

# LINEAR REGRESSION

Idea

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- Linear regression is a linear approach to modeling the relationship between a scalar response and one or more explanatory variables.

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- Linear regression is a linear approach to modeling the relationship between a scalar response and one or more explanatory variables.
- Type of linear regression :
  - ▶ Simple linear regression :  $\hat{y} = xw + b$   
where  $\hat{y}, x, w, b$  is a scalar variable.
  - ▶ Multivariate linear regression :  $\hat{y} = xw + b = \bar{x}w$   
where  $w, x$  are vectors,  $\hat{y}, b$  is a scalar number.

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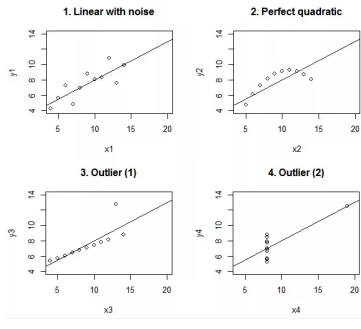
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- ▶ Quick
- ▶ Easy to implement

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- ▶ Essential to extract data to **independent** features
- ▶ May drop relations between explanatories
- ▶ Hard to scale with complicated, unlinear data



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## Model of Linear regression

Mathematics model :

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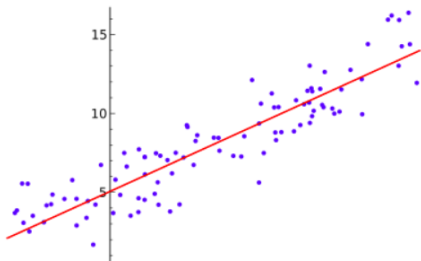
- Training data set :  $Y$  and  $X$

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## Model of Linear regression

Mathematics model :

- Training data set :  $Y$  and  $X$
- Need to find a matrix  $W$  where  $\hat{y} = Wx$  such that  $\hat{y}$  most fit to  $y$  in  $Y$



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## Model of Linear regression

- Lost function :

$$\mathbb{L} = \frac{1}{2} \sum_{i=1}^N (y_i - \hat{y})^2 = \frac{1}{2} \sum_{i=1}^N (y_i - \hat{x}_i w)^2 = \frac{1}{2} \|y - \bar{X}w\|_2^2$$

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- Derivative :

$$\frac{\partial \mathbb{L}(w)}{\partial w} = \bar{X}^T (\bar{X}w - y)$$

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- Minimum at :

$$w = (\bar{X}^T \bar{X})^{-1} \bar{X}^T y$$