B TREE

SEMINAR REPORT

Vu Nham Nguyen Nguyen

Data Structure and Algorithm

Table of content

I. INTRODUCTION

B-Tree is a self-balancing search tree. In most of the other self-balancing search trees (like AVL and Red-Black Trees), it is assumed that everything is in main memory. To understand the use of B-Trees, we must think of the huge amount of data that cannot fit in main memory. When the number of keys is high, the data is read from disk in the form of blocks. Disk access time is very high compared to the main memory access time. The main idea of using B-Trees is to reduce the number of disk accesses.

II. HISTORY

B-trees were invented by Rudolf Bayer and Edward M. McCreight while working at Boeing Research Labs, for the purpose of efficiently managing index pages for large random access files. The basic assumption was that indexes would be so voluminous that only small chunks of the tree could fit in main memory. Bayer and McCreight's paper, Organization and maintenance of large ordered indices, was first circulated in July 1970 and later published in Acta Informatica

Bayer and McCreight never explained what, if anything, the B stands for: Boeing, balanced, broad, bushy, and Bayer have been suggested. McCreight has said that "the more you think about what the B in B-trees means, the better you understand B-trees."

III. OPERATION

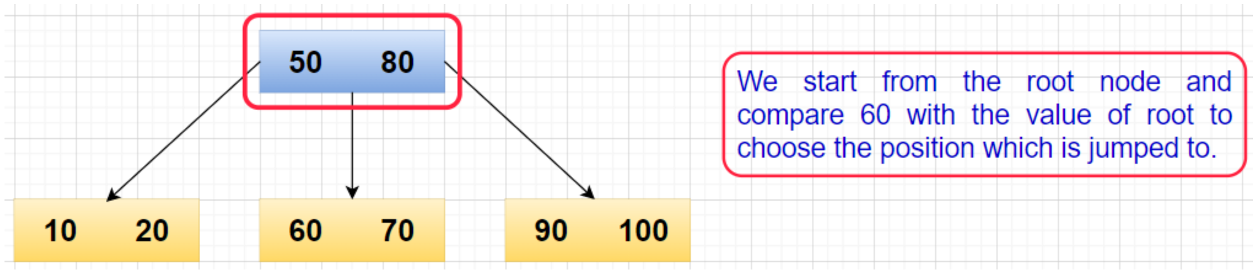
1. Search Operation in B – Tree

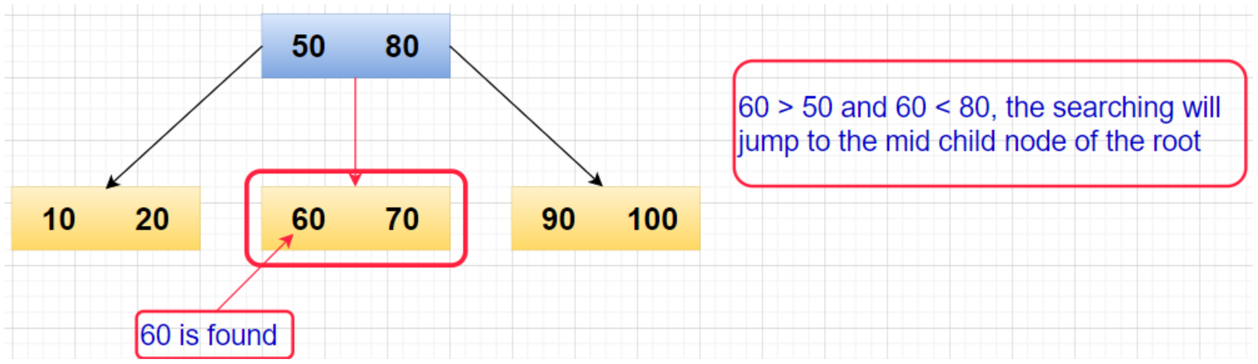
Searching in B – Tree is similar to the search in Binary Search Trees. If the key is found, we will print out "The key exists". Otherwise, if the current position is leaf node and does not contain the key which we need, we just inform "The key does not exist".

Algorithm: You want to find x.

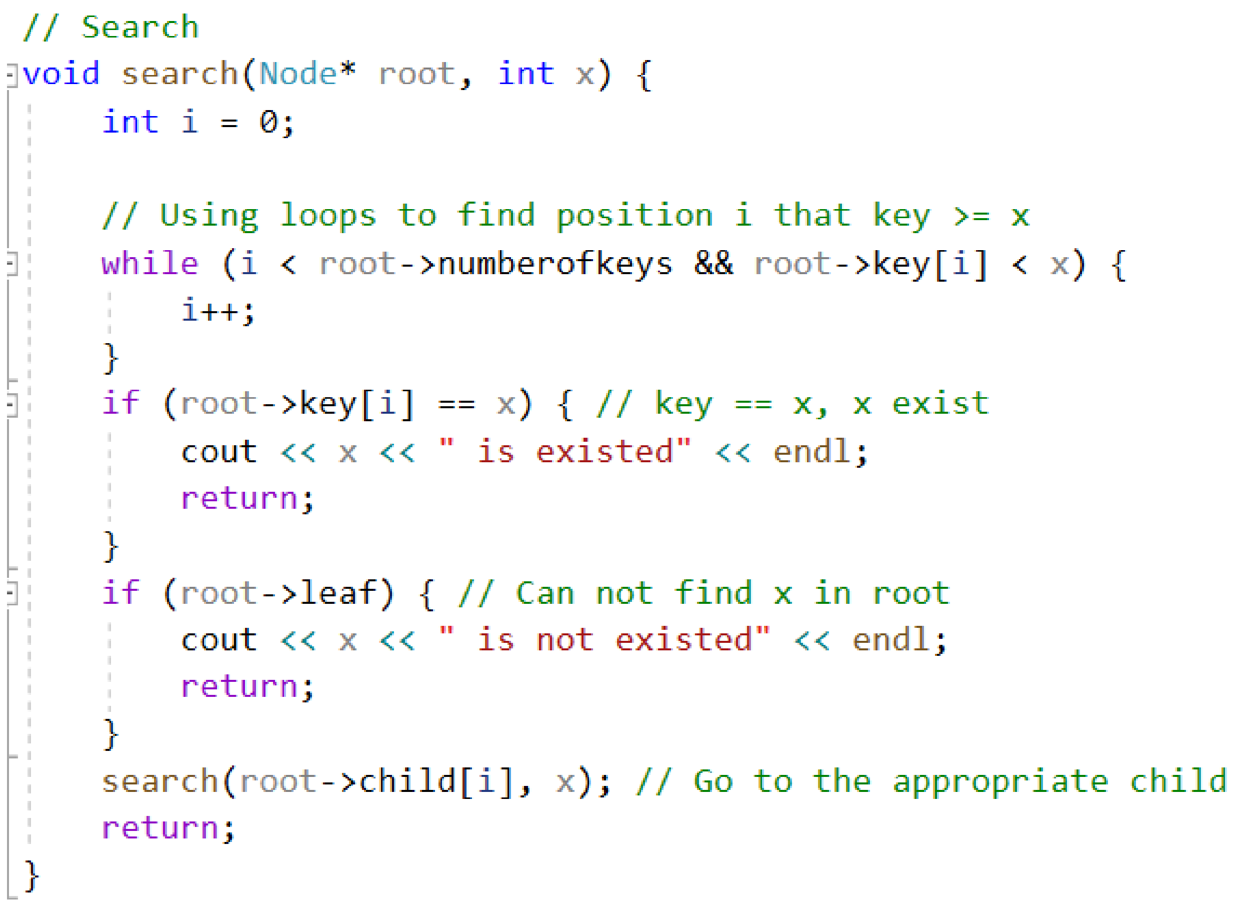
- Start searching from the root and recursively traverse down the tree.
- Within a node having n-1 values and n children, indexed as value[**0**], value[**1**], … up to value[**n-2**] , using loop:
  - Find the position I that key >= value[i]
  - If value[i] = key, print out the result.
  - If value[i] not equal to key, traverse down the child node of value[i]
- If k is not found in the tree, print out nothing.

Example and demo: Search 60 in B – Tree


50 80
10 20
60 70
90 100
We start from the root node and compare 60 with the value of root to choose the position which is jumped to.

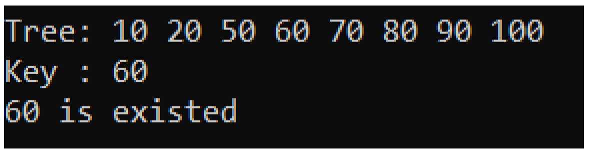

50 80
10 20
60 70
90 100
60 > 50 and 60 < 80, the searching will jump to the mid child node of the root
60 is found

Code



```
// Search
void search(Node* root, int x) {
    int i = 0;

    // Using loops to find position i that key >= x
    while (i < root->numberofkeys && root->key[i] < x) {
        i++;
    }
    if (root->key[i] == x) { // key == x, x exist
        cout << x << " is existed" << endl;
        return;
    }
    if (root->leaf) { // Can not find x in root
        cout << x << " is not existed" << endl;
        return;
    }
    search(root->child[i], x); // Go to the appropriate child
    return;
}
```

Run code:



```
Tree: 10 20 50 60 70 80 90 100
Key : 60
60 is existed
```

2. Insert Operation in B – tree

Insertion is an operation to add a new value to the tree. During the insertion, the tree can grow in height if it satisfies the conditions. Unlike Binary Search Trees, we have a predefined range on the number of keys that a node can contain. Therefore, insert operation consists of two cases:

- Case 1: The node is not full.
- Case 2: If the node is full then split it before inserting (Preemptive Split).The operation splits the node in half and moves a key up; hence this is the reason why B – tree grows up and Binary Search Trees grow down.

3. Preemptive Split

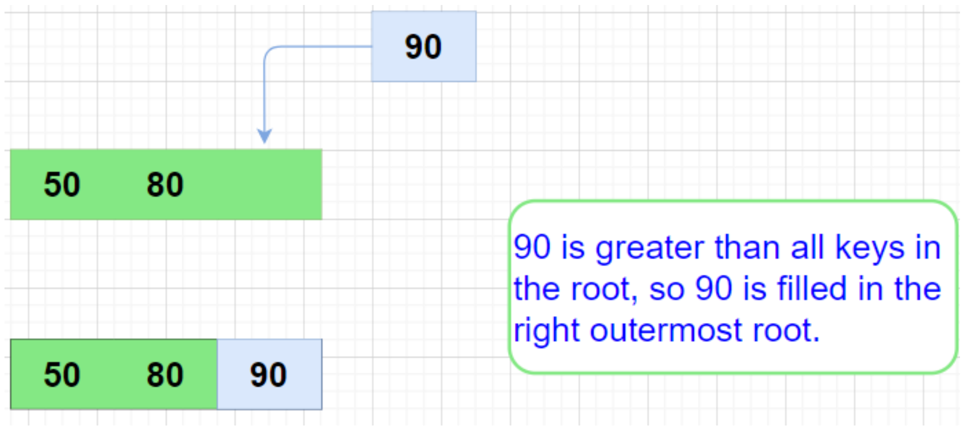
Algorithm

To insert a new key x, we do these following steps:

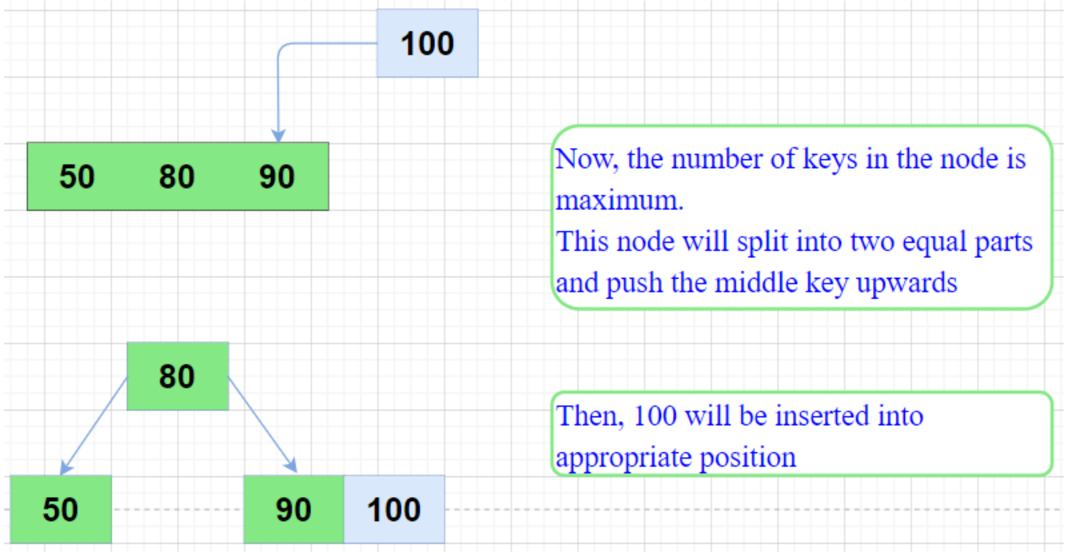
- Check if root is null, we initialize x as root and then return. Otherwise, checking the number of keys in the root whether it is full or not, if it is full, we will split the root. On the other hand, we search for an appropriate child node to insert.
- Update number of keys after inserting.
- After searching an appropriate child node, we check the number of keys in that node. If the node is full, we will split this node into two halves and one middle key.
- Push the middle key upwards and add it to the parent node. Then, we check these two new children nodes and insert the key into the appropriate child.
- If the node is not full, we insert the key into the node in ascending order.

Let us understand this algorithm with an example tree of minimum degree "t" as 2 (maximum degree is 4), therefore, the maximum number of keys a node is 2 * t – 1 which is 3.

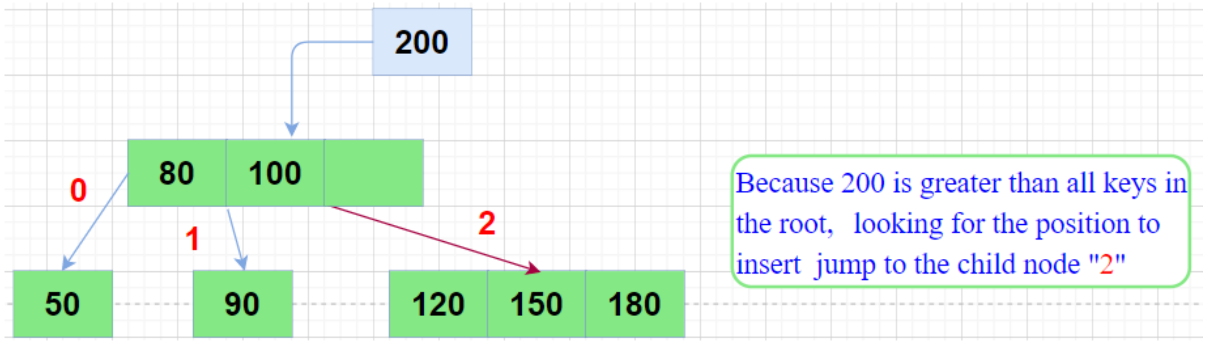
Ex1: Insert 90


90
50 80
90 is greater than all keys in the root, so 90 is filled in the right outermost root.
50 80 90

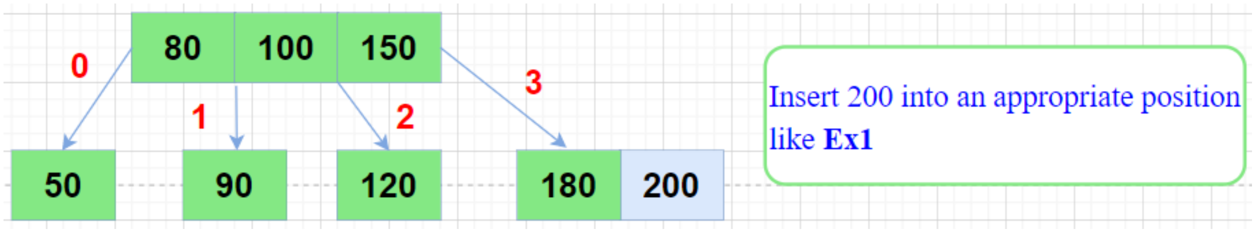
Ex2: Insert 100


100
50 80 90
Now, the number of keys in the node is maximum.
This node will split into two equal parts and push the middle key upwards
80
50
90 100
Then, 100 will be inserted into appropriate position

Ex3: Insert 200

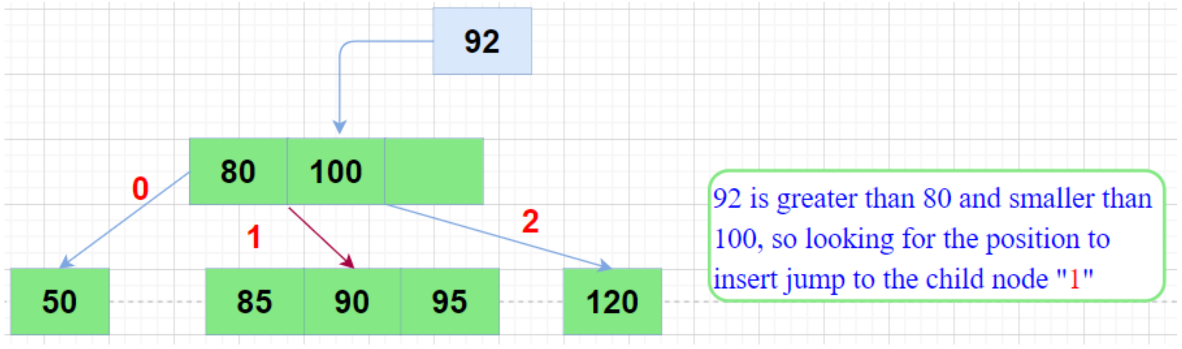

200
80 100
0
1
2
50
90
120 150 180
Because 200 is greater than all keys in the root, looking for the position to insert jump to the child node "2"

Now, this child node "**2**" already has the maximum number of keys. We need to split this node into two parts and the middle key "**150**" will go up to the parent node and be filled in key[**2**] of the root node. Making these two parts will create two new child nodes.



Ex4: Insert 92

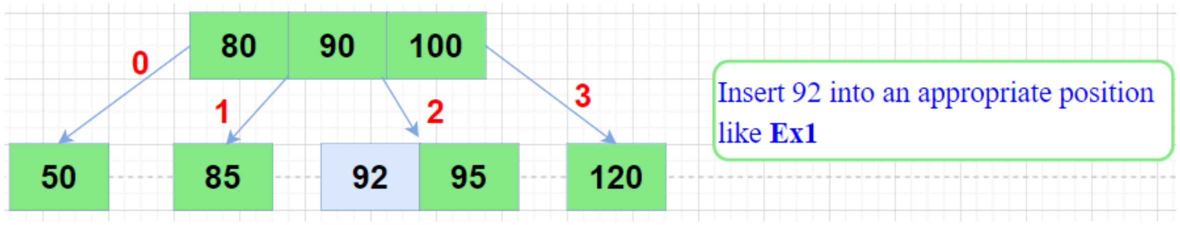
Now, this child node "1" already have the maximum number of keys. We need to split this node into two parts and the middle key "**90**" will go up to

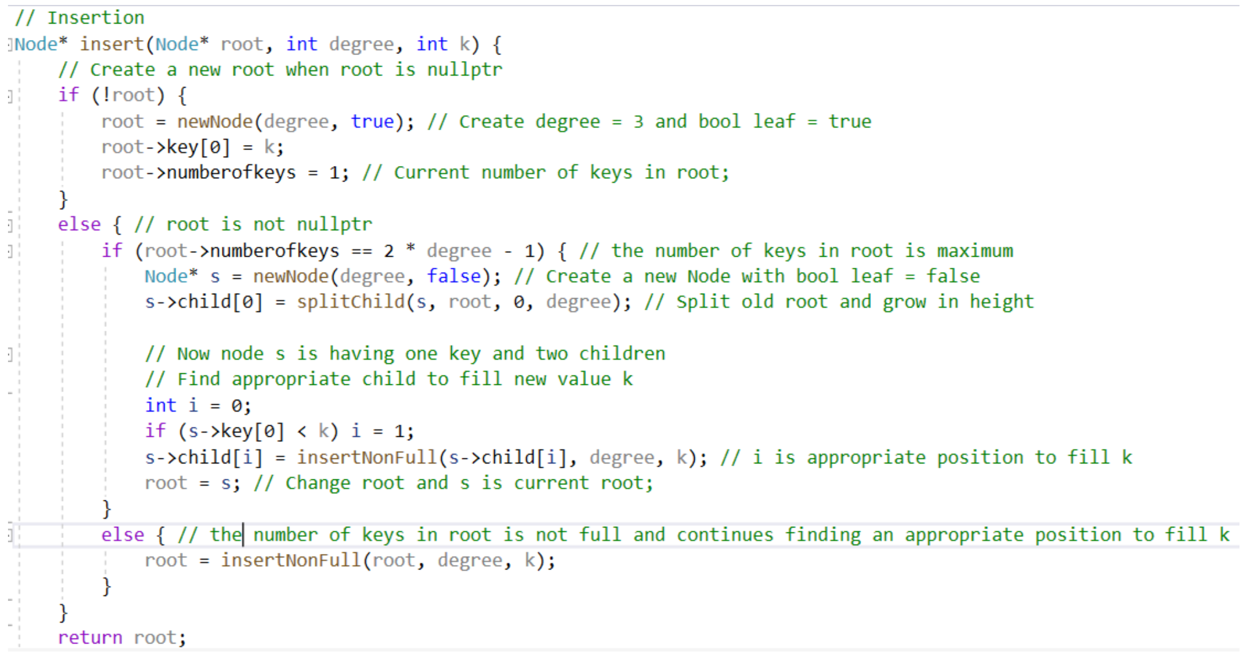

92
80
100
0
1
2
50
85
90
95
120
92 is greater than 80 and smaller than 100, so looking for the position to insert jump to the child node "1"

the parent node.

The keys behind the position "1" are moved to the right node step by step. Then, the children behind position "1" are also moved to the end. Making these two parts will create two new child nodes.

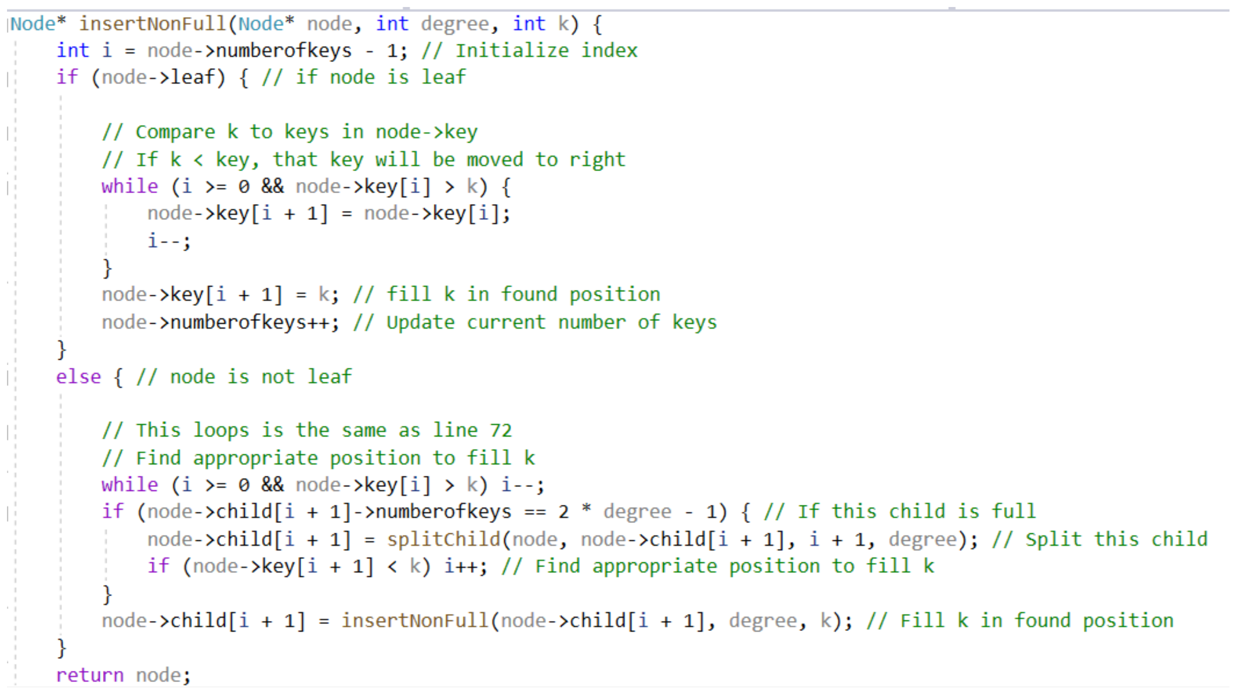
The key "**90**" is filled in key[1] of the root node.



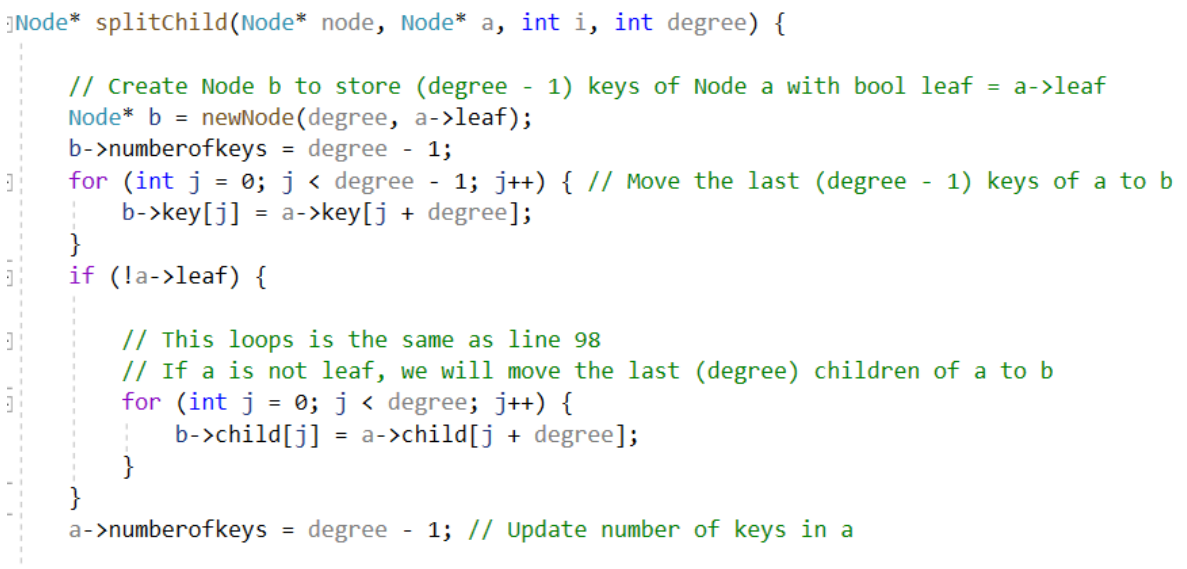


```
// Insertion
Node* insert(Node* root, int degree, int k) {
    // Create a new root when root is nullptr
    if (!root) {
        root = newNode(degree, true); // Create degree = 3 and bool leaf = true
        root->key[0] = k;
        root->numberofkeys = 1; // Current number of keys in root;
    }
    else { // root is not nullptr
        if (root->numberofkeys == 2 * degree - 1) { // the number of keys in root is maximum
            Node* s = newNode(degree, false); // Create a new Node with bool leaf = false
            s->child[0] = splitChild(s, root, 0, degree); // Split old root and grow in height

            // Now node s is having one key and two children
            // Find appropriate child to fill new value k
            int i = 0;
            if (s->key[0] < k) i = 1;
            s->child[i] = insertNonFull(s->child[i], degree, k); // i is appropriate position to fill k
            root = s; // Change root and s is current root;
        }
        else { // the number of keys in root is not full and continues finding an appropriate position to fill k
            root = insertNonFull(root, degree, k);
        }
    }
    return root;
```

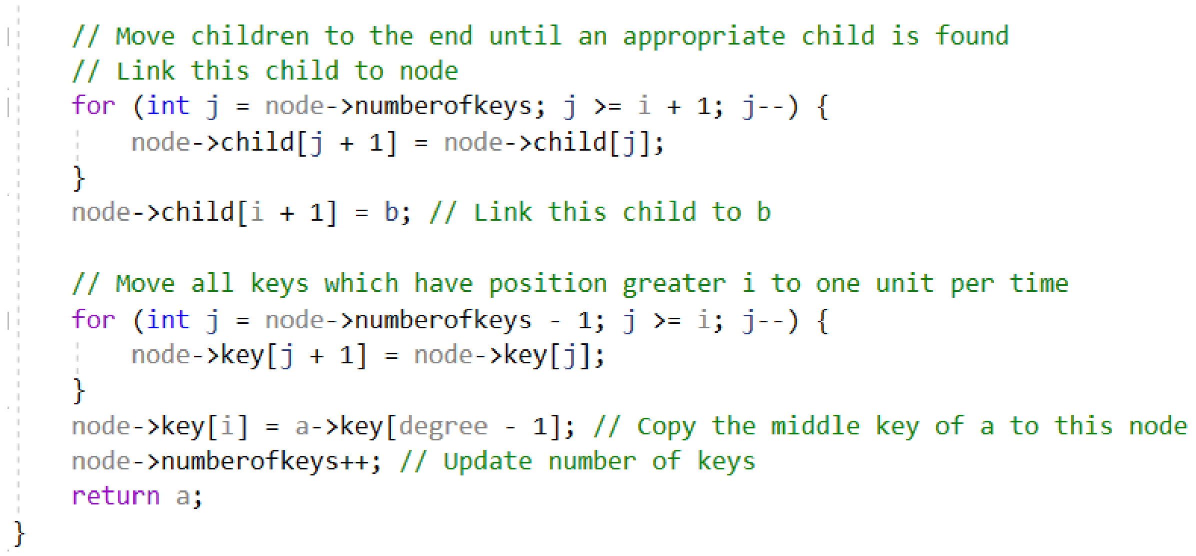
Code



```
Node* insertNonFull(Node* node, int degree, int k) {
    int i = node->numberofkeys - 1; // Initialize index
    if (node->leaf) { // if node is leaf

        // Compare k to keys in node->key
        // If k < key, that key will be moved to right
        while (i >= 0 && node->key[i] > k) {
            node->key[i + 1] = node->key[i];
            i--;
        }
        node->key[i + 1] = k; // fill k in found position
        node->numberofkeys++; // Update current number of keys
    }
    else { // node is not leaf

        // This loops is the same as line 72
        // Find appropriate position to fill k
        while (i >= 0 && node->key[i] > k) i--;
        if (node->child[i + 1]->numberofkeys == 2 * degree - 1) { // If this child is full
            node->child[i + 1] = splitChild(node, node->child[i + 1], i + 1, degree); // Split this child
            if (node->key[i + 1] < k) i++; // Find appropriate position to fill k
        }
        node->child[i + 1] = insertNonFull(node->child[i + 1], degree, k); // Fill k in found position
    }
    return node;
```



```
Node* splitChild(Node* node, Node* a, int i, int degree) {

    // Create Node b to store (degree - 1) keys of Node a with bool leaf = a->leaf
    Node* b = newNode(degree, a->leaf);
    b->numberofkeys = degree - 1;
    for (int j = 0; j < degree - 1; j++) { // Move the last (degree - 1) keys of a to b
        b->key[j] = a->key[j + degree];
    }
    if (!a->leaf) {

        // This loops is the same as line 98
        // If a is not leaf, we will move the last (degree) children of a to b
        for (int j = 0; j < degree; j++) {
            b->child[j] = a->child[j + degree];
        }
    }
    a->numberofkeys = degree - 1; // Update number of keys in a
```

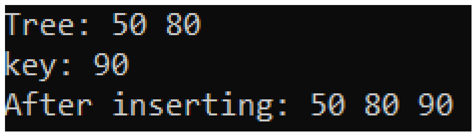


```
    // Move children to the end until an appropriate child is found
    // Link this child to node
    for (int j = node->numberofkeys; j >= i + 1; j--) {
        node->child[j + 1] = node->child[j];
    }
    node->child[i + 1] = b; // Link this child to b

    // Move all keys which have position greater i to one unit per time
    for (int j = node->numberofkeys - 1; j >= i; j--) {
        node->key[j + 1] = node->key[j];
    }
    node->key[i] = a->key[degree - 1]; // Copy the middle key of a to this node
    node->numberofkeys++; // Update number of keys
    return a;
}
```

**** Run code**

Ex1: Input: 50, 80

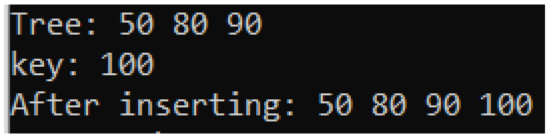
Insert: 90



```
Tree: 50 80
key: 90
After inserting: 50 80 90
```

Ex2: Input: 50, 80, 90

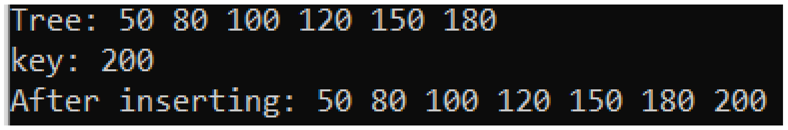
Insert: 100



```
Tree: 50 80 90
key: 100
After inserting: 50 80 90 100
```

Ex3: Input: 50, 80, 100, 120, 150, 180

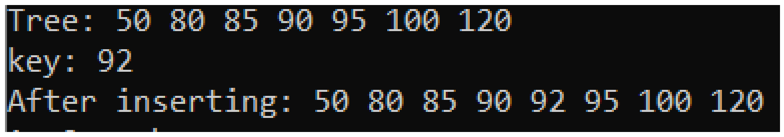
Insert: 200



```
Tree: 50 80 100 120 150 180
key: 200
After inserting: 50 80 100 120 150 180 200
```

Ex4: Input: 50 80 90 100 120 85 95

Insert: 92



```
Tree: 50 80 85 90 95 100 120
key: 92
After inserting: 50 80 85 90 92 95 100 120
```

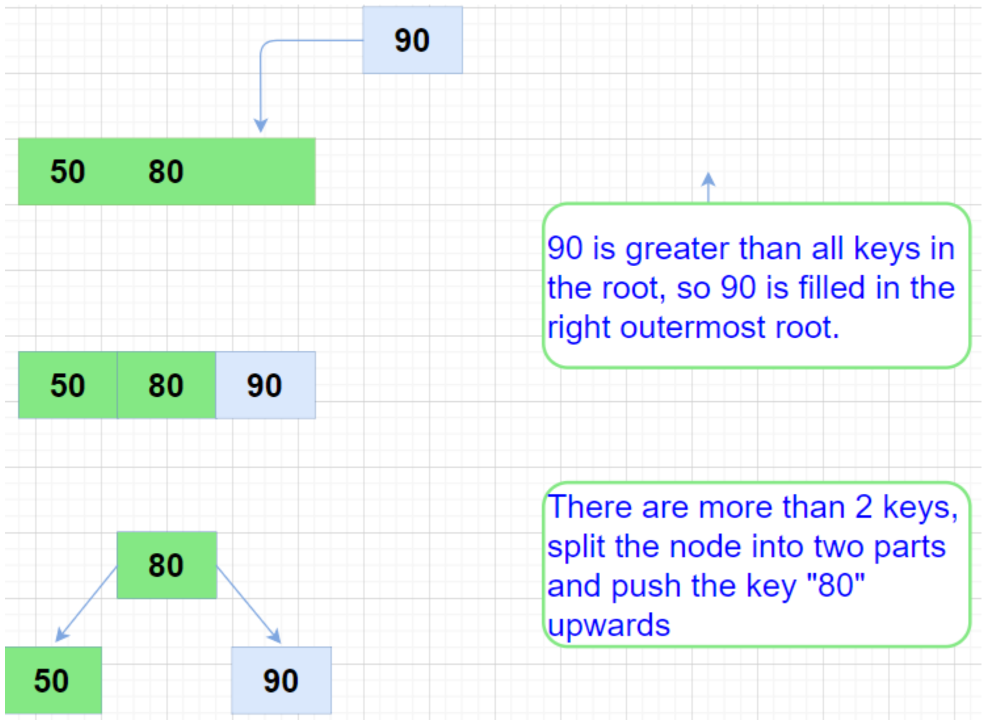
****Not Preemptive Split: a special case for 2-3 tree (B – tree order 3)**

Algorithm

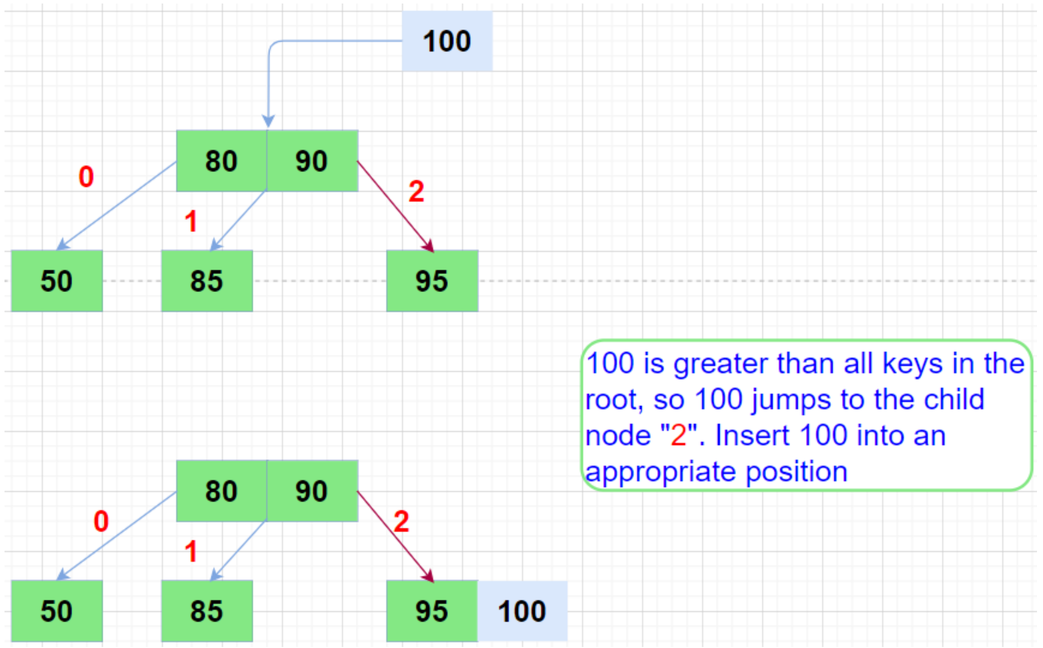
- The same as the above insertion algorithm. But after inserting, we will check the number of keys in the node and decide to split or not.
- If there are more than the maximum number of keys, we will split the node into two parts and one middle key. If the maximum degree is odd, two parts that are separated will be equal. Conversely, the left part or the right part can contain more keys.
- Push the middle key upwards and add it to the parent node. Then, we make these two new children nodes. If there is overflow in the parent node, we will continue to split until the overflow disappears.
- If there are fewer or equal to the maximum number of keys, the program finishes.

With some examples there is a tree of maximum degree "t" as 3. The maximum number of keys a node is t – 1 which is 2.

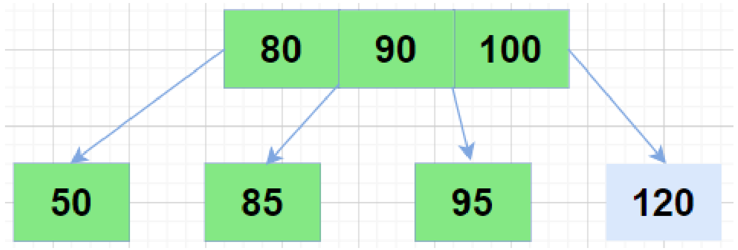
Ex1: Insert 90


90
50 80
90 is greater than all keys in the root, so 90 is filled in the right outermost root.
50 80 90
There are more than 2 keys, split the node into two parts and push the key "80" upwards
80
50
90

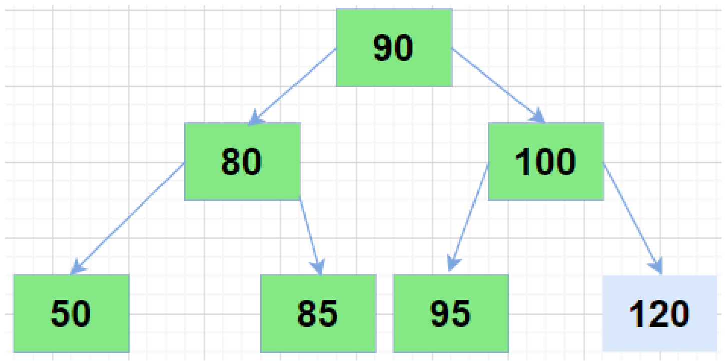
Ex2: Insert 100


100
80
90
0
1
2
50
85
95
100 is greater than all keys in the root, so 100 jumps to the child node "2". Insert 100 into an appropriate position
80
90
0
1
2
50
85
95
100

Insert 120. Since the child node “2” is full. After inserting 120, it overflows and it will first split into two parts and push the middle key “100” (95 < 100 < 120) upwards. Then, add 100 to the root node and make these two new nodes as its child (95 and 120).



Now, the root is overflowing. It will continue to split until the overflow disappears.



IV. COMPLEXITY

1. Space complexity

The complexity of space in a B-tree is O(n), which n is the number of elements in the tree.

2. Time complexity

The complexity of time of searching, insertion, deletion operations in B-tree is O(logn) in average case, and O(logn) in worst case, which n is the number of elements in the tree.

Let's explain why they are O(logn). We come first with the searching function. Firstly, we need to define some variable, 'm' is the max degree of a B-tree; 'n' is the number of elements in a tree. Overall, big-O notation considers the worst case scenario, the searching operation depends on the height of the tree. Then we have to consider the worst height of tree which is the highest height = $log_{\frac{m}{2}}(\frac{n+1}{2})$. And then we perform the binary search in one node in each height, so that is logm. Multiply two of them we have the total of logm*$log_{\frac{m}{2}}(\frac{n+1}{2})$, but the m is a constant so logm is constant too. Which means the big-O notation is O(logn). We can do similarly with the insertion and deletion operations.

3. Comparison

As we have learned that the Binary Search Tree has space and time complexity is better than 2-3 trees but just slightly better. Because the number of comparisons in 2-3 trees is more and then redundancy in each node makes the space bigger. So in terms of efficiency, a Binary Search Tree is better than 2-3 trees. Then 2-3 trees are better than 2-3-4 trees with the similar above explanation. Moreover, if you didn't know, the 2-3 tree is B-tree order of 3 and 2-3-4 tree is B-tree order of 4. So B-tree is not better than Binary Search Tree just in terms of efficiency. But in real life problems when you need to access a disk then B-tree is better because of its minimum height possible.

4. *Example*

Because B-tree is suitable for storage systems that read and write large blocks of data. They are particularly well suited to on-disk storage. The searching, insertion, deletion all take the logarithmic time.

It is common to use in file systems. So why is B-tree a common use for file systems? Because it was designed flat, having many values in a node which also means it is having a low height. Therefore, it reduces the amount of data that has to be sent (I/O operations). Reducing tree depth also implies limiting I/O, because each node traversing in a binary tree is an I/O operation. Because each node does not have to be completely filled, the tradeoff is complexity and storage inefficiency. As a result, B-trees are suitable for file systems.

What about real life applications? We have known the advantages of B-tree, so it will be well suitable for 'store', maybe just at this moment with all we have learned. Just imagine how you modify the tree, for example, a store of food. For restaurants, they can 'insert' the dishes they serve while , for customers, they can search for food that they want. So it will have the searching and inserting for food quickly, but also it depends how you modify the tree too, such as a node can be a type of food or a restaurant.

IV. ADVANTAGE AND DISADVANTAGE

1. Advantages of B-tree

- In every node of B-tree, it keeps the keys sorted in order (ascending or descending).
- Due to its flat structure, we can easily and efficiently traverse through the data while storing a million items.
- It is also a balanced tree then why the worst case is the same as the average case.
- Compared to hash tables, B-Tree allows for ordered sequential access, which makes it more efficient overall.
- B-tree and its index are stored in a file so B-tree doesn't have to be rebuilt every time when needed in a program.

2. Disadvantages of B-tree

- When executing a search, a B-tree that does not fill RAM requires more disk searches on average than a hash table.
- The implementation of a B-tree is far more difficult than that of a hash table.
- When the keys are dense, an array's random access is quicker and uses less memory than a b-tree. The implementation of a B-tree is significantly more difficult than that of an array.
- When compared to other balanced trees, B-trees are inefficient when stored in RAM. Many keys/values must be moved around while inserting or removing items. If a b-tree has a low branching factor and a node fits a cache line, it has a possibility of being efficient in RAM. Cache misses may be reduced as a result of this.

V. APPLICATION

B-Tree algorithms are good for accessing **pages** (or blocks) of stored information which are then copied into main memory for processing. In the worst case, they are designed to do dynamic set operations in O(log n) time because of their high **"branching factor"** (hundreds or thousands of keys on any node). It is this branching factor that makes B-Trees so efficient for **block storage/retrieval,** since a large branching factor greatly reduces the height of the tree and thus the number of disk accesses needed to find any key! The total disk accesses for most of the operations are reduced significantly compared to balanced Binary Search Trees like AVL Tree, Red-Black Tree, ... Also, B-Tree is the default index type for most storage engines.

For example: a B-Tree with a branching factor of 1001 and a height of 2 can store over 1 Billion keys! And since the root node can be kept in main memory, you only need 2 disk accesses to find any key in this tree!

B-Tree is used to **index the data** in multilevel layers and provides fast access to the actual data stored in a self-balancing fashion on the disks since the access to value stored in a large database is very time consuming.

Each node in the B-Tree stores not only keys, but also a record pointer for each key to the actual data being stored. (Could also potentially store the record in the B-Tree itself). To find the data you want, search the B-Tree using the key and then use the pointer to retrieve the data (No additional disk access is required if the record is stored in the node) process.

Searching an un-indexed and unsorted database containing n key values needs O(n) running time in the worst case. List indexing and sparse-array indexing have serious performance disadvantages. However, if we use B-Tree to index this database, it will be searched in O(log n) time in the worst case. The only serious competitor with B-Tree indexing could be hashing.

Databases and file systems (SQL SERVER, QUERIES…): It is a useful algorithm for databases and file systems for handling a bulky amount of data.

- Dictionary - range queries, say if you want to find all entries where the key is between two values in a sorted order. For range queries like between XX and YY.
- Map folders/directories.
- Recording and checking a list of keys, spellcheck: You want to read a list of correct words, and then check every word typed against those words. Storing all the words in a tree of some sort makes the lookups quicker - scanning through an entire list each time would be slow (scaling with the number of words), while a look-up in a tree scales with the logarithm of the number - roughly speaking. In a similar way, you could use a tree to compare two lists - put the first in a B-Tree, then check each item in the other against the tree.

VI. REFERENCE

Geeksforgeeks

StackOverflow

Jenny’s lectures CS/IT NET&JRF

Nargish Gupta

saurabhschool

Wikipedia

Oracle

Viska Gupta

DZone

Javatpoint

Quora

CodeProject

https://cs.stackexchange.com