Project 4: Answer Sheet

Question 1

a) $-\frac{5}{4}$

(Debug) In[∘]:= `Limit[(x^2 - x - 6) / (x^2 - 10 x + 21), x → 3]`

(Debug) Out[∘]= $-\frac{5}{4}$

b) $\frac{2}{5}$

(Debug) In[∘]:= $\text{Limit}\left[\frac{2\,e^{x}+3\,e^{-x}}{5\,e^{x}-7\,e^{-x}},\ x\to\infty\right]$

(Debug) Out[∘]= $\boxed{\frac{2}{5}\ \ \text{if}\ \ \text{Log}[e]>0}$

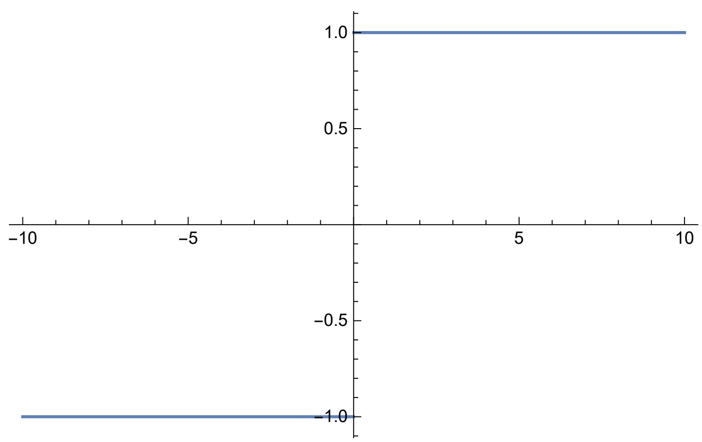
c) Limit DNE. Neither.

In[∘]:= $\text{Limit}\left[\frac{\text{Abs}[x]}{x},\ x\to 0\right]$

Out[∘]= Indeterminate

In[∘]:= $\text{Plot}\left[\frac{\text{Abs}[x]}{x},\ \{x,\ -10,\ 10\}\right]$

Out[∘]=



Question 2

a) $\sec^2(x)$

In[1]:= `D[Tan[x], x]`

Out[1]= $\text{Sec}[x]^2$

b) $-\frac{e^2}{x^2} - \frac{1}{2\,x^{3/2}} - 4\,x^3 + 3\,\sqrt{5}\,x^{-1+\sqrt{5}} + 2\,e^x\,\text{Log}[e]$

In[3]:= $D\left[2\,e^x + \frac{1}{\sqrt{x}} - x^4 + 3\,x^{\sqrt{5}} + \pi^3 + \frac{e^2}{x},\ x\right]$

Out[3]= $-\frac{e^2}{x^2} - \frac{1}{2\,x^{3/2}} - 4\,x^3 + 3\,\sqrt{5}\,x^{-1+\sqrt{5}} + 2\,e^x\,\text{Log}[e]$

c) $4\left(7\,\text{Cos}[7\,x] - \frac{\text{Sin}[x]}{2}\right)\left(\frac{\text{Cos}[x]}{2} + \text{Sin}[7\,x]\right)^3$

In[2]:= $D\left[\left(\text{Sin}[7\,x] + \frac{1}{2}\,\text{Cos}[x]\right)^4,\ x\right]$

Out[2]= $4\left(7\,\text{Cos}[7\,x] - \frac{\text{Sin}[x]}{2}\right)\left(\frac{\text{Cos}[x]}{2} + \text{Sin}[7\,x]\right)^3$

Question 3

1) Domain and Range

In[4]:= $\text{FunctionDomain}\left[\frac{x^3}{x-2},\ x\right]$

Out[4]= `x < 2 || x > 2`

In[5]:= $\text{FunctionRange}\left[\frac{x^3}{x-2},\ x,\ y\right]$

Out[5]= True

Domain is $x < 2, x > 2$.

Range is unrestricted. $(-\infty, \infty)$

2) y - intercepts and x - intercepts

Out[6]= $\text{Intercepts}\left[\frac{x^3}{-2+x},\ \{x, y\}\right]$

There is an intercept at point (0, 0)

3) Symmetry

Determine if the function is odd or even.

In[7]:=
```
myEvenFunction[x_] := x^2/(x - 2)
Equal[myEvenFunction[x], myEvenFunction[-x]]
```

Out[8]= $\frac{x^2}{-2+x} == \frac{x^2}{-2-x}$

In[9]:=
```
myOddFunction[x_] := x^2/(x - 2)
FullSimplify[ForAll[x, myOddFunction[x] == myOddFunction[-x]]]
```

Out[10]= False

f (x) is an even function since f (-x) = f (x). Therefore, it is symmetric about the y - axis.

4) Vertical and Horizontal Asymptote(s)

Based on the domain, there is a vertical asymptote at x = 2.

5) First Derivative: Increase/Decrease and Relative Extrema

In[11]:=
```
Dt[x^2/(x - 2), x]
```

Out[11]= $\frac{2x}{-2+x} - \frac{x^2}{(-2+x)^2}$

$f'(x) = \frac{2x}{-2+x} - \frac{x^2}{(-2+x)^2}$

At f'(x) = 0, the critical point is at x = 2.

For interval (- ∞, 2), choose value x = 1. For interval (2, ∞), choose value x = 5. At x = 1, the function is decreasing since -3 < 0 . At x = 5, the function is increasing since $\frac{5}{9} > 0$.

In[12]:=
```
f3[x_] := 2 x/(-2 + x) - x^2/(-2 + x)^2
f3[1]
```

Out[13]= -3

In[15]:=
```
f3[5]
```

Out[15]= $\frac{5}{9}$

6) Second Derivative: Concavity and Inflection Points

In[25]:=
```
f3'[x]
```

Out[25]= $\frac{2}{-2+x}-\frac{4\,x}{(-2+x)^2}+\frac{2\,x^2}{(-2+x)^3}$

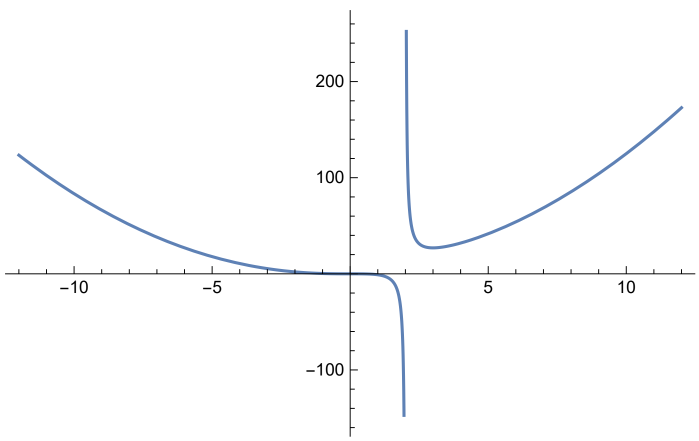
In[26]:=
```
Solve[f3'[x] == 0, x]
```

Out[26]= { }

7) Graph the Function

In[17]:= Plot$\left[\frac{x^3}{x-2}, \{x, -12, 12\}\right]$

Out[17]=



Question 4

a) $-\frac{2\,x+y}{x+2\,y}$

In[36]:=
```
dy/dx of x^2 + xy + y^2 = 3
ImplicitD[x^2 + x*y + y^2 == 3, D[y, x]]
```

Out[36]= $-\frac{2\,x+y}{x+2\,y}$

b) y = -2

In[37]:=
$$f4[x\_, y\_] := -\frac{2x+y}{x+2y}$$

```
f4[1, -2]
```

Out[38]= 0

The line tangent to the curve at the point (1,-2) is y = -2. At this point, the gradient is 0. Therefore, the slope equals 0, which meant that the line is y= -2.

Question 5

a) x = 3, x = -3

In[71]:=
$$f5[x\_] := 3x + \frac{27}{x}$$

```
f5'[x]
```

Out[72]= $3 - \frac{27}{x^2}$

In[73]:=
```
Solve[f5'[x] == 0, x]
```

Out[73]= {{x → -3}, {x → 3}}

There are two critical points at x = -3 and x = 3.

b) Absolute Minimum at x = 3, y = 18. No local minimum or maximum.

In[44]:=
```
Dt[f5'[x], x]
```

Out[44]= $\frac{54}{x^3}$

In[69]:=
$$f[x\_] := \frac{54}{x^3}$$

```
f[3]
```

Out[70]= 2

In[74]:=
```
f5'[3]
```

Out[74]= 0

Since at f’’(3) = 2 which is greater than f’(3) = 0, there is an absolute minimum at x = 3.

Question 6

Equation for Perimeter: 2(x+y) = 30 → y = 15 - x

Equation for Area: xy → x(15 - x)

In[80]:=
```
Dt[15 x - x^2, x]
```

Out[80]= $15 - 2x$

In[83]:=
```
Dt[15 - 2 x, x]
```

Out[83]= -2

To be the maximum value possible, dA/dx must equal 0.
Since - 2 < 0, the area is at its maximum.
Therefore, 15 - 2x = 0, which meant x = 15/2 , and y = 15/2.

In[84]:=
```
(15 / 2) ^2
```

Out[84]= $\frac{225}{4}$

The maximum possible area is $\frac{225}{4}$ cm^2.

Question 7

a)

In[87]:=
```
f7[x_] := x^3 + 5 x + 7
Solve[f7[x] == 0, x]
```

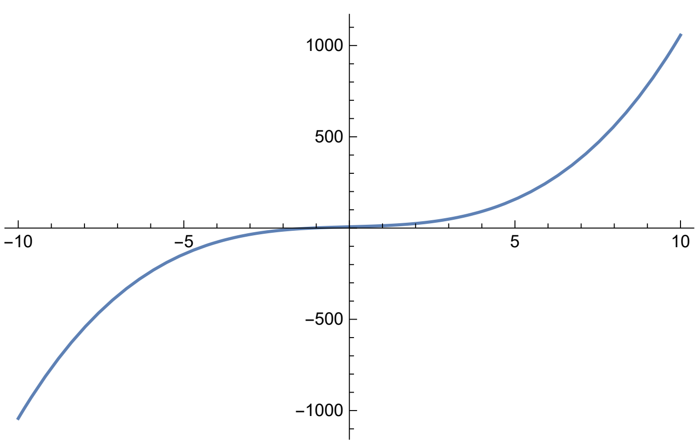
Out[88]= {{x → -1.12...}, {x → 0.560... - 2.44... ⅈ}, {x → 0.560... + 2.44... ⅈ}}

In[90]:=
```
N[{{x → Root[7 + 5 #1 + #1^3 &, 1, 0]},
  {x → Root[7 + 5 #1 + #1^3 &, 2, 0]}, {x → Root[7 + 5 #1 + #1^3 &, 3, 0]}}]
```

Out[90]= {{x → -1.11944}, {x → 0.559719 - 2.43718 ⅈ}, {x → 0.559719 + 2.43718 ⅈ}}

In[89]:=
```
Plot[f7[x], {x, -10, 10}]
```

Out[89]=



There is one value where x0 = 0. This occur as an x - intercept at x = -1.11944.

In[91]:= `f7[-2]`

Out[91]= -11

In[92]:= `f7[-1]`

Out[92]= 1

To show that there is a number x0 which equals 0, two values of x can be used to narrow down the number. Since the function is continuous, there is a number that exists between (-2, 1) such that f(x0) = 0.

b)

In[93]:= `f7[-1.5]`

Out[93]= -3.875

In[94]:= `f7[-1]`

Out[94]= 1

Between the interval of [-1.5, -1] lies a number such that f(x0) = 0.

c)

In[95]:= `Dt[f7[x], x]`

Out[95]= $5+3x^2$

Since the values of f'(x) is greater than 0 for all real value of x, the function is increasing. Therefore, there is only only one intercept.

Question 8

a)

In[97]:= $\text{Dt}\left[\sqrt{1-x},\, x\right]$

Out[97]= $-\frac{1}{2\sqrt{1-x}}$

Linear approximation of a f(x) = f(a) + f'(a)(x-a)

In[101]:= $\sqrt{1-0}+\left(-\frac{1}{2\sqrt{1-0}}\right)(x-0)$

Out[101]= $1-\frac{x}{2}$

b)

In[106]:=
```
f8[x_] := 1 - x/2
f8[0.1]
```
Out[107]= 0.95

In[108]:=
```
f8[0.01]
```
Out[108]= 0.995

$\sqrt{0.9}$ is approximately 0.95.
$\sqrt{0.99}$ is approximately 0.995.

Question 9

There are 4 equal intervals. $\left[\frac{-1}{2}, 0\right]$, $\left[0, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 1\right]$, and $\left[1, \frac{3}{2}\right]$.

$\triangle x = \frac{b-a}{n} = \frac{\frac{3}{2}-\left(\frac{1}{2}\right)}{4} = \frac{1}{2}$

In[109]:=
```
f9[x_] := 1/(x^4 + 1)
(1/2) (f9[-1/2] + f9[0] + f9[1/2] + f9[1])
```
Out[110]= $\frac{115}{68}$

In[111]:=
```
N[115/68]
```
Out[111]= 1.69118

The integral is approximately equal to $\frac{115}{68}$ or 1.69118.

Question 10

a) $-x^2$

In[112]:= $D\left[\int_x^1 t^2 \, dt, x\right]$

Out[112]= $-x^2$

b) $\frac{1}{4}\left(4\,x + 4\,x\,\text{Cos}\left[2\,x^2\right]\right)$

In[113]:=
```
D[∫_0^(x^2) (Cos[t])^2 ⅆt, x]
```

Out[113]= $\frac{1}{4}\left(4\,x + 4\,x\,\text{Cos}\left[2\,x^2\right]\right)$

Question 11

a) $\frac{2\,x^{5/2}}{5} + \frac{x^3}{3} + c$

In[114]:=
```
∫x^2 (1 + 1/√x) ⅆx
```

Out[114]= $\frac{2\,x^{5/2}}{5} + \frac{x^3}{3}$

b) 0

In[115]:=
```
∫_-1^1 x^2 Sin[x^3] ⅆx
```

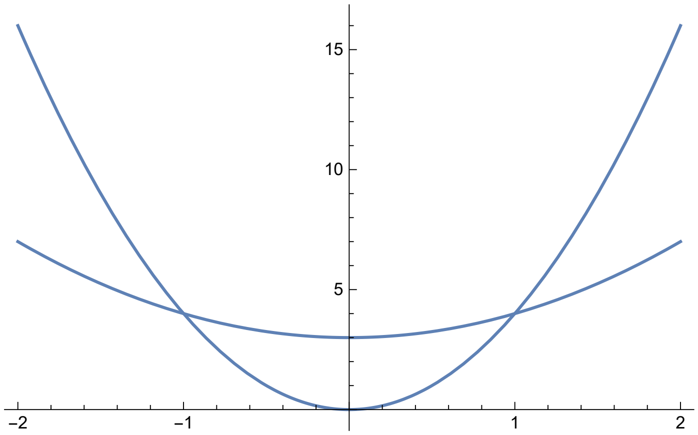
Out[115]= 0

Question 12

In[127]:=
```
p1 = Plot[4 x^2, {x, -2, 2}]
p2 = Plot[(x^2) + 3, {x, -2, 2}]
```

In[146]:=
```
Show[p1, p2]
```

Out[146]=



In[149]:=
```
f1[x_] := 4 x^2;
g1[x_] := (x^2) + 3;
```

In[151]:=
```
sol = NSolve[f1[x] == g1[x], x]
```

Out[151]=
```
{{x → -1.}, {x → 1.}}
```

In[152]:=
```
{a, b} = x /. sol
```

Out[152]=
```
{-1., 1.}
```

In[157]:=
```
Integrate[g1[x] - f1[x], {x, a, b}]
```

Out[157]=
```
4.
```

The area bounded by the two curves is equal to 4.