Univariate times series

In practice

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Box and Jenkins methodology

1. stationarity (if needed),
2. order selection,
3. estimation,
4. diagnostic checking,
5. final order selection,
6. forecasting,
7. ex post analysis.

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Methods

- trend-seasonality decomposition,
- differencing transformations,
- Box-Cox transformation.

Differencing transformations

The two most common cases:

- Time series with a d-degree polynomial: ∇^d.
- Time series with a s-period seasonality: ∇_s^D.

Box-Cox transformation

In order to remove a “variance problem”, the Box-Cox transformation is sometimes used:

$$\frac{X_t^\lambda - 1}{\lambda}$$

with $\lambda \in \mathbb{R}$.
Note that:

$$\frac{X_t^\lambda - 1}{\lambda} \xrightarrow{\lambda \to 0} \ln\left(X_t\right).$$

Remark

One can find some stationarity tests that aren't comprehensive.

In practice

- A very slowly decrease of the time series "ACF" suggests differencing at lag 1.
- A very slowly decrease of the time series "ACF" every s multiple lags suggests differencing at lag s.

Differencing is iteratively done, and generally : $d \leq 2$ and $D \leq 2$.

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Methodology
Stationarity
Order selection
Estimation
Diagnostic checking
Final order selection
Forecasting
Ex post analysis

Methods

We consider now that the transformed (or not) time series can potentially be fitted by a zero-mean ARMA model.
To select the model order, one can use:

- confidence intervals,
- corner method,
- information criterium,
- heuristic.

Use of confidence intervals

Idea: empirically establish maximum values for p and q.

- For a AR(p) process:

$$\forall h > p : \sqrt{n}\widehat{r}(h) \xrightarrow{\mathcal{L}} \mathcal{N}(0,1).$$

  One can define a confidence interval of level 95% and search the number of lags for which 95% of the $\widehat{r}(h)$ are in $\left[-\frac{1.96}{\sqrt{n}}, \frac{1.96}{\sqrt{n}}\right]$.

- For a MA(q) process:

$$\forall h > q : \sqrt{n}\widehat{\rho}(h) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, 1 + 2\sum_{k=1}^{q}\rho^2(k)\right).$$

  One can define a confidence interval of level 95% and search the number of lags for which 95% of the $\widehat{\rho}(h)$ are in:

$$\left[-\frac{1.96}{\sqrt{n}}\left(1 + 2\sum_{k=1}^{q}\widehat{\rho}^2(k)\right)^{\frac{1}{2}}, \frac{1.96}{\sqrt{n}}\left(1 + 2\sum_{k=1}^{q}\widehat{\rho}^2(k)\right)^{\frac{1}{2}}\right].$$

Information criterium

Aim : search a model for which a chosen information criterium is minimal.

Heuristic

In order to draw hypothesis, find "significant" autocorrelations:

- For the AR part: partial autocorrelations.
- For the MA part: (simple) autocorrelations.

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Estimation

Use of the maximum likelihood estimation after a preliminary estimation.

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Aim

- significance test of the parameters,
- whiteness and normality of the residuals.

Significance test of the parameters

For example, we consider the following test for φ_p:

$$\begin{cases} H_0 : \text{the process order is ARMA}\,(p-1,q)\ (\varphi_p = 0) \\ H_1 : \text{the process oder is ARMA}(p,q)\ (\varphi_p \neq 0) \end{cases} .$$

We use the Student statistic:

$$t = \frac{|\widehat{\varphi}_p|}{\sqrt{\widehat{\text{Var}}\,(\widehat{\varphi}_p)}}.$$

We reject H_0 at the 5% level if $|t| > 1.96$.

Whiteness and normality of the residuals

- Whiteness: LjungBox test.
- Normality: Shapiro-Wilk.

Note that if the normality of the residuals is rejected, it could be useful to add an ARCH or GARCH part...

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Final order selection

There are two main ways to finally select a model:

- Information criterium (e.g. AIC or BIC)).
- Predictive power.

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Forecasting

The forecasting and the forecasting interval are obtained using the autoregressive and moving average representations.

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Ex post analysis

Idea: truncate the time series and measure the forecasting error with an indictor such as the Root Mean Square Error (RMSE) or the Mean Average Percentage Error (MAPE):

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t - \widehat{x}_t)^2},$$

$$\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{x_t - \widehat{x}_t}{x_t}\right|.$$