

# Precourse 3

---

## Problem 1

### List

#### Node

The `Node` class need to implement constructor:

```
data = value;
next = nullptr;
prev = nullptr;
```

The `nullptr` has type `std::nullptr_t`, which can implicitly converted to any pointer type, we shouldn't assign `next = NULL` or `prev = NULL` because it will cause ambiguity in overloaded function in this case:

```
f(int);
f(foo *);
```

#### Iterator

`iterator(Node *node) :`

This function will assign the attribute `current` to the node by

```
this->current = node;
```

`iterator::operator*():`

This function return the data in the current node of iterator, implement this by

```
return this->current->data;
```

`iterator::operator++():`

This function overrides prefix increment of iterator, corresponding to `++x`, which means increase value of `x` then assign it to other variable.

```
current = current->next;
return *this;
```

```
iterator::operator++(int):
```

This function overrides postfix increment of iterator, corresponding to `x++`, which means assign value of `x` first, then increases `x`.

```
auto temp = iterator (current);  
current = current->next;  
return temp;
```

```
operator!=(const iterator &other):
```

This function will compare 2 iterator to decide whether it point to the same node:

```
return this->current != other.current;
```

### list member function

```
list():
```

This is an empty constructor of list, need to point head, tail to `nullptr` and set `size_ = 0`.

```
this->head = nullptr;  
this->tail = nullptr;  
this->size_ = 0;
```

```
list(const list &other):
```

This is constructor from another list, firstly need to init empty list, then for each element of input list, use `push_back` to append value to new list.

```
new (this) list();  
this->size_ = 0;  
for(iterator it = other.cbegin(); it != other.cend() ; ++it){  
    this->push_back(*it);  
}
```

```
list(std::initializer_list<T> initList):
```

This is constructor from an array or a object has datatype that can implicitly convert to type `std::initializer_list`.

```

this->size_ = 0;
new (this) list();
for (auto it = initList.begin(); it!=initList.end(); ++it){
    this->push_back(*it);
}

```

`~list():`

This is destructor of a list, just simply call `clear()` and set `size_ = 0`.

```

this->clear();
this->size_ = 0;

```

`operator=(const list &other):`

This is implementation of operator `=`, the logic will be the same with constructor from another list, but need to return pointer to current list

```

this->clear();
this->size_ = 0;
for(iterator it = other.cbegin(); it != other.cend() ; it++){
    this->push_back(*it);
}
return *this;

```

`push_back(const T &value):`

Append value to the end of a list, this process requires create new Node, chain the new node to the end of list and point the `tail` to new node. Don't forget to increase the size by 1.

```

Node* temp = new Node(value);
if (this->size_ == 0){
    this->head = temp;
    this->tail = temp;
}
else{
    temp->prev = this->tail;
    this->tail->next = temp;
}
this->tail=temp;
this->size_ += 1;

```

`push_front(const T &value):`

This is the same logic with `push_back` unless it insert to the head of list.

#### `pop_back()`:

Remove the last element of a list, this function only executes when list is not empty. If the size of list is 1, need to point both `head` and `tail` to `nullptr`. Else, delete the last node and modify the second last point to `nullptr` and point `tail` to second last.

```
if (this->size_ > 0){
    if (this->size_ == 1){
        delete this->head;
        this->head = nullptr;
        this->tail = nullptr;
        this->size_ --;
    }
    else{
        Node * second_last = this->tail->prev;
        delete second_last->next;
        this->tail = second_last;
        second_last->next = nullptr;
        this->size_ --;
    }
}
```

#### `pop_front()`:

The logic will be the same with `pop_back`.

#### `emplace_back(T &&value)`:

This function will use the `push_back`.

```
this->push_back(value);
```

#### `emplace_front(T &&value)`:

This function will use the `push_front`.

```
this->push_front(value);
```

#### `size()` :

This function will return the size of list

```
return this->size_
```

### print():

This function will print all value of a list from head to tail.

```
for(iterator it = this->cbegin(); it != this->cend() ; it++){
    std::cout << *it << " ";
}
```

### clear():

The logic of this function is: while the head still point to an not null pointer, do the **pop\_front**.

```
while(this->head){
    this->pop_front();
}
this->size_ = 0;
```

### begin():

This function return iterator point to head of a list.

```
return iterator(this->head);
```

### end():

This function return iterator point to tail of a list - always a null pointer.

```
return iterator(nullptr);
```

### cbegin():

This function return a constant iterator point to head of a list. A constant iterator won't allow to modify the data.

```
const auto temp = iterator(this->head)
return temp;
```

### cend():

This function return a constant iterator point to tail of a list. Because tail always null pointer so it will be like the **end()**.

```
return iterator(nullptr);
```

After finish implementation, the result output in terminal.

```
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem1/list$ make clean && make
rm -rf main main.o list.o
mpic++ -std=c++14 -O3 -Wall -march=native -I/usr/local/include -c -o main.o main.cpp
mpic++ -std=c++14 -O3 -Wall -march=native -I/usr/local/include -c -o list.o list.cpp
cc -std=c++14 -O3 -Wall -march=native -I/usr/local/include -o main main.o list.o -L/usr/local/lib -lstdc++
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem1/list$ ./main
Test 1: Default constructor - size: 0
List elements: []
-----
Test 2: Initializer list constructor - size: 5
List elements: [1 2 3 4 5 ]
-----
Test 3: Copy constructor - size: 5
List elements: [1 2 3 4 5 ]
-----
Test 4: push_back - size: 6
List elements: [1 2 3 4 5 6 ]
-----
Test 5: emplace_back - size: 7
List elements: [1 2 3 4 5 6 7 ]
-----
Test 6: push_front - size: 8
List elements: [0 1 2 3 4 5 6 7 ]
-----
Test 7: emplace_front - size: 9
List elements: [-1 0 1 2 3 4 5 6 7 ]
-----
Test 8: pop_back - size: 8
List elements: [-1 0 1 2 3 4 5 6 ]
-----
Test 9: pop_front - size: 7
List elements: [0 1 2 3 4 5 6 ]
-----
Test 10: Iterators - values: 0 1 2 3 4 5 6
-----
Test 11: Clear - size: 0
List elements: []
All tests passed successfully!
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem1/list$ ]
```

## vector

**vector():**

This is constructor for empty vector, in this section, init **size** and **capacity** is 0 and **data\_ = nullptr**.

```
this->data_ = nullptr;
this->size = 0;
this->capacity = 0;
```

**~vector():**

This is destructor of vector, need to delete array of **data\_**, set **size** and **capacity** to 0 and point **data\_** to a **nullptr**.

```
delete [] this->data_;
this->data_ = nullptr;
```

```
this->size = 0;
this->capacity = 0;
```

### vector(const vector &other):

This is a constructor vector from other constant vector. Need to init empty vector first, then for each element in other vector and push\_back the value.

```
new (this) vector();
for(auto it = other.cbegin(); it != other.cend() ; it++){
    this->push_back(*it);
}
```

### vector(vector &&other):

The logic of this part is the same with constructor with another constant vector.

**operator=(const vector &other):** This is = operator, can treat this operator is the same with a constructor from another vector but need to clear all data first.

```
delete [] this->data_;
this->size = 0;
this->capacity = 0;
this->data_ = nullptr;
for(auto it = other.cbegin(); it != other.cend() ; it++){
    this->push_back(*it);
}
return *this;
```

### operator=(vector &&other):

This logic is the same with above = operator.

### at(size\_t index):

This function return data value at position **index**. Need to throw error when index is out of array.

```
if(index>= this->size){
    throw std::out_of_range( "Index out of range" );
}
return this->data_[index];
```

### operator[](size\_t index):

Can reuse **at** function when override this operator.

```
return this->at(index);
```

**front():**

Return the first element of vector.

```
return this->data_[0];
```

**back():**

Return the last element of vector.

```
return this->data_[this->size -1];
```

**data():**

Return the array of data.

```
return this->data_;
```

**getSize():**

Return **size** of vector.

**push\_back(const T &value):** Append new value to the end of vector. If the **capacity** is 0, init the vector with only 1 element with **value**, **size** and **capacity** is set to 1. Else if the **size==capacity**, mean the allocated memory is not enough, now need to increase the **capacity** by 2 times and copy data to new array and insert new value, at last need to delete all data from old array, avoid memory leak.

```
if (this->capacity==0){
    this->size = this->capacity = 1;
    this->data_ = new T[1];
    this->data_[0] = value;
}
else if (this->size == this->capacity){

    T* temp = new T[2*this->capacity];
    for (size_t i = 0; i < this->capacity; i++) {
        temp[i] = this->data_[i];
    }
    temp[this->capacity] = value;
    this->size += 1;
    this->capacity *=2;
    delete [] this->data_;
```



```

        this->data_ = temp;
    }
    else{
        this->data_[this->size] = value;
        this->size++;
    }

```

### emplace\_back(Args &&... args):

This function is used when an element in vector is not a common type like `int` or `float`, but a `struct` or `class` instead. To implement this, we need to `push_back` the object created by input arguments.

```

this->push_back(T(std::forward<Args>(args)...));

```

### append\_range(InputIt first, InputIt last):

This function will append a range of input iterator.

```

for(InputIt it=first;it!=last;it++){
    this->push_back(*it);
}

```

### pop\_back():

Decrease the `size` by 1 if `size > 0`.

```

this->size = this->size > 0 ? this->size - 1 : 0;

```

### reserve(size\_t newCapacity):

Reserve a fix amount of data. If the `newCapacity` smaller than current `size`, only keep the first elements.

```

if (this->size==0){
    this->capacity = 0;
}
T* temp = new T[newCapacity];
unsigned int temp_size = newCapacity<this->size ? newCapacity : this->size;
for (unsigned int i=0;i<temp_size;i++){
    temp[i] = this->data_[i];
}
this->capacity = newCapacity;
delete [] this->data_;

```

```
this->data_ = temp;  
this->size = temp_size;
```

**begin():**

Return the pointer to the head of data

```
return this->data_;
```

**cbegin():**

Return constant pointer to the head of data

```
const T* temp = this->data_;  
return temp;
```

**end():**

Return pointer to the tail of vector

```
return this->data_ + this->size;
```

**cend():**

Return constant pointer to the tail of vector

```
const T* temp = this->data_ + this->size;  
return temp;
```

**rbegin():**

Return reverse iterator point to the end of data

```
return std::reverse_iterator<T *> (this->end());
```

**crbegin():**

Return constant reverse iterator point to the end of data

```
std::reverse_iterator<const T *> temp = std::reverse_iterator<const T *>  
(this->cend());
```

```
return temp;
```

**rend():**

Return reverse iterator point to the head of data

```
return std::reverse_iterator<T *> (this->begin());
```

**crend():**

Return constant reverse iterator point to the head of data

```
std::reverse_iterator<const T *> temp = std::reverse_iterator<const T *>
(this->cbegin());
return temp;
```

After finish implementation, the output result in the terminal will look like this

```
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem1/vector$ make clean && make
rm -rf main main.o vector.o
mpic++ -std=c++14 -O3 -Wall -march=native -I/usr/local/include -c -o main.o main.cpp
mpic++ -std=c++14 -O3 -Wall -march=native -I/usr/local/include -c -o vector.o vector.cpp
cc -std=c++14 -O3 -Wall -march=native -I/usr/local/include -o main main.o vector.o -L/usr/local/lib -lstdc++
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem1/vector$ ./main
Test 1: Constructor, push_back
Vector elements using operator[]: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
-----
Test 2: operator[]
Vector elements using operator[]: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
-----
Test 3: at (including exception situation)
Accessing elements using at(): [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, Exception: Index out of range]
-----
Test 4: front, back
Front element: 1
Back element: 10
-----
Test 5: data
Vector elements using data(): [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
-----
Test 6: Iterators
Vector elements using iterators: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
-----
Test 7: Reverse Iterators
Vector elements using reverse iterators: [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
-----
Test 8: emplace_back
Vector elements after modifications: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
-----
Test 9: pop_back
Vector elements after pop_back: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
-----
Test 10: reserve
Vector elements after reserve: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
-----
All tests completed.
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem1/vector$
```

## Problem 2

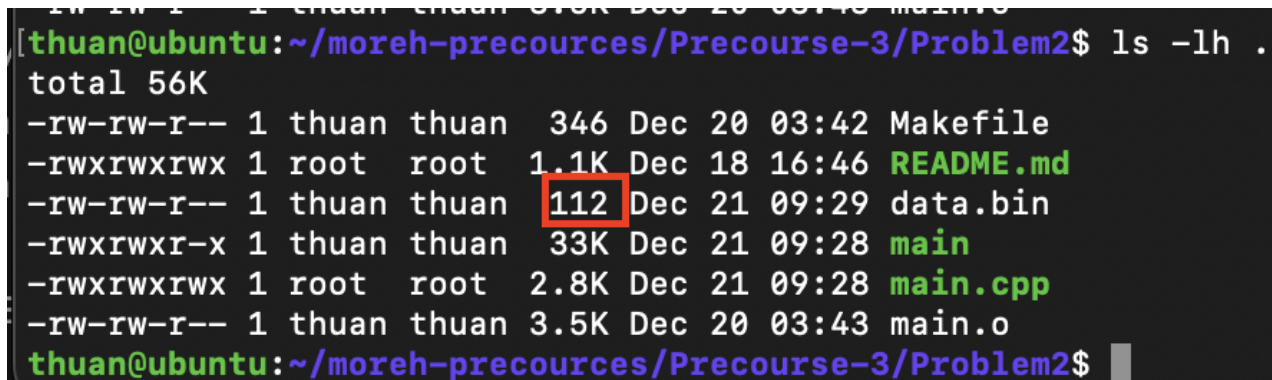
Identify problem

This is the implementation of `serialize` and `deserialize` function

```
void serialize(char* path, library* lib, int number_of_books){
    /*
        pseudo implement, please remove
        write(path, lib->books);
    */
    FILE* fp = fopen(path,"w");
    fwrite(lib->books , number_of_books , sizeof(lib->books[0]) , fp );
    fclose(fp);
    std::cout<<"size of a book when call serialize whole object: "
    <<sizeof(lib->books[0])<<std::endl;
}

// Function to load the books data of the library
void deserialize(char* path, library *lib, int number_of_books){
    /*
        pseudo implement, please remove
        read(path, lib->books);
    */
    FILE* fp = fopen(path,"r");
    std::size_t s = fread (lib->books, sizeof(lib->books[0]),
    number_of_books, fp);
    fclose(fp);
}
```

When check the size of `data.bin` it show that it take 112 bytes on disk



```
thuan@ubuntu:~/moreh-precources/Precourse-3/Problem2$ ls -lh .
total 56K
-rw-rw-r-- 1 thuan thuan 346 Dec 20 03:42 Makefile
-rwxrwxrwx 1 root  root  1.1K Dec 18 16:46 README.md
-rw-rw-r-- 1 thuan thuan 112 Dec 21 09:29 data.bin
-rwxrwxr-x 1 thuan thuan 33K Dec 21 09:28 main
-rwxrwxrwx 1 root  root  2.8K Dec 21 09:28 main.cpp
-rw-rw-r-- 1 thuan thuan 3.5K Dec 20 03:43 main.o
thuan@ubuntu:~/moreh-precources/Precourse-3/Problem2$
```

The output terminal show this

## Print All Contents of The Library

```
-----  
Title : The Songs of Stardust:Harmony  
Author : John Bolton  
ISBN : 15952557  
volume_number : 1  
-----  
-----
```

```
Title : Whispers Secrets are Unveiled  
Author : donald trump  
ISBN : 67652241  
volume_number : 99  
-----  
-----
```

```
size of a book when call serialize whole object: 56
```

## Print All Contents of The Library

```
-----  
Title : The Songs of Stardust:Harmony  
Author : John Bolton  
ISBN : 15952557  
volume_number : 1  
-----  
-----
```

```
Title : Whispers Secrets are Unveiled  
Author : donald trump  
ISBN : 67652241  
volume_number : 99  
-----  
-----
```

When call `sizeof(lib->books[0])` it return 56 instead of 54. So the size of file on disk is  $56 * 2 = 112$  bytes. This is behavior of compiler to avoid alignment issue. The compiler will pad the struct to fit 8-byte boundary. So it will pad 54 bytes to 56 bytes.

To solve this problem, we need to write each attribute of a book to file instead of whole book object.

### Solve the problem

This is implementation to write attributes of each book to file.

```
void serialize(char* path, library* lib, int number_of_books){  
    /*  
        pseudo implement, please remove  
        write(path, lib->books);  
    */  
}
```

```
FILE* fp = fopen(path,"w");
for (int idx = 0 ; idx < number_of_books;idx++){
    fwrite(&lib->books[idx].title,1,sizeof(lib->books[idx].title),fp);
    fwrite(&lib->books[idx].author,1,sizeof(lib->books[idx].author),fp);
    fwrite(&lib->books[idx].volume_number,1,sizeof(lib->books[idx].volume_number),fp);
    fwrite(&lib->books[idx].ISBN,1,sizeof(lib->books[idx].ISBN),fp);
}
fclose(fp);
std::cout<<"done"<<std::endl;
}

// Function to load the books data of the library
void deserialize(char* path, library *lib, int number_of_books){
    /*
    pseudo implement, please remove
    read(path, lib->books);
    */
    FILE* fp = fopen(path,"r");
    for (int idx = 0 ; idx < number_of_books;idx++){
        fread(&lib->books[idx].title,sizeof(lib->books[idx].title),1,fp);
        fread(&lib->books[idx].author,sizeof(lib->books[idx].author),1,fp);
        fread(&lib->books[idx].volume_number,sizeof(lib->books[idx].volume_number),1,fp);
        fread(&lib->books[idx].ISBN,sizeof(lib->books[idx].ISBN),1,fp);
    }
    fclose(fp);
}
```

When execute the program it will output like this

```
thuan@ubuntu:~/moreh-precources/Precourse-3/Problem2$ ./main

Print All Contents of The Library
-----
Title : The Songs of Stardust:Harmony
Author : John Bolton
ISBN : 15952557
volume_number : 1
-----
-----
Title : Whispers Secrets are Unveiled
Author : donald trump
ISBN : 67652241
volume_number : 99
-----
done

Print All Contents of The Library
-----
Title : The Songs of Stardust:Harmony
Author : John Bolton
ISBN : 15952557
volume_number : 1
-----
-----
Title : Whispers Secrets are Unveiled
Author : donald trump
ISBN : 67652241
volume_number : 99
-----
```

The size of `data.bin` on disk is decrease to 108 bytes

```
thuan@ubuntu:~/moreh-precources/Precourse-3/Problem2$ ls -lh .
total 60K
-rw-rw-r-- 1 thuan thuan 346 Dec 20 03:42 Makefile
-rwxrwxrwx 1 root root 1.1K Dec 18 16:46 README.md
-rw-rw-r-- 1 thuan thuan 108 Dec 21 14:07 data.bin
-rwxrwxr-x 1 thuan thuan 36K Dec 21 14:07 main
-rwxrwxrwx 1 root root 3.8K Dec 21 14:06 main.cpp
-rw-rw-r-- 1 thuan thuan 5.5K Dec 21 13:24 main.o
thuan@ubuntu:~/moreh-precources/Precourse-3/Problem2$
```

## Problem 3

In this problem we need to implement an interface using pybind11 to use the `matmul` function in python. The interface need to take numpy array as input, then request buffer to get pointer to data and then call the `matmul` function in C to do mathematic process.

This is implementation of interface



```

void matmul_c_interface(py::array_t<float> A, py::array_t<float> B,
py::array_t<float> C,int M, int N, int K){
    py::buffer_info A_buf = A.request(), B_buf = B.request(), C_buf =
C.request();
    float *ptr_A = static_cast<float *>(A_buf.ptr);
    float *ptr_B = static_cast<float *>(B_buf.ptr);
    float *ptr_C = static_cast<float *>(C_buf.ptr);

    matmul_c(ptr_A,ptr_B,ptr_C,M,N,K);
}

PYBIND11_MODULE(matmul_c, m) {
    m.doc() = "pybind11 example plugin"; // optional module docstring

    m.def("matmul_c_interface", &matmul_c_interface, "A function that adds
two numbers");
}

```

After finishing implementation we need to build the interface

```

$ export Python_Standard_Library_PATH=$(python3 -c "import sys;
print(':' .join(sys.path))")

$ g++ -O3 -Wall -shared -std=c++11 -fPIC -I$Python_Standard_Library_PATH \
$(python3 -m pybind11 --includes)
matmul.cpp \
-o matmul_c$(python3-config --
extension-suffix)

```

When execute `python3 main.py` it show to terminal like this

```

[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem3$ python3 main.py
Calculation in C : 0.10987353324890137 sec
Calculation in python : 77.59704399108887 sec
thuan@ubuntu:~/moreh-precources/Precourse-3/Problem3$ █

```

Because I use ubuntu VM with only 1 core, so the python execute for very long. When using c++ interface it the execution time reduce significantly.

## Problem 4

To solve this problem, I add a vector `std::vector<int> points_cluster;` to save the cluster that point[i] belongs to. I also change the return type of function `void updateCentroids() -> double updateCentroids()`, this function will return the total distance change of centroids between 2 iterations. This is the `KMeans.h` after modification



```

class KMeans {
private:
    std::vector<Point> data_points;
    std::vector<Point> centroids;
    std::vector<int> points_cluster;
    int k;

public:
    KMeans(int num_points, int k);
    void run(int max_iterations);

private:
    void initializeCentroids();
    void generateRandomDataPoints(int num_points);
    void assignToClusters();
    double updateCentroids();
    double calculateDistance(const Point& p1, const Point& p2);
};

```

## Function implementation

**KMeans::generateRandomDataPoints(int num\_points)**: Firstly, we need to set random seed fix by call `srand(42)`, then enter a for loop with number of step equal to `num_points`, each step we randomly generate point and push back to `this->data_points`.

```

double fRand(double fMin, double fMax)
{
    double f = (double)rand() / RAND_MAX;
    return fMin + f * (fMax - fMin);
}

void KMeans::generateRandomDataPoints(int num_points) {
    // TODO: Implement this function
    // Generate a given number of random data points within a specified
    range ( 0< x, y<50)
    srand(42);
    double min = 0, max = 50;
    std::vector<Point> vec;
    for (int i = 0 ; i< num_points;i++){
        Point p = Point(fRand(min,max),fRand(min,max));
        vec.push_back(p);
    }
    this->data_points = vec;
    // this->epsilon = 1e-3; // minimum centroids distance change between
    2 iteration to decide stop algorithm
    // this->stop = false; // decide to stop algorithm or not
}

```

The `fRand` is a helper function to generate a double number from `fMin` to `fMax`.

`KMeans::initializeCentroids()`: Because we generate data points randomly with uniform distribution, so we can chose the first `k` points in the data as initalize centroids. We also need to assign each point to a cluster at this step, I decide to assign all point to centroid 0 coressponding to cluster 0, this won't affect anything, just to make every points has its own initial cluster.

```
void KMeans::initializeCentroids() {
    // TODO: Implement this function
    // Initialize the centroids of clusters randomly.
    srand(42);
    std::vector<Point> vec;
    for(int i=0;i<k;i++){
        vec.push_back(this->data_points[i]);
    }
    this->centroids = vec;
    std::size_t l = this->data_points.size();
    for (std::size_t i =0;i<l;i++){
        this->points_cluster.push_back(0);
    }
}
```

`KMeans::assignToClusters()`: The logic of this function is, for each point, we will calculate the distance of it to every centroids, then we chose the closest centroid and assign this point to this centroid.

```
void KMeans::assignToClusters() {
    // TODO: Implement this function
    // Assign each point to a cluster of the nearest centroid
    for (std::size_t i =0; i <this->data_points.size();i++){
        double min_dis = 1e9;
        int cluster = 0;
        for (size_t j=0 ;j<this->centroids.size();j++){
            double dist = this->calculateDistance(this-
>data_points[i],this->centroids[j]);
            if (dist < min_dis){
                min_dis = dist;
                cluster = j;
            }
        }
        this->points_cluster[i] = (int)cluster;
    }
}
```

`KMeans::updateCentroids()`: This function will update the centroid of each cluster, new centroid will be the center of all point belong to corresponding cluster. We also need to observe the total distance of new centroids to old one for each cluster, if the total distance doesn't change significantly, stop the algorithm

```
double KMeans::updateCentroids() {
    // TODO: Implement this function
    // Update the centroids of clusters based on the current assignment of
    data points.
    double sum = 0;
    for (int i=0;i<this->k;i++){
        double sum_x =0, sum_y=0,total=0;
        for (std::size_t j =0;j<this->data_points.size();j++){
            if(points_cluster[j] == i){
                sum_x+= this->data_points[j].x;
                sum_y+= this->data_points[j].y;
                total+=1;
            }
        }
        Point p = Point(sum_x/total, sum_y/total);
        sum += this->calculateDistance(p,this->centroids[i]);
        this->centroids[i] = p;
    }
    return sum;
}
```

**KMeans::run(int max\_iterations)**: The logic of this function will do a for loop to **max\_iterations**, each step call to assign each point to exact cluster, then update the new centroids. If total distance of centroids between 2 iteration is small than an epsilon, (in the algorithm, I set epsilon is  $1e-3$  by hard code), then stop algorithm.

```
void KMeans::run(int max_iterations) {
    // TODO: Implement K-means algorithm and print the coordinates of each
    cluster centroid After the maximum number of iterations
    double sum;
    for (int i=0;i< max_iterations;i++){
        this->assignToClusters();
        sum = this->updateCentroids();
        std::cout<<"Iteration "<<i + 1<<"\n";
        std::cout<<"Centroid points :[";
        for (auto p :this->centroids){
            std::cout<< "("<< p.x<<","<<p.y <<"), ";
        }
        std::cout<<"]\n";
        std::cout<<"-----\n";
        if(sum <1e-3){
            std::cout<< "centroids doesn't change significantly, stop
algorithm!\n";
            break;
        }
    }
}
```

The output result of terminal show like this

```
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem4$ make clean && make
rm -f main.o KMeans.o Point.o kmeans
g++ -Wall -Wextra -std=c++11 -c main.cpp -o main.o
g++ -Wall -Wextra -std=c++11 -c KMeans.cpp -o KMeans.o
g++ -Wall -Wextra -std=c++11 -c Point.cpp -o Point.o
g++ -Wall -Wextra -std=c++11 -o kmeans main.o KMeans.o Point.o
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem4$ ./kmeans
Iteration 1
Centroid points :[(29.9879,6.01468), (22.0138,38.3931), (9.50702,13.6509), (43.4596,26.181), ]

-----

Iteration 2
Centroid points :[(35.2469,6.99629), (22.0138,38.3931), (10.2061,12.4531), (43.3106,29.5284), ]

-----

Iteration 3
Centroid points :[(37.844,9.62585), (22.0138,38.3931), (10.2061,12.4531), (43.4661,37.896), ]

-----

Iteration 4
Centroid points :[(37.844,9.62585), (21.448,37.9513), (10.2061,12.4531), (42.1993,39.0627), ]

-----

Iteration 5
Centroid points :[(37.844,9.62585), (21.448,37.9513), (10.2061,12.4531), (42.1993,39.0627), ]

-----

centroids doesn't change significantly, stop algorithm!
[thuan@ubuntu:~/moreh-precources/Precourse-3/Problem4$ ]
```