2 Linear Algebra

Vector Objects

- Geometric vectors: $\vec{x}, \vec{y}, \lambda\vec{x}$ scala
- Polynomials:
- Audio signals: Series of number

Add, scale

- Elements of R^n (tuple of n real nums)

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in R^3$$

$a + b = c \in R^3$

$\lambda * a = \lambda a \in R^3$ (λ is scala)

- Closure Concept: $\forall a, b \in S, a + b \in S$

Systems of Linear Equations

Simply $x \to f(x)$ (find f)

$$\begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} x_1 + \begin{bmatrix} a_{12} \\ \vdots \\ a_{m2} \end{bmatrix} x_2 + \cdots \begin{bmatrix} a_{1n} \\ a_{mn} \end{bmatrix} x_n$$

$$= \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \Leftrightarrow \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_m \end{bmatrix}$$

Matrices

$$A + B = \begin{bmatrix} a_{11} + b_{12} & \cdots & a_{1n} + b_{1n} \\ \vdots & & \\ a_{m1} + b_{m1} & \cdots & a_{mn} + b_{mn} \end{bmatrix}$$

For $A \in R^{m \times n}$, $B \in R^{n \times k}$, c_{ij} of $C = AB \in R^{m \times k}$ are computed as:

$$c_{ij} = \sum_{l=1}^{n} a_{il} \cdot b_{lj}, \quad i = 1, \dots, m$$
$$j = 1, \dots, k$$

Choose what is n at row and n at column

Identity Matrix

$$I_n = \begin{bmatrix} 1 & \cdots & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 \end{bmatrix} \in R^{n \times n}$$

Property of matrix

- Associativity:
  $\forall A \in R^{m \times n}, B \in R^{n \times p}, C \in R^{m \times p}: (AB)C = A(BC)$
- Distributivity:
  $\forall A, B \in R^{m \times n}, C, D \in R^{n \times p}$:
  $(A+B)C = AC + BC$
  $A(C+D) = AC + AD$

* $\forall A \in R^{m \times n}: I_m A = A I_n = A$

Inverse and Transpose

$A \cdot A^{-1} = I_n \Rightarrow A^{-1} = I_n \cdot A$

Calculate inverse: $[A \mid I_n] \rightsquigarrow \dots \rightsquigarrow [I_n \mid A^{-1}]$

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[A \mid I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} d_3 - d_1 \\ d_2 - d_1 \\ d_4 - d_1 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -2 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} d_3 - 2d_2 \\ d_4 - d_2 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$d_3 - d_4 \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} d_1 - 2d_3 \\ d_2 + 2d_3 \\ d_4 - d_3 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 2 & -2 & 2 \\ 0 & 1 & 0 & 0 & 1 & -1 & 2 & -2 \\ 0 & 0 & 1 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & -1 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 2 & -2 & 2 \\ 1 & -1 & 2 & -2 \\ 1 & -1 & 1 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

Vector Spaces

Groups

$G := (\mathcal{G}, \otimes)$ is called a group if the follow hold:

1. Closure of $\mathcal{G}$ under $\otimes$: $\forall x, y \in \mathcal{G}$: $x \otimes y \in \mathcal{G}$