Problem. *Find the least constant k, such that*

$$a+b+c+k\cdot\frac{(a-c)^2}{a+c}\geqslant\sqrt{3(a^2+b^2+c^2)}$$

for all $a\geqslant b\geqslant c$ be positive real numbers.

(Vasile Cirtoaje, Leonard Giugiuc)

Solution. (*Nguyen Van Huyen*) Let $a=5, b=c=1$ we get $k\geqslant\frac{3}{4}$. We will show that

$$a+b+c+\frac{3}{4}\cdot\frac{(a-c)^2}{a+c}\geqslant\sqrt{3\left(a^2+b^2+c^2\right)},$$

or

$$7(c^2+a^2)+4b(c+a)+2ca\geqslant 4(c+a)\sqrt{3(a^2+b^2+c^2)}.$$

Using the AM-GM inequality, we have

$$3(c+a)\sqrt{3(a^2+b^2+c^2)}\leqslant\frac{9}{4}(c+a)^2+3(a^2+b^2+c^2).$$

We need to prove

$$7(c^2+a^2)+4b(c+a)+2ca\geqslant\frac{4}{3}\left[\frac{9}{4}(c+a)^2+3(a^2+b^2+c^2)\right].$$

Expand and simplify to

$$4(a-b)(b-c)\geqslant 0.$$

Which is true. The equality holds for $a=b=c$ or $a=5b=5c$. $\square$