

**Problem.** Let  $a, b, c$  be positive real numbers such that  $a + b + c = 3$ . Prove that

$$\frac{a(b+c)}{a^2+2bc+3} + \frac{b(c+a)}{b^2+2ca+3} + \frac{c(a+b)}{c^2+2ab+3} \leq 1.$$

**Solution.** (Nguyen Van Huyen) Using the Cauchy-Schwarz inequality and AM-GM inequality, we have

$$a^2 + 3 = a^2 + \frac{(a+b+c)^2}{3} \geq \frac{(a+a+b+c)^2}{1+3} = \frac{(2a+b+c)^2}{4} \geq 2a(b+c).$$

So

$$a^2 + 2bc + 3 \geq 2(ab + bc + ca),$$

or

$$\frac{a(b+c)}{a^2+2bc+3} \leq \frac{a(b+c)}{2(ab+bc+ca)}.$$

Therefore

$$\sum \frac{a(b+c)}{a^2+2bc+3} \leq \sum \frac{a(b+c)}{2(ab+bc+ca)} = 1.$$

The proof is completed. □