**Problem.** *Let $a, b, c$ be three distinct real numbers. Prove that*

$$\frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2} \geqslant 2. \tag{1}$$

*(Dao Hai Long)*

**Solution 1.** (*Nguyen Van Huyen*) Let

$$x=\frac{a}{b-c}, \quad y=\frac{b}{c-a}, \quad z=\frac{c}{a-b}.$$

Then $xy+yz+zx=1$. The inequality (1) becomes

$$x^2+y^2+z^2 \geqslant 2.$$

Since $xy+yz+zx=1$, we obtain $z=-\frac{1+xy}{x+y}$. We need to prove

$$x^2+y^2+\left(\frac{1+xy}{x+y}\right)^2 \geqslant 2,$$

or

$$(x+y)^2+\left(\frac{1+xy}{x+y}\right)^2 \geqslant 2(1+xy).$$

Using the AM-GM inequality, we have

$$(x+y)^2+\left(\frac{1+xy}{x+y}\right)^2 \geqslant 2\sqrt{(x+y)^2\cdot\left(\frac{1+xy}{x+y}\right)^2}=2|1+xy| \geqslant 2(1+xy).$$

The proof is completed. □

**Solution 2.** Note that

$$\sum a(2a-b-c)=2(a^2+b^2+c^2-ab-bc-ca),$$

and

$$\sum (b-c)^2(2a-b-c)^2=2(a^2+b^2+c^2-ab-bc-ca)^2.$$

Therefore, according to the Cauchy-Schwarz inequality, we have

$$\sum \frac{a^2}{(b-c)^2} \geqslant \frac{[a(2a-b-c)+b(2b-c-a)+c(2c-a-b)]^2}{(b-c)^2(2a-b-c)^2+(c-a)^2(2b-c-a)^2+(a-b)^2(2c-a-b)^2}=2.$$

The proof is completed. □