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Energy-Efficient RSSI-based Localization for Wireless Sensor Networks

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***Abstract*—Sensor positioning is a fundamental block in many location-dependent applications of wireless sensor networks. Although the main objective in localization is primarily enhancing the positioning accuracy, the importance of the energy consumption and localization accuracy pose new challenges. The localization is usually assisted with some self-known position sensors called anchor nodes. In this letter, optimal power allocation for the anchor nodes in a sense of minimizing the energy consumption considering estimation errors is investigated. To have a better estimation of the relative distance between the anchor and unknown nodes using received signal strength indicator (RSSI), average energy of the received beacon is introduced as a new decision metric. Based on this, squared position error bound as an accuracy parameter is derived, and then an optimization problem is proposed to maximize the localization performance. More specifically, the optimal power allocation policy is first derived for the case that the anchor nodes estimate their own locations with no error. Since there are unavoidable errors in anchor nodes positions, the optimization problem is then modified by including uncertainty in the anchor nodes positions. The results show that a substantial reduction in power consumption can be achieved by optimally allocation of the transmission power.**

***Index Terms*—Localization, energy efficiency, squared position error bound, wireless sensor networks.**

I. Introduction

Localization techniques play a critical role in most of wireless sensor network (WSN) applications such as coverage calculation, event detection, object tracking, and location aware routing [1]. In such applications, sensor nodes are categorized into anchor nodes (ANs) and unknown nodes (UNs). The main difference between them is that the ANs know their locations, for instance with the help of GPS, whereas they are unknown for the UNs [1]. A localization scheme tries to localize the UNs using the information extracted from the signaling between the ANs and UNs. The information can manifest itself in the form of received signal strength indicator (RSSI), time of arrival, angel of arrival, and time differential of arrival [1]. Among them, RSSI-based localization schemes are the most prevalent one due to easier implementation and less complexity [1]. In this method, the distance between the ANs and UNs is estimated using a signal propagation model.

Besides localization accuracy, the energy consumed in localization procedure is another remarkable performance metric, especially for energy-constrained WSNs. In general, transmission powers of the ANs play an important role in the network localization, they directly affect the localization accuracy as well as network lifetime [2]–[5].

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Recently, the problem of designing energy efficient localization frameworks has gained lots of interest. In [2], optimal power allocation policy with aim at minimizing squared position error bound (SPEB) for a given power budget is studied, and a distributed algorithm is proposed in [3] to achieve optimal set of powers for the TOA-based localization. The TOA-based localization problem is also modeled as a noncooperative game in [4], and the authors investigate minimization of positioning energy cost. Similar problem is studied in [5], wherein a coalition formation game is developed to optimize the energy consumption by maximizing sleep times of a cluster of ANs, localizing an UN with a certain accuracy.

In contrast to the aforementioned studies, in this letter, we investigate the optimal power allocation problem for RSSI-based localization. Due to the dynamic nature of wireless environments, estimating the relative distance by comparing the transmitted and received power of one sample is not be reliable. In order to have a more accurate estimation, the average received energy of $2m$ consecutive samples is considered as a new decision metric. We derive SPEB, as a measure of localization accuracy [2], [3], [6], for this novel test statistic and show that SPEB depends on ANs transmission powers. Then, an optimization problem is formulated to minimize the average energy consumption while keeping SPEB bounded. We observe that by knowing a priori density function of UN position, it is not necessary for all ANs to participate in the localization procedure. The achievements, however, is not reliable in practical scenarios, since it is assumed that an AN does not have any error in its location estimation [2]–[5]. In order to improve the model, we include error in each AN position and then formulate a robust optimal power allocation problem. Although the problem of optimal power allocation with imperfect localization parameters such as uncertainties in distance, angular, and channel parameters is studied in [2], [3], to the best of our knowledge, it is the first time that the error in the ANs position is considered in the optimal power allocation problem. Finally, we provide guidlines for the extension of the studies to a network with multiple UNs.

The rest of the paper is organized as follows. Section II discusses the system model considered. Optimization problem is formulated in Section III. Finally, efficiencies of the proposed scheme is evaluated by numerical results in Section IV, followed by concluding remarks provided in Section V.

II. System Model

The scenario of interest consists of a WSN deployed in a certain geographical area, with N ANs and one UN. We assume that $\theta_j^a = \left[x_j^a, y_j^a\right], j = 1, 2, \ldots, N$, and $\theta = [x, y]$ are coordinates of the ANs and UN, respectively. The UN communicates with the ANs to localize itself. Let $s_j\left(k\right)$

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denote the k-th sample of a beacon transmitted by j-th AN. Moreover, let $v(k)$ represent the k-th sample of a zero mean complex-valued Gaussian noise, which is independent and identically distributed. If only j-th AN transmits its beacon, the signal received by UN is

$$y_j(k) = h_j s_j(k) + v(k) \,, \tag{1}$$

where $y_j(k)$ is the k-th sample of received signal, and h_j is the gain of the channel between the AN j and the UN. In addition, we consider block fading channels with Rayleigh distribution. Hence, the channel gain does not substantially change during the beacon interval. In order to mitigate the uncertainty problem associated with the RSSI, the received energy, averaged over $2m$ consecutive samples, is used for the localization. Thus, we have

$$P_{Rj} = \frac{1}{2m} \sum_{k=0}^{2m-1} |y_j(k)|^2 \,, \tag{2}$$

where P_{Rj} is the received power at the j-th AN and represents as a metric for the localization. Let σ_v^2 be the variance of the Gaussian noise in (1). We define $\eta_j = \mathbb{E}\{|h_j|^2\} p_j / \sigma_v^2$ as the received SNR due to the beacon of the j-th AN at the UN side, where $\mathbb{E}\{\cdot\}$ denotes the expectation operation. In addition, The path-loss can be expressed as $\mathbb{E}\{|h_j|^2\} = k d_j^{-\alpha}$, where d_j denotes the Euclidean distance between the UN and the AN j, α represents the path loss exponent, and k is depending on the operating frequency. It is shown that P_{R_j}/σ_v^2 follows a non-central chi-square distribution with $2m$ degrees of freedom, which can be well approximated by a Gaussian distribution for large m. That is, P_{Rj}/σ_v^2 can be described by a Gaussian distribution with mean $\overline{P}_{Rj} = \sigma_v^2(1+\eta_j)$ and variance $\sigma_j^2 = \frac{\sigma_v^4}{m}(1+2\eta_j)$ [7]. Hence, The probability density function (PDF) of the received power can be calculated as

$$f\left(P_{Rj}\,|\theta\right) \sim \mathcal{N}\left(\sigma_v^2(1+\eta_j), \frac{\sigma_v^4}{m}(1+2\eta_j)\right) . \tag{3}$$

III. Performance Evaluation

The Cramo-Rao bound (CRB) matrix provides a lower bound on the covariance of any unbiased location estimator and is equal to inverse of fisher information matrix (FIM) [1]. CRB asserts that $\mathrm{var}^2(\|\theta - \hat{\theta}\|^2) \geq \mathrm{tr}(\mathbf{I}^{-1}(\theta))$, where $\hat{\theta}$ is an estimation of the UN position θ, and $\mathbf{I}(\theta)$ is the FIM [8]. SPEB is defined as $\mathrm{tr}\left(\mathbf{I}^{-1}(\theta)\right)$ and is extensively used for evaluating the estimation accuracy [2], [3]. Similarly, in this letter, we focus on SPEB instead of CRB, since it is a tractable performance measure for designing optimal power allocation. With the analysis provided in [9], the following lemma has a straightforward proof that is omitted.

Lemma I: Let $\widehat{\mathbf{x}}$ be the estimation of the parameter vector $\mathbf{x}$ based on observation vector $\mathbf{r}$. We denote a priori density function of $\mathbf{x}$ by $f(\mathbf{x})$ and joint PDF of $\mathbf{r}$ and $\mathbf{x}$ by $f(\mathbf{r};\mathbf{x}) = f(\mathbf{r}|\mathbf{x})\, f(\mathbf{x})$. FIM is calculated as

$$\mathbf{I}(\mathbf{x}) = \mathbf{Q} + \mathbb{E}_{f(\mathbf{x})}\{\mathbf{H}\} \,, \tag{4}$$

where

$$[\mathbf{Q}]_{k,l} = -\mathbb{E}_{f(\mathbf{x})}\left[\frac{\partial^2 \ln f(\mathbf{x})}{\partial x_k \partial x_l}\right] , \quad k,l = 1,2,\ldots,|\mathbf{x}| \,, \tag{5}$$

$$[\mathbf{H}]_{k,l} = -\mathbb{E}_{f(\mathbf{r}|\mathbf{x})}\left[\frac{\partial^2 \ln f(\mathbf{r}|\mathbf{x})}{\partial x_k \partial x_l}\right] , k,l = 1,2,\ldots,|\mathbf{x}| \,, \tag{6}$$

where $|\mathbf{x}|$ is the cardinality of the vector $\mathbf{x}$, and x_k is k-th element of the vector $\mathbf{x}$. $\mathbf{Q}$ reflects a priori information of $\mathbf{x}$, and $\mathbf{H}$ represents the information of the observation vector $\mathbf{r}$. As $\mathbb{E}_{f(\mathbf{x})}\{\mathbf{H}\}$ cannot be calculated analytically, it is practical to approximate it with [5]

$$\mathbb{E}_{f(\mathbf{x})}\{\mathbf{H}\} \approx \mathbf{H}\{\mathbb{E}\{\mathbf{x}\}\} \,. \tag{7}$$

This approximation gets tight when the variance of a priori position of the unknown node is relatively smaller than distance between unknown and anchor nodes [8].

A. No error in anchor nodes locations

First, it is assumed that the ANs know their exact locations. Similar to [10], a priori density function of UN location is supposed to be Gaussian. The location of the UN can be defined as $x = \bar{x} + \varepsilon_x$ and $y = \bar{y} + \varepsilon_y$, where $\bar{x}$ and $\bar{y}$ are the mean of the UN location variables and $\varepsilon = [\varepsilon_x, \varepsilon_y]^T$ are independent zero-mean Gaussian processes with covariance matrix

$$\mathbf{C} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix} . \tag{8}$$

In order to find $\mathbf{I}(\theta)$, we first note that $f(P_R;\theta) = f(P_R|\theta)\, f(\theta)$, and from *Lemma I*, FIM can be divided into two part, i.e., $\mathbf{I} = \mathbf{I_D} + \mathbf{I_P}$, where $\mathbf{I_D}$ indicates the information of the received energy, and $\mathbf{I_P}$ indicates the information about the previous location of the UN. Assume that the information about the received power from each AN is independent of other ANs [10]. Hence,

$$f(P_R|\theta) = \prod_{j=1}^{N} f(P_{Rj}|\theta) \,. \tag{9}$$

Now, by substituting (3) into (9) and the results into (6)–(7), and after some algebraic manipulations, $\mathbf{I}_D$ is derived as

$$\mathbf{I_D} = \sum_{j=1}^{N} \xi_j \mathbf{J}(\varphi_\mathbf{j}) \,, \tag{10}$$

where φ_j is angle between the UN and AN j, $\xi_j = \frac{m\alpha^2\sigma_v^2\eta_j^2}{(1+2\eta_j)d_j^2}$, and

$$\mathbf{J}(\varphi_\mathbf{j}) = \begin{bmatrix} \cos^2\varphi_\mathbf{j} & -\cos\varphi_\mathbf{j}\sin\varphi_\mathbf{j} \\ -\cos\varphi_\mathbf{j}\sin\varphi_\mathbf{j} & \sin^2\varphi_\mathbf{j} \end{bmatrix} . \tag{11}$$

Also, it is proved in [5] that $\mathbf{I_P} = \mathbf{C}^{-1}$. Then, the FIM can be easily computed. The expression in (10) shows that the SPEB of the proposed decision metric depends on the ANs transmission powers. In fact, the localization accuracy can be controlled by different power allocations. As a result, an optimization problem can be formulated in order to minimize the power consumption assuring a certain level of localization accuracy. To this end, the following optimization problem is

$$P_\theta \geq \left[\left(\mathbf{J}_{\theta\theta} + \frac{1}{(1-\rho^2)\sigma_x^2} \right) - \left(\mathbf{J}_{\theta\theta^a} - \frac{\rho}{(1-\rho^2)\sigma_x\sigma_y} \right) \left(\mathbf{J}_{\theta^a\theta^a} + \gamma + \frac{1}{(1-\rho^2)\sigma_y^2} \right)^{-1} \left(\mathbf{J}_{\theta^a\theta} - \frac{\rho}{(1-\rho^2)\sigma_x\sigma_y} \right) \right]^{-1} \tag{20}$$

formulated as

$$\arg\min_{p_j} \quad \sum_{j=1}^{N} p_j, \tag{12}$$

$$\text{s.t.} \quad \text{SPEB} \leq T, \tag{C1}$$

$$\sum_{j=1}^{N} p_j \leq P_{\max}\,, \tag{C2}$$

where T is maximum tolerable error in the localization, and $P_{\max}$ is the total power available for whole networks to localize UN. Although (12) gives us the optimal power distribution, the results might be useless in realistic situations, where each AN knows its location with some level of error. To deal with this problem, we extend the above formulations by involving errors in the location of the ANs.

B. Error in anchor nodes locations

Since ANs usually obtain their locations by GPS, it is reasonable to account the error in their locations. Thus, in the rest of the paper, the errors exist in the location of the ANs. Let $\theta^a = [x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N]^T$ be the set of the ANs locations. The errors in the location of ANs are assumed to be additive white Gaussian noise with zero mean and the covariance matrix of [10]

$$\mathbf{C}^a = \text{diag}_{2N\times 2N}\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2, \sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2\right)\,. \tag{13}$$

For simplicity, it is assumed that the error in the location of each AN is independent of the others. Furthermore, the variance of the error in x axis is the same as y axis. Even though we consider the same error variance, the formulations presented in the paper can be easily extended to the general case which is left for future studies. In this case, we have

$$f\left(P_R; \theta, \theta^a\right) = f\left(P_R|\theta, \theta^a\right) f\left(\theta^a\right) f\left(\theta\right)\,, \tag{14}$$

where $f(\theta^a)$ is the PDF of the ANs locations and $f(\theta)$ shows a priori density function of the UN. Thanks to *Lemma I*, the FIM associated with $f(P_R; \theta, \theta^a)$ consists of three distinct elements: the information of received signal $\mathbf{I_D}$, a priori information of the UN location $\mathbf{I_P}$, and a priori information of the ANs' locations $\mathbf{I_A}$. Then, $\mathbf{I} = \mathbf{I_D} + \mathbf{I_P} + \mathbf{I_A}$. Each element can be calculated by substituting its PDF into (4)–(6). Hence, $\mathbf{I_P}$ and $\mathbf{I_A}$ are easily calculated as

$$\mathbf{I_P} = \begin{bmatrix} 0_{2N\times 2N} & 0_{2N\times 2} \\ 0_{2\times 2N} & \lambda_{2\times 2} \end{bmatrix}, \quad \lambda = \mathbf{C}^{-1}\,, \tag{15}$$

$$\mathbf{I_A} = \begin{bmatrix} \gamma_{2N\times 2N} & 0_{2N\times 2} \\ 0_{2\times 2N} & 0_{2\times 2} \end{bmatrix}, \quad \gamma_{2N\times 2N} = \left(\mathbf{C}^a\right)^{-1}\,, \tag{16}$$

where $\mathbf{C}$ and $\mathbf{C}_a$ are covariance matrices defined in (8) and (13), respectively. If we define $\mathbf{\Theta} = [\theta, \theta^a]$, the information matrix can be rewritten as [8]

$$\mathbf{I_D} = \begin{bmatrix} \mathbf{I}_{\theta^a\theta^a} & \mathbf{I}_{\theta^a\theta} \\ \mathbf{I}_{\theta\theta^a} & \mathbf{I}_{\theta\theta} \end{bmatrix}\,, \tag{17}$$

where

$$[\mathbf{I_{xy}}]_{kl} = -\mathbb{E}_{f(P_R|\bar{\Theta})}\left[\frac{\partial^2 \ln f\left(P_R|\bar{\Theta}\right)}{\partial x_k \partial y_l}\right] \quad \begin{array}{l} k = 1, 2, ..., |\mathbf{x}|\,, \\ l = 1, 2, ... |\mathbf{y}|\,. \end{array} \tag{18}$$

Note that we use (7) to compute right hand side of (18), where $\bar{\Theta}$ is the expected value of Θ. The final FIM is calculated by adding $\mathbf{I_P}$, $\mathbf{I_D}$, and $\mathbf{I_A}$

$$\mathbf{I} = \begin{bmatrix} \mathbf{I}_{\theta^a\theta^a} + \gamma & I_{\theta^a\theta} \\ \mathbf{I}_{\theta\theta^a} & I_{\theta\theta} + \lambda \end{bmatrix}\,. \tag{19}$$

From (19), we can compute SPEB and thereby an lower bound for the variance of the localization error. It can be shown that the covariance of estimation of θ is lower bounded by (20). In this case the optimization problem is achieved by replacing the new SPEB from (20) into optimization problem.

We can extend the formulation for a network with K UNs, with the aim at minimizing the total power. In this case, each UN l has a positioning error bound SPEB_l that should be less than the corresponding threshold T_l. Altogether, we have

$$\arg\min_{p_j} \quad \sum_{j=1}^{N} p_j\,, \tag{21}$$

$$\text{s.t.} \quad \text{SPEB}_l \leq T_l\,, \qquad l = 1, 2, \ldots, K\,,$$

$$\sum_{j=1}^{N} p_j \leq K P_{\max}\,.$$

Two optimization problems presented in this letter are complicated to solve analytically. However, these two problems mathematically are similar to those presented in [4], [5], in which the authors use the game theoretical approaches, and [3] where the optimization problem transforms to the second-order cone programs.

IV. Numerical Results

In this section, the performance of the proposed scheme is evaluated to demonstrate the effects of various parameters introduced throughout the paper. In order to justify the efficiency of the optimal power allocation, we compare our results with uniform power allocation scheme used in [5], wherein all ANs transmit their beacons with the same power $P_{\max}/N$. To set up a simulation environment, the values of $\alpha = 3.5$ and $\sigma_v^2 = 6dB$ are adopted from Sensor Network Localization Explorer package [11]. The value of $m = 50$ is chosen according to [7], so that the received energy would be well approximated by a Gaussian distribution. An environment with one UN and 4 ANs, distributed in an area with dimensions of 250×250 square meters, is simulated. First, we find the

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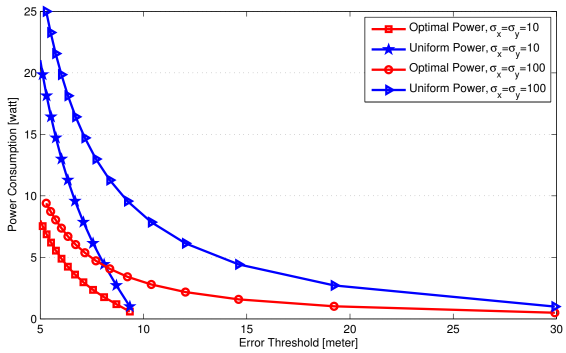

Fig. 1. Impact of variance of a priori density of UN location on power consumption and localization error threshold T. We assume that $\rho = 0.01$.

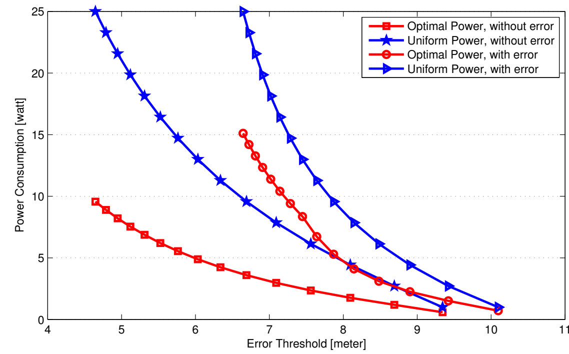

Fig. 2. Power consumption versus localization error threshold T. Optimal power allocation outperforms uniform one even for the case of having error in locations of the ANs. We assume that $\rho = 0.01$ and $\sigma_x = \sigma_y = 10$.

error threshold T_ℓ when all ANs transmit with the same power $P_{\max}/N$ (uniform power scheme). With this threshold, the minimum power consumption is then computed by solving (12). To this end, we use a genetic algorithm package with population size of 20 of MATLAB optimization toolbox.

Fig. 1 depicts the plot of power consumption versus required localization accuracy for two different values of $\sigma_x = \sigma_y = 10, 100$. It expresses the efficiency of our proposed method compared to the equal power allocation. From the figure, the total power consumption decreases by an increment in error threshold T; because each AN (or a group of ANs) should invest more energy to reduce SPEB, as can be verified by mathematical derivations provided in Section III. Another observation made from this figure is that the error threshold in which the minimum power consumption achieved is approximately equal to the variance of a priori density function of the UN location. Besides, to reach the same accuracy, more power consumption is required when we have more uncertainty in the UN location.

Fig. 2 illustrates the effect of the ANs errors in the power allocation. Two sets of curves are provided in the figure corresponding to two conditions: 1) When the ANs have no errors in their locations estimation, labeled by *without error*; 2) When they know their locations by some levels of errors, labeled by *with error*. The error is modeled by (13) for $\mathbf{C}^a = \mathrm{diag}_{8\times 8}(5, 5, \ldots, 5)$. Similar to Fig. 1, the optimal power still outperforms equal power allocation policy, as shown in *without error* and *with error* curves. Specifically, for the case of $T = 8$, the optimal design consumes about 60.1% and 47.7% less power than the corresponding uniform power allocation for without and with error, respectively. To justify the smaller gain in the reduction of the power consumption in *with error* case, we should note that the optimal solution tries to allocate higher powers to the ANs close to the UN. However, the optimal power policy does not necessarily allocate more power to the nearest ANs when they have error in their locations, and prefers to choose another AN with lower error in its position, leading to higher power consumption to meet the same accuracy compare to the *without error* case. However, increasing the power investment does not necessarily lead to the desired localization accuracy, due to the error in the estimations of the ANs' locations.

V. Conclusion

In this paper, we investigated an energy efficient localization scheme in wireless sensor networks. Squared position error bound was derived for a novel decision metric, based on the average energy received from the anchor nodes' beacons, and an optimization problem was formulated to minimize power consumption for achieving a certain localization accuracy. Optimal power allocation policy was formulated for two cases, i.e., without and with uncertainty in the anchor nodes positions. The results showed that optimal resource allocation can dramatically reduce the power consumption without sacrificing the localization accuracy in both cases. Moreover, though higher power consumption would be undeniable if there is an error in an anchor node position.

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