Problem. *Find the least constant k, such that*

$$a + b + c + k \cdot \frac{(a-c)^2}{a+c} \geqslant \sqrt{3(a^2 + b^2 + c^2)}$$

for all $a \geqslant b \geqslant c$ be positive real numbers.

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(Vasile Cirtoaje, Leonard Giugiuc)

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Solution. (*Nguyen Van Huyen*) Let $a = 5, b = c = 1$ we get $k \geqslant \frac{3}{4}$. We will show that

$$a + b + c + \frac{3}{4} \cdot \frac{(a-c)^2}{a+c} \geqslant \sqrt{3\left(a^2 + b^2 + c^2\right)},$$

or

$$7(c^2 + a^2) + 4b(c+a) + 2ca \geqslant 4(c+a)\sqrt{3(a^2 + b^2 + c^2)}.$$

Using the AM-GM inequality, we have

$$3(c+a)\sqrt{3(a^2 + b^2 + c^2)} \leqslant \frac{9}{4}(c+a)^2 + 3(a^2 + b^2 + c^2).$$

We need to prove

$$7(c^2 + a^2) + 4b(c+a) + 2ca \geqslant \frac{4}{3}\left[\frac{9}{4}(c+a)^2 + 3(a^2 + b^2 + c^2)\right].$$

Expand and simplify to

$$4(a-b)(b-c) \geqslant 0.$$

Which is true.   □