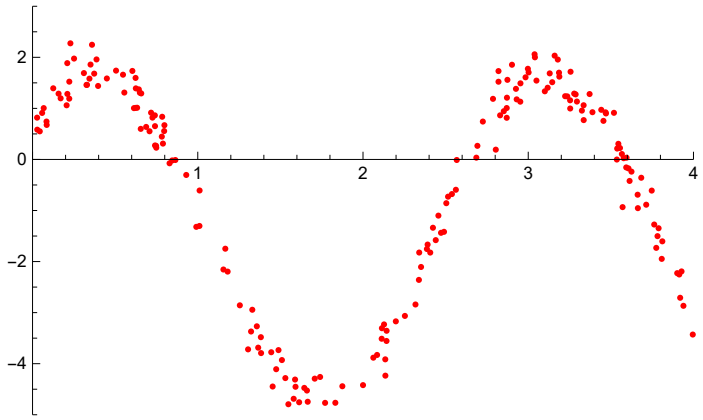
Project 9: Regression via Optimization

Note: Perhaps this should be called parameter estimation via optimization.

Consider the following data.

In[309]:=
```
numPoints = 200;
X = RandomVariate[UniformDistribution[{0, 4}], numPoints];
error = RandomVariate[NormalDistribution[0, 0.3], numPoints];
g[s_] := 3.15 * Sin[2.3 * s + 0.7] - 1.5;
Y = g[X] + error;
data = Transpose[{X, Y}];
ListPlot[data, PlotStyle → Red, PlotRange → {{0, 4}, {-5, 3}}]
```

Out[315]=



The aim of this project is to use the Steepest Descent algorithm acting on the "spring potential energy" function associated to the data above, and the family of functions given by

In[316]:=
```
h[s] = a * Sin[b * s + c] + d;
```

... Set: Tag Real in 0.01[s] is Protected.

where a, b, c, and d are unknown constants, which become the arguments of the "spring potential energy" function. You can (and should) copy and edit code from class to achieve this. Add this code below, and export this notebook to a pdf and submit it to GradeScope. Your completed work should contain a working Steepest Descent algorithm which starts at the function h(s)=0.5 sin(x); that is, corresponding to a=0.5, b=c=d=0. Additionally, you must plot the optimizing function your code obtains, overlayed on the above data, to show that it is a good fit.

In[317]:=
```
model[a_, b_, c_, d_, s_] := a * Sin[b * s + c] + d;
F[a_, b_, c_, d_] := Module[{}, y[z_] := model[a, b, c, d, z];
  Total[(Y - y[X])^2] / numPoints]
dF[a_, b_, c_, d_] = {D[Expand[F[a, b, c, d]], a],
  D[Expand[F[a, b, c, d]], b], D[Expand[F[a, b, c, d]], c], D[Expand[F[a, b, c, d]], d]}

maxGradAllowed = 1.0;
x = {2, 2, 2.5, -1};
h = 0.1;
threshold = 0.1;
grad = Apply[dF, x];

While[Sqrt[grad.grad] > threshold,
 If[Sqrt[grad.grad] > maxGradAllowed, grad = maxGradAllowed * grad / Sqrt[grad.grad]];
 xNew = x - h * grad;
 If[Apply[F, xNew] < Apply[F, x], x = xNew, h = h / 10];
 grad = Apply[dF, x];]
Print[{x, Apply[F, x]}];

f[s_] := (x[[1]]) * Sin[x[[2]] * s + (x[[3]])] + (x[[4]]);
A = ListPlot[data, PlotStyle → Red, PlotRange → {{0, 4}, {-7, 5}}];
B = Plot[f[s], {s, 0, 4}, PlotRange → {{0, 4}, {-7, 5}}];
Show[B, A]
```

Out[319]=

$$\left\{-0.00818665 \sin[0.0276254\, b + c] + \frac{1}{100} d \sin[0.0276254\, b + c] + \frac{1}{100} a \sin[0.0276254\, b + c]^2 - 0.00583157 \sin[0.0286347\, b + c] + \frac{1}{100} d \sin[0.0286347\, b + c] + \text{\texttt{··· 698 ···}} + \frac{1}{100} d \sin[3.93927\, b + c] + \frac{1}{100} a \sin[3.93927\, b + c]^2 + 0.0342931 \sin[3.99572\, b + c] + \frac{1}{100} d \sin[3.99572\, b + c] + \frac{1}{100} a \sin[3.99572\, b + c]^2,\ \text{\texttt{··· 1 ···}},\ \text{\texttt{··· 1 ···}},\ 0.979059 + 2\, d + \text{\texttt{··· 199 ···}} + \frac{1}{100} a \sin[3.99572\, b + c]\right\}$$

large output | show less | show more | show all | set size limit...

```
{{3.08646, 2.30765, 0.686258, -1.45279}, 0.0894578}
```

Out[330]=