

APPENDIX D: SHORT TABLE OF INTEGRALS

Elementary forms

BASIC RULES

1. **Constant rule*** $\int 0 \, du = 0 + C$
2. **Power rule** $\int u^n \, du = \frac{u^{n+1}}{n+1}; \quad n \neq -1$
 $\int u^n \, du = \ln |u|; \quad n = -1$
3. **Natural exponential rule** $\int e^u \, du = e^u$
4. **Logarithmic rule** $\int \ln |u| \, du = u \ln |u| - u$

TRIGONOMETRIC RULES

5. $\int \sin u \, du = -\cos u$
6. $\int \cos u \, du = \sin u$
7. $\int \tan u \, du = -\ln |\cos u| = \ln |\sec u|$
8. $\int \cot u \, du = \ln |\sin u|$
9. $\int \sec u \, du = \ln |\sec u + \tan u|$
10. $\int \csc u \, du = -\ln |\csc u + \cot u|$
11. $\int \sec^2 u \, du = \tan u$
12. $\int \csc^2 u \, du = -\cot u$
13. $\int \sec u \tan u \, du = \sec u$
14. $\int \csc u \cot u \, du = -\csc u$

EXPONENTIAL RULE

15. $\int a^u \, du = \frac{a^u}{\ln a} \quad a > 0, a \neq 1$

HYPERBOLIC RULES

16. $\int \cosh u \, du = \sinh u$
17. $\int \sinh u \, du = \cosh u$
18. $\int \tanh u \, du = \ln \cosh u$
19. $\int \coth u \, du = \ln |\sinh u|$
20. $\int \operatorname{sech} u \, du = \tan^{-1}(\sinh u) \text{ or } 2 \tan^{-1} e^u$
21. $\int \operatorname{csch} u \, du = \ln \left| \tanh \frac{u}{2} \right|$

INVERSE RULES

22. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a}$
23. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1} \frac{u}{a}$
24. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$
25. $\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \frac{u}{a} & \text{if } \left| \frac{u}{a} \right| < 1 \\ \frac{1}{a} \coth^{-1} \frac{u}{a} & \text{if } \left| \frac{u}{a} \right| > 1 \end{cases}$
26. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right|$
27. $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \left| \frac{u}{a} \right|$
 $= -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right|$

*Notice that this formula shows the addition of a constant, C . When using an integral table or technology, the constant will usually not be shown, so it is important that you remember to insert the constant of integration each time you evaluate an integral, even when getting it from a table or from technology.

$$28. \int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) \quad 29. \int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right|$$

$$= \sinh^{-1} \frac{u}{a} \quad = -\frac{1}{a} \operatorname{csch}^{-1} \left| \frac{u}{a} \right|$$

Linear and quadratic forms

INTEGRALS INVOLVING $au + b$

$$30. \int (au + b)^n du = \frac{(au + b)^{n+1}}{(n+1)a}$$

$$31. \int u(au + b)^n du = \frac{(au + b)^{n+2}}{(n+2)a^2} - \frac{b(au + b)^{n+1}}{(n+1)a^2}$$

$$32. \int u^2(au + b)^n du = \frac{(au + b)^{n+3}}{(n+3)a^3} - \frac{2b(au + b)^{n+2}}{(n+2)a^3} + \frac{b^2(au + b)^{n+1}}{(n+1)a^3}$$

$$33. * \int u^m(au + b)^n du = \begin{cases} \frac{u^{m+1}(au + b)^n}{m+n+1} + \frac{nb}{m+n+1} \int u^m(au + b)^{n-1} du & \text{or} \\ \frac{u^m(au + b)^{n+1}}{(m+n+1)a} - \frac{mb}{(m+n+1)a} \int u^{m-1}(au + b)^n du & \text{or} \\ \frac{-u^{m+1}(au + b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b} \int u^m(au + b)^{n+1} du \end{cases}$$

$$34. \int \frac{du}{au + b} = \frac{1}{a} \ln |au + b|$$

$$35. \int \frac{u du}{au + b} = \frac{u}{a} - \frac{b}{a^2} \ln |au + b|$$

$$36. \int \frac{u^2 du}{au + b} = \frac{(au + b)^2}{2a^3} - \frac{2b(au + b)}{a^3} + \frac{b^2}{a^3} \ln |au + b|$$

$$37. \int \frac{u^3 du}{au + b} = \frac{(au + b)^3}{3a^4} - \frac{3b(au + b)^2}{2a^4} + \frac{3b^2(au + b)}{a^4} - \frac{b^3}{a^4} \ln |au + b|$$

INTEGRALS INVOLVING $u^2 + a^2$

$$38. \int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} \quad 39. \int \frac{u du}{u^2 + a^2} = \frac{1}{2} \ln(u^2 + a^2)$$

$$40. \int \frac{u^2 du}{u^2 + a^2} = u - a \tan^{-1} \frac{u}{a} \quad 41. \int \frac{u^3 du}{u^2 + a^2} = \frac{u^2}{2} - \frac{a^2}{2} \ln(u^2 + a^2)$$

$$42. \int \frac{du}{u(u^2 + a^2)} = \frac{1}{2a^2} \ln \left(\frac{u^2}{u^2 + a^2} \right) \quad 43. \int \frac{du}{u^2(u^2 + a^2)} = -\frac{1}{a^2 u} - \frac{1}{a^3} \tan^{-1} \frac{u}{a}$$

$$44. \int \frac{du}{u^3(u^2 + a^2)} = -\frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln \left(\frac{u^2}{u^2 + a^2} \right)$$

INTEGRALS INVOLVING $u^2 - a^2$, $u^2 > a^2$

$$45. \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| \quad \text{or} \quad -\frac{1}{a} \coth^{-1} \frac{u}{a}$$

$$46. \int \frac{u du}{u^2 - a^2} = \frac{1}{2} \ln |u^2 - a^2|$$

*When the integral is given as another integral, then do not add the constant until the form no longer involves an integration.

$$\begin{aligned}
47. \int \frac{u^2 du}{u^2 - a^2} &= u + \frac{a}{2} \ln \left| \frac{u-a}{u+a} \right| & 48. \int \frac{u^3 du}{u^2 - a^2} &= \frac{u^2}{2} + \frac{a^2}{2} \ln |u^2 - a^2| \\
49. \int \frac{du}{u(u^2 - a^2)} &= \frac{1}{2a^2} \ln \left| \frac{u^2 - a^2}{u^2} \right| & 50. \int \frac{du}{u^2(u^2 - a^2)} &= \frac{1}{a^2 u} + \frac{1}{2a^3} \ln \left| \frac{u-a}{u+a} \right| \\
51. \int \frac{du}{u^3(u^2 - a^2)} &= \frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln \left| \frac{u^2}{u^2 - a^2} \right|
\end{aligned}$$

INTEGRALS INVOLVING $a^2 - u^2, u^2 < a^2$

$$\begin{aligned}
52. \int \frac{du}{a^2 - u^2} &= \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| \text{ or } \frac{1}{a} \tanh^{-1} \frac{u}{a} \\
53. \int \frac{u du}{a^2 - u^2} &= -\frac{1}{2} \ln |a^2 - u^2| \\
54. \int \frac{u^2 du}{a^2 - u^2} &= -u + \frac{a}{2} \ln \left| \frac{a+u}{a-u} \right| \\
55. \int \frac{u^3 du}{a^2 - u^2} &= -\frac{u^2}{2} - \frac{a^2}{2} \ln |a^2 - u^2| \\
56. \int \frac{du}{u(a^2 - u^2)} &= \frac{1}{2a^2} \ln \left| \frac{u^2}{a^2 - u^2} \right| \\
57. \int \frac{du}{u^2(a^2 - u^2)} &= -\frac{1}{a^2 u} + \frac{1}{2a^3} \ln \left| \frac{a+u}{a-u} \right| \\
58. \int \frac{du}{u^3(a^2 - u^2)} &= -\frac{1}{2a^2 u^2} + \frac{1}{2a^4} \ln \left| \frac{u^2}{a^2 - u^2} \right| \\
59. \int \frac{du}{(a^2 - u^2)^2} &= \frac{u}{2a^2(a^2 - u^2)} + \frac{1}{4a^3} \ln \left| \frac{a+u}{a-u} \right| \\
60. \int \frac{u du}{(a^2 - u^2)^2} &= \frac{1}{2(a^2 - u^2)} \\
61. \int \frac{u^2 du}{(a^2 - u^2)^2} &= \frac{u}{2(a^2 - u^2)} - \frac{1}{4a} \ln \left| \frac{a+u}{a-u} \right| \\
62. \int \frac{u^3 du}{(a^2 - u^2)^2} &= \frac{a^2}{2(a^2 - u^2)} + \frac{1}{2} \ln |a^2 - u^2| \\
63. \int \frac{du}{u(a^2 - u^2)^2} &= \frac{1}{2a^2(a^2 - u^2)} + \frac{1}{2a^4} \ln \left| \frac{u^2}{a^2 - u^2} \right| \\
64. \int \frac{du}{u^2(a^2 - u^2)^2} &= -\frac{1}{a^4 u} + \frac{u}{2a^4(a^2 - u^2)} + \frac{3}{4a^5} \ln \left| \frac{a+u}{a-u} \right| \\
65. \int \frac{du}{u^3(a^2 - u^2)^2} &= -\frac{1}{2a^4 u^2} + \frac{1}{2a^4(a^2 - u^2)} + \frac{1}{a^6} \ln \left| \frac{u^2}{a^2 - u^2} \right|
\end{aligned}$$

INTEGRALS INVOLVING $au^2 + bu + c$

$$\begin{aligned}
66. \int \frac{du}{au^2 + bu + c} &= \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2au + b}{\sqrt{4ac - b^2}} \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2au + b - \sqrt{b^2 - 4ac}}{2au + b + \sqrt{b^2 - 4ac}} \right| \end{cases} \\
67. \int \frac{u du}{au^2 + bu + c} &= \frac{1}{2a} \ln |au^2 + bu + c| - \frac{b}{2a} \int \frac{du}{au^2 + bu + c} \\
68. \int \frac{u^2 du}{au^2 + bu + c} &= \frac{u}{a} - \frac{b}{2a^2} \ln |au^2 + bu + c| + \frac{b^2 - 2ac}{2a^2} \int \frac{du}{au^2 + bu + c} \\
69. \int \frac{u^m du}{au^2 + bu + c} &= \frac{u^{m-1}}{(m-1)a} - \frac{c}{a} \int \frac{u^{m-2} du}{au^2 + bu + c} - \frac{b}{a} \int \frac{u^{m-1} du}{au^2 + bu + c}
\end{aligned}$$

$$\begin{aligned}
 70. \quad \int \frac{du}{u(au^2 + bu + c)} &= \frac{1}{2c} \ln \left| \frac{u^2}{au^2 + bu + c} \right| - \frac{b}{2c} \int \frac{du}{au^2 + bu + c} \\
 71. \quad \int \frac{du}{u^2(au^2 + bu + c)} &= \frac{b}{2c^2} \ln \left| \frac{au^2 + bu + c}{u^2} \right| - \frac{1}{cu} + \frac{b^2 - 2ac}{2c^2} \int \frac{du}{au^2 + bu + c} \\
 72. \quad \int \frac{du}{u^n(au^2 + bu + c)} &= -\frac{1}{(n-1)cu^{n-1}} - \frac{b}{c} \int \frac{du}{u^{n-1}(au^2 + bu + c)} - \frac{a}{c} \int \frac{du}{u^{n-2}(au^2 + bu + c)} \\
 73. \quad \int \frac{du}{(au^2 + bu + c)^2} &= \frac{2au + b}{(4ac - b^2)(au^2 + bu + c)} + \frac{2a}{4ac - b^2} \int \frac{du}{au^2 + bu + c} \\
 74. \quad \int \frac{u \, du}{(au^2 + bu + c)^2} &= -\frac{bu + 2c}{(4ac - b^2)(au^2 + bu + c)} - \frac{b}{4ac - b^2} \int \frac{du}{au^2 + bu + c} \\
 75. \quad \int \frac{u^2 \, du}{(au^2 + bu + c)^2} &= \frac{(b^2 - 2ac)u + bc}{a(4ac - b^2)(au^2 + bu + c)} + \frac{2c}{4ac - b^2} \int \frac{du}{au^2 + bu + c} \\
 76. \quad \int \frac{u^m \, du}{(au^2 + bu + c)^n} &= \frac{-u^{m-1}}{(2n - m - 1)a(au^2 + bu + c)^{n-1}} \\
 &\quad - \frac{(n - m)b}{(2n - m - 1)a} \int \frac{u^{m-1} \, du}{(au^2 + bu + c)^n} \\
 &\quad + \frac{(m - 1)c}{(2n - m - 1)a} \int \frac{u^{m-2} \, du}{(au^2 + bu + c)^n}
 \end{aligned}$$

Radical forms

INTEGRALS INVOLVING $\sqrt{au + b}$

$$\begin{aligned}
 77. \quad \int \frac{du}{\sqrt{au + b}} &= \frac{2\sqrt{au + b}}{a} \\
 78. \quad \int \frac{u \, du}{\sqrt{au + b}} &= \frac{2(au - 2b)}{3a^2} \sqrt{au + b} \\
 79. \quad \int \frac{u^2 \, du}{\sqrt{au + b}} &= \frac{2(3a^2u^2 - 4abu + 8b^2)}{15a^3} \sqrt{au + b} \\
 80. \quad \int \frac{du}{u\sqrt{au + b}} &= \begin{cases} \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{au + b} - \sqrt{b}}{\sqrt{au + b} + \sqrt{b}} \right| \\ \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{au + b}{-b}} \end{cases} \\
 81. \quad \int \frac{du}{u^2\sqrt{au + b}} &= -\frac{\sqrt{au + b}}{bu} - \frac{a}{2b} \int \frac{du}{u\sqrt{au + b}} \\
 82. \quad \int \sqrt{au + b} \, du &= \frac{2\sqrt{(au + b)^3}}{3a} \\
 83. \quad \int u\sqrt{au + b} \, du &= \frac{2(3au - 2b)}{15a^2} \sqrt{(au + b)^3} \\
 84. \quad \int u^2\sqrt{au + b} \, du &= \frac{2(15a^2u^2 - 12abu + 8b^2)}{105a^3} \sqrt{(au + b)^3}
 \end{aligned}$$

INTEGRALS INVOLVING $\sqrt{u^2 + a^2}$

$$\begin{aligned}
 85. \quad \int \sqrt{u^2 + a^2} \, du &= \frac{u\sqrt{u^2 + a^2}}{2} + \frac{a^2}{2} \ln(u + \sqrt{u^2 + a^2}) \\
 86. \quad \int u\sqrt{u^2 + a^2} \, du &= \frac{(u^2 + a^2)^{3/2}}{3} \\
 87. \quad \int u^2\sqrt{u^2 + a^2} \, du &= \frac{u(u^2 + a^2)^{3/2}}{4} - \frac{a^2u\sqrt{u^2 + a^2}}{8} - \frac{a^4}{8} \ln(u + \sqrt{u^2 + a^2})
 \end{aligned}$$

88. $\int u^3 \sqrt{u^2 + a^2} du = \frac{(u^2 + a^2)^{5/2}}{5} - \frac{a^2(u^2 + a^2)^{3/2}}{3}$
89. $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2}) \text{ or } \sinh^{-1} \frac{u}{a}$
90. $\int \frac{u du}{\sqrt{u^2 + a^2}} = \sqrt{u^2 + a^2}$
91. $\int \frac{u^2 du}{\sqrt{u^2 + a^2}} = \frac{u\sqrt{u^2 + a^2}}{2} - \frac{a^2}{2} \ln(u + \sqrt{u^2 + a^2})$
92. $\int \frac{u^3 du}{\sqrt{u^2 + a^2}} = \frac{(u^2 + a^2)^{3/2}}{3} - a^2 \sqrt{u^2 + a^2}$
93. $\int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right|$
94. $\int \frac{du}{u^2 \sqrt{u^2 + a^2}} = -\frac{\sqrt{u^2 + a^2}}{a^2 u}$
95. $\int \frac{du}{u^3 \sqrt{u^2 + a^2}} = -\frac{\sqrt{u^2 + a^2}}{2a^2 u^2} + \frac{1}{2a^3} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right|$
96. $\int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right|$
97. $\int \frac{\sqrt{u^2 + a^2}}{u^2} du = -\frac{\sqrt{u^2 + a^2}}{u} + \ln(u + \sqrt{u^2 + a^2})$

INTEGRALS INVOLVING $\sqrt{u^2 - a^2}$, $a > 0$

98. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}|$
99. $\int \frac{u du}{\sqrt{u^2 - a^2}} = \sqrt{u^2 - a^2}$
100. $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u\sqrt{u^2 - a^2}}{2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}|$
101. $\int \frac{u^3 du}{\sqrt{u^2 - a^2}} = \frac{(u^2 - a^2)^{3/2}}{3} + a^2 \sqrt{u^2 - a^2}$
102. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right|$
103. $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u}$
104. $\int \frac{du}{u^3 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{2a^2 u^2} + \frac{1}{2a^3} \sec^{-1} \left| \frac{u}{a} \right|$
105. $\int \frac{\sqrt{u^2 - a^2}}{u} du = \frac{u\sqrt{u^2 - a^2}}{2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}|$
106. $\int u \sqrt{u^2 - a^2} du = \frac{(u^2 - a^2)^{3/2}}{3}$
107. $\int u^2 \sqrt{u^2 - a^2} du = \frac{u(u^2 - a^2)^{3/2}}{4} + \frac{a^2 u \sqrt{u^2 - a^2}}{8} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}|$
108. $\int u^3 \sqrt{u^2 - a^2} du = \frac{(u^2 - a^2)^{5/2}}{5} + \frac{a^2(u^2 - a^2)^{3/2}}{3}$
109. $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \sec^{-1} \left| \frac{u}{a} \right|$

INTEGRALS INVOLVING $\sqrt{a^2 - u^2}$, $a > 0$

$$110. \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a}$$

$$111. \int \frac{u du}{\sqrt{a^2 - u^2}} = -\sqrt{a^2 - u^2}$$

$$112. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u\sqrt{a^2 - u^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a}$$

$$113. \int \frac{u^3 du}{\sqrt{a^2 - u^2}} = \frac{(a^2 - u^2)^{3/2}}{3} - a^2 \sqrt{a^2 - u^2}$$

$$114. \int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| \quad \text{or} \quad -\frac{1}{a} \operatorname{sech}^{-1} \left| \frac{u}{a} \right|$$

$$115. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u}$$

$$116. \int \frac{du}{u^3 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{2a^2 u^2} - \frac{1}{2a^3} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right|$$

$$117. \int \sqrt{a^2 - u^2} du = \frac{u\sqrt{a^2 - u^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a}$$

$$118. \int u \sqrt{a^2 - u^2} du = -\frac{(a^2 - u^2)^{3/2}}{3}$$

$$119. \int u^2 \sqrt{a^2 - u^2} du = -\frac{u(a^2 - u^2)^{3/2}}{4} + \frac{a^2 u \sqrt{a^2 - u^2}}{8} + \frac{a^4}{8} \sin^{-1} \frac{u}{a}$$

$$120. \int u^3 \sqrt{a^2 - u^2} du = \frac{(a^2 - u^2)^{5/2}}{5} - \frac{a^2 (a^2 - u^2)^{3/2}}{3}$$

$$121. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right|$$

Trigonometric forms

INTEGRALS INVOLVING $\cos au$

$$122. \int \cos au du = \frac{\sin au}{a}$$

$$123. \int u \cos au du = \frac{\cos au}{a^2} + \frac{u \sin au}{a}$$

$$124. \int u^2 \cos au du = \frac{2u}{a^2} \cos au + \left(\frac{u^2}{a} - \frac{2}{a^3} \right) \sin au$$

$$125. \int u^3 \cos au du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4} \right) \cos au + \left(\frac{u^3}{a} - \frac{6u}{a^3} \right) \sin au$$

$$126. \int u^n \cos au du = \frac{u^n \sin au}{a} - \frac{n}{a} \int u^{n-1} \sin au du$$

$$127. \int \cos^2 au du = \frac{u}{2} + \frac{\sin 2au}{4a}$$

$$128. \int u \cos^2 au du = \frac{u^2}{4} + \frac{u \sin 2au}{4a} + \frac{\cos 2au}{8a^2}$$

$$129. \int \cos^3 au du = \frac{\sin au}{a} - \frac{\sin^3 au}{3a}$$

$$130. \int \cos^4 au du = \frac{3u}{8} + \frac{\sin 2au}{4a} + \frac{\sin 4au}{32a}$$

INTEGRALS INVOLVING $\sin au$

131. $\int \sin au \, du = -\frac{\cos au}{a}$
132. $\int u \sin au \, du = \frac{\sin au}{a^2} - \frac{u \cos au}{a}$
133. $\int u^2 \sin au \, du = \frac{2u}{a^2} \sin au + \left(\frac{2}{a^3} - \frac{u^2}{a}\right) \cos au$
134. $\int u^3 \sin au \, du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4}\right) \sin au + \left(\frac{6u}{a^3} - \frac{u^3}{a}\right) \cos au$
135. $\int u^n \sin au \, du = -\frac{u^n \cos au}{a} + \frac{n}{a} \int u^{n-1} \cos au \, du$
136. $\int \sin^2 au \, du = \frac{u}{2} - \frac{\sin 2au}{4a}$
137. $\int u \sin^2 au \, du = \frac{u^2}{4} - \frac{u \sin 2au}{4a} - \frac{\cos 2au}{8a^2}$
138. $\int \sin^3 au \, du = -\frac{\cos au}{a} + \frac{\cos^3 au}{3a}$
139. $\int \sin^4 au \, du = \frac{3u}{8} - \frac{\sin 2au}{4a} + \frac{\sin 4au}{32a}$

INTEGRALS INVOLVING $\sin au$ and $\cos au$

140. $\int \sin au \cos au \, du = \frac{\sin^2 au}{2a}$
141. $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)}$
142. $\int \sin^n au \cos au \, du = \frac{\sin^{n+1} au}{(n+1)a}$
143. $\int \cos^n au \sin au \, du = -\frac{\cos^{n+1} au}{(n+1)a}$
144. $\int \sin^2 au \cos^2 au \, du = \frac{u}{8} - \frac{\sin 4au}{32a}$
145. $\int \frac{du}{\sin au \cos au} = \frac{1}{a} \ln |\tan au|$
146. $\int \frac{du}{\sin^2 au \cos au} = \frac{1}{a} \ln \left| \tan \left(\frac{\pi}{4} + \frac{au}{2} \right) \right| - \frac{1}{a \sin au}$
147. $\int \frac{du}{\sin au \cos^2 au} = \frac{1}{a} \ln \left| \tan \frac{au}{2} \right| + \frac{1}{a \cos au}$

INTEGRALS INVOLVING $\tan au$

148. $\int \tan au \, du = -\frac{1}{a} \ln |\cos au| \text{ or } \frac{1}{a} \ln |\sec au|$
149. $\int \tan^2 au \, du = \frac{\tan au}{a} - u$
150. $\int \tan^3 au \, du = \frac{\tan^2 au}{2a} + \frac{1}{a} \ln |\cos au|$
151. $\int \tan^n au \, du = \frac{\tan^{n-1} au}{(n-1)a} - \int \tan^{n-2} au \, du$
152. $\int \tan^n au \sec^2 au \, du = \frac{\tan^{n+1} au}{(n+1)a}$

INTEGRALS INVOLVING $\cot au$

153. $\int \cot au \, du = \frac{1}{a} \ln |\sin au|$
154. $\int \cot^2 au \, du = -\frac{\cot au}{a} - u$
155. $\int \cot^3 au \, du = -\frac{\cot^2 au}{2a} - \frac{1}{a} \ln |\sin au|$
156. $\int \cot^n au \, du = -\frac{\cot^{n-1} au}{(n-1)a} - \int \cot^{n-2} au \, du$
157. $\int \cot^n au \csc^2 au \, du = -\frac{\cot^{n+1} au}{(n+1)a}$

INTEGRALS INVOLVING $\sec au$

158. $\int \sec au \, du = \frac{1}{a} \ln |\sec au + \tan au| = \frac{1}{a} \ln \left| \tan \left(\frac{au}{2} + \frac{\pi}{4} \right) \right|$
159. $\int \sec^2 au \, du = \frac{\tan au}{a}$
160. $\int \sec^3 au \, du = \frac{\sec au \tan au}{2a} + \frac{1}{2a} \ln |\sec au + \tan au|$
161. $\int \sec^n au \, du = \frac{\sec^{n-2} au \tan au}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} au \, du$
162. $\int \sec^n au \tan au \, du = \frac{\sec^n au}{na}$

INTEGRALS INVOLVING $\csc au$

163. $\int \csc au \, du = \frac{1}{a} \ln |\csc au - \cot au| = \frac{1}{a} \ln \left| \tan \frac{au}{2} \right|$
164. $\int \csc^2 au \, du = -\frac{\cot au}{a}$
165. $\int \csc^3 au \, du = -\frac{\csc au \cot au}{2a} + \frac{1}{2a} \ln \left| \tan \frac{au}{2} \right|$
166. $\int \csc^n au \, du = -\frac{\csc^{n-2} au \cot au}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} au \, du$
167. $\int \csc^n au \cot au \, du = -\frac{\csc^n au}{na}$

Inverse trigonometric forms

INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS, $a > 0$

168. $\int \cos^{-1} \frac{u}{a} \, du = u \cos^{-1} \frac{u}{a} - \sqrt{a^2 - u^2}$
169. $\int u \cos^{-1} \frac{u}{a} \, du = \left(\frac{u^2}{2} - \frac{a^2}{4} \right) \cos^{-1} \frac{u}{a} - \frac{u \sqrt{a^2 - u^2}}{4}$
170. $\int u^2 \cos^{-1} \frac{u}{a} \, du = \frac{u^3}{3} \cos^{-1} \frac{u}{a} - \frac{(u^2 + 2a^2) \sqrt{a^2 - u^2}}{9}$
171. $\int \frac{\cos^{-1} \frac{u}{a}}{u} \, du = \frac{\pi}{2} \ln |u| - \int \frac{\sin^{-1} \frac{u}{a}}{u} \, du$
172. $\int \frac{\cos^{-1} \frac{u}{a}}{u^2} \, du = -\frac{\cos^{-1} \frac{u}{a}}{u} + \frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right|$

173. $\int \left(\cos^{-1} \frac{u}{a}\right)^2 du = u \left(\cos^{-1} \frac{u}{a}\right)^2 - 2u - 2\sqrt{a^2 - u^2} \cos^{-1} \frac{u}{a}$
174. $\int \sin^{-1} \frac{u}{a} du = u \sin^{-1} \frac{u}{a} + \sqrt{a^2 - u^2}$
175. $\int u \sin^{-1} \frac{u}{a} du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right) \sin^{-1} \frac{u}{a} + \frac{u\sqrt{a^2 - u^2}}{4}$
176. $\int u^2 \sin^{-1} \frac{u}{a} du = \frac{u^3}{3} \sin^{-1} \frac{u}{a} + \frac{(u^2 + 2a^2)\sqrt{a^2 - u^2}}{9}$
177. $\int \frac{\sin^{-1} \frac{u}{a}}{u} du = \frac{u}{a} + \frac{\left(\frac{u}{a}\right)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 \left(\frac{u}{a}\right)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \left(\frac{u}{a}\right)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots$
178. $\int \frac{\sin^{-1} \frac{u}{a}}{u^2} du = -\frac{\sin^{-1} \frac{u}{a}}{u} - \frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right|$
179. $\int \left(\sin^{-1} \frac{u}{a}\right)^2 du = u \left(\sin^{-1} \frac{u}{a}\right)^2 - 2u + 2\sqrt{a^2 - u^2} \sin^{-1} \frac{u}{a}$
180. $\int \tan^{-1} \frac{u}{a} du = u \tan^{-1} \frac{u}{a} - \frac{a}{2} \ln(u^2 + a^2)$
181. $\int u \tan^{-1} \frac{u}{a} du = \frac{1}{2} (u^2 + a^2) \tan^{-1} \frac{u}{a} - \frac{au}{2}$
182. $\int u^2 \tan^{-1} \frac{u}{a} du = \frac{u^3}{3} \tan^{-1} \frac{u}{a} - \frac{au^2}{6} + \frac{a^3}{6} \ln(u^2 + a^2)$

Exponential and logarithmic forms

INTEGRALS INVOLVING e^{au}

183. $\int e^{au} du = \frac{e^{au}}{a}$
184. $\int ue^{au} du = \frac{e^{au}}{a} \left(u - \frac{1}{a}\right)$
185. $\int u^2 e^{au} du = \frac{e^{au}}{a} \left(u^2 - \frac{2u}{a} + \frac{2}{a^2}\right)$
186. $\int u^n e^{au} du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} du$
187. $\int \frac{e^{au}}{u} du = \ln |u| + \frac{au}{1 \cdot 1!} + \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^3}{3 \cdot 3!} + \dots$
188. $\int \frac{e^{au}}{u^n} du = \frac{-e^{au}}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{e^{au}}{u^{n-1}} du$
189. $\int \frac{du}{p + qe^{au}} = \frac{u}{p} - \frac{1}{ap} \ln |p + qe^{au}|$
190. $\int \frac{du}{(p + qe^{au})^2} = \frac{u}{p^2} + \frac{1}{ap(p + qe^{au})} - \frac{1}{ap^2} \ln |p + qe^{au}|$
191. $\int \frac{du}{pe^{au} + qe^{-au}} = \begin{cases} \frac{1}{a\sqrt{pq}} \tan^{-1} \left(\sqrt{\frac{p}{q}} e^{au} \right), & p > 0, q > 0 \\ \frac{1}{2a\sqrt{-pq}} \ln \left| \frac{e^{au} - \sqrt{-\frac{q}{p}}}{e^{au} + \sqrt{-\frac{q}{p}}} \right|, & p > 0, q < 0 \end{cases}$
192. $\int e^{au} \sin bu du = \frac{e^{au}(a \sin bu - b \cos bu)}{a^2 + b^2}$
193. $\int e^{au} \cos bu du = \frac{e^{au}(a \cos bu + b \sin bu)}{a^2 + b^2}$

$$194. \int u e^{au} \sin bu \, du = \frac{u e^{au} (a \sin bu - b \cos bu)}{a^2 + b^2} - \frac{e^{au} [(a^2 - b^2) \sin bu - 2ab \cos bu]}{(a^2 + b^2)^2}$$

$$195. \int u e^{au} \cos bu \, du = \frac{u e^{au} (a \cos bu + b \sin bu)}{a^2 + b^2} - \frac{e^{au} [(a^2 - b^2) \cos bu + 2ab \sin bu]}{(a^2 + b^2)^2}$$

INTEGRALS INVOLVING $\ln |u|$

$$196. \int \ln |u| \, du = u \ln |u| - u$$

$$197. \int (\ln |u|)^2 \, du = u (\ln |u|)^2 - 2u \ln |u| + 2u$$

$$198. \int (\ln |u|)^n \, du = u (\ln |u|)^n - n \int (\ln |u|)^{n-1} \, du$$

$$199. \int u \ln |u| \, du = \frac{u^2}{2} \left(\ln |u| - \frac{1}{2} \right)$$

$$200. \int u^m \ln |u| \, du = \frac{u^{m+1}}{m+1} \left(\ln |u| - \frac{1}{m+1} \right)$$