Problem. *If $a,\ b,\ c$ be positive numbers, then*

$$\frac{a^2}{(a+b)^2}+\frac{b^2}{(b+c)^2}+\frac{c^2}{(c+a)^2}\geqslant\frac{3}{4}.$$

Solution. (*Nguyen Van Huyen*) We have

$$4a^2(a+b+c)^2-(4a^2-b^2-bc+7ca)(a+b)^2=c(a+b)(a-b)^2+(b^2+ab-2ca)^2\geqslant 0,$$

thus

$$\frac{a^2}{(a+b)^2}\geqslant\frac{4a^2-b^2-bc+7ca}{4(a+b+c)^2}.$$

Which gives

$$\sum\frac{a^2}{(a+b)^2}\geqslant\sum\frac{4a^2-b^2-bc+7ca}{4(a+b+c)^2}=\frac{3}{4}.$$

The proof is completed. □