Preference modelling

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Introduction

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Lemma

- if you have no preferences...
- there is no need to worry about decisions!

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$\rightarrow$ We need to have concepts to represent preferences

- in a variety of disciplines:
- economics, psychology, political science, operational research, multiple criteria decision making...

$\rightarrow$ preference modelling

Binary relation

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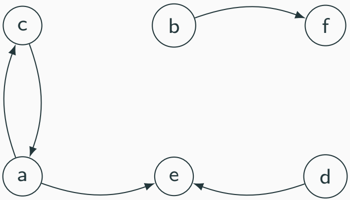
- A possible interpretation of aRb is: a is preferred to b
- Another one:
  - $A = \{$alan, bonnie, clara, diana, eddy, fanny$\}$
  - $R =$ "wants to see tonight"
  - $R = \{(a, c), (c, a), (d, e), (b, f), (a, e)\}$

Representation of a binary relation

- Matrix representation:

$\mathcal{R}$	a	b	c	d	e	f
a	0	0	1	0	1	0
b	0	0	0	0	0	1
c	1	0	0	0	0	0
d	0	0	0	0	1	0
e	0	0	0	0	0	0
f	0	0	0	0	0	0

- Graphical representation:



Let R and T be two binary relations on the same set A:

- Inclusion: $R \subseteq T$ iff $aRb \Rightarrow aTb$
- Union: $a(R \cup T)b$ iff aRb or (inclusive) aTb
- Intersection: $a(R \cap T)b$ iff aRb and aTb
- Relative Product: $a(R.T)b$ iff $\exists c \in A$ s.t. aRc and cTb

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- complete iff $aRb \vee bRa$
- weakly complete iff $a \neq b \Rightarrow aRb \vee bRa$

Preference structures

Preference structures

Preference structure

A preference structure is a collection of binary relations defined on the set A and such that:

- $\forall a, b \in A$, at least one relation is satisfied
- $\forall a, b \in A$, if one relation is satisfied, another one cannot be satisfied

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A preference structure defines **a partition** of the set $A \times A$.

- Each preference relation in a preference structure is uniquely characterized by its properties (symmetry, transitivity...)
- Any preference structure can be characterized by a unique binary relation R (called characteristic relation)

Strict preferences, indifference and incomparability

Strict preferences: P

- There are clear and positive reasons for a significant preference for one of the two options,
- P is asymmetric

Indifference: I

- There are clear and positive reasons for an equivalence between the two options,
- I is symmetric and reflexive

Incomparability: J

- There are no clear and positive reasons for one of the above situations,
- J is symmetric and irreflexive

Preference structure

- $\{P, I, J\}$ is a preference structure if:
  - P is asymmetric
  - I is symmetric and reflexive
  - J is symmetric and irreflexive,
  - $P \cup I \cup J$ is complete,
  - P, I and J are exclusives
- Example:
  - $A = \{a, b, c, d, e\}$
  - $P = \{(b, a), (b, c), (b, d), (b, e), (d, c), (e, c)\}$
  - $I = \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, c), (c, a)\}$
  - $J = \{(a, e), (e, a), (a, d), (d, a), (d, e), (e, d)\}$

Characterisation

Characterisation

Every preference structure is characterized by the relation $R = P \cup I$

$$(a, b) \in R \Leftrightarrow (a, b) \in P \text{ or } (a, b) \in I$$

- We have:

$$\begin{aligned}
(a, b) \in P &\quad\Leftrightarrow\quad (a, b) \in R \text{ and } (b, a) \notin R \\
(a, b) \in I &\quad\Leftrightarrow\quad (a, b) \in R \text{ and } (b, a) \in R \\
(a, b) \in J &\quad\Leftrightarrow\quad (a, b) \notin R \text{ and } (b, a) \notin R
\end{aligned}$$

- R is the characteristic relation of the preference structure $\{P, I, J\}$
- $(a, b) \in R$ means that: "*a is at least as good as b*"

Preference models

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- The preference structure $\{P, I, J\}$ satisfies the following properties:
  - No incomparability ($J = \emptyset$)
  - P is transitive
  - I is transitive

Total preorder

- Numerical representation of a total preorder:

$$\begin{cases} (a,b) \in P \Leftrightarrow g(a) > g(b) \\ (a,b) \in I \Leftrightarrow g(a) = g(b) \end{cases}$$

- The characteristic relation R is represented by:

$$(a,b) \in R \Leftrightarrow g(a) \geq g(b)$$

- Whenever a decision problem is reduced to the comparison of "profit", the underlying preference structure is a preorder.

Total (pre)order

- In a total preorder:
    - I is an equivalence relation: reflexive, symmetric and transitive
    - P is a weak order: asymmetric and negatively transitive
    - Knowing P is enough to know all the structure

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Total Order

A total order is a total preorder without equally-ranked candidates

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Total order

R is an order iff it satisfies the following properties:

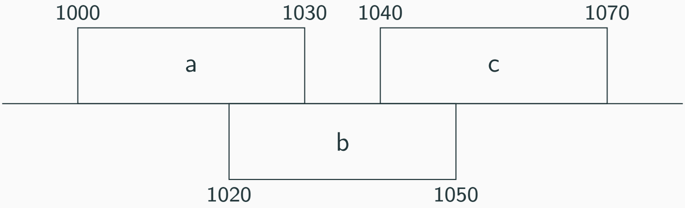
- R is complete
- R is transitive
- R is antisymmetric

Preference models

Semiorder

Considering a threshold

Let $a, b, c \in A$ 3 elements, such that $g(a) = 1000$, $g(b) = 1020$ and $g(c) = 1040$.
If we assume that we have a threshold $q = 30$, we will have $(a, b) \in I$, $(b, c) \in I$ and $(c, a) \in P$:



Considering a threshold

- A discrimination threshold aims to consider small differences as not significant
- The transitivity of the indifference relation is not compatible with the existence of such a threshold,
- Any preference structure underlying a threshold model verifies:

$$\begin{cases} (a,b) \notin J \quad \text{(that is } J = \emptyset) \\ (a,b) \in P,\ (b,c) \in I,\ (c,d) \in P \Rightarrow (a,d) \in P \\ (a,d) \in I,\ (a,b) \in P,\ (b,c) \in P \Rightarrow (d,c) \in P \end{cases}$$

- Any preference structure that verifies the properties above can be represented by a threshold model (if A is finite or countable)

Semiorder

Semiorder

A reflexive relation $R = \langle P, I \rangle$, defined on A , is a semiorder if there exists a function g with values in $\mathbb{R}$, and a non-negative constant q such that $\forall a, b \in A$,

$$\begin{cases} (a,b) \in P & \Leftrightarrow \quad g(a) > g(b) + q, \\ (a,b) \in I & \Leftrightarrow \quad |g(a) - g(b)| \leq q. \end{cases}$$

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Semiorder

R is a semiorder iff it satisfies the following properties:

- R is complete
- R is Ferrers
- R is semi-transitive

Preference models

Interval Order

What if the threshold is variable?

- One may want to vary the threshold according to the level of the scale
- We introduce a variable threshold such that:

$$\begin{cases} (a,b) \in P & \Leftrightarrow \quad g(a) > g(b) + q(g(b)) \\ (a,b) \in I & \Leftrightarrow \quad \begin{cases} g(a) \leq g(b) + q(g(b)), \\ g(b) \leq g(a) + q(g(a)) \end{cases} \end{cases}$$

Consistency condition

- Consistency Condition:

$$g(a) > g(b) \Rightarrow g(a) + q(g(a)) > g(b) + q(g(b))$$

- If the consistency condition is satisfied, then the **underlying preference structure is a semiorder**. The problem can be reduced (by transforming the functions g and q) to a model where the threshold is constant (with, for example, $q(g(a)) = \alpha g(a) + \beta$)
- If the consistency condition is not satisfied, then the underlying preference structure has to satisfy:

$$\begin{cases} (a, b) \notin J \quad \text{(that is } J = \emptyset) \\ (a, b) \in P,\ (b, c) \in I,\ (c, d) \in P \Rightarrow (a, d) \in P \end{cases}$$

- A preference structure is an interval order if it can be represented by a **variable threshold model**

Interval orders

Interval order

R is an interval order iff it satisfies the following properties:

- R is complete
- R is Ferrers

Interval-actions

- It is sometimes difficult to translate the consequences of decisions by a precise numerical assessment
- The evaluation of each action can be apprehended by an interval of possible values for $g(a) : [l_a, u_a]$

$\rightarrow$ How can we compare such interval-actions?

Interval-actions

Model 1:

- The intervals have to be disjoint in order to mark a preference:

$$\begin{cases} (a,b) \in P & \Rightarrow \quad l_a > u_b \\ (a,b) \in I & \Rightarrow \quad (a,b) \notin P \text{ and } (b,a) \notin P \end{cases}$$

- It is then an interval order structure

  $\rightarrow$ with $l_1 = g(a)$ and $u_a = g(a) + q(g(a))$, it is a **variable threshold model**

- When the intervals are of identical length, it is a semiorder structure (the length of the intervals corresponds to a constant threshold)

Model 2:

- There is a preference as soon as an interval impinge on the other:

$$\begin{cases} (a,b) \in P & \Rightarrow \quad l_a > l_b \text{ and } u_a > u_b \\ (a,b) \in I & \Rightarrow \quad (a,b) \notin P \text{ and } (b,a) \notin P \end{cases}$$

- In this case P is a partial order, and I is the complementary relation

Preference models

Pseudo-orders

Taking two thresholds into account

- It may seem arbitrary to determine a value below which there is an indifference, and above which there is a strict preference
- There is often a hesitation area
- We introduce a preference threshold (**in addition to the indifference threshold**) beyond which there is a strict preference
- Between the indifference threshold and the preference threshold exists an ambiguous zone in which the decision maker hesitates between indifference and preference

Double threshold order

Double threshold order

Let $R = \langle P, Q, I \rangle$ be a relation on a finite set A. R is a double threshold order iff, $\forall a, b \in A$,

$$\begin{cases} (a,b) \in P & \Leftrightarrow \quad g(a) > g(b) + p(g(b)) \\ (a,b) \in Q & \Leftrightarrow \quad g(b) + p(g(b)) \geq g(a) > g(b) + q(g(b)) \\ (a,b) \in I & \Leftrightarrow \quad \begin{cases} g(b) + q(g(b)) \geq g(a) \\ g(a) + q(g(a)) \geq g(b) \end{cases} \end{cases}$$

- Q represents a "weak" preference relation, where one is hesitant between an indifference or a preference relation

Pseudo-order

- A pseudo-order is a particular case of double threshold order, such that the thresholds fulfil a coherence condition

Pseudo-order

Let $R = \langle P, Q, I \rangle$ be a relation on a finite set A. R is a pseudo-order iff, $\forall a, b \in A$,

$$\begin{cases} R \text{ is a double threshold order} \\ g(a) > g(b) \Leftrightarrow \begin{cases} g(a) + q(g(a)) > g(b) + q(g(b)) \\ g(a) + p(g(a)) > g(b) + p(g(b)) \end{cases} \end{cases}$$

Interval comparison

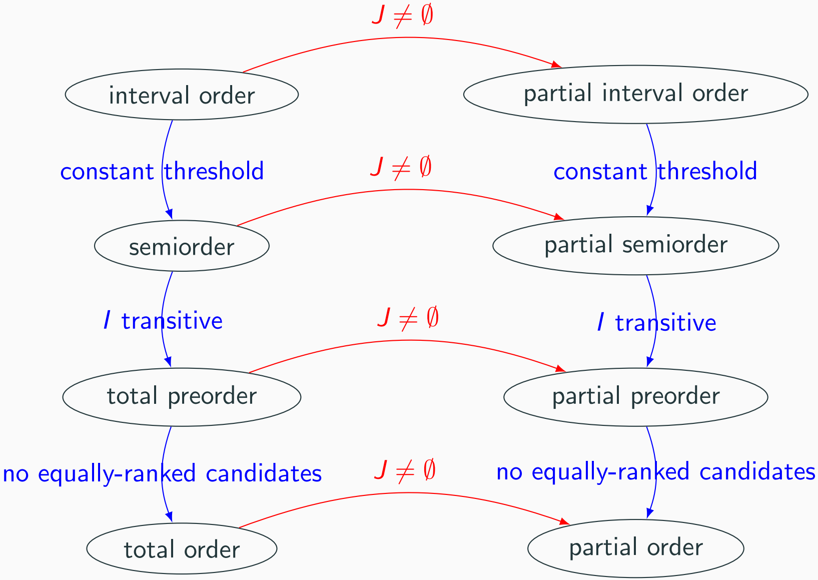
A three-way structure appears when we conpare intervals as follows:

$$\begin{cases} (a,b) \in P & \Leftrightarrow \quad l_a > u_b \\ (a,b) \in Q & \Leftrightarrow \quad u_a > u_b > l_a > l_b \\ (a,b) \in I & \Leftrightarrow \quad [l_a, u_a] \subseteq [l_b, u_b] \text{ or } [l_b, u_b] \subseteq [l_a, u_a] \end{cases}$$

Preference models

Incomparability

Partial models



Valued structures

Fuzzy preferences

- The preference models we have seen until now assume that the preference relation is unique (for $a, b \in A$)
- Each $(a, b) \in P$ can be associated to a value $v(a, b)$ representing the **“degree”** or the **“validity”** of the preference of a over b
- Let $v(a, b) \in [0, 1]$ such that
  - $v(a, b) = 1$: the degree of the preference of a over b is maximum,
  - $v(a, b) = 0$: the degree of the preference of a over b is minimum.
- Useful when a and b are compared several times during votes, polls, ...

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- $v(a, c) = v(a, b) + v(b, c) \longrightarrow$ additivity (notion of intensity of the preferences)
- α-cut of a valued relation: we keep only the couples (a, b) satisfying $v(a, b) \geq \alpha$
- A valued relation is max-min-transitive iff all α-cuts are transitive

Comparison of preferences gap

- It is possible to compare the gap between preferences
- Let $a, b, c, d \in A$ such that $(a, b) \in P$ and $(c, d) \in P$
- We consider the statement *"The preference of a over b is stronger (less strong, equivalent, incomparable) than the preference of c over d"*

$\Rightarrow$ Defines a preference structure over $A \times A$

Comparison of preferences gap

- Example: the **additive model**
- 2 preferences structures:
  - (I, P) over A
  - $(\sim, \succ)$ over $A \times A$
- Defined by the function g:

$$\begin{cases} (a,b) \in P & \Leftrightarrow \quad g(a) > g(b) \\ (a,b) \in I & \Leftrightarrow \quad g(a) = g(b) \\ (a,b) \succ (c,d) & \Leftrightarrow \quad g(a) - g(b) > g(c) - g(d) \\ (a,b) \sim (c,d) & \Leftrightarrow \quad g(a) - g(b) = g(c) - g(d) \end{cases}$$

Conclusion

To conclude

- Brief survey of classical preference structures
- Vast and complex literature
- Some important questions we did not ask here:
  - the question of the approximation of preference structure by another one
  - the way to collect and validate preference information in a given context
  - the links between preference modeling and the question of meaningfulness in measurement theory
  - the statistical analysis of preference data
  - questions on the links between value systems and preferences