Problem (Vasile Cirtoaje). *If a, b, c are positive real numbers, then*

$$\frac{a^2}{a^2 + ab + b^2} + \frac{b^2}{b^2 + bc + c^2} + \frac{c^2}{c^2 + ca + a^2} \geqslant 1.$$
 (1)

Solution. (Nguyen Van Huyen) Since

$$\sum \frac{a(ab-c^2)}{a+b+c} = 0,$$

and

$$\sum \frac{b^2 + ab}{a^2 + b^2 + c^2 + ab + bc + ca} = 1.$$

We see that the inequality (1) follows by adding the inequality below and two similar inequalities

$$\frac{b^2}{b^2 + bc + c^2} \geqslant \frac{b^2 + ab + \frac{a(ab - c^2)}{a + b + c}}{a^2 + b^2 + c^2 + ab + bc + ca}.$$
 (2)

Indeed, we can rewrite the inequality (2) as

$$\frac{b^2}{b^2 + bc + c^2} - \frac{b^2 + ab}{a^2 + b^2 + c^2 + ab + bc + ca} \geqslant \frac{\frac{a(ab - c^2)}{a + b + c}}{a^2 + b^2 + c^2 + ab + bc + ca},$$

or

$$\frac{ab(ab - c^2)}{b^2 + bc + c^2} \geqslant \frac{a(ab - c^2)}{a + b + c},$$

which is equivalent to

$$\frac{a(ab - c^2)^2}{(a+b+c)(b^2 + bc + c^2)} \geqslant 0.$$

Thus, the inequality (2) is proven.