A METHOD FOR IMPROVING THE EFFICIENCY OF EXTERNAL SORTING ALGORITHMS USING THE DISTRIBUTION METHOD

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There are two general methods forming the basis for all known varieties of external sorting algorithms using magnetic tapes: merging and distribution (the terminology used here corresponds basically to that of [3-5]). The distinguishing characteristic of the first of these methods is that in the process of sorting, the sizes of the internally ordered groups of words are increased (and their number is decreased) until complete merging into a single ordered file has been achieved. In the distribution method the converse is true: the sizes of the externally ordered groups are reduced until an ordered sequence of groups, each of unit size, is obtained. The fact that in both cases the smallest groups formed in the process of proper external sorting are usually considerably larger than unity (due to the possibility of using fast methods of internal sorting for ordering groups of words smaller than the core storage), does not change the essence of the matter.

External sorting algorithms using various modifications of the merging method have many good features, have a wide field of effective application, and have been the subject of a number of studies described in the literature both here and abroad. The distribution method has been given much less attention, even though it has specific advantages making it more efficient in some practical applications, and sometimes even the only possible method (for example in the case of only two magnetic tapes [4, 5]).

This paper is a generalization of certain results obtained in [4, 5] for approaching the problem of more efficient external sorting algorithms,based on a variety of distribution methods from a unified viewpoint.

Irrespective of the criteria used in the distribution process (whether it is the condition that a specific digit of the key should coincide with a given value, or that the complete value of the key should be within a specified interval), the process can be interpreted as a plane graph taking the form of a tree, in which the nodes are allocated according to definite distribution levels. An example of such a tree is given in [4]. The nodes of the tree correspond to the groups arising in the distribution of the unordered file, while the branches emerging from the nodes correspond to the process of distributing the groups over a series of subgroups of a higher level (the process of growing the tree). Such a tree is characterized by the property that one and only one branch terminates in each node (apart from the root node) from a next lower level (having a smaller number of nodes), and an identical number of branches go from each node, excepting the nodes of the highest level, to the next higher level.

We formulate the following definitions.

1. The modulus of the distribution μ will be the number of branches originating from each node of the tree.

2. A phase of the distribution φ_{ij} is the process of growing the tree from the j-th node of the (i−1)-th level to the corresponding μ nodes of the i-th level.

3. The cost c_{ij} of executing a phase is some quantity depending on the method used in growing the tree, and in general determines the time spent on executing a phase of the distribution.

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TABLE 1

p	1	2	3	4	5	....
K_0	4	2	2	1	1	....

For external sorting by means of magnetic tapes, where the basic expenditure of time is related to the transfer of information between tapes, we can write

$$c_{ij} = a \cdot N_{i-1,j},$$

where a is a constant for a given method of distribution showing how many times during the execution of phase φ_{ij} each of the $N_{i-1,j}$ words of the distributed group are transferred between core storage and the tapes.

The cost of growing the tree from the zeroth to the h-th level, which is the interpretation of the complete distribution of the original file, will obviously be

$$C = a \sum_{i=1}^{h} \sum_{j=1}^{v_i} N_{i-1,j} \tag{1}$$

where v_i is the number of nodes at the i-th level.

In the case of a uniform distribution M of the words of the original file with respect to the distribution criteria $\sum_{j=1}^{v_i} N_{i-1,j} = M$ for all $i \leq h$, we have $[\log_\mu m]_{ni}$, and then the specific cost of growing the tree is

$$\eta = \frac{C}{M} = a\,[\log_\mu m]_{ni}, \tag{2}$$

where $m = v_h$ is the number of nodes in the highest level of the distribution; $[\log_\mu m]_{ni}$ is the nearest integer (ni) greater than $\log_\mu m$.

Let us consider the method of executing a phase φ_{ij} in which the distribution is carried out according to the modulus $\mu = Kp$, where p denotes the number of output tapes, and K is some integer called the characteristic number. In the special case when $K=1$, we have the distribution method used in a number of papers [1, 2], in which the group to be distributed is scanned only once during the execution of φ_{ij}, so that consequently $\mu(1) = p$, $a(1) = 2$, and the specific cost of growing the tree is

$$\eta(1) = 2\,[\log_p m]_{ni}. \tag{3}$$

It follows from (3) that if the minimum number of tapes used is equal to 2 (one intput and one output tape), the method of distribution for $K=1$ cannot in general be realized (which is also true for the method of merging).

A K-fold increase in the modulus can be obtained by scanning the original groups K times. During the scanning, each word of the group to be distributed is read K times from the input tape and is written once on the output tape, so that $a(K) = K+1$ and

$$\eta(K) = (K+1)\,[\log_{pK} m]_{ni}. \tag{4}$$

Let us determine the value $K = K_0$ of the characteristic number at which $\eta(K)$ takes its minimum value. Assuming for simplicity that $[\log_{pK} m]_{ni} = \log_{pK} m$, from the condition $\frac{d[(K+1)\log_{pK} m]}{dK}$ we have

$$1 - \frac{K_0 + 1}{K_0 \ln pK_0} = 0. \tag{5}$$

The integer function of the integer argument $K_0 = f(p)$ satisfying (5) with an accuracy up to its region of definition is represented in Table 1.

Let us illustrate the method described above by considering one of the sorting algorithms, chosen for its simplicity (although it is of independent interest, since it can be used effectively with certain values of the sorting parameters).