Image Filtering

Lowpass Filter: blurring image

* Ideal LowPass Filter - ILPF) : the transfer functions is described as:

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

where $D_0$is the cut-off frequency with non-negative value, $D(u,v)$is the distance from $(u,v)$to the filter center. Orbit of $D(u,v) = D_0$is a circle.

For example:

$$f(x,y) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad F(u,v) = \begin{bmatrix} 4 & -2-j2 & 0 & -2+j2 \\ -2-j2 & j2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2+2j & 2 & 0 & -j2 \end{bmatrix}$$

The original image f and the Fourier coefficient image.

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The pixel positions of the original image.

$$f(x,y)=\begin{bmatrix} 0,0 & 0,1 & 0,2 & 0,3 \\ 1,0 & 1,1 & 1,2 & 1,3 \\ 2,0 & 2,1 & 2,2 & 2,3 \\ 3,0 & 3,1 & 3,2 & 3,3 \end{bmatrix}$$

Convert the center of image (u-H/2, v-W/2), in this case H=W=4, (u-2, v-2).

$$f_c(x,y)=\begin{bmatrix} -2,-2 & -2,-1 & -2,0 & -2,1 \\ -1,-2 & -1,-1 & -1,0 & -1,1 \\ 0,-2 & 0,-1 & 0,0 & 0,1 \\ 1,-2 & 1,-1 & 1,0 & 1,1 \end{bmatrix}$$

The centered image.

$$F(u,v)=\begin{bmatrix} 4 & -2-2j & 0 & -2+2j \\ -2-2j & +j2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2+2j & 2 & 0 & -j2 \end{bmatrix}$$

$$F_c(u,v)=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -j2 & -2+2j & 2 \\ 0 & -2+2j & 4 & -2-2j \\ 0 & 2 & -2-2j & j2 \end{bmatrix}$$

The centered Fourier image.

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Sqrt(u^2+v^2)<=D_0; Assume that D_0=H/2=4/2=2

$$D(x,y)=\begin{bmatrix} -2,-2 & -2,-1 & -2,0 & -2,1 \\ -1,-2 & -1,-1 & -1,0 & -1,1 \\ 0,-2 & 0,-1 & 0,0 & 0,1 \\ 1,-2 & 1,-1 & 1,0 & 1,1 \end{bmatrix} \qquad H(u,v)=\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The filtered image.

$$G(u,v)=H(u,v)*F_c(u,v)=\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -j2 & -2+2j & 2 \\ 0 & -2+2j & 4 & -2-2j \\ 0 & 2 & -2-2j & j2 \end{bmatrix}$$

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The image after filtering

g =

0.25 + 0.1250i	-0.125 - 0.25i	0.1250i	-0.125
-0.125 - 0.25i	0.75 + 0.375i	-0.625	-0.1250i
0.125i	-0.625	0.75 - 0.375i	-0.125 + 0.25i
-0.125	-0.1250i	-0.125 + 0.25i	0.25 - 0.125i

g_real =

0.25	-0.125	0	-0.125
-0.125	0.75	-0.625	0
0	-0.625	0.75	-0.125
-0.125	0	-0.125	0.25

g_non-negative integer =

0	0	0	0
0	1	0	0
0	0	1	0
0	0	0	0

>=0.5 ; 1 and <0.5; 0