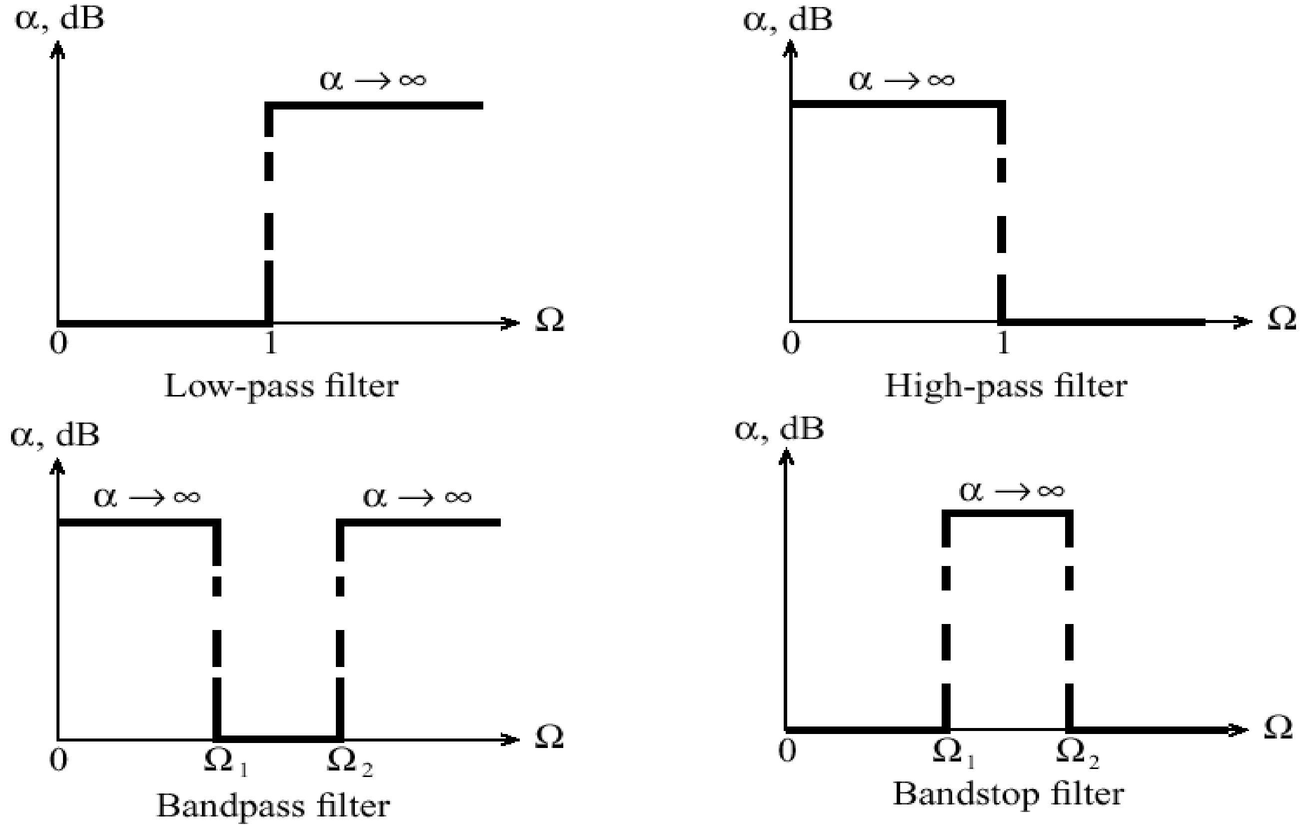
Analog Electronics for Communications

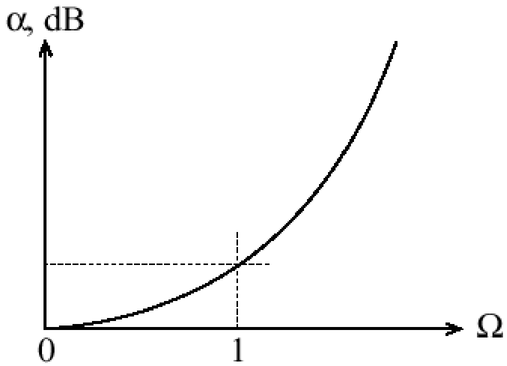
Semester 20192

Lecture 6

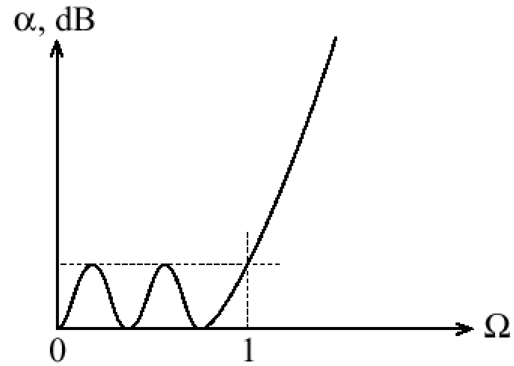
RF Filter Design – Basic Filter Types



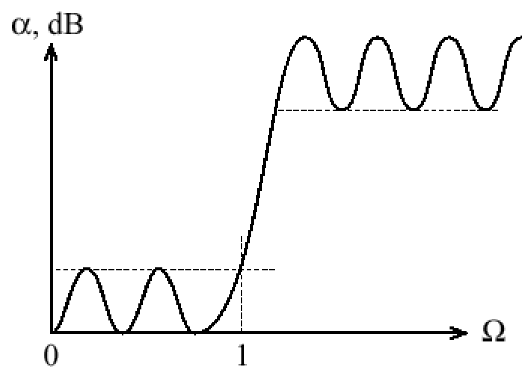
Filter Attenuation Profiles



Binominal filter



Chebyshev filter

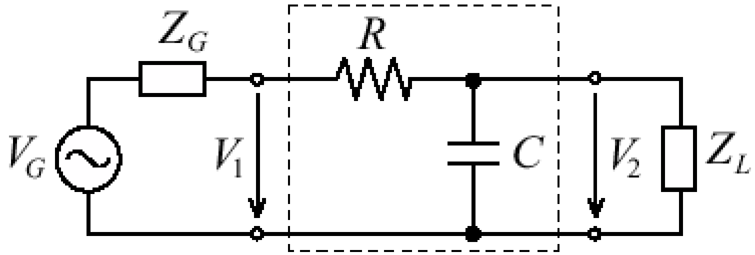


Elliptic filter

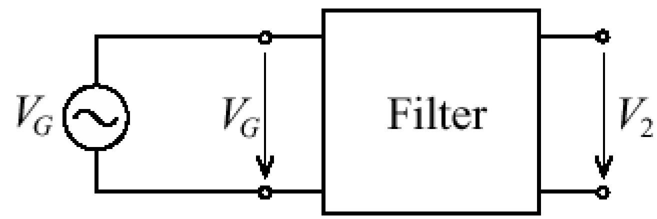
RF Filter Parameters

- Insertion Loss: $IL = 10\,log\,\frac{P_{in}}{P_L} = -10\,log\left(1 - \left|\Gamma_{in}\right|^2\right)$
- Ripple
- Bandwidth: $BW^{\,3\mathrm{dB}} = f_u^{\,3\mathrm{dB}} - f_L^{\,3\mathrm{dB}}$
- Shape Factor: $SF = \frac{BW_{A_{min}}}{BW_{A_{max}}}$
- Rejection

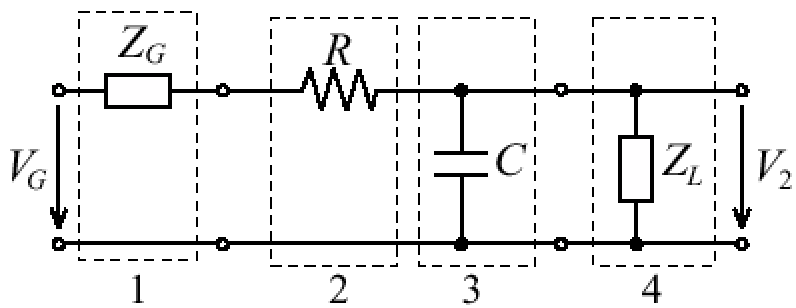
Low-Pass Filter



(a) Low-pass filter with load resistance.



(b) Network with input/output voltages



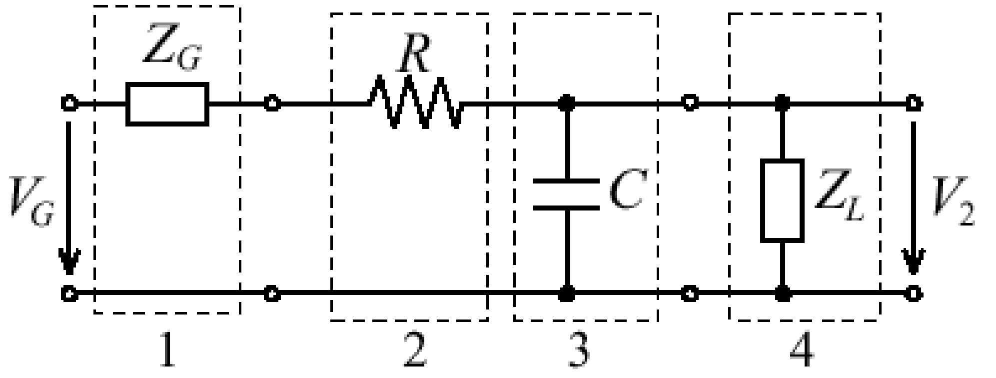
Cascading four ABCD-networks.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & R_G \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R_L & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \left(R + R_G\right)\left(j\omega C + \dfrac{1}{R_L}\right) & R_G + R_L \\ j\omega C + \dfrac{1}{R_L} & 1 \end{bmatrix}$$

RF Filter Parameters

Cascading four ABCD-networks.



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & R_G \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R_L & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+\left(R+R_G\right)\left(j\omega C+\dfrac{1}{R_L}\right) & R_G+R_L \\ j\omega C+\dfrac{1}{R_L} & 1 \end{bmatrix}$$

Low-Pass Filter Frequency Response

- Frequency Response from the ABCD Definitions:

$$A = \left.\frac{v_1}{v_2}\right|_{i_2=0}$$

- So the **Transfer Function** is Simply:

$$\frac{1}{A} = H(\omega) = \frac{1}{1 + j\omega\left(R_G + R\right)C}$$

Low-Pass Filter Frequency Response

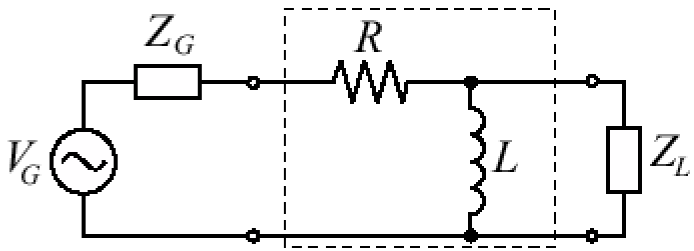
- Corresponding Phase is:

$$\phi(\omega) = tan^{-1}\left(\frac{Im\left\{H(\omega)\right\}}{Re\left\{H(\omega)\right\}}\right)$$

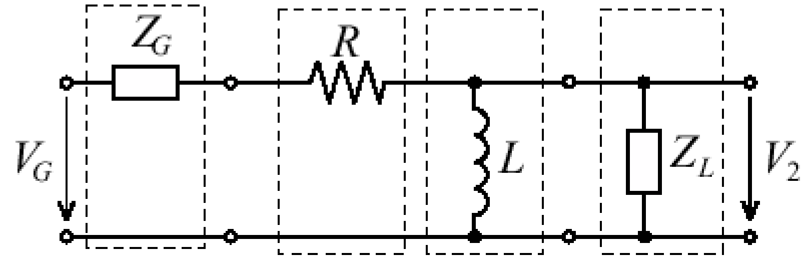
- Group Delay:

$$t_g = \frac{d\phi(\omega)}{d\omega}$$

High-Pass Filter



(a) High-pass filter with load resistance



(b) Network and input/output voltages

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & R_G \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{j\omega L} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_L} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \left(R + R_G\right)\left(\dfrac{1}{j\omega L} + \dfrac{1}{R_L}\right) & R_G + R_L \\ \dfrac{1}{j\omega L} + \dfrac{1}{R_L} & 1 \end{bmatrix}$$

High-Pass Filter Frequency Response

- Frequency Response from the ABCD Definitions:

$$A = \left. \frac{v_1}{v_2} \right|_{i_2 = 0}$$

- So the **Transfer Function** is Simply:

$$\frac{1}{A} = H(\omega) = \frac{1}{1 + \left(R_G + R\right)\left(\frac{1}{j\omega L} + \frac{1}{R_L}\right)}$$

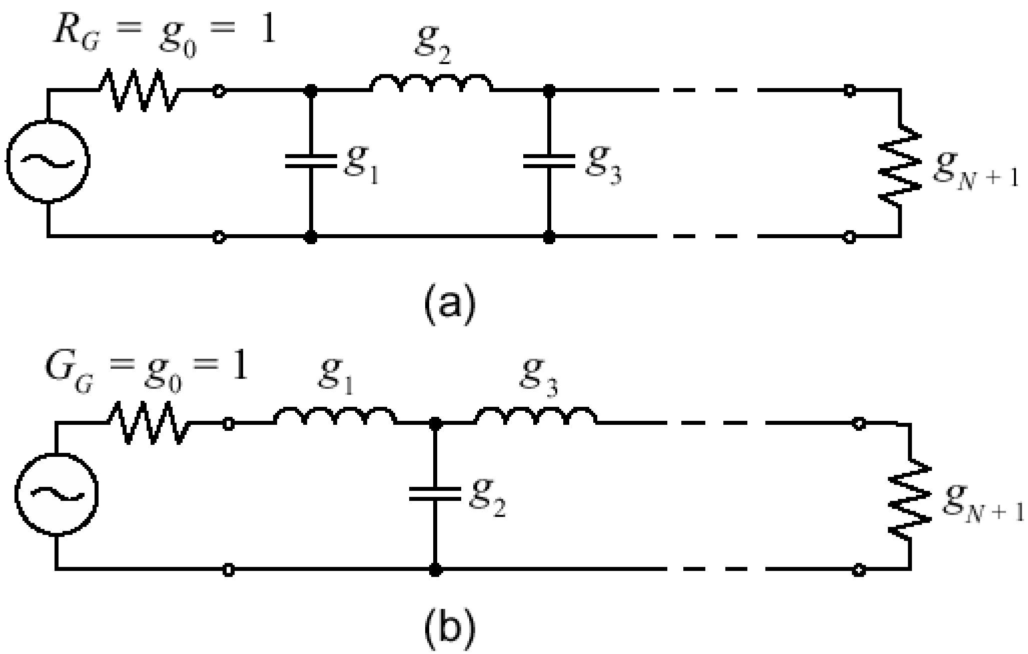
High-Pass Filter Frequency Response

- For $\omega \to \infty$:

$$\frac{V_2}{V_G} = \frac{1}{1 + \dfrac{\left(R + R_G\right)}{R_L}} = \frac{R_L}{R_L + R_G + R}$$

- Inductive Influence Can Be Neglected

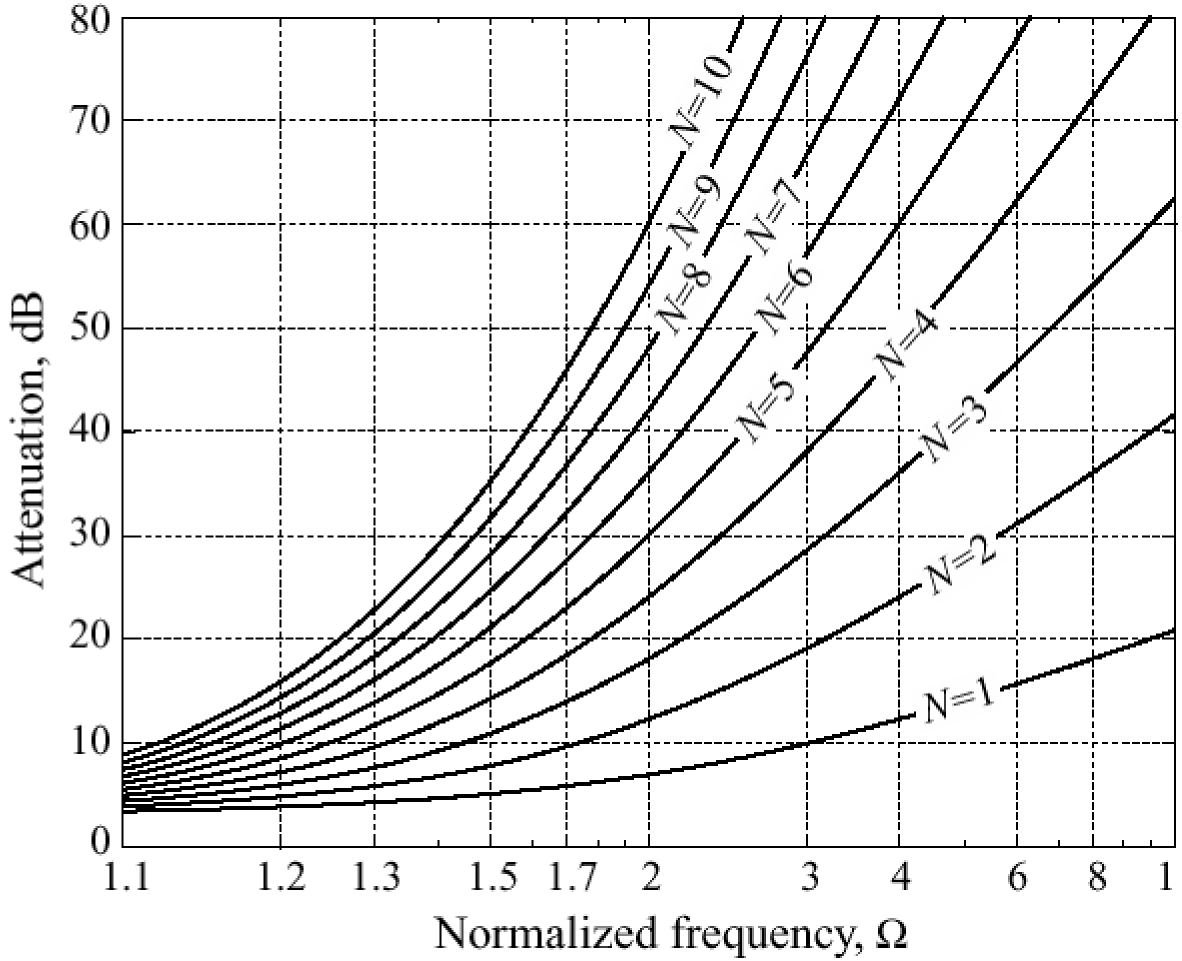
Low-Pass Filter Realizations



Low-Pass Butterworth Filter Coefficients

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

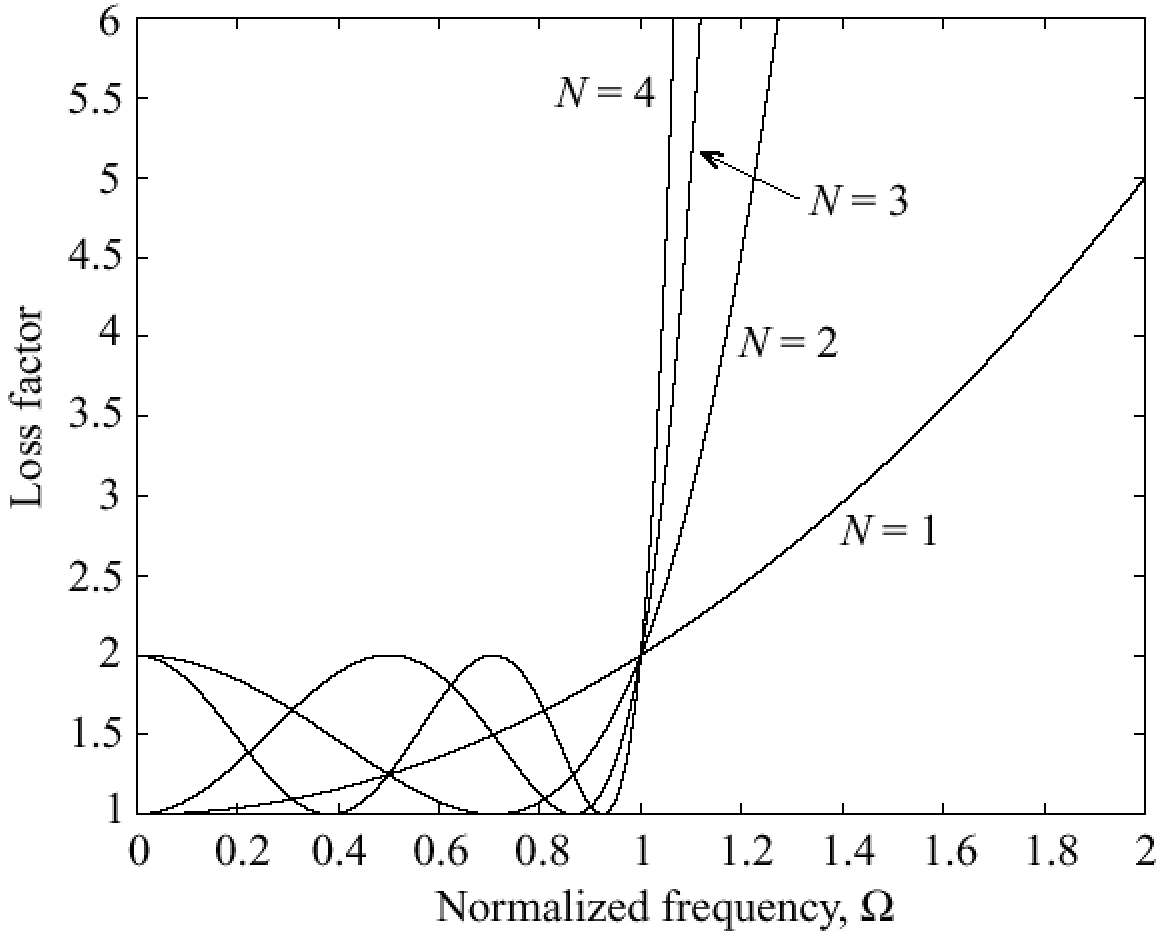
Low-Pass Butterworth Filter Attenuation



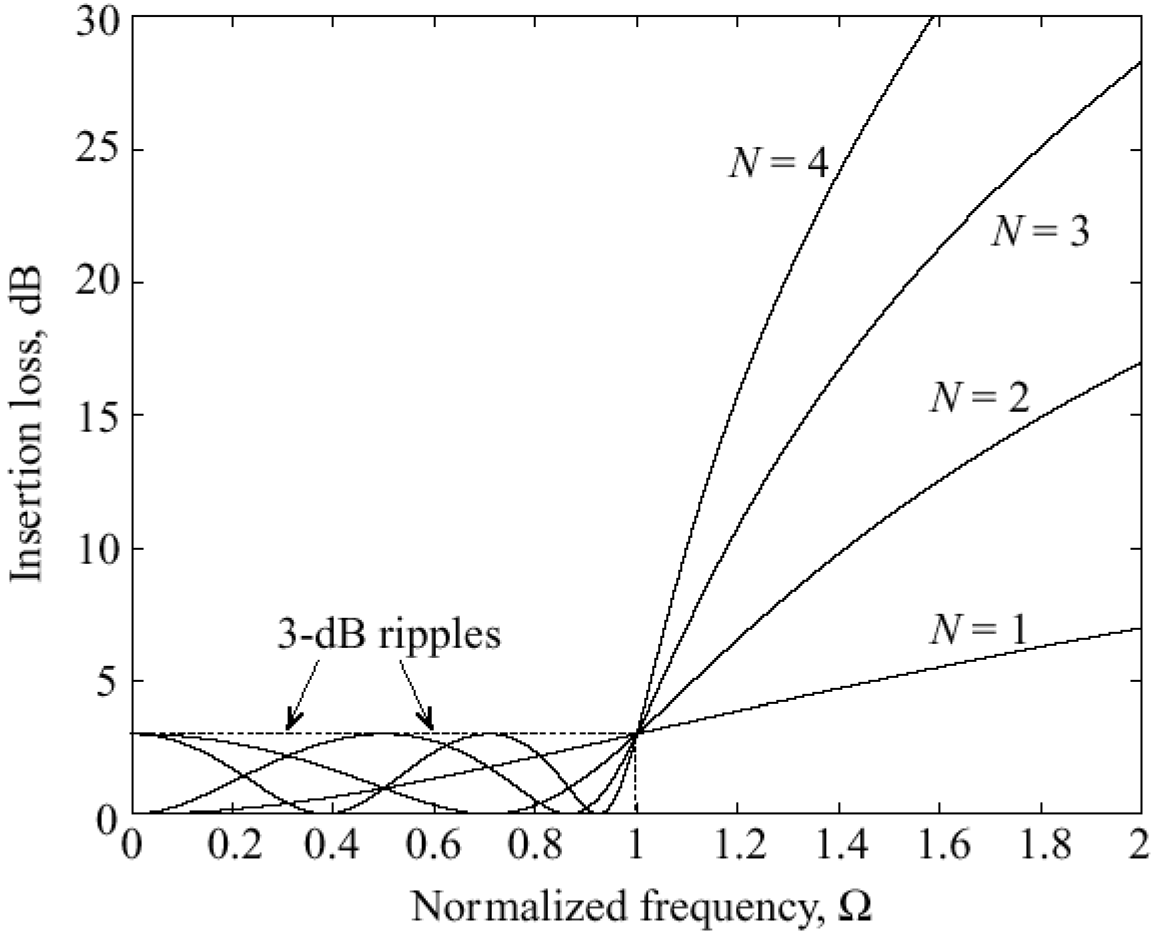
Low-Pass Linear-Phase Filter Coefficients

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

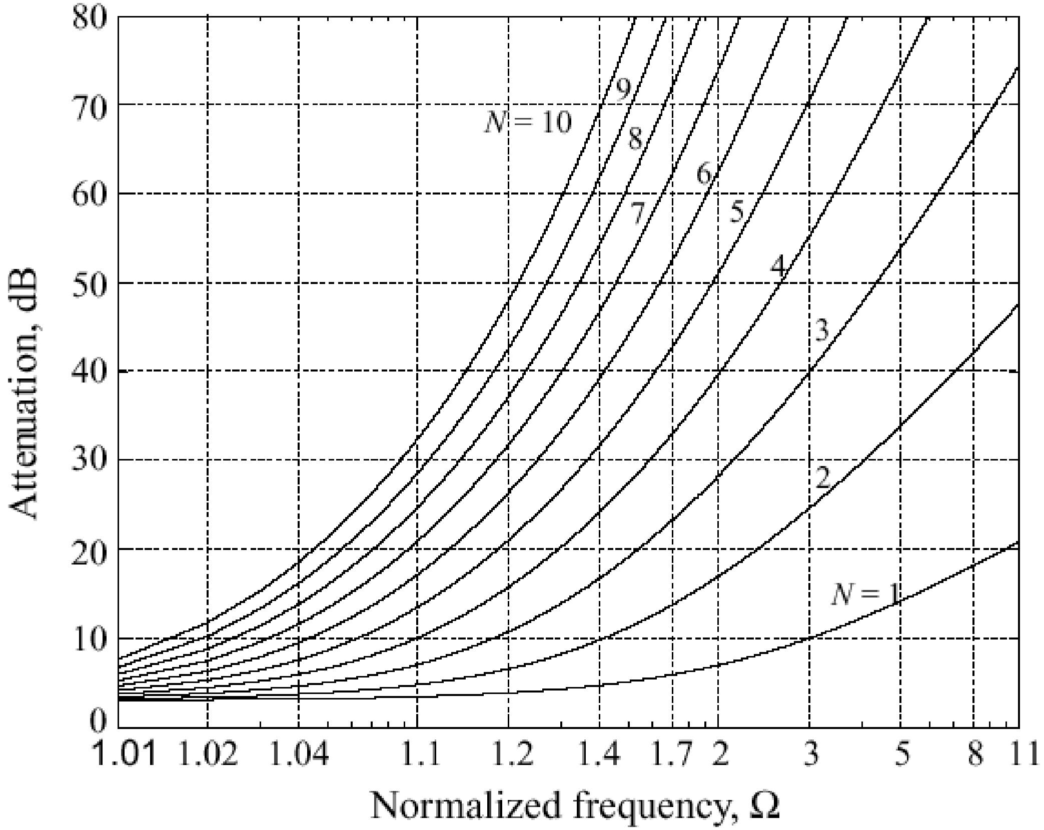
Chebyshev-Type Filters



Chebyshev-Type Filters

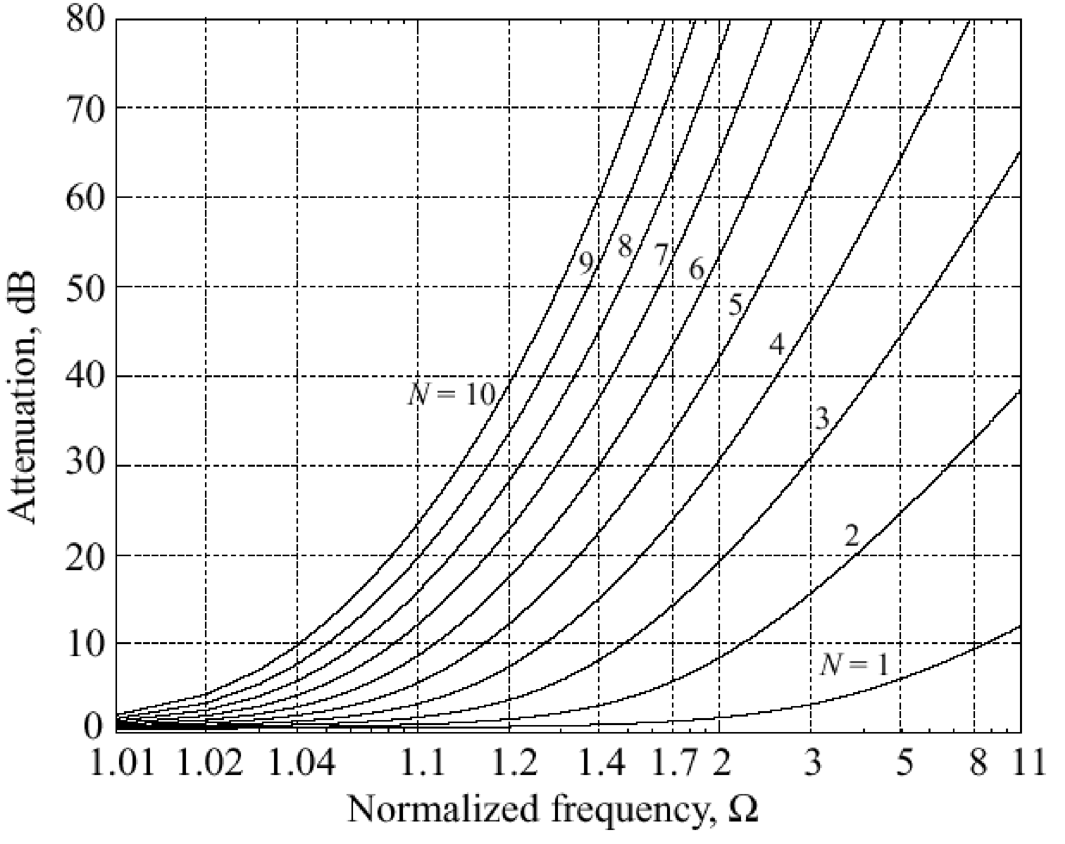


Chebyshev-Type Filter Response



Response for 3 dB ripple Chebyshev LPF

Chebyshev-Type Filter Response



Response for 0.5 dB ripple Chebyshev LPF

Low-Pass Chebysev Filter Coefficients – 3 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Low-Pass Chebysev Filter Coefficients – 0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7939	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

Standard Low-Pass Filter Design

- The normalized inductors and capacitors ($g_1, g_2, \ldots, g_N$) are denormalized using:

$$C = \frac{C_n}{2\pi f_C R} \qquad \text{and} \qquad L = \frac{L_n R}{2\pi f_C}$$

where C_n, L_n, are the g_n normalized values from the tables

Low-Pass Filter Design Example

- Design a Low-Pass Filter with cut-off frequency of 900 MHz and a stop band attenuation of 18 dB @1.8 GHz.
- From the Butterworth Nomograph, A_{max} = 1 and A_{min} = 18. A_{max} = 1 since unity gain. And the order of the filter is N = 3.
- From Butterworth Tables, $g_1 = g_3$=1.0 and g_2 = 2.

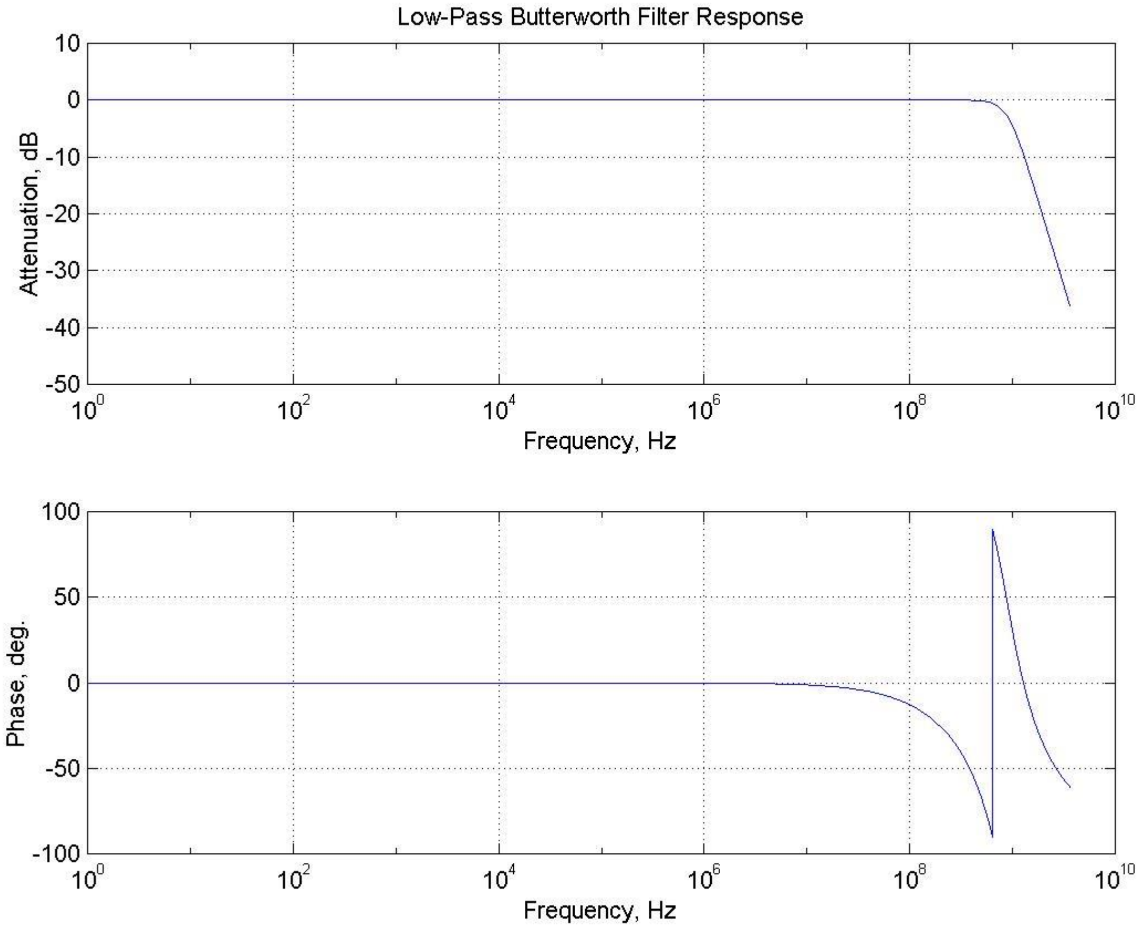
Low-Pass Filter Design Example

- De-Normalized Values For the Tee-Configuration Low-Pass Filter Are:

$$L_1 = L_2 = \frac{g_1 R_L}{2\pi\left(900\times10^6\right)} = 8.8\,nH$$

$$C_1 = \frac{g_2}{2\pi\left(900\times10^6\right)R_L} = 7\ pF$$

Low-Pass Filter Design Example



Low- To High-Pass Transformation

- Transform the Low-Pass Filter Normalized Component Values to the Normalized High-Pass Values
- Inductors in Low-Pass Configuration Become Capacitors in High-Pass.
- Capacitors in Low-Pass Configuration Become Inductors in High-Pass
- $C_{HP\_norm} = \dfrac{1}{\omega_c L_{LP\_norm}}\ ;\quad L_{HP\_norm} = \dfrac{1}{\omega_c C_{LP\_norm}}$

RF Filter Parameters

- Insertion Loss: $IL = 10\,log\,\dfrac{P_{in}}{P_L} = -10\,log\left(1 - \left|\Gamma_{in}\right|^2\right)$
- Ripple
- Bandwidth: $BW^{\text{3dB}} = f_u^{\text{3dB}} - f_L^{\text{3dB}}$
- Shape Factor: $SF = \dfrac{BW_{A_{min}}}{BW_{A_{max}}}$
- Rejection

De-Normalizing Filter Component Values

- All Normalized Component Values Are De-Normalized Using the Following:

$$C_{actual} = \frac{C_{normalized}}{R_g}$$

and

$$L_{actual} = L_{normalized} R_g$$

Transformation From Low-Pass Filter

Table 5-5 Transformation between normalized low-pass filter and actual bandpass and bandstop filter $(BW = \omega_U - \omega_L)$

Low-pass prototype	Low-pass	High-pass	Bandpass	Bandstop
$L = g_k$	$\frac{L}{\omega_c}$	$\frac{1}{\omega_c L}$	$\frac{L}{BW}$, $\frac{BW}{\omega_0^2 L}$	$\frac{1}{(BW)L}$, $\frac{(BW)L}{\omega_0^2}$
$C = g_k$	$\frac{C}{\omega_c}$	$\frac{1}{\omega_c C}$	$\frac{C}{BW}$, $\frac{BW}{\omega_0^2 C}$	$\frac{1}{(BW)C}$, $\frac{(BW)C}{\omega_0^2}$

Normalized Low- to Band-Pass Filter Transformation

- Normalized Band-Pass Shunt Elements from Shunt Low-Pass Capacitor:

$$L_{BP\_norm\_shunt} = \frac{\omega_{upper} - \omega_{lower}}{\omega_o^2 C_{LP\_norm}}$$

$$C_{BP\_norm\_shunt} = \frac{C_{LP\_norm}}{\omega_{upper} - \omega_{lower}}$$

Normalized Low- to Band-Pass Filter Transformation

- Normalized Band-Pass Series Elements from Series Low-Pass Inductor:

$$L_{BP\_norm\_series} = \frac{L_{LP\_norm}}{\omega_{upper} - \omega_{lower}}$$

$$C_{BP\_norm\_series} = \frac{\omega_{upper} - \omega_{lower}}{\omega_o^2 L_{LP\_norm}}$$

Normalized Low- to Band-Stop Filter Transformation

- Normalized Band-Stop Shunt Component Values from Low-Pass Shunt Capacitor:

$$L_{Stop\_norm\_shunt} = \frac{1}{\left(\omega_{upper} - \omega_{lower}\right) C_{LP\_norm}}$$

$$C_{Stop\_norm\_shunt} = \frac{\left(\omega_{upper} - \omega_{lower}\right) C_{LP\_norm}}{\omega_o^2}$$

Normalized Low- to Band-Stop Filter Transformation

- Normalized Band-Stop Series Component Values from Low-Pass Series Inductor:

$$L_{Stop\_norm\_series} = \frac{\left(\omega_{upper} - \omega_{lower}\right) L_{LP\_norm}}{\omega_o^2}$$

$$C_{Stop\_norm\_series} = \frac{1}{\left(\omega_{upper} - \omega_{lower}\right) L_{LP\_norm}}$$