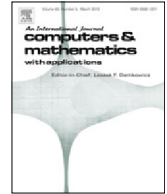




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Efficient and fast numerical method for pricing discrete double barrier option by projection method

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ABSTRACT

In this paper, we introduce a new and considerably fast numerical method based on projection method in pricing discrete double barrier option. According to the Black–Scholes framework, the price of option in each monitoring dates is the solution of well-known partial differential equation that can be expressed recursively upon the heat equation solution. These recursive solutions are approximated by projection method and expressed in operational matrix form. The most important advantage of this method is that its computational time is nearly fixed against monitoring dates increase. Afterward, in implementing projection method we use Legendre polynomials as an orthogonal basis. Finally, the numerical results show the validity and efficiency of presented method in comparison with some others.

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1. Introduction

Option pricing is one of the most interesting problems in mathematical finance. This field of study is being investigated from both theoretical and practical point of view by many researchers. Barrier options are one of the most applicable types of exotic derivatives in the financial market. As a description, a knock-out double barrier option is one that is deactivated when the price of underlying asset touches each of two predetermined barriers before the expiry date. According to the way of how the underlying asset price is monitoring, there are two types of barrier option, namely continuous and discrete. Discrete barrier options which we especially concern are those that the price of underlying asset is monitored at the specific dates, for example daily, weekly or monthly. Barrier options have been investigated by many researchers over the two past decades. Kamrad and Ritchken [1] applied the standard trinomial tree method and also Kwok [2] used the binomial and trinomial trees for pricing path-dependent options. Dai and Lyuu [3] introduced bino-trinomial tree method for pricing barrier options. In [4], the adaptive mesh model and its special case (AMM 8) were implemented and the quadrature methods QUAD K20 and QUAD K30 were proposed in [5]. An analytical solution for single barrier option based on Z-transform was driven by Fusai in [6]. A numerical solution for discrete barrier options based on combination of quadrature method and interpolation procedure is presented in [7]. Milev and Tagliani [8] presented a numerical algorithm for pricing discrete double barrier options. Golbabai et al. [9] applied finite element method for discrete double barrier option pricing. Farnoosh et al. [10,11] provided algorithms for pricing discrete single and double barrier options that are viable even for case of time-dependent parameters. Yoon and Kim [12] priced options under a stochastic interest rate by double Mellin transform. In [13], the

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Mellin transform is applied for pricing options with time-dependent parameters and discontinuous payoff. Fusai et al. [14] presented a method based on Wiener–Hopf factorization for pricing discrete exotic options.

The projection method is one of the most important applicable numerical methods that is implemented vastly for solving problems that arise in science, engineering, applied mathematics and especially in mathematical finance (see [15–17]). For example Galerkin, Petrov–Galerkin and collocation method are all projection methods that are widely used for solving integral equations, ordinary differential equations and partial differential equations. Projection operators, which have been investigated in functional analysis, are useful in discussing projection methods. In this article we present a numerical algorithm for pricing discrete double barrier options based on projection operators in general form.

This article is organized as follows. In Section 2, the Black–Scholes model and its corresponding partial differential equation for pricing discrete double barrier option are explained. Afterward by taking some well-known transformations, the problem is reduced to heat equation. Next the solution is driven as a recursive formula. A numerical approximation method based on projection operator is presented for pricing discrete double barrier option in Section 3. In Section 4, we use Legendre polynomials as an orthogonal basis in implementing projection method. Finally numerical results are given that demonstrate the proposed method is comparable with other numerical methods.

2. The pricing model

In this paper we assume that the stock price is accorded to prominent process, geometric Brownian motion, as follows:

$$dS_t = rS_t dt + \sigma S_t dB_t,$$

with initial stock price S_0 , where coefficients r and σ are the risk-free rate and the volatility respectively. As mentioned in Section 1, we concern in pricing knock-out discrete double barrier call option on stock, i.e. a call option that becomes worthless if the stock price hits lower or upper barrier at the specific monitoring dates $0 = t_0 < t_1 < \dots < t_M = T$. According to the well-known Black–Scholes framework, the price of discretely monitored double barrier call option as a function of stock price at time $t \in (t_{m-1}, t_m)$, namely $\mathcal{P}(S, t, m-1)$, is obtained from forward solving the following partial differential equations [18]

$$-\frac{\partial \mathcal{P}}{\partial t} + rS \frac{\partial \mathcal{P}}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \mathcal{P}}{\partial S^2} - r\mathcal{P} = 0, \quad (1)$$

with the following initial conditions:

$$\begin{aligned} \mathcal{P}(S, t_0, 0) &= (S - E) \mathbf{1}_{(\max(E, L) \leq S \leq U)}, \\ \mathcal{P}(S, t_m, 0) &= \mathcal{P}(S, t_m, m-1) \mathbf{1}_{(L \leq S \leq U)}; \quad m = 1, 2, \dots, M-1, \end{aligned}$$

where $\mathcal{P}(S, t_m, m-1) := \lim_{t \rightarrow t_m} \mathcal{P}(S, t, m-1)$. Also the constants E, L and U are exercise price, lower barrier and upper barrier respectively. Following simple change of variable is performed in two steps. At first we define function $C(z, t, m)$ as below:

$$C(z, t, m) := \mathcal{P}(S, t, m),$$

where $z = \ln\left(\frac{S}{L}\right)$; $E^* = \ln\left(\frac{E}{L}\right)$; $\mu = r - \frac{\sigma^2}{2}$; $\theta = \ln\left(\frac{U}{L}\right)$ and $\delta = \max\{E^*, 0\}$. Then the partial differential equation (1) and its initial condition are changed into:

$$\begin{aligned} -C_t + \mu C_z + \frac{\sigma^2}{2} C_{zz} &= rC, \\ C(z, t_0, 0) &= L(e^z - e^{E^*}) \mathbf{1}_{(\delta \leq z \leq \theta)}, \\ C(z, t_m, m) &= C(z, t_m, m-1) \mathbf{1}_{(0 \leq z \leq \theta)}; \quad m = 1, 2, \dots, M-1. \end{aligned} \quad (2)$$

As a second step we use the following transformation:

$$C(z, t_m, m) = e^{\alpha z + \beta t} h(z, t, m),$$

where

$$\alpha = -\frac{\mu}{\sigma^2}; \quad c^2 = -\frac{\sigma^2}{2}; \quad \beta = \alpha\mu + \alpha^2 \frac{\sigma^2}{2} - r.$$

Therefore, the partial differential equation (2) and its initial condition are led to:

$$\begin{aligned} -h_t + c^2 h_{zz} &= 0, \\ h(z, t_0, 0) &= L e^{-\alpha z} (e^z - e^{E^*}) \mathbf{1}_{(\delta \leq z \leq \theta)}; \quad m = 0, \\ h(z, t_m, m) &= h(z, t_m, m-1) \mathbf{1}_{(0 \leq z \leq \theta)}; \quad m = 1, \dots, M-1. \end{aligned}$$

The resulted partial differential equation is known as heat equation that could be solved analytically, see e.g [19];

$$h(z, t, m) = \begin{cases} L \int_{\delta}^{\theta} k(z - \xi, t) e^{-\alpha \xi} (e^{\xi} - e^{E^*}) d\xi; & m = 0 \\ \int_0^{\theta} k(z - \xi, t - t_m) h(\xi, t_m, m - 1) d\xi; & m = 1, 2, \dots, M - 1, \end{cases}$$

where

$$k(z, t) = \frac{1}{\sqrt{4\pi c^2 t}} e^{-\frac{z^2}{4c^2 t}}. \quad (3)$$

We consider monitoring dates equally spaced, i.e. $t_m = m\tau$ where $\tau = \frac{T}{M}$. In this way $h(z, t_m, m - 1)$ is a function of two variables z, m . By defining $f_m(z) := h(z, t_m, m - 1)$, the following relations are obtained:

$$f_1(z) = \int_0^{\theta} k(z - \xi, \tau) f_0(\xi) d\xi, \quad (4)$$

$$f_m(z) = \int_0^{\theta} k(z - \xi, \tau) f_{m-1}(\xi) d\xi; \quad m = 2, 3, \dots, M; \quad (5)$$

where

$$f_0(\xi) = L e^{-\alpha z} (e^z - e^{E^*}) \mathbf{1}_{(\delta \leq \xi \leq \theta)}.$$

3. Pricing by projection method

Let $L^2[0, \theta]$ be space of all square integrable functions on $[0, \theta]$ and $\{\phi_i\}_{i=0}^{\infty}$ be a basis of $L^2[0, \theta]$. Now, we define operator $\mathcal{K} : L^2[0, \theta] \rightarrow L^2[0, \theta]$ as follows:

$$\mathcal{K}(f)(z) := \int_0^{\theta} \kappa(z - \xi, \tau) f(\xi) d\xi \quad (6)$$

where κ is defined in (3). Because κ is a continuous function, \mathcal{K} is a bounded linear compact operator on $L^2[0, \theta]$ [20,21]. According to the definition of operator \mathcal{K} , Eqs. (4) and (5) can be rewritten as below:

$$f_1 = \mathcal{K}f_0, \quad (7)$$

$$f_m = \mathcal{K}f_{m-1}; \quad m = 2, 3, \dots, M. \quad (8)$$

Definition 1. Let X be a linear space. Linear operator $P : X \rightarrow X$ is called projection operator if $P^2 = P$.

Let $X_n = \text{span}\{\phi_0, \phi_1, \dots, \phi_{d_n}\}$ be sequence of subspaces of $L^2[0, \theta]$ where $d_n < d_{n+1}$. Consider $P_n : L^2[0, \theta] \rightarrow X_n$ be a sequence of continuous projection operators. Then for any $f \in L^2[0, \theta]$ we have:

$$P_n(f) = \sum_{i=0}^{d_n} a_i \phi_i(z). \quad (9)$$

We denote $\tilde{f}_{m,n}, m \geq 1$ as follows:

$$\tilde{f}_{1,n} = P_n \mathcal{K}(f_0), \quad (10)$$

$$\tilde{f}_{m,n} = P_n \mathcal{K}(\tilde{f}_{m-1,n}) = (P_n \mathcal{K})^m(f_0), \quad m \geq 2, \quad (11)$$

where $P_n \mathcal{K}$ is defined as below:

$$(P_n \mathcal{K})(f) = P_n(\mathcal{K}(f)).$$

If P_n is chosen appropriately then f_m could be approximated by $\tilde{f}_{m,n}$ properly. Especially, if continuous projection operators P_n converge pointwise to identity operator I , i.e.

$$\forall f \in L^2[0, \theta] \quad \lim_{n \rightarrow \infty} \|P_n(f) - f\| = 0,$$

then operator $P_n \mathcal{K}$ is also a compact operator and we have (see [22])

$$\lim_{n \rightarrow \infty} \|P_n \mathcal{K} - \mathcal{K}\| = 0. \quad (12)$$

With the aid of the following inequality

$$\|(P_n \mathcal{K})^m - \mathcal{K}^m\| \leq \|(P_n \mathcal{K})\| \|(P_n \mathcal{K})^{m-1} - \mathcal{K}^{m-1}\| + \|P_n \mathcal{K} - \mathcal{K}\| \|\mathcal{K}\|^{m-1} \quad (13)$$

and relation (12) by induction we obtain

$$\lim_{n \rightarrow \infty} \|(P_n \mathcal{K})^m - \mathcal{K}^m\| = 0. \quad (14)$$

Therefore the following convergence relation is yielded:

$$\|\tilde{f}_{m,n} - f_m\| = \|(P_n \mathcal{K})^m(f_0) - \mathcal{K}^m(f_0)\| \leq \|(P_n \mathcal{K})^m - \mathcal{K}^m\| \|f_0\| \rightarrow 0 \quad \text{as } n \rightarrow \infty. \quad (15)$$

Considering that each $\tilde{f}_{m,n} \in X_n$, we arrive at

$$\tilde{f}_{m,n} = \sum_{j=0}^{d_n} a_{mj} \phi_j(z) = \Phi_n' F_m,$$

where $F_m = [a_{m0}, a_{m1}, \dots, a_{md_n}]'$ and $\Phi_n = [\phi_0, \phi_1, \dots, \phi_{d_n}]'$. From Eq. (11) we obtain

$$\tilde{f}_{m,n} = (P_n \mathcal{K})^{m-1}(\tilde{f}_{1,n}). \quad (16)$$

Because X_n is a finite dimensional space, linear operator $P_n \mathcal{K}$ on X_n reduces to a $d_n \times d_n$ matrix K . So Eq. (16) can be written as the following matrix operator form

$$\tilde{f}_{m,n} = \Phi_n' K^{m-1} F_1. \quad (17)$$

For computing option price by (17), it is enough to calculate the matrix operator K and the vector F_1 . Therefore the complexity of our algorithm is $\mathcal{O}(n^2)$.

In order to obtain the matrix K , for any y in the form of

$$y = \sum_{j=0}^{d_n} a_j \phi_j(z),$$

we have

$$\mathcal{K}(y)(z) = \sum_{j=0}^{d_n} a_j \int_0^\theta \kappa(z - \xi, \tau) \phi_j(\xi) d\xi, \quad (18)$$

and the above relation can be written in the following matrix form

$$\mathcal{K}(y) = [\mathcal{K}\phi_0, \mathcal{K}\phi_1, \dots, \mathcal{K}\phi_{d_n}] \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{d_n} \end{bmatrix},$$

so we get

$$P_n \mathcal{K}(y) = [P_n \mathcal{K}\phi_0, P_n \mathcal{K}\phi_1, \dots, P_n \mathcal{K}\phi_{d_n}] \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{d_n} \end{bmatrix}.$$

Also each $P_n \mathcal{K}\phi_j$ can be written in the form

$$P_n \mathcal{K}\phi_j = \sum_{i=0}^{d_n} k_{ij} \phi_i(z).$$

Finally, if we define $K = (k_{ij})_{d_n \times d_n}$, then Eq. (18) is expressed as

$$P_n \mathcal{K}(y) = \Phi_n' K \vec{a},$$

where $\vec{a} = [a_0, a_1, \dots, a_{d_n}]'$.

4. Pricing by Legendre polynomial

In this section, we apply orthogonal projection, among different types of the projection, that is based on Legendre polynomials. The Legendre polynomials are defined with the following recurrence formula:

$$p_i(x) = xp_{i-1}(x) + \left(\frac{i-1}{i}\right)(xp_{i-1}(x) - p_{i-2}(x)),$$

where $p_0(x) = 1$ and $p_1(x) = x$. The $\{p_i(x)\}_{i=0}^{\infty}$ is an orthogonal basis for $L^2[-1, 1]$. If we define $\tilde{p}_i(x) := \sqrt{w_i^{-1}} p_i\left(\frac{2}{\theta}x - 1\right)$ where

$$w_i = \frac{\theta}{2i+1},$$

then $\{\tilde{p}_i(x)\}_{i=0}^{\infty}$ is an orthonormal basis for $L^2[0, \theta]$. Now, consider $\Pi_n = \text{span} \{\tilde{p}_i(x)\}_{i=0}^n$ be the space of all polynomials with degrees less than or equal to n and $P_n : L^2[0, \theta] \rightarrow \Pi_n$ be orthogonal projection operator, that is defined as follows:

$$\forall f \in L^2[0, \theta] \quad P_n(f) = \sum_{i=0}^n \langle f, \tilde{p}_i(x) \rangle \tilde{p}_i(x) \quad (19)$$

where $\langle \cdot, \cdot \rangle$ indicates the inner product. Because the projection operators P_n , that is defined above, converge pointwise to identity operator I , the relation (15) holds. According to the orthogonal projection (19) Φ_n , F_1 and K in (17) are simply obtained as follows:

$$\begin{aligned} \Phi_n &= [\tilde{p}_0(x), \tilde{p}_1(x), \dots, \tilde{p}_n(x)]' \\ F_1 &= [a_{10}, a_{11}, \dots, a_{1n}]', \quad K = (k_{ij})_{(n+1) \times (n+1)} \end{aligned}$$

where

$$\begin{aligned} a_{1i} &= \int_0^\theta \int_\delta^\theta \tilde{p}_i(\eta) \kappa(\eta - \xi, \tau) f_0(\xi) d\xi d\eta, \quad 0 \leq i \leq n, \\ k_{ij} &= \int_0^\theta \int_0^\theta \tilde{p}_{i-1}(\eta) \tilde{p}_{j-1}(\xi) \kappa(\eta - \xi, \tau) d\xi d\eta, \quad 1 \leq i, j \leq n+1. \end{aligned}$$

Therefore, the price of knock-out discrete double barrier option can be approximated as follows:

$$\mathcal{P}(S_0, t_M, M-1) \simeq e^{\alpha z_0 + \beta t} \tilde{f}_{M,n}(z_0) \quad (20)$$

where $z_0 = \log\left(\frac{S_0}{L}\right)$ and $\tilde{f}_{M,n}$ from (17). Here we emphasize that the matrix form of relation (17) provides nearly invariant computational time against monitoring dates increase.

5. Numerical result

In current section, the presented method in the previous section for pricing knock-out call discrete double barrier option is compared with some other methods in the following examples. The numerical results are obtained from relation (20) with n Legendre basis functions. The source code of this method was written in MATLAB 2014 on a 3.2 GHz Intel Core i5 PC with 8 GB RAM.

Example 1. In this example we consider pricing knock-out call discrete double barrier option with the risk-free rate $r = 0.05$, volatility $\sigma = 0.25$, the maturity time $T = 0.5$, spot price $S_0 = 100$, strike price $E = 100$, upper barrier $U = 120$ and lower barriers $L = 80, 90, 95, 99, 99.5$. In Table 1, the numerical results of the presented method with some different ones in [8,23] are compared for various numbers of monitoring dates. The benchmark as accurate value is relevant to quadrature method QUAD-K200. As one can see in Table 1, we obtain the benchmark values with 16 basis functions in less than one second that shows the efficiency and accuracy of presented method in comparison with others. Furthermore, we can see that CPU time of our method increases insignificantly against increases of the monitoring dates.

Table 1Double barrier option pricing of [Example 1](#): $T = 0.5$, $r = 0.05$, $\sigma = 0.25$, $S_0 = 100$, $E = 100$.

M	L	Presented method ($n = 16$)	Milev (200)	Milev (400)	Trinomial	Quad-K20	Quad-K30	AMM-8	Benchmark
5	80	2.4499	–	–	2.4439	2.4499	2.4499	2.4499	2.4499
	90	2.2028	–	–	2.2717	2.2028	2.2028	2.2027	2.2028
	95	1.6831	1.6831	1.6831	1.6926	1.6831	1.6831	1.6830	1.6831
	99	1.0811	1.0811	1.0811	0.3153	1.0811	1.0811	1.0811	1.0811
	99.9	0.9432	0.9432	0.9432	–	0.9432	0.9432	0.9433	0.9432
CPU		0.52 s	1 s	5 s					
25	80	1.9420	–	–	1.9490	1.9420	1.9420	1.9419	1.9420
	90	1.5354	–	–	1.5630	1.5354	1.5354	1.5353	1.5354
	95	0.8668	0.8668	0.8668	0.8823	0.8668	0.8668	0.8668	0.8668
	99	0.2931	0.2931	0.2931	0.3153	0.2931	0.2931	0.2932	0.2931
	99.9	0.2023	0.2023	0.2023	–	0.2023	0.2023	0.2024	0.2023
CPU		0.54 s	8 s	30 s					
125	80	1.6808	–	–	1.7477	1.6803	1.6808	1.6807	1.6808
	90	1.2029	–	–	1.2370	1.2026	1.2029	1.2028	1.2029
	95	0.5532	0.5528	0.5531	0.5699	0.5531	0.5532	0.5531	0.5532
	99	0.1042	0.1042	0.1042	0.1201	0.1042	0.1042	0.1043	0.1042
	99.9	0.0513	0.0513	0.0513	–	0.0513	0.0513	0.0513	0.0513
CPU		0.59 s	35 s	150 s					
250	80	1.6165	–	–	1.8631	1.8581	1.6164	1.6163	1.6165
	90	1.1237	–	–	1.2334	1.1234	1.1237	1.1236	1.1237
	95	0.4867	–	–	0.5148	0.4864	0.4867	0.4867	0.4867
	99	0.0758	–	–	0.0772	0.0758	0.0758	0.0759	0.0758
	99.9	0.0311	–	–	–	0.0311	0.0311	0.0311	0.0311
CPU		0.64 s	–	–					

Table 2Double barrier option pricing of [Example 2](#): $T = 0.5$, $r = 0.05$, $\sigma = 0.25$, $E = 100$, $U = 110$ and $L = 95$.

s_0	Presented method $n = 16$	Crank–Nicolson (1000)	Milev (200)	Milev (1000)	MC (st.error) 10^7 path
95	0.174498	0.1656	0.174503	0.174498	–
95.0001	0.174499	≈ 0.1656	0.174501	0.174499	0.17486 (0.00064)
95.5	0.182428	0.1732	0.182429	0.182428	0.18291 (0.00066)
99.5	0.229349	0.2181	0.229356	0.229349	0.22923 (0.00073)
100	0.232508	0.2212	0.232514	0.232508	0.23263 (0.00036)
100.5	0.234972	0.2236	0.234978	0.234972	0.23410 (0.00073)
109.5	0.174462	0.1658	0.174463	0.174462	0.17426 (0.00063)
109.9999	0.167394	≈ 0.1591	0.167399	0.167394	0.16732 (0.00062)
110	0.167393	0.1591	0.167398	0.167393	–
CPU		Minutes	1 s	39 s	Hundred sec

Example 2. The parameters of the second example are similar to [Example 1](#) but we consider upper and lower barriers as $U = 110$ and $L = 95$ respectively. In [Table 2](#) option price for different spot prices is evaluated and compared with Milev numerical algorithm [8], Crank–Nicholson [24] and Monte Carlo (MC) method with 10^7 simulations in [25]. Also the standard error of the Monte Carlo method is presented in [Table 2](#). The numerical results in [Table 2](#) show that our method is successful to achieve the accuracy of Milev method with 1000 grid points in less than one second that shows the efficiency of our algorithm.

Example 3. Because the probability that the stock price hits upper barrier when $U \geq 2E$ is negligible, so discrete single down-and-out call option can be estimated by double ones with upper barrier bigger than $2E$ (see [8]). For $r = 0.1$, volatility $\sigma = 0.2$, the maturity time $T = 0.5$, spot price $S_0 = 100$, strike price $E = 100$ and lower barriers $L = 95, 99.5, 99.9$ the price, the Delta and the Gamma for a discrete single down-and-out call option are estimated by double ones with $U = 2.5E$. The numerical results are shown in [Table 3](#) and compared with analytical Fusai formula in [6] and Monte Carlo (MC) method with 10^8 simulations in [26] that verify the validity of our method in this case.

6. Conclusion and remarks

In this article, we have applied projection method for pricing discrete double barrier options. In [Section 3](#) we achieved to obtain a matrix relation (17) for solving this problem. Against other mentioned methods, increase of computational time

Table 3

The price, the Delta and the Gamma of single barrier option in Example 3: $T = 0.5$, $r = 0.1$, $\sigma = 0.2$, $S_0 = 100$, $E = 100$, $U = 250$.

L	M	Presented method ($n = 32$)	Presented method ($n = 64$)	Fusai analytical method (IR 17)	MC (st.error) 10^8 path
Option price					
95	25	6.63155	6.63156	6.63156	6.63204 (0.0009)
99.5	25	3.35553	3.35558	3.35558	3.35584 (0.00068)
99.9	25	3.00891	3.00887	3.00887	3.00918 (0.00064)
95	125	6.16798	6.16864	6.16864	6.16879 (0.00088)
99.5	125	1.95966	1.9613	1.9613	1.96142 (0.00053)
99.9	125	1.51116	1.51021	1.51068	1.5105 (0.00046)
Option delta					
95	25	0.92905	0.92903	0.92912	0.92906 (0.00031)
99.5	25	1.07109	1.07106	1.07115	1.07118 (0.00027)
99.9	25	1.03741	1.03748	1.03757	1.03755 (0.00027)
95	125	0.98984	0.98954	0.98963	0.98889 (0.00070)
99.5	125	1.27353	1.27361	1.27373	1.27368 (0.00044)
99.9	125	1.16636	1.16550	1.165562	1.16572 (0.00043)
Option gamma					
95	25	-0.01275	-0.0128	-0.01277	-0.01285 (0.00035)
99.5	25	0.12270	0.12272	0.12274	0.12274 (0.00015)
99.9	25	0.14890	0.14825	0.14827	0.14824 (0.00015)
95	125	-0.02035	-0.02081	-0.02078	-0.02040 (0.00078)
99.5	125	0.26387	0.26069	0.26073	0.26078 (0.00054)
99.9	125	0.37391	0.39294	0.39302	0.39297 (0.00053)
CPU	11 s		27 s		

is negligible when number of monitoring dates increases. In fact, computational time of this method depends essentially on the number of basis functions that we use in expansion (9).

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