# MATH 140 Final Exam

Spring 2030

Name: ______________________________ ID: ____________

Instructor: ______________________________

**Instructions:**

1. This is a closed book exam.
2. Show your answers and arguments for your answers in the space provided.
3. Answers without proper justification **will not** receive full credit.
4. Put your final answers in the provided answer boxes.
5. **Calculators and electronic devices are not allowed.**

**Question 1. (24 points)** Evaluate the limit or show that it does not exist. If it does not exist, determine whether the limit is $\infty$, -$\infty$, or neither.

(a) $lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 10x + 21}$

(b) $lim_{x \to \infty} \dfrac{2e^x + 3e^{-x}}{5e^x - 7e^{-x}}$

(c) $lim_{x \to 0} \dfrac{|x|}{x}$

**Question 2. (24 points)** Differentiate the following functions.

(a) $f(x) = \tan(x)$

(b) $f(x) = 2e^x + \frac{1}{\sqrt{x}} - x^4 + 3x^{\sqrt{5}} + \pi^3 + \frac{e^2}{x}$

(c) $f(x) = \big(\sin(7x) + \frac{1}{2}\cos(x)\big)^4$

**Question 3. (24 points)** Use the guidelines for curve sketching to sketch the graph of the following function:

$$f(x) = \frac{x^3}{x-2}$$

**Question 4. (24 points)** Let $x$ and $y$ be selected via the equation $x^2+xy+y^2 = 3$

(a) Using implicit differentiation, express the derivative $\frac{\partial y}{\partial x}$ in terms of $x$ and $y$.

(b) Find the equation of the line tangent to the curve given above at the point $(1, -2)$.

**Question 5. (24 points)** Let $f(x) = 3x + \frac{27}{x}$.

(a) Find the critical points of $f$.

(b) For each critical point, determine whether $f$ has a local maximum, local minimum, or neither at that point.

**Question 6. (24 points)** Determine the maximum possible area of a rectangle of perimeter 30cm. (For full credit, you need to prove that your answer is the maximum possible, using calculus.)

**Question 7. (24 points)** In this problem, you must do part (a), however for full credit you only need to complete either part (b) or part (c), whichever you choose.

(a) Consider the function $f(x) = x^3 + 5x + 7$. Show that there exists a number $x_0$ such that $f(x_0) = 0$

(b) Find an interval of length $\frac{1}{2}$ or less containing the number $x_0$ from part (a). (Your answer must be explicit, like $7.2 \leq x \leq 7.7$, answers like $x_0 - \frac{1}{4} \leq x \leq x_0 + \frac{1}{4}$ will not receive credit.)

(c) Prove that $x = x_0$ from part (a) is the only real number for which $f(x) = 0$.

**Question 8. (24 points)**
(a) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at the point $a = 0$.

(b) Use the above approximation to estimate $\sqrt{0.9}$ and $\sqrt{0.99}$.

**Question 9. (24 points)** Estimate the integral

$$\int_{-1/2}^{3/2} \frac{1}{x^4+1} dx$$

Using a Riemann sum with four equal width intervals, and by choosing the sample point to be the left endpoint of each interval.

**Question 10. (24 points)** Find the indicated derivative using the Fundamental Theorem of Calculus.

(a) $\frac{\partial}{\partial x}\left(\int_x^1 t^2 \, dt\right)$

(b) $\frac{\partial}{\partial x}\left(\int_0^{x^2} \cos^2(t) \, dt\right)$

**Question 11. (24 points)** Evaluate the following integrals:

(a) $\int x^2\left(1+\frac{1}{\sqrt{x}}\right) dx$

(b) $\int_{-1}^{1} x^2 \sin(x^3) \, dx$

**Question 12. (24 points)** Find the area of the region bounded by the curves $y = 4x^2$ and $y = x^2 + 3$.