



# Quanto Credit Default Swaps

## Theory, Pricing & Practice



Nicholas Burgess (2023)

## PART ONE: Theory

### Quanto CDS

- Introduction to Quanto CDS
- CDS Pricing & Par Spread
- The Quanto Market Data Problem
- Quanto FX Risks
- Quanto Market Data Adjustments
- Quanto FX Jump Risk Explained
- Quanto Hazard Rates

## PART TWO: Pricing & Practice

### Case Studies

- **Eurozone**: Quanto Effect
- **Italy**: CDS Par Spreads, EUR vs USD
- **Japan**: Sovereign & Corporate Basis Spreads
- **Brazil**: Quanto CDS & Basis
- **Brazil**: Corporate Implied FX Jumps
- **Brazil**: Quick Rules of Thumb

Quanto CDS Research & Support

<https://github.com/nburgessx/QuantResearch/blob/main/Quanto%20CDS>

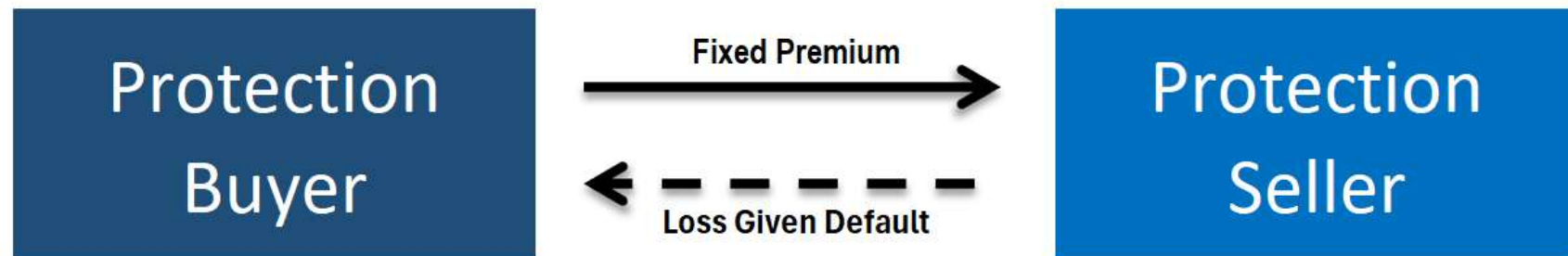
## PART ONE - THEORY

Quanto CDS

# Quanto CDS – Introduction to Quanto CDS

## Credit Default Swap

- An insurance contract to protect against losses should the reference bond default
- The protection buyer pays regular **Fixed Premiums** in exchange for the **Loss Given Default**,  $LGD = N(1 - RR)$



## Liquid vs Contractual Currency

- The **Liquid Currency** for a CDS contract is USD, regardless of bond domicile & issue currency
- When the CDS **Contractual Currency** is not the liquid currency, the CDS is considered a **Quanto CDS**



# Credit Default Swaps – Pricing



Probability of Default at  
End of Period  
 $= \lambda Q(t_i)$

Probability of Survival  
 $= Q(t_i)$

**Present Value** is the sum of **Risky Discounted** cash flows

$$CDS PV^1 = \underbrace{\sum_{i=1}^n N(1 - R) \lambda Q(t_i) P(0, t_i)}_{\text{Protection Leg}} - \underbrace{\sum_{i=1}^n N s \tau_i Q(t_i) P(0, t_i)}_{\text{Premium Leg}} \quad (1)$$

<sup>1</sup> There are other ways to compute conditional default probabilities, here we use  $\lambda Q(t_i)$  to simplify the par spread calculation.

# Credit Default Swaps – Par Spread

## Quotation

- CDS contracts typically quote as a **Par Spread, s**
- This is the spread that makes the premium and protection leg PVs equal.

## Par Spread Formula

- Setting the PV in equation (1) to zero and rearranging gives (2),

$$s = \frac{\sum_{i=1}^n N(1-R) \lambda Q(t_i) P(0, t_i)}{\sum_{i=1}^n N \tau_i Q(t_i) P(0, t_i)} \quad (2)$$

## Par Spread Rule of Thumb

- Terms in red cancel and assuming annual premiums i.e.,  $\tau = 1$  gives a simple rule of thumb,

$$s \approx (1-R) \lambda \quad (3)$$

# Quanto CDS – The Quanto Market Data Problem

## The Problem – Market Data in a Different Currency (USD)

- We want to price a BRL CDS on a BRL Sovereign Bond.
- However, only USD CDS market data quotes are available.

## The Solution – Quanto Market Data Adjustments

- Brigo, Pede and Petrelli (2018) suggest we make a **Quanto Adjustment** to the USD market data.
- Naming conventions, they call BRL the **Contract currency** ( or Quanto) and USD the **Liquid currency**.
- They derive a model to convert USD hazard rates into BRL equivalent rates.
- The quanto adjustment incorporates FX crash risk and FX covariance risk.

# Quanto CDS – Quanto FX Risks

## Quanto FX Risk

- We want to trade BRL CDS to buy/sell protection on a BRL Bond
- Typically, the BRL Quanto CDS will be illiquid, so we might use liquid USD CDS market data as a proxy.
- When using USD market data to price Quanto CDS we need to adjust for Quanto FX risk

$$\text{Quanto FX Risk} = \text{Crash Risk} + \text{Covariance Risk}$$

## FX Crash Risk

- FX devaluation or crash risk on default
- Quanto CDS Spreads cheapen relative to USD

## FX Covariance Risk

- FX covariance with credit spreads
- As default intensity increases, Quanto currency weakens
- Quanto protection becomes cheap relative to USD



# Quanto CDS – Quanto Market Data Adjustments

## Quanto Market Data Adjustment

- **Li, Mercurio & Resnick (2018)** present a quanto market data adjustment formula in [1], see equation (25)

$$\underbrace{\lambda^{BRL}}_{\text{Contract Currency}} = (1 + \gamma) \left( 1 + \underbrace{\frac{1}{2} \rho_{(C,FX)} \sigma_C \sigma_{FX} t}_{\text{Covariance Term}} \right) \underbrace{\lambda^{USD}}_{\text{Liquid Currency}} \quad (4)$$

- The FX jump parameter,  $\gamma$  captures a single FX jump at default in %
- The FX jump dominates and covariance is a second order effect
- Setting the covariance term to zero we get a useful approximation or ‘rule of thumb’.

$$\lambda^{BRL} = (1 + \gamma) \lambda^{USD} \quad (5)$$

- Equivalently, using (3) gives an expression in terms of CDS par spreads,  $s$

$$s^{BRL} = (1 + \gamma) s^{USD} \quad (6)$$

# Credit Default Swaps – FX Jump on Default Explained

## Sovereign CDS - Brazil Case Study

- Consider Brazil Sovereign CDS in USD (Liquid Currency)
- and Brazil CDS in EUR (Quanto Contractual Currency)
- Let's define an FX process as  $Z = \text{Quanto}/\text{USD}$
- The FX process  $Z = \text{BRL}/\text{USD}$  i.e., how many USD needed to buy 1 BRL?
- We can measure the jump  $\gamma\%$  in the FX rate if there is a sovereign default

## Credit Default and Impact of FX Devaluation

- Should Brazil default we expect BRL will weaken versus USD
- That is the amount of USD to buy 1 BRL to jump down i.e., Negative Jump,  $\gamma\%$
- Therefore, Quanto protection payments in BRL are worth less relative to USD
- Par spreads measure protection payment value.
- Consequently, we expect Lower Quanto Par Spreads in BRL relative to USD

### Key Points

- Negative FX Jump on Default  
i.e. BRL Devaluation
- Quanto Protection Cheapens  
Relative to USD
- Lower Quanto CDS Spreads  
USD CDS > BRL CDS

# Credit Default Swaps – Quanto Hazard Rates

Consider Brazil CDS in BRL (Quanto) and USD (Liquid)

- $Z = \text{BRL/USD}$  (Quanto/USD)
- On Default Expect **Negative FX Jump  $\gamma$**
- Quanto Protection Cheaper Relative to USD

Cheaper Quanto Protection

- BRL CDS Spread < USD CDS Spread
- BRL Hazard Rates < USD Hazard Rates, since  $s \approx (1 - R) \lambda$

## Quanto Rules of Thumb

- $s^{Quanto} = (1 + \gamma)s^{Liquid}$
- $\lambda^{Quanto} = (1 + \gamma)\lambda^{Liquid}$

Quanto Hazard Rates & Market Data

- Intuitively, we can deduce that  $s^{Quanto} = (1 + \gamma)s^{Liquid}$
- Furthermore,  $s \approx (1 - R) \lambda$  implies  $\lambda^{Quanto} = (1 + \gamma)\lambda^{Liquid}$
- Therefore, we should not use raw USD hazard Rates (Liquid) to price BRL CDS (Quanto)
- We must scale hazard rates for Quanto CDS pricing and instead use  $\lambda^{Quanto}$

# Credit Default Swaps – Mathematics of Quanto Hazard Rates I

- Consider a default process  $D(t) = 1\{\tau < t\}$  for default time  $\tau$
- Now  $D(t)$  is not a martingale, so we apply a compensator to create a Martingale process,  $M(t)$
- The compensator removes the predictable drift from the  $D(t)$  process
- Here the compensator term represents expected defaults over the time-period  $(0, t)$

$$M(t) = D(t) - \int_0^t (1 - D(s))\lambda(s)ds$$

or

$$dM(t) = dD(t) - (1 - D(t))\lambda(t)dt(7)$$

## Credit Default Swaps – Mathematics of Quanto Hazard Rates II

- The Quanto Martingale process,  $M^Q(t)$  can be computed using the Girsanov theorem to change to Quanto probability measure. Given a Radon-Nikodym derivative  $R(t)$  we have,

$$\begin{aligned}dM^Q(t) &= dM(t) - \frac{dD(t), R(t)}{R(t)} \\&= dM(t) - d(D(t), \gamma D(t)) \\&= dM(t) - (1 - D(t))\gamma\lambda(t)\end{aligned}$$

- Substituting (7) for  $dM(t)$  gives,

$$dM^Q(t) = dD(t) - (1 - D(t)) \underbrace{(1 + \gamma)\lambda(t)}_{= \lambda^Q(t)} dt \quad (8)$$

- Comparing  $dM^Q(t)$  with  $dM(t)$  we can see the Quanto hazard rate process is given by,

$$\lambda^Q = (1 + \gamma)\lambda \quad (9)$$

## PART TWO – PRICING & PRACTICE

### Case Studies Quanto Credit Default Swaps



# Eurozone Case Study – Quanto Effect on CDS Spreads

## Eurozone FX Crash Risk

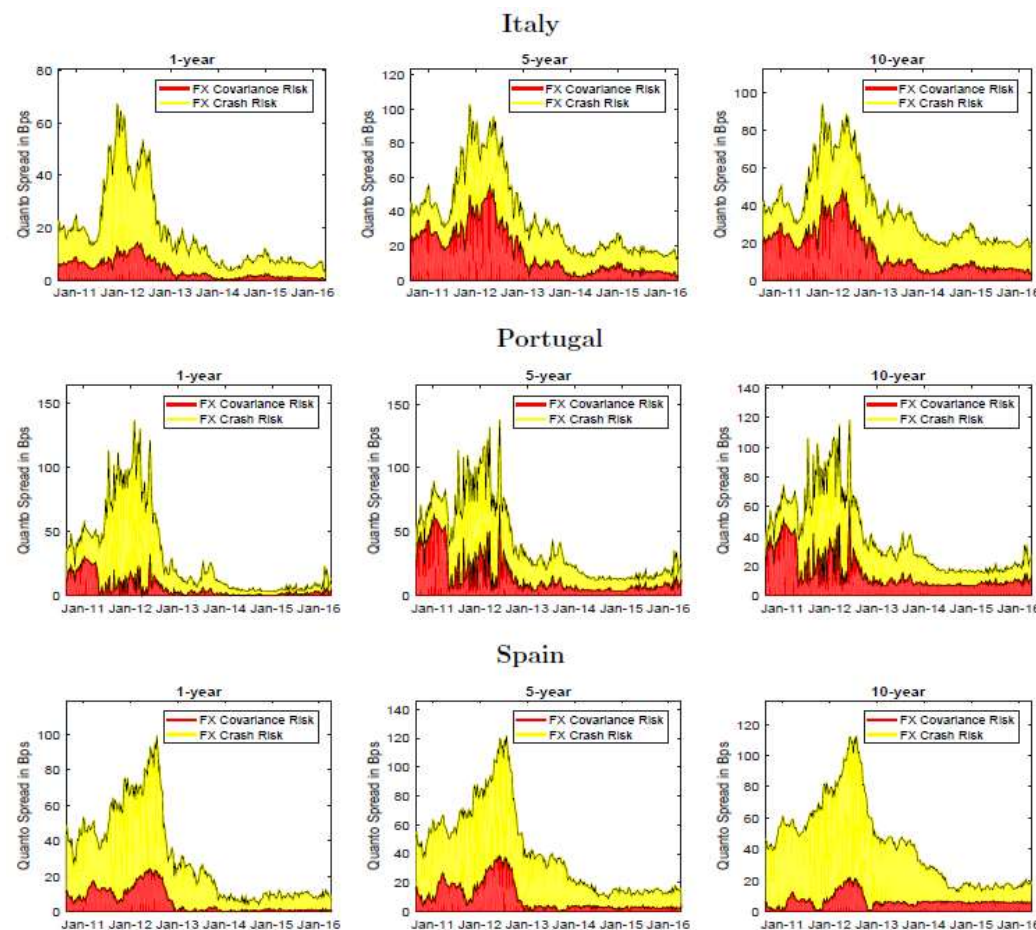
- Varies depending on credit regime & market distress
- On average accounts for ca. 10 bps of the Quanto Basis

## Eurozone FX Covariance

- Similar to FX crash risk, covariance risk of ca. 10 bps
- Covariance risk grows with CDS maturity

## Eurozone Quanto Basis

- Combining FX Crash and Covariance Risk
- Total Quanto Basis  $\approx 20$  bps
- i.e. USD CDS Spread  $>$  EUR CDS Spread



Source: Lando & Nielson (2018)

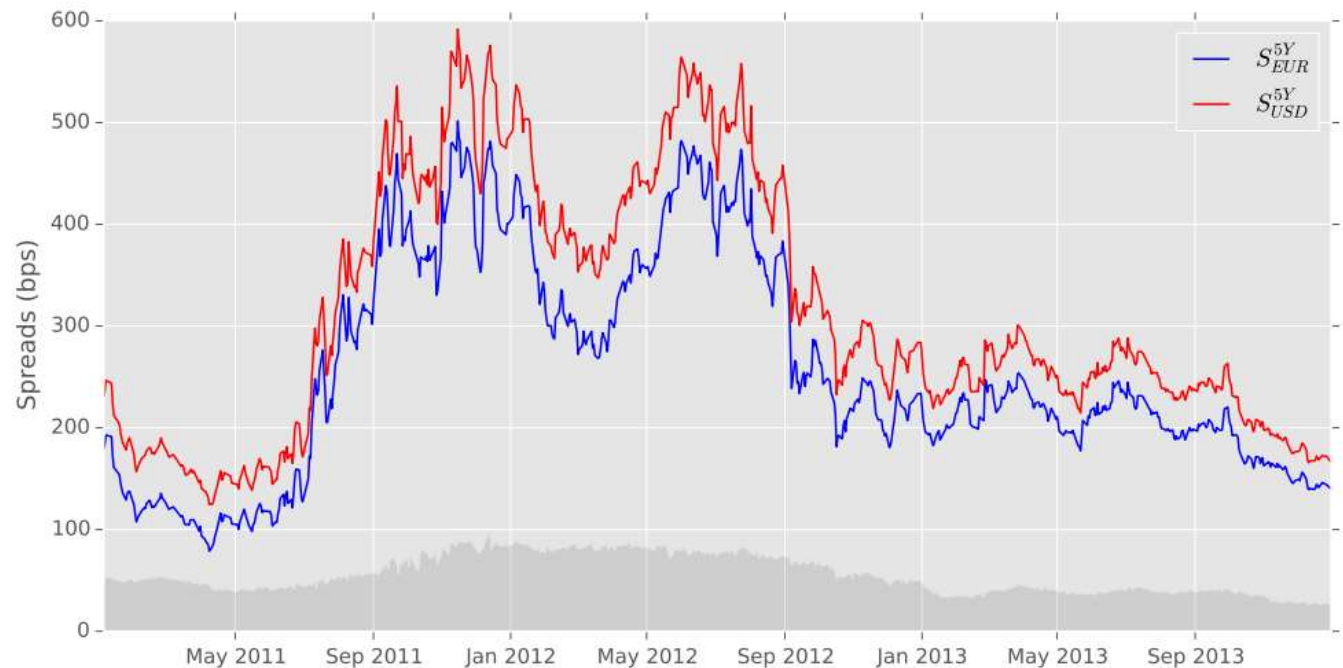
# Italy Case Study – CDS Par Spreads EUR vs USD

## Quanto Par Spreads

- Typically, Quanto CDS spreads are lower than the liquid currency CDS in USD

## Italy CDS Spreads: EUR vs USD

- Italy CDS spreads in EUR (Quanto) are lower than those in USD by ca. 20 bps



Italy CDS Spreads, Source: Brigo (2016) – Quant Summit Europe

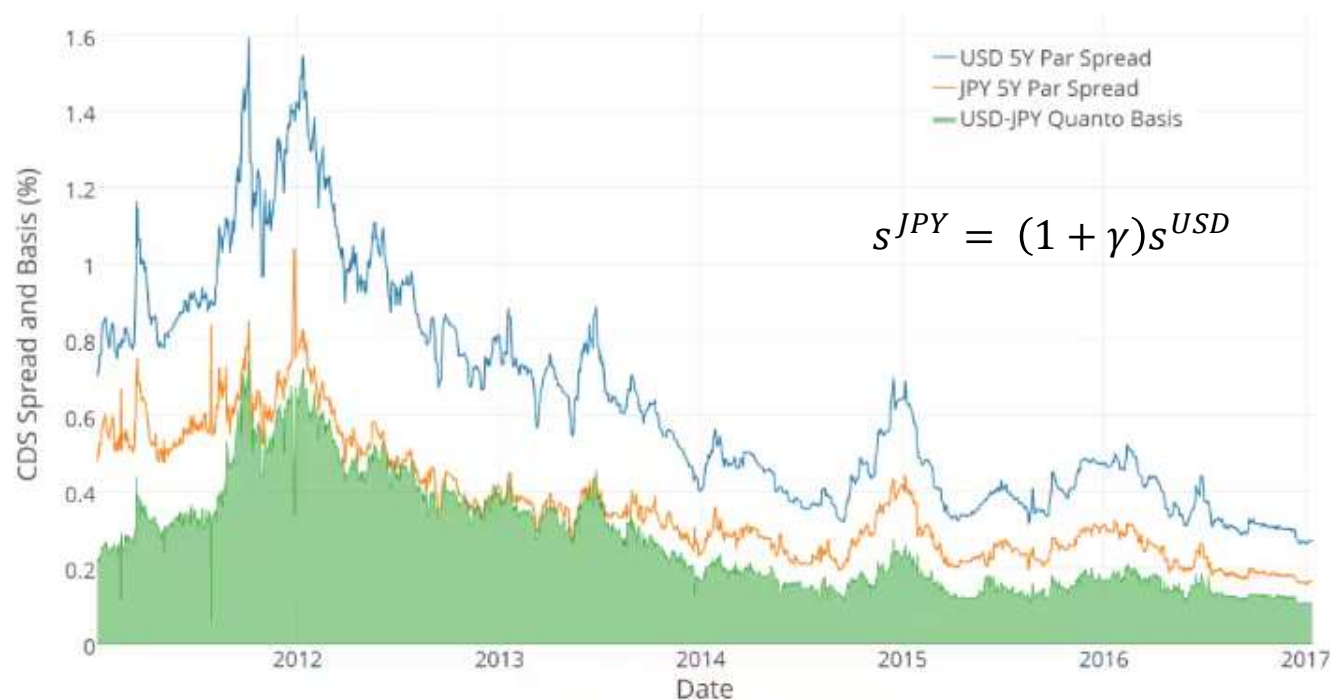
# Japan Case Study – Sovereign Quanto Basis Spread

## Quanto Basis

- The difference between USD and Quanto CDS Spreads is called the **Quanto Basis**

## Japan Sovereign CDS Spreads

- Japan CDS spreads in JPY (Quanto) are lower than those in USD by ca. 20 bps

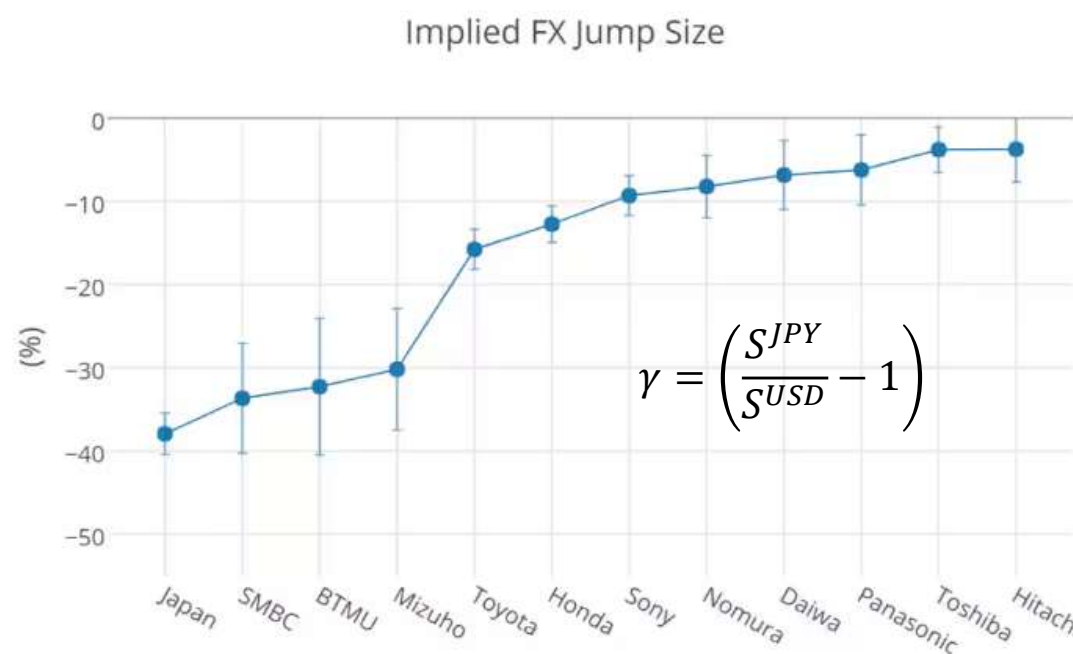


Japan Sovereign CDS Spreads, Source: S&P Global (2018)

# Japan Case Study – Corporate Quanto Basis

## How to Imply FX Jump Sizes and Quanto Basis for Corporates?

- In Japan corporate CDS market, significant **negative FX Jumps** (JPY Devaluation) priced in.
- FX jump sizes are strongly correlated with **systemic importance** of corporate
- There is evidence of similar FX jump sizes across industry groups. Suggests possibility of building **sector basis curves**



Japan Corporate Implied FX Jump Sizes, Source: S&P Global (2018)

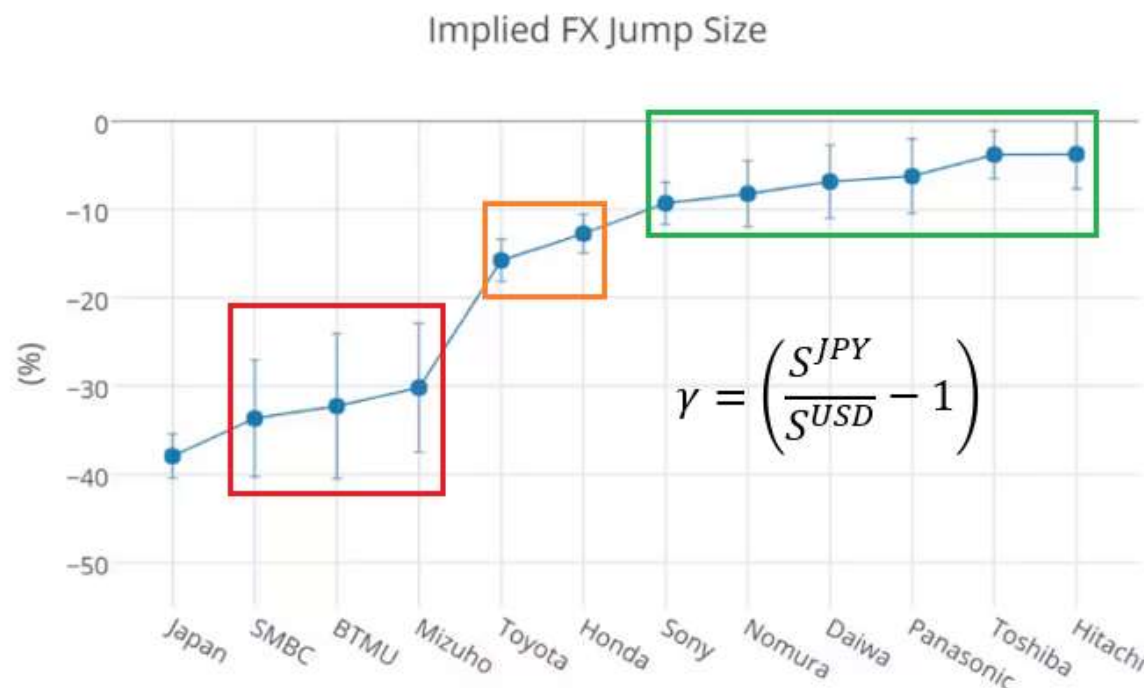
# Japan Case Study – Industry Sector Implied FX Jump Sizes

## Sector Basis Curves

There is evidence of similar FX jump sizes across industry groups. Jump sizes correlated to **systemic importance**.

## JPY Implied Jump Sizes by Sector

- JPY Sovereign -40%
  - JPY Banking Sector -35%
  - JPY Automotive Sector -15%
- (Automotive sector constitutes 90% of total GDP!)
- JPY Large Corporates -10%
  - JPY Mid-Small Corporates -5%



JPY Implied FX Jump Sizes by Sector, Source: S&P Global (2018)



# Brazil Case Study – Quanto CDS & Quanto Basis

## Brazil Sovereign CDS Spreads (BRL)

- Consider an FX jump of -40%
- CDS spreads in BRL (Quanto) would be lower than those in USD by ca. 71 bps

### CDS Trade Parameters & Price

BuyOrSellProtection	BUY	
BuySellIndicator, $\phi$	1	
Notional, N	10,000,000	
HazardRate, $\lambda^{\text{Domestic CCY}}$	2.2947%	
RecoveryRate, R	25.00%	Spread (bps)
CreditSpread, s	1.0000%	100.00
PremiumFrequency	QUARTERLY	
PremiumYearFraction, $\Delta t$	0.2500	
Maturity, years	5.0000	
ZeroRate, z	14.00%	

### Quanto Parameters

FX Jump on Default, $J_{FX}$	-40.00%	
Correlation, $\rho_{C,FX}$	-0.25	
Credit Volatility, $\sigma_C$	25.00%	
FX Volatility, $\sigma_{FX}$	10.00%	Lambda Adj
HazardRate, $\lambda^{\text{Quanto CCY}}$	1.3553%	-0.9394%

$$\lambda^{\text{Quanto}}(t) = (1 + J_{FX}) \left( 1 + \frac{1}{2} \rho_{C,FX} \sigma_C \sigma_{FX} t \right) \lambda^{\text{Domestic}}(t)$$

Present Value (PV)	Without Adj	With Adj	+/-
CreditDefaultSwap	242,273	5,648	-236,626
PremiumLeg	-335,060	-342,377	-7,316
AccruedInterest	-964	-581	383
ProtectionLeg	578,297	348,605	-229,692

Par CDS Rate	Without Adj	With Adj	+/-	Quanto Basis
Par CDS Rate, p	1.7231%	1.0165%	-0.7066%	70.66
ProtectionPV - AccruedInterest	-577,334	-348,024	229,309	
RiskyFixedAnnuity	-33,506,024	-34,237,674	-731,650	

## What-If Analysis

Quanto Basis		FX Jump on Default						
70.66		0.00%	-5.00%	-10.00%	-20.00%	-30.00%	-40.00%	-50.00%
Correlation	1.00	-10.80	-1.62	7.56	25.91	44.25	62.58	80.89
	0.75	-8.10	0.94	9.99	28.07	46.14	64.19	82.24
	0.50	-5.40	3.51	12.42	30.23	48.02	65.81	83.58
	0.25	-2.70	6.07	14.85	32.38	49.91	67.43	84.93
	0.00	0.00	8.64	17.28	34.54	51.80	69.04	86.28
	-0.25	2.70	11.20	19.70	36.70	53.68	70.66	87.62
	-0.50	5.40	13.77	22.13	38.86	55.57	72.27	88.97
	-0.75	8.10	16.33	24.56	41.01	57.46	73.89	90.32
	-1.00	10.80	18.89	26.99	43.17	59.34	75.51	91.66



# Brazil Case Study – Corporate Implied FX Jumps

## Brazil Implied FX Jump (Estimates)

- Sovereign -40%
- Banking Sector -35%
- Large Corporates -10%
- Mid-Small Corporates -5%

Can be implied from Direct Market Quotes  
or Sector Quanto Basis Curves

$$\gamma = \left( \frac{S^{JPY}}{S^{USD}} - 1 \right)$$

## What-If Analysis

	Quanto Basis	FX Jump on Default			
		MID CORP	LARGE CORP	BANKS	SOVEREIGN
Correlation	70.66	-5.00%	-10.00%	-35.00%	-40.00%
	1.00	-1.62	7.56	44.25	62.58
	0.75	0.94	9.99	46.14	64.19
	0.50	3.51	12.42	48.02	65.81
	0.25	6.07	14.85	49.91	67.43
	0.00	8.64	17.28	51.80	69.04
	-0.25	11.20	19.70	53.68	70.66
	-0.50	13.77	22.13	55.57	72.27
	-0.75	16.33	24.56	57.46	73.89
	-1.00	18.89	26.99	59.34	75.51

# Brazil Case Study – Quanto CDS & Quick Rule of Thumb

## Brazil Implied FX Jump (Estimates)

- Sovereign -40%
- Banking Sector -35%
- Large Corporates -10%
- Mid-Small Corporates -5%

## BRL Par Spreads: As a Multiple of USD CDS

- Sovereign 60%
- Banking Sector 55%
- Large Corporates 90%
- Mid-Small Corporates 95%

Rule of Thumb:  $S^{BRL} = (1 + \gamma)S^{USD}$



**Example:** BRL CDS(Sovereign)  $\approx$  (1 - 40%) USD CDS

$$= 60\% \times \text{USD CDS}$$

$$= 60\% \times 172.10\text{bps} = 103.26 \text{ bps}$$

# References

- Bloomberg (2016), Pricing Default-Contingent Contracts with Deterministic Credit
- Brigo D., Pede N. and Petrelli A. (2018), Multi Currency Credit Default Swaps: Quanto Effects & FX Devaluation Jumps
- Itkin A., Shcherbakov V. and Veygman A. (2019), A New Model for Pricing Quant Credit Default Swaps
- Lando D. and Nielsen A. (2018), Quanto CDS Spreads
- Li M., Mercurio F. and Resnick S. (2018), The GARCH linear SDE: Explicit formulas and the pricing of a quanto CDS

Have questions or want further info?

## Contact

LinkedIn: [www.linkedin.com/in/nburgessx](https://www.linkedin.com/in/nburgessx)