That is, for a zero-mean white-noise input, the cross-correlation between input and output of a linear system is proportional to the impulse response of the system. Similarly, the power spectrum of a white-noise input is

$$\Phi_{xx}(e^{j\omega}) = \sigma_x^2, \qquad -\pi \le \omega \le \pi. \tag{2.203}$$

Thus, from Eq. (2.201),

$$\Phi_{xy}(e^{j\omega}) = \sigma_x^2 H(e^{j\omega}). \tag{2.204}$$

In other words, the cross power spectrum is in this case proportional to the frequency response of the system. Equations (2.202) and (2.204) may serve as the basis for estimating the impulse response or frequency response of a linear time-invariant system if it is possible to observe the output of the system in response to a white-noise input.

2.11 SUMMARY

In this chapter, we have considered a number of basic definitions relating to discrete-time signals and systems. We considered the definition of a set of basic sequences, the definition and representation of linear time-invariant systems in terms of the convolution sum, and some implications of stability and causality. The class of systems for which the input and output satisfy a linear constant-coefficient difference equation with initial rest conditions was shown to be an important subclass of linear time-invariant systems. The recursive solution of such difference equations was discussed and the classes of FIR and IIR systems defined.

An important means for the analysis and representation of linear time-invariant systems lies in their frequency-domain representation. The response of a system to a complex exponential input was considered, leading to the definition of the frequency response. The relation between impulse response and frequency response was then interpreted as a Fourier transform pair.

We called attention to many properties of Fourier transform representations and discussed a variety of useful Fourier transform pairs. Tables 2.1 and 2.2 summarize the properties and theorems, and Table 2.3 contains some useful Fourier transform pairs.

The chapter concludes with an introduction to discrete-time random signals. These basic ideas and results will be developed further and used in later chapters.

Although the material in this chapter was presented without direct reference to continuous-time signals, an important class of discrete-time signal-processing problems arises from sampling such signals. In Chapter 4 we consider the relationship between continuous-time signals and sequences obtained by periodic sampling.

PROBLEMS

Basic Problems with Answers

2.1. For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, (4) time invariant, and (5) memoryless:

(a) $T(x[n]) = g[n]x[n]$ with $g[n]$ given
(b) $T(x[n]) = \sum_{k=n_0}^{n} x[k]$
(c) $T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$
(d) $T(x[n]) = x[n - n_0]$
(e) $T(x[n]) = e^{x[n]}$
(f) $T(x[n]) = ax[n] + b$
(g) $T(x[n]) = x[-n]$
(h) $T(x[n]) = x[n] + 3u[n + 1]$

2.2. **(a)** The impulse response $h[n]$ of a linear time-invariant system is known to be zero, except in the interval $N_0 \leq n \leq N_1$. The input $x[n]$ is known to be zero, except in the interval $N_2 \leq n \leq N_3$. As a result, the output is constrained to be zero, except in some interval $N_4 \leq n \leq N_5$. Determine N_4 and N_5 in terms of N_0, N_1, N_2, and N_3.
(b) If $x[n]$ is zero, except for N consecutive points, and $h[n]$ is zero, except for M consecutive points, what is the maximum number of consecutive points for which $y[n]$ can be nonzero?

2.3. By direct evaluation of the convolution sum, determine the step response of a linear time-invariant system whose impulse response is

$$h[n] = a^{-n}u[-n], \qquad 0 < a < 1.$$

2.4. Consider the linear constant-coefficient difference equation

$$y[n] - \tfrac{3}{4}y[n-1] + \tfrac{1}{8}y[n-2] = 2x[n-1].$$

Determine $y[n]$ for $n \geq 0$ when $x[n] = \delta[n]$ and $y[n] = 0, n < 0$.

2.5. A causal linear time-invariant system is described by the difference equation

$$y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1].$$

(a) Determine the homogeneous response of the system, i.e., the possible outputs if $x[n] = 0$ for all n.
(b) Determine the impulse response of the system.
(c) Determine the step response of the system.

2.6. **(a)** Find the frequency response $H(e^{j\omega})$ of the linear time-invariant system whose input and output satisfy the difference equation

$$y[n] - \tfrac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2].$$

(b) Write a difference equation that characterizes a system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}.$$

2.7. Determine whether each of the following signals is periodic. If the signal is periodic, state its period.
(a) $x[n] = e^{j(\pi n/6)}$
(b) $x[n] = e^{j(3\pi n/4)}$
(c) $x[n] = [\sin(\pi n/5)]/(\pi n)$
(d) $x[n] = e^{j\pi n/\sqrt{2}}$

2.8. An LTI system has impulse response $h[n] = 5(-1/2)^n u[n]$. Use the Fourier transform to find the output of this system when the input is $x[n] = (1/3)^n u[n]$.

2.9. Consider the difference equation

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = \frac{1}{3}x[n-1].$$

(a) What are the impulse response, frequency response, and step response for the causal LTI system satisfying this difference equation.
(b) What is the general form of the homogeneous solution of the difference equation?
(c) Consider a different system satisfying the difference equation that is neither causal nor LTI, but that has $y[0] = y[1] = 1$. Find the response of this system to $x[n] = \delta[n]$.

2.10. Determine the output of a linear time-invariant system if the impulse response $h[n]$ and the input $x[n]$ are as follows:
(a) $x[n] = u[n]$ and $h[n] = a^n u[-n-1]$, with $a > 1$.
(b) $x[n] = u[n-4]$ and $h[n] = 2^n u[-n-1]$.
(c) $x[n] = u[n]$ and $h[n] = (0.5)2^n u[-n]$.
(d) $h[n] = 2^n u[-n-1]$ and $x[n] = u[n] - u[n-10]$
Use your knowledge of linearity and time invariance to minimize the work in Parts (b)–(d).

2.11. Consider an LTI system with frequency response

$$H(e^{j\omega}) = \frac{1 - e^{-j2\omega}}{1 + \frac{1}{2}e^{-j4\omega}}, \qquad -\pi < \omega \leq \pi.$$

Determine the output $y[n]$ for all n if the input $x[n]$ for all n is

$$x[n] = \sin\left(\frac{\pi n}{4}\right).$$

2.12. Consider a system with input $x[n]$ and output $y[n]$ that satisfy the difference equation

$$y[n] = ny[n-1] + x[n].$$

The system is causal and satisfies initial-rest conditions; i.e., if $x[n] = 0$ for $n < n_0$, then $y[n] = 0$ for $n < n_0$.
(a) If $x[n] = \delta[n]$, determine $y[n]$ for all n.
(b) Is the system linear? Justify your answer.
(c) Is the system time invariant? Justify your answer.

2.13. Indicate which of the following discrete-time signals are eigenfunctions of stable, linear time-invariant discrete-time systems:
(a) $e^{j2\pi n/3}$
(b) 3^n
(c) $2^n u[-n-1]$
(d) $\cos(\omega_0 n)$
(e) $(1/4)^n$
(f) $(1/4)^n u[n] + 4^n u[-n-1]$

2.14. A single input–output relationship is given for each of the following three systems:
(a) System A: $x[n] = (1/3)^n$, $y[n] = 2(1/3)^n$.
(b) System B: $x[n] = (1/2)^n$, $y[n] = (1/4)^n$.
(c) System C: $x[n] = (2/3)^n u[n]$, $y[n] = 4(2/3)^n u[n] - 3(1/2)^n u[n]$.
Based on this information, pick the strongest possible conclusion that you can make about each system from the following list of statements:
(i) The system cannot possibly be LTI.
(ii) The system must be LTI.
(iii) The system can be LTI, and there is only one LTI system that satisfies this input–output constraint.
(iv) The system can be LTI, but cannot be uniquely determined from the information in this input–output constraint.
If you chose option (iii) from this list, specify either the impulse response $h[n]$ or the frequency response $H(e^{j\omega})$ for the LTI system.

2.15. Consider the system illustrated in Figure P2.15-1. The ouput of an LTI system with an impulse response $h[n] = \left(\frac{1}{4}\right)^n u[n+10]$ is multiplied by a unit step function $u[n]$ to yield the output of the overall system. Answer each of the following questions, and briefly justify your answers:

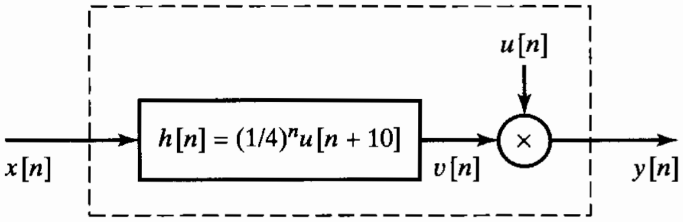


Figure P2.15-1

(a) Is the overall system LTI?
(b) Is the overall system causal?
(c) Is the overall system stable in the BIBO sense?

2.16. Consider the following difference equation:

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n].$$

(a) Determine the general form of the homogeneous solution to this difference equation.
(b) Both a causal and an anticausal LTI system are characterized by this difference equation. Find the impulse responses of the two systems.
(c) Show that the causal LTI system is stable and the anticausal LTI system is unstable.
(d) Find a particular solution to the difference equation when $x[n] = (1/2)^n u[n]$.

2.17. **(a)** Determine the Fourier transform of the sequence

$$r[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right], & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

Sketch $w[n]$ and express $W(e^{j\omega})$, the Fourier transform of $w[n]$, in terms of $R(e^{j\omega})$, the Fourier transform of $r[n]$. (*Hint:* First express $w[n]$ in terms of $r[n]$ and the complex exponentials $e^{j(2\pi n/M)}$ and $e^{-j(2\pi n/M)}$.)
(c) Sketch the magnitude of $R(e^{j\omega})$ and $W(e^{j\omega})$ for the case when $M = 4$.

2.18. For each of the following impulse responses of LTI systems, indicate whether or not the system is causal:
(a) $h[n] = (1/2)^n u[n]$
(b) $h[n] = (1/2)^n u[n-1]$
(c) $h[n] = (1/2)^{|n|}$
(d) $h[n] = u[n+2] - u[n-2]$
(e) $h[n] = (1/3)^n u[n] + 3^n u[-n-1]$

2.19. For each of the following impulse responses of LTI systems, indicate whether or not the system is stable:
(a) $h[n] = 4^n u[n]$
(b) $h[n] = u[n] - u[n-10]$
(c) $h[n] = 3^n u[-n-1]$

(d) $h[n] = \sin(\pi n/3)u[n]$
(e) $h[n] = (3/4)^{|n|}\cos(\pi n/4 + \pi/4)$
(f) $h[n] = 2u[n+5] - u[n] - u[n-5]$

2.20. Consider the difference equation representing a causal LTI system

$$y[n] + (1/a)y[n-1] = x[n-1].$$

(a) Find the impulse response of the system, $h[n]$, as a function of the constant a.
(b) For what range of values of a will the system be stable?

Basic Problems

2.21. Consider an arbitrary linear system with input $x[n]$ and output $y[n]$. Show that if $x[n] = 0$ for all n, then $y[n]$ must also be zero for all n.

2.22. For each of the pairs of sequences in Figure P2.22-1, use discrete convolution to find the response to the input $x[n]$ of the linear time-invariant system with impulse response $h[n]$.

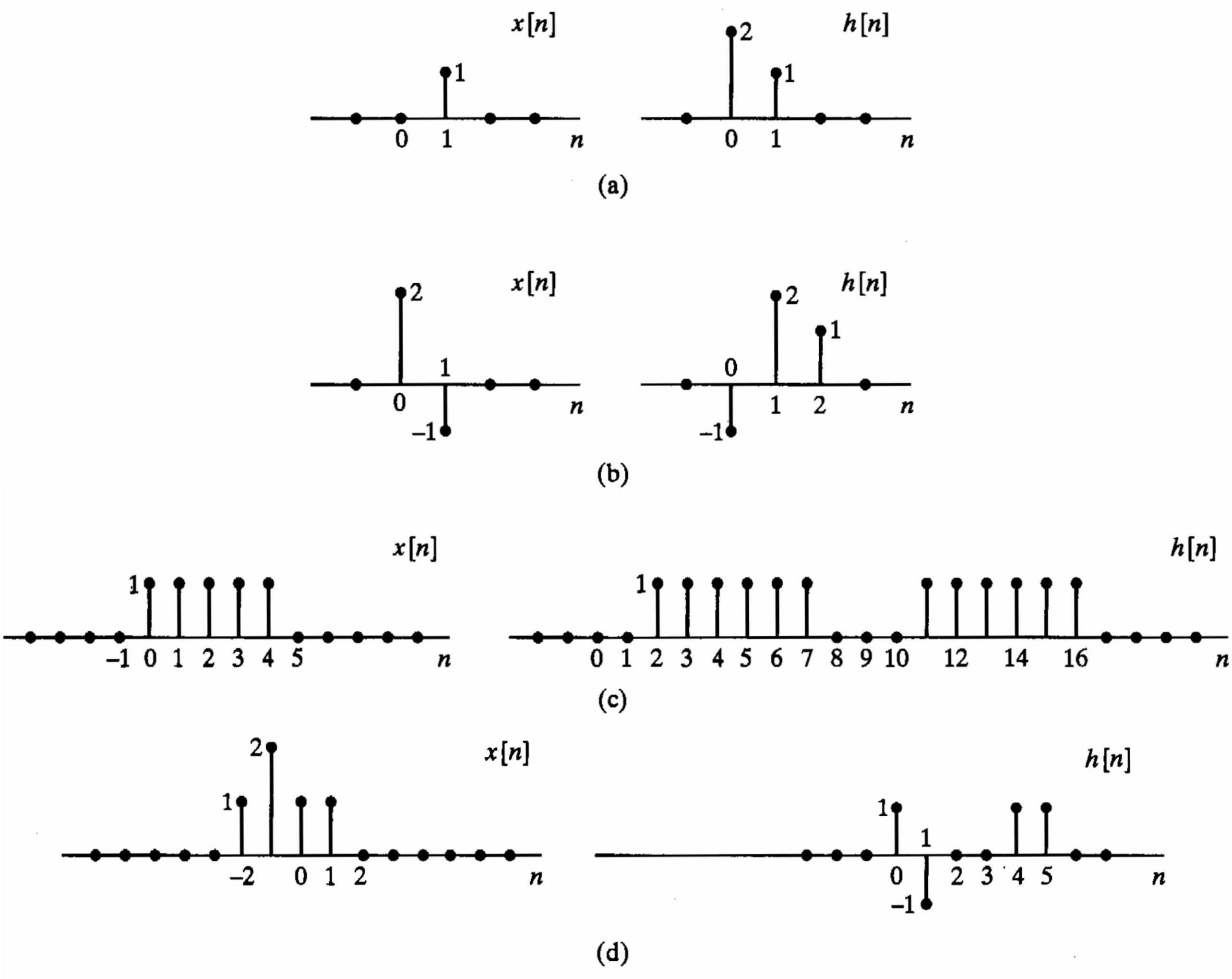


Figure P2.22-1

2.23. Using the definition of linearity (Eqs. (2.26a)–(2.26b)), show that the ideal delay system (Example 2.3) and the moving-average system (Example 2.4) are both linear systems.

2.24. The impulse response of a linear time-invariant system is shown in Figure P2.24-1. Determine and carefully sketch the response of this system to the input $x[n] = u[n-4]$.

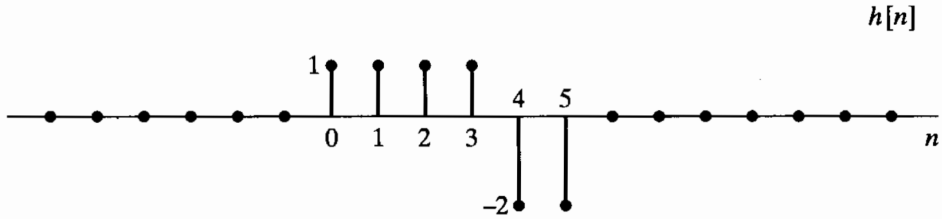


Figure P2.24-1

2.25. A linear time-invariant system has impulse response $h[n] = u[n]$. Determine the response of this system to the input $x[n]$ shown in Figure P2.25-1 and described as

$$x[n] = \begin{cases} 0, & n < 0, \\ a^n, & 0 \le n \le N_1, \\ 0, & N_1 < n < N_2, \\ a^{n-N_2}, & N_2 \le n \le N_2 + N_1, \\ 0, & N_2 + N_1 < n, \end{cases}$$

where $0 < a < 1$.

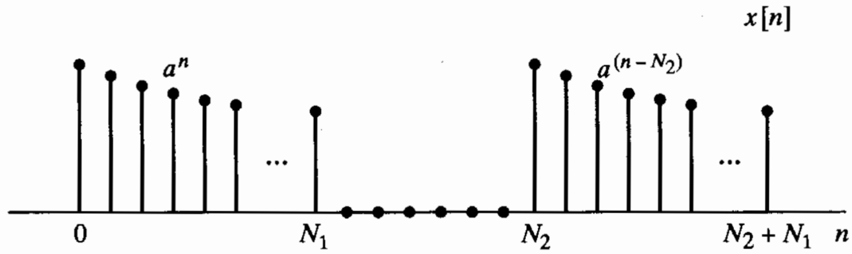


Figure P2.25-1

2.26. Which of the following discrete-time signals could be eigenfunctions of any stable LTI system?

(a) $5^n u[n]$
(b) $e^{j2\omega n}$
(c) $e^{j\omega n} + e^{j2\omega n}$
(d) 5^n
(e) $5^n \cdot e^{j2\omega n}$

2.27. Three systems A, B, and C have the inputs and outputs indicated in Figure P2.27-1. Determine whether each system could be LTI. If your answer is yes, specify whether there could be more than one LTI system with the given input–output pair. Explain your answer.

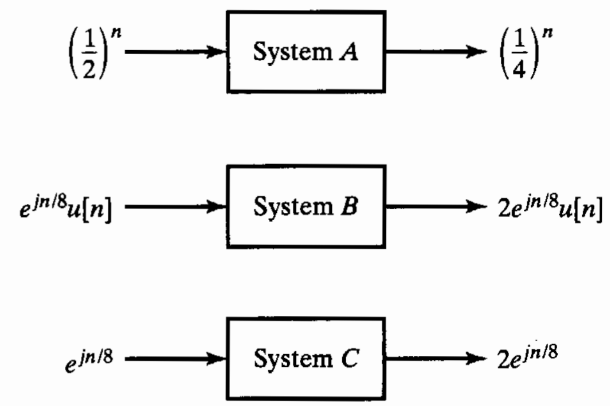


Figure P2.27-1

2.28. Determine which of the following signals is periodic. If a signal is periodic, determine its period.

(a) $x[n] = e^{j(2\pi n/5)}$
(b) $x[n] = \sin(\pi n/19)$
(c) $x[n] = ne^{j\pi n}$
(d) $x[n] = e^{jn}$

2.29. A discrete-time signal $x[n]$ is shown in Figure P2.29-1.

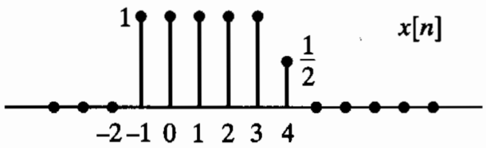


Figure P2.29-1

Sketch and label carefully each of the following signals:

(a) $x[n-2]$
(b) $x[4-n]$
(c) $x[2n]$
(d) $x[n]u[2-n]$
(e) $x[n-1]\delta[n-3]$

2.30. For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time invariant.

(a) $T(x[n]) = (\cos \pi n)x[n]$
(b) $T(x[n]) = x[n^2]$
(c) $T(x[n]) = x[n]\sum_{k=0}^{\infty} \delta[n-k]$
(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

2.31. Consider the difference equation

$$y[n] + \frac{1}{15}y[n-1] - \frac{2}{5}y[n-2] = x[n].$$

(a) Determine the general form of the homogeneous solution to this equation.
(b) Both a causal and an anticausal LTI system are characterized by the given difference equation. Find the impulse responses of the two systems.
(c) Show that the causal LTI system is stable and the anticausal LTI system is unstable.
(d) Find a particular solution to the difference equation when $x[n] = (3/5)^n\, u[n]$.

2.32. Consider an LTI system with frequency response

$$H(e^{j\omega}) = e^{-j(\omega - \frac{\pi}{4})}\left(\frac{1 + e^{-j2\omega} + 4e^{-j4\omega}}{1 + \frac{1}{2}e^{-j2\omega}}\right), \qquad -\pi < \omega \le \pi.$$

Determine the output $y[n]$ for all n if the input for all n is

$$x[n] = \cos\left(\frac{\pi n}{2}\right).$$

2.33. Consider an LTI system with $|H(e^{j\omega})| = 1$, and let $\arg[H(e^{j\omega})]$ be as shown in Figure P2.33-1. If the input is

$$x[n] = \cos\left(\frac{3\pi}{2}n + \frac{\pi}{4}\right),$$

determine the output $y[n]$.

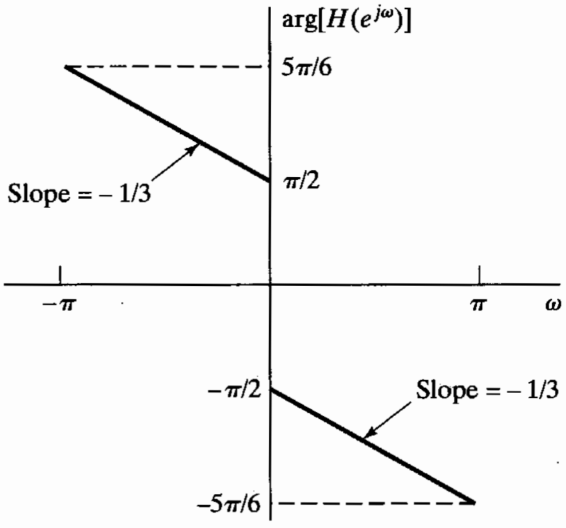


Figure P2.33-1

2.34. The input–output pair shown in Figure P2.34-1 is given for a stable LTI system.

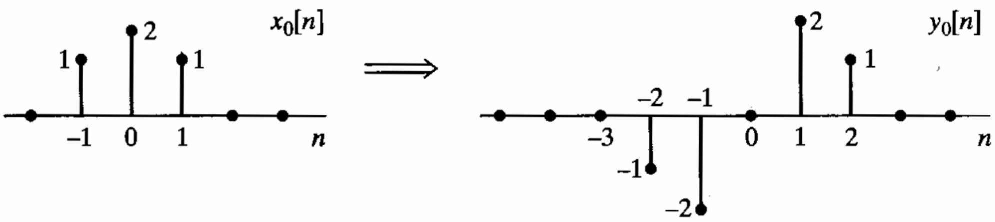


Figure P2.34-1

(a) Determine the response to the input $x_1[n]$ in Figure P2.34-2.

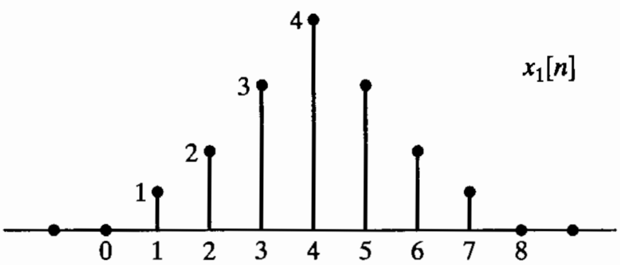


Figure P2.34-2

(b) Determine the impulse response of the system.

Advanced Problems

2.35. The system T in Figure P2.35-1 is known to be *time invariant*. When the inputs to the system are $x_1[n]$, $x_2[n]$, and $x_3[n]$, the responses of the system are $y_1[n]$, $y_2[n]$, and $y_3[n]$, as shown.

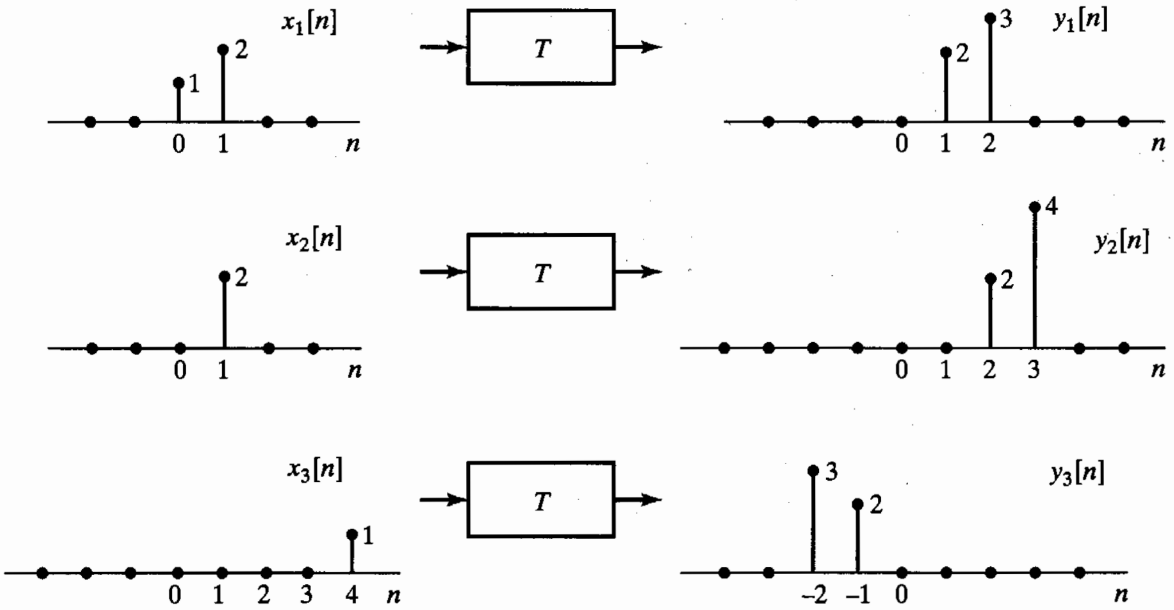


Figure P2.35-1

(a) Determine whether the system T could be linear.
(b) If the input $x[n]$ to the system T is $\delta[n]$, what is the system response $y[n]$?
(c) What are all possible inputs $x[n]$ for which the response of the system T can be determined from the given information alone?

2.36. The system L in Figure P2.36-1 is known to be *linear*. Shown are three output signals $y_1[n]$, $y_2[n]$, and $y_3[n]$ in response to the input signals $x_1[n]$, $x_2[n]$, and $x_3[n]$, respectively.

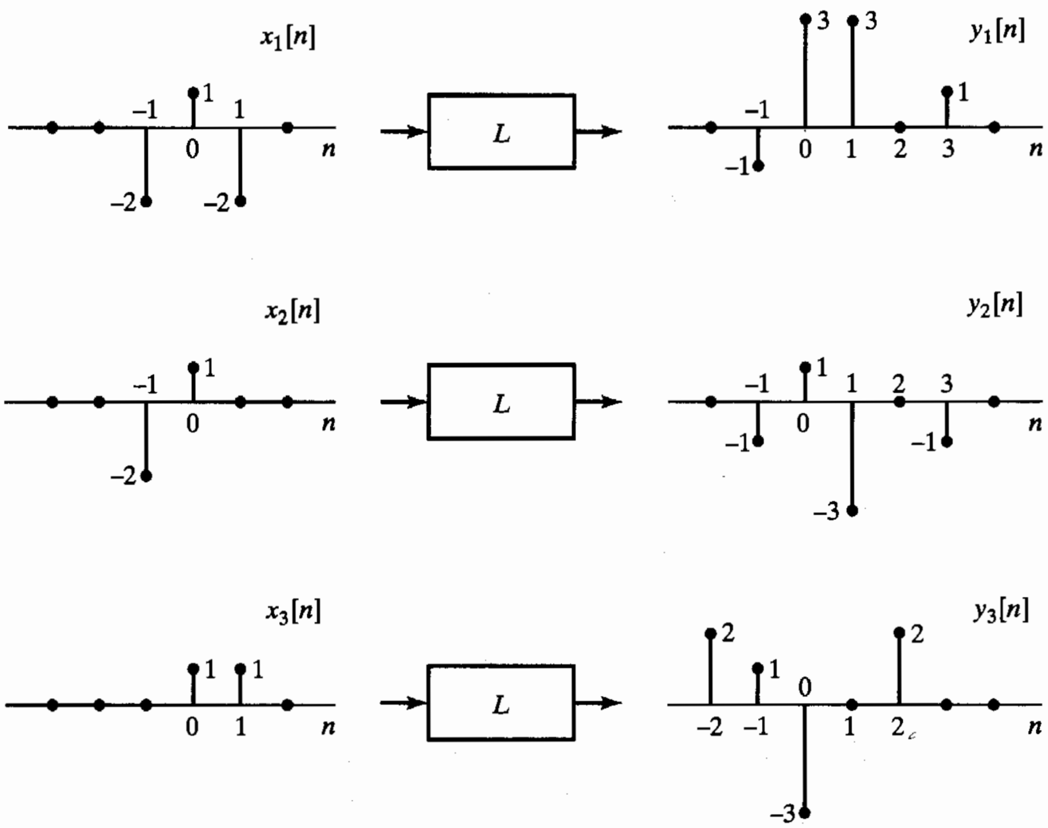


Figure P2.36-1

(a) Determine whether the system L could be time invariant.
(b) If the input $x[n]$ to the system L is $\delta[n]$, what is the system response $y[n]$?

2.37. Consider a discrete-time linear time-invariant system with impulse response $h[n]$. If the input $x[n]$ is a periodic sequence with period N (i.e., if $x[n] = x[n+N]$), show that the output $y[n]$ is also a periodic sequence with period N.

2.38. In Section 2.5, we stated that the solution to the homogeneous difference equation

$$\sum_{k=0}^{N} a_k y_h[n-k] = 0 \tag{P2.38-1}$$

is of the form

$$y_h[n] = \sum_{m=1}^{N} A_m z_m^n, \tag{P2.38-2}$$

with the A_m's arbitrary and the z_m's the N roots of the polynomial

$$\sum_{k=0}^{N} a_k z^{-k} = 0; \tag{P2.38-3}$$

i.e.,

$$\sum_{k=0}^{N} a_k z^{-k} = \prod_{m=1}^{N} (1 - z_m z^{-1}). \tag{P2.38-4}$$

(a) Determine the general form of the homogeneous solution to the difference equation

$$y[n] - \tfrac{3}{4}y[n-1] + \tfrac{1}{8}y[n-2] = 2x[n-1]. \tag{P2.38-5}$$

(b) Determine the coefficients A_m in the homogeneous solution if $y[-1] = 1$ and $y[0] = 0$.
(c) Now consider the difference equation

$$y[n] - y[n-1] + \tfrac{1}{4}y[n-2] = 2y[n-1]. \tag{P2.38-6}$$

If the homogeneous solution contains only terms of the form of Eq. (P2.38-2), show that the initial conditions $y[-1] = 1$ and $y[0] = 0$ cannot be satisfied.
(d) If Eq. (P2.38-3) has two roots that are identical, then, in place of Eq. (P2.38-2), $y_h[n]$ will take the form

$$y_h[n] = \sum_{m=1}^{N-1} A_m z_m^n + nB_1 z_1^n, \tag{P2.38-7}$$

where we have assumed that the double root is z_1. Using Eq. (P2.38-7), determine the general form of $y_h[n]$ for Eq. (P2.38-6). Verify explicitly that your answer satisfies Eq. (P2.38-6) with $x[n] = 0$.
(e) Determine the coefficients A_1 and B_1 in the homogeneous solution obtained in Part (d) if $y[-1] = 1$ and $y[0] = 0$.

2.39. Consider a system with input $x[n]$ and output $y[n]$. The input–output relation for the system is defined by the following two properties:

1. $y[n] - ay[n-1] = x[n]$,
2. $y[0] = 1$.

(a) Determine whether the system is time invariant.
(b) Determine whether the system is linear.

(c) Assume that the difference equation (property 1) remains the same, but the value $y[0]$ is specified to be zero. Does this change your answer to either Part (a) or Part (b)?

2.40. Consider the linear time-invariant system with impulse response

$$h[n] = \left(\frac{j}{2}\right)^n u[n], \qquad \text{where } j = \sqrt{-1}.$$

Determine the steady-state response, i.e., the response for large n, to the excitation

$$x[n] = \cos(\pi n)u[n].$$

2.41. A linear time-invariant system has frequency response

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega 3}, & |\omega| < \frac{2\pi}{16}\left(\frac{3}{2}\right), \\ 0, & \frac{2\pi}{16}\left(\frac{3}{2}\right) \le |\omega| \le \pi. \end{cases}$$

The input to the system is a periodic unit-impulse train with period $N = 16$; i.e.,

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k].$$

Find the output of the system.

2.42. Consider the system in Figure P2.42-1.

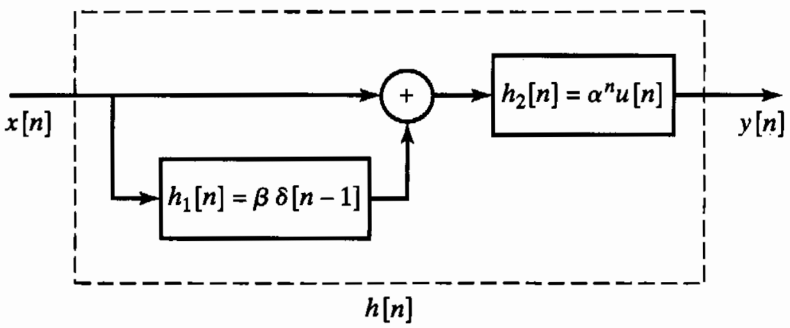


Figure P2.42-1

(a) Find the impulse response $h[n]$ of the overall system.
(b) Find the frequency response of the overall system.
(c) Specify a difference equation that relates the output $y[n]$ to the input $x[n]$.
(d) Is this system causal? Under what condition would the system be stable?

2.43. For $X(e^{j\omega}) = 1/(1 - ae^{-j\omega})$, with $-1 < a < 0$, determine and sketch the following as a function of ω:
(a) $\mathcal{R}e\{X(e^{j\omega})\}$
(b) $\mathcal{J}m\{X(e^{j\omega})\}$
(c) $|X(e^{j\omega})|$
(d) $\sphericalangle X(e^{j\omega})$

2.44. Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ shown in Figure P2.44-1. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:

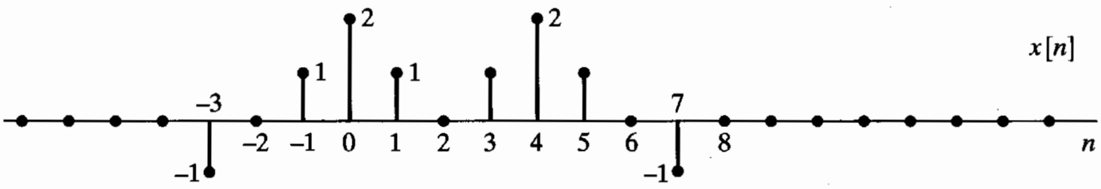
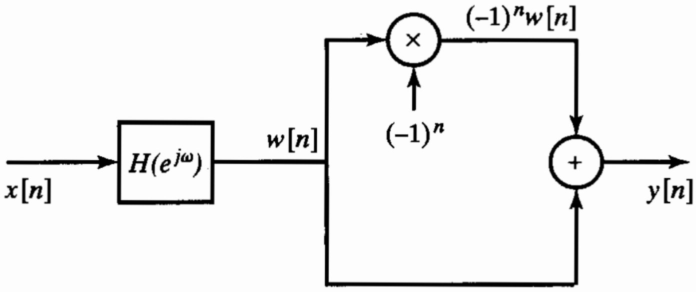


Figure P2.44-1

(a) Evaluate $X(e^{j\omega})|_{\omega=0}$.
(b) Evaluate $X(e^{j\omega})|_{\omega=\pi}$.
(c) Find $\sphericalangle X(e^{j\omega})$.
(d) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.
(e) Determine and sketch the signal whose Fourier transform is $X(e^{-j\omega})$.
(f) Determine and sketch the signal whose Fourier transform is $\mathcal{R}e\{X(e^{j\omega})\}$.

2.45. For the system in Figure P2.45-1, determine the output $y[n]$ when the input $x[n]$ is $\delta[n]$ and $H(e^{j\omega})$ is an ideal lowpass filter as indicated, i.e.,

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/2, \\ 0, & \pi/2 < |\omega| \le \pi. \end{cases}$$



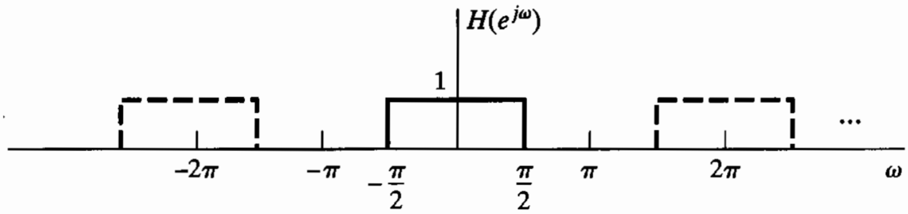


Figure P2.45-1

2.46. A sequence has the discrete-time Fourier transform

$$X(e^{j\omega}) = \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})}, \qquad |a| < 1.$$

(a) Find the sequence $x[n]$.
(b) Calculate $\int_{-\pi}^{\pi} X(e^{j\omega})\cos(\omega)d\omega/2\pi$.

2.47. A linear time-invariant system is described by the input–output relation

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

(a) Determine $h[n]$, the impulse response of the system.
(b) Is this a stable system?
(c) Determine $H(e^{j\omega})$, the frequency response of the system. Use trigonometric identities to obtain a simple expression for $H(e^{j\omega})$.

(d) Plot the magnitude and phase of the frequency response.
(e) Now consider a new system whose frequency response is $H_1(e^{j\omega}) = H(e^{j(\omega+\pi)})$. Determine $h_1[n]$, the impulse response of the new system.

2.48. Let the real discrete-time signal $x[n]$ with Fourier transform $X(e^{j\omega})$ be the input to a system with the output defined by

$$y[n] = \begin{cases} x[n], & \text{if } n \text{ is even,} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the discrete-time signal $s[n] = 1 + \cos(\pi n)$ and its (generalized) Fourier transform $S(e^{j\omega})$.
(b) Express $Y(e^{j\omega})$, the Fourier transform of the output, as a function of $X(e^{j\omega})$ and $S(e^{j\omega})$.
(c) You would like to approximate $x[n]$ by the interpolated signal $w[n] = y[n] + (1/2)(y[n+1] + y[n-1])$. Determine the Fourier transform $W(e^{j\omega})$ as a function of $Y(e^{j\omega})$.
(d) Sketch $X(e^{j\omega})$, $Y(e^{j\omega})$, and $W(e^{j\omega})$ for the case when $x[n] = \sin(\pi n/a)/(\pi n/a)$ and $a > 1$. Under what conditions is the proposed interpolated signal $w[n]$ a good approximation for the original $x[n]$.

2.49. Consider a discrete-time LTI system with frequency response $H(e^{j\omega})$ and corresponding impulse response $h[n]$.
(a) We are first given the following three clues about the system:
(i) The system is causal.
(ii) $H(e^{j\omega}) = H^*(e^{-j\omega})$.
(iii) The DTFT of the sequence $h[n+1]$ is real.
Using these three clues, show that the system has an impulse response of finite duration.
(b) In addition to the preceding three clues, we are now given two more clues:
(iv) $\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} H(e^{j\omega})d\omega = 2.$
(v) $H(e^{j\pi}) = 0.$
Is there enough information to identify the system uniquely? If so, determine the impulse response $h[n]$. If not, specify as much as you can about the sequence $h[n]$.

2.50. Consider the three sequences

$$\begin{aligned} v[n] &= u[n] - u[n-6], \\ w[n] &= \delta[n] + 2\delta[n-2] + \delta[n-4], \\ q[n] &= v[n] * w[n]. \end{aligned}$$

(a) Find and sketch the sequence $q[n]$.
(b) Find and sketch the sequence $r[n]$ such that $r[n] * v[n] = \sum_{k=-\infty}^{n-1} q[k]$.
(c) Is $q[-n] = v[-n] * w[-n]$? Justify your answer.

2.51. A linear time-invariant system has impulse response $h[n] = a^n u[n]$.
(a) Determine $y_1[n]$, the response of the system to the input $x_1[n] = e^{j(\pi/2)n}$.
(b) Use the result of Part (a) to help to determine $y_2[n]$, the response of the system to the input $x_2[n] = \cos(\pi n/2)$.
(c) Determine $y_3[n]$, the response of the system to the input $x_3[n] = e^{j(\pi/2)n}u[n]$.
(d) Compare $y_3[n]$ with $y_1[n]$ for large n.

2.52. The frequency response of an LTI system is

$$H(e^{j\omega}) = e^{-j\omega/4}, \qquad -\pi < \omega \le \pi.$$

Determine the output of the system, $y[n]$, when the input is $x[n] = \cos(5\pi n/2)$. Express your answer in as simple a form as you can.

2.53. Consider the cascade of LTI discrete-time systems shown in Figure P2.53-1.

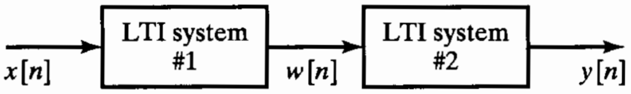


Figure P2.53-1

The first system is described by the equation

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.5\pi, \\ 0, & 0.5\pi \le |\omega| < \pi, \end{cases}$$

and the second system is described by the equation

$$y[n] = w[n] - w[n-1].$$

The input to this system is

$$x[n] = \cos(0.6\pi n) + 3\delta[n-5] + 2.$$

Determine the output $y[n]$. With careful thought, you will be able to use the properties of LTI systems to write down the answer by inspection.

2.54. Consider an LTI system with frequency response

$$H(e^{j\omega}) = e^{-j[(\omega/2)+(\pi/4)]}, \qquad -\pi < \omega \le \pi.$$

Determine $y[n]$, the output of this system, if the input is

$$x[n] = \cos\left(\frac{15\pi n}{4} - \frac{\pi}{3}\right)$$

for all n.

2.55. For the system shown in Figure P2.55-1, System 1 is a memoryless nonlinear system. System 2 determines the value of A according to the relation

$$A = \sum_{n=0}^{100} y[n].$$

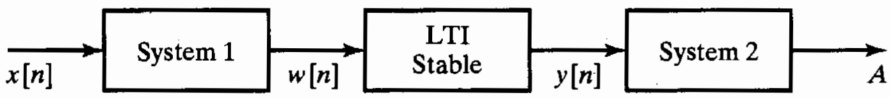


Figure P2.55-1

Specifically, consider the class of inputs of the form $x[n] = \cos(\omega n)$, with ω a real finite number. Varying the value of ω at the input will change A; i.e., A will be a function of ω. In general, will A be periodic in ω? Justify your answer.

2.56. Consider a system S with input $x[n]$ and output $y[n]$ related according to the block diagram in Figure P2.56-1.

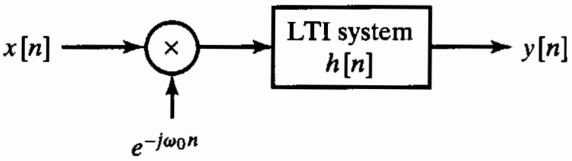


Figure P2.56-1

The input $x[n]$ is multiplied by $e^{-j\omega_0 n}$, and the product is passed through a stable LTI system with impulse response $h[n]$.

(a) Is the system S linear? Justify your answer.

(b) Is the system S time invariant? Justify your answer.

(c) Is the system S stable? Justify your answer.

(d) Specify a system C such that the block diagram in Figure P2.56-2 represents an alternative way of expressing the input–output relationship of the system S. (*Note:* The system C does not have to be an LTI system.)

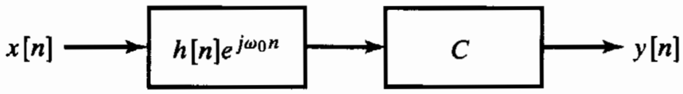


Figure P2.56-2

2.57. An ideal lowpass filter with zero delay has impulse response $h_{\text{lp}}[n]$ and frequency response

$$H_{\text{lp}}(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.2\pi, \\ 0, & 0.2\pi \le |\omega| \le \pi. \end{cases}$$

(a) A new filter is defined by the equation $h_1[n] = (-1)^n h_{\text{lp}}[n] = e^{j\pi n} h_{\text{lp}}[n]$. Determine an equation for the frequency response of $H_1(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(b) A second filter is defined by the equation $h_2[n] = 2h_{\text{lp}}[n]\cos(0.5\pi n)$. Determine the equation for the frequency response $H_2(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

(c) A third filter is defined by the equation

$$h_3[n] = \frac{\sin(0.1\pi n)}{\pi n} h_{\text{lp}}[n].$$

Determine the equation for the frequency response $H_3(e^{j\omega})$, and plot the equation for $|\omega| < \pi$. What kind of filter is this?

2.58. The LTI system

$$H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0, \end{cases}$$

is referred to as a 90° phase shifter and is used to generate what is referred to as an analytic signal $w[n]$ as shown in Figure P2.58-1. Specifically, the analytic signal $w[n]$ is a complex-valued signal for which

$$\mathcal{Re}\{w[n]\} = x[n],$$

$$\mathcal{Im}\{w[n]\} = y[n].$$

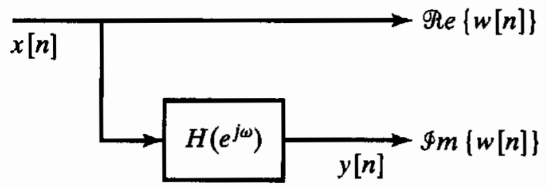


Figure P2.58-1

If $X(e^{j\omega})$ is as shown in Figure P2.58-2, determine and sketch $W(e^{j\omega})$, the Fourier transform of the analytic signal $w[n] = x[n] + jy[n]$.

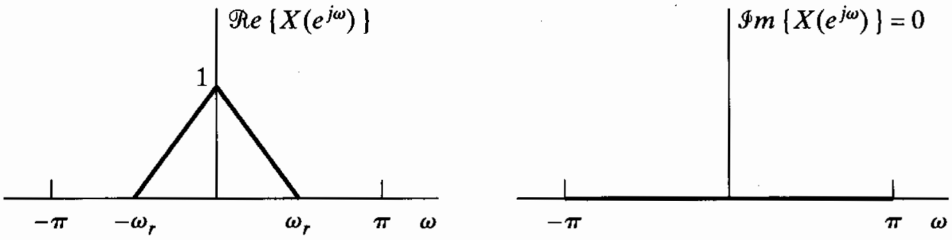


Figure P2.58-2

2.59. The autocorrelation sequence of a signal $x[n]$ is defined as

$$R_x[n] = \sum_{k=-\infty}^{\infty} x^*[k]x[n+k].$$

(a) Show that for an appropriate choice of the signal $g[n]$, $R_x[n] = x[n] * g[n]$, and identify the proper choice for $g[n]$.

(b) Show that the Fourier transform of $R_x[n]$ is equal to $|X(e^{j\omega})|^2$.

2.60. The signals $x[n]$ and $y[n]$ shown in Figure P2.60-1 are the input and corresponding output for an LTI system.

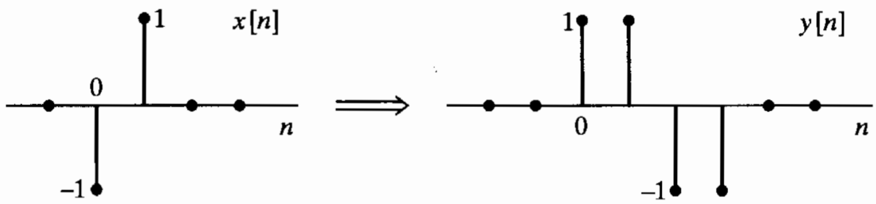


Figure P2.60-1

(a) Find the response of the system to the sequence $x_2[n]$ in Figure P2.60-2.

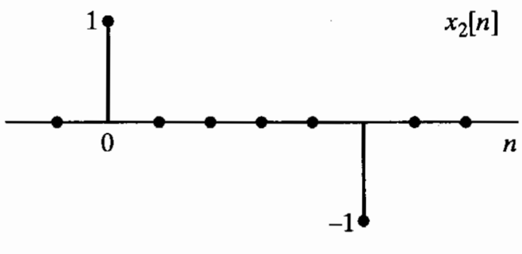


Figure P2.60-2

(b) Find the impulse response $h[n]$ for this LTI system.

2.61. Consider a system for which the input $x[n]$ and output $y[n]$ satisfy the difference equation

$$y[n] - \tfrac{1}{2}y[n-1] = x[n]$$

and for which $y[-1]$ is constrained to be zero for every input. Determine whether or not the system is stable. If you conclude that the system is stable, show your reasoning. If you conclude that the system is **not** stable, give an example of a bounded input that results in an unbounded output.

Extension Problems

2.62. The causality of a system was defined in Section 2.2.4. From this definition, show that, for a linear time-invariant system, causality implies that the impulse response $h[n]$ is zero for $n < 0$. One approach is to show that if $h[n]$ is *not* zero for $n < 0$, then the system *cannot* be causal. Show also that if the impulse response is zero for $n < 0$, then the system will necessarily be causal.

2.63. Consider a discrete-time system with input $x[n]$ and output $y[n]$. When the input is

$$x[n] = \left(\frac{1}{4}\right)^n u[n],$$

the output is

$$y[n] = \left(\frac{1}{2}\right)^n \qquad \text{for all } n.$$

Determine which of the following statements is correct:

- The system must be LTI.
- The system could be LTI.
- The system cannot be LTI.

If your answer is that the system must or could be LTI, give a possible impulse response. If your answer is that the system could not be LTI, explain clearly why not.

2.64. Consider an LTI system whose frequency response is

$$H(e^{j\omega}) = e^{-j\omega/2}, \qquad |\omega| < \pi.$$

Determine whether or not the system is causal. Show your reasoning.

2.65. In Figure P2.65-1, two sequences $x_1[n]$ and $x_2[n]$ are shown. Both sequences are zero for all n outside the regions shown. The Fourier transforms of these sequences are $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$, which, in general, can be expected to be complex and can be written in the form

$$X_1(e^{j\omega}) = A_1(\omega)e^{j\theta_1(\omega)},$$
$$X_2(e^{j\omega}) = A_2(\omega)e^{j\theta_2(\omega)},$$

where $A_1(\omega)$, $\theta_1(\omega)$, $A_2(\omega)$, and $\theta_2(\omega)$ are all real functions chosen so that both $A_1(\omega)$ and $A_2(\omega)$ are nonnegative at $\omega = 0$, but otherwise can take on both positive and negative values. Determine appropriate choices for $\theta_1(\omega)$ and $\theta_2(\omega)$, and sketch these two phase functions in the range $0 < \omega < 2\pi$.

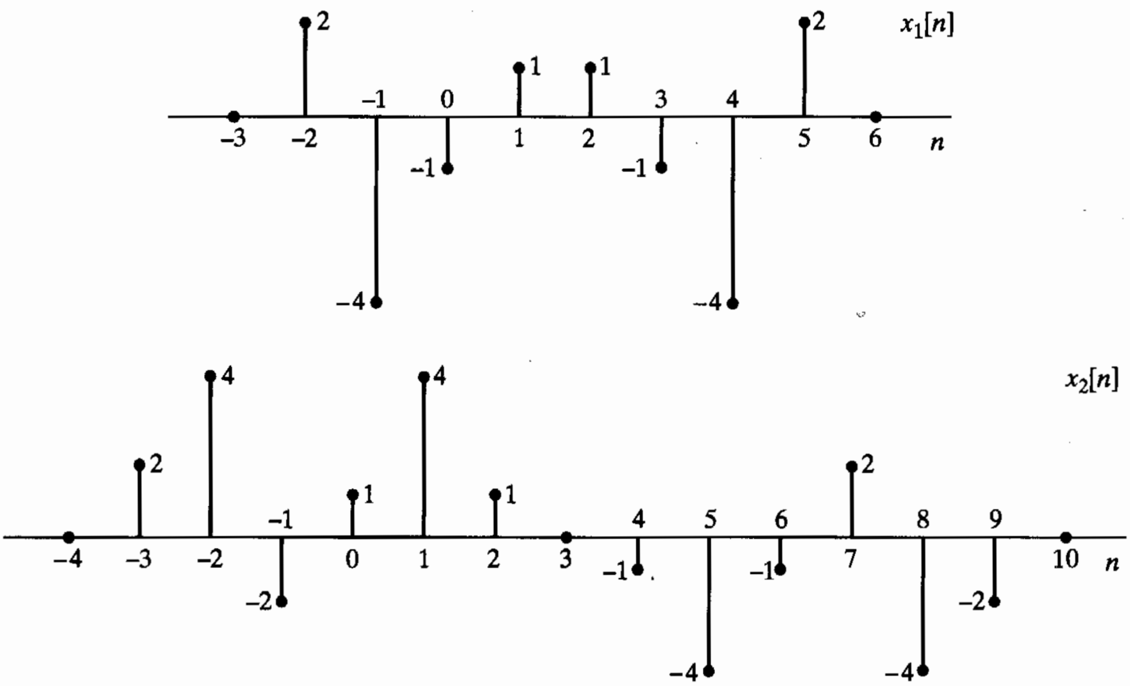


Figure P2.65-1

2.66. Consider the cascade of discrete-time systems in Figure P1.66-1. The time-reversal systems are defined by the equations $f[n] = e[-n]$ and $y[n] = g[-n]$. Assume throughout the problem that $x[n]$ and $h_1[n]$ are real sequences.

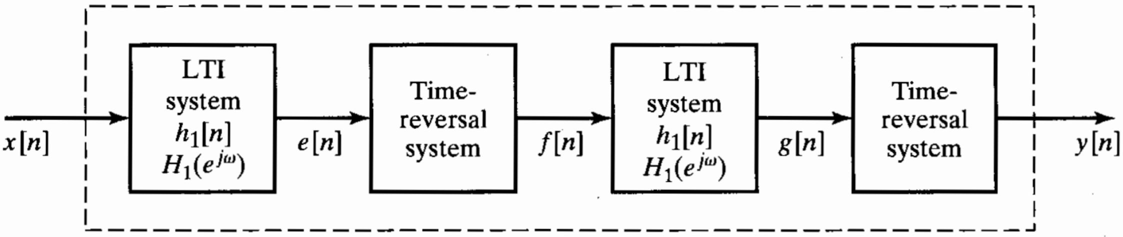


Figure P2.66-1

(a) Express $E(e^{j\omega})$, $F(e^{j\omega})$, $G(e^{j\omega})$, and $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$ and $H_1(e^{j\omega})$.

(b) The result from Part (a) should convince you that the overall system is LTI. Find the frequency response $H(e^{j\omega})$ of the overall system.

(c) Determine an expression for the impulse response $h[n]$ of the overall system in terms of $h_1[n]$.

2.67. The overall system in the dotted box in Figure P1.67-1 can be shown to be linear and time invariant.

(a) Determine an expression for $H(e^{j\omega})$, the frequency response of the overall system from the input $x[n]$ to the output $y[n]$, in terms of $H_1(e^{j\omega})$, the frequency response of the internal LTI system. Remember that $(-1)^n = e^{j\pi n}$.

(b) Plot $H(e^{j\omega})$ for the case when the frequency response of the internal LTI system is

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \le \pi. \end{cases}$$

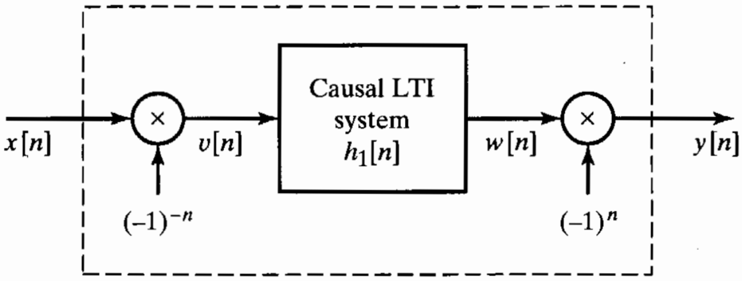


Figure P2.67-1

2.68. Figure P1.68-1 shows the input–output relationships of Systems A and B, while Figure P1.68-2 contains two possible cascade combinations of these systems.

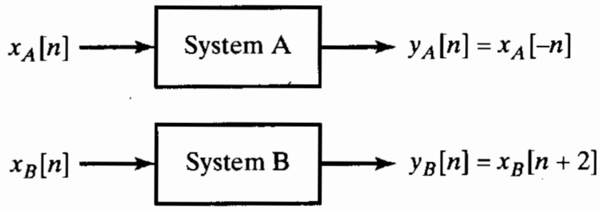


Figure P2.68-1

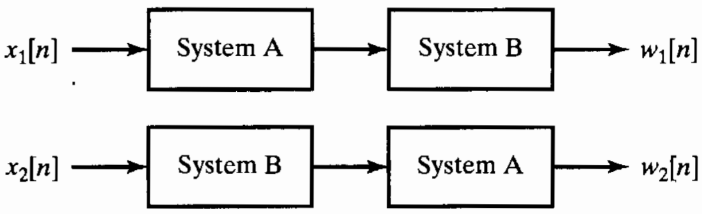


Figure P2.68-2

If $x_1[n] = x_2[n]$, will $w_1[n]$ and $w_2[n]$ necessarily be equal? If your answer is *yes,* clearly and concisely explain why and demonstrate with an example. If your answer is *not necessarily,* demonstrate with a counterexample.

2.69. Consider the system in Figure P2.69-1, where the subsystems S_1 and S_2 are LTI.

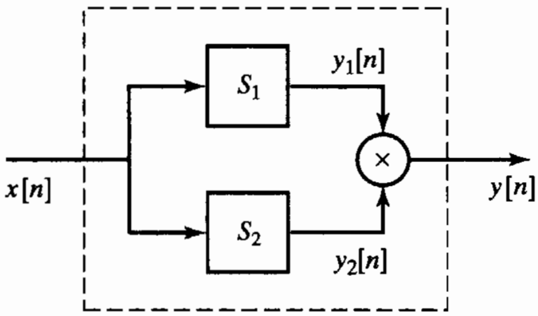


Figure P2.69-1

(a) Is the overall system enclosed by the dashed box, with input $x[n]$ and output $y[n]$ equal to the product of $y_1[n]$ and $y_2[n]$, guaranteed to be an LTI system? If so, explain your reasoning. If not, provide a counterexample.

(b) Suppose S_1 and S_2 have frequency responses $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ that are known to be zero over certain regions. Let

$$H_1(e^{j\omega}) = \begin{cases} 0, & |\omega| \le 0.2\pi, \\ \text{unspecified}, & 0.2\pi < |\omega| \le \pi, \end{cases}$$

$$H_2(e^{j\omega}) = \begin{cases} \text{unspecified}, & |\omega| \le 0.4\pi, \\ 0, & 0.4\pi < |\omega| \le \pi. \end{cases}$$

Suppose also that the input $x[n]$ is known to be bandlimited to 0.3π, i.e.,

$$X(e^{j\omega}) = \begin{cases} \text{unspecified}, & |\omega| < 0.3\pi, \\ 0, & 0.3\pi \le |\omega| \le \pi. \end{cases}$$

Over what region of $-\pi \le \omega < \pi$ is $Y(e^{j\omega})$, the DTFT of $y[n]$, guaranteed to be zero?

2.70. A commonly used numerical operation called the *first backward difference* is defined as

$$y[n] = \nabla(x[n]) = x[n] - x[n-1],$$

where $x[n]$ is the input and $y[n]$ is the output of the first-backward-difference system.

(a) Show that this system is linear and time invariant.

(b) Find the impulse response of the system.

(c) Find and sketch the frequency response (magnitude and phase).

(d) Show that if

$$x[n] = f[n] * g[n],$$

then

$$\nabla(x[n]) = \nabla(f[n]) * g[n] = f[n] * \nabla(g[n]),$$

where $*$ denotes discrete convolution.

(e) Find the impulse response of a system that could be cascaded with the first-difference system to recover the input; i.e., find $h_i[n]$, where

$$h_i[n] * \nabla(x[n]) = x[n].$$

2.71. Let $H(e^{j\omega})$ denote the frequency response of an LTI system with impulse response $h[n]$, where $h[n]$ is, in general, complex.

(a) Using Eq. (2.109), show that $H^*(e^{-j\omega})$ is the frequency response of a system with impulse response $h^*[n]$, where * denotes complex conjugation.

(b) Show that if $h[n]$ is real, the frequency response is conjugate symmetric, i.e., $H(e^{-j\omega}) = H^*(e^{j\omega})$.

2.72. Let $X(e^{j\omega})$ denote the Fourier transform of $x[n]$. Using the Fourier transform synthesis or analysis equations (Eqs. (2.133) and (2.134)), show that

(a) the Fourier transform of $x^*[n]$ is $X^*(e^{-j\omega})$,

(b) the Fourier transform of $x^*[-n]$ is $X^*(e^{j\omega})$.

2.73. Show that for $x[n]$ real, property 7 in Table 2.1 follows from property 1 and that properties 8–11 follow from property 7.

2.74. In Section 2.9, we stated a number of Fourier transform theorems without proof. Using the Fourier synthesis or analysis equations (Eqs. (2.133) and (2.134)), demonstrate the validity of Theorems 1–5 in Table 2.2.

2.75. In Section 2.9.6, it was argued intuitively that

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}), \tag{P2.75-1}$$

when $Y(e^{j\omega})$, $H(e^{j\omega})$, and $X(e^{j\omega})$ are, respectively, the Fourier transforms of the output $y[n]$, impulse response $h[n]$, and input $x[n]$ of a linear time-invariant system; i.e.,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]. \tag{P2.75-2}$$

Verify Eq. (P2.75-1) by applying the Fourier transform to the convolution sum given in Eq. (P2.75-2).

2.76. By applying the Fourier synthesis equation (Eq. (2.133)) to Eq. (2.172) and using Theorem 3 in Table 2.2, demonstrate the validity of the modulation theorem (Theorem 7, Table 2.2).

2.77. Let $x[n]$ and $y[n]$ denote complex sequences and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective Fourier transforms.

(a) By using the convolution theorem (Theorem 6 in Table 2.2) and appropriate properties from Table 2.2, determine, in terms of $x[n]$ and $y[n]$, the sequence whose Fourier transform is $X(e^{j\omega})Y^*(e^{j\omega})$.

(b) Using the result in Part (a), show that

$$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega. \tag{P2.77-1}$$

Equation (P2.77-1) is a more general form of Parseval's theorem, as given in Section 2.9.5.

(c) Using Eq. (P2.77-1), determine the numerical value of the sum

$$\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/4)}{2\pi n}\frac{\sin(\pi n/6)}{5\pi n}.$$

2.78. Let $x[n]$ and $X(e^{j\omega})$ represent a sequence and its Fourier transform, respectively. Determine, in terms of $X(e^{j\omega})$, the transforms of $y_s[n]$, $y_d[n]$, and $y_e[n]$. In each case, sketch $Y(e^{j\omega})$ for $X(e^{j\omega})$ as shown in Figure P2.78-1.

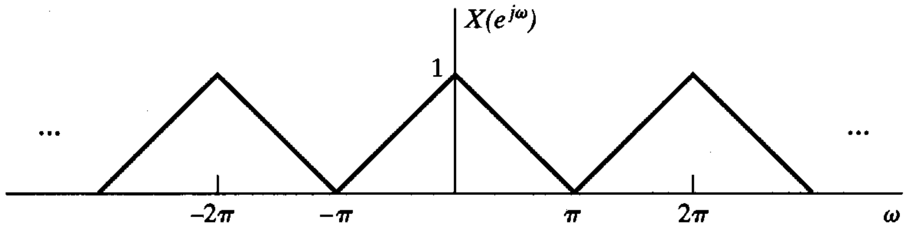


Figure P2.78-1

(a) Sampler:

$$y_s[n] = \begin{cases} x[n], & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

Note that $y_s[n] = \frac{1}{2}\{x[n] + (-1)^n x[n]\}$ and $-1 = e^{j\pi}$.

(b) Compressor:

$$y_d[n] = x[2n].$$

(c) Expander:

$$y_e[n] = \begin{cases} x[n/2], & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

2.79. The two-frequency correlation function $\Phi_x(N, \omega)$ is often used in radar and sonar to evaluate the frequency and travel-time resolution of a signal. For discrete-time signals, we define

$$\Phi_x(N, \omega) = \sum_{n=-\infty}^{\infty} x[n+N]x^*[n-N]e^{-j\omega n}.$$

(a) Show that

$$\Phi_x(-N, -\omega) = \Phi_x^*(N, \omega).$$

(b) If

$$x[n] = Aa^n u[n], \qquad 0 < a < 1,$$

find $\Phi_x(N, \omega)$. (Assume that $N \geq 0$.)

(c) The function $\Phi_x(N, \omega)$ has a frequency domain dual. Show that

$$\Phi_x(N, \omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{j[v+(\omega/2)]}\right) X^*\left(e^{j[v-(\omega/2)]}\right) e^{j2vN} dv.$$

2.80. Let $x[n]$ and $y[n]$ be stationary, uncorrelated random signals. Show that if

$$w[n] = x[n] + y[n],$$

then

$$m_w = m_x + m_y \quad \text{and} \quad \sigma_w^2 = \sigma_x^2 + \sigma_y^2.$$

2.81. Let $e[n]$ denote a white-noise sequence, and let $s[n]$ denote a sequence that is uncorrelated with $e[n]$. Show that the sequence

$$y[n] = s[n]e[n]$$

is white, i.e., that

$$E\{y[n]y[n+m]\} = A\delta[m],$$

where A is a constant.

2.82. Consider a random signal $x[n] = s[n] + e[n]$, where both $s[n]$ and $e[n]$ are independent zero-mean stationary random signals with autocorrelation functions $\phi_{ss}[m]$ and $\phi_{ee}[m]$ respectively.

(a) Determine expressions for $\phi_{xx}[m]$ and $\Phi_{xx}(e^{j\omega})$.

(b) Determine expressions for $\phi_{xe}[m]$ and $\Phi_{xe}(e^{j\omega})$.

(c) Determine expressions for $\phi_{xs}[m]$ and $\Phi_{xs}(e^{j\omega})$.

2.83. Consider an LTI system with impulse response $h[n] = a^n u[n]$ with $|a| < 1$.

(a) Compute the deterministic autocorrelation function $\phi_{hh}[m]$ for this impulse response.

(b) Determine the energy density function $|H(e^{j\omega})|^2$ for the system.

(c) Use Parseval's theorem to evaluate the integral

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega$$

for the system.

2.84. The input to the first-backward-difference system (Example 2.10) is a zero-mean white-noise signal whose autocorrelation function is $\phi_{xx}[m] = \sigma_x^2\delta[m]$.

(a) Determine and plot the autocorrelation function and the power spectrum of the corresponding output of the system.

(b) What is the average power of the output of the system?

(c) What does this problem tell you about the first backward difference of a noisy signal?

2.85. Let $x[n]$ be a real, stationary, white-noise process, with zero mean and variance σ_x^2. Let $y[n]$ be the corresponding output when $x[n]$ is the input to a linear time-invariant system with impulse response $h[n]$. Show that

(a) $E\{x[n]y[n]\} = h[0]\sigma_x^2$,

(b) $\sigma_y^2 = \sigma_x^2 \sum_{n=-\infty}^{\infty} h^2[n]$.

2.86. Let $x[n]$ be a real stationary white-noise sequence, with zero mean and variance σ_x^2. Let $x[n]$ be the input to the cascade of two causal linear time-invariant discrete-time systems, as shown in Figure P1.86-1.

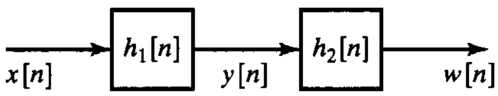


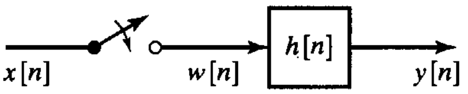
Figure P2.86-1

(a) Is $\sigma_y^2 = \sigma_x^2 \sum_{k=0}^{\infty} h_1^2[k]$?

(b) Is $\sigma_w^2 = \sigma_y^2 \sum_{k=0}^{\infty} h_2^2[k]$?

(c) Let $h_1[n] = a^n u[n]$ and $h_2[n] = b^n u[n]$. Determine the impulse response of the overall system in Figure P1.86-1, and, from this, determine σ_w^2. Are your answers to parts (b) and (c) consistent?

2.87. Sometimes we are interested in the statistical behavior of a linear time-invariant system when the input is a suddenly applied random signal. Such a situation is depicted in Figure P1.87-1.



(switch closed at $n = 0$) **Figure P2.87–1**

Let $x[n]$ be a stationary white-noise process. The input to the system, $w[n]$, given by

$$w[n] = \begin{cases} x[n], & n \geq 0, \\ 0, & n < 0, \end{cases}$$

is a nonstationary process, as is the output $y[n]$.

(a) Derive an expression for the mean of the output in terms of the mean of the input.
(b) Derive an expression for the autocorrelation sequence $\phi_{yy}[n_1, n_2]$ of the output.
(c) Show that, for large n, the formulas derived in parts (a) and (b) approach the results for stationary inputs.
(d) Assume that $h[n] = a^n u[n]$. Find the mean and mean-square values of the output in terms of the mean and mean-square values of the input. Sketch these parameters as a function of n.

2.88. Let $x[n]$ and $y[n]$ respectively denote the input and output of a system. The input–output relation of a system sometimes used for the purpose of noise reduction in images is given by

$$y[n] = \frac{\sigma_s^2[n]}{\sigma_x^2[n]}(x[n] - m_x[n]) + m_x[n],$$

where

$$\sigma_x^2[n] = \frac{1}{3}\sum_{k=n-1}^{n+1}(x[k] - m_x[n])^2,$$

$$m_x[n] = \frac{1}{3}\sum_{k=n-1}^{n+1} x[k],$$

$$\sigma_s^2[n] = \begin{cases} \sigma_x^2[n] - \sigma_w^2, & \sigma_x^2[n] \geq \sigma_w^2, \\ 0, & \text{otherwise}, \end{cases}$$

and σ_w^2 is a known constant proportional to the noise power.
(a) Is the system linear?
(b) Is the system shift invariant?
(c) Is the system stable?
(d) Is the system causal?
(e) For a fixed $x[n]$, determine $y[n]$ when σ_w^2 is very large (large noise power) and when σ_w^2 is very small (small noise power). Does $y[n]$ make sense for these extreme cases?

2.89. Consider a random process $x[n]$ that is the response of the linear time-invariant system shown in Figure P2.89-1. In the figure, $w[n]$ represents a real zero-mean stationary white-noise process with $E\{w^2[n]\} = \sigma_w^2$.

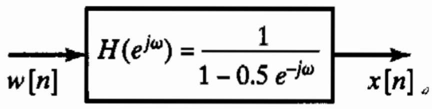


Figure P2.89-1

(a) Express $\mathcal{E}\{x^2[n]\}$ in terms of $\phi_{xx}[n]$ or $\Phi_{xx}(e^{j\omega})$.
(b) Determine $\Phi_{xx}(e^{j\omega})$, the power density spectrum of $x[n]$.
(c) Determine $\phi_{xx}[n]$, the correlation function of $x[n]$.

2.90. Consider a linear time-invariant system whose impulse response is real and is given by $h[n]$. Suppose the responses of the system to the two inputs $x[n]$ and $v[n]$ are, respectively, $y[n]$ and $z[n]$, as shown in Figure P2.90-1.

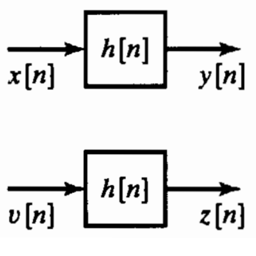


Figure P2.90-1

The inputs $x[n]$ and $v[n]$ in the figure represent real zero-mean stationary random processes with autocorrelation functions $\phi_{xx}[n]$ and $\phi_{vv}[n]$, cross-correlation function $\phi_{xv}[n]$, power spectra $\Phi_{xx}(e^{j\omega})$ and $\Phi_{vv}(e^{j\omega})$, and cross power spectrum $\Phi_{xv}(e^{j\omega})$.

(a) Given $\phi_{xx}[n], \phi_{vv}[n], \phi_{xv}[n], \Phi_{xx}(e^{j\omega}), \Phi_{vv}(e^{j\omega})$, and $\Phi_{xv}(e^{j\omega})$, determine $\Phi_{yz}(e^{j\omega})$, the cross power spectrum of $y[n]$ and $z[n]$, where $\Phi_{yz}(e^{j\omega})$ is defined by

$$\phi_{yz}[n] \stackrel{\mathcal{F}}{\longleftrightarrow} \Phi_{yz}(e^{j\omega}),$$

with $\phi_{yz}[n] = E\{y[k]z[k-n]\}$.

(b) Is the cross power spectrum $\Phi_{xv}(e^{j\omega})$ always nonnegative; i.e., is $\Phi_{xv}(e^{j\omega}) \geq 0$ for all ω? Justify your answer.

2.91. Consider the LTI system shown in Figure P2.91-1. The input to this system, $e[n]$, is a stationary zero-mean white-noise signal with average power σ_e^2. The first system is a backward-difference system as defined in Eq. 2.45 with $f[n] = e[n] - e[n-1]$. The second system is an ideal lowpass filter with frequency response

$$H_2(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \leq \pi. \end{cases}$$

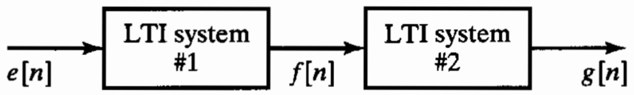


Figure P2.91-1

(a) Determine an expression for $\Phi_{ff}(e^{j\omega})$, the power spectrum of $f[n]$, and plot this expression for $-2\pi < \omega < 2\pi$.

(b) Determine an expression for $\phi_{ff}[m]$, the autocorrelation function of $f[n]$.

(c) Determine an expression for $\Phi_{gg}(e^{j\omega})$, the power spectrum of $g[n]$, and plot this expression for $-2\pi < \omega < 2\pi$.

(d) Determine an expression for σ_g^2, the average power of the output.