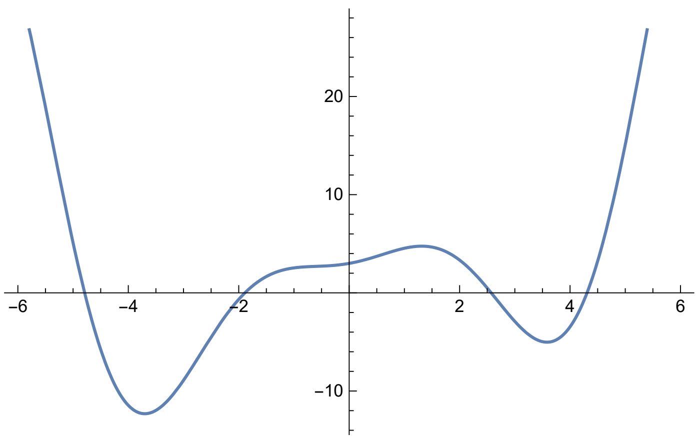
Project 5 - Bisection and Newton’s Method

Bisection Method

In[682]:=
```
F[x_] := x^2 Cos[x] + x + 3;
Plot[F[x], {x, -6, 6}]
```

Out[683]=



In[684]:=
```
a = -4;
b = 2;
```

In[686]:=
```
F[a]
N[F[a]]
```

Out[686]= $-1 + 16 \text{Cos}[4]$

Out[687]= -11.4583

In[688]:=
```
F[b]
N[F[b]]
```

Out[688]= $5 + 4 \text{Cos}[2]$

Out[689]= 3.33541

Since F[a]*F[b] < 0. There is a difference in signs, meaning that the function is discontinuous.

In[690]:=
```
N[F[a] * F[b]] < 0
```

Out[690]= True

In[691]:=
```
For[i = 0, i < 10, i++, Print[StringForm["Loop #: ``", i + 1]];
 c = (a + b) / 2;
 If[F[c] < 0, a = c];
 If[F[c] > 0, b = c];
 c = (a + b) / 2;
 Print[N[{"a = ", a, ", b = ", b, ", c = ", c, ", F[c] = ", F[c]}]]]
```

```
Loop #: 1
{a = , -4., , b = , -1., , c = , -2.5, , F[c] = , -4.50715}
Loop #: 2
{a = , -2.5, , b = , -1., , c = , -1.75, , F[c] = , 0.704121}
Loop #: 3
{a = , -2.5, , b = , -1.75, , c = , -2.125, , F[c] = , -1.50142}
Loop #: 4
{a = , -2.125, , b = , -1.75, , c = , -1.9375, , F[c] = , -0.283426}
Loop #: 5
{a = , -1.9375, , b = , -1.75, , c = , -1.84375, , F[c] = , 0.239846}
Loop #: 6
{a = , -1.9375, , b = , -1.84375, , c = , -1.89063, , F[c] = , -0.0144502}
Loop #: 7
{a = , -1.89063, , b = , -1.84375, , c = , -1.86719, , F[c] = , 0.114541}
Loop #: 8
{a = , -1.89063, , b = , -1.86719, , c = , -1.87891, , F[c] = , 0.0505051}
Loop #: 9
{a = , -1.89063, , b = , -1.87891, , c = , -1.88477, , F[c] = , 0.0181423}
Loop #: 10
{a = , -1.89063, , b = , -1.88477, , c = , -1.8877, , F[c] = , 0.00187477}
```

To find the estimate of c where F(c) = 0, we take the absolute value of the difference of a and b for all ten loops. Only one loop exist where the lowest absolute value of the difference was less than 0.01. That value of c = -1.8877 will result in the lowest value of of F[c] in the 10 resulting loops, which is F[c] = 0.0018.

In[692]:=
```
If[Abs[a - b] ≤ 0.01, Print[N[c]], Exit[]]
```

```
-1.8877
```

Newton’s Method

In[693]:=
```
a = -3;
```

```
In[694]:= For[i = 1, i ≤ 5, i++,
            a = a - F[a]/F'[a];];
          Print@Row[{"c = ", N[a], ", F[c] = ", N[F[a]]}]
```

c = -1.88803, F[c] = -2.22045×10^{-16}

Using Newton's Method, we were able to achieve a more accurate value of c where F[c] is closer to 0 than the Bisection method. In the loop of 5 iterates, c = -1.8803 has a F[c] = -2.22045 * 10^{-16} which is closer than the values found using the Bisection method where c = -1.8877 and F[c] = 0.00187477.

Numerical Newton's Method

```
In[696]:= h = 0.001;
          g[x_] := (F[x + h] - F[x])/h;

In[698]:= For[i = 1, i ≤ 5, i++,
            a = a - F[a]/g[a];];
          Print@Row[{"c = ", N[a], ", F[c] = ", N[F[a]]}]
```

c = -1.88803, F[c] = 0.

This version of the algorithm using the derivative of F was able to find c where F[c] = 0. Whereas, the other methods were only able to approximate.

Building a Function Call

```
In[700]:= Clear[bisectionMethod];
          bisectionMethod[f_, a_, b_, n_] := Module[{a0, b0, c, y},
            a0 = a;
            b0 = b;
            c = (a0 + b0) / 2;
            For[i = 0, i < n, i++, Print[StringForm["Loop #: ``", i + 1]];
             y = f[c];
             If[y < 0, a0 = c, If[y > 0, b0 = c, Break[]]];
             c = (a0 + b0) / 2;
             x = f[a0];
             z = f[b0];
             Print [{{N[a0], N[b0]}, {N[x], N[z]}}]
            ]
           ]
```

Problems with Newton's Method

In[702]:=
```
Clear[x0, x1, x, H, k]
H[x_] := ArcTan[x];
k[x_] := (H[x + h] - H[x]) / h;
x0 = 0.5;
h = 0.001;
For[i = 1, i ≤ 10, i + 1, Print["Loop #:", i++];
 x = x0 - H[x0] / k[x0];
 x0 = x;
 Print[{x, H[x]}]]
```

Loop #:1

{-0.0797915, -0.0796228}

Loop #:2

{0.000331914, 0.000331914}

Loop #:3

$\{-2.45182\times10^{-10}, -2.45182\times10^{-10}\}$

Loop #:4

$\{8.17273\times10^{-17}, 8.17273\times10^{-17}\}$

Loop #:5

$\{-2.72424\times10^{-23}, -2.72424\times10^{-23}\}$

Loop #:6

$\{9.08081\times10^{-30}, 9.08081\times10^{-30}\}$

Loop #:7

$\{-3.02693\times10^{-36}, -3.02693\times10^{-36}\}$

Loop #:8

$\{1.00898\times10^{-42}, 1.00898\times10^{-42}\}$

Loop #:9

$\{-3.36326\times10^{-49}, -3.36326\times10^{-49}\}$

Loop #:10

$\{1.12109\times10^{-55}, 1.12109\times10^{-55}\}$

In[708]:=
```
Clear[x0, x1, x, H, k]
H[x_] := ArcTan[x];
k[x_] := (H[x + h] - H[x])/h;
x0 = 2.0;
h = 0.001;
For[i = 1, i ≤ 10, i + 1, Print["Loop #:", i++];
 x = x0 - H[x0]/k[x0];
 x0 = x;
 Print[{x, H[x]}]]
```

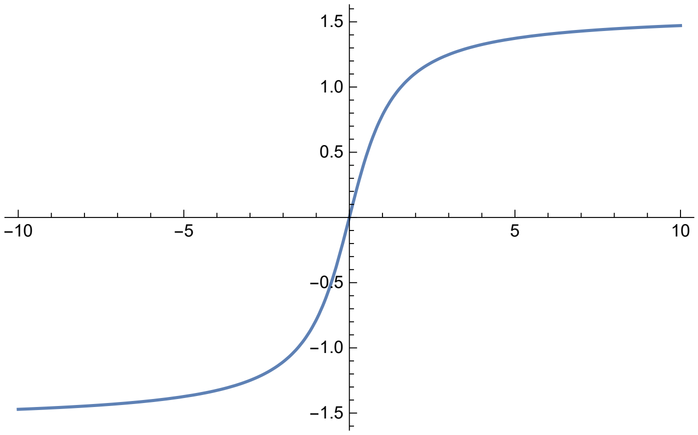
```
Loop #:1
{-3.53796, -1.29533}
Loop #:2
{13.9667, 1.49932}
Loop #:3
{-280.022, -1.56723}
Loop #:4
{122611., 1.57079}
Loop #:5
{-2.36594×10^10, -1.5708}
Loop #:6
```

Power: Infinite expression $\frac{1}{0.}$ encountered.

```
{ComplexInfinity, Indeterminate}
Loop #:7
{Indeterminate, Indeterminate}
Loop #:8
{Indeterminate, Indeterminate}
Loop #:9
{Indeterminate, Indeterminate}
Loop #:10
{Indeterminate, Indeterminate}
```

In[714]:=
```
Plot[H[x], {x, -10, 10}]
```

Out[714]=

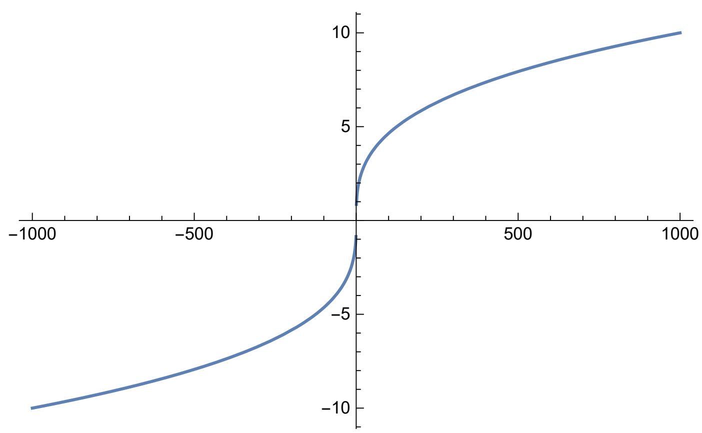


Arctan(x) with initial condition at x = 2 is indeterminate after the fifth loop and there is an error in recursion. This occurred because Newton's method generate a sequence {x_n} with alternating signs that are the same distance from the origin. (x_n+1 = -x_n). However, after the fifth loop, it fails because x_n+1 > x_n and the derivative of G is 0. Dividing by G'(x) = 0 will result in an indeterminate answer. This can be seen in the graph where the graph flattens out, creating a horizontal tangent line.

In[715]:=
```
Clear[x0, x1, x, G, l, h]
G[x_] := Sign[x] * (Abs[x]) ^ (1 / 3);
l[x_] := (G[x + h] - G[x]) / h;
x0 = 1.0;
h = 0.001;
For[i = 1, i ≤ 10, i + 1, Print["Loop #:", i++];
 x = x0 - G[x0] / l[x0];
 x0 = x;
 Print[{x, G[x]}]]
```

```
Loop ⌗:1
{-2.001, -1.26013}
Loop ⌗:2
{4.001, 1.58753}
Loop ⌗:3
{-8.003, -2.00025}
Loop ⌗:4
{16.005, 2.5201}
Loop ⌗:5
{-32.011, -3.17517}
Loop ⌗:6
{64.021, 4.00044}
Loop ⌗:7
{-128.043, -5.04025}
Loop ⌗:8
{256.085, 6.35031}
Loop ⌗:9
{-512.171, -8.00089}
Loop ⌗:10
{1024.34, 10.0805}
```

In[724]:=
```
Plot[G[x], {x, -1000, 1000}]
```

Out[724]=


The function is growing without converging at a root. The sequence for this function is x_n = ((-1)^n)*(2^(n-1)). This mean that no matter what the initial point x0 is, the function is rapidly increasing, moving further away from the root and diverges.