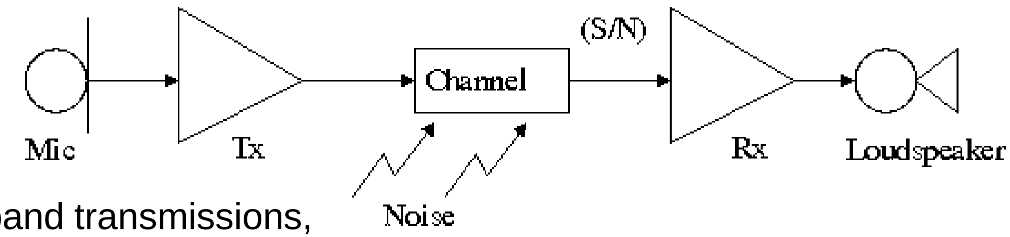
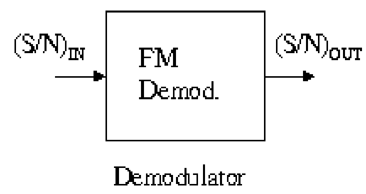
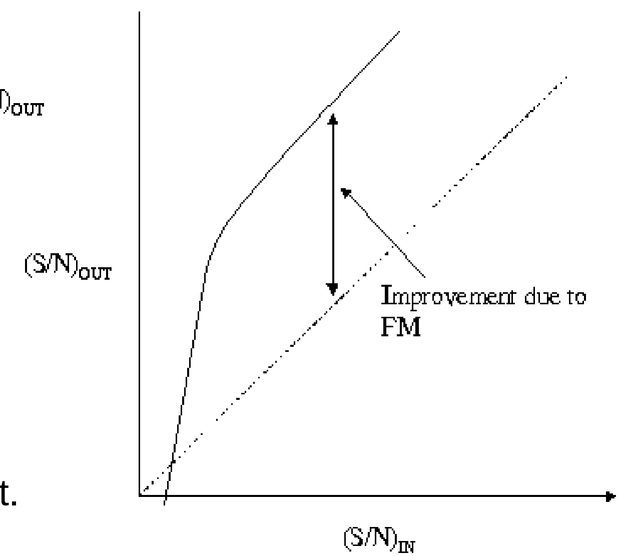
Introduction to Modulation and Demodulation

The purpose of a communication system is to transfer information from a source to a destination.



In practice, problems arise in baseband transmissions, the major cases being:



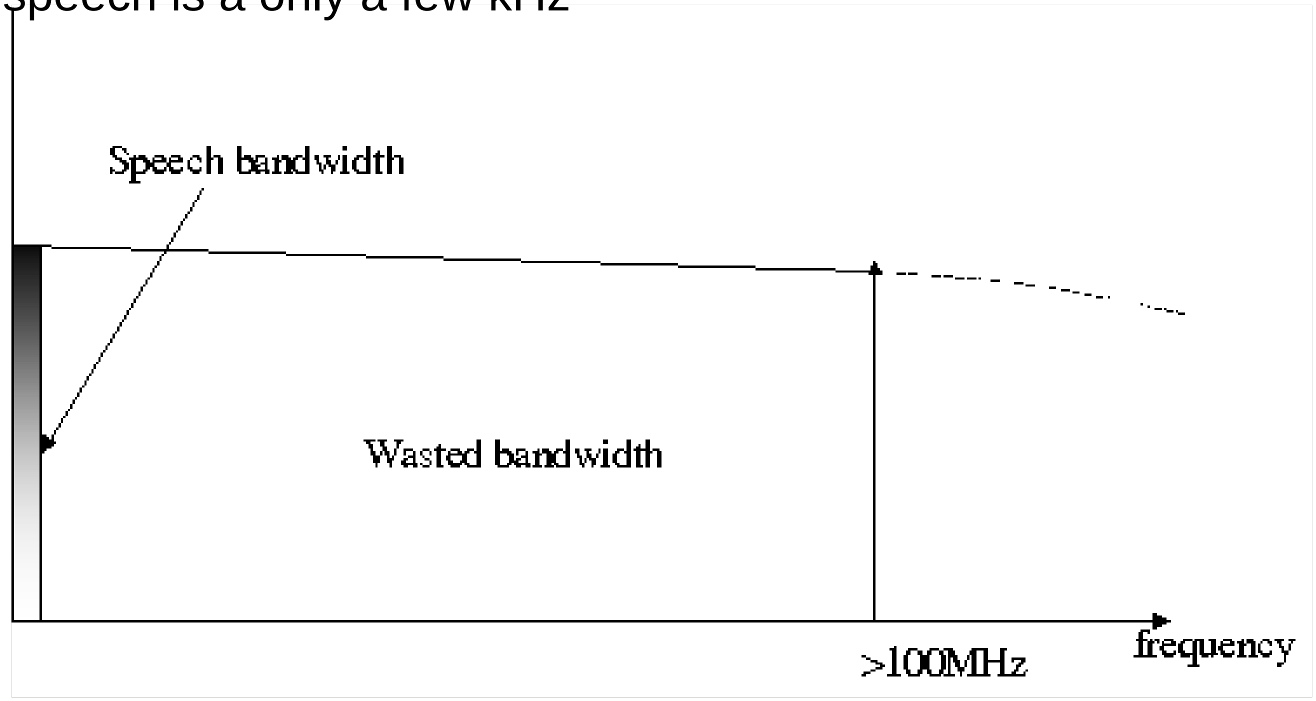


- Noise in the system – external noise and circuit noise reduces the signal-to-noise (S/N) ratio at the receiver (Rx) input and hence reduces the quality of the output.

- Such a system is not able to fully utilise the available bandwidth, for example telephone quality speech has a bandwidth ≃ 3kHz, a co-axial cable has a bandwidth of 100's of Mhz.
- Radio systems operating at baseband frequencies are very difficult.
- Not easy to network.

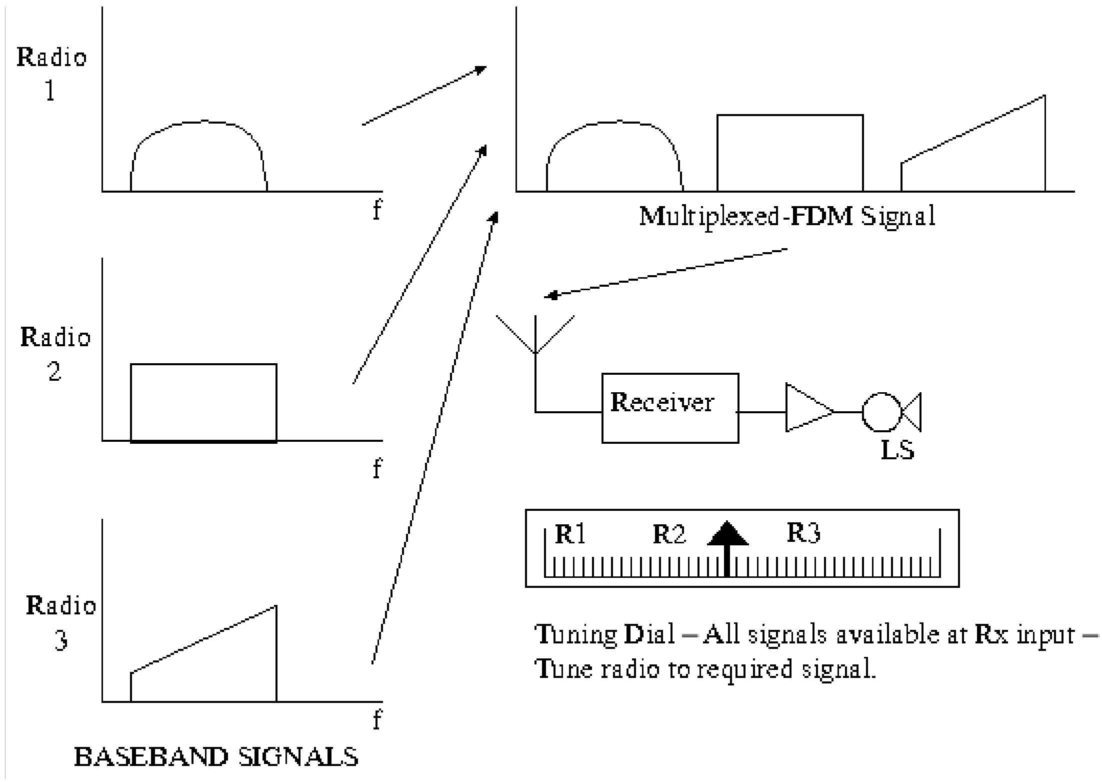
Multiplexing

Multiplexing is a modulation method which improves channel bandwidth utilisation.
For example, a co-axial cable has a bandwidth of 100's of Mhz.
Baseband speech is a only a few kHz



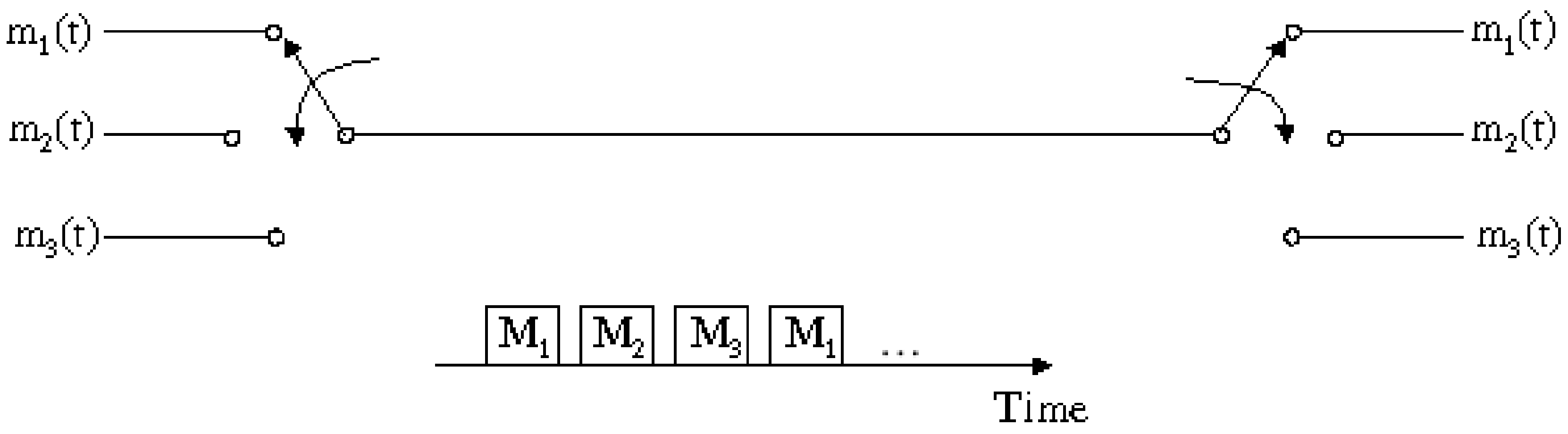
1) Frequency Division Multiplexing FDM

This allows several 'messages' to be translated from baseband, where they are all in the same frequency band, to adjacent but non overlapping parts of the spectrum. An example of FDM is broadcast radio (long wave LW, medium wave MW, *etc.*)



2) Time Division Multiplexing TDM

TDM is another form of multiplexing based on sampling which is a modulation technique. In TDM, samples of several analogue message symbols, each one sampled in turn, are transmitted in a sequence, *i.e.* the samples occupy adjacent time slots.



Radio Transmission

- Aerial dimensions are of the same order as the wavelength, λ, of the signal (*e.g.* quarter wave $\lambda/4$, $\lambda/2$ dipoles).

λ is related to frequency by $\boxed{\lambda = \frac{c}{f}}$ where *c* is the velocity of an electromagnetic wave, and *c* = $3x10^8$ m/sec in free space.

For baseband speech, with a signal at 3kHz, ($3x10^3$Hz) $\boxed{\lambda = \frac{3x10^8}{3x10^3}}$ = 10^5 metres or 100km.

- Aerials of this size are impractical although some transmissions at Very Low Frequency (VLF) for specialist applications are made.

- A modulation process described as 'up-conversion' (similar to FDM) allows the baseband signal to be translated to higher 'radio' frequencies.

- Generally 'low' radio frequencies 'bounce' off the ionosphere and travel long distances around the earth, high radio frequencies penetrate the ionosphere and make space communications possible.
  The ability to 'up convert' baseband signals has implications on aerial dimensions and design, long distance terrestrial communications, space communications and satellite communications. Background 'radio' noise is also an important factor to be considered.

- In a similar content, optical (fibre optic) communications is made possible by a modulation process in which an optical light source is modulated by an information source.

Networks

- A baseband system which is essentially point-to-point could be operated in a network. Some forms of access control (multiplexing) would be desirable otherwise the performance would be limited. Analogue communications networks have been in existence for a long time, for example speech radio networks for ambulance, fire brigade, police authorities *etc.*

- For example, 'digital speech' communications, in which the analogue speech signal is converted to a digital signal via an analogue-to-digital converter give a form more convenient for transmission and processing.

What is Modulation?

In modulation, a message signal, which contains the information is used to control the parameters of a carrier signal, so as to impress the information onto the carrier.

The Messages

The message or modulating signal may be either:
analogue – denoted by $m(t)$
digital – denoted by $d(t)$ – *i.e.* sequences of 1's and 0's
The message signal could also be a multilevel signal, rather than binary; this is not considered further at this stage.

The Carrier

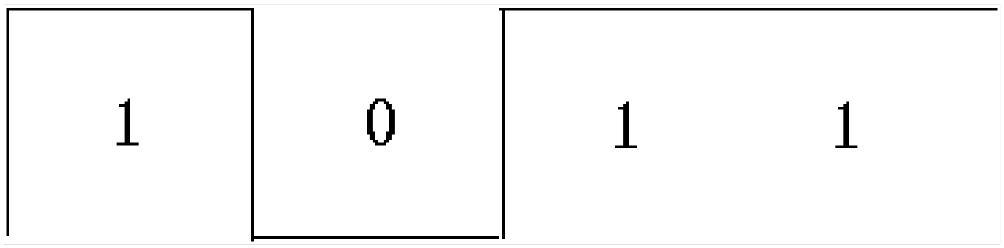
The carrier could be a 'sine wave' or a 'pulse train'.

Consider a 'sine wave' carrier:

$$v_c(t) = V_c \cos(\omega_c t + \varphi_c)$$

- If the message signal $m(t)$ controls amplitude – gives AMPLITUDE MODULATION AM
- If the message signal $m(t)$ controls frequency – gives FREQUENCY MODULATION FM
- If the message signal $m(t)$ controls phase- gives PHASE MODULATION PM or ϕM

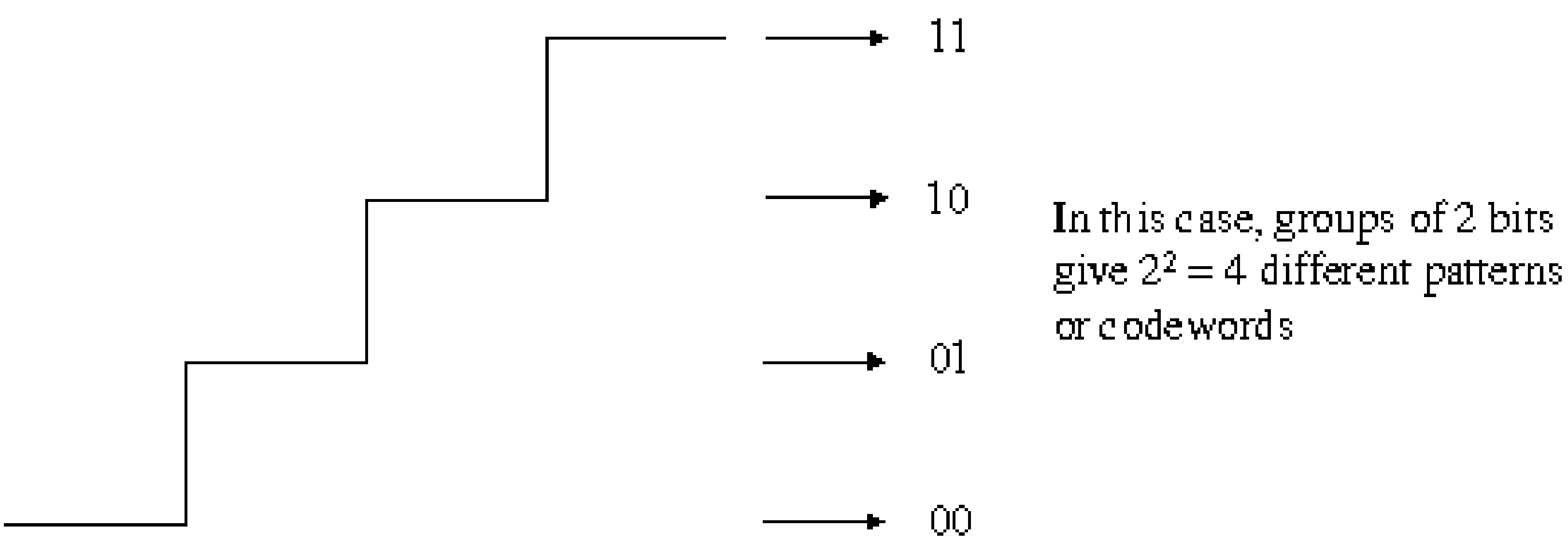
- Considering now a digital message *d*(*t*):
  If the message *d*(*t*) controls amplitude – gives **AMPLITUDE SHIFT KEYING ASK**.
  As a special case it also gives a form of Phase Shift Keying (PSK) called **PHASE REVERSAL KEYING PRK**.
- If the message *d*(*t*) controls frequency – gives **FREQUENCY SHIFT KEYING FSK.**
- If the message *d*(*t*) controls phase – gives **PHASE SHIFT KEYING PSK**.
- In this discussion, *d*(*t*) is a binary or 2 level signal representing 1's and 0's



- The types of modulation produced, *i.e.* ASK, FSK and PSK are sometimes described as binary or 2 level, *e.g.* Binary FSK, BFSK, BPSK, *etc.* or 2 level FSK, 2FSK, 2PSK *etc.*
- Thus there are 3 main types of Digital Modulation:
  ASK, FSK, PSK.

Multi-Level Message Signals

As has been noted, the message signal need not be either analogue (continuous) or binary, 2 level. A message signal could be multi-level or *m* levels where each level would represent a discrete pattern of 'information' bits. For example, *m* = 4 levels

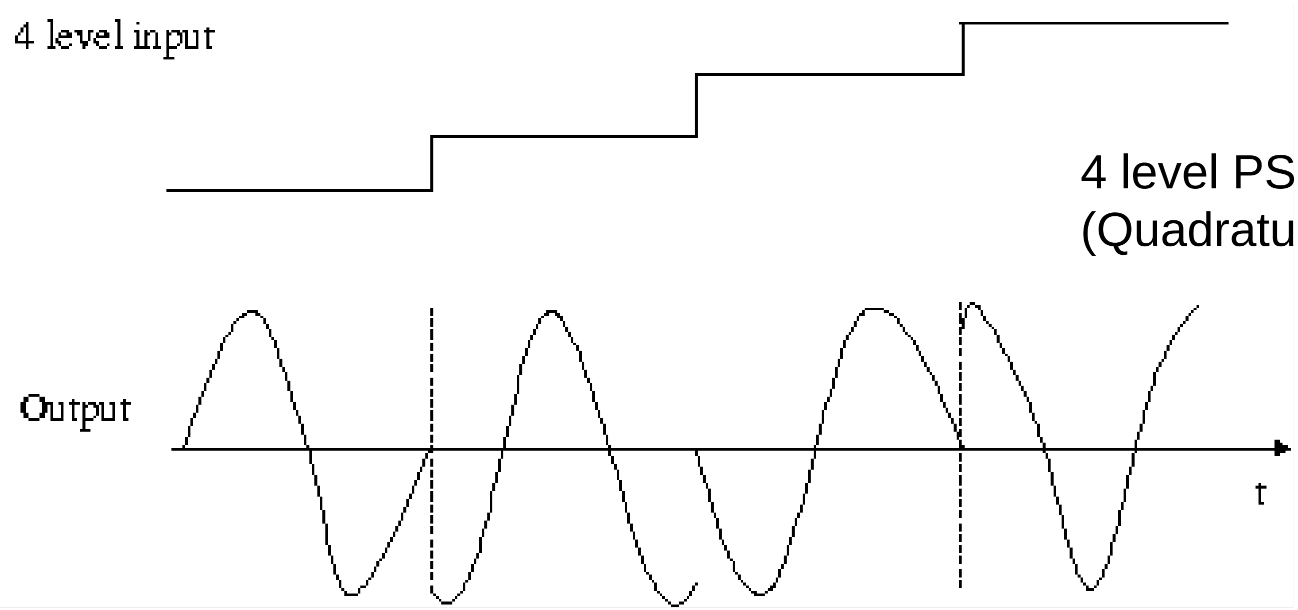


- In general *n* bits per codeword will give 2*n* = *m* different patterns or levels.
- Such signals are often called *m*-ary (compare with binary).
- Thus, with *m* = 4 levels applied to:

  Amplitude gives 4ASK or *m*-ary ASK

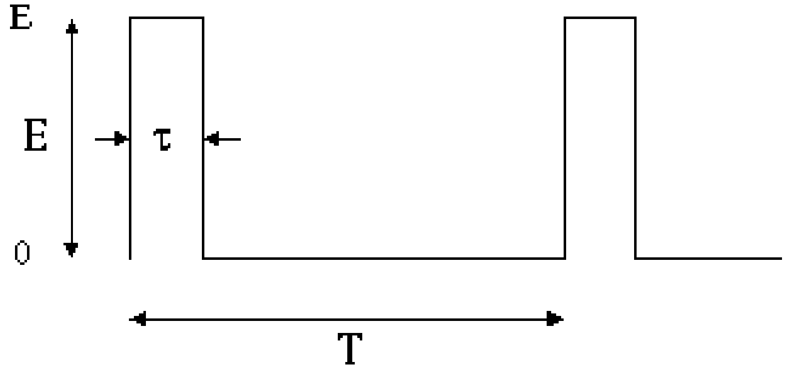
  Frequency gives 4FSK or *m*-ary FSK

  Phase gives 4PSK or *m*-ary PSK



4 level PSK is also called QPSK (Quadrature Phase Shift Keying).

Consider Now A Pulse Train Carrier



where $$p(t)=E,0<t<\tau$$ $$p(t)=0,\tau<t<T$$

and $$p(t)=\frac{E\tau}{T}+\frac{2E\tau}{T}\sum_{n=1}^{\infty} sinc(\frac{n\omega\tau}{2})\cos(n\omega\tau)$$

- The 3 parameters in the case are:
  Pulse Amplitude E
  Pulse width vt
  Pulse position T

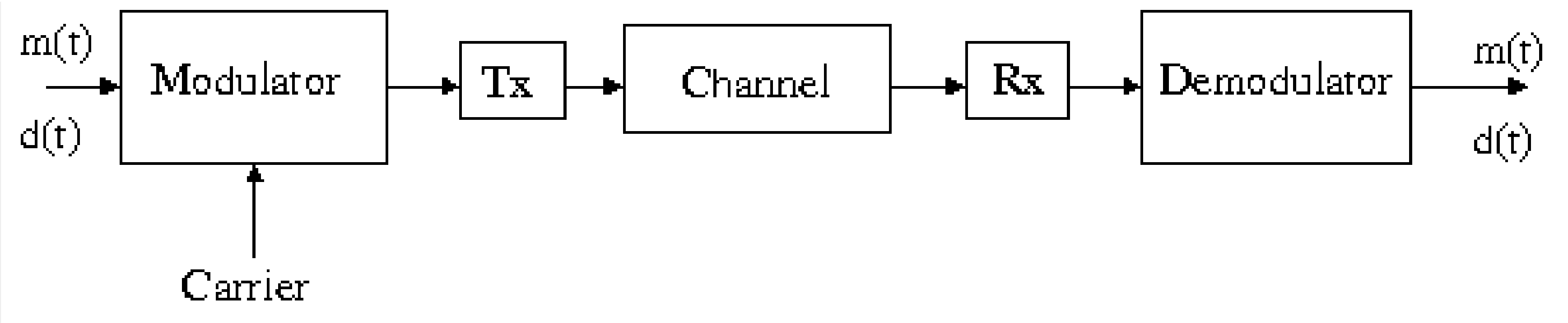
Hence:

- If *m*(*t*) controls E – gives PULSE AMPLITUDE MODULATION PAM
- If *m*(*t*) controls t - gives PULSE WIDTH MODULATION PWM
- If *m*(*t*) controls T - gives PULSE POSITION MODULATION PPM

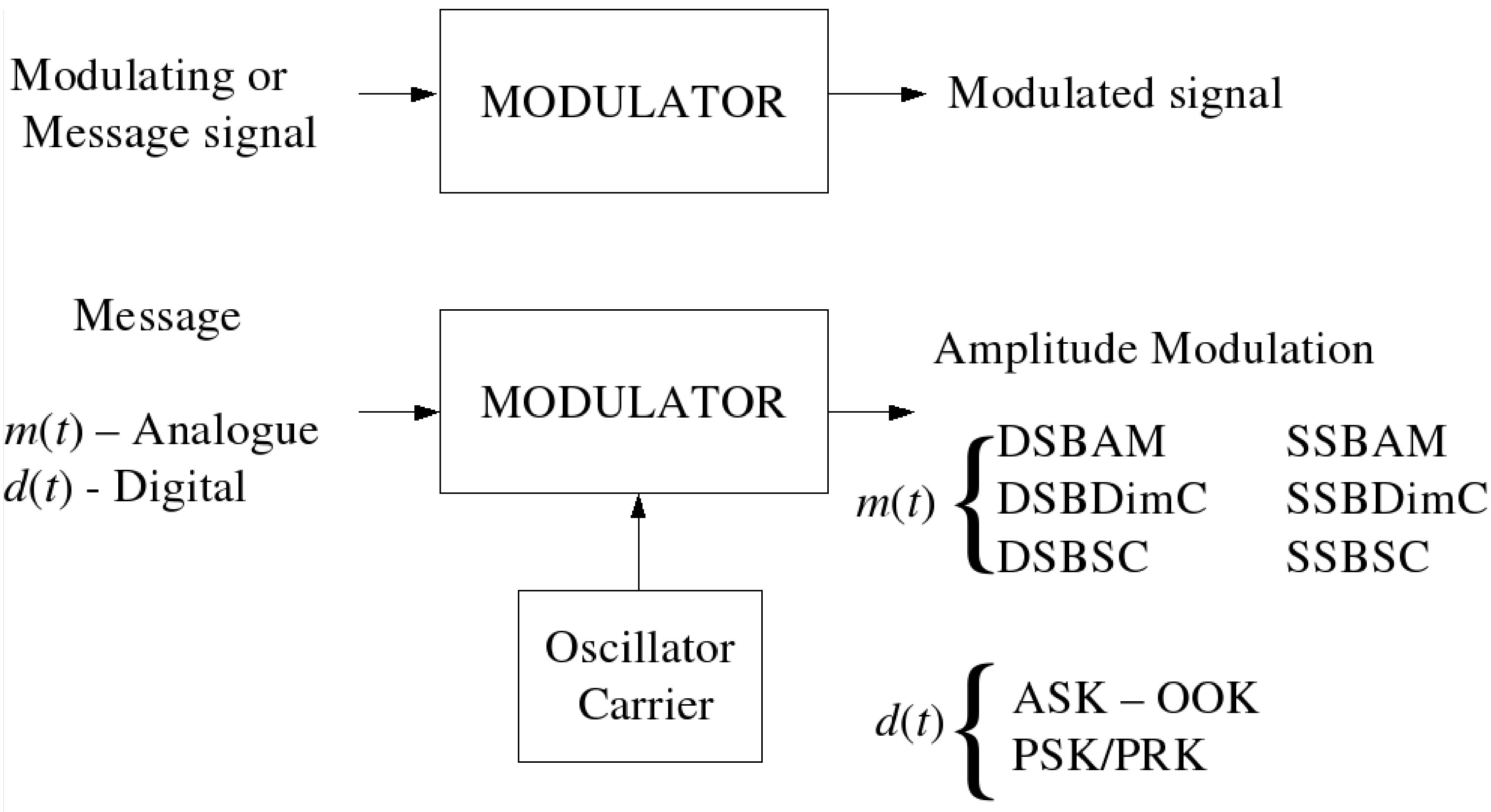
In principle, a digital message *d*(*t*) could be applied but this will not be considered further.

What is Demodulation?

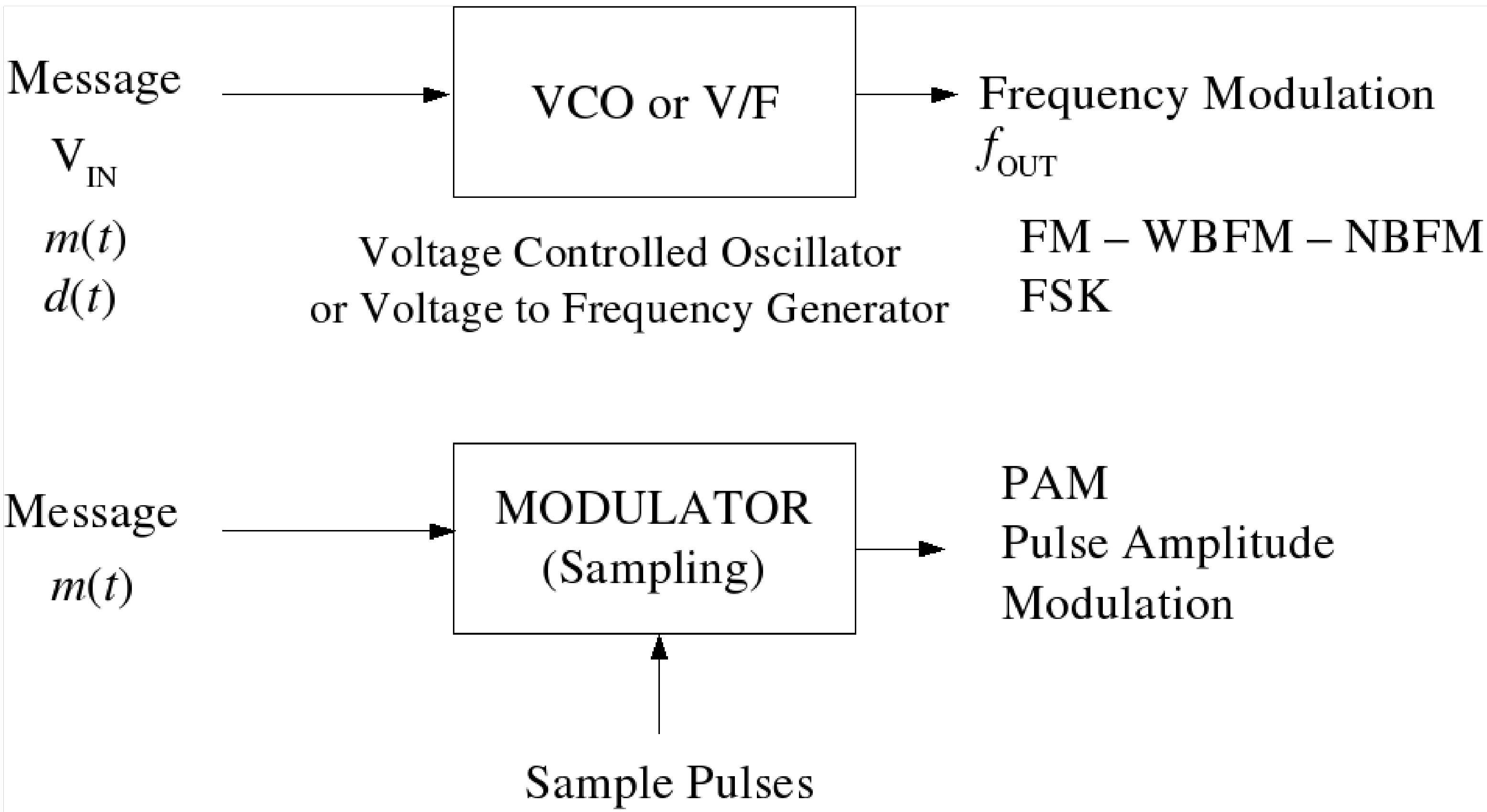
Demodulation is the reverse process (to modulation) to recover the message signal *m*(*t*) or *d*(*t*) at the receiver.



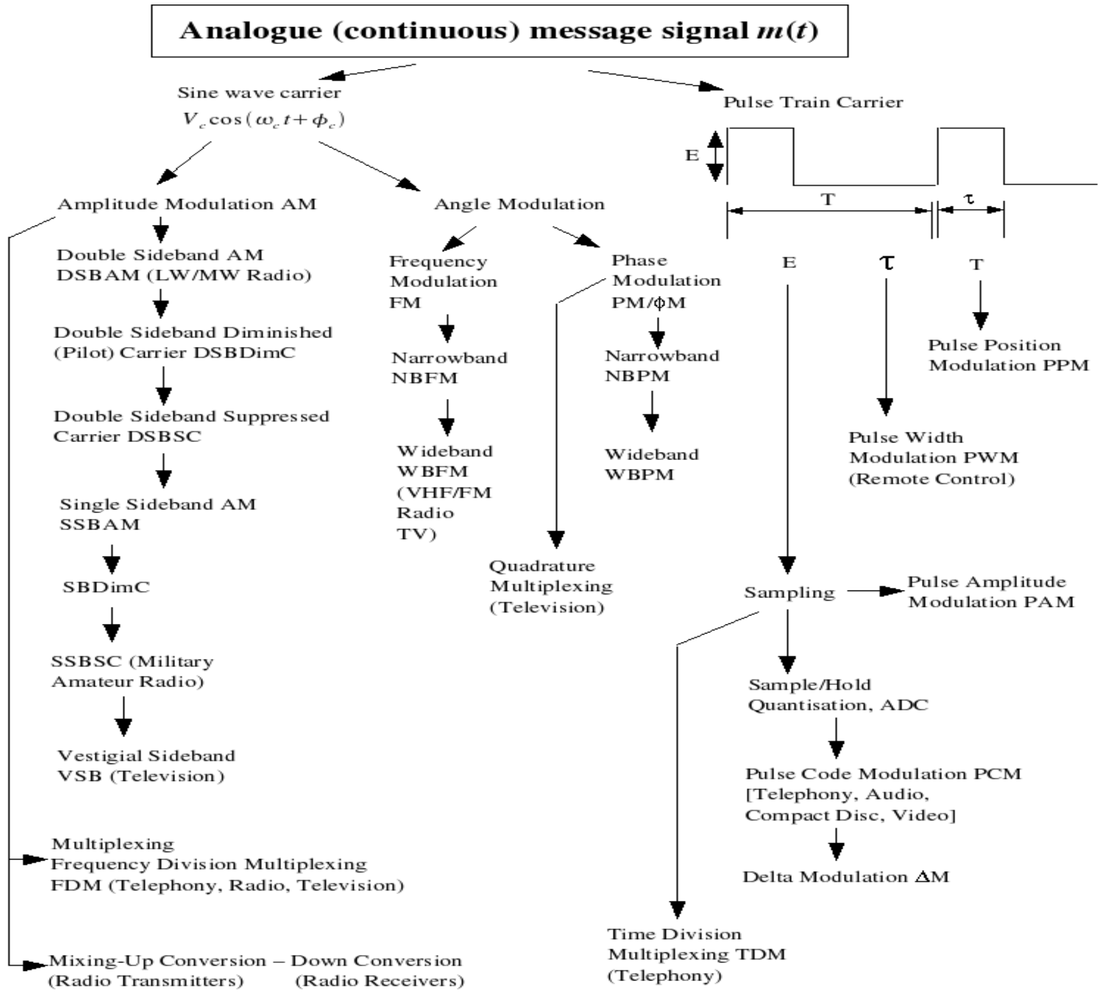
Summary of Modulation Techniques 1



Summary of Modulation Techniques 2



Summary of Modulation Techniques with some Derivatives and Familiar Applications



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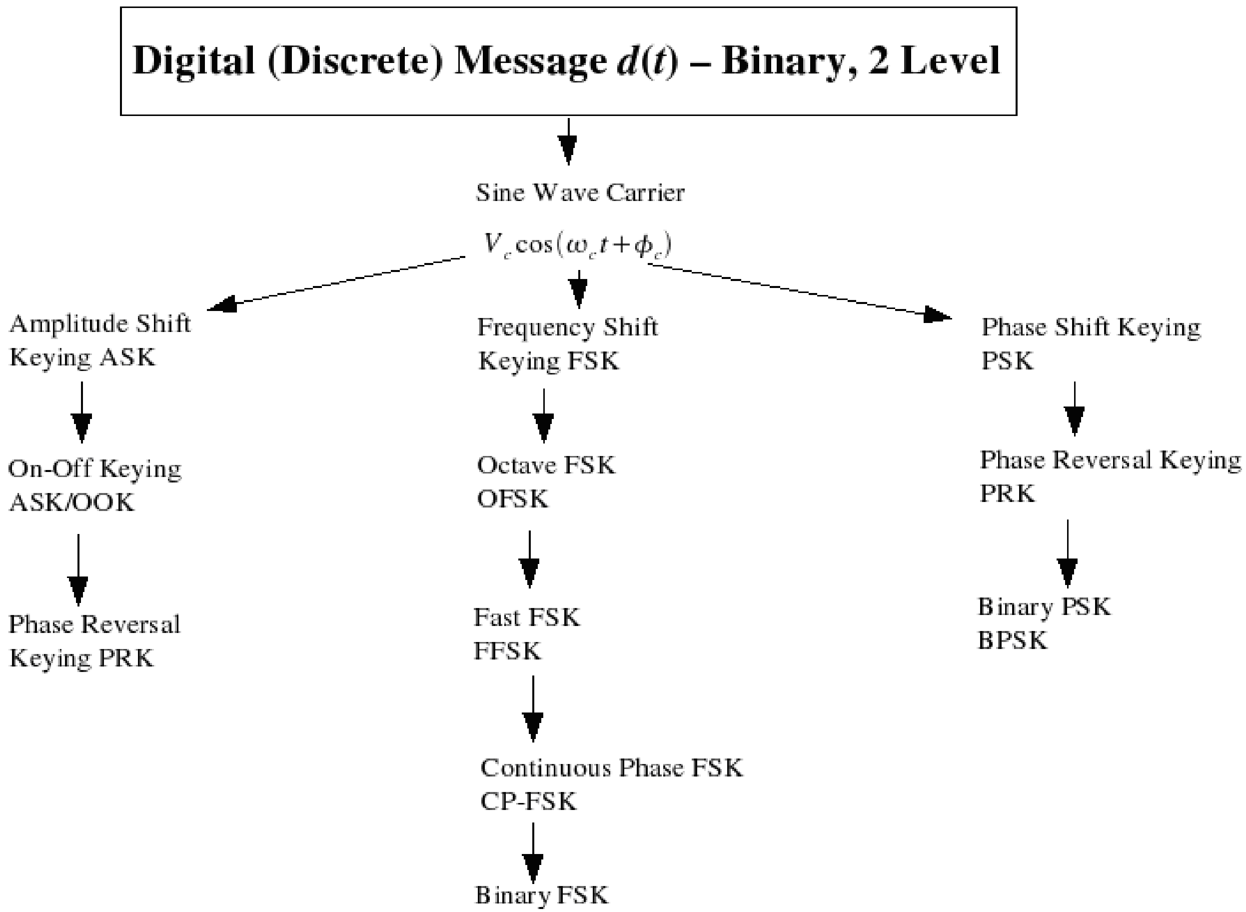
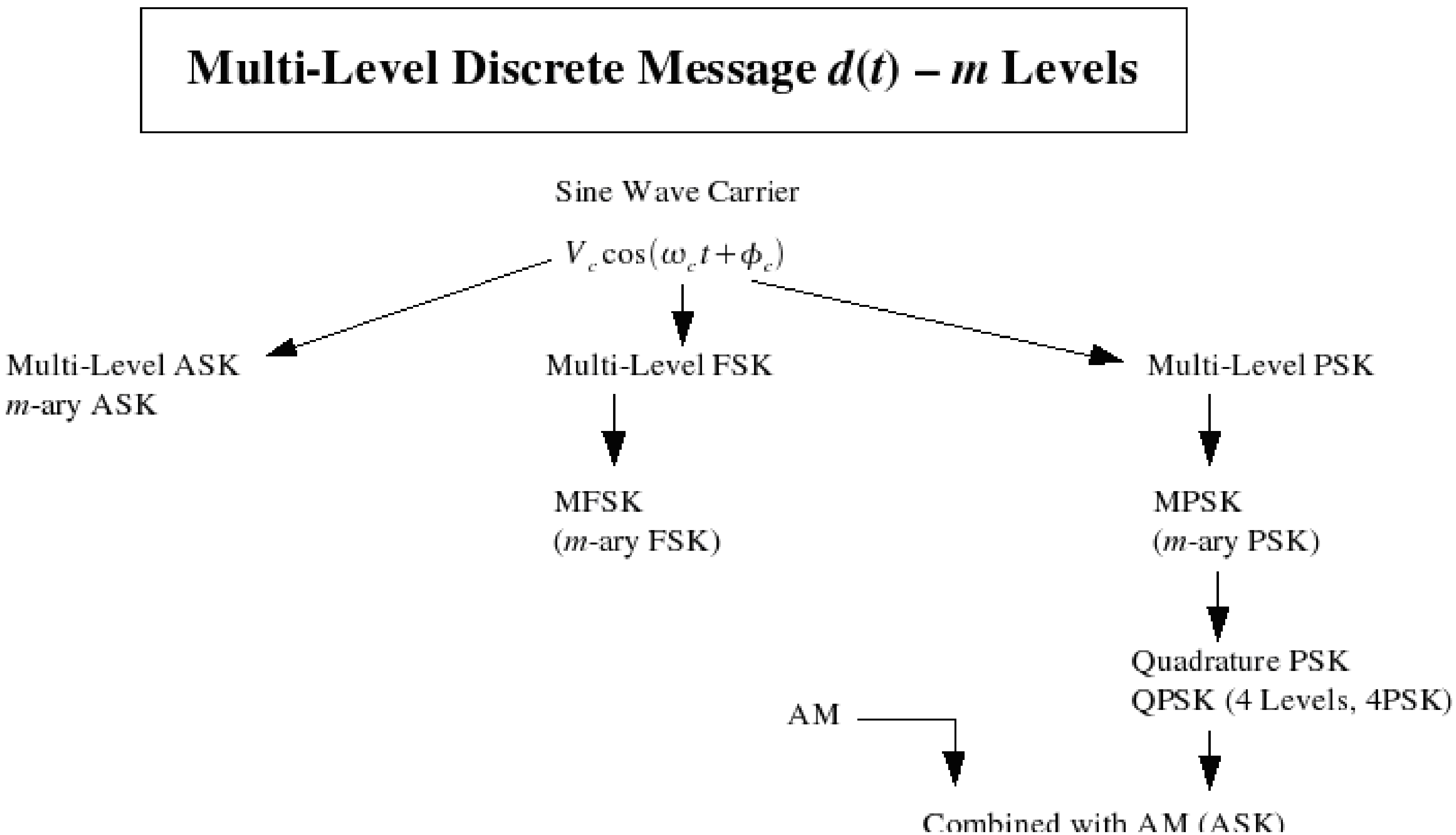
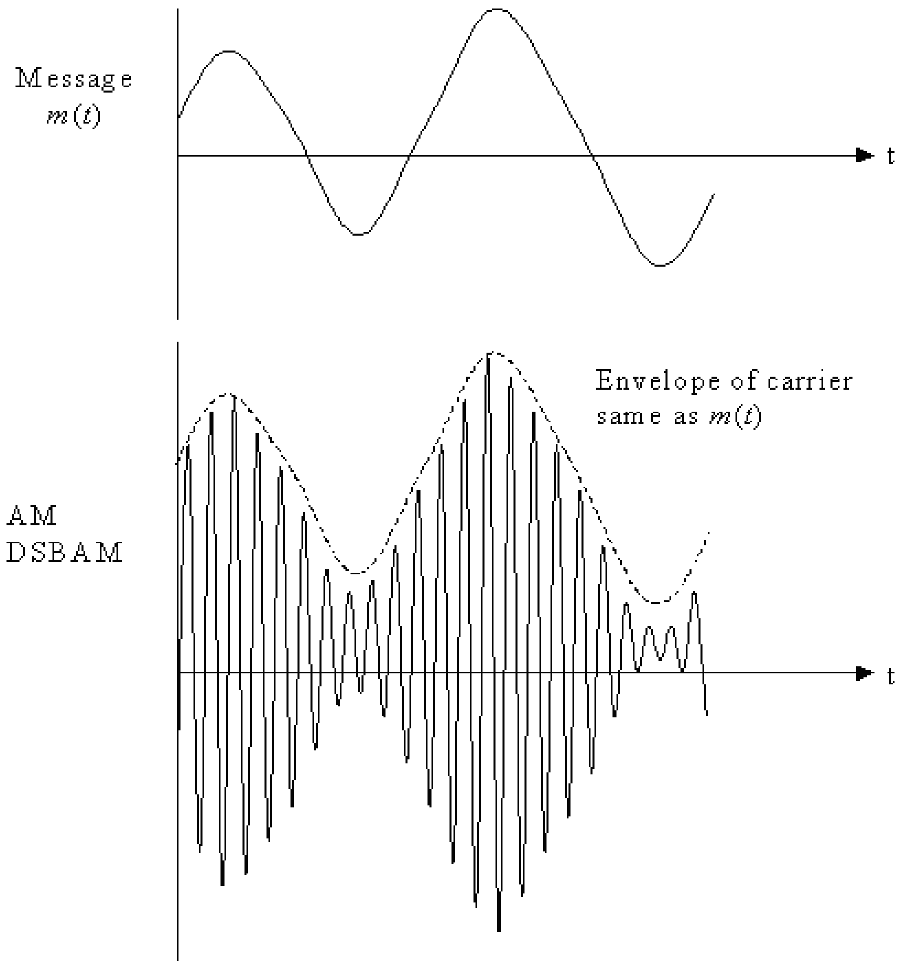


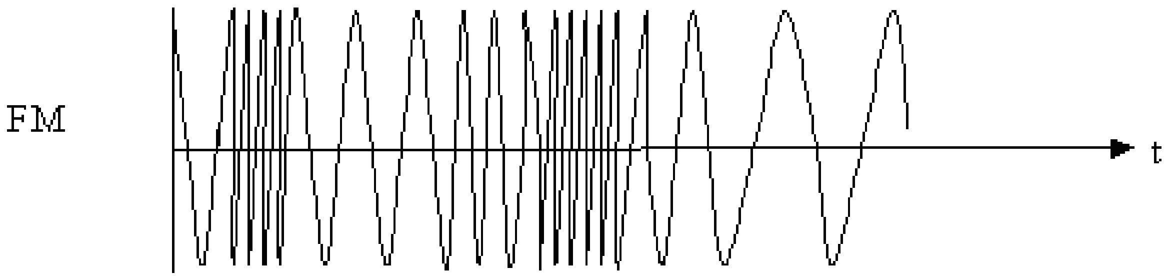
Summary of Modulation Techniques with some Derivatives and Familiar Applications 2

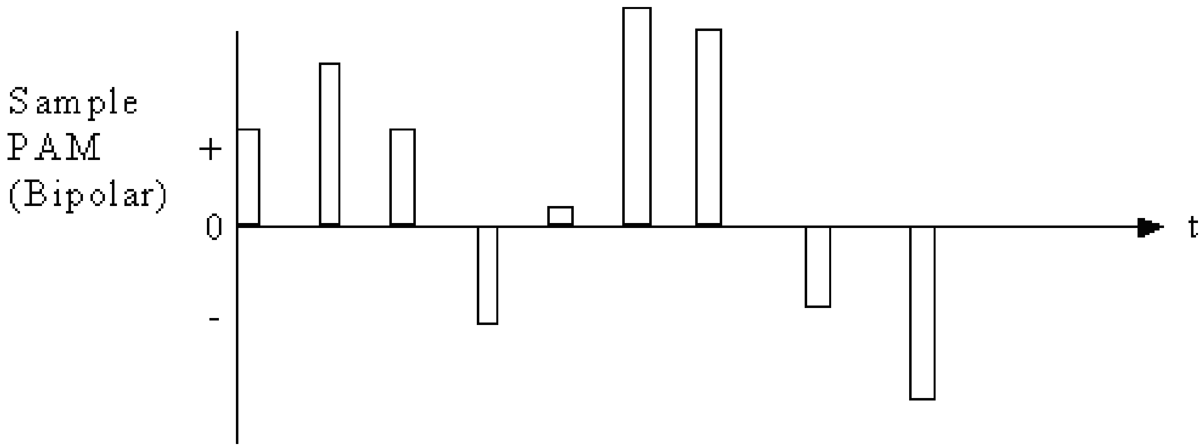


Modulation Types AM, FM, PAM

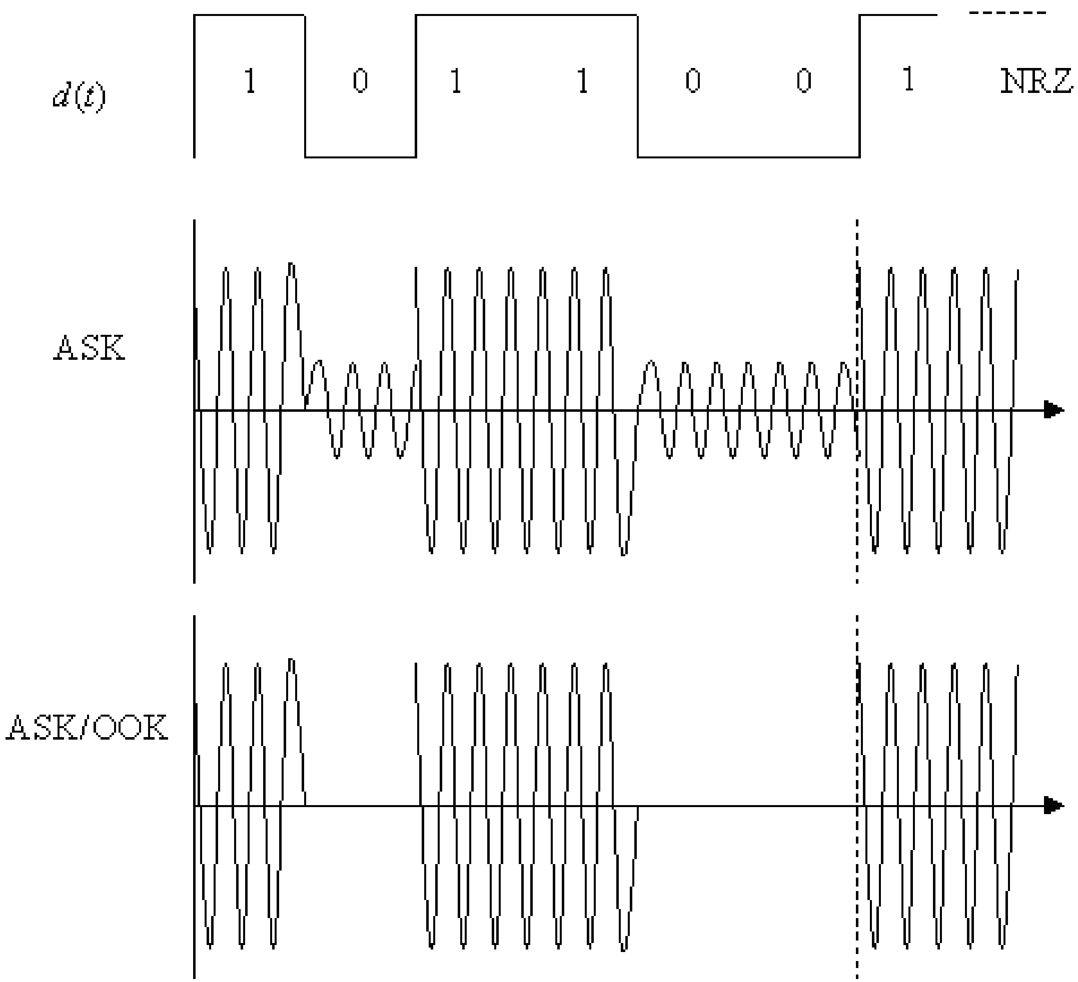


Modulation Types AM, FM, PAM 2

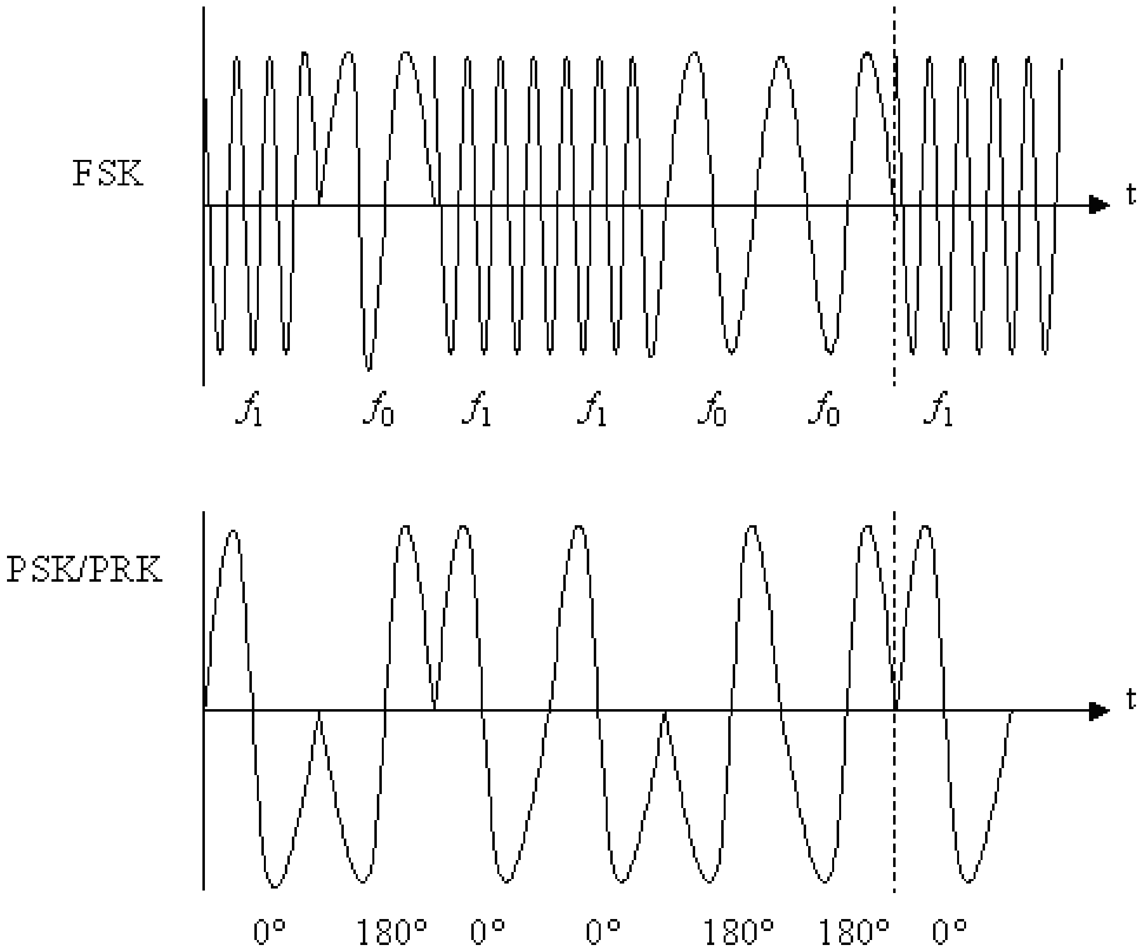




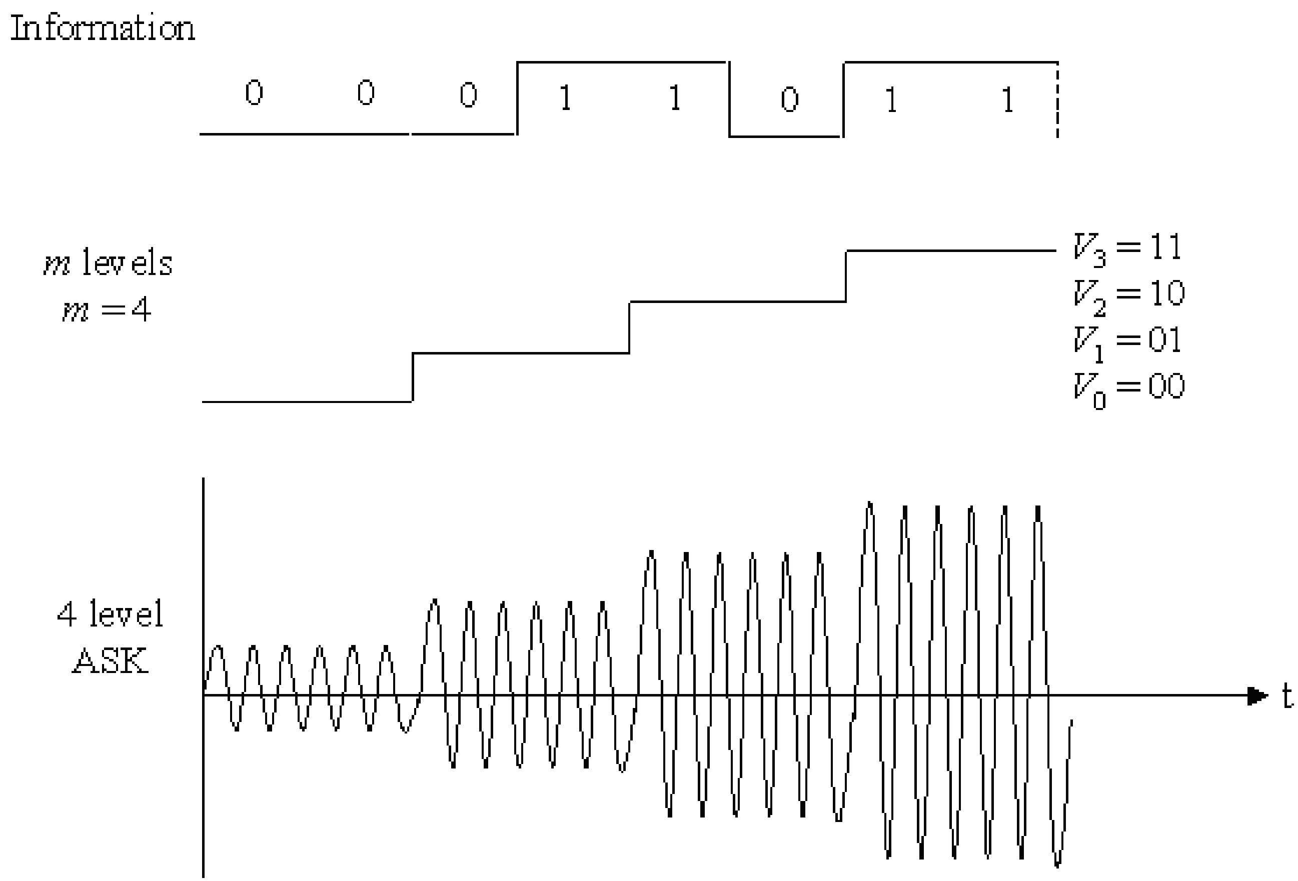
Modulation Types (Binary ASK, FSK, PSK)



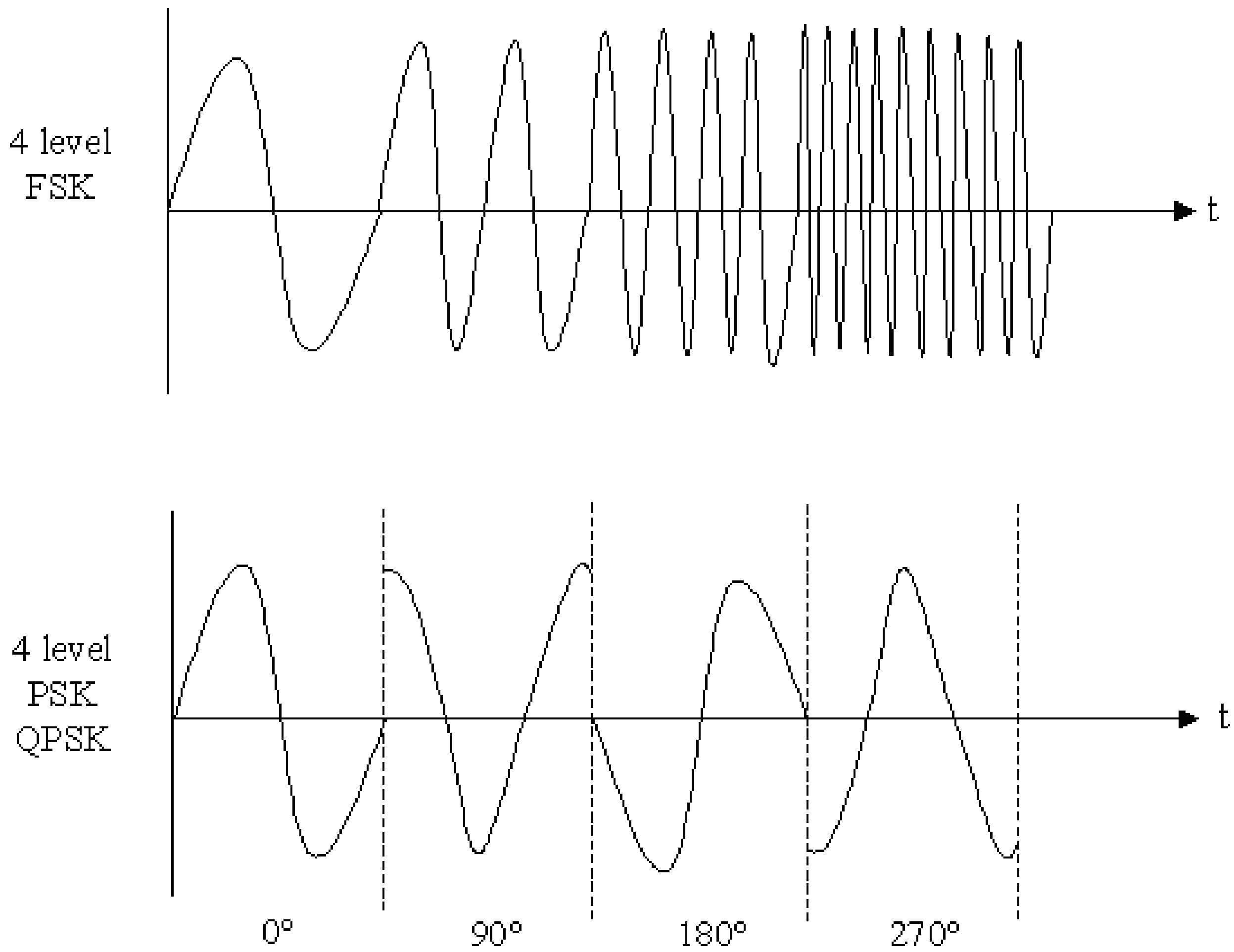
Modulation Types (Binary ASK, FSK, PSK) 2



Modulation Types – 4 Level ASK, FSK, PSK

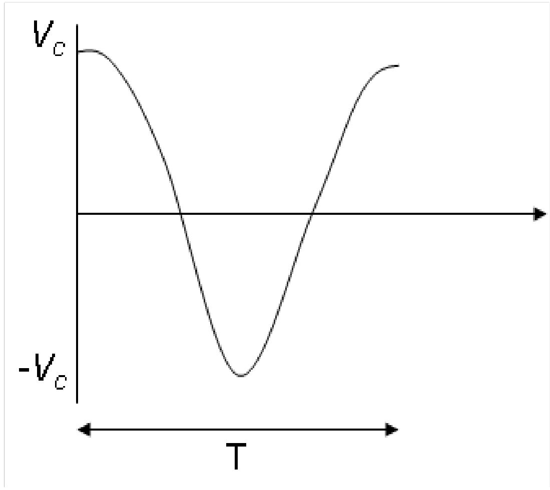


Modulation Types – 4 Level ASK, FSK, PSK 2



Analogue Modulation – Amplitude Modulation

Consider a 'sine wave' carrier.



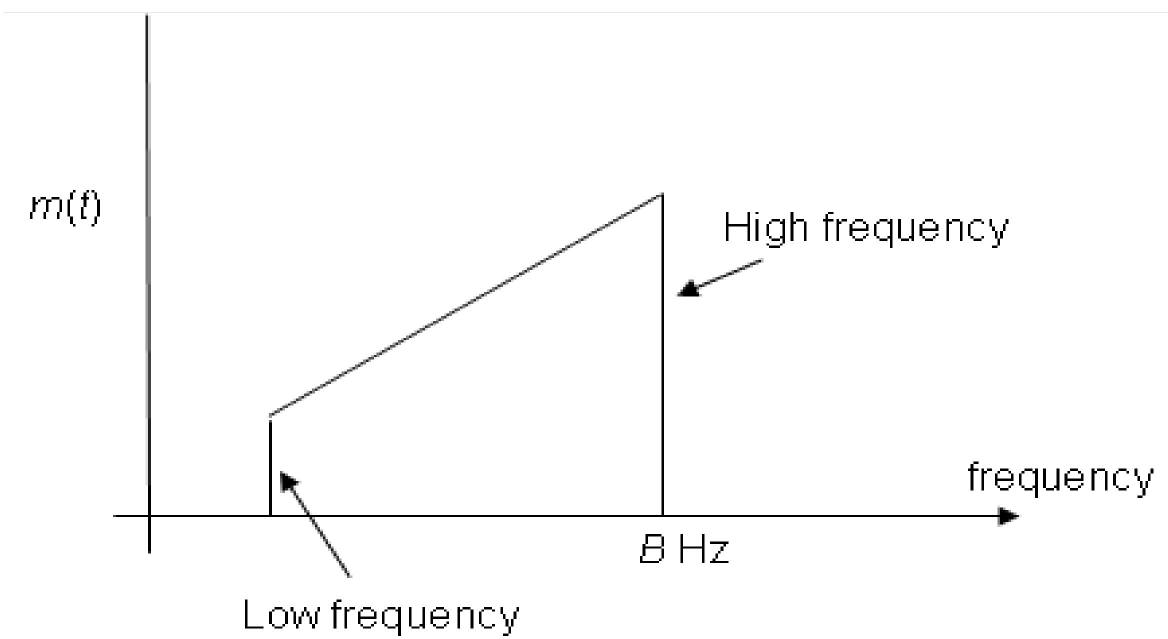
$v_c(t) = V_c\ cos(\omega_c t)$, peak amplitude = V_c, carrier frequency ω_c radians per second. Since $\omega_c = 2\pi f_c$, frequency = f_c Hz where $f_c = 1/T$.

Amplitude Modulation AM

In AM, the modulating signal (the message signal) $m(t)$ is 'impressed' on to the amplitude of the carrier.

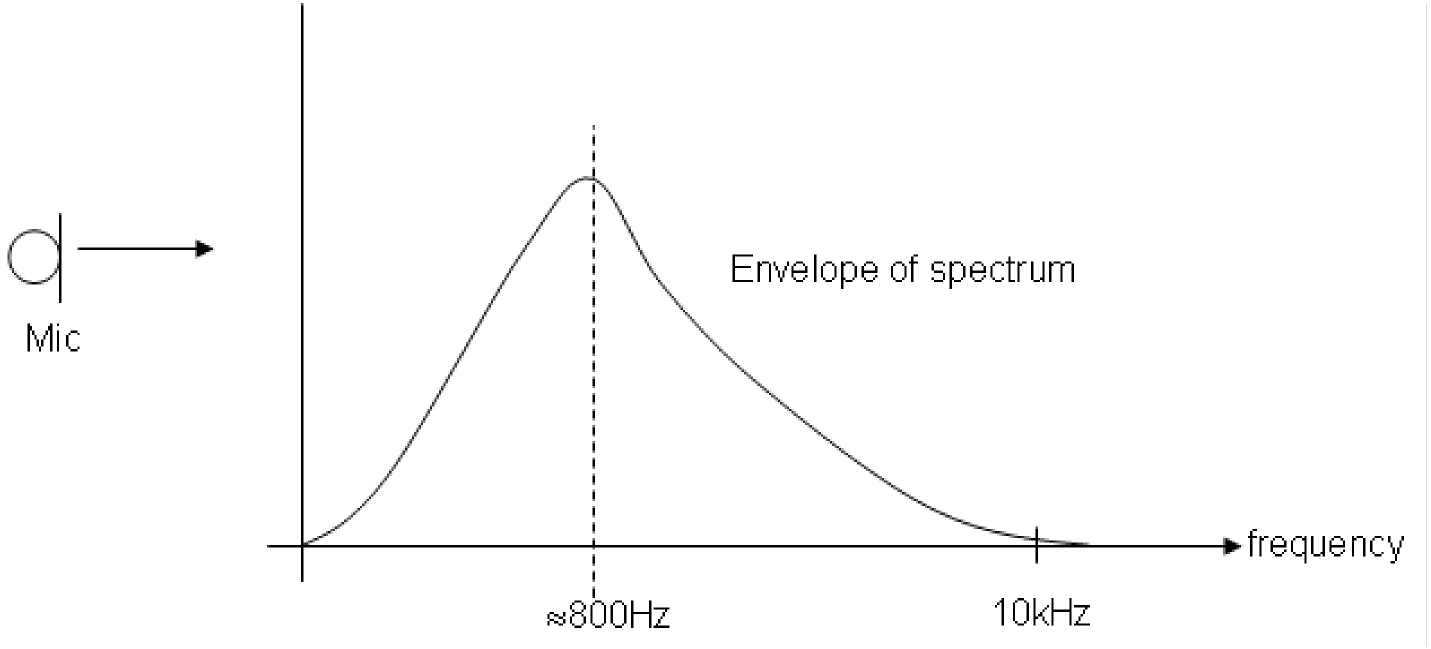
Message Signal *m*(*t*)

In general $m(t)$ will be a band of signals, for example speech or video signals. A notation or convention to show baseband signals for $m(t)$ is shown below

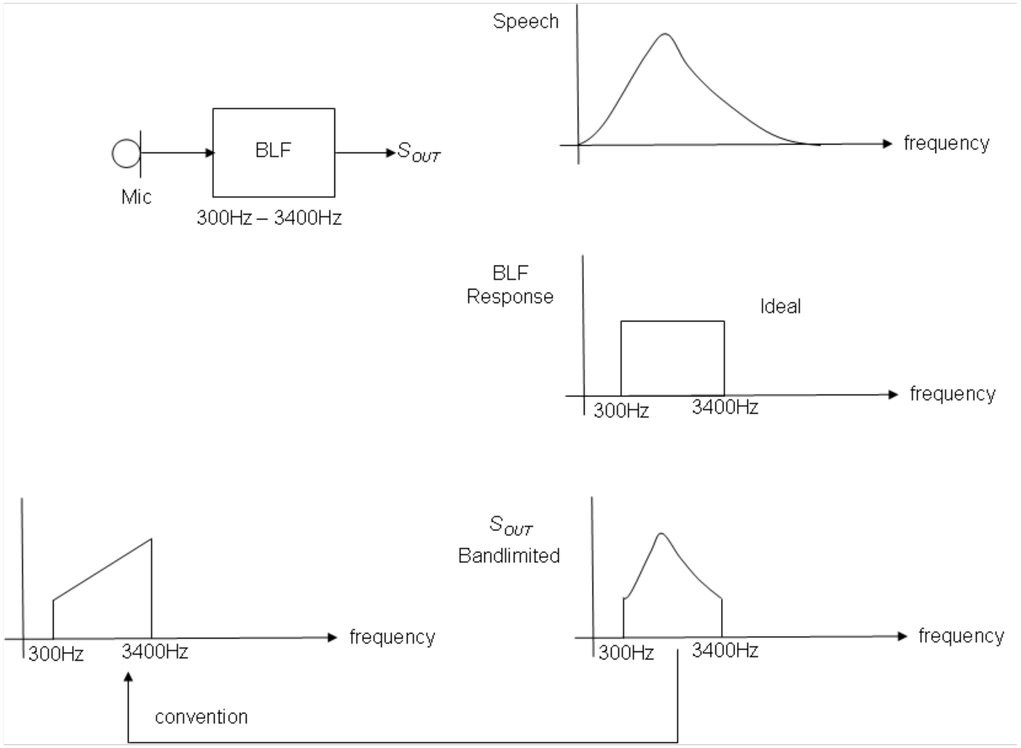


Message Signal *m*(*t*)

In general $m(t)$ will be band limited. Consider for example, speech via a microphone. The envelope of the spectrum would be like:



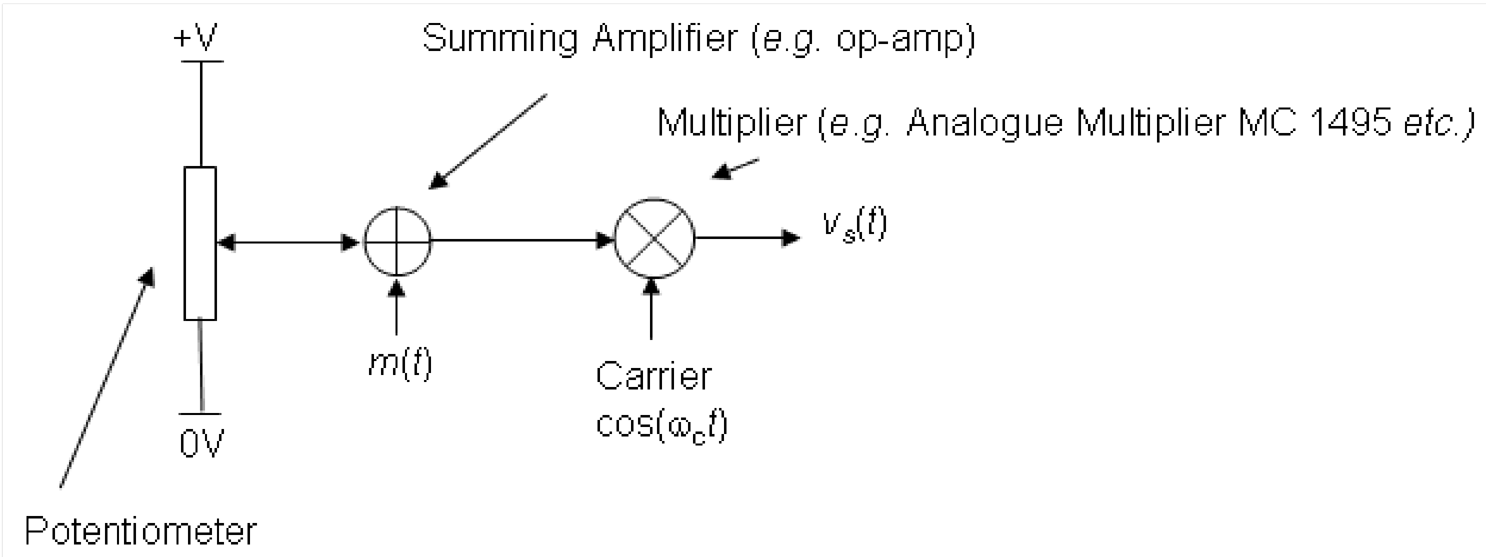
Message Signal *m*(*t*)



In order to make the analysis and indeed the testing of AM systems easier, it is common to make *m*(*t*) a test signal, *i.e.* a signal with a constant amplitude and frequency given by

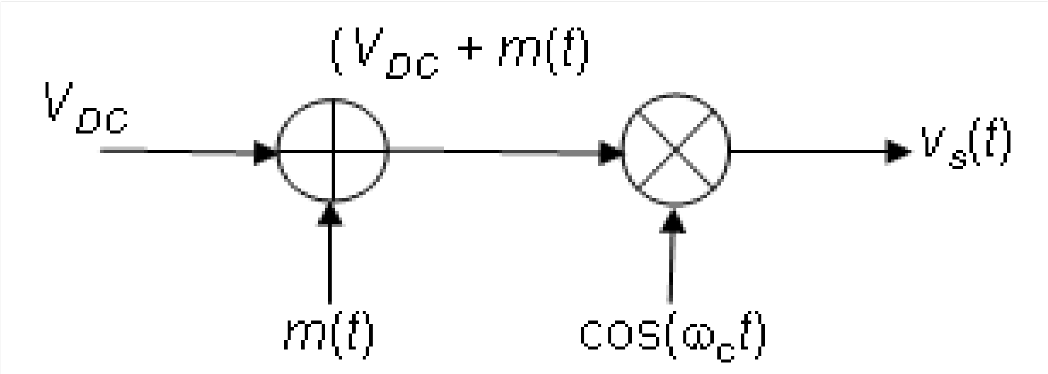
$$\boxed{m(t) = V_m \cos(\omega_m t)}$$

Schematic Diagram for Amplitude Modulation



V_{DC} is a variable voltage, which can be set between 0 Volts and +V Volts. This schematic diagram is very useful; from this all the important properties of AM and various forms of AM may be derived.

Equations for AM



From the diagram $v_s(t) = (V_{DC} + m(t))\cos(\omega_c t)$ where V_{DC} is the DC voltage that can be varied. The equation is in the form Amp cos $\omega_c t$ and we may 'see' that the amplitude is a function of $m(t)$ and V_{DC}. Expanding the equation we get:

$$v_s(t) = V_{DC}\cos(\omega_c t) + m(t)\cos(\omega_c t)$$

Equations for AM

Now let $m(t) = V_m \cos \omega_m t$, *i.e.* a 'test' signal, $v_s(t) = V_{DC}\cos(\omega_c t) + V_m \cos(\omega_m t)\cos(\omega_c t)$

Using the trig identity $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

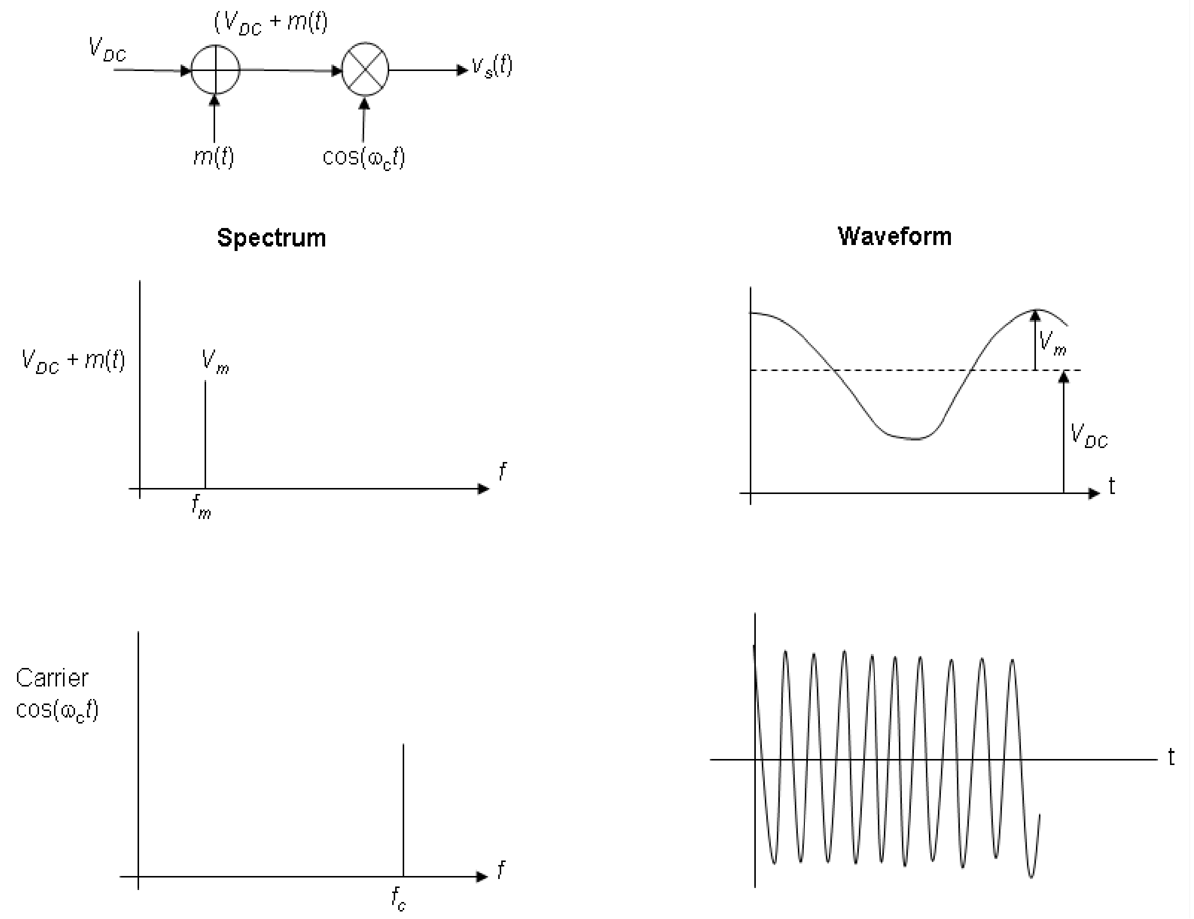
we have $v_s(t) = V_{DC}\cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t) + \frac{V_m}{2}\cos((\omega_c - \omega_m)t)$

Components:	**Carrier**	**upper sideband USB**	**lower sideband LSB**
Amplitude:	V_{DC}	$V_m/2$	$V_m/2$
Frequency:	ω_c f_c	$\omega_c + \omega_m$ $f_c + f_m$	$\omega_c - \omega_m$ $f_c + f_m$

This equation represents **Double Amplitude Modulation – DSBAM**

Spectrum and Waveforms

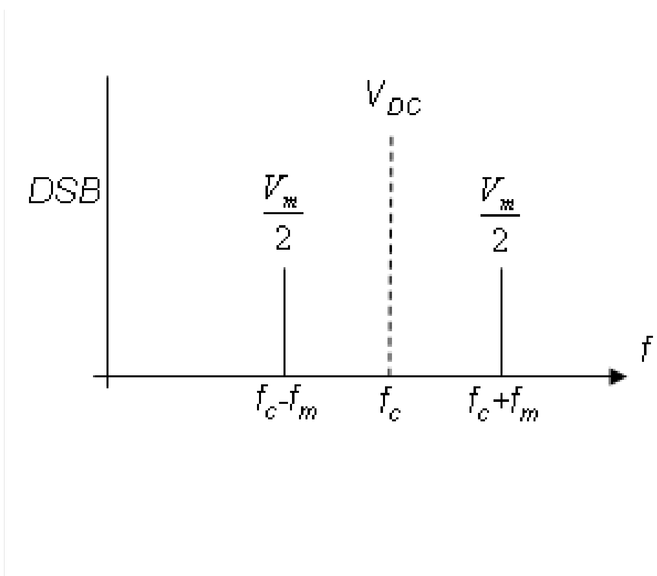
The following diagrams represent the spectrum of the input signals, namely (V_{DC} + $m(t)$), with $m(t) = V_m \cos \omega_m t$, and the carrier $\cos \omega_c t$ and corresponding waveforms.

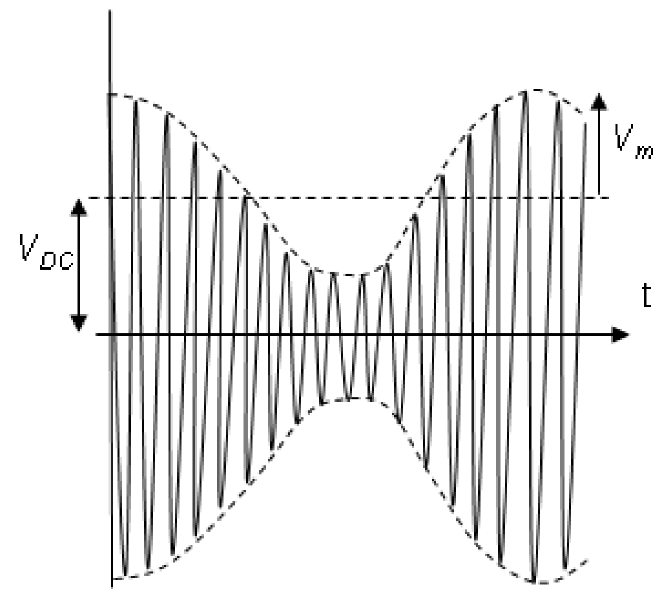


Spectrum and Waveforms

The above are input signals. The diagram below shows the spectrum and corresponding waveform of the <u>output</u> signal, given by

$$v_s(t) = V_{DC}\cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c+\omega_m)t) + \frac{V_m}{2}\cos((\omega_c-\omega_m)t)$$



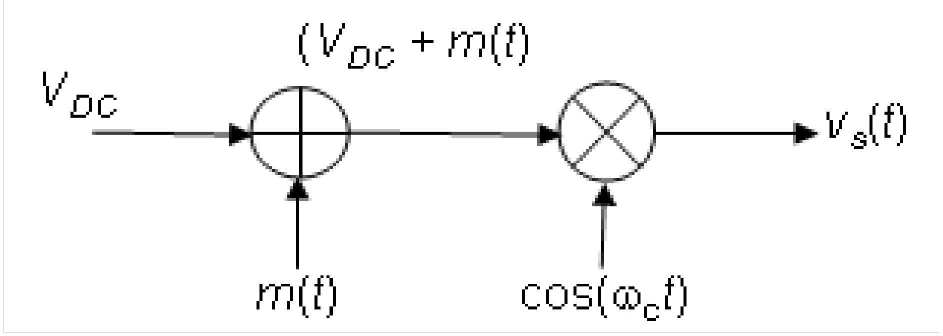


Double Sideband AM, DSBAM

The component at the output at the carrier frequency f_c is shown as a broken line with amplitude V_{DC} to show that the amplitude depends on V_{DC}. The structure of the waveform will now be considered in a little more detail.

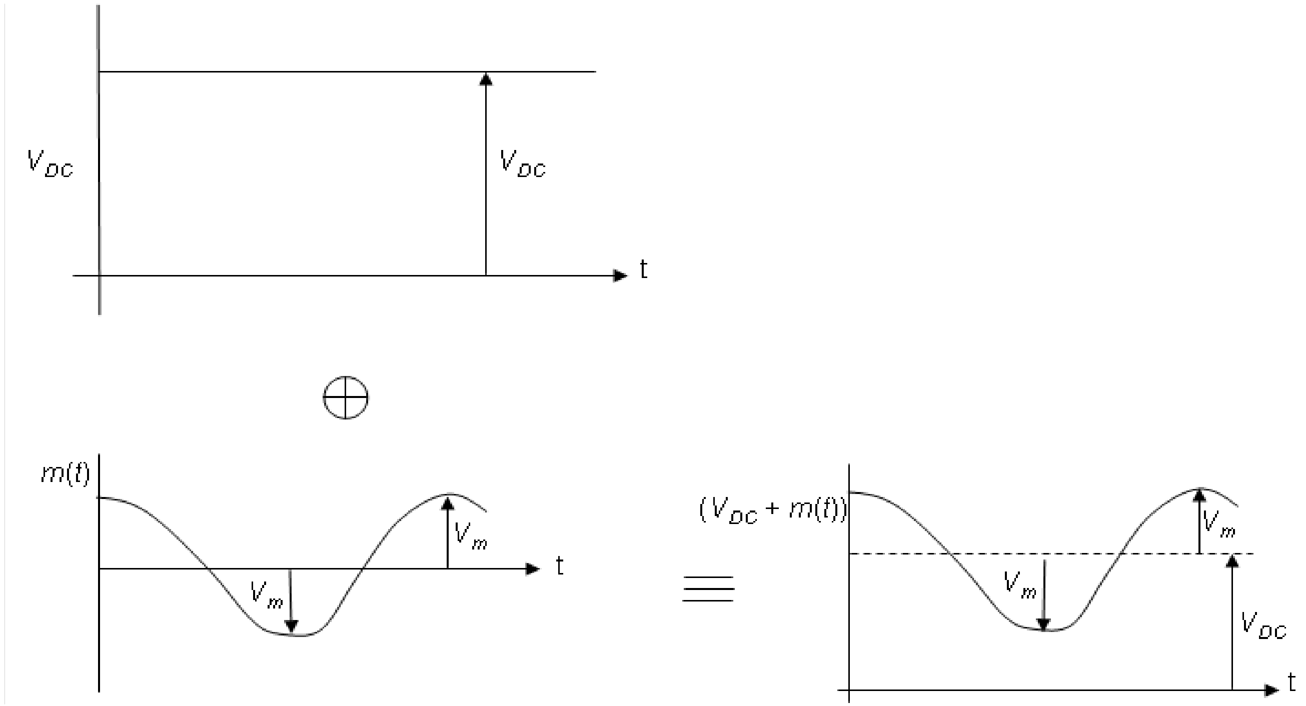
Waveforms

Consider again the diagram



V_{DC} is a variable DC offset added to the message; $m(t) = V_m \cos \omega_m t$

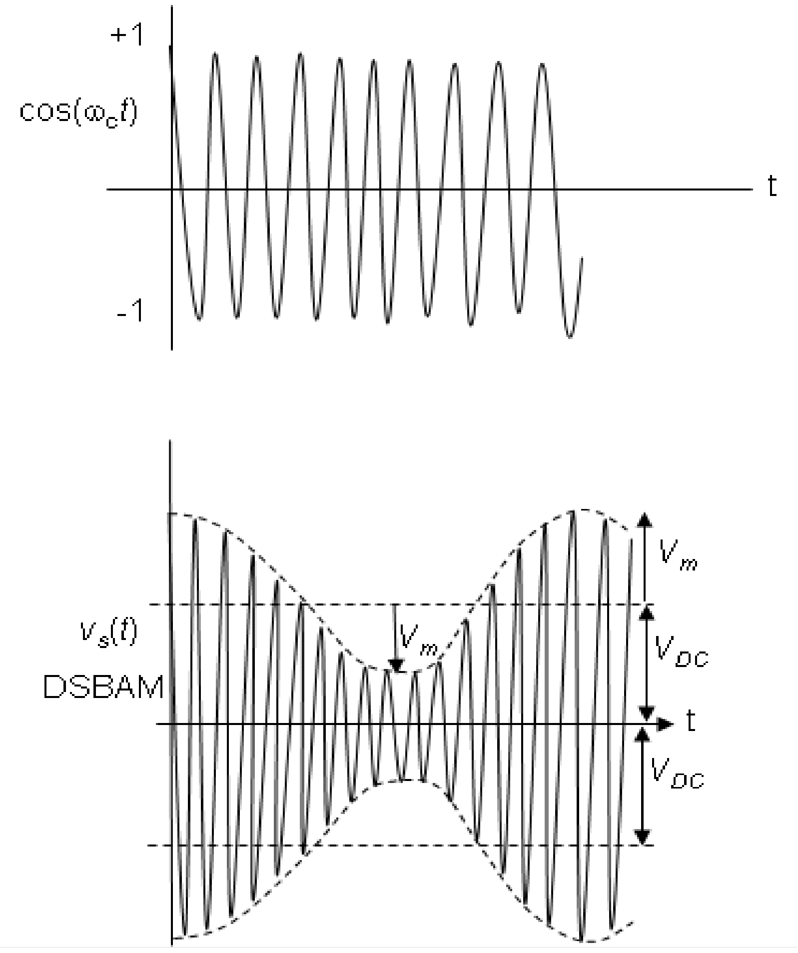
Double Sideband AM, DSBAM



This is multiplied by a carrier, cos $\omega_c t$. We effectively multiply (V_{DC} + $m(t)$) waveform by +1, -1, +1, -1, ...

The product gives the output signal $\boxed{v_s(t) = (V_{DC} + m(t))\cos(\omega_c t)}$

Double Sideband AM, DSBAM



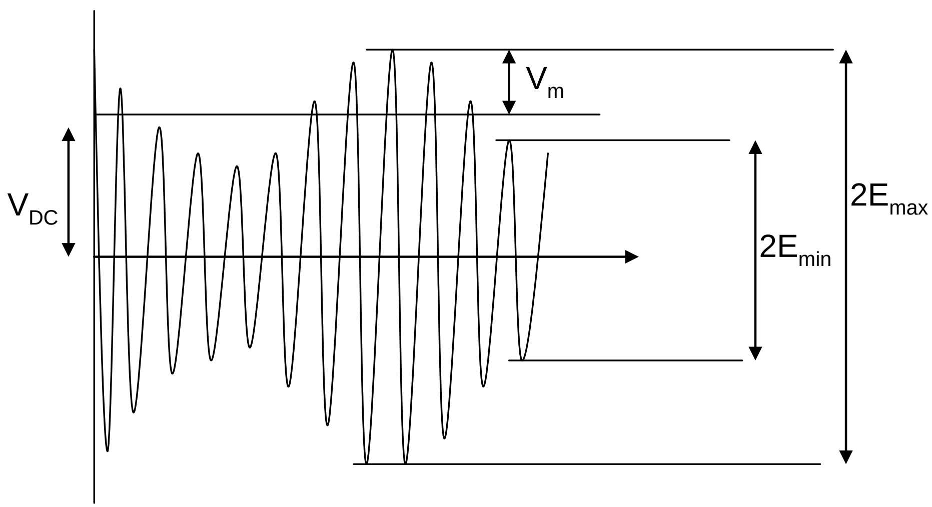
Modulation Depth

Consider again the equation $v_s(t) = (V_{DC} + V_m \cos(\omega_m t))\cos(\omega_c t)$, which may be written as

$$v_s(t) = V_{DC}\left(1 + \frac{V_m}{V_{DC}}\cos(\omega_m t)\right)\cos(\omega_c t)$$

The ratio is $\frac{V_m}{V_{DC}}$ defined as the **modulation depth**, **m**, *i.e.* Modulation Depth $m = \frac{V_m}{V_{DC}}$

From an oscilloscope display the modulation depth for Double Sideband AM may be determined as follows:



Modulation Depth 2

$2E_{max}$ = maximum peak-to-peak of waveform
$2E_{min}$ = minimum peak-to-peak of waveform

Modulation Depth $$m = \frac{2E_{max} - 2E_{min}}{2E_{max} + 2E_{min}}$$

This may be shown to equal $\frac{V_m}{V_{DC}}$ as follows:

$$2E_{max} = 2(V_{DC} + V_m) \qquad 2E_{min} = 2(V_{DC} - V_m)$$

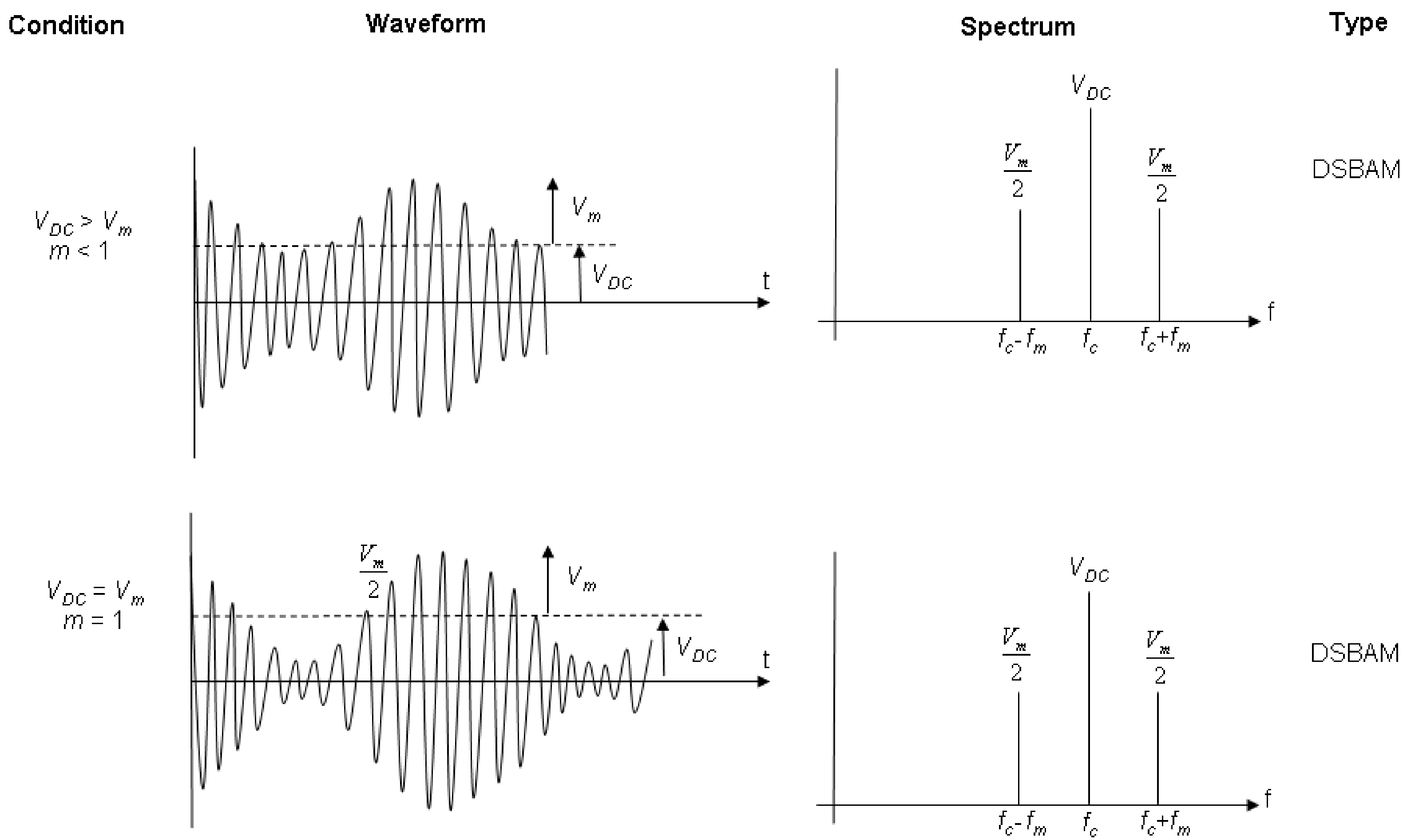
$$m = \frac{2V_{DC} + 2V_m - 2V_{DC} + 2V_m}{2V_{DC} + 2V_m + 2V_{DC} - 2V_m} = \frac{4V_m}{4V_{DC}} = \frac{V_m}{V_{DC}}$$

Double Sideband Modulation 'Types'

There are 3 main types of DSB

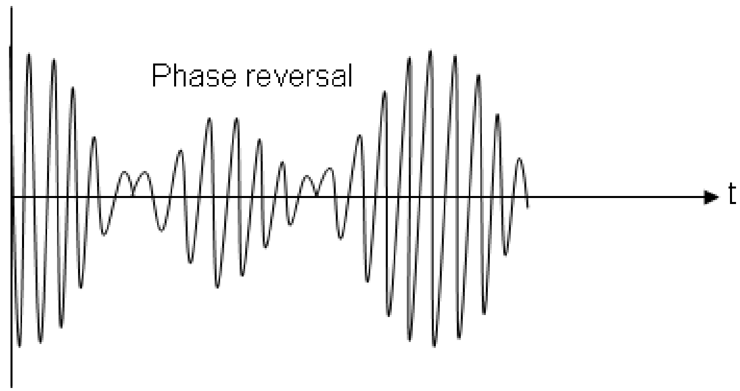
- Double Sideband Amplitude Modulation, DSBAM – with carrier
- Double Sideband Diminished (Pilot) Carrier, DSB Dim C
- Double Sideband Suppressed Carrier, DSBSC
- The type of modulation is determined by the modulation depth, which for a fixed $m(t)$ depends on the DC offset, V_{DC}. Note, when a modulator is set up, V_{DC} is fixed at a particular value. In the following illustrations we will have a fixed message, $V_m \cos \omega_m t$ and vary V_{DC} to obtain different types of Double Sideband modulation.

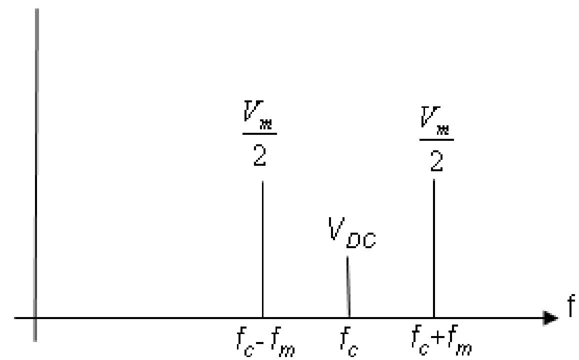
Graphical Representation of Modulation Depth and Modulation Types.



Graphical Representation of Modulation Depth and Modulation Types 2.

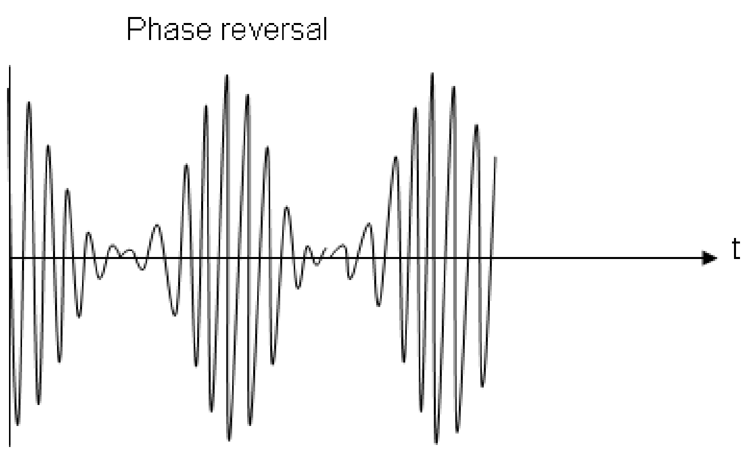
$V_{DC} < V_m$
$m > 1$

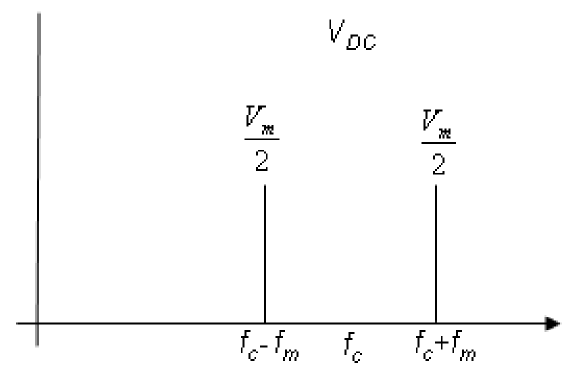




DSBDimC

$V_{DC} = 0$
$m = \infty$





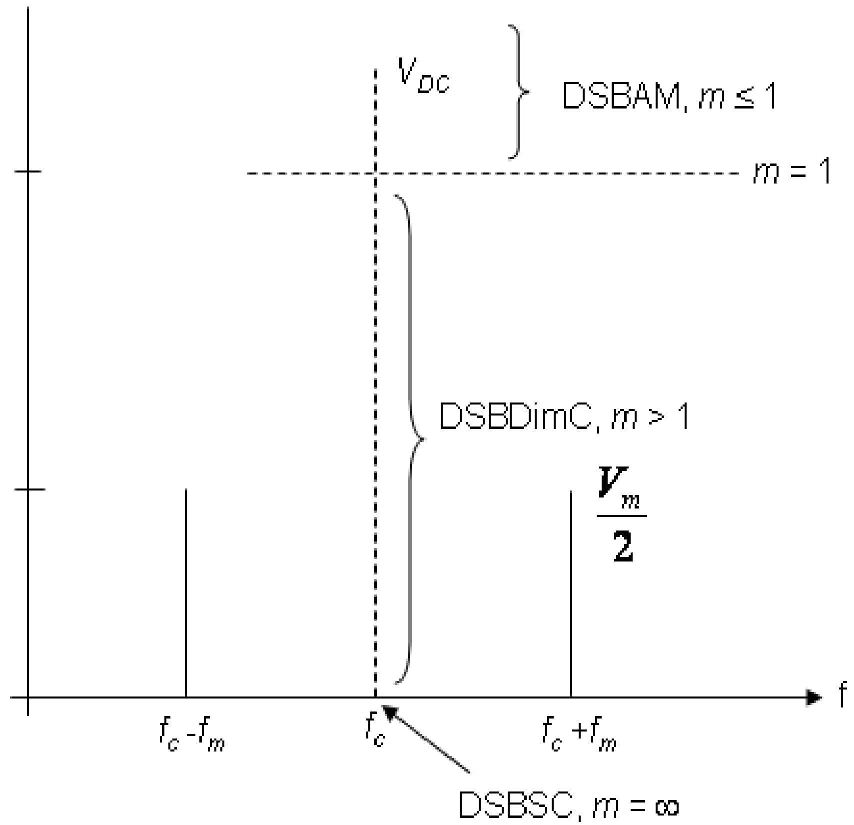
DSBSC

Graphical Representation of Modulation Depth and Modulation Types 3

Note then that V_{DC} may be set to give the modulation depth and modulation type.

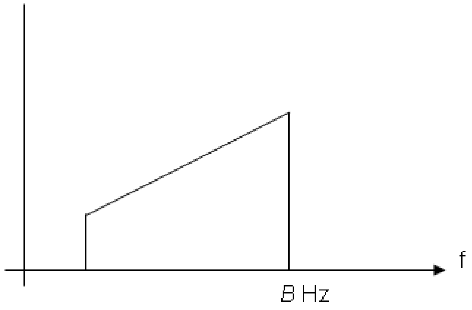
DSBAM $V_{DC} >> V_m$, $m \leq 1$
DSB Dim C $0 < V_{DC} < V_m$,
$m > 1$ $(1 < m < \infty)$
DSBSC $V_{DC} = 0$, $m = \infty$

The spectrum for the 3 main types of amplitude modulation are summarised

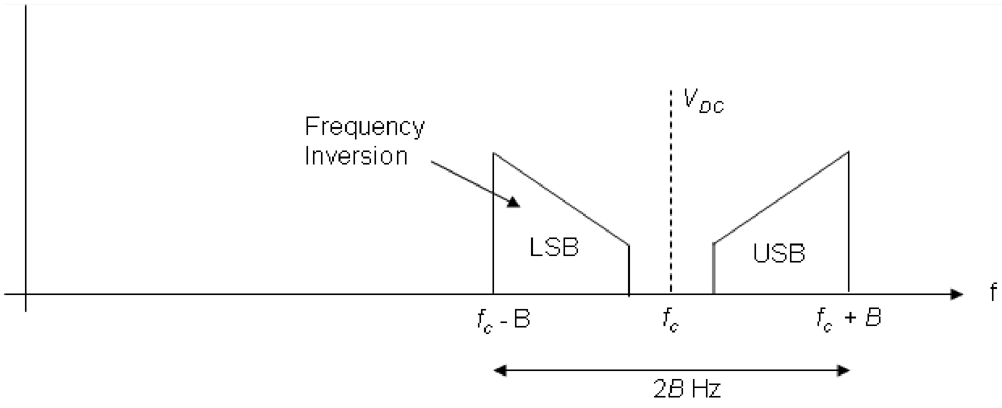


Bandwidth Requirement for DSBAM

In general, the message signal *m*(*t*) will not be a single 'sine' wave, but a band of frequencies extending up to *B* Hz as shown



Remember – the 'shape' is used for convenience to distinguish low frequencies from high frequencies in the baseband signal.



Bandwidth Requirement for DSBAM

Amplitude Modulation is a linear process, hence the principle of superposition applies. The output spectrum may be found by considering each component cosine wave in $m(t)$ separately and summing at the output.

Note:

- Frequency inversion of the LSB
- the modulation process has effectively shifted or frequency translated the baseband $m(t)$ message signal to USB and LSB signals centred on the carrier frequency f_c
- the USB is a frequency shifted replica of $m(t)$
- the LSB is a frequency inverted/shifted replica of $m(t)$
- both sidebands each contain the same message information, hence either the LSB or USB could be removed (because they both contain the same information)
- the bandwidth of the DSB signal is $2B$ Hz, *i.e.* twice the highest frequency in the baseband signal, $m(t)$
- The process of multiplying (or mixing) to give frequency translation (or up-conversion) forms the basis of radio transmitters and frequency division multiplexing which will be discussed later.

Power Considerations in DSBAM

Remembering that Normalised Average Power = $(V_{RMS})^2 = \left(\frac{V_{pk}}{\sqrt{2}}\right)^2$

we may tabulate for AM components as follows:

$$v_s(t) = V_{DC}\cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t) + \frac{V_m}{2}\cos((\omega_c - \omega_m)t)$$

Component	***Carrier***	***USB***	***LSB***
Amplitude pk	V_{DC}	$\frac{V_m}{2}$	$\frac{V_m}{2}$
Power	$\frac{V_{DC}^2}{2}$	$\left(\frac{V_m}{2\sqrt{2}}\right)^2 = \frac{V_m^2}{8}$	$\left(\frac{V_m}{2\sqrt{2}}\right)^2 = \frac{V_m^2}{8}$
Power	$\frac{V_{DC}^2}{2}$	$\frac{m^2 V_{DC}^2}{8}$	$\frac{m^2 V_{DC}^2}{8}$

Total Power P_T = Carrier Power P_c + P_{USB} + P_{LSB}

Power Considerations in DSBAM

From this we may write two equivalent equations for the total power P_T, in a DSBAM signal

$$P_T = \frac{V_{DC}^{\ 2}}{2} + \frac{V_m^{\ 2}}{8} + \frac{V_m^{\ 2}}{8} = \frac{V_{DC}^{\ 2}}{2} + \frac{V_m^{\ 2}}{4}$$

and

$$P_T = \frac{V_{DC}^{\ 2}}{2} + \frac{m^2 V_{DC}^{\ 2}}{8} + \frac{m^2 V_{DC}^{\ 2}}{8}$$

The carrier power $P_c = \frac{V_{DC}^{\ 2}}{2}$ *i.e.* $P_T = P_c + P_c \frac{m^2}{4} + P_c \frac{m^2}{4}$ or $P_T = P_c \left(1 + \frac{m^2}{2} \right)$

Either of these forms may be useful. Since both USB and LSB contain the same information a useful ratio which shows the proportion of 'useful' power to total power is

$$\frac{P_{USB}}{P_T} = \frac{P_c \frac{m^2}{4}}{P_c \left(1 + \frac{m^2}{2} \right)} = \frac{m^2}{4 + 2m^2}$$

Power Considerations in DSBAM

For DSBAM ($m \leq 1$), allowing for $m(t)$ with a dynamic range, the average value of m may be assumed to be $m = 0.3$

Hence, $$\boxed{\frac{m^2}{4+2m^2} = \frac{(0.3)^2}{4+2(0.3)^2} = 0.0215}$$

Hence, on average only about 2.15% of the total power transmitted may be regarded as 'useful' power. ($\approx$ 95.7% of the total power is in the carrier!)

Even for a maximum modulation depth of $m = 1$ for DSBAM the ratio $\boxed{\dfrac{m^2}{4+2m^2} = \dfrac{1}{6}}$

i.e. only 1/6th of the total power is 'useful' power (with 2/3 of the total power in the carrier).

Example

Suppose you have a portable (for example you carry it in your ' back pack') DSBAM transmitter which needs to transmit an average power of 10 Watts in each sideband when modulation depth m = 0.3. Assume that the transmitter is powered by a 12 Volt battery. The total power will be

$$P_T = P_c + P_c \frac{m^2}{4} + P_c \frac{m^2}{4}$$

where $P_c \frac{m^2}{4}$ = 10 Watts, *i.e.* $P_c = \frac{4(10)}{m^2} = \frac{40}{(0.3)^2}$ = 444.44 Watts

Hence, total power P_T = 444.44 + 10 + 10 = 464.44 Watts.

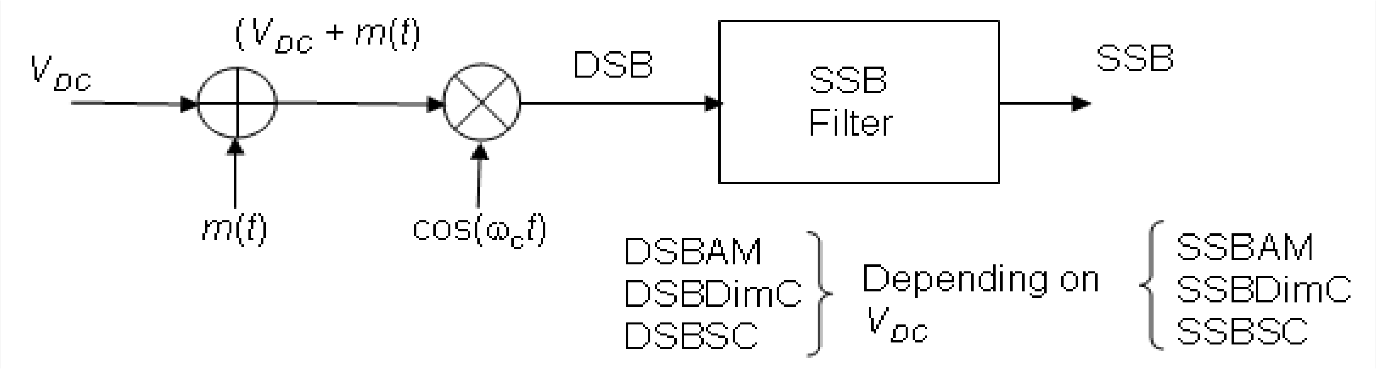
Hence, battery current (assuming ideal transmitter) = Power / Volts = $\frac{464.44}{12}$ amps!

i.e. a large and heavy 12 Volt battery.

Suppose we could remove one sideband and the carrier, power transmitted would be 10 Watts, *i.e.* 0.833 amps from a 12 Volt battery, which is more reasonable for a portable radio transmitter.

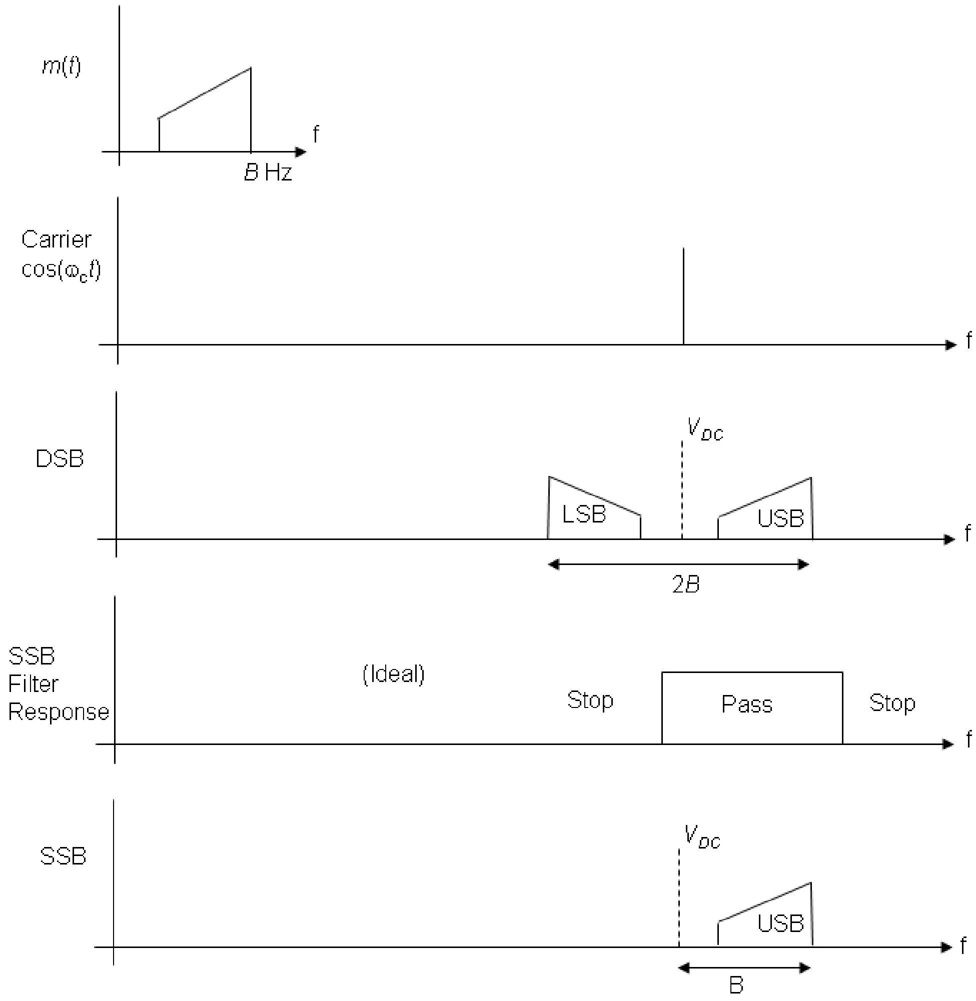
Single Sideband Amplitude Modulation

One method to produce signal sideband (SSB) amplitude modulation is to produce DSBAM, and pass the DSBAM signal through a band pass filter, usually called a single sideband filter, which passes one of the sidebands as illustrated in the diagram below.



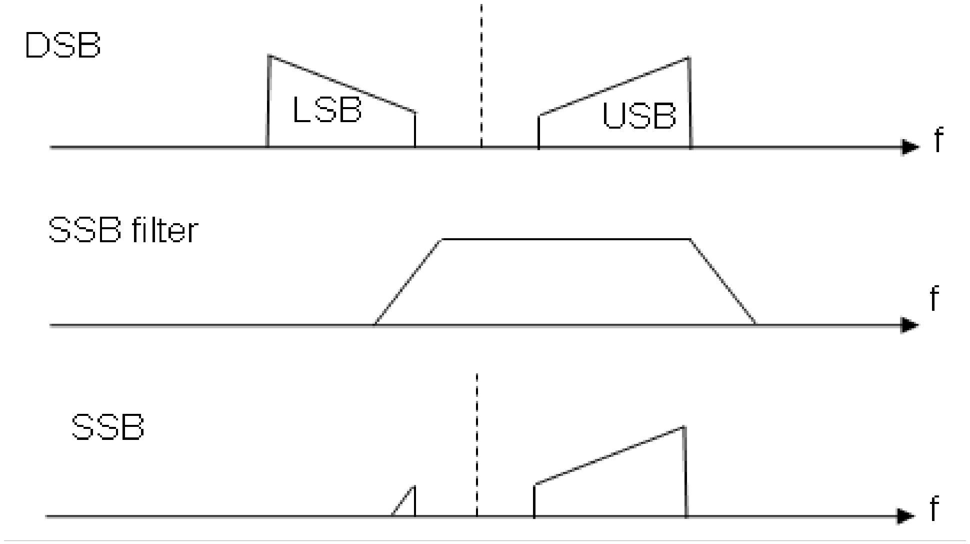
The type of SSB may be SSBAM (with a 'large' carrier component), SSBDimC or SSBSC depending on V_{DC} at the input. A sequence of spectral diagrams are shown on the next page.

Single Sideband Amplitude Modulation



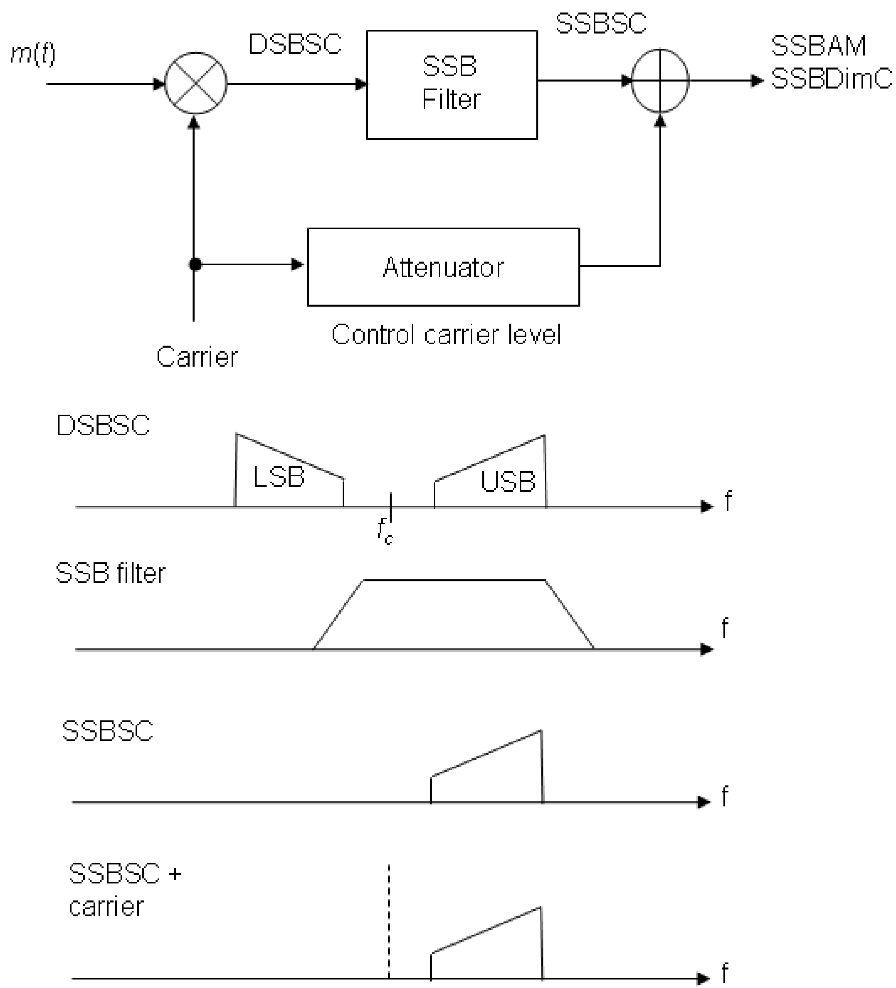
Single Sideband Amplitude Modulation

Note that the bandwidth of the SSB signal *B* Hz is half of the DSB signal bandwidth. Note also that an ideal SSB filter response is shown. In practice the filter will not be ideal as illustrated.

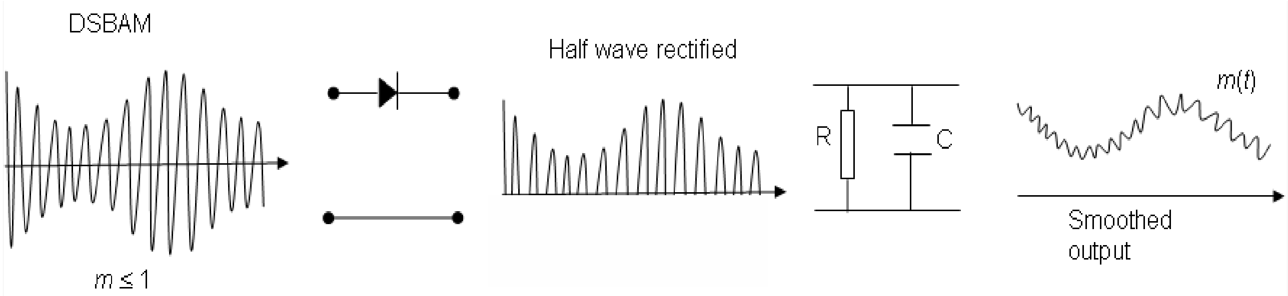


As shown, with practical filters some part of the rejected sideband (the LSB in this case) will be present in the SSB signal. A method which eases the problem is to produce SSBSC from DSBSC and then add the carrier to the SSB signal.

Single Sideband Amplitude Modulation



Single Sideband Amplitude Modulation



with $m(t) = V_m \cos \omega_m t$, we may write:

$$v_s(t) = V_{DC}\cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t) + \frac{V_m}{2}\cos((\omega_c - \omega_m)t)$$

The SSB filter removes the LSB (say) and the output is

$$v_s(t) = V_{DC}\cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t)$$

Again, note that the output may be
SSBAM, V_{DC} large
SSBDimC, V_{DC} small
SSBSC, $V_{DC} = 0$

For SSBSC, output signal =

$$v_s(t) = \frac{V_m}{2}\cos((\omega_c + \omega_m)t)$$

Power in SSB

From previous discussion, the total power in the DSB signal is $\boxed{P_T = P_c\left(1+\frac{m^2}{2}\right)}$

= $\boxed{P_T = P_c + P_c\frac{m^2}{4} + P_c\frac{m^2}{4}}$ for DSBAM.

Hence, if P_c and m are known, the carrier power and power in one sideband may be determined. Alternatively, since SSB signal =

$$\boxed{v_s(t) = V_{DC}\cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t)}$$

then the power in SSB signal (Normalised Average Power) is

$$\boxed{P_{SSB} = \frac{{V_{DC}}^2}{2} + \left(\frac{V_m}{2\sqrt{2}}\right)^2 = \frac{{V_{DC}}^2}{2} + \frac{{V_m}^2}{8}}$$

Power in SSB signal = $\boxed{\frac{{V_{DC}}^2}{2} + \frac{{V_m}^2}{8}}$

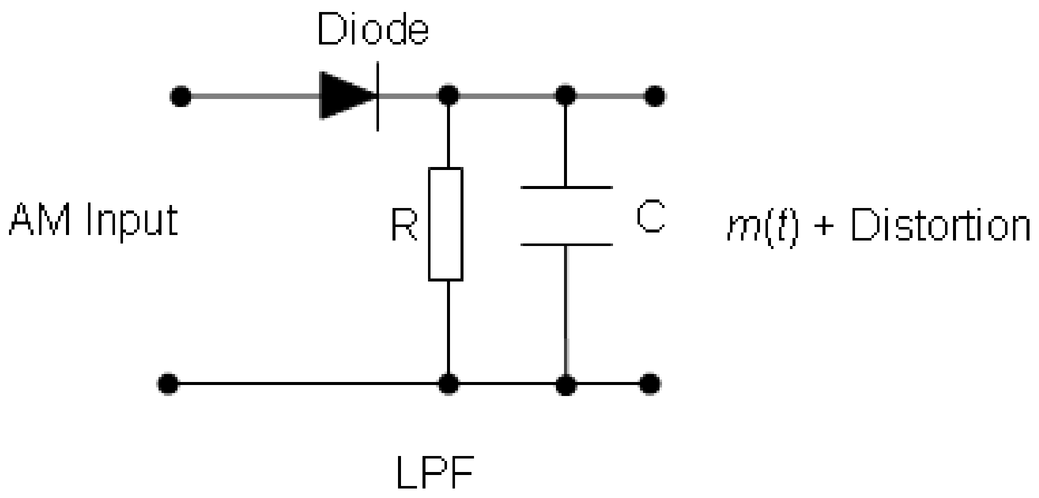
Demodulation of Amplitude Modulated Signals

There are 2 main methods of AM Demodulation:

- **Envelope or non-coherent Detection/Demodulation.**
- **Synchronised or coherent Demodulation.**

Envelope or Non-Coherent Detection

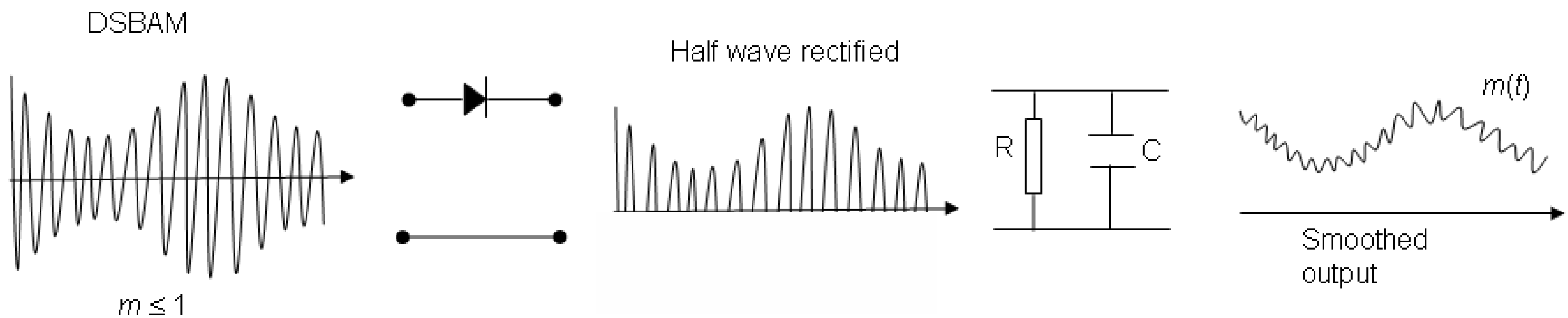
An envelope detector for AM is shown below:



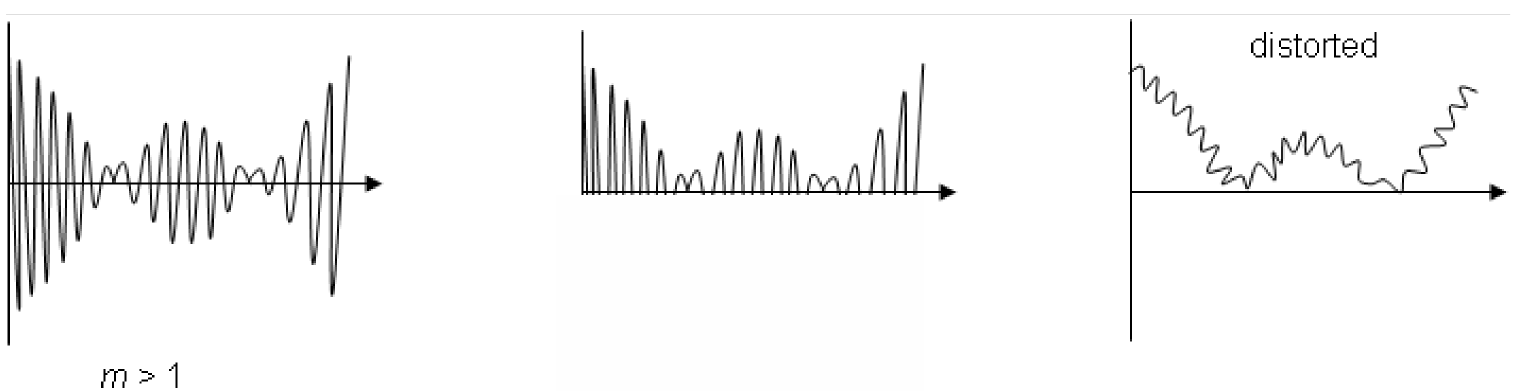
This is obviously simple, low cost. But the AM input <u>must be</u> DSBAM with *m* << 1, *i.e.* it does not demodulate DSBDimC, DSBSC or SSBxx.

Large Signal Operation

For large signal inputs, (≈ Volts) the diode is switched *i.e.* forward biased ≡ ON, reverse biased ≡ OFF, and acts as a half wave rectifier. The 'RC' combination acts as a 'smoothing circuit' and the output is *m*(*t*) plus 'distortion'.

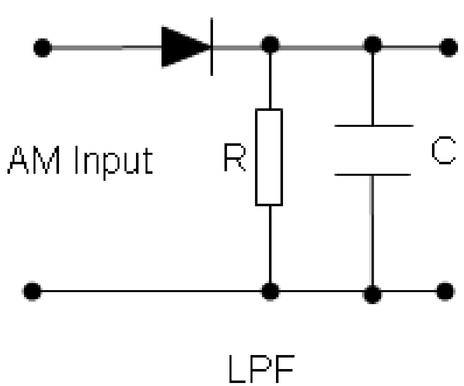


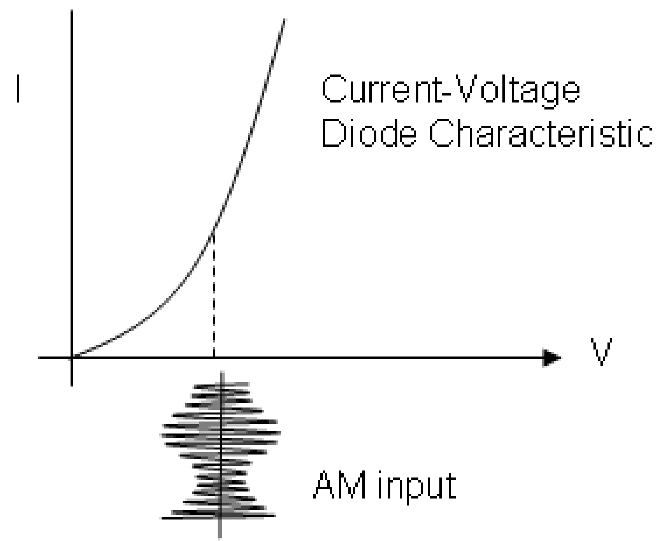
If the modulation depth is > 1, the distortion below occurs



Small Signal Operation – Square Law Detector

For small AM signals (~ millivolts) demodulation depends on the diode square law characteristic.





The diode characteristic is of the form $i(t) = av + bv2 + cv3 + \ldots$, where

$$\boxed{v = \left(V_{DC} + m(t)\right)\cos(\omega_c t)}$$ *i.e.* DSBAM signal.

Small Signal Operation – Square Law Detector

i.e. $$\boxed{a\left(V_{DC}+m(t)\right)\cos(\omega_c t)+b\left(\left(V_{DC}+m(t)\right)\cos(\omega_c t)\right)^2+\ldots}$$

$$=\boxed{aV_{DC}+am(t)\cos(\omega_c t)+b\left(V_{DC}{}^2+2V_{DC}m(t)+m(t)^2\right)\cos^2(\omega_c t)+\ldots}$$

$$=\boxed{aV_{DC}+am(t)\cos(\omega_c t)+\left(bV_{DC}{}^2+2bV_{DC}m(t)+bm(t)^2\right)\left(\frac{1}{2}+\frac{1}{2}\cos(2\omega_c t)\right)}$$

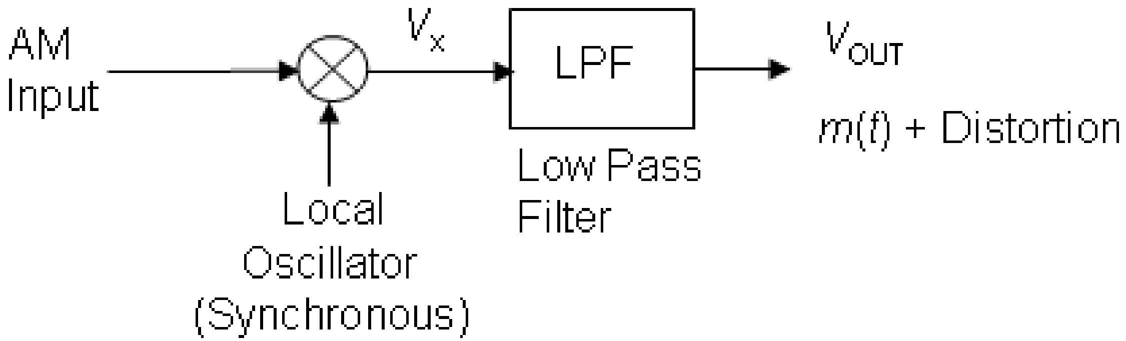
$$=\boxed{aV_{DC}+am(t)\cos(\omega_c t)+\frac{bV_{DC}{}^2}{2}+\frac{2bV_{DC}m(t)}{2}+\frac{bm(t)^2}{2}+b\frac{V_{DC}{}^2}{2}\cos(2\omega_c t)+\ldots}$$

'LPF' removes components.

Signal out = $\boxed{aV_{DC}+\frac{bV_{DC}{}^2}{2}+bV_{DC}m(t)}$ *i.e.* the output contains $m(t)$

Synchronous or Coherent Demodulation

A synchronous demodulator is shown below

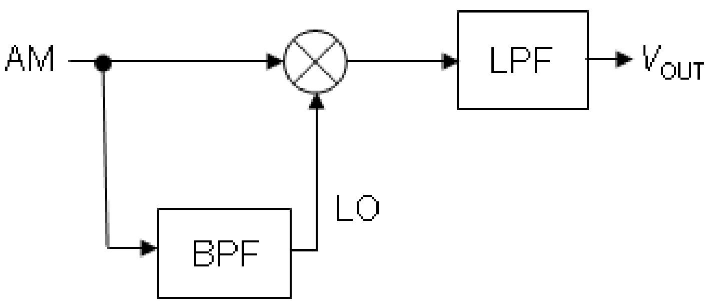


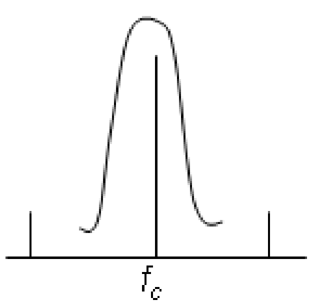
This is relatively more complex and more expensive. The Local Oscillator (LO) must be synchronised or coherent, *i.e.* at the same frequency and in phase with the carrier in the AM input signal. This additional requirement adds to the complexity and the cost.

However, the AM input may be <u>any</u> form of AM, *i.e.* DSBAM, DSBDimC, DSBSC or SSBAM, SSBDimC, SSBSC. (Note – this is a 'universal' AM demodulator and the process is similar to correlation – the LPF is similar to an integrator).

Synchronous or Coherent Demodulation

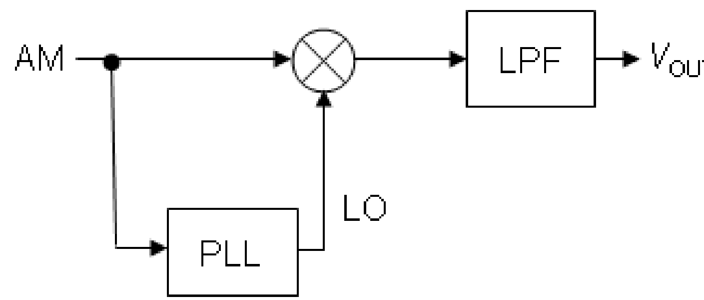
If the AM input contains a small or large component at the carrier frequency, the LO may be derived from the AM input as shown below.





BPF – tuned to f_c

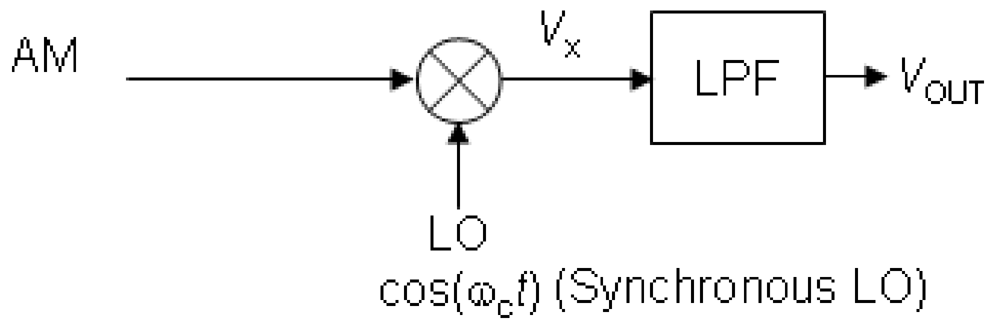
OR



Phase Locked Loop locked at f_c – regenerating a LO

Synchronous (Coherent) Local Oscillator

If we assume zero path delay between the modulator and demodulator, then the ideal LO signal is $\cos(\omega_c t)$. Note – in general the will be a path delay, say τ, and the LO would then be $\cos(\omega_c(t - \tau))$, *i.e.* the LO is synchronous with the carrier implicit in the received signal. Hence for an ideal system with zero path delay



Analysing this for a DSBAM input = $(V_{DC} + m(t))\cos(\omega_c t)$

Synchronous (Coherent) Local Oscillator

V_X = AM input x LO

$$= \boxed{\left(V_{DC} + m(t)\right)\cos^2(\omega_c t)}$$

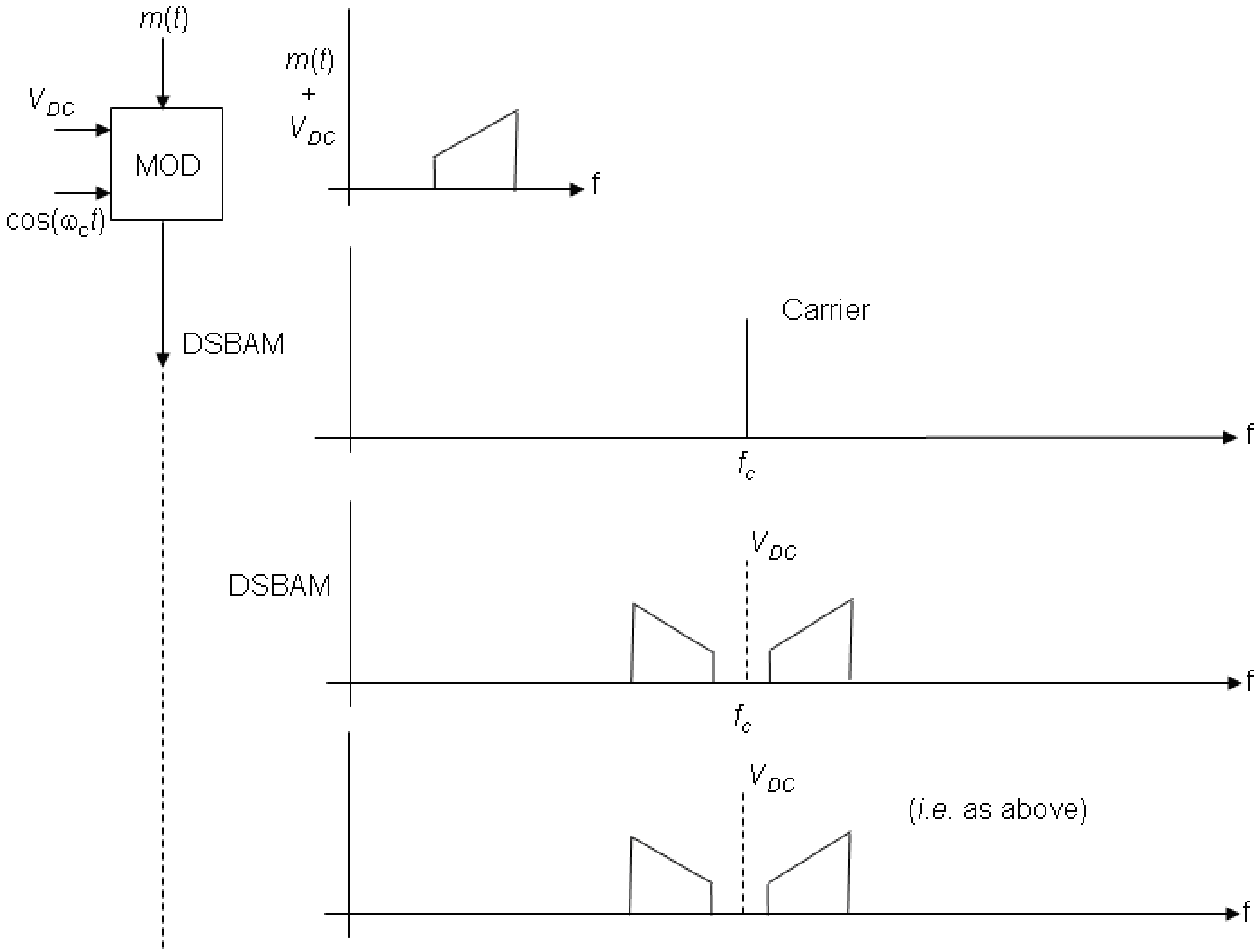
$$= \boxed{\left(V_{DC} + m(t)\right)\cos(\omega_c t) * \cos(\omega_c t)}$$

$$= \boxed{\left(V_{DC} + m(t)\right)\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)}$$

$$\boxed{V_x = \frac{V_{DC}}{2} + \frac{V_{DC}}{2}\cos(2\omega_c t) + \frac{m(t)}{2} + \frac{m(t)}{2}\cos(2\omega_c t)}$$

We will now examine the signal spectra from 'modulator to V_X'

Synchronous (Coherent) Local Oscillator



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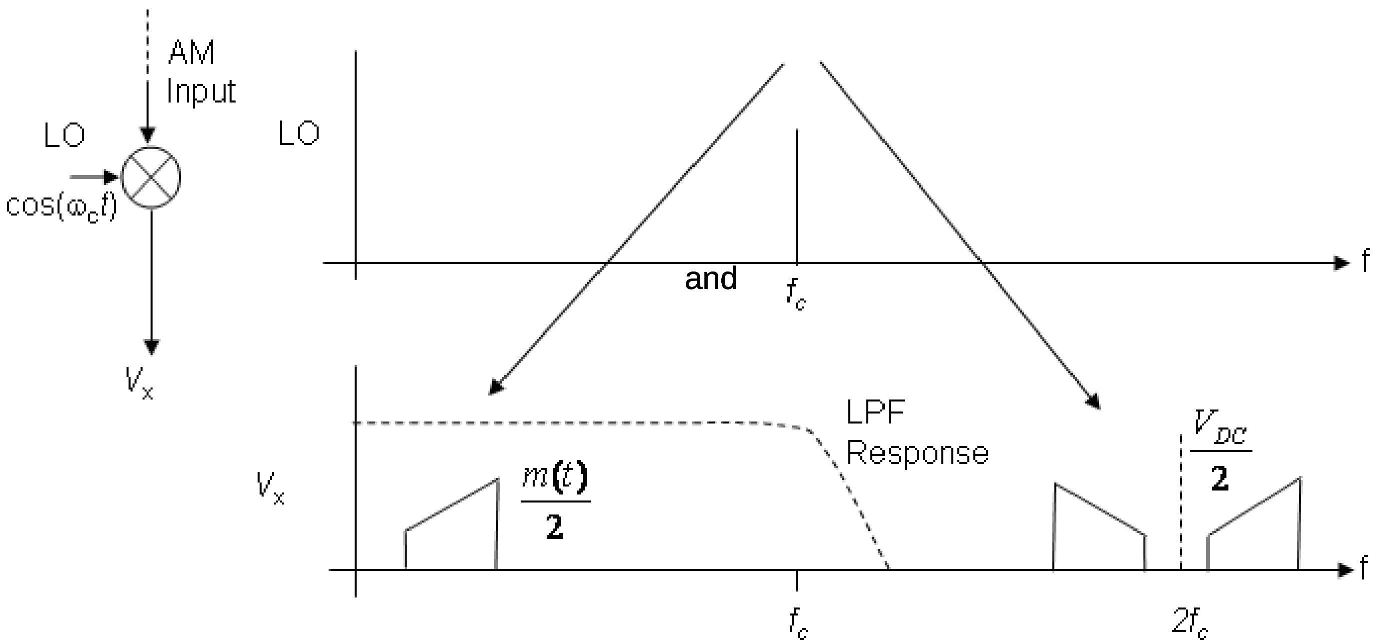
Synchronous (Coherent) Local Oscillator



Note – the AM input has been 'split into two' – 'half' has moved or shifted up to

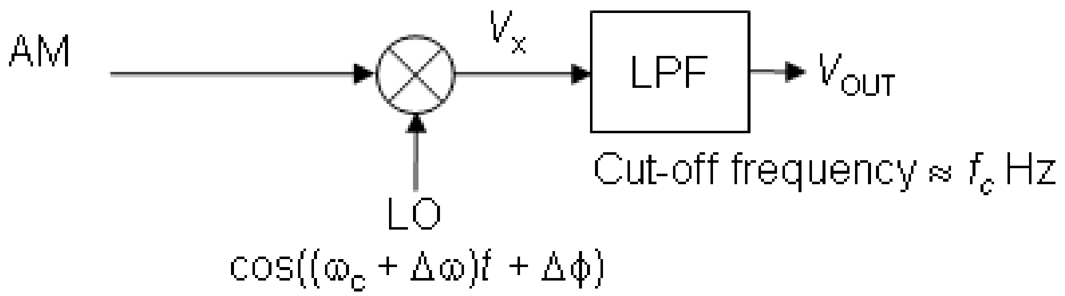
$\boxed{2f_c\left(\frac{m(t)}{2}\cos(2\omega_c t)+V_{DC}\cos(2\omega_c t)\right)}$ and half shifted down to baseband, $\boxed{\frac{V_{DC}}{2}}$ and $\boxed{\frac{m(t)}{2}}$

Synchronous (Coherent) Local Oscillator

The LPF with a cut-off frequency ≈ f_c will pass only the baseband signal i.e.

$$V_{out} = \frac{V_{DC}}{2} + \frac{m(t)}{2}$$

In general the LO may have a frequency offset, $\Delta\omega$, and/or a phase offset, $\Delta\phi$, i.e.



The AM input is essentially either:

- DSB (DSBAM, DSBDimC, DSBSC)
- SSB (SSBAM, SSBDimC, SSBSC)

1. Double Sideband (DSB) AM Inputs

The equation for DSB is $\left(V_{DC} + m(t)\right)\cos(\omega_c t)$ where VDC allows full carrier (DSBAM), diminished carrier or suppressed carrier to be set.

Hence, Vx = AM Input x LO $V_x = \left(V_{DC} + m(t)\right)\cos(\omega_c t).\cos\left((\omega_c + \Delta\omega)t + \Delta\varphi\right)$

Since $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$$V_x = \frac{\left(V_{DC} + m(t)\right)}{2}\left[\cos\left((\omega_c + \omega_c + \Delta\omega)t + \Delta\varphi\right) + \cos\left((\omega_c + \Delta\omega)t + \Delta\varphi - \omega_c t\right)\right]$$

$$V_x = \left(\frac{V_{DC}}{2} + \frac{m(t)}{2}\right)\left[\cos\left((2\omega_c + \Delta\omega)t + \Delta\varphi\right) + \cos(\Delta\omega t + \Delta\varphi)\right]$$

$$V_x = \frac{V_{DC}}{2}\cos\left((2\omega_c + \Delta\omega)t + \Delta\varphi\right) + \frac{V_{DC}}{2}\cos(\Delta\omega t + \Delta\varphi)$$
$$+ \frac{m(t)}{2}\cos\left((2\omega_c + \Delta\omega)t + \Delta\varphi\right) + \frac{m(t)}{2}\cos(\Delta\omega t + \Delta\varphi)$$

1. Double Sideband (DSB) AM Inputs

The LPF with a cut-off frequency ≈ f_c Hz will remove the components at $2\omega_c$ (i.e. components above ω_c) and hence

$$V_{out} = \frac{V_{DC}}{2}\cos(\Delta\omega t + \Delta\varphi) + \frac{m(t)}{2}\cos(\Delta\omega t + \Delta\varphi)$$

Obviously, if $\Delta\omega = 0$ and $\Delta\varphi = 0$ we have, as previously $V_{out} = \frac{V_{DC}}{2} + \frac{m(t)}{2}$

Consider now if $\Delta\omega$ is equivalent to a few Hz offset from the ideal LO. We may then say

$$V_{out} = \frac{V_{DC}}{2}\cos(\Delta\omega t) + \frac{m(t)}{2}\cos(\Delta\omega t)$$

The output, if speech and processed by the human brain may be intelligible, but would include a low frequency 'buzz' at $\Delta\omega$, and the message amplitude would fluctuate. The requirement $\Delta\omega = 0$ is necessary for DSBAM.

1. Double Sideband (DSB) AM Inputs

Consider now if $\Delta\omega$ is equivalent to a few Hz offset from the ideal LO. We may then say

$$V_{out} = \frac{V_{DC}}{2}\cos(\Delta\omega t) + \frac{m(t)}{2}\cos(\Delta\omega t)$$

The output, if speech and processed by the human brain may be intelligible, but would include a low frequency 'buzz' at $\Delta\omega$, and the message amplitude would fluctuate. The requirement $\Delta\omega = 0$ is necessary for DSBAM.

Consider now that $\Delta\omega = 0$ but $\Delta\phi \neq 0$, i.e. the frequency is correct at ω_c but there is a phase offset. Now we have

$$V_{out} = \frac{V_{DC}}{2}\cos(\Delta\varphi) + \frac{m(t)}{2}\cos(\Delta\varphi)$$

'cos($\Delta\phi$)' causes fading (i.e. amplitude reduction) of the output.

1. Double Sideband (DSB) AM Inputs

The 'V_{DC}' component is not important, but consider for m(t),

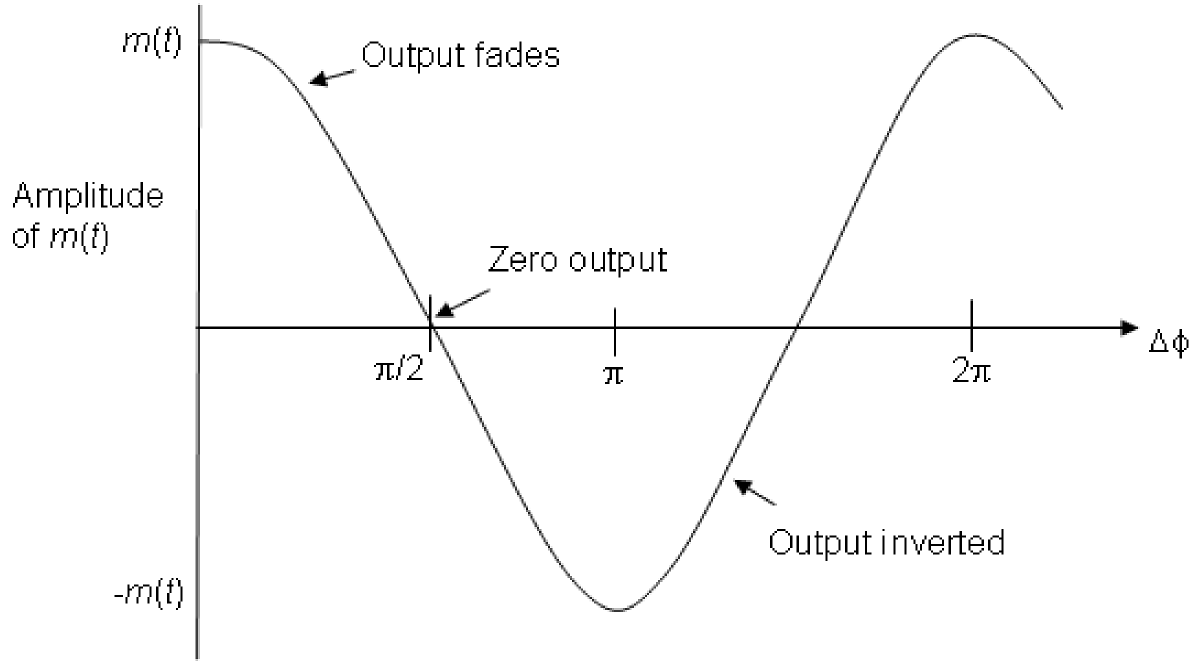
- if $\Delta\varphi = \frac{\pi}{2}$ (90^0), $\cos\left(\frac{\pi}{2}\right) = 0$ i.e. $V_{out} = \frac{m(t)}{2}\cos\left(\frac{\pi}{2}\right) = 0$
- if $\Delta\varphi = \frac{\pi}{2}$ (180^0), $\cos(\pi) = -1$ i.e. $V_{out} = \frac{m(t)}{2}\cos(\pi) = -m(t)$

The phase inversion if $\Delta\phi = \pi$ may not be a problem for speech or music, but it may be a problem if this type of modulator is used to demodulate PRK

However, the major problem is that as $\Delta\phi$ increases towards $\frac{\pi}{2}$ the signal strength output gets weaker (fades) and at $\frac{\pi}{2}$ the output is zero

1. Double Sideband (DSB) AM Inputs

If the phase offset varies with time, then the signal fades in and out. The variation of amplitude of the output, with phase offset $\Delta\phi$ is illustrated below



Thus the requirement for $\Delta\omega = 0$ and $\Delta\phi = 0$ is a 'strong' requirement for DSB amplitude modulation.

2. Single Sideband (SSB) AM Input

The equation for SSB with a carrier depending on V_{DC} is

$$V_{DC}\cos(\omega_c t)+\frac{V_m}{2}\cos(\omega_c+\omega_m t)$$

i.e. assuming $m(t)=V_m\cos(\omega_m t)$

Hence $$V_x=\left(V_{DC}\cos(\omega_c t)+\frac{V_m}{2}\cos(\omega_c+\omega_m)t\right)\cos((\omega_c+\Delta\omega)t+\Delta\varphi)$$

$$=\frac{V_{DC}}{2}\cos((2\omega_c+\Delta\omega)t+\Delta\varphi)+\frac{V_{DC}}{2}\cos(\Delta\omega t+\Delta\varphi)$$
$$+\frac{V_m}{4}\cos((2\omega_c+\omega_m+\Delta\omega)t+\Delta\varphi)+\frac{V_m}{4}\cos((\omega_m-\Delta\omega)t-\Delta\varphi)$$

2. Single Sideband (SSB) AM Input

The LPF removes the $2\omega_c$ components and hence

$$\boxed{\frac{V_{DC}}{2}\cos(\Delta\omega t+\Delta\varphi)+\frac{V_m}{4}\cos((\omega_m-\Delta\omega)t-\Delta\varphi)}$$

Note, if $\Delta\omega = 0$ and $\Delta\phi = 0$, $\boxed{\frac{V_{DC}}{2}+\frac{V_m}{4}\cos(\omega_m t)}$,i.e. $\boxed{m(t)=V_m\cos(\omega_m t)}$ has been recovered.

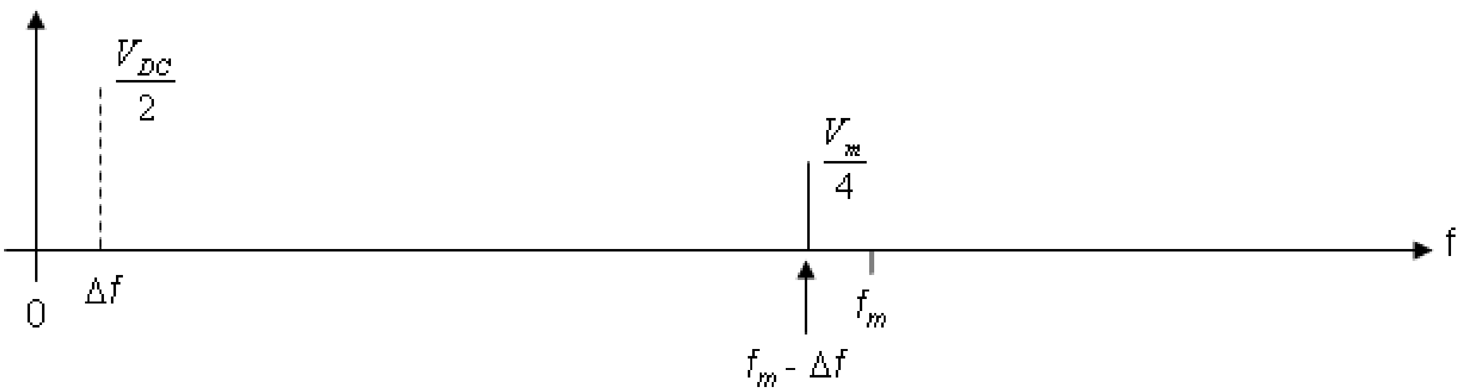
Consider first that $\Delta\omega \neq 0$, e.g. an offset of say 50Hz. Then

$$\boxed{V_{out}=\frac{V_{DC}}{2}\cos(\Delta\omega t)+\frac{V_m}{4}\cos((\omega_m-\Delta\omega)t)}$$

If m(t) is a signal at say 1kHz, the output contains a signal a 50Hz, depending on V_{DC} and the 1kHz signal is shifted to 1000Hz - 50Hz = 950Hz.

2. Single Sideband (SSB) AM Input

The spectrum for V_{out} with $\Delta\omega$ offset is shown



Hence, the effect of the offset $\Delta\omega$ is to shift the baseband output, up or down, by $\Delta\phi$. For speech, this shift is not serious (for example if we receive a 'whistle' at 1kHz and the offset is 50Hz, you hear the whistle at 950Hz ($\Delta\omega$ = +ve) which is not very noticeable. Hence, small frequency offsets in SSB for speech may be tolerated. Consider now that $\Delta\omega = 0$, $\Delta\phi = 0$, then

$$\boxed{V_{out} = \frac{V_{DC}}{2}\cos(\Delta\varphi) + \frac{V_m}{4}\cos(\omega_m t - \Delta\varphi)}$$

2. Single Sideband (SSB) AM Input

- This indicates a fading V_{DC} and a phase shift in the output. If the variation in $\Delta\phi$ with time is relatively slow, thus phase shift variation of the output is not serious for speech.

- Hence, for SSB small frequency and phase variations in the LO are tolerable. The requirement for a coherent LO is not as a stringent as for DSB. For this reason, SSBSC (suppressed carrier) is widely used since the receiver is relatively more simple than for DSB and power and bandwidth requirements are reduced.

Comments

- In terms of 'evolution', early radio schemes and radio on long wave (LW) and medium wave (MW) to this day use DSBAM with $m < 1$. The reason for this was the reduced complexity and cost of 'millions' of receivers compared to the extra cost and power requirements of a few large LW/MW transmitters for broadcast radio, *i.e.* simple envelope detectors only are required.

- Nowadays, with modern integrated circuits, the cost and complexity of synchronous demodulators is much reduced especially compared to the additional features such as synthesised LO, display, FM *etc.* available in modern receivers.

Amplitude Modulation forms the basis for:

- Digital Modulation – Amplitude Shift Keying ASK
- Digital Modulation – Phase Reversal Keying PRK
- Multiplexing – Frequency Division Multiplexing FDM
- Up conversion – Radio transmitters
- Down conversion – Radio receivers