Instructions:

Your J-number = 00266502

Keep every 0 in your J-number and replace every non-zero number in your J-number by 1 to get

Your (truth-value) **T-number**:= 00111101

Define $\forall k \in 1..8$,

$T_k :=$ the k[th] entry in your **truth-value J-number** counting from the left.

Therefore,

$T_1 :=$ 0

$T_2 :=$ 0

$T_3 :=$ 1

$T_4 :=$ 1

$T_5 :=$ 1

$T_6 :=$ 1

$T_7 :=$ 0

$T_8 :=$ 1

Next construct an expression $G(w, x, y, z)$ as follows:

$\alpha(T_1, T_2, T_3, T_4)$

$= \quad (T_1)(w) + (T_2)(x') + (T_3)(y) + (T_4)(z')$

$= \quad (0)w + (0)x' + (1)y + (1)z'$

$= \quad y + z'$

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$\alpha(T_1, T_2, T_3, T_4)$

$= (T_1)(w) + T_2(x') + (T_3)(y) + T_4(z')$

$= (0)w + (0)x' + (1)y + (1)z'$

$= 0 + 0 + y + z'$

$= y + z'$

$\beta(T_3, T_4, T_5, T_6)$

$= \left(\left(((T_3)w)'\right)\left(((T_4)x)'\right)\left(((T_5)y)'\right)\left(((T_6)z)'\right)\right)'$

$= \left(\left(((1)w)'\right)\left(((1)x)'\right)\left(((1)y)'\right)\left(((1)z)'\right)\right)'$

$= (w')' + (x')' + (y')' + (z')'$

$= w + x + y + z$

$\gamma(T_5, T_6, T_7, T_8)$

$= (T_5)(w') + (T_6)(x) + (T_7)(y') + (T_8)(z)$

$= (1)(w') + (1)x + (0)(y') + (1)z$

$= w' + x + z$

$G(w, x, y, z)$

$= (\alpha(T_1, T_2, T_3, T_4))(\beta(T_3, T_4, T_5, T_6))(\gamma(T_5, T_6, T_7, T_8))$

$= (y + z')(w + x + y + z)(w' + x + z)$

$\beta(T_3, T_4, T_5, T_6)$

$= \left(\left(((T_3)w)' \right) \left(((T_4)x)' \right) \left(((T_5)y)' \right) \left(((T_6)z)' \right) \right)'$

$= \left(\left(((1)w)' \right) \left(((1)x)' \right) \left(((1)y)' \right) \left(((1)z)' \right) \right)'$

$= \left((w') (x') (y') (z') \right)'$

$= (w')' + (x')' + (y')' + (z')'$

$= w + x + y + z$

$\gamma(T_5, T_6, T_7, T_8)$

$= (T_5)(w') + (T_6)(x) + (T_7)(y') + (T_8)(z)$

$= (1)(w') + (1)x + (0)(y') + (1)z$

$= w' + x + z$

$G(w, x, y, z)$

$= (\alpha(T_1, T_2, T_3, T_4))(\beta(T_3, T_4, T_5, T_6))(\gamma(T_5, T_6, T_7, T_8))$

$= (y + z')(w + x + y + z)(w' + x + z)$

6 (1a) Find $\varphi(p,q,r)$ in terms of: p, q and r if $\varphi(p,q,r)$ has the following truth table:

p	q	r	$\varphi(p,q,r)$	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	0	$T_1 =$ 0	0	0	0	0	0			
0	0	1	$T_2 =$ 0	0	0	0	0	0			
0	1	0	$T_3 =$ 1	0	0	0	0	1			
0	1	1	$T_4 =$ 1	0	0	0	1	0			
1	0	0	$T_5 =$ 1	0	0	1	0	0			
1	0	1	$T_6 =$ 1	0	1	0	0	0			
1	1	0	$T_7 =$ 0	0	0	0	0	0			
1	1	1	$T_8 =$ 1	1	0	0	0	0			

4(i) $\varphi(p,q,r)$ $= (\varphi_1) \vee (\varphi_2) \vee (\varphi_3) \vee (\varphi_4) \vee (\varphi_5)$

$= (p \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r))$

1(ii) Find the version of $F(A,B,C)$ of $\varphi(p,q,r)$ in Sets:

$F(A,B,C)$ $= (A \cap B \cap C) \cup (A \cap (B^c) \cap C) \cup (A \cap (B^c) \cap (C^c)) \cup ((A^c) \cap B \cap C) \cup ((A^c) \cap B \cap (C^c))$

1(ii) Find the version of $E(x,y,z)$ of $\varphi(p,q,r)$ in BA:

$E(x,y,z)$ $= xyz + xy'z + xy'z' + x'yz + x'yz'$

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$\varphi(p, q, r)$

$$
\begin{aligned}
&= (\varphi_1) + (\varphi_2) + (\varphi_3) + (\varphi_4) + (\varphi_5) \\
&= (p \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r))
\end{aligned}
$$

$F(A, B, C)$

$$
= (A \cap B \cap C) \cup (A \cap (B^c) \cap C) \cup (A \cap (B^c) \cap (C^c)) \cup ((A^c) \cap B \cap C) \cup ((A^c) \cap B \cap (C^c))
$$

$E(x, y, z)$

$$
\begin{aligned}
&= (x * y * z) + (x * (y') * z) + (x * (y') * (z')) + ((x') * y * z) + ((x') * y * (z')) \\
&= xyz + xy'z + xy'z' + x'yz + x'yz'
\end{aligned}
$$

6 (1b) Use the consensus method to find the following for your answer from (1a):

$E(x,y,z) = xyz + xy'z + xy'z' + x'yz + x'yz'$

3 (i) the prime implicants of E: $PI(E) = \{xy', xz, x'y, yz\}$

3 (i) the minimal sum-of-products form: $MSOP(E) = xy' + x'y + yz$

$\cancel{Cns(xy', xz)}$

$\cancel{= xz}$

(i) E

$= xyz + xy'z + xy'z' + x'yz + x'yz' \quad (Cns(xyz, xy'z) = xz)$

$= xyz + xy'z + xy'z' + x'yz + x'yz' + xz \quad (xz \subseteq xyz,\ xz \subseteq xy'z)$

$= xy'z' + x'yz + x'yz' + xz \quad (Cns(xy'z', xz) = xy')$

$= xy'z' + x'yz + x'yz' + xz + xy' \quad (xy' \subseteq xy'z')$

$= x'yz + x'yz' + xz + xy' \quad (Cns(x'yz, x'yz') = x'y)$

$= x'yz + x'yz' + xz + xy' + x'y \quad (x'y \subseteq x'yz,\ x'y \subseteq x'yz')$

$= xz + xy' + x'y \quad (Cns(xz, x'y) = yz)$

$= xz + xy' + x'y + yz$

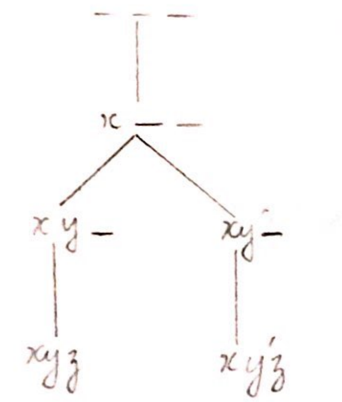
$= xy' + xz + x'y + yz$

$PI(E) = \{xy', xz, x'y, yz\}$

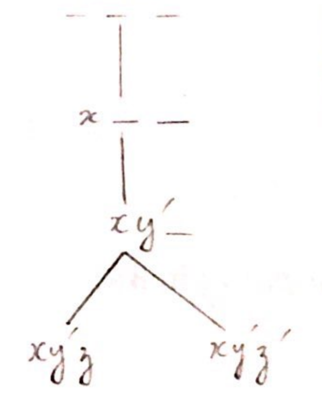
(ii) E

$= xy'$	$= xy'(1)$	$= xy'(z+z')$	$= \underset{①}{xy'z} + xy'z'$	$= xy'$	$= MSOP_1(E) =$	xy'	$= MSOP_2(E)$
$+ xz$	$+ x(1)z$	$+ x(y+y')z$	$+ \underset{②}{xyz} + \underset{①}{xy'z}$			$+ xz$	
$+ x'y$	$+ x'y(1)$	$+ x'y(z+z')$	$+ \underset{③}{x'yz} + x'yz'$	$+ x'y$		$+ x'y$	
$+ yz$	$+ (1)yz$	$+ (x+x')yz$	$+ \underset{②}{xyz} + \underset{③}{x'yz}$	$+ yz$			
				$L_2 \subseteq L_1 \cup L_3 \cup L_4$		$L_4 \subseteq L_1 \cup L_2 \cup L_3$	

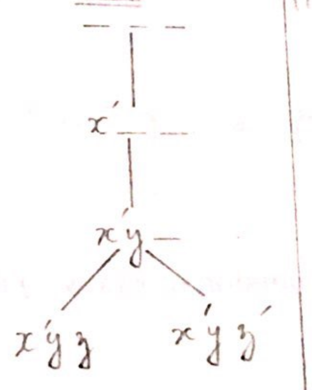
(ii) Tree



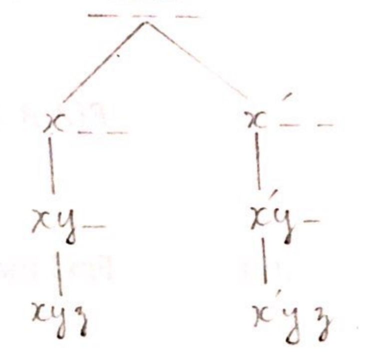
(ii) Tree



(ii) Tree



(ii) Tree



5 (1c) From (1b) $MSOP\ (E) =$ $xy' + x'y + yz$

1 (i) Find the version $\psi(p,q,r)$ of $MSOP\ (E)$ in PC

$$\psi(p,q,r) = (p \wedge (\neg q)) \vee ((\neg p) \wedge q) \vee (q \wedge r)$$

4 (ii) **Prove in PC** that your answer from (1a) $\varphi(p,q,r) = \psi(p,q,r)$

5

(iv) $\varphi(p,q,r)$

$= (p\wedge q\wedge r)\vee(p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q\wedge r)\vee((\neg p)\wedge q\wedge(\neg r))$

$= ((p\wedge r)\wedge q)\vee((p\wedge r)\wedge(\neg q))\vee(p\wedge(\neg q)\wedge(\neg r))\vee(((\neg p)\wedge q)\wedge r)\vee(((\neg p)\wedge q)\wedge(\neg r))$

$= ((p\wedge r)\wedge(q\vee(\neg q)))\vee(p\wedge(\neg q)\wedge(\neg r))\vee(((\neg p)\wedge q)\wedge(r\vee(\neg r)))$

$= ((p\wedge r)\wedge T)\vee(p\wedge(\neg q)\wedge(\neg r))\vee(((\neg p)\wedge q)\wedge T)$

$= (p\wedge r)\vee(p\wedge((\neg q)\wedge(\neg r)))\vee((\neg p)\wedge q)$

$= (p\wedge(r\vee((\neg q)\wedge(\neg r))))\vee((\neg p)\wedge q)$

$= (p\wedge((r\vee(\neg q))\wedge(r\vee(\neg r))))\vee((\neg p)\wedge q)$

$= (p\wedge((r\vee(\neg q))\wedge T))\vee((\neg p)\wedge q)$

$= (p\wedge(r\vee(\neg q)))\vee((\neg p)\wedge q)$

$= ((p\wedge r)\vee(p\wedge(\neg q)))\vee((\neg p)\wedge q)$

$= ((p\wedge r)\vee((p\wedge r)\wedge q))\vee(p\wedge(\neg q))\vee((\neg p)\wedge q)\vee(((\neg p)\wedge q)\wedge r)$

$= ((p\wedge r)\wedge T)\vee((p\wedge r)\wedge q)\vee((p\wedge(\neg q))\wedge T)\vee((\neg p)\wedge q)\vee((\neg p)\wedge q\wedge r)$

$= ((p\wedge r)\wedge(q\vee(\neg q)))\vee(p\wedge r\wedge q)\vee((p\wedge(\neg q))\wedge(r\vee(\neg r)))\vee((\neg p)\wedge q)\vee((\neg p)\wedge q\wedge r)$

$= (((p\wedge r)\wedge q)\vee((p\wedge r)\wedge(\neg q)))\vee(p\wedge r\wedge q)\vee(((p\wedge(\neg q))\wedge r)\vee((p\wedge(\neg q))\wedge(\neg r)))\vee((\neg p)\wedge q)\vee((\neg p)\wedge q\wedge r)$

$= (p\wedge r\wedge q)\vee(p\wedge r\wedge q)\vee(p\wedge r\wedge(\neg q))\vee(p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q)\vee((\neg p)\wedge q\wedge r)$

$= ((p\wedge q\wedge r)\vee(p\wedge q\wedge r))\vee((p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge r))\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q)\vee((\neg p)\wedge q\wedge r)$

$= (p\wedge q\wedge r)\vee(p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q)\vee((\neg p)\wedge q\wedge r)$

$= (p\wedge q\wedge r)\vee((\neg p)\wedge q\wedge r)\vee(p\wedge(\neg q)\wedge r)\vee(p\wedge(\neg q)\wedge(\neg r))\vee((\neg p)\wedge q)$

$= ((q\wedge r)\wedge(p\vee(\neg p)))\vee((p\wedge(\neg q))\wedge(r\vee(\neg r)))\vee((\neg p)\wedge q)$

$= ((q\wedge r)\wedge T)\vee((p\wedge(\neg q))\wedge T)\vee((\neg p)\wedge q)$

$= (q\wedge r)\vee(p\wedge(\neg q))\vee((\neg p)\wedge q)$

$= (p\wedge(\neg q))\vee((\neg p)\wedge q)\vee(q\wedge r)$

$\varphi(p,q,r) = (p\wedge(\neg q))\vee((\neg p)\wedge q)\vee(q\wedge r) = \psi(p,q,r)$

$\varphi(p,q,r) = \psi(p,q,r)$

6 (2a) For the expression from page 53 below:

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+z)$$

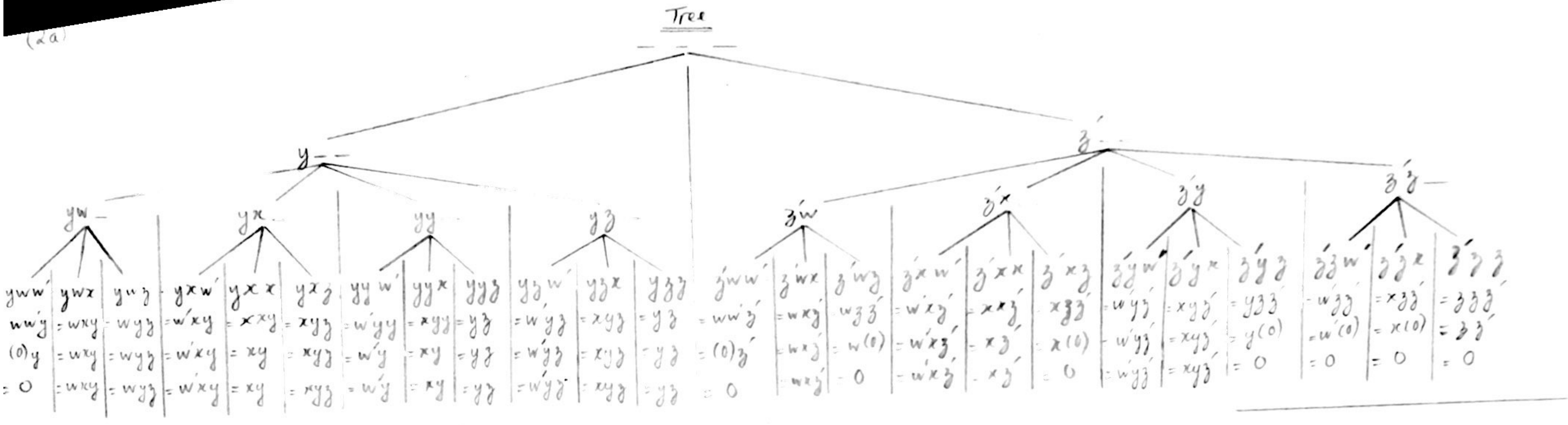
find a sum-of-products form, $SOP(G)$

$$SOP(G) = w'y + xy + xz' + yz$$

Show the tree on the facing side.

2

(2a)



(2a)

G

$= wxy + wyz + w'xy + xy + xyz + w'y + xy + yz + w'yz + xyz + yz + wxz' + w'xz' + xz' + w'yz' + xyz'$ $\quad (s + s = s)$

$= wxy + wyz + w'xy + xy + xyz + w'y + yz + w'yz + wxz' + w'xz' + xz' + w'yz' + xyz'$ $\quad (s + (s * t) = s)$

$= wyz + xy + w'y + yz + w'yz + wxz' + w'xz' + xz' + w'yz'$ $\quad (s + (s * t) = s)$

$= xy + w'y + yz + wxz' + w'xz' + xz' + w'yz'$ $\quad (s + (s * t) = s)$

$= xy + w'y + yz + wxz' + w'xz' + xz'$ $\quad (s + (s * t) = s)$

$= xy + w'y + yz + xz'$

$= w'y + xy + xz' + yz$

$=$ SOP(G)

6 (2b) For the expression below **(which is the same as in (2a))**:

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+z)$$

find the complete sum-of-products form, $CSOP(G)$

$$CSOP(G) = wxyz + wxyz' + wxy'z' + wx'yz + w'xyz + w'xyz' + w'xy'z' + w'x'yz + w'x'yz'$$

6

(2b)

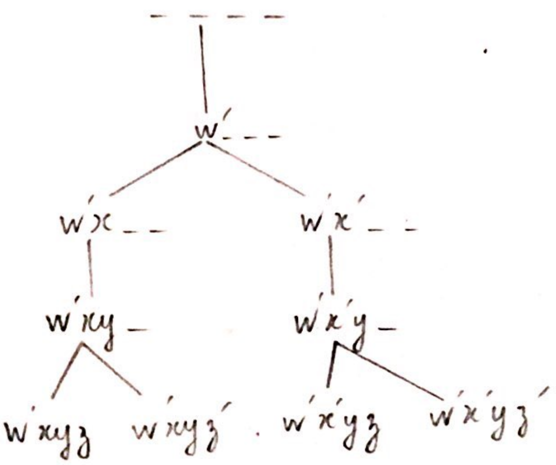
SOP(G)

$$
\begin{aligned}
&= w'y + xy + xz' + yz \\
&= w'(1)y(1) + (1)xy(1) + (1)x(1)z' + (1)(1)yz \\
&= w'(x+x')y(z+z') + (w+w')xy(z+z') + (w+w')x(y+y')z' + (w+w')(x+x')yz \\
&= \underbrace{w'xyz}_{1} + \underbrace{w'xyz'}_{2} + \underbrace{w'x'yz}_{3} + w'x'yz' \\
&\quad + \underbrace{wxyz}_{4} + \underbrace{wxyz'}_{5} + \underbrace{w'xyz}_{1} + \underbrace{w'xyz'}_{2} \\
&\quad + \underbrace{wxyz'}_{5} + wxy'z' + \underbrace{w'xyz'}_{2} + w'xy'z' \\
&\quad + \underbrace{wxyz}_{4} + wx'yz + \underbrace{w'xyz}_{1} + \underbrace{w'x'yz}_{3}
\end{aligned}
$$

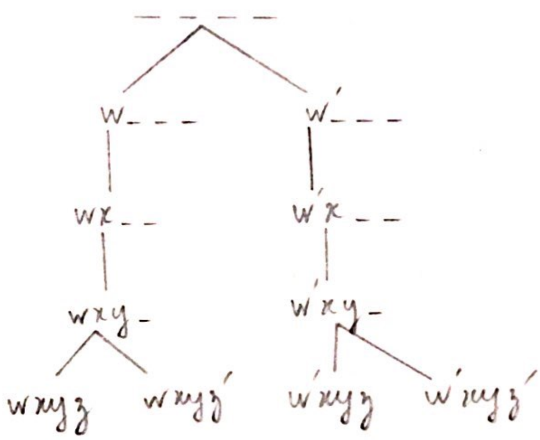
$$
\begin{aligned}
&= w'xyz + w'xyz' + w'x'yz + w'x'yz' \\
&\quad + wxyz + wxyz' + \quad + \\
&\quad + \quad + wxy'z' + \quad + w'xy'z' \\
&\quad + \quad + wx'yz + \quad +
\end{aligned}
$$

$$
\begin{aligned}
&= wxyz + wxyz' + wxy'z' + wx'yz + w'xyz + w'xyz' + w'xy'z' + w'x'yz + w'x'yz' \\
&= CSOP(E)
\end{aligned}
$$

(2b) Tree



(2b) Tree



5 (2c) For the expression below **which is the same as in (2a) and (2b):**

$$G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+z)$$

use your answer from (2a) and (2b) to find the following:

$PI(G)$ and $MSOP\ (G)$

Answer from (2a) $SOP(G) = w'y + xy + xz' + yz$

Answer from (2b) $CSOP\ (G) = wxyz + wxyz' + wxy'z' + wx'yz$
$+ w'xyz + w'xyz' + w'xy'z' + w'x'yz + w'x'yz'$

2 (i) $PI(G) = w'y + xy + xz' + yz$

3 (ii) $MSOP\ (G) = w'y + xz' + yz$

(i) $SOP(G)$

$= w'y + xy + xz' + yz$

$PI(G) = \{w'y, xy, xz', yz\}$

(i)

$$\left[\begin{array}{ll} w'y \not\subseteq xy & xy \not\subseteq w'y \\ w'y \not\subseteq xz' & xz' \not\subseteq w'y \\ w'y \not\subseteq yz & yz \not\subseteq w'y \\ xy \not\subseteq xz' & xz' \not\subseteq xy \\ xy \not\subseteq yz & yz \not\subseteq xy \\ xz' \not\subseteq yz & yz \not\subseteq xz' \\ \multicolumn{2}{c}{Cns(xz', yz) = xy} \end{array}\right]$$

(ii)

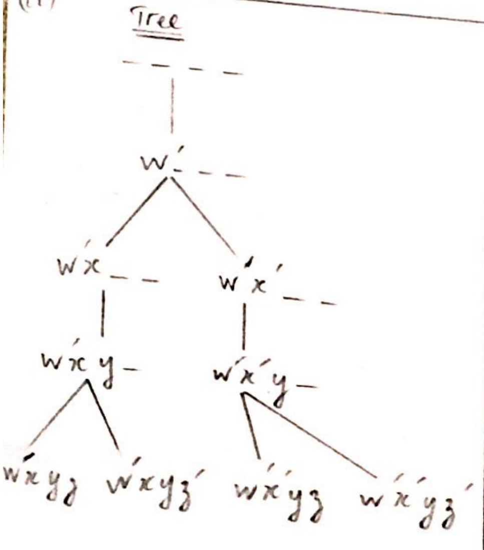
$SOP(G)$

$$\begin{aligned}
= {} & w'y = w'(1)y(1) = w'(x+x')y(z+z') = \underset{①}{w'xyz} + \underset{②}{w'xyz'} + \underset{③}{w'x'yz} + w'x'yz' \\
& + xy \quad + (1)xy(1) \quad + (w+w')xy(z+z') \quad + \underset{④}{wxyz} + \underset{⑤}{wxyz'} + \underset{①}{w'xyz} + \underset{②}{w'xyz'} \\
& + xz' \quad + (1)x(1)z' \quad + (w+w')x(y+y')z' \quad + \underset{⑤}{wxyz'} + wxy'z' + \underset{②}{w'xyz'} + w'xy'z' \\
& + yz \quad + (1)(1)yz \quad + (w+w')(x+x')yz \quad + \underset{④}{wxyz} + wx'yz + \underset{①}{w'xyz} + \underset{③}{w'x'yz} \\
& = w'y + xz' + yz = MSOP(G)
\end{aligned}$$

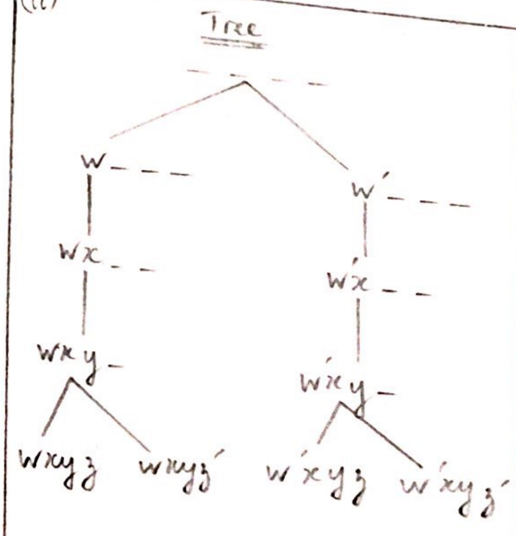
L_2

CL, UL_2, UL_3

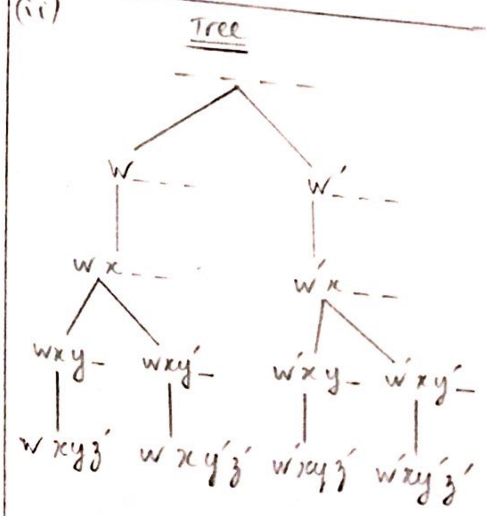
(ii) Tree



(ii) Tree



(ii) Tree



6 (3a) We define:

$A := SOP(G(w,x,y,z))=$ $w'y + xy + xz' + yz$

$B = MSOP(G(w,x,y,z))=$ $w'y + xz' + yz$.

Copy your answers from (1a)

Compute:

1 (i) $A_S = 4$ $A_L = 8$. $B_S = 3$ $B_L = 6$

1 (ii) On the basis of your answers to (i), circle the correct choice below.

Smplr (A,B) *Smplr*(B,A) Neither

Why?

4 (iii) Write down the version of $G(w,x,y,z)$ in PC below and find its **formation-tree on the facing side.**

Version of $G(w,x,y,z)$in PC

$= (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee s)$

at back

6

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(i) $Smnd(A) = \{w'y, xy, xz', yz\}$	(i) $A_S = \nu(Smnd(A)) = 4$	(i) $MSLtr(A) = \{w':1, x:2, y:3, z:1, z':1\}$	(i) $A_L = \nu(MSLtr(A)) = 8$
(i) $Smnd(B) = \{w'y, xz', yz\}$	(i) $B_S = \nu(Smnd(B)) = 3$	(i) $MSLtr(B) = \{w':1, x:1, y:2, z:1, z':1\}$	(i) $B_L = \nu(MSLtr(B)) = 6$

(ii)

$$\cfrac{\cfrac{(B_L = 6 < 8 = A_L) \wedge (B_S = 3 \leq 8 = A_L)}{(B_L < A_L) \wedge (B_S \leq A_L)}}{Smplr(B, A)}$$

(iii)

$$\cfrac{\cfrac{G(w,x,y,z) = (y + z')(w + x + y + z)(w' + x + z)}{G(w,x,y,z) = (y + (z')) * (w + x + y + z) * ((w') + x + z)}}{G_{PC}(p,q,r,s) = (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee s)}$$

(iii) $G(w,x,y,z)$

$= (r \vee (\neg s)) \wedge (p \vee q \vee r \vee s) \wedge ((\neg p) \vee q \vee s)$

$= \boxed{\boxed{r \vee \boxed{\neg s}} \wedge \boxed{\boxed{\boxed{p \vee q} \vee r} \vee s}} \wedge \boxed{\boxed{\boxed{\neg p} \vee q} \vee s}$

2 (3b) Find the versions: $H(A,B,C,D)$ of $G(w,x,y,z)$

and $K(A,B,C,D)$ of $MSOP\big(G(w,x,y,z)\big)$

in Sets.

1 (i) $H(A,B,C,D)$

$$= (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup D)$$

1 (i) $K(A,B,C,D)$

$$= ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$$

(i) $G(w,x,y,z) = (y+z')(w+x+y+z)(w'+x+z)$

$G(w,x,y,z) = (y+(z')) * (w+x+y+z) * ((w')+x+z)$

$G_{sets}(A,B,C,D) = (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup D)$

(i) $MSOP(G(w,x,y,z)) = w'y + xz' + yz$

$MSOP(G(w,x,y,z)) = ((w') * y) + (x * (z')) + (y * z)$

$MSOP_{Sets}(G(A,B,C,D)) = ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$

9 (3c) Prove in Sets that:

$$H(A, B, C, D) = K(A, B, C, D)$$

9

(3c) $H(A,B,C,D)$

$= (C \cup (D^c)) \cap (A \cup B \cup C \cup D) \cap ((A^c) \cup B \cup D)$

$= ((C \cup (D^c)) \cap (A \cup B \cup C \cup D)) \cap ((A^c) \cup B \cup D)$ $\quad (E \cap F = F \cap E)$

$= ((A \cup B \cup C \cup D) \cap (C \cup (D^c))) \cap ((A^c) \cup B \cup D)$ $\quad (E \cap (F \cup G) = (E \cap F) \cup (E \cap G))$

$= (((A \cup B \cup C \cup D) \cap C) \cup ((A \cup B \cup C \cup D) \cap (D^c))) \cap ((A^c) \cup B \cup D)$ $\quad (E \cap F = F \cap E)$

$= ((C \cap (A \cup B \cup C \cup D)) \cup ((D^c) \cap (A \cup B \cup C \cup D))) \cap ((A^c) \cup B \cup D)$ $\quad (E \cap (F \cup G) = (E \cap F) \cup (E \cap G))$

$= ((C \cap A) \cup (C \cap B) \cup (C \cap C) \cup (C \cap D)) \cup ((D^c) \cap A) \cup ((D^c) \cap B) \cup ((D^c) \cap C) \cup ((D^c) \cap D))) \cap ((A^c) \cup B \cup D)$ $\quad (E \cap E = E,\ E \cap (E^c) = \{\})$

$= ((C \cap A) \cup (C \cap B) \cup C \cup (C \cap D)) \cup (((D^c) \cap A) \cup ((D^c) \cap B) \cup ((D^c) \cap C) \cup \{\})) \cap ((A^c) \cup B \cup D)$ $\quad (E \cup \{\} = E)$

$= (((C \cap A) \cup (C \cap B) \cup C \cup (C \cap D)) \cup (((D^c) \cap A) \cup ((D^c) \cap B) \cup ((D^c) \cap C))) \cap ((A^c) \cup B \cup D)$ $\quad (E \cup (E \cap F) = E)$

$= (C \cup ((D^c) \cap A) \cup ((D^c) \cap B)) \cap ((A^c) \cup B \cup D)$. $\quad (E \cap (F \cup G) = (E \cap F) \cup (E \cap G))$

$= ((C \cup ((D^c) \cap A) \cup ((D^c) \cap B)) \cap (A^c)) \cup ((C \cup ((D^c) \cap A) \cup ((D^c) \cap B)) \cap B) \cup ((C \cup ((D^c) \cap A) \cup ((D^c) \cap B)) \cap D)$ $\quad (E \cap F = F \cap E)$

$= ((A^c) \cap (C \cup ((D^c) \cap A) \cup ((D^c) \cap B))) \cup (B \cap (C \cup ((D^c) \cap A) \cup ((D^c) \cap B))) \cup (D \cap (C \cup ((D^c) \cap A) \cup ((D^c) \cap B)))$ $\quad (E \cap (F \cup G) = (E \cap F) \cup (E \cap G))$

$= (((A^c) \cap C) \cup ((A^c) \cap ((D^c) \cap A)) \cup ((A^c) \cap ((D^c) \cap B))) \cup ((B \cap C) \cup (B \cap ((D^c) \cap A)) \cup (B \cap ((D^c) \cap B))) \cup ((D \cap C) \cup (D \cap ((D^c) \cap A)) \cup (D \cap ((D^c) \cap B)))$ $\quad (E \cap E = E,\ E \cap (E^c) = \{\})$

$= ((A^c) \cap C) \cup (\{\} \cap (D^c)) \cup ((A^c) \cap (D^c) \cap B) \cup (B \cap C) \cup (B \cap (D^c) \cap A) \cup (B \cap (D^c)) \cup (D \cap C) \cup (\{\} \cap A) \cup (\{\} \cap B)$ $\quad (E \cap \{\} = \{\})$

$= ((A^c) \cap C) \cup \{\} \cup ((A^c) \cap (D^c) \cap B) \cup (B \cap C) \cup (B \cap (D^c) \cap A) \cup (B \cap (D^c)) \cup (D \cap C) \cup \{\} \cup \{\}$ $\quad (E \cup \{\} = E,\ E \cap F = F \cap E)$

$= ((A^c) \cap C) \cup ((A^c) \cap B \cap (D^c)) \cup (B \cap C) \cup (A \cap B \cap (D^c)) \cup (B \cap (D^c)) \cup (C \cap D)$ $\quad (E \cup (E \cap F) = E)$

$= ((A^c) \cap C) \cup (B \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$ $\quad (E = E \cap \mathcal{U})$

$= ((A^c) \cap C) \cup (\mathcal{U} \cap (B \cap C)) \cup (B \cap (D^c)) \cup (C \cap D)$ $\quad (\mathcal{U} = E \cup E^c)$

$= ((A^c) \cap C) \cup ((A \cup (A^c)) \cap (B \cap C)) \cup (B \cup (D^c)) \cup (C \cap D)$ $\quad (E \cap (F \cup G) = (E \cap F) \cup (E \cap G))$

(3c)

$$= ((A^c) \cap C) \cup (A \cap (B \cap C)) \cup ((A^c) \cap (B \cap C))) \cup (B \cup (D^c)) \cup (C \cap D) \qquad (E = E \cap \mathcal{U})$$

$$= ((A^c) \cap C) \cup ((A \cap B \cap C) \cup ((A^c) \cap B \cap C)) \cap \mathcal{U}) \cup (B \cup (D^c)) \cup (C \cap D) \qquad (\mathcal{U} = E \cup (E^c))$$

$$= ((A^c) \cap C) \cup (((A \cap B \cap C) \cup ((A^c) \cap B \cap C)) \cap (D \cup (D^c))) \cup (B \cup (D^c)) \cup (C \cap D) \qquad (E \cap (F \cup G) = (E \cap F) \cup (E \cap G))$$

$$= ((A^c) \cap C) \cup ((A \cap B \cap C) \cap D) \cup ((A \cap B \cap C) \cap (D^c)) \cup (((A^c) \cap B \cap C) \cap D) \cup (((A^c) \cap B \cap C) \cap (D^c))) \cup (B \cup (D^c)) \cup (C \cap D) \qquad (E = E \cup (E \cap F))$$

$$= ((A^c) \cap C) \cup \underbrace{((A^c) \cap B \cap C \cap D)}_{①} \cup \underbrace{((A^c) \cap B \cap C \cap (D^c))}_{②}) \cup \underbrace{(A \cap B \cap C \cap D)}_{③} \cup \underbrace{(A \cap B \cap C \cap (D^c))}_{④} \cup \underbrace{((A^c) \cap B \cap C \cap D)}_{①} \cup \underbrace{((A^c) \cap B \cap C \cap (D^c))}_{②} \cup ((B \cup (D^c)) \cup \underbrace{(A \cap B \cap C \cap (D^c))}_{④})$$

$$\cup ((C \cap D) \cup \underbrace{(A \cap B \cap C \cap D)}_{③}) \qquad (E \cup E = E)$$

$$= ((A^c) \cap C) \cup ((A^c) \cap B \cap C \cap D) \cup ((A^c) \cap B \cap C \cap (D^c)) \cup (B \cup (D^c)) \cup (A \cap B \cap C \cap (D^c)) \cup (C \cap D) \cup (A \cap B \cap C \cap D) \qquad (E \cup (E \cap F) = E)$$

$$= ((A^c) \cap C) \cup (B \cup D^c) \cup (A \cap B \cap C \cap (D^c)) \cup (C \cap D) \cup (A \cap B \cap C \cap D) \qquad (E \cup (E \cap F) = E)$$

$$= ((A^c) \cap C) \cup (B \cup (D^c)) \cup (C \cap D) \cup (A \cap B \cap C \cap D) \qquad (E \cup (E \cap F) = E)$$

$$= ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D)$$

$$H(A,B,C,D) = ((A^c) \cap C) \cup (B \cap (D^c)) \cup (C \cap D) = K(A,B,C,D)$$

$$H(A,B,C,D) = K(A,B,C,D)$$

(1c)
(i)

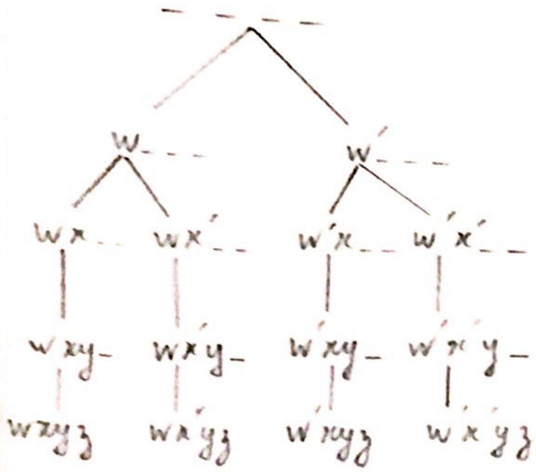
$$MSOP(E(x,y,z)) = xy' + x'y + yz$$

$$MSOP(E(x,y,z)) = (x * (y')) + ((x') * y) + (y * z)$$

$$MSOP_{PL}(E(p,q,r)) = (p \wedge (\neg q)) \vee ((\neg p) \wedge q) \vee (q \wedge r)$$

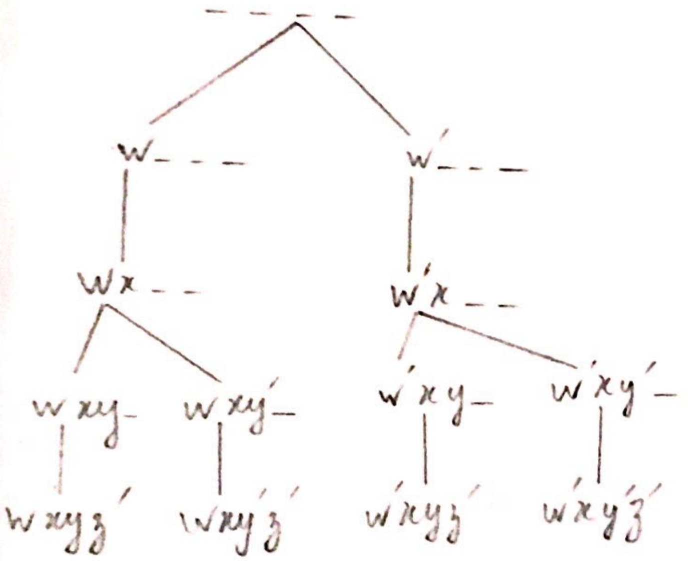
(2c)
(ii)

Tree



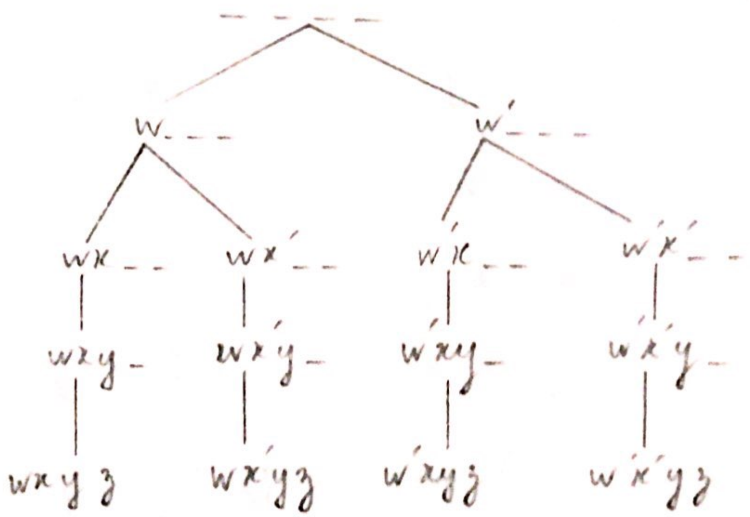
(2b)

Tree



(2b)

Tree



(1b) (i)

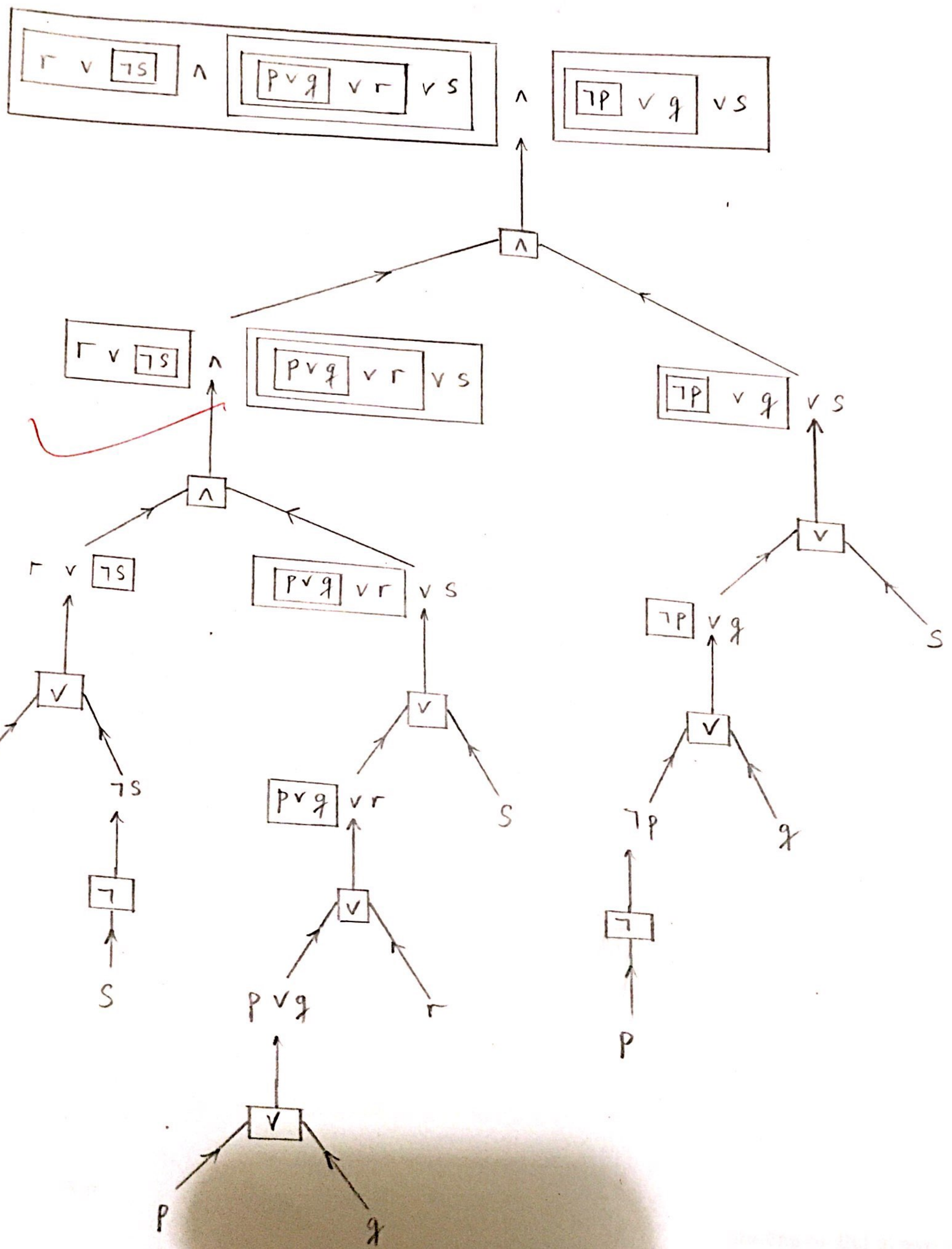
$$\begin{bmatrix} xy' \nleq xz & xz \nleq xy' \\ xy' \nleq x'y & x'y \nleq xy' \\ xy' \nleq yz & yz \nleq xy' \\ xz \nleq x'y & x'y \nleq xz \\ xz \nleq yz & yz \nleq xz \\ x'y \nleq yz & yz \nleq x'y \\ \mathrm{Cns}(xy', yz) = xz & \\ \mathrm{Cns}(xz, x'y) = yz & \end{bmatrix}$$

(2a)

$$\begin{bmatrix} w'y \nleq xy & xy \nleq w'y \\ w'y \nleq xz' & xz' \nleq w'y \\ w'y \nleq yz & yz \nleq w'y \\ xy \nleq xz' & xz' \nleq xy \\ xy \nleq yz & yz \nleq xy \\ xz' \nleq yz & yz \nleq xz' \end{bmatrix}$$

You are free to talk to anyone

(3a)
(iii)



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