

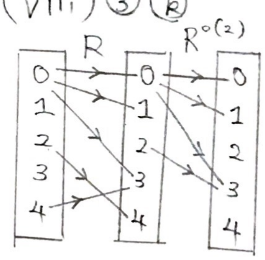
(viii) ③ ⓙ

$R^{\circ(3)}$

$= R^{\circ(2+1)}$

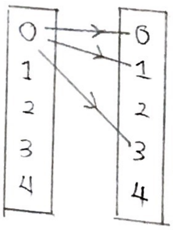
$= R^{\circ(2)} \circ R$

(viii) ③ ⓚ



$AD(R^{\circ(2)} \circ R)$

(viii) ③ ⓛ



$AD(R^{\circ(3)})$

(viii) ③ ⓜ

$R^{\circ(3)}$

$= \{(0,0),(0,1),(0,3)\}$

(viii) ③ ⓠ

$R^{\circ(4)} = \{(0,0),(0,1),(0,3)\}$

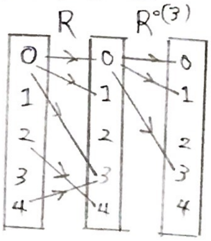
(viii) ③ ⓝ

$R^{\circ(4)}$

$= R^{\circ(3+1)}$

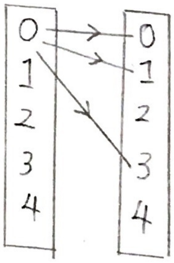
$= R^{\circ(3)} \circ R$

(viii) ③ ⓞ



$AD(R^{\circ(3)} \circ R)$

(viii) ③ ⓟ



$AD(R^{\circ(4)})$

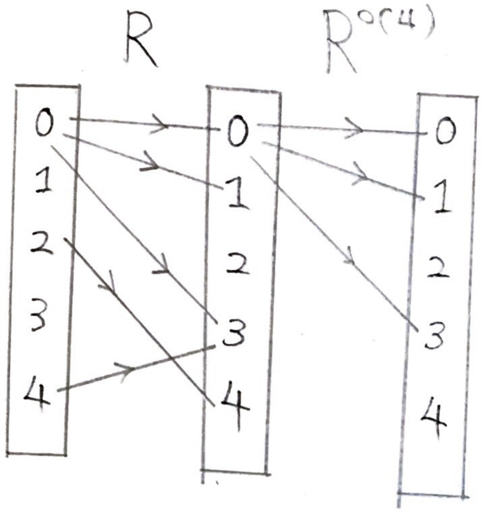
(viii) ③ ⓡ

$R^{\circ(5)}$

$= R^{\circ(4+1)}$

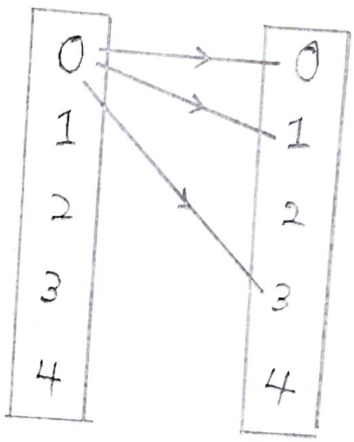
$= R^{\circ(4)} \circ R$

(viii) ③ ⓢ



$AD(R^{\circ(4)} \circ R)$

(viii) ③ ⓣ

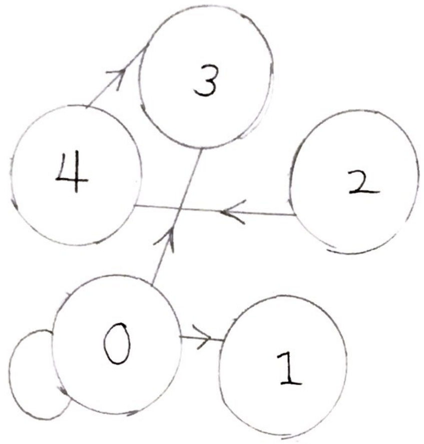


$AD(R^{\circ(5)})$

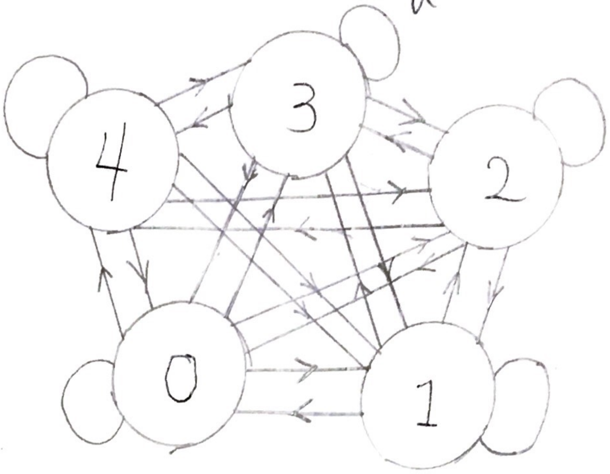
(viii) ③ ⓤ

$R^{\circ(5)} = \{(0,0), (0,1), (0,3)\}$

(viii) ⓪ DG(R)



(viii) ① DG(EqvClsr(R))



(viii) ②

$$EqvClsr(R) = \left\{ \begin{array}{l} (0,0), (0,1), (0,2), (0,3), (0,4), \\ (1,0), (1,1), (1,2), (1,3), (1,4), \\ (2,0), (2,1), (2,2), (2,3), (2,4), \\ (3,0), (3,1), (3,2), (3,3), (3,4), \\ (4,0), (4,1), (4,2), (4,3), (4,4) \end{array} \right\}$$

(viii) ③

	0	1	2	3	4
0	1	1	1	1	1
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1

$$M(EqvClsr(R)) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(viii) ④ ⓐ

$EqvClsr(0, EqvClsr(R)) = \{0, 1, 2, 3, 4\}$

$EqvClsr(1, EqvClsr(R)) = \{0, 1, 2, 3, 4\}$

$EqvClsr(2, EqvClsr(R)) = \{0, 1, 2, 3, 4\}$

$EqvClsr(3, EqvClsr(R)) = \{0, 1, 2, 3, 4\}$

$EqvClsr(4, EqvClsr(R)) = \{0, 1, 2, 3, 4\}$

(viii) ④ ⓑ

$$\frac{\{0, 1, 2, 3, 4\}}{EqvClsr(R)} = \{\{0, 1, 2, 3, 4\}\}$$

17 (1abc) Carry out the following instructions

1 (viii) On the **facing side**

draw: $DG(R)$, $DG\left(EqvClsr(R)\right)$ and

find: $EqvClsr(R)$, $M\left(EqvClsr(R)\right)$

Record on **this side**:

$$\left\{EqvCls(x,R) \in \mathcal{P}(0..4) \,\middle|\, x \in \{0,1,2,3,4\}\right\}$$

$$= \frac{\{0,1,2,3,4\}}{EqvClsr(R)}$$

$$= \{\{0,1,2,3,4\}\}$$

1

17 (1abc) Carry out the following instructions

5 (ix) Find a general formula for: $EqvClsr(R)$ and prove your assertion.

Use your formula to find $M\left(EqvClsr(R)\right)$ and check that it agrees with your answer on (viii)

(ix) 22

$$\bigcup_{k=0}^{5}\left(\left(R\cup(R^{op})\right)^{ok}\right)$$

$$= \Delta_{0..4} \cup \left(\left(R\cup(R^{op})\right)^{o(1)}\right) \cup \left(\left(R\cup(R^{op})\right)^{o(2)}\right)$$

$$\cup \left(\left(R\cup(R^{op})\right)^{o(3)}\right) \cup \left(\left(R\cup(R^{op})\right)^{o(4)}\right)$$

$$\cup \left(\left(R\cup(R^{op})\right)^{o(5)}\right)$$

$$= \left\{\begin{matrix} (0,0),(0,1),(0,2),(0,3),(0,4), \\ (1,0),(1,1),(1,2),(1,3),(1,4), \\ (2,0),(2,1),(2,2),(2,3),(2,4), \\ (3,0),(3,1),(3,2),(3,3),(3,4), \\ (4,0),(4,1),(4,2),(4,3),(4,4) \end{matrix}\right\}$$

(ix) 23

	0	1	2	3	4
0	1	1	1	1	1
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1

2

$$M\left(\bigcup_{k=0}^{5}\left(\left(R\cup(R^{op})\right)^{ok}\right)\right) = \begin{bmatrix} 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \end{bmatrix}$$

(ix) 24

$$M\left(EqvClsr(R)\right) = \begin{bmatrix} 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \end{bmatrix} = M\left(\bigcup_{k=0}^{5}\left(\left(R\cup(R^{op})\right)^{ok}\right)\right)$$

(ix)⓪

$\mathrm{EquClsr}(R)$

$= \mathrm{RflxClsr}(R) \cup (\mathrm{TrnsClsr}(\mathrm{SymClsr}(R)))$

$= \bigcup_{k=0}^{5} \left((R \cup R^{op})^{\circ k} \right)$

Proof!

(ix)①

$(R \cup (R^{op}))^{\circ(0)}$

$= \Delta_{1..4}$

$= \{(0,0),(1,1),(2,2),(3,3),(4,4)\}$

(ix)②

$(R \cup (R^{op}))^{\circ(1)}$

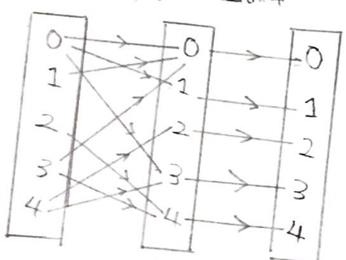
$= (R \cup (R^{op}))^{\circ(0+1)}$

$= (R \cup (R^{op}))^{\circ(0)} \circ (R \cup (R^{op}))$

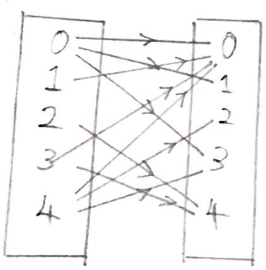
$= \Delta_{0..4} \circ (R \cup (R^{op}))$

(ix)③

$R \cup (R^{op})$ $\Delta_{0..4}$



(ix)④ $AD(\Delta_{0..4} \circ (R \cup (R^{op})))$



(ix)⑤

$(R \cup (R^{op}))^{\circ(1)} = \{(0,0),(0,1),(0,3),(1,0),(1,4),(2,4),(3,0),(3,4),(4,2),(4,3)\}$

(ix)⑨

$(R \cup (R^{op}))^{\circ(2)} = \{(0,0),(0,1),(0,3),(0,4),(1,0),(1,1),(1,3),(1,4),(2,2),(2,3),(3,0),(3,1),(3,3),(3,4),(4,0),(4,1),(4,3)\}$

(ix)⑬

$(R \cup (R^{op}))^{\circ(3)} = \{(0,0),(0,1),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,2),(2,3),(2,4),(3,0),(3,1),(3,3),(4,0),(4,1),(4,3)\}$

(ix)⑰

$(R \cup (R^{op}))^{\circ(4)} = \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,2),(2,3),(2,4),(3,0),(3,1),(3,2),(3,3),(4,0),(4,1),(4,3)\}$

(ix)㉑

$(R \cup (R^{op}))^{\circ(4)} = \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,3),(3,0),(3,1),(3,2),(3,3),(4,0),(4,1),(4,2),(4,3),(4,4)\}$

(ix)⑩ $(R \cup (R^{op}))^{\circ(3)}$

$= (R \cup (R^{op}))^{\circ(2)} \circ (R \cup (R^{op}))$

(ix)⑭ $(R \cup (R^{op}))^{\circ(4)}$

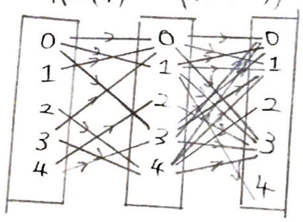
$= (R \cup (R^{op}))^{\circ(3)} \circ (R \cup (R^{op}))$

(ix)⑱ $(R \cup (R^{op}))^{\circ(5)}$

$= (R \cup (R^{op}))^{\circ(4)} \circ (R \cup (R^{op}))$

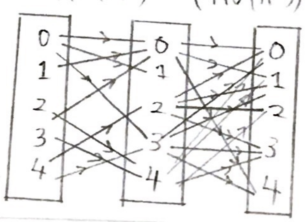
(ix)⑪

$R \cup (R^{op})$ $(R \cup (R^{op}))^{\circ(2)}$

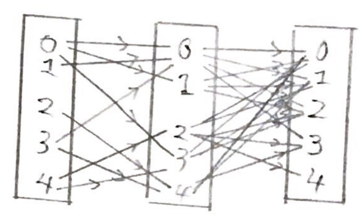


(ix)⑮

$R \cup (R^{op})$ $(R \cup (R^{op}))^{\circ(3)}$



(ix)⑲

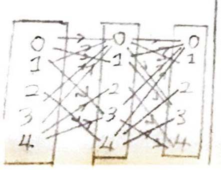


(ix)⑥ $(R \cup (R^{op}))^{\circ(2)}$

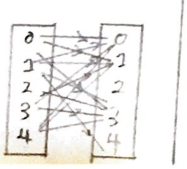
$= (R \cup (R^{op}))^{\circ(1)} \circ (R \cup (R^{op}))$

(ix)⑦

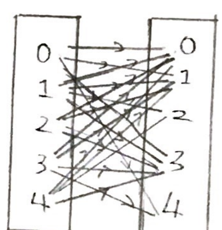
$R \cup (R^{op})$ $(R \cup (R^{op}))^{\circ(1)}$



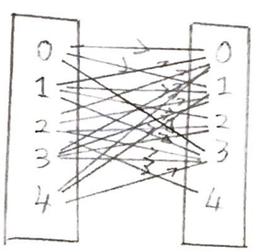
(ix)⑧ $AD(R \cup (R^{op}))^{\circ(2)})$



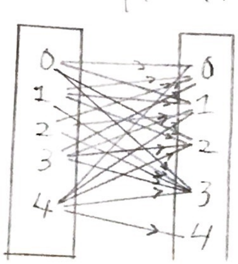
(ix)⑫ $AD((R \cup (R^{op}))^{\circ(3)})$



(ix)⑯ $AD((R \cup (R^{op}))^{\circ(4)})$



(ix)⑳ $AD((R \cup (R^{op}))^{\circ(5)})$



6 (2a) **A, B are as defined on page 202.** Circle the correct choices and provide an answer. Express your answers as arrow-diagrams and include the source and target as sets with elements. Otherwise prove

1(i) $\exists \alpha \in Fnc(A,B)\left(\left(Inj(\alpha)\right) \wedge \left(Srj(\alpha)\right)\right)$ Y N Pf W

$$\alpha := \begin{tikzcd} 0 \arrow[r] & 1 \\ 8 \arrow[r] & 4 \end{tikzcd} : \{0,8\} \to \{1,4\}$$

1(ii) $\exists \beta \in Fnc(A,B)\left(\left(\cancel{Inj}(\beta)\right) \wedge \left(Srj(\beta)\right)\right)$ Y N Pf W

$$\frac{\dfrac{\nu(A) = 2 \not> 2 = \nu(B)}{\nu(A) \leq \nu(B)}}{Inj(\beta) \neq \{\}}$$

1(iii) $\exists \gamma \in Fnc(A,B)\left(\left(Inj(\gamma)\right) \wedge \left(\cancel{Srj}(\gamma)\right)\right)$ Y N Pf W

$$\frac{\dfrac{\nu(A) = 2 \not< 2 = \nu(B)}{\nu(A) \geq \nu(B)}}{Srj(\gamma) \neq \{\}}$$

1(iii) $\cancel{\exists \gamma}$ δ $\in Fnc(A,B)\left(\left(\cancel{Inj}(\delta)\right) \wedge \left(\cancel{Srj}(\delta)\right)\right)$ Y N Pf W

$$\delta := \begin{tikzcd} 0 \arrow[r] & 1 \\ 8 \arrow[ur] & 4 \end{tikzcd} : \{0,8\} \to \{1,4\}$$

1 (iv) Find: $Im(\delta) = \{1\}$

1 (v) **On the facing page,** draw arrow-diagrams for **every** adjusted maximal bijective restriction of $\delta : A \longrightarrow B$ and count the total number of **such restrictions:** 3

6

(1) (iv)

$$\delta\Big|^{Im} := \begin{tikzcd} 0 \arrow[dr] & \\ & 1 \\ 8 \arrow[ur] & \end{tikzcd} : \{0, 8\} \to \{1\}$$

---

(1) (V)

① $$\delta\Big|^{Im}_{\{\}} := \quad : \{\} \to \{\}$$

---

(1) (V)

② $$\delta\Big|^{Im}_{\{0\}} := \boxed{0} \longrightarrow \boxed{1} : \{0\} \to \{1\}$$

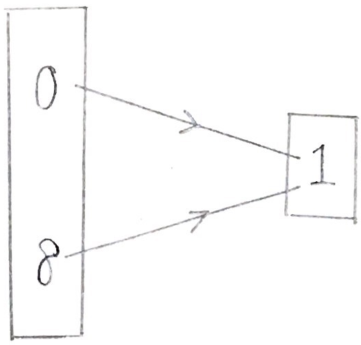
---

③ $$\delta\Big|^{Im}_{\{8\}} := \boxed{8} \longrightarrow \boxed{1} : \{8\} \to \{1\}$$

---

6 (2b) Carry out the following instructions:

1 (i) Draw: $AD\left(\delta\Big|^{Im} : A \longrightarrow Im(\delta)\right)$ below.



4 (ii) Define the relation $S \subseteq A \times A$ as follows:

$$\forall x, y \in A \quad \left(xSy\right) :\Leftrightarrow \left(\delta(x) \neq \delta(y)\right)$$

Draw $DG(S)$ below.

1 (iii) $Eqv(S)$, that is, is S an equivalence relation?

Y N Pf W

$\cancel{Rflx}(S)$

8

5 (2c) **Circle the correct choice** on the right and **prove your assertion** accordingly:

$\forall \varphi \in Fnc(S,T) \quad Eqv(R(\varphi))$ Y N Pf W

where the relation

$$R(\varphi) := \left\{(u\,,v) \in S \times S \,\middle|\, \varphi(u) \neq \varphi(v)\right\} \subseteq S \times S$$

$$\frac{\dfrac{\dfrac{\forall u,v \in R(\varphi) \quad \varphi(u) \neq \varphi(v)}{\varphi(u) \neq \varphi(v)}}{\cancel{Sym}(R(\varphi))}}{\cancel{Eqv}(R(\varphi))}$$

6

6 (3a) Define the relations $P \subseteq J \times J$ and $Q \subseteq E \times E$ as follows:

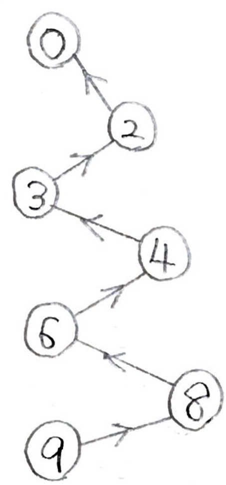
$$P := \left\{ (j\,,k) \in J \times J \,\middle|\, j \nless k \right\} \subseteq \quad J \times J$$

$$Q := \left\{ (e\,,f) \in E \times E \,\middle|\, e \ngtr f \right\} \subseteq \quad E \times E$$

and answer the following questions.

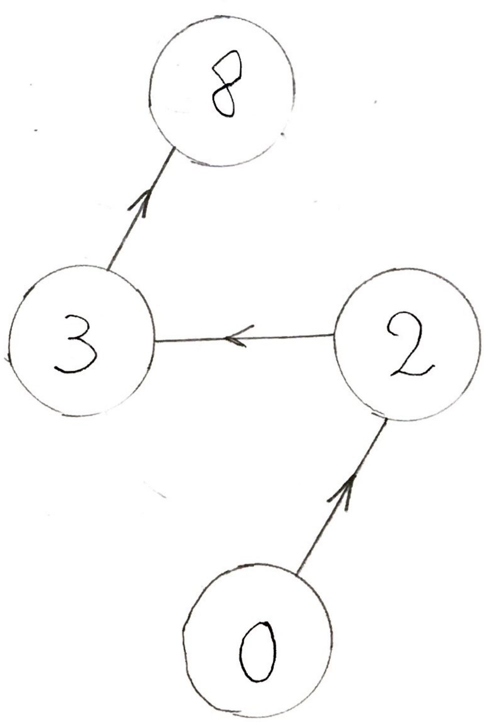
3 (i) Prove that $PO(P)$ on the facing side and draw $HsDgrm(P)$ on this side.

$HsDgrm(P)$:



3 (ii) Prove that $PO(Q)$ on the facing side and draw $HsDgrm(Q)$ on this side.

$HsDgrm(Q)$:



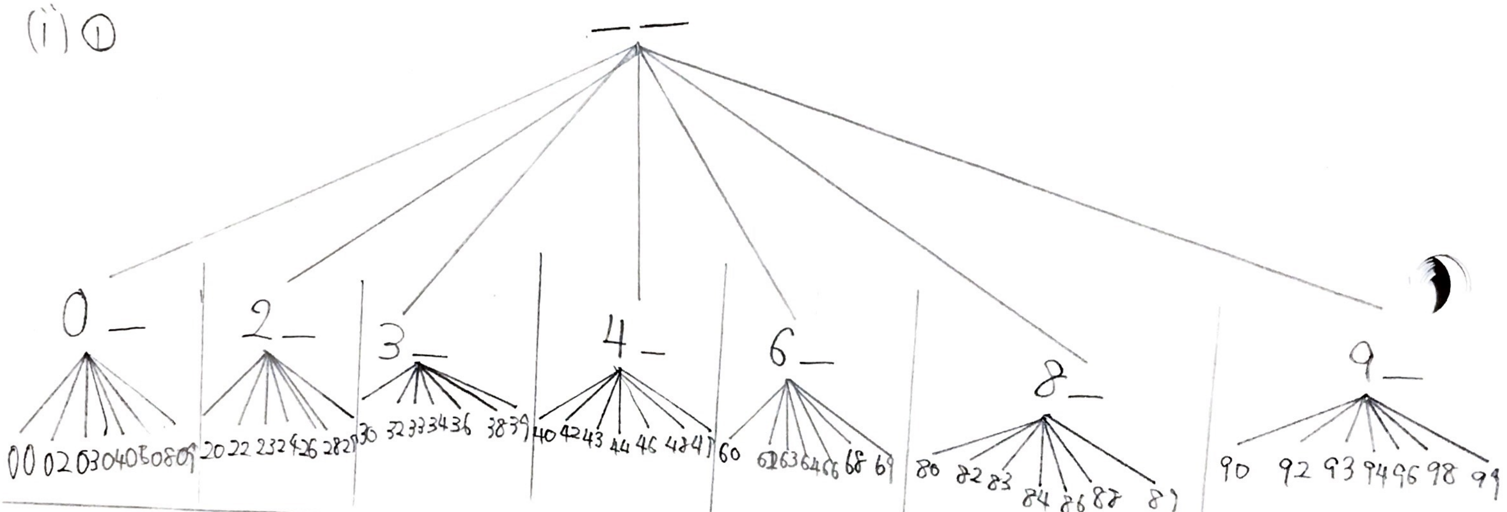
6

(i)⓪ $J \times J$

$$= \{0,2,3,4,6,8,9\} \times \{0,2,3,4,6,8,9\}$$

$$= \left\{ \begin{array}{l} (0,0),(0,2),(0,3),(0,4),(0,6),(0,8),(0,9), \\ (2,0),(2,2),(2,3),(2,4),(2,6),(2,8),(2,9), \\ (3,0),(3,2),(3,3),(3,4),(3,6),(3,8),(3,9), \\ (4,0),(4,2),(4,3),(4,4),(4,6),(4,8),(4,9), \\ (6,0),(6,2),(6,3),(6,4),(6,6),(6,8),(6,9), \\ (8,0),(8,2),(8,3),(8,4),(8,6),(8,8),(8,9), \\ (9,0),(9,2),(9,3),(9,4),(9,6),(9,8),(9,9) \end{array} \right\}$$

(i)①

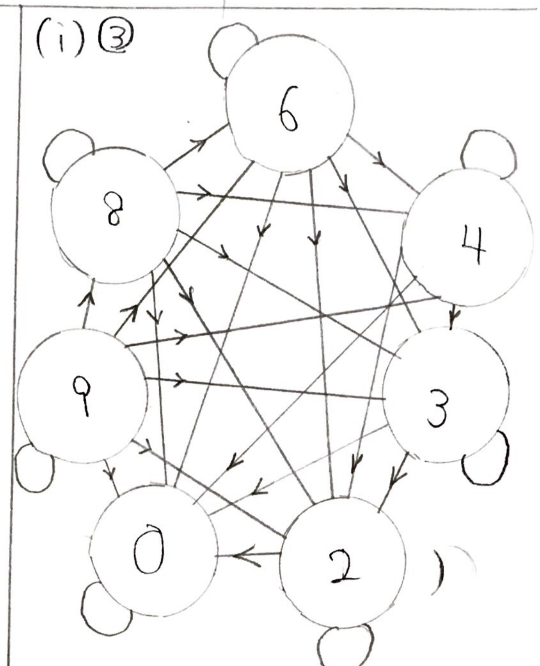


(i)②

P

$$= \left\{ \begin{array}{l} (0,0),(2,0),(2,2),(3,0),(3,2),(3,3), \\ (4,0),(4,2),(4,3),(4,4),(6,0),(6,2), \\ (6,3),(6,4),(6,6),(8,0),(8,2),(8,3), \\ (8,4),(8,6),(8,8),(9,0),(9,2),(9,3), \\ (9,4),(9,6),(9,8),(9,9) \end{array} \right\}$$

(i)③



DG(P)

(i) ④

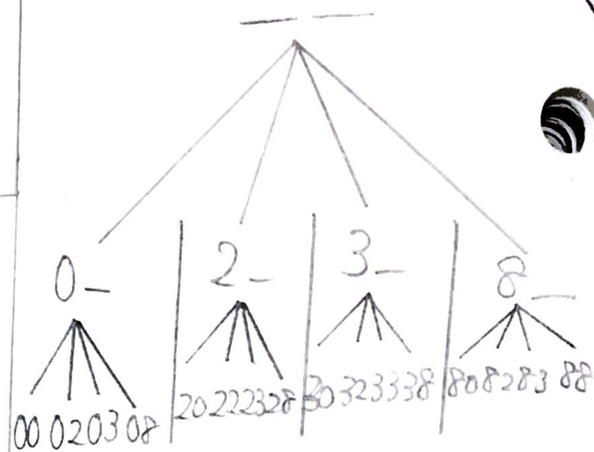
$$\frac{(Rflx(P)) \wedge (AntSym(P)) \wedge (Trns(P))}{PO(P)}$$

(ii) ⓪ $E \times E$

$= \{0,2,3,8\} \times \{0,2,3,8\}$

$= \left\{ \begin{array}{l} (0,0),(0,2),(0,3),(0,8), \\ (2,0),(2,2),(2,3),(2,8), \\ (3,0),(3,2),(3,3),(3,8), \\ (8,0),(8,2),(8,3),(8,8) \end{array} \right\}$

(ii) ①

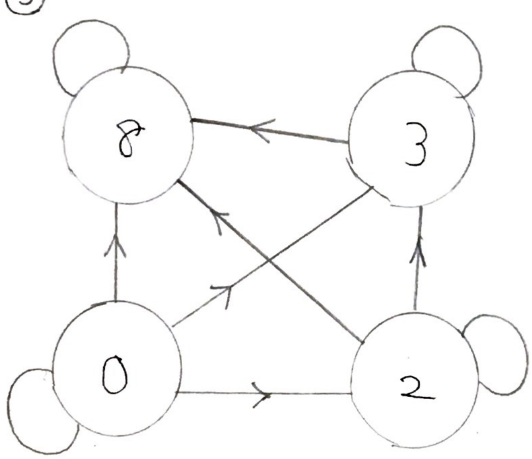


(ii) ②

Q

$= \left\{ \begin{array}{l} (0,0),(0,2),(0,3),(0,8), \\ (2,2),(2,3),(2,8),(3,3), \\ (3,8),(8,8) \end{array} \right\}$

(ii) ③



(ii) ④

$$\frac{(Rflx(Q)) \wedge (AntSym(Q)) \wedge (Trns(Q))}{PO(Q)}$$

6 (3b) Define the relations $P \vee Q,\ P \wedge Q \subseteq (J \times E) \times (J \times E)$ as follows:

$\forall (j, e) \in (J \times E), \forall (k, f) \in (J \times E)$

$$(j, e)\left(P \vee Q\right)(k, f) :\Leftrightarrow \left((jPk)\right) \vee \left(eQf\right)$$

$$(j, e)\left(P \wedge Q\right)(k, f) :\Leftrightarrow \left(jPk\right) \wedge \left((eQf)\right)$$

Circle the correct choices on the right and **provide an answer** accordingly **on the facing side:**

3 (i) $PO\left(P \vee Q\right)$ Y N Pf W

If yes draw $HsDgrm\left(P \vee Q\right)$ on this side.

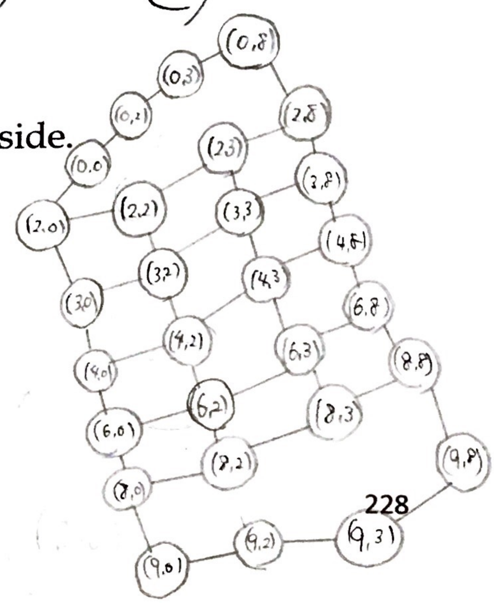
$HsDgrm\left(P \vee Q\right)$:

0

3 (ii) $PO\left(P \wedge Q\right)$ Y N Pf W

If yes draw $HsDgrm\left(P \wedge Q\right)$ on this side.

$HsDgrm\left(P \wedge Q\right)$:



(i)

$$\cfrac{\cfrac{\cfrac{(9,3)\,(P \vee Q)\,(2,0) \qquad (2,0)\,(P \vee Q)\,(8,8)}{(9,3)\,\cancel{(P \vee Q)}\,(8,8)}}{\cancel{\mathrm{Trns}}\,(P \vee Q)}}{\cancel{\mathrm{PO}}\,(P \vee Q)}$$

(ii)①

$$\cfrac{\cfrac{\cfrac{\cfrac{\mathrm{Rflx}(P) \qquad \mathrm{Rflx}(Q)}{\forall j \in J \;\; {}_jP_j \qquad \forall e \in E \;\; {}_eQ_e}}{\forall (j,e) \in J \times E \;\; ({}_jP_j) \wedge ({}_eQ_e)}}{\forall (j,e) \in J \times E \;\; (j,e)(P \wedge Q)(j,e)}}{\mathrm{Rflx}(P \wedge Q)}$$

(ii)①

$$\cfrac{\cfrac{\cfrac{\mathrm{Trns}(P) \qquad \mathrm{Trns}(Q)}{\forall r,s,t \in J \left(\cfrac{({}_rP_s) \wedge ({}_sP_t)}{{}_rP_t}\right) \quad \forall a,b,c \in Q \left(\cfrac{({}_aQ_b) \wedge ({}_bQ_c)}{{}_aQ_c}\right)}}{\forall (r,a)(s,b)(t,c) \in J \times E \left(\cfrac{(({}_rP_s) \wedge ({}_aQ_b)) \wedge (({}_sP_t) \wedge ({}_bQ_c))}{({}_rP_t) \wedge ({}_aQ_c)}\right)}}{\cfrac{\forall (r,a),(s,b),(t,c) \in J \times E \left(\cfrac{(({}_rP_s)(P \wedge Q)({}_aQ_b)) \wedge (({}_sP_t)(P \wedge Q)({}_bQ_c))}{({}_rP_t)(P \wedge Q)({}_aQ_c)}\right)}{\mathrm{Trns}(P \wedge Q)}}$$

(ii)②

$$\cfrac{\cfrac{\cfrac{\cfrac{\mathrm{AntSym}(P) \qquad \mathrm{AntSym}(Q)}{\forall r,s \in J \left(\cfrac{({}_rP_s) \wedge ({}_sP_r)}{r=s}\right) \quad \forall a,b \in E \left(\cfrac{({}_aP_b) \wedge ({}_bQ_a)}{a=b}\right)}}{\forall (r,a),(s,b), \in J \times E \left(\cfrac{(({}_rP_s) \wedge ({}_aP_b)) \wedge (({}_sP_r) \wedge ({}_bQ_a))}{(r=s)\,(a=b)}\right)}}{\forall (r,a)(s,b) \in J \times E \left(\cfrac{((r,a)(P \wedge Q)(s,b)) \wedge ((s,b)(P \wedge Q)(r,a))}{(r=s)\;\;(a=b)}\right)}}{\mathrm{AntSym}(P \wedge Q)}$$

(ii)③

$$\cfrac{(\mathrm{Rflx}(P \wedge Q)) \wedge (\mathrm{Trns}(P \wedge Q)) \wedge (\mathrm{AntSym}(P \wedge Q))}{\mathrm{Po}(P \wedge Q)}$$

5 (3c) Define: $Prtn(S) :=$ the set of partitions of S and

$EqvRln(S) :=$ the set of equivalence relations on S

Draw the digraphs of equivalence relations on B on the facing side and give them names: E_n where $n \in \mathbb{N}$ using as many n's as you need and list these names on this side.

Determine the following:

2 (i) $\nu(Prtn(B)) = 2$

$Prtn(B)$

$= \{\{\{1\},\{4\}\},\{\{1,4\}\}\}$

3 (ii) $\nu(EqnRln(B)) = 2$

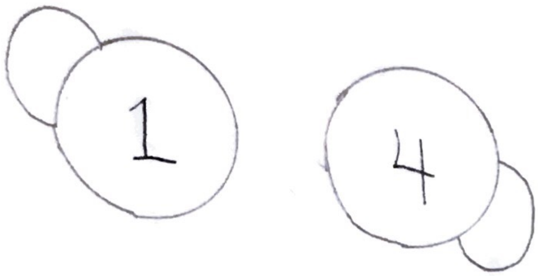
$EqnRln(\boldsymbol{B})$

8

$= \{E_1, E_2\}$

You may talk to anyone

(i)① $DG(E_1)$



(i)① $DG(E_2)$