Idea

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- Type of linear regression :
  - ► Simple linear regression :  $\hat{y} = xw + b$  where  $\hat{y}, x, w, b$  is a scalar variable.
  - ► Multivariate linear regression :  $\hat{y} = xw + b = \bar{x}w$  where w, x are vectors,  $\hat{y}, b$  is a scalar number.

Pros and Cons

Pros

- Pros
  - Quick

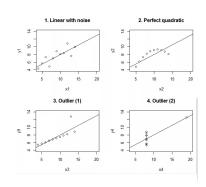
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  - Hard to scale with complicated, unlinear data



Model of Linear regression

Mathematics model:

Model of Linear regression

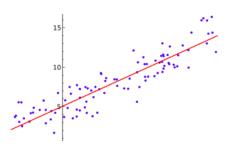
#### Mathematics model:

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- $\bullet$  Training data set : Y and X
- Need to find a matrix W where ŷ = Wx such that ŷ most fit to y in Y



#### Model of Linear regression

Lost function :

$$\mathbb{L} = \frac{1}{2} \sum_{i=1}^{N} (y_i - \hat{y})^2 = \frac{1}{2} \sum_{i=1}^{N} (y_i - \hat{x}_i w)^2 = \frac{1}{2} \|y - \bar{X}w\|_2^2$$

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• Derivative :

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• Minimum at :

$$w = \left(\bar{X}^T \bar{X}\right)^{-1} \bar{X}^T y$$