General Considerations: Units in RF Design

$$
\begin{aligned}
A_V|_{\text{dB}} &= 20\log\frac{V_{out}}{V_{in}} \\
A_P|_{\text{dB}} &= 10\log\frac{P_{out}}{P_{in}}.
\end{aligned}
$$

- **This relationship between Power and Voltage only holds when the *input and output impedance are equal***

$$
\begin{aligned}
A_P|_{\text{dB}} &= 10\log\frac{\dfrac{V_{out}^2}{R_0}}{\dfrac{V_{in}^2}{R_0}} \\
&= 20\log\frac{V_{out}}{V_{in}} \\
&= A_V|_{\text{dB}},
\end{aligned}
$$

$$P_{sig}|_{\text{dBm}} = 10\log(\frac{P_{sig}}{1\text{ mW}})$$

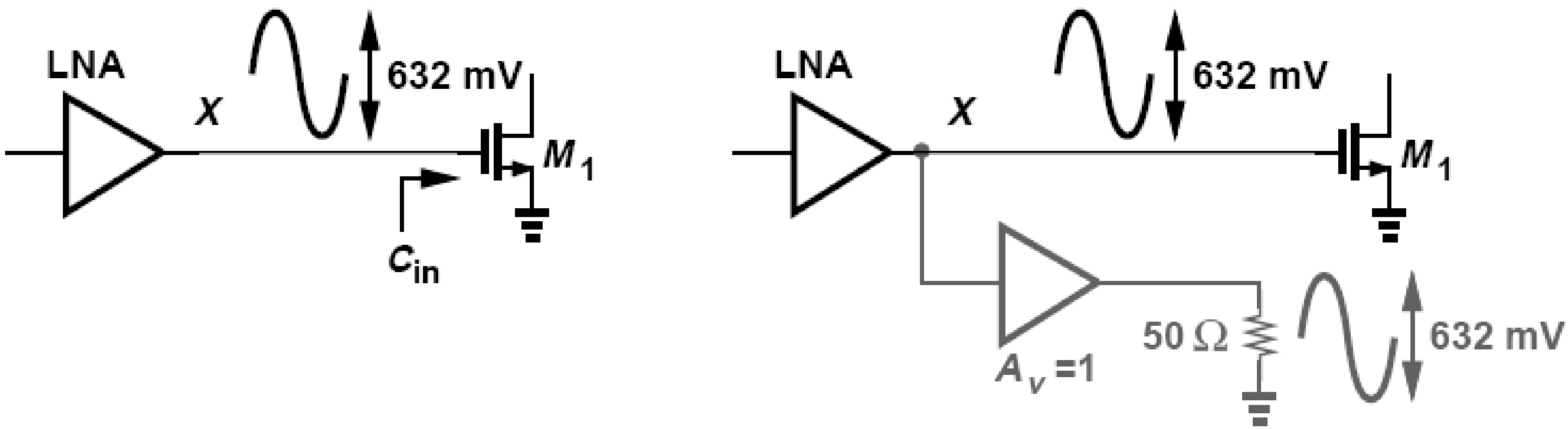
An amplifier senses a sinusoidal signal and delivers a power of 0 dBm to a load resistance of 50 Ω. Determine the peak-to-peak voltage swing across the load.

Example of Units in RF

A GSM receiver senses a narrowband (modulated) signal having a level of -100 dBm. If the front-end amplifier provides a voltage gain of 15 dB, calculate the peak-to-peak voltage swing at the output of the amplifier.

- **Output *voltage* of the amplifier is of interest in this example**

dBm Used at Interfaces Without Power Transfer



- dBm can be used at interfaces that do not necessarily entail power transfer
- We mentally attach an ideal voltage buffer to node *X* and drive a 50-Ω load. We then say that the signal at node *X* has a level of 0 dBm, tacitly meaning that *if* this signal were applied to a 50-Ω load, *then* it would deliver 1 mW.

General Considerations: Time Variance

- **A system is linear if its output can be expressed as a linear combination (superposition) of responses to individual inputs.**

$$y_1(t) = f[x_1(t)]$$

$$y_2(t) = f[x_2(t)]$$

$$ay_1(t) + by_2(t) = f[ax_1(t) + bx_2(t)].$$

- **A system is time-invariant if a time shift in its input results in the same time shift in its output.**

If $\boldsymbol{y(t) = f\,[x(t)]}$

then $\boldsymbol{y(t-\tau) = f\,[x(t-\tau)]}$

Nonlinearity: Memoryless and Static System

$$y(t) = \alpha x(t),$$

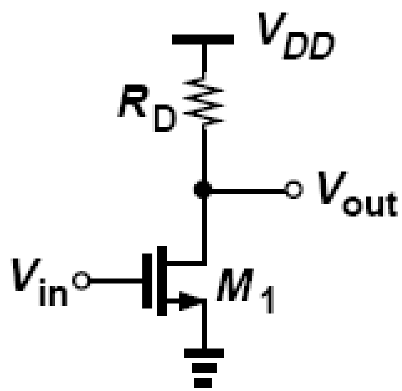
linear

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

nonlinear

- The input/output characteristic of a memoryless nonlinear system can be approximated with a polynomial

V_{out}



- In this idealized case, the circuit displays only second-order nonlinearity

Effects of Nonlinearity: Harmonic Distortion

$$y(t) = h(t) * x(t)$$

Linear Time-invariant

$$y(t) = h(t, \tau) * x(t)$$

Linear Time-variant

- **The output of a "dynamic" system depends on the past values of its input/output**

$$\begin{aligned} y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\ &= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t. \end{aligned}$$

DC | **Fundamental** | **Second Harmonic** | **Third Harmonic**

- **Even-order harmonics result from α_j with even j**
- **nth harmonic grows in proportion to A^n**

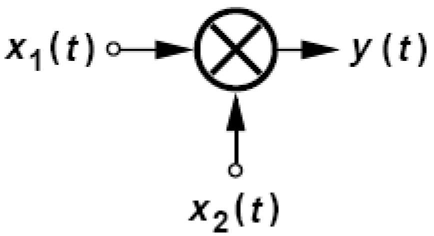
Example of Harmonic Distortion in Mixer

An analog multiplier "mixes" its two inputs below, ideally producing $y(t)$ = $kx_1(t)x_2(t)$, where k is a constant. Assume $x_1(t) = A_1 \cos \omega_1 t$ and $x_2(t) = A_2 \cos \omega_2 t$.

(a)If the mixer is ideal, determine the output frequency components.

(b) If the input port sensing $x_2(t)$ suffers from third-order nonlinearity, determine the output frequency components.

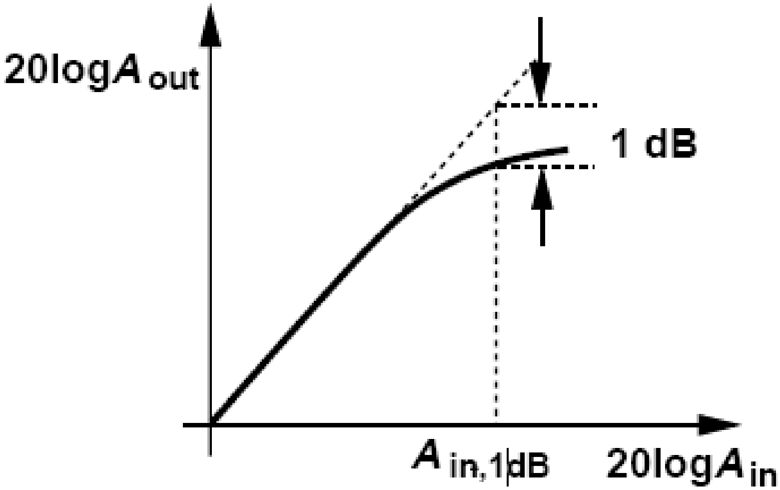
Solution:



Example of Harmonics on GSM Signal

The transmitter in a 900-MHz GSM cellphone delivers 1 W of power to the antenna. Explain the effect of the harmonics of this signal.

Gain Compression: 1-dB Compression Point

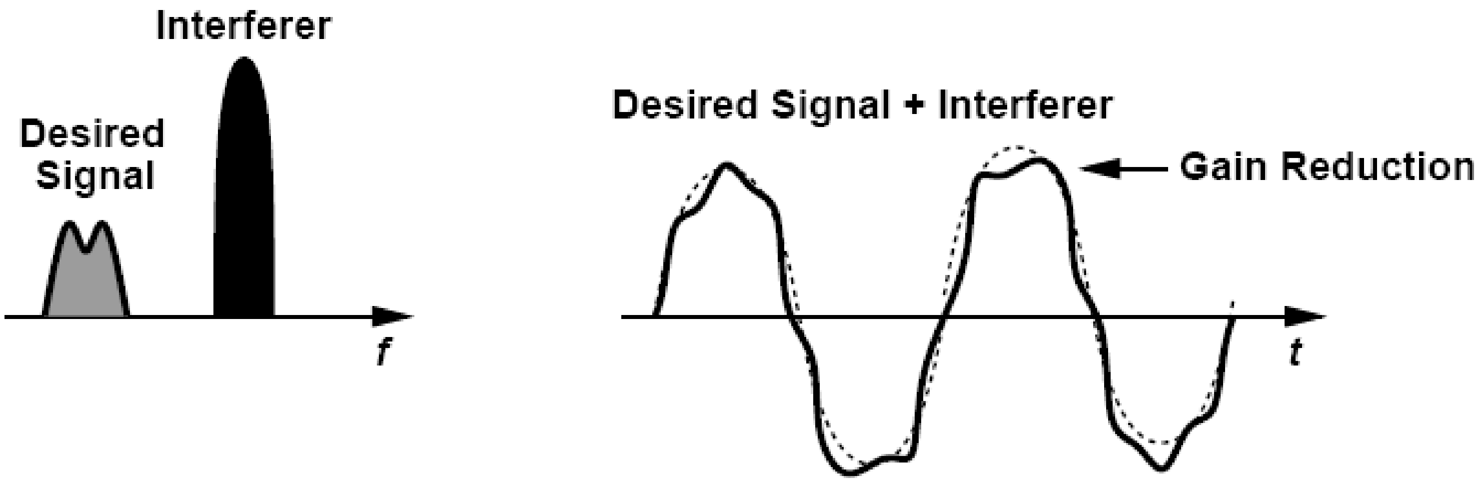


$$20\log\left|\alpha_1+\frac{3}{4}\alpha_3 A_{in,1dB}^2\right| = 20\log|\alpha_1| - 1\text{ dB}.$$

$$A_{in,1dB}=\sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

- **Output falls below its ideal value by 1 dB at the 1-dB compression point**
- **Peak value instead of peak-to-peak value**

Gain Compression: Desensitization



$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots.$$

For $A_1 << A_2$

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

- **Desensitization: the receiver gain is reduced by the large excursions produced by the interferer even though the desired signal itself is small.**

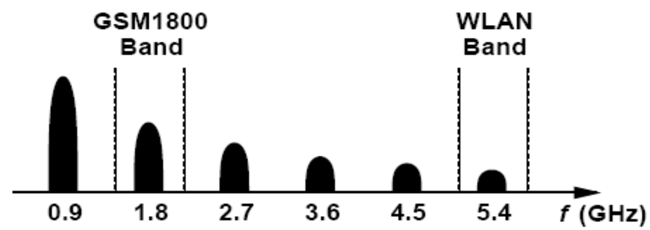
Example of Gain Compression

A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it does not desensitize a 1.8-GHz receiver having P_{1dB} = -25 dBm? Assume the receiver is 1 m away and the 1.8-GHz signal is attenuated by 10 dB as it propagates across this distance.

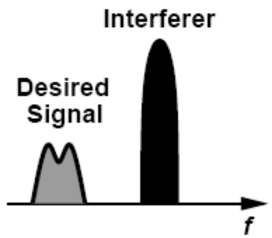
Effects of Nonlinearity: Intermodulation— Recall Previous Discussion

So far we have considered the case of:

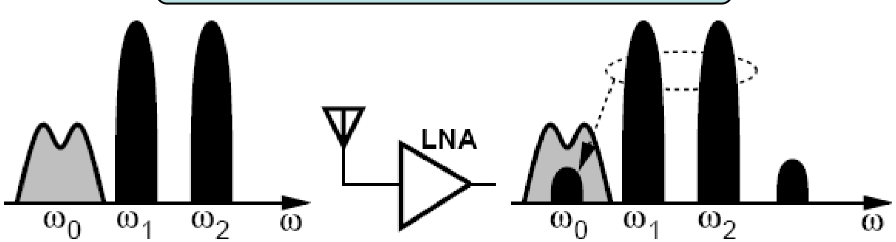
- **Single Signal** → **Harmonic distortion**



- **Signal + one large interferer** → **Desensitization**



- **Signal + two large interferers** → **Intermodulation**



Effects of Nonlinearity: Intermodulation

assume $$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

Thus

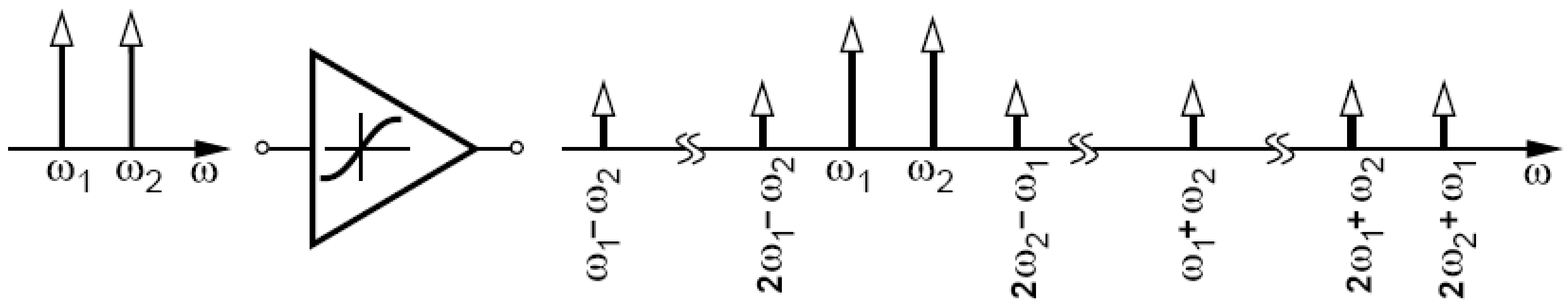
$$y(t) = \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Intermodulation products:

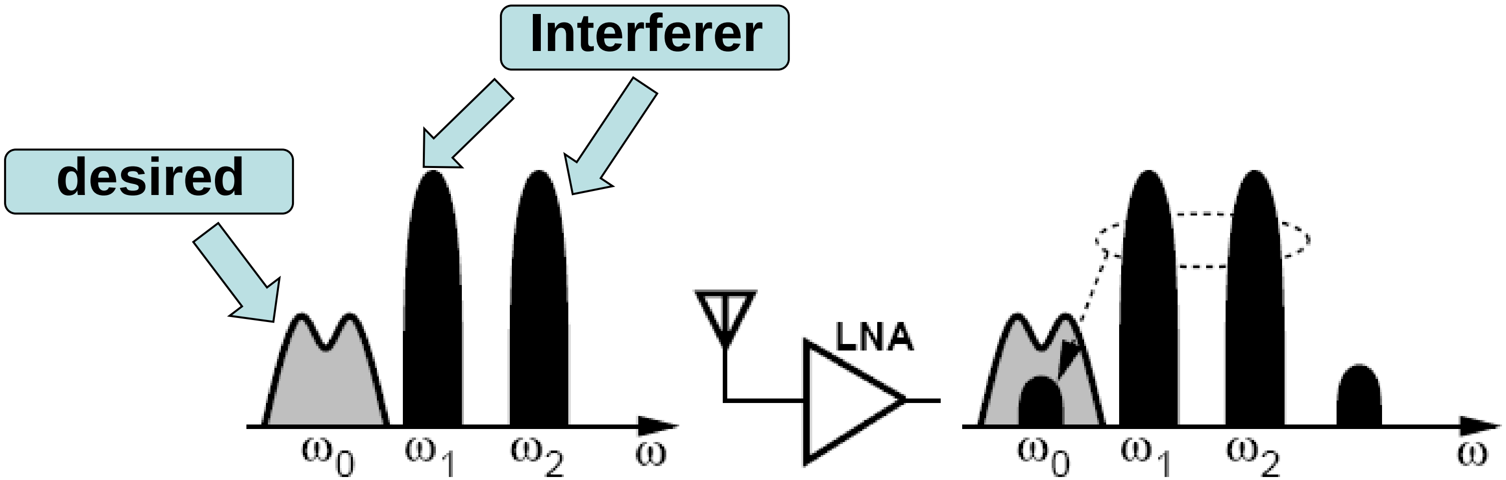
$$\begin{aligned}
\omega &= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t \\
\omega &= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 - \omega_1)t
\end{aligned}$$

Fundamental components:

$$\omega = \omega_1,\ \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right) \cos \omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right) \cos \omega_2 t$$



Intermodulation Product Falling on Desired Channel



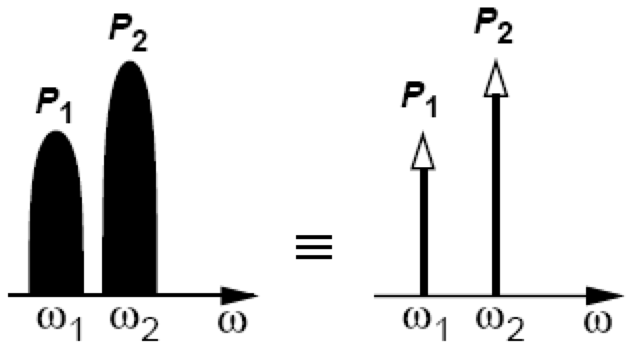
$$2\omega_1 - \omega_2 = \omega_0$$

- **A received small desired signal along with two large interferers**
- **Intermodulation product falls onto the desired channel, corrupts signal.**

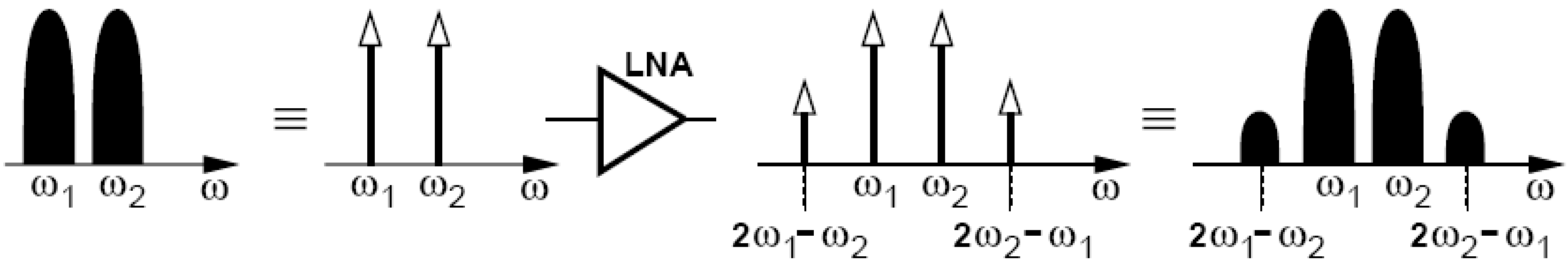
Example of Intermodulation

Suppose four Bluetooth users operate in a room as shown in figure below. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz. At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.

Intermodulation: Tones and Modulated Interferers



(a)



- **In intermodulation Analyses:**

  (a) approximate the interferers with tones

  (b) calculate the level of intermodulation products at the output

  (c) mentally convert the intermodulation tones back to modulated components so as to see the corruption.

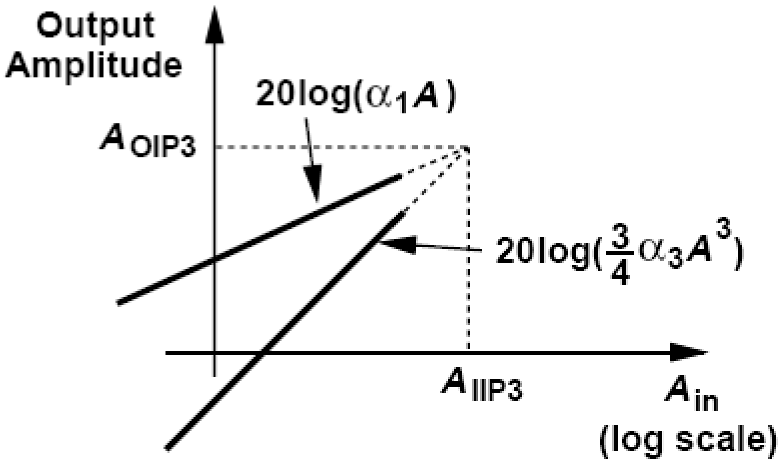
Example of Gain Compression and Intermodulation

A Bluetooth receiver employs a low-noise amplifier having a gain of 10 and an input impedance of 50 Ω. The LNA senses a desired signal level of -80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz. For simplicity, assume the LNA drives a 50-Ω load.

(a) Determine the value of α_3 that yields a P_{1dB} of -30 dBm.

(b) If each interferer is 10 dB below P_{1dB}, determine the corruption experienced by the desired signal at the LNA output.

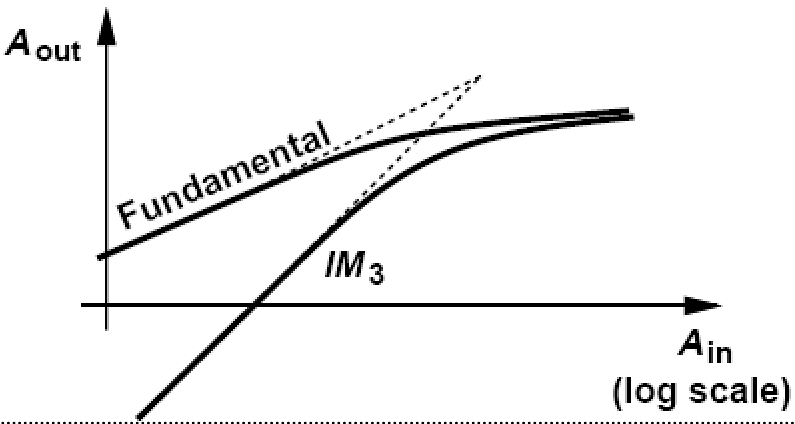
Intermodulation: Third Intercept Point

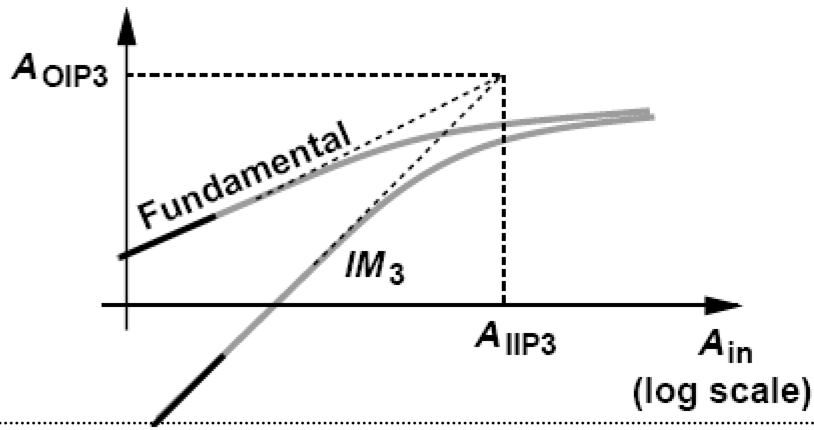


$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}}$$
$$\approx 9.6\ \text{dB}.$$



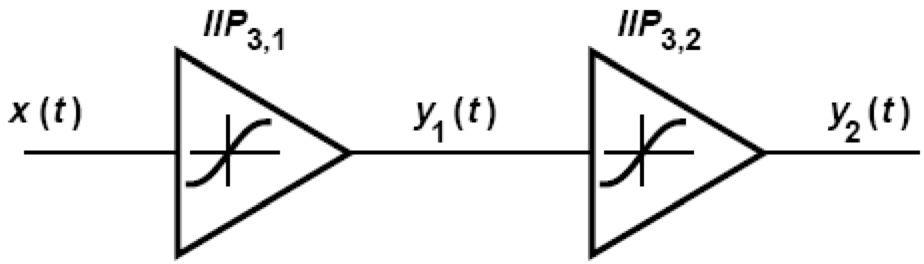


- **IP3 is not a directly measureable quantity, but a point obtained by extrapolation**

Example of Third Intercept Point

A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What IIP_3 is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.

Effects of Nonlinearity: Cascaded Nonlinear Stages



$$\begin{aligned} y_1(t) &= \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \\ y_2(t) &= \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t) \end{aligned}$$

$$\begin{aligned} y_2(t) &= \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 \\ &+ \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3. \end{aligned}$$

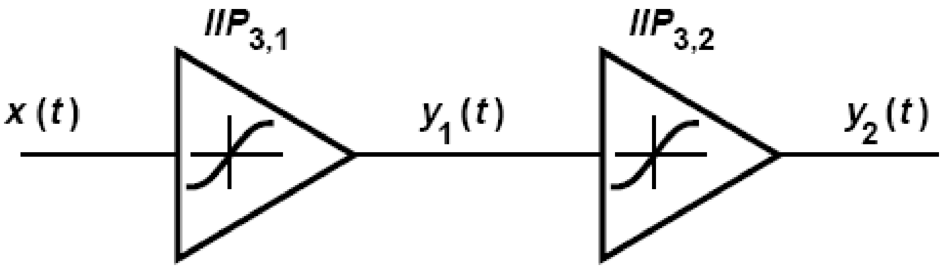
Considering only the first- and third-order terms, we have:

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots$$

Thus,

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}.$$

Cascaded Nonlinear Stages: Intuitive results



$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\left|\frac{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}{\alpha_1\beta_1}\right|$$

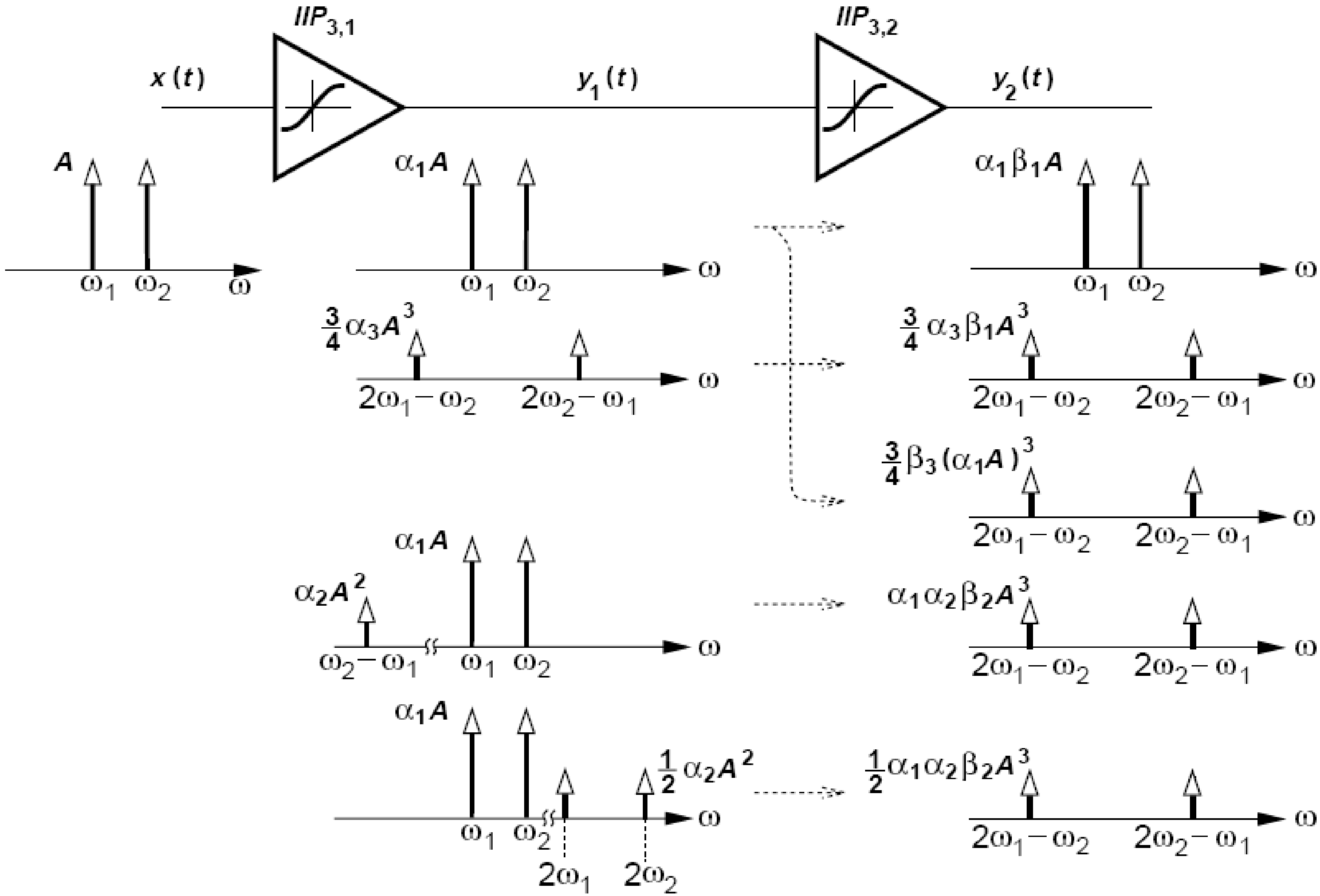
$$= \frac{3}{4}\left|\frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2\beta_2}{\beta_1} + \frac{\alpha_1^2\beta_3}{\beta_1}\right|$$

$$= \left|\frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}\right|$$

- **To "refer" the IP_3 of the second stage to the input of the cascade, we must divide it by α_1. Thus, the higher the gain of the first stage, the more nonlinearity is contributed by the second stage.**

IM Spectra in a Cascade (I)

Let us assume $x(t) = A\cos\omega_1 t + A\cos\omega_2 t$ and identify the IM products in a cascade:



IM Spectra in a Cascade (Ⅱ)

Adding the amplitudes of the IM products, we have

$$y_2(t) = \alpha_1\beta_1 A(\cos\omega_1 t + \cos\omega_2 t)$$
$$+ \left(\frac{3\alpha_3\beta_1}{4} + \frac{3\alpha_1^3\beta_3}{4} + \frac{3\alpha_1\alpha_2\beta_2}{2}\right) A^3[\cos(\omega_1 - 2\omega_2)t + \cos(2\omega_2 - \omega_1)t] + \cdots$$

- **Add in phase as worst-case scenario**
- **Heavily attenuated in narrow-band circuits**

For more stages:

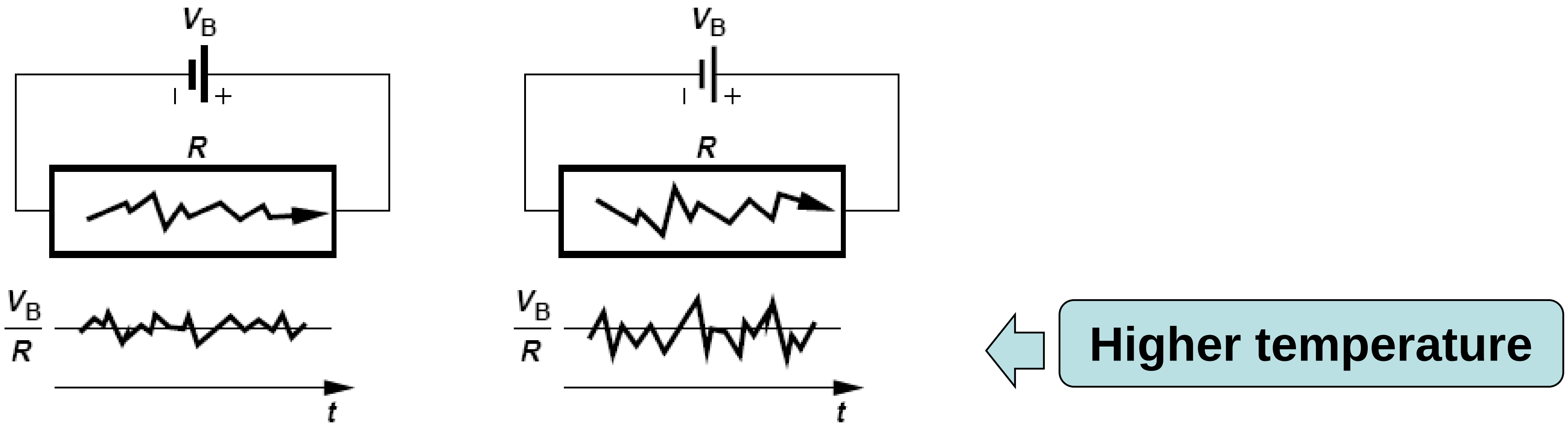
$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$
$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$

- **Thus, if each stage in a cascade has a gain greater than unity, the nonlinearity of the latter stages becomes increasingly more critical because the IP3 of each stage is equivalently scaled down by the total gain preceding that stage.**

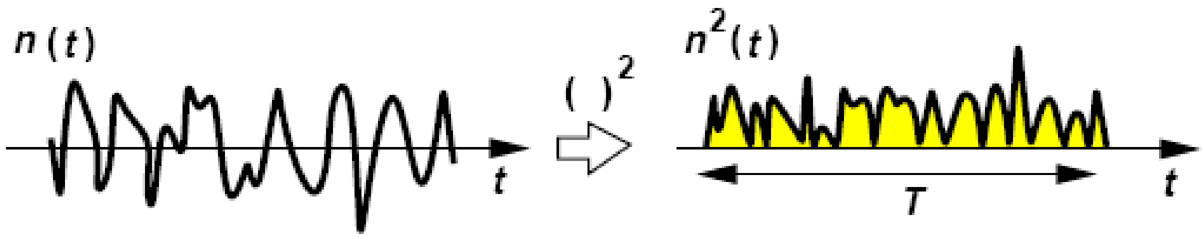
Example of Cascaded Nonlinear Stages

A low-noise amplifier having an input IP_3 of -10 dBm and a gain of 20 dB is followed by a mixer with an input IP_3 of +4 dBm. Which stage limits the IP_3 of the cascade more?

Noise: Noise as a Random Process



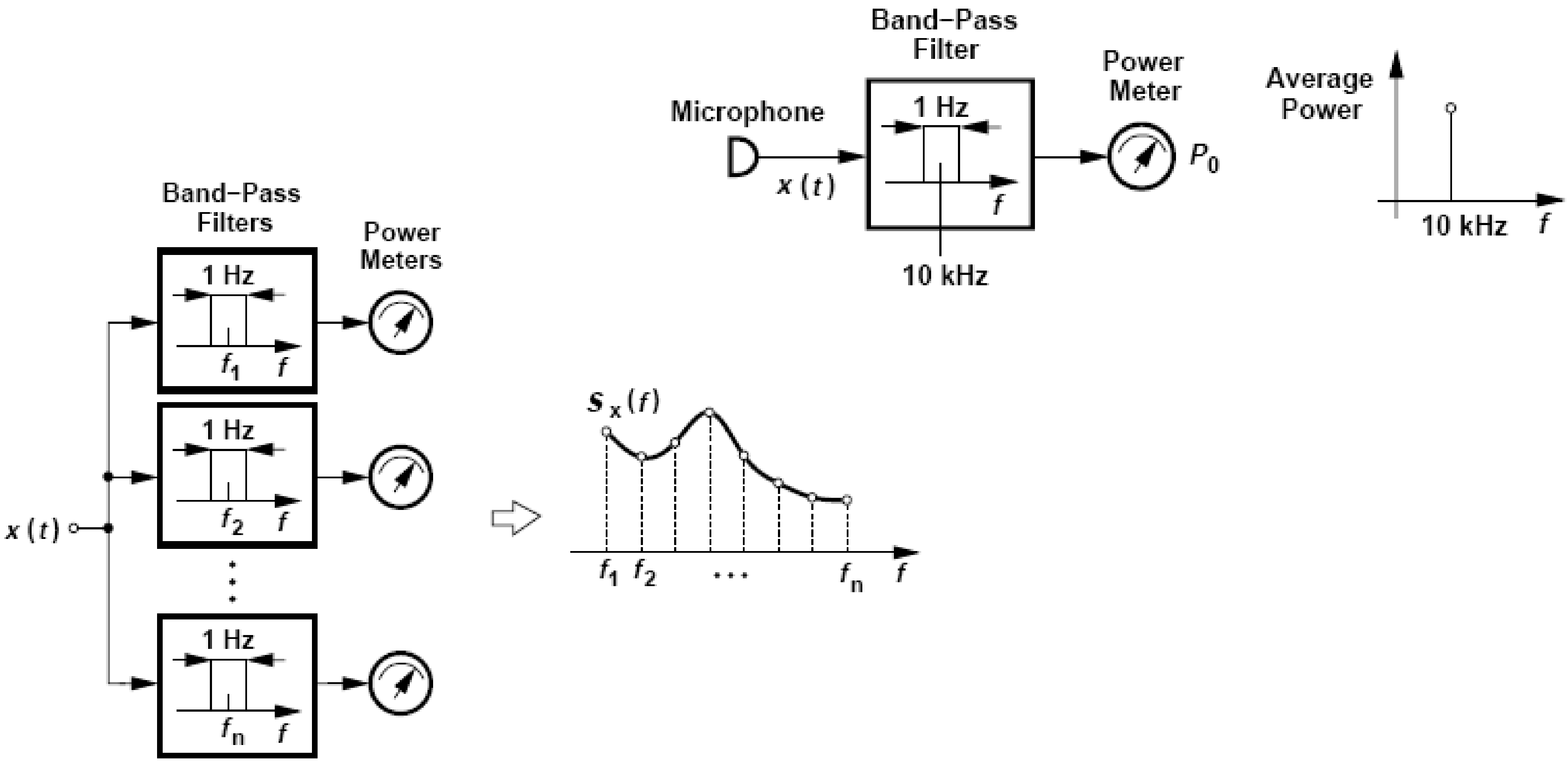
- The average current remains equal to V_B/R but the instantaneous current displays random values



$$P_n = \lim_{T\to\infty} \frac{1}{T} \int_0^T n^2(t)dt$$

- *T* must be long enough to accommodate several cycles of the lowest frequency.

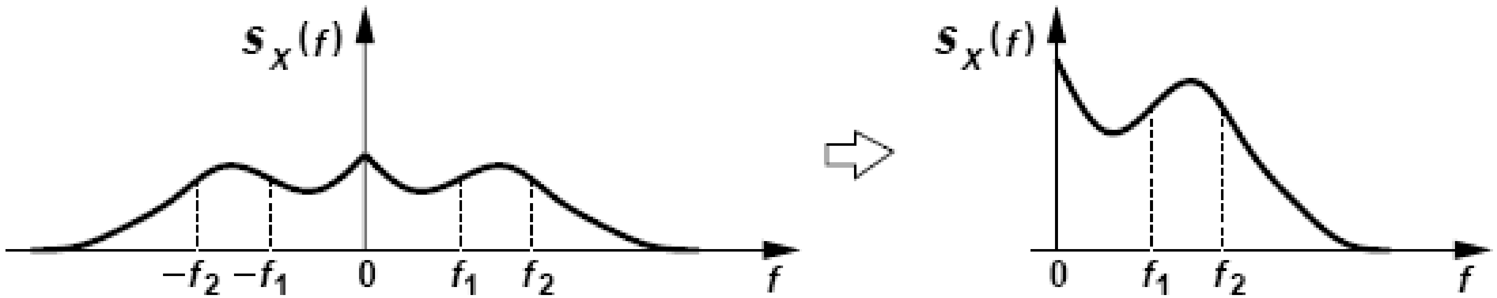
Measurement of Noise Spectrum



- To measure signal's frequency content at 10 kHz, we need to filter out the remainder of the spectrum and measure the average power of the 10-kHz component.

Noise Spectrum: Power Spectral Density (PSD)

Two-Sided | **One-Sided**



$$\int_0^{\infty} S_x(f)df = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t)dt$$

- **Total area under $S_x(f)$ represents the average power carried by $x(t)$**

Example of Noise Spectrum

A resistor of value R_1 generates a noise voltage whose one-sided PSD is given by

$$S_v(f) = 4kTR_1$$

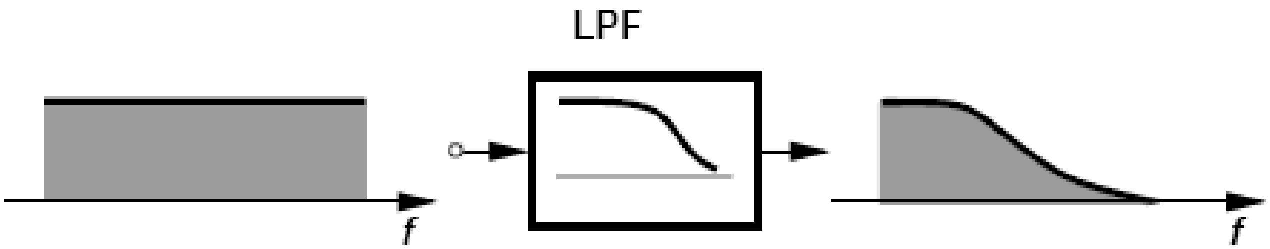
where $k = 1.38 \times 10^{-23}$ J/K denotes the Boltzmann constant and T the absolute temperature. Such a flat PSD is called "white" because, like white light, it contains all frequencies with equal power levels.

(a) What is the total average power carried by the noise voltage?

(b) What is the dimension of $S_v(f)$?

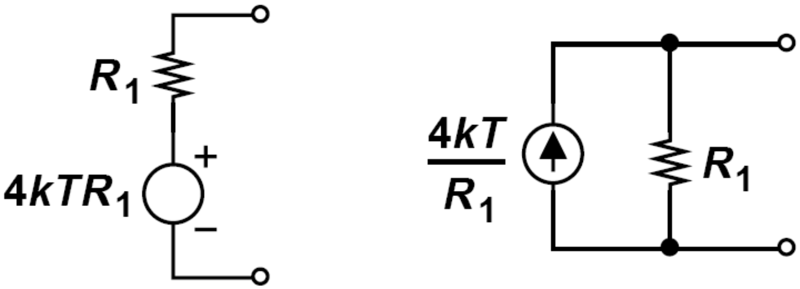
(c) Calculate the noise voltage for a 50-Ω resistor in 1 Hz at room temperature.

Effect of Transfer Function on Noise/ Device Noise



$$S_y(f) = S_x(f)|H(f)|^2$$

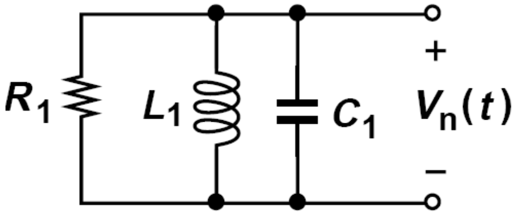
- Define PSD to allow many of the frequency-domain operations used with deterministic signals to be applied to random signals as well.



- Noise can be modeled by a series voltage source or a parallel current source
- Polarity of the sources is unimportant but must be kept same throughout the calculations

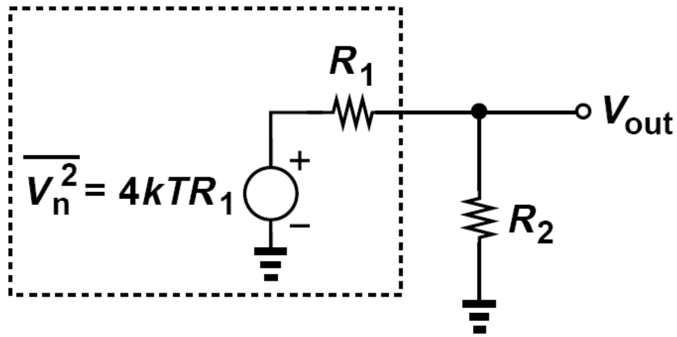
Example of Device Noise

Sketch the PSD of the noise voltage measured across the parallel RLC tank depicted in figure below.



Can We Extract Energy from Resistor?

Suppose R_2 is held at T = 0 K (No noise)

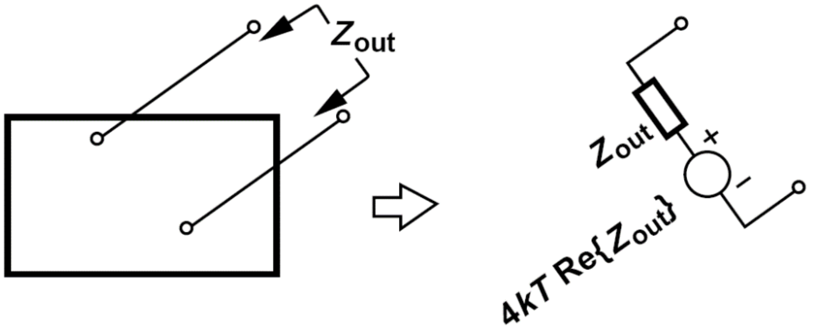


$$
\begin{aligned}
P_{R2} &= \frac{\overline{V_{out}^2}}{R_2} \\
&= \overline{V_n^2}\left(\frac{R_2}{R_1+R_2}\right)^2 \frac{1}{R_2} \\
&= 4kT\frac{R_1R_2}{(R_1+R_2)^2}.
\end{aligned}
$$

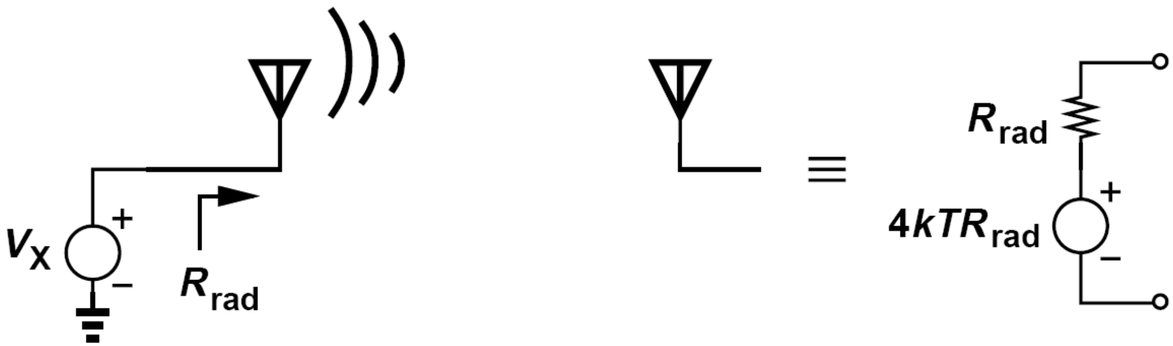
This quantity reaches a maximum if $R_2 = R_1$:
$P_{R2,max} = kT$

Available noise power

A Theorem about Lossy Circuit



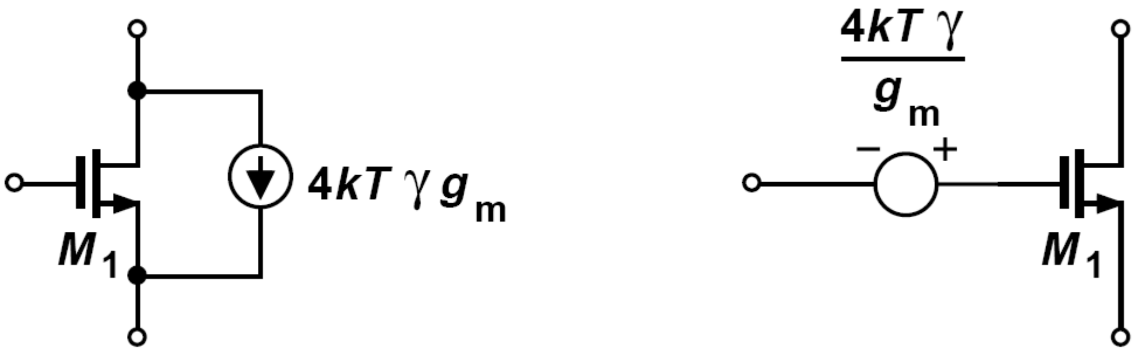
- If the real part of the impedance seen between two terminals of a passive (reciprocal) network is equal to *Re{Z_{out}}*, then the PSD of the thermal noise seen between these terminals is given by *4kTRe{Z_{out}}*



$$\overline{V_{n,ant}^2} = 4kTR_{rad}.$$

- An example of transmitting antenna, with radiation resistance R_{rad}

Noise in MOSFETS



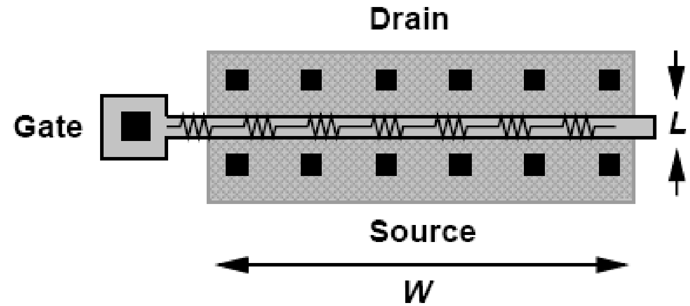
$$\overline{I_n^2} = 4kT\gamma g_m \qquad \overline{V_n^2} = 4kT\gamma / g_m$$

- Thermal noise of MOS transistors operating in the saturation region is approximated by a current source tied between the source and drain terminals, or can be modeled by a voltage source in series with gate.

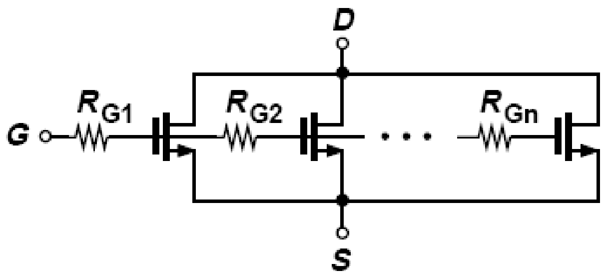
Gate-induced Noise Current

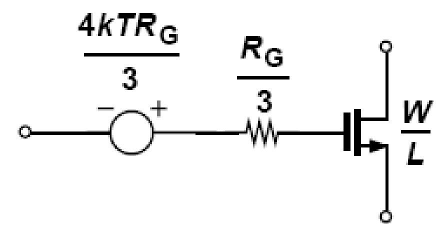
$$R_G = \frac{W}{L} R_{\square}$$

$$4kT\frac{R_G}{3} \ll \frac{4kT\gamma}{g_m}$$

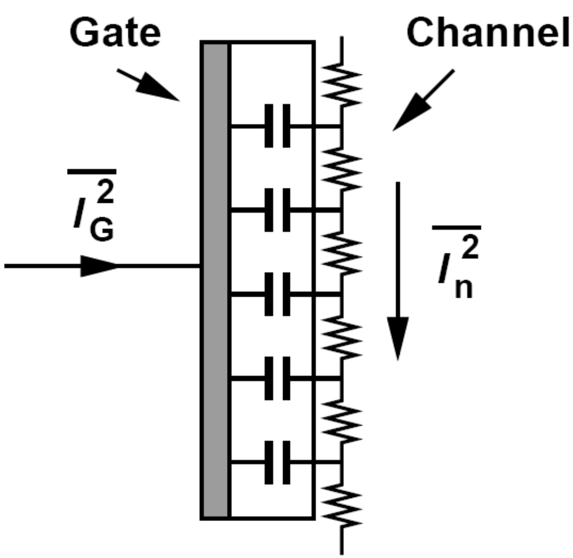


(a)



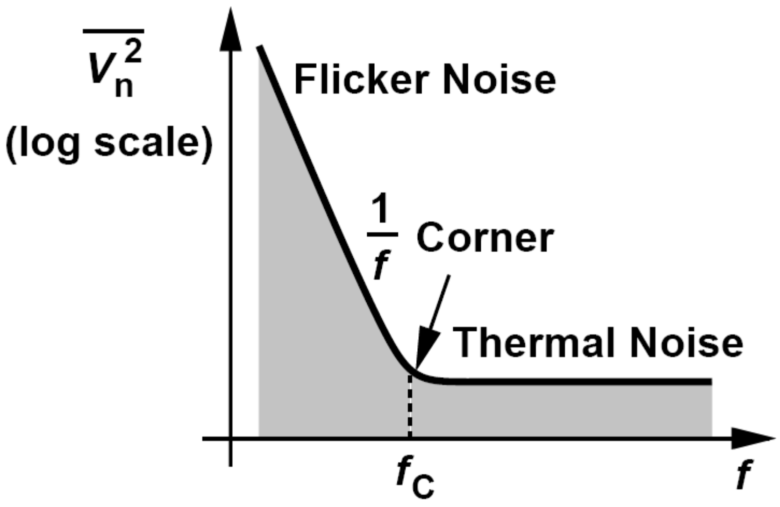


$$R_{G1} + R_{G2} + \cdots + R_{Gn} = R_G$$



- At very high frequencies thermal noise current flowing through the channel couples to the gate capacitively

Flicker Noise and An Example



$$\overline{V_n^2} = \frac{K}{WLC_{ox}} \frac{1}{f}$$

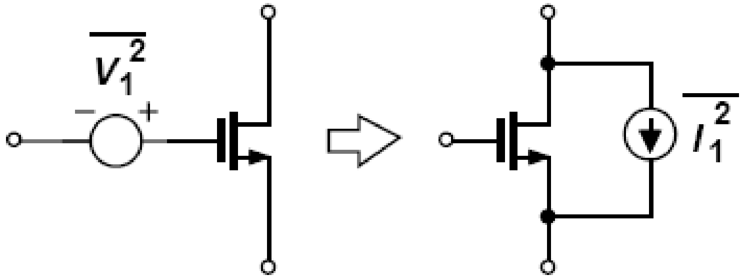
$$\frac{K}{WLC_{ox}} \frac{1}{f_c} g_m^2 = 4kT\gamma g_m$$

$$f_c = \frac{K}{WLC_{ox}} \frac{g_m}{4kT\gamma}$$

Can the flicker noise be modeled by a current source?

Yes, a MOSFET having a small-signal voltage source of magnitude V_1 in series with its gate is equivalent to a device with a current source of value $g_m V_1$ tied between drain and source. Thus,

$$\overline{I_1^2} = g_m^2 \frac{K}{WLC_{ox}} \frac{1}{f}$$



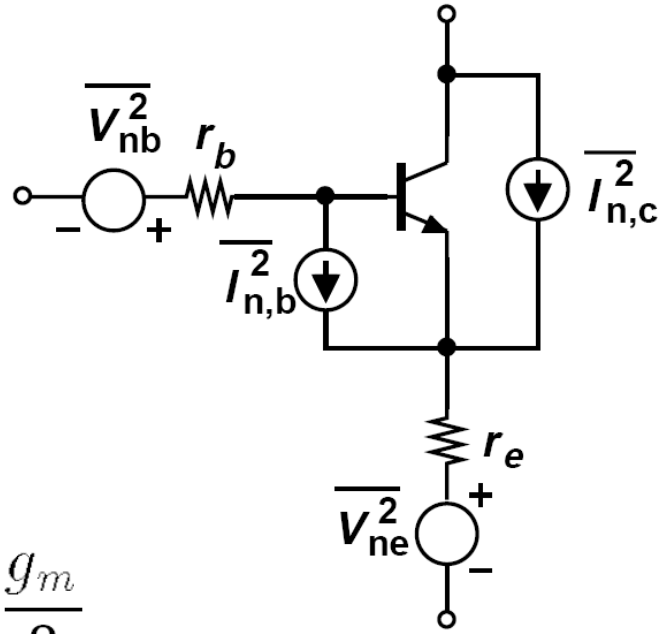
Noise in Bipolar Transistors

Bipolar transistors contain physical resistances in their base, emitter, and collector regions, all of which generate thermal noise. Moreover, they also suffer from “shot noise” associated with the transport of carriers across the base-emitter junction.

$$\overline{I_{n,b}^2} = 2qI_B = 2q\frac{I_C}{\beta}$$

$$\overline{I_{n,c}^2} = 2qI_C,$$

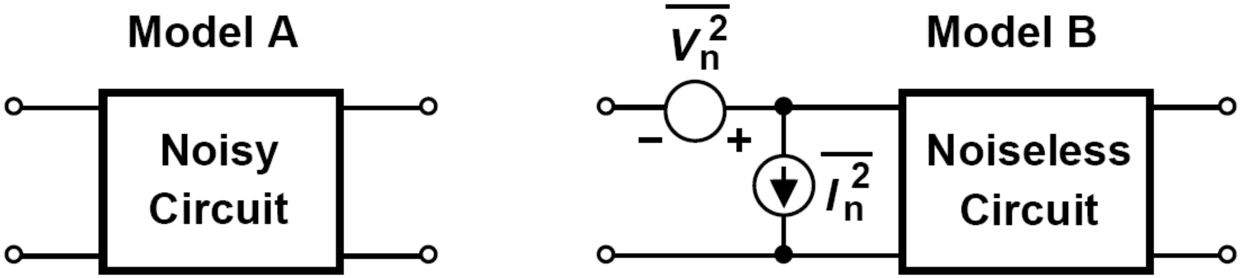
$$g_m = I_C/(kT/q)$$



$$\overline{I_{n,c}^2} = 4kT\frac{g_m}{2}$$

- **In low-noise circuits, the base resistance thermal noise and the collector current shot noise become dominant.**

Representation of Noise in Circuits
Input-Referred Noise

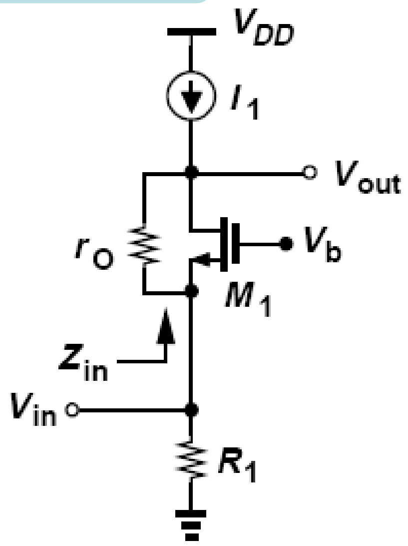


- **Voltage source: short the input port of models A and B and equate their output noise voltage**
- **Current source: leave the input ports open and equate the output noise voltage**

Example of Input-Referred Noise

Calculate the input-referred noise of the common-gate stage depicted in figure below (left). Assume I_1 is ideal and neglect the noise of R_1.

Solution:



Noise Figure

Noise Factor $$\text{NF} = \frac{SNR_{in}}{SNR_{out}}$$

Noise Figure $$\text{NF}|_{\text{dB}} = 10 \log \frac{SNR_{in}}{SNR_{out}}.$$

- **Depends on not only the noise of the circuit under consideration but the SNR provided by the preceding stage**
- **If the input signal contains no noise, NF=∞**

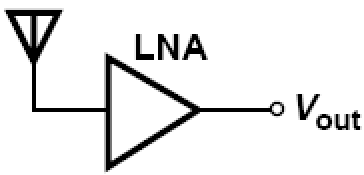
Calculation of Noise Figure

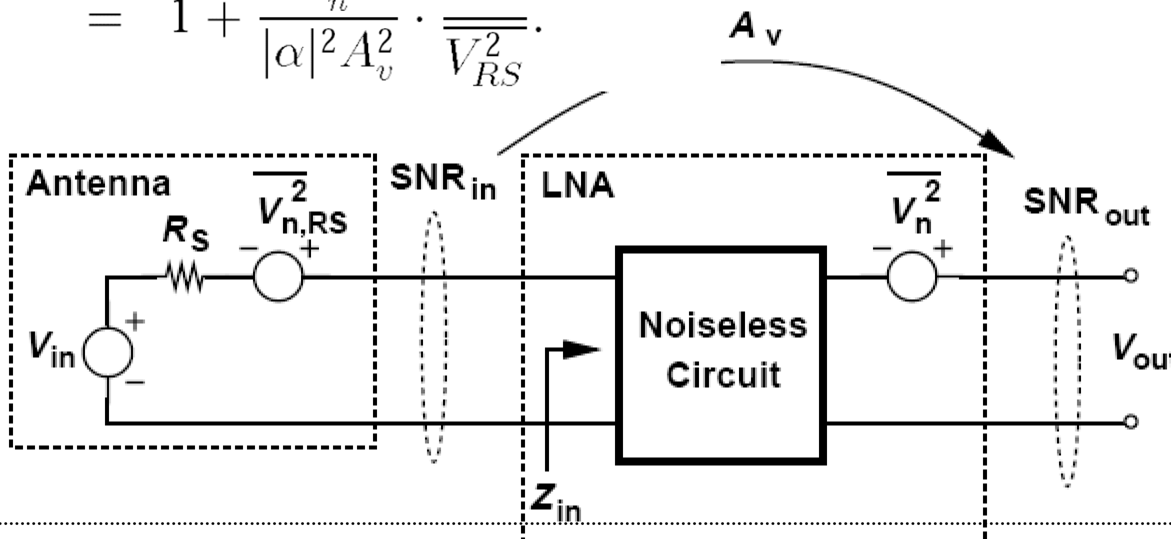
$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}$$

$$\alpha = Z_{in}/(Z_{in} + R_S)$$

$$\begin{aligned} \text{NF} &= \frac{V_{in}^2}{4kTR_S} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{V_{in}^2 |\alpha|^2 A_v^2} \\ &= \frac{1}{\overline{\overline{V_{RS}^2}}} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{|\alpha|^2 A_v^2} \\ &= 1 + \frac{\overline{V_n^2}}{|\alpha|^2 A_v^2} \cdot \frac{1}{\overline{\overline{V_{RS}^2}}}. \end{aligned}$$





- NF must be specified with respect to a source impedance-typically 50 Ω
- Reduce the right hand side to a simpler form:

$$\text{NF} = \frac{1}{4kTR_S} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2} \qquad A_0 = |\alpha| A_v$$

Calculation of NF: Summary

Calculation of NF

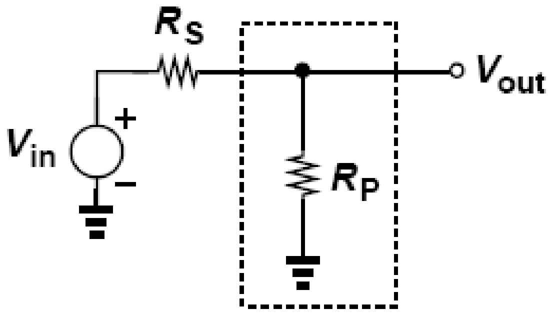
- Divide total output noise by the gain from V_{in} to V_{out} and normalize the result to the noise of R_s

- Calculate the output noise due to the amplifier, divide it by the gain, normalize it to $4kTR_s$ and add 1 to the result

Example of Noise Figure Calculation

Compute the noise figure of a shunt resistor R_P with respect to a source impedance R_S

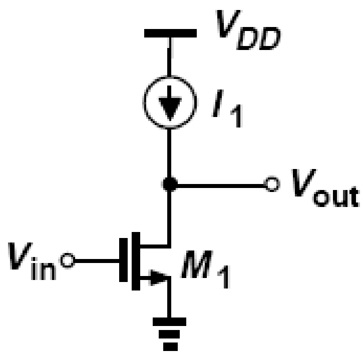
Solution:

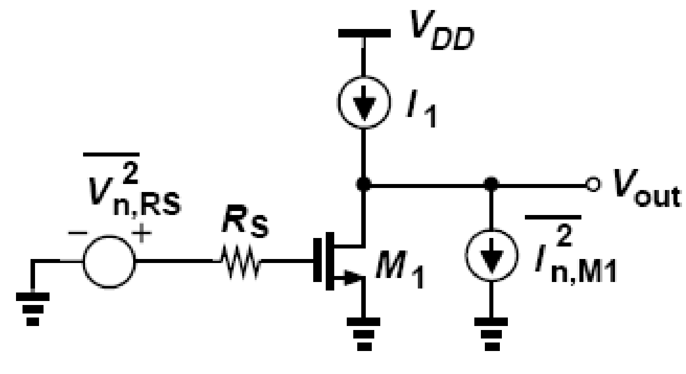


Another Example of Noise Figure Calculation

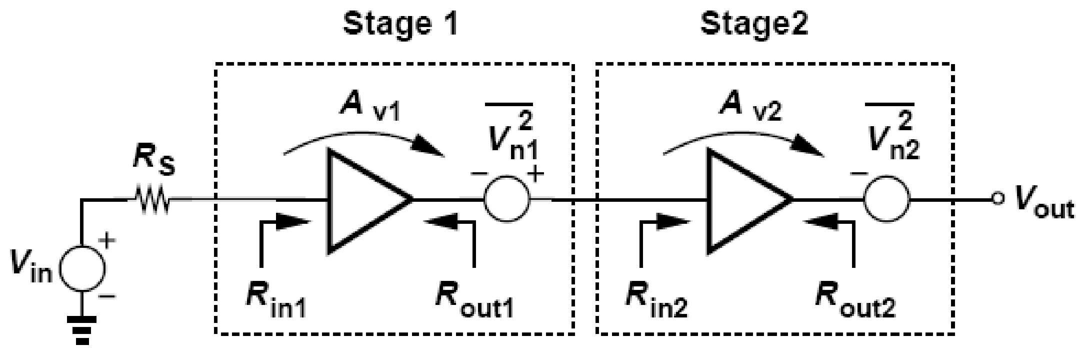
Determine the noise figure of the common-source stage shown in below (left) with respect to a source impedance R_S. Neglect the capacitances and flicker noise of M_1 and assume I_1 is ideal.

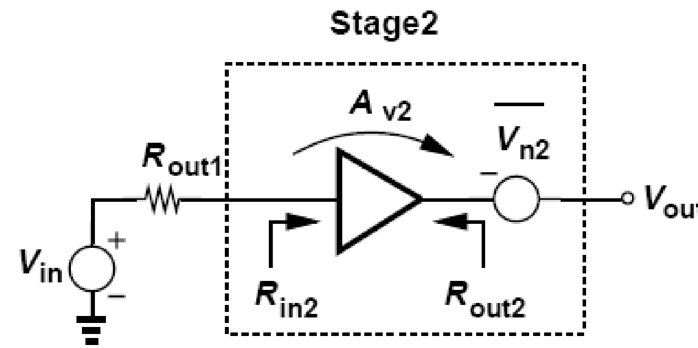
Solution:





Noise Figure of Cascaded Stages





$$A_0 = \frac{V_{out}}{V_{in}} = \frac{R_{in1}}{R_{in1}+R_S} A_{v1} \frac{R_{in2}}{R_{in2}+R_{out1}} A_{v2}$$

$$\overline{V_{n,out}^2} = \overline{V_{n2}^2} + \overline{V_{n1}^2} \frac{R_{in2}^2}{(R_{in2}+R_{out1})^2} A_{v2}^2$$

$$\mathrm{NF}_{\mathrm{tot}} = 1 + \frac{\overline{V_{n,out}^2}}{A_0^2} \cdot \frac{1}{4kTR_S}$$

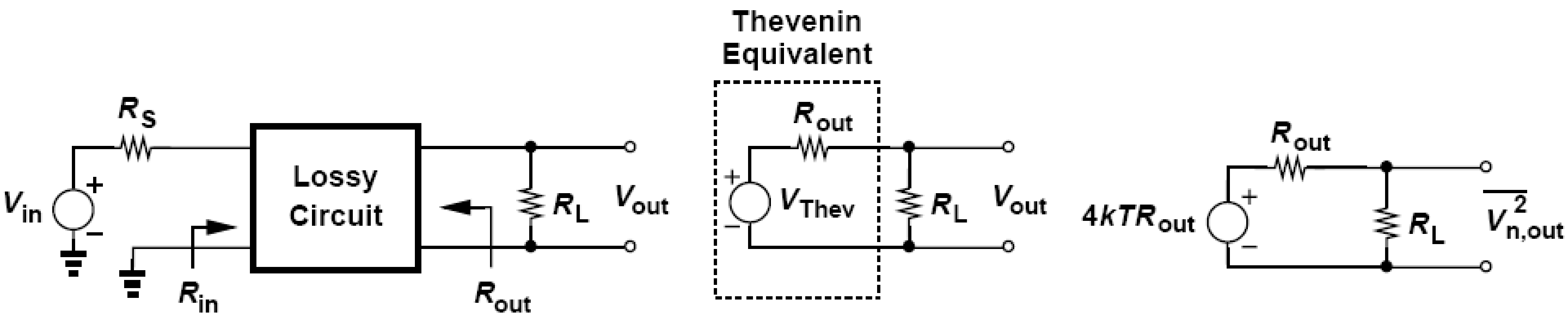
$$NF_2 = 1 + \frac{\overline{V_{n2}^2}}{\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2} A_{v2}^2} \frac{1}{4kTR_{out1}}$$

$$NF_{tot} = NF_1 + \frac{NF_2 - 1}{A_{P1}}$$

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1} \cdots A_{P(m-1)}}.$$

Called “Friis’ equation”, stage increases, implying that the first few stages in a cascade are the most critical. this result suggests that the noise contributed by each stage decreases as the total gain preceding that

Noise Figure of Lossy Circuits



The power loss is calculated as:

$$L = P_{in}/P_{out}$$

$$L = \frac{V_{in}^2}{V_{Thev}^2}\frac{R_{out}}{R_S}$$

$$\overline{V^2_{n,out}} = 4kTR_{out}\frac{R_L^2}{(R_L+R_{out})^2}$$

$$A_0 = \frac{V_{Thev}}{V_{in}}\frac{R_L}{R_L+R_{out}}$$

$$\begin{aligned} NF &= 4kTR_{out}\frac{V_{in}^2}{V_{Thev}^2}\frac{1}{4kTR_S} \\ &= L. \end{aligned}$$

Example of Noise Figure of Lossy Circuits

The receiver shown below incorporates a front-end band-pass filter (BPF) to suppress some of the interferers that may desensitize the LNA. If the filter has a loss of *L* and the LNA a noise figure of NF_{LNA}, calculate the overall noise figure. For example, if L = 1.5 dB and NFLNA = 2 dB