Social Choice Theory

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Introduction on Social choice theory

Social choice theory

Social choice theory

Research of a **mechanism** (electoral system or aggregation method) allowing to aggregate in a **reasonable** way the opinion expressed during an **election** by several **voter** concerning different candidates, in order to determine a **winner** (elected candidate), or in order to **rank** by order of preferences the different candidates.

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$\Rightarrow$ Problem of the **preference aggregation**

Social choice theory

- Some works:
  - originally: Borda (1781) and Condorcet(1785)
  - and after: results from Arrow (1951), May (1952), Black (1958)

  $\Rightarrow$ a huge literature (see Kelly (1991))
- Some fundamental results:
  - economy, politics, applied mathematics, operational research, management, artificial intelligence
  - two Nobel prizes: Kenneth Joseph Arrow (1972), Amartya Kumar Sen (1998)

Social choice theory: applications

- Political elections

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- Some other elections: university, company, etc.

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- Aggregation :
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  - Several nature states with different consequences
  - Several criteria, ...

Social choice theory: vocabulary

- Group ⇒ Society
- Group's member ⇒ Voter
- Alternatives ⇒ Candidates
- Partial preferences ⇒ Individual preferences
- Global preferences ⇒ Collective preferences
- Problems :
  - Choice of an unique candidate (or a subset of candidates considered to be equivalent): Uninominal elections
  - Ranking of the candidates : Elections by ranking

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Slides inspired from :

- Meltem Öztürk : `www.lamsade.dauphine.fr/ozturk`
- Sébastien Konieczny : `www.cril.univ-artois.fr/ konieczny`

Work hypothesis

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Problem presentation

Problem presentation

- The choice of the candidate(s) will affect all the society
- Taking into account the opinion of all members of the society

$\Rightarrow$ Individual preferences within the group: **democracy**

$\Rightarrow$ Decision making within the group: **elections**

$\Rightarrow$ **Majority**

Problem presentation

- Philosophical problems: majority vs minority
- Political problems
    - Direct or indirect democracy
    - How are we voting?
    - Who can be a candidate?
    - etc
- Technical problems

Problem presentation

Majority decision: if a larger number of people vote for a than for b, then a has to be preferred to b

- no problem if there is only two candidates, this rule has good properties
- how can we extend this idea with several (more than two) candidates?

$\Rightarrow$ Several methods

Work hypothesis

- All the voters are sincere
- All the voters are able to compare two candidates, and to rank them in a preorder.

Work hypothesis

Types of voting procedures

Voting systems

How can we vote?

- Uninominal elections : each voter votes for the candidate that he ranks in first position
- Ranking systems: each voter ranks the candidates
- Other systems: acceptable or non-acceptable candidates; veto; ideal candidates; etc...

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How can we aggregate?

- How can we put together the individual preferences?
- How can we extract the best candidate, or a ranking of candidates, from the result of the election?

Uninominal elections

Uninominal elections

One-stage systems

Uninominal election with one stage (plurality voting)

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- Aggregation method: the candidate getting a simple majority of votes is elected

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  - Respect of majority in the British system

Uninominal elections

Two-stage systems

Two-stage French system

- Uninominal election
- First stage: if a candidate has the absolute majority, he is elected. Otherwise, only the two candidates who have the highest scores remain in the second stage.
- Second stage: the winner is the candidate that get the more votes.

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  - Participation
  - Separability

Two-stage French system

To sum up:

- The two-stage French system is not monotonic
- The two-stage French system does not always encourage participation
- The two-stage French system is manipulable
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Are there some better methods?

Uninominal elections

Sequential voting

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- Is it possible to apply majority for n candidates?
- Compare 2 candidates, then compare the winner with the 3rd candidate, and so on until the last of the n candidates

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More complex methods?

Election by rankings

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The Condorcet criterion

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Condorcet's Paradox

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Condorcet principle

A candidate that is preferred to all other candidates using the Condorcet criterion is called a Condorcet winner. This candidate, if he exists, should be elected.

It can be shown that there is never more than one Condorcet winner.

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- Is the British system a Condorcet method?
- Is the two-stage French system a Condorcet method?

Election by rankings

Non ranking voting rules

Non ranking voting rules

- Let m candidates. A nonranking voting rule is a not empty subset of $\{1, 2, \ldots, m-1\}$.
- Represents the number of candidates for which a voter can vote
- The candidate who has the more votes is elected.
- Examples:

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  Approval voting is the nonranking voting rule which is the less manipulable [Fishburn, 81]

Election by rankings

Scoring voting rules

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- Let m candidates. A scoring voting rule is defined by:
  - A non decreasing sequence of integers: $s_0 \leq s_1 \leq \dots s_{m-1}$ such that $s_0 < s_{m-1}$
  - Each voter gives s_0 points to the candidate he ranks in last, s_1 points to the candidate he ranks in next to last...
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- Examples:
  - $s_0 = s_1 = \dots = s_{m-2} < s_{m-1}$: British system
  - $s_0 = 0,\ s_1 = 1,\ \dots,\ s_{m-1} = m - 1$: Borda's rule
    - Always give one (or several) winner(s)
    - Give a ranking of all candidates

Scoring voting rules: properties

Each scoring voting rule satisfies the following properties:

- Monotonicity
- Separability
- Encourage participation
- They are not Condorcet method

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  - Example of Borda's rule

Election by rankings

Condorcet method

Condorcet method

- Copeland's rule : Give the following score to each candidate a: for each candidate $b \neq a$,
  - $+1$ if a majority prefers a to b,
  - -1 if a majority prefers b to a,
  - 0 otherwise
  - The winner is the candidate who has the higher Copeland's score
- Kramer-Simpson's rule : Give the following score to each candidate a:
  - for each candidate $b \neq a$, compute $N(a, b)$, which is the number of voter who prefer a to b
  - Simpson's score of candidate a is the minimum of $N(a, b)$
  - The winner is the candidate who has the higher Simpson's score

Condorcet method

- Copeland and Kramer-Simpson's rules are monotonic
- No Condorcet method satisfies the separability
- No Condorcet method encourages participation

Election by rankings

Multi-stage ranking systems

Single transferrable vote: Hare quota

- At least two seats to be filled
- Each voter ranks all the candidates
- Counting:

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    Let v be the number of voters, and n the number of seats to be filled
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    - If no candidate reaches this quota, the candidate with the lowest number of votes is eliminated and his/her votes are moved to the second of the list in each ballot where he/she was at the top of the list.

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    - When one or more candidates reach the quota, they are definitely elected. If they have exceeded the quota, the surplus votes are distributed equally to each second of his/her lists.

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    - When one or more candidates reach the quota, they are definitely elected. If they have exceeded the quota, the surplus votes are distributed equally to each second of his/her lists.
    - If all the seats are filled, we stop.

Alternative vote (or Instant run-off voting)

- Each voter ranks all the candidates
- The first preference of each voter is counted
- If one candidate holds a majority, that candidate wins
- Otherwise the candidate who holds the fewest first preferences is eliminated
- If there is an exact tie for last place in numbers of votes, various tie-breaking rules determine which candidate to eliminate
- Ballots assigned to eliminated candidates are recounted and assigned to one of the remaining candidates based on the next preference on each ballot
- The process repeats until one candidate achieves a majority

Coombs' rule

- Each voter ranks all the candidates
- The first preference of each voter is counted
- If one candidate holds a majority, that candidate wins
- Otherwise, the candidate ranked last by the largest number of voters is eliminated
- The process repeats until one candidate achieves a majority

Election by rankings

Summary

What are we looking for?

A democratic method:

- which always give a result
- which chooses the Condorcet winner if he exists (Condorcet method)
- which is not manipulable
- which is monotonic, separable, encourages participation, ...

$\Rightarrow$ Need to axiomatize!

Axiomatization

Axiomatization

Arrow's impossibility theorem

Hypothesis

- Use majority if $n < 3$ (less than three candidates)
- m voters (finite number of voters)
- The voters rank all the candidates
- Problem: find a method which satifies a set of given conditions

Which conditions?

Arrow's properties

- Universality: Every configuration of candidates is possible

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- Independance of Irrelevant Alternatives (IIA): the social preference of A compared to B should be independent of preferences for other alternatives.

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- Unanimity: If every individual prefers a certain option to another, then so must the resulting societal preference order
- Independance of Irrelevant Alternatives (IIA): the social preference of A compared to B should be independent of preferences for other alternatives.
- Non-dictatorship: The social welfare function should account for the wishes of multiple voters. It should not depend only upon the preferences of one individual, i.e. the dictator

Arrow' theorem (1951)

Arrow's theorem

For elections with more than 2 candidates, no voting procedure satisfies simultaneously Universality, Transitivity, Unanimity, Independance of Irrelevant Alternatives and Non-dictatorship.

Arrow' theorem (1951)

Arrow's theorem

For elections with more than 2 candidates, no voting procedure satisfies simultaneously Universality, Transitivity, Unanimity, Independance of Irrelevant Alternatives and Non-dictatorship.

This is a negative result: there are fundamental limits to democratic decision making!

Axiomatization

Examples

Two-stage system

- Universality?
- Transitivity?
- Unanimity?
- Independance of Irrelevant Alternatives?
- Non-dictatorship?

Two-stage system

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$$
\begin{array}{lcl}
10 & : & b \succ a \succ c \succ d \\
6 & : & c \succ a \succ d \succ b \\
5 & : & a \succ d \succ b \succ c
\end{array}
$$

  b is elected. 6 voters change their preferences

- Non-dictatorship?

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Condorcet

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$$\begin{array}{ccc} 1 & : & a \succ b \succ c \\ 1 & : & b \succ c \succ a \\ 1 & : & c \succ a \succ b \end{array}$$

  - For $\{a, b\}$, a (2) is preferred to b (1)
  - For $\{a, c\}$, c (2) is preferred to a (1)
  - For $\{b, c\}$, b (2) is preferred to c (1)

  Non transitive relation
- Unanimity?
- Independance of Irrelevant Alternatives?
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- Independance of Irrelevant Alternatives? No

$$\begin{array}{lll} 2 & : & b \succ a \succ c \succ d \\ 1 & : & a \succ c \succ d \succ b \end{array}$$

  a is elected (a : 7; b : 6; c : 4; d : 1).
  If c and d step down, b is elected (a : 1, b : 2)
- Non-dictatorship?

Borda

- Universality? OK
- Transitivity? OK
- Unanimity? OK
- Independance of Irrelevant Alternatives? No
- Non-dictatorship? OK

Strategic manipulation

Is it possible to manipulate an election?

- From voters:
  - Voters can lie about their preferences
  - Voter can move house
- From candidates:
  - Bring some false candidates
- From the authority:
  - Choose the “best” voting procedure
  - Choose the “good” constituencies

Manipulation from voters

Gibbard-Satthertwaite theorem [Gibbard 73, Satthertwaite 75]

The only non-manipulable voting method satisfying the Pareto property for elections with more than 2 candidates is a dictatorship.

- In other words, every "realistic" voting method is prey to strategic manipulation...
- But Gibbard-Satterthwaite only tells us that manipulation is possible in principle
  - It does not give any indication of how to misrepresent preferences.

Conclusion

Conclusion

- A lot of voting procedures
- Arrow's impossibility theorem
- Need to study the properties of the chosen method
- Many desirable properties
- Many possibilities of manipulation
- Is democraty = vote?