CSCI 2270 – Data Structures

Recitation 5, Summer 2020

Graph and Graph Traversal Algorithms

Objectives

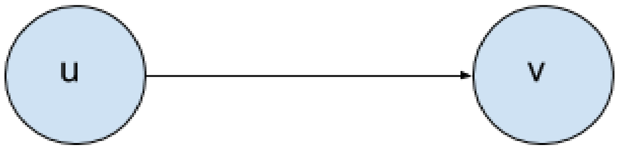
1. What is a Graph?
2. Depth-First Search
3. Breadth-First Search
4. Exercise
    a. Challenge 1 (Silver Badge)
    b. Challenge 2 (Gold Badge)

In the real world, many problems are represented in terms of objects and connections between them. For example, in an airline route map, we might be interested in questions like: “What’s the fastest way to go from Denver to San Francisco” or “What is the cheapest way to go from Denver to San Francisco” To answer these questions we need information about connections between objects. Graphs are data structures used for solving these kinds of problems.

Graph: A graph is a pair (V, E), where V is a set of nodes, called vertices and E is a collection of pairs of vertices, called edges.

Directed Edge

- Ordered pair of vertices (u, v)
- The first vertex u is the origin
- Second vertex v is the destination
- Example: one-way road traffic



Undirected Edge:

- Unordered pair of vertices (u, v)
- Example: Railway lines

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Applications of Graphs

- Representing relationships between components in electronic circuits.
- Transportation networks: Highway network, Flight network
- Computer networks: Local area network, Internet web

Graph Representation

As in other Abstract Data types, to manipulate graphs we need to represent them in some useful form. Basically, there are two ways of doing this:

- Adjacency Matrix
- Adjacency Lists

Graph Traversals

To solve problems on graphs, we need a mechanism for traversing the graphs. Graph traversal algorithms are also called graph search algorithms. Like tree traversal algorithms (Inorder, Preorder, Postorder traversals), graph search algorithms can be thought of as starting at some source vertex in a graph, and 'search' the graph by going through the edges and making the vertices. Next week we will discuss two such algorithms for traversing the graphs.

- Depth First Search (DFS)
- Breadth-First Search (BFS)

Depth First Search (DFS)

The strategy followed by depth-first search is, as its name implies, to search “deeper” in the graph whenever possible.

The DFS algorithm is a recursive algorithm that uses the idea of backtracking. The idea is to travel as deep as possible from neighbor to neighbor before backtracking. What determines how deep it is possible is that you must follow edges, and you don't visit any vertex twice. To do this properly we need to keep track of which vertices have already been visited, plus how we got to where we currently are so that we can backtrack.
Here, the word backtrack means that when you are moving forward and there are no more nodes along the current path, you move backward on the same path to find nodes to traverse.

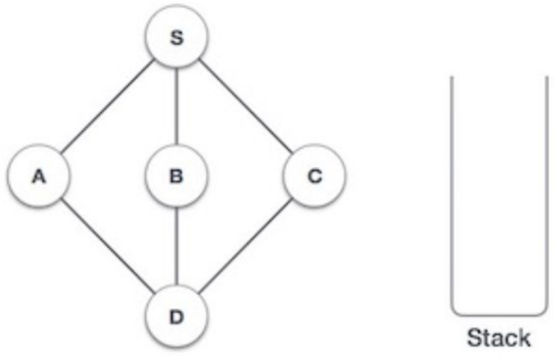
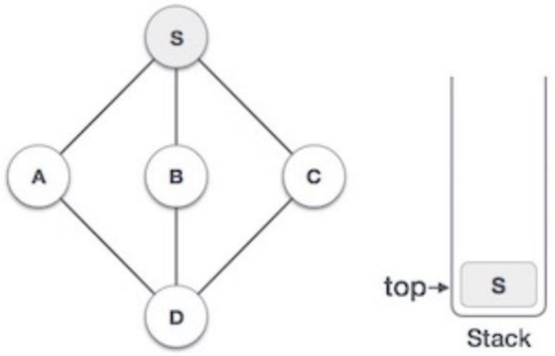
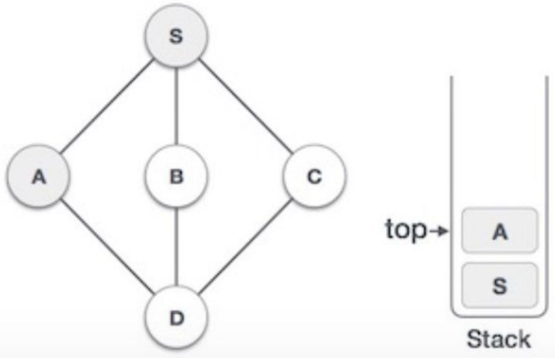
All the nodes on the current path are visited, after which the next path will be selected. We would be using the Adjacency lists to get the neighbors of any vertex.

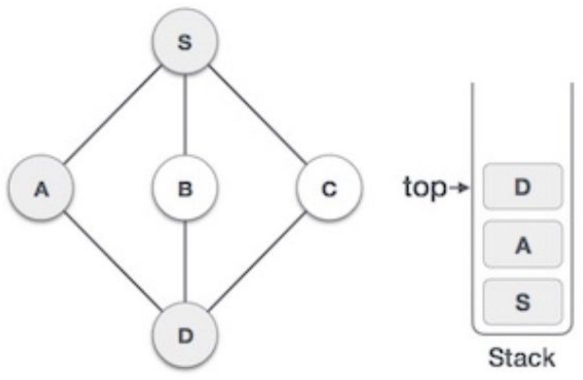
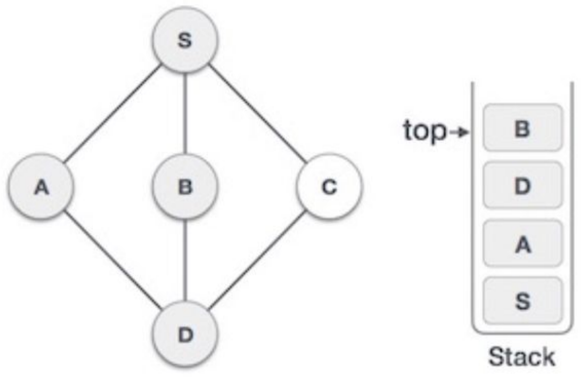
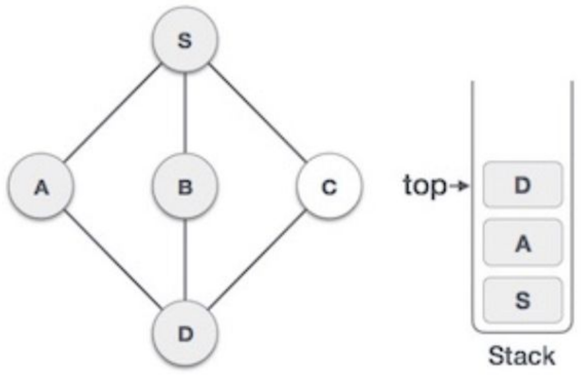
Here’s a pseudocode of this recursive approach:

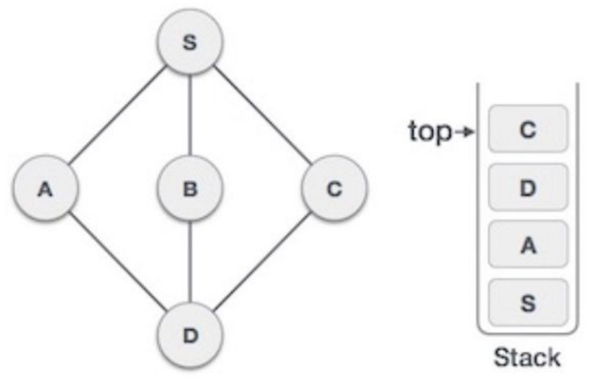
```
DFS(G, u)
    u.visited = true
    for each v ∈ G.Adj[u]
        if v.visited == false
            DFS(G,v)

init() {
    for each u ∈ G //u are nodes in the graph
        u.visited = false
     for each u ∈ G
        If u.visited==false
            DFS(G, u)
}
```

Make sure that the nodes that are visited are marked. This will prevent you from visiting the same node more than once. If you do not mark the nodes that are visited and you visit the same node more than once, you may end up in an infinite loop.
This recursive behavior (which uses a system stack) can be simulated by an iterative algorithm explicitly using a stack. So, DFS uses a stack to push nodes we mean to visit onto that.
We gave pseudocode for DFS traversal using recursion and we will be giving a visualization of DFS traversal using stacks to help you understand better. The following will give you the basic idea of Stack implementation.

Step	Traversal	Description
1		Initialize the stack.
2		Mark **S** as visited and put it onto the stack. Explore any unvisited adjacent node from **S**. We have three nodes and we can pick any of them. For this example, we shall take the node in an alphabetical order.
3		Mark **A** as visited and put it onto the stack. Explore any unvisited adjacent node from A. Both **S** and **D** are adjacent to **A** but we are concerned for unvisited nodes only.

4		Visit **D** and mark it as visited and put onto the stack. Here, we have **B** and **C** nodes, which are adjacent to **D** and both are unvisited. However, we shall again choose in an alphabetical order.
5		We choose **B**, mark it as visited and put onto the stack. Here **B** does not have any unvisited adjacent node. So, we pop **B** from the stack.
6		We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find **D** to be on the top of the stack.

7		Only unvisited adjacent node is from **D** is **C** now. So we visit **C**, mark it as visited and put it onto the stack.

As C does not have any unvisited adjacent node so we keep popping the stack until we find a node that has an unvisited adjacent node. In this case, there's none and we keep popping until the stack is empty.

Applications of DFS

- Finding connected components in an undirected graph
  (a variant of DFS is used for doing the same in directed graphs - Kosaraju algorithm)
- Solving puzzles, such as mazes (DFS helps to reach the goal faster)

Breadth-First Search (BFS)

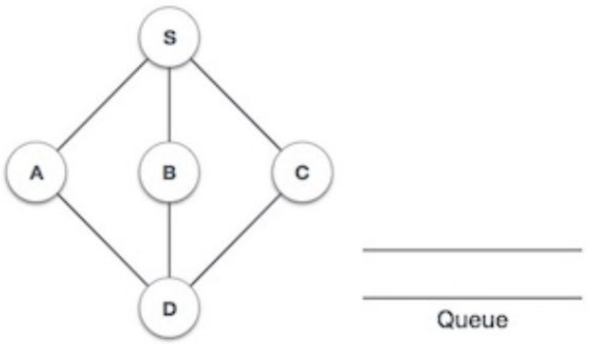
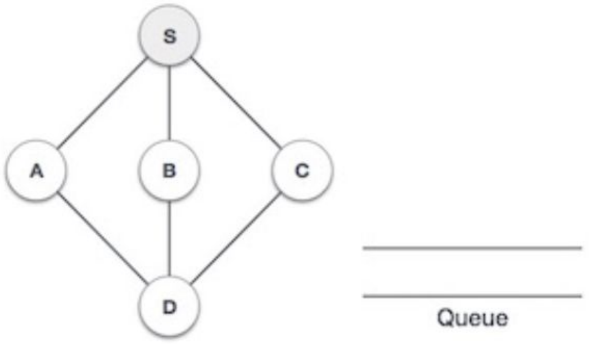
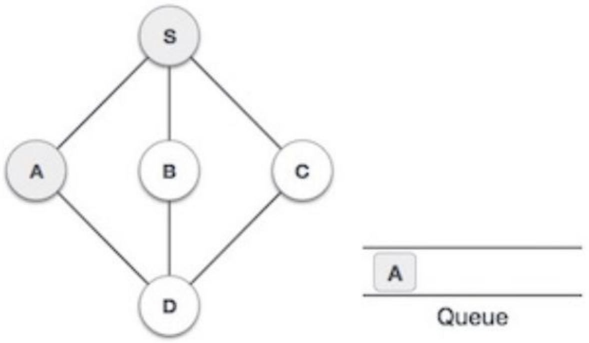
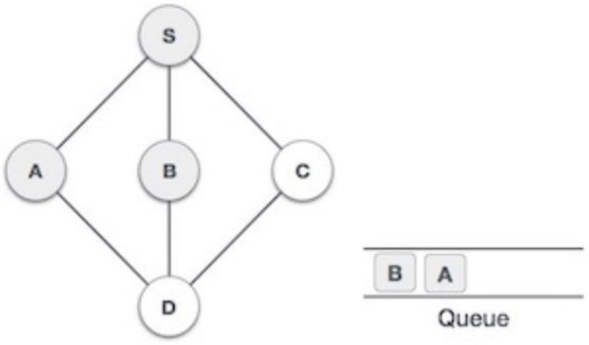
Breadth-first search is one of the simplest algorithms for searching a graph and the archetype for many important graph algorithms.

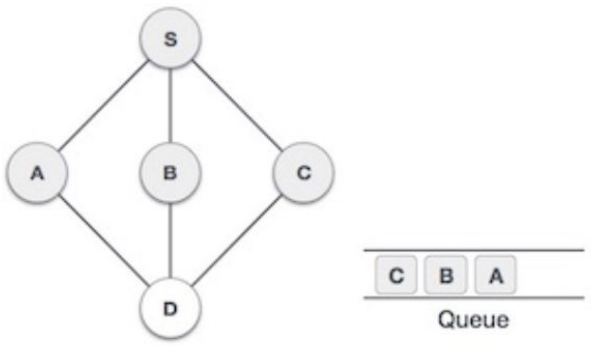
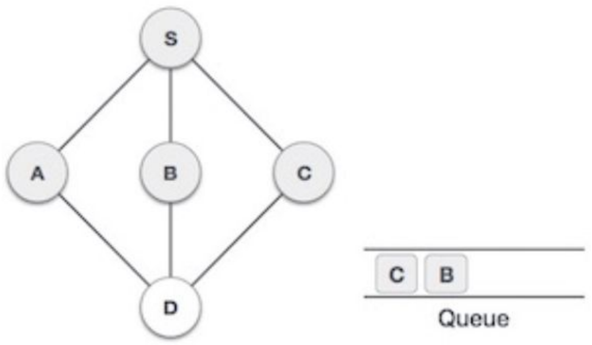
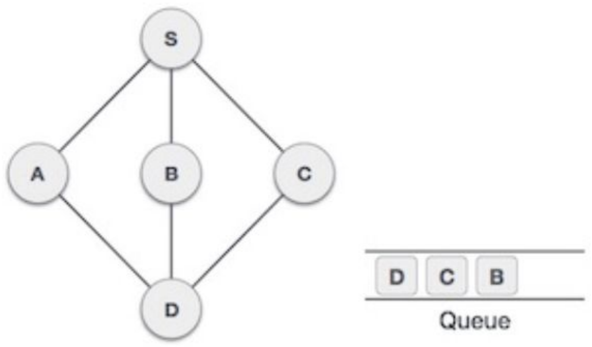
BFS works level by level. Initially, BFS starts at a given vertex, which is at level 0. In the first stage, it visits all vertices at level 1. In the second stage, it visits all vertices at the second level. These new vertices are those that are adjacent to level 1 vertices.

BFS continues this process until all the levels of the graph are completed. Generally, queue data structure is used for storing the vertices of a level. As similar to DFS, assume that initially all vertices are marked as unvisited. Vertices that have been processed and removed from the queue are marked visited. We use a queue to represent the visited set as it will keep the vertices in order of when they were first visited.

Breadth-first search is so named because it expands the frontier between discovered and undiscovered vertices uniformly across the breadth of the frontier. That is, the algorithm discovers all vertices at distance k from s before discovering any vertices at distance k + 1.

Similarly, we have an example for BFS as well using queues.

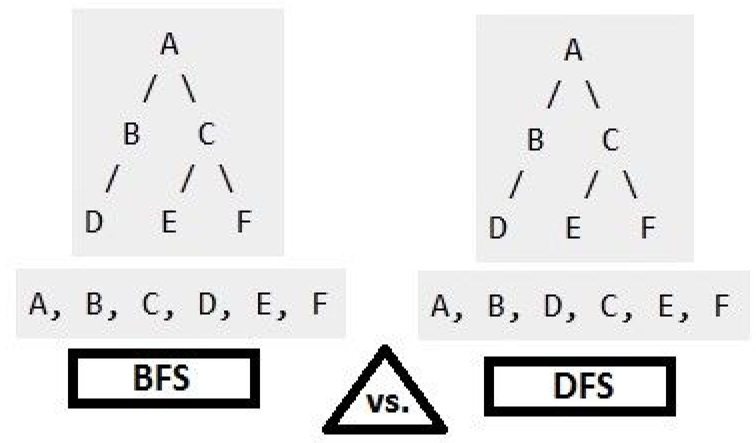
Step	Traversal	Description
1		Initialize the queue.
2		We start from visiting **S** (starting node), and mark it as visited.
3		We then see an unvisited adjacent node from **S**. In this example, we have three nodes but alphabetically we choose **A**, mark it as visited and enqueue it.
4		Next, the unvisited adjacent node from **S** is **B**. We mark it as visited and enqueue it.

5		Next, the unvisited adjacent node from **S** is **C**. We mark it as visited and enqueue it.
6		Now, **S** is left with no unvisited adjacent nodes. So, we dequeue and find **A**.
7		From **A** we have **D** as unvisited adjacent node. We mark it as visited and enqueue it.

Applications of BFS

- Finding all connected components in an undirected graph.
- Finding the shortest path between two nodes.

Comparison of BFS vs DFS on a simple graph:



Exercise

A. Silver Badge Problem (Mandatory)

The silver problem has two parts this time and both of them are mandatory.

1. Download the Lab5 folder from Moodle.
2. **For Part 1:**
   a. Follow the TODOs in Part1_Graph.cpp and complete the printGraph() function which prints all the graph vertices with their adjacency list.
3. **For Part 2:**
   a. Follow the TODOs and in Part2_Graph.cpp, complete the **findShortestPath()** function.
   b. This function finds the length of the shortest path between a source and destination vertex in an undirected and unweighted graph.

B. Gold Badge Problem (Not Mandatory)

1. In Part2_Graph.cpp, complete the **printPath()** function after modifying the **findShortestPath()** function so as to print the shortest path after finding it's length.