

#1 (24 points) Evaluate the limit or show that it does not exist. If it does not exist, determine whether the limit is ∞ , $-\infty$, or neither.

a) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 10x + 21}$

b) $\lim_{x \rightarrow \infty} \frac{2e^x + 3e^{-x}}{5e^x - 7e^{-x}}$

c) $\lim_{x \rightarrow 0} \frac{|x|}{x}$

#2 (24 points) Differentiate the following functions.

a) $f(x) = \tan(x)$

b) $f(x) = 2e^x + \frac{1}{\sqrt{x}} - x^4 + 3x^{\sqrt{5}} + \pi^3 + \frac{e^2}{x}$

c) $f(x) = \left(\sin(7x) + \frac{1}{2} \cos(x) \right)^4$

#3 (24 points) Use the guidelines for curve sketching to sketch the graph of the following function: $f(x) = \frac{x^3}{x-2}$.

#4 (24 points) Let x and y be related via the equation $x^2 + xy + y^2 = 3$.

a) using implicit differentiation, express the derivative $\frac{dy}{dx}$ in terms of x and y .

b) Find the equation of the line tangent to the curve given above at the point $(1, -2)$.

#5 (24 points) Let $f(x) = 3x + \frac{27}{x}$.

a) Find the critical points of f .

b) For each critical point, determine whether f has a local maximum, local minimum, or neither at that point.

#6 (24 points) Determine the maximum possible area of a rectangle of perimeter 30cm. (For full credit, you need to prove that your answer is the maximum possible, using calculus.)

#7 (24 points) In this problem, you must do part (a), however for full credit you only need to complete either part (b) or part (c), whichever you choose.

(a) consider the function $f(x) = x^3 + 5x + 7$. Show that there exists a number x_0 such that

$$f(x_0) = 0.$$

b) Find an interval of length $\frac{1}{2}$ or less containing the number x_0 from part (a). (Your answer must be explicit, like $7.2 \leq x \leq 7.7$; answers like $x_0 - \frac{1}{4} \leq x \leq x_0 + \frac{1}{4}$ will not receive credit.)

c) Prove that $x = x_0$ from part (a) is the only real number for which $f(x) = 0$.

#8 (24 points)

a) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at the point $a=0$.

b) Use the above approximation to estimate $\sqrt{0.9}$ and $\sqrt{0.99}$.

#9 (24 points) Estimate the integral

$$\int_{-1/2}^{3/2} \frac{1}{x^4+1} dx$$

Using a Riemann sum with four equal width intervals, and by choosing the sample point to be the left endpoint of each interval.

#10 (24 points) Find the indicated derivative using the Fundamental Theorem of Calculus.

(a) $\frac{d}{dx} \left(\int_x^1 t^2 dt \right)$

(b) $\frac{d}{dx} \left(\int_0^{x^2} \cos^2(t) dt \right)$

#11 (24 points) Evaluate the following integrals

(a) $\int x^2 \left(1 + \frac{1}{\sqrt{x}}\right) dx$

(b) $\int_{-1}^1 x^2 \sin(x^3) dx$

#12 (24 points) Find the area of the region bounded by the curves $y = 4x^2$ and $y = x^2 + 3$.