Social Choice Theory

Exercice 1:(Scoring voting rules)

A **scoring voting rule** is defined by :

— A non decreasing sequence of integers : $s_0 \leq s_1 \leq \ldots s_{m-1}$ such that $s_0 < s_{m-1}$

— Each voter gives s_0 points to the candidate he ranks in last, s_1 points to the candidate he ranks in next to last...

— The candidate who gets the more points is elected

1. Let the following voters. Give the result obtained by each candidate

voters	preferences
3	$c > a > b$
2	$a > b > c$
1	$a > c > b$
1	$b > c > a$

2. Is there a Condorcet winner ? If yes, can he be elected (and be the only winner) by a scoring method ? If he can, give the value of s_1, s_2 and s_3 ; otherwise explain why
3. What are your conclusions ?

Exercice 2:(Straffin [1980])

Let the following profile :

voters	preferences
1	$a > b > c > d > e$
4	$c > d > b > e > a$
1	$e > a > d > b > c$
3	$e > a > b > d > c$

1. Who is the Copeland winner ? The Kramer-Simpson winner ? How can you deduct that there is no Condorcet winner from these results ?
2. Who is the Borda winner ?
3. Is there a scoring method which elects only c ? Only b ? Only d ?

Exercice 3:(Separability)

Let V_1 and V_2 be two disjoint groups of voters over the same set of candidates A. Let B_1 be the subset of candidates elected by V_1, and B_2 be the candidates elected by V_2.

Thus, if $B_1 \cap B_2 \neq \emptyset$, then the set of voters $V_1 \cup V_2$ should elect $B_1 \cap B_2$.

1. Do you know a method which does not satisfy this property ? Which one ?
2. Prove that scoring methods satisfy this property

Exercice 4:(Voting methods)

Let A be the finite set of candidates, and V be the finite set of voters. We consider the following voting rule :

1. Each voter presents a total order over the set of candidates A
2. Each candidate ranked in first position gets 4 points
3. Each candidate ranked in second position gets 2 points
4. Each candidate ranked in third position gets 1 point
5. All the other candidates get 0 point
6. The candidate who gets the more points is elected

Analyze this procedure with respect to Arrow's theorem.

Exercice 5:(Board meeting)

The CEO of a firm, Mr LeChef, wants to see his candidate, d, elected against the three other candidates a, b and c. He knows that the 20 members of the board meeting have the following preferences :

voters	preferences
3	$b \succ c \succ a \succ d$
8	$a \succ d \succ c \succ b$
5	$c \succ d \succ a \succ b$
4	$d \succ c \succ a \succ b$

What voting method could choose Mr LeChef in order to ensure the election of d ?