University of Technology and Education

Faculty of Electrical & Electronic Engineering

Lecture:

IMAGE PROCESSING

Chapter 3:

Image Transforms_Fourier

Nguyen Thanh Hai, PhD

Image Transform

Image transforms

- **Fourier**
- **Cosine**
- **Wavelet**
- Unique
- Sine
- Hartley
- Hadamard
- Harr
- Daubechies
- Karhunen-loeve
- Slant
- Hotelling

… and many other transforms

Fourier Transform

Introduction

- Fourier Transform (FT): an image in the spatial is transformed to consider in the frequency domain. One can use the FT in filtering image or others

- FT of two-dimensional (2D)

- Properties of Discrete Fourier Transform

- Applications

• *Extracting feature of image based on frequency*

• *Filtering noise*

• *Determining frequencies in an image*

Fourier Transform

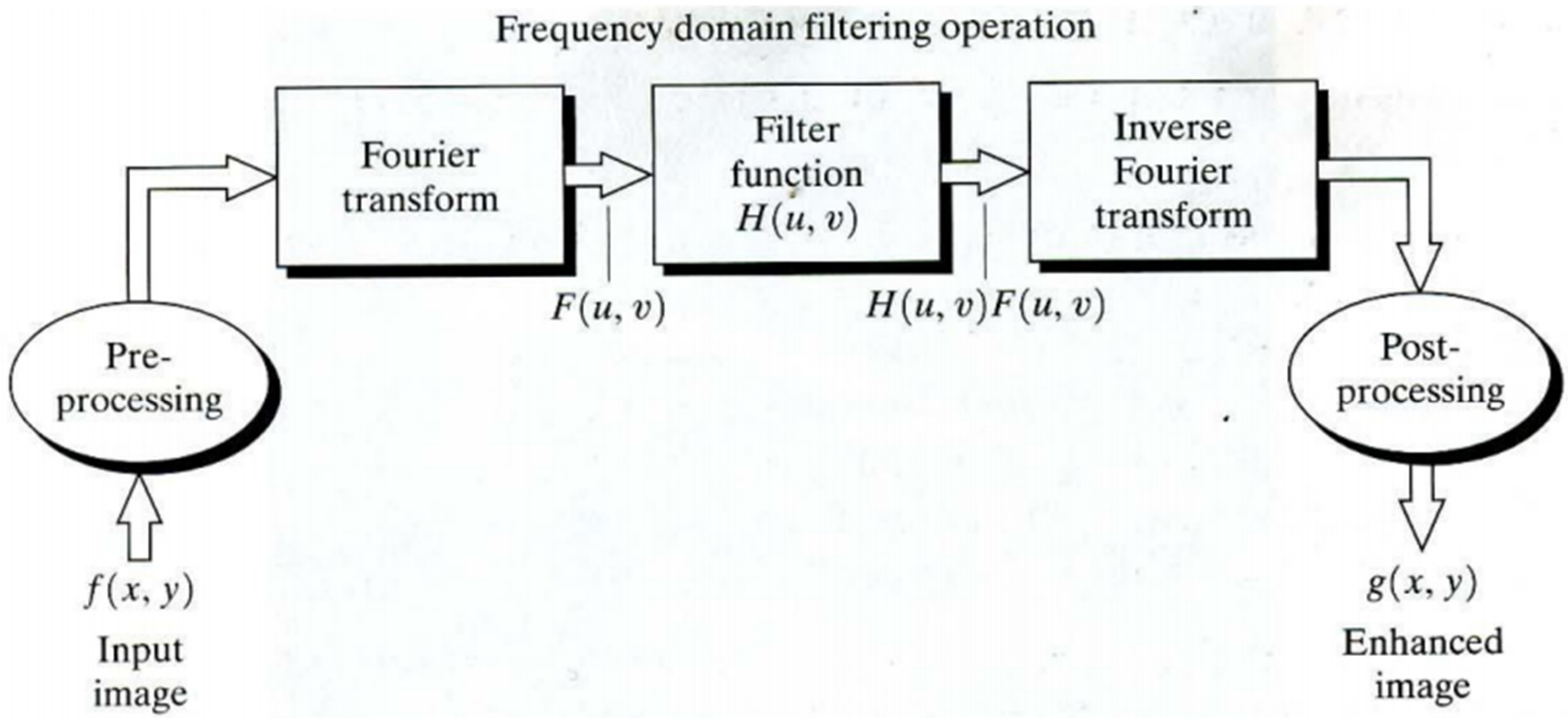
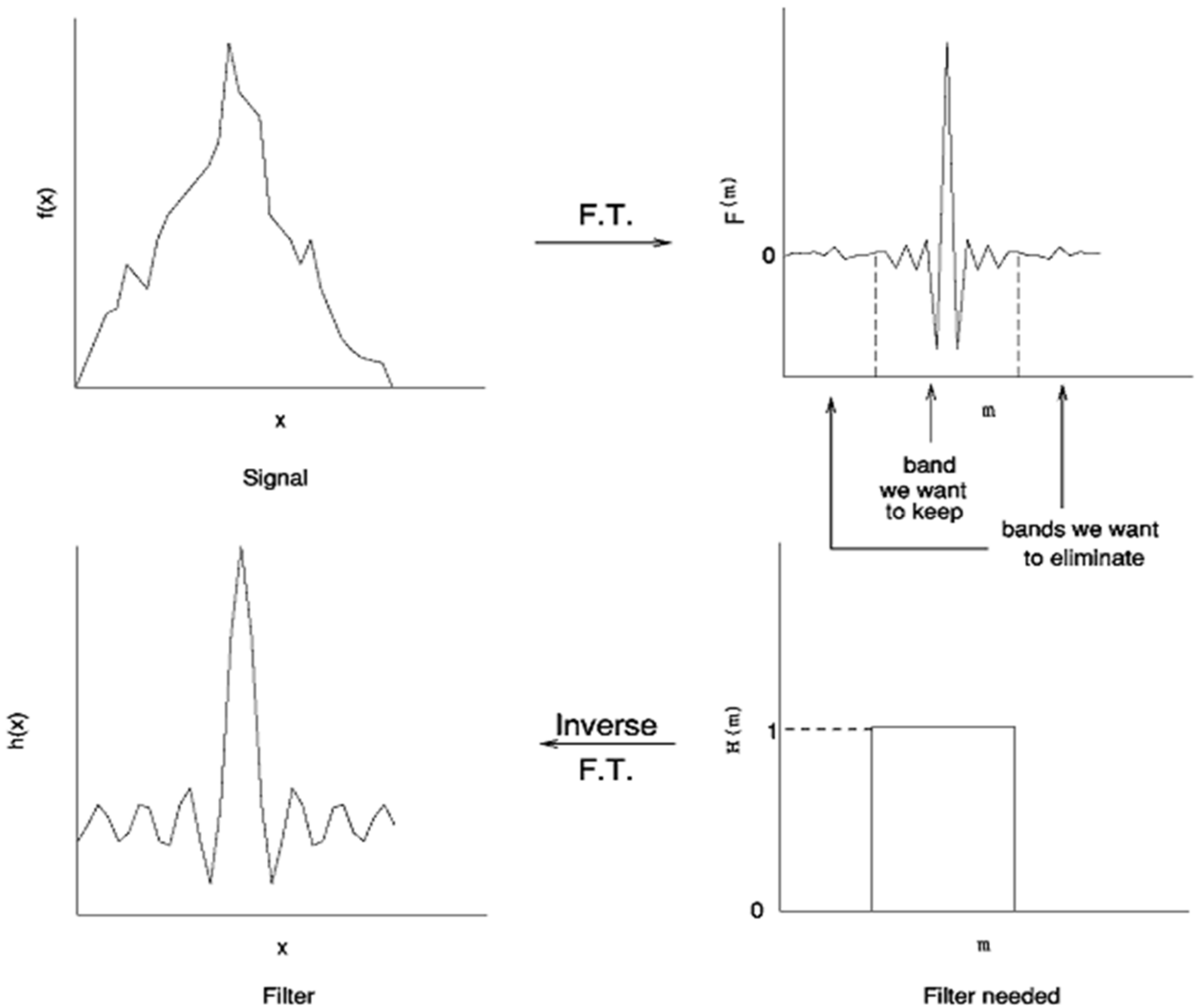


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Fourier Transform

1D Fourier Transform



Top row: a signal and its Fourier transform. Middle row: the unit sample response function of a filter on the left, and the filter's system function on the right

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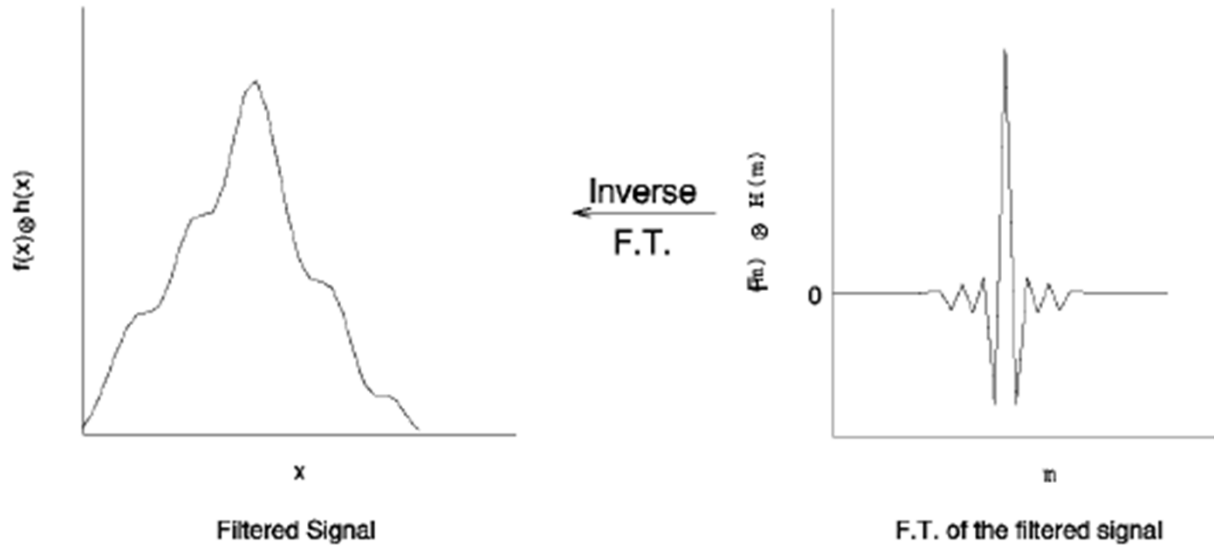
Fourier Transform



Bottom row: On the left the filtered signal that can be obtained by convolving the signal at the top with the filter in the middle. On the right the Fourier transform of the filtered signal obtained by multiplying the Fourier transform of the signal at the top, with the Fourier transform (system function) of the filter in the middle

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Image Transforms

One-Dimensional Discrete Fourier Transform

The Fourier transform of a discrete function of one variable, $f(x)$, $x = 0, 1, 2, \ldots, M-1$, is given by the equation

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad \text{for } u = 0, 1, 2, \ldots, M-1. \qquad (4.2\text{-}5)$$

This *discrete Fourier transform* (DFT) is the foundation for most of the work in this chapter. Similarly, given $F(u)$, we can obtain the original function back using the inverse DFT:

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad \text{for } x = 0, 1, 2, \ldots, M-1. \qquad (4.2\text{-}6)$$

The $1/M$ multiplier in front of the Fourier transform sometimes is placed in front of the inverse instead. Other times (not as often) both equations are multiplied by $1/\sqrt{M}$. The location of the multiplier does not matter. If two multipliers are used, the only requirement is that their product be equal to $1/M$.

One-Dimensional Discrete Fourier Transform

The concept of the frequency domain, mentioned numerous times in this chapter and in Chapter 3, follows directly from Euler's formula:

for $u = 0, 1, 2, \ldots, M - 1$. Thus, we see that *each* term of the Fourier transform [that is, the value of $F(u)$ for each value of u] is composed of the sum of *all* values of the function $f(x)$. The values of $f(x)$, in turn, are multiplied by sines and cosines of various frequencies. The domain (values of u) over which the values of $F(u)$ range is appropriately called the *frequency domain*, because u determines the frequency of the components of the transform. (The x's also affect the frequencies, but they are summed out and they all make the same contributions for each value of u.) Each of the M terms of $F(u)$ is called a *frequency component* of the transform. Use of the terms *frequency domain* and *frequency components* is really no different from the terms *time domain* and *time components*, which we would use to express the domain and values of $f(x)$ if x were a time variable.

Image Transforms

One-Dimensional Discrete Fourier Transform

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Image Transforms

One-Dimensional Discrete Fourier Transform

In general, we see from Eqs. (4.2-5) or (4.2-8) that the components of the Fourier transform are complex quantities. As in the analysis of complex numbers, we find it convenient sometimes to express $F(u)$ in polar coordinates:

$$F(u) = |F(u)|e^{-j\phi(u)}$$

$$|F(u)| = \left[R^2(u) + I^2(u)\right]^{1/2}$$

is called the *magnitude* or *spectrum* of the Fourier transform, and

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

is called the *phase angle* or *phase spectrum* of the transform.

$R(u)$ and $I(u)$ are the real and imaginary parts of $F(u)$

power spectrum, defined as the square of the Fourier spectrum:

$$\begin{aligned} P(u) &= |F(u)|^2 \\ &= R^2(u) + I^2(u). \end{aligned}$$

The term *spectral density* also is used to refer to the power spectrum.

Image Transforms

One-Dimensional Discrete Fourier Transform

EXAMPLE 4.1:
Fourier spectra of two simple 1-D functions.

Before proceeding, it will be helpful to consider a simple one-dimensional example of the DFT. Figure 4.2(a) shows a function and Fig. 4.2(b) shows its Fourier spectrum. Both $f(x)$ and $F(u)$ are discrete quantities, but the points in the plots are linked to make them easier to follow visually. In this example, $M = 1024$, $A = 1$, and K is only 8 points. Also note that the spectrum is centered at $u = 0$. As shown in the following section, this is accomplished by multiplying $f(x)$ by $(-1)^x$ before taking the transform. The next two figures depict basically the same thing, but with $K = 16$ points. The important features to note are that (1) the height of the spectrum doubled as the area under the curve in the x-domain doubled, and (2) the number of zeros in the spectrum in the same interval doubled as the length of the function doubled. This "reciprocal" nature of the Fourier transform pair is most useful in interpreting results of image processing in the frequency domain.

Image Transforms

One-Dimensional Discrete Fourier Transform

In the discrete transform of Eq. (4.2-5), the function $f(x)$ for $x = 0, 1, 2, \dots, M - 1$, represents M samples from its continuous counterpart. It is important to keep in mind that these samples are *not* necessarily always taken at integer values of x in the interval $[0, M - 1]$. They are taken at equally spaced, but otherwise arbitrary, points. This is usually represented by letting x_0 denote the first (arbitrarily located) point in the sequence. The first value of the sampled function is then $f(x_0)$. The next sample has taken a fixed interval Δx units away to give $f(x_0 + \Delta x)$. The kth sample gives us $f(x_0 + k\Delta x)$, and the final sample is $f(x_0 + [M - 1]\Delta x)$. Thus, in the discrete case, when we write $f(k)$, it is understood that we are utilizing shorthand notation that really means $f(x_0 + k\Delta x)$. In terms of the notation we have used thus far, $f(x)$ is then understood to mean

$$f(x) \triangleq f(x_0 + x\Delta x)$$

One-Dimensional Discrete Fourier Transform

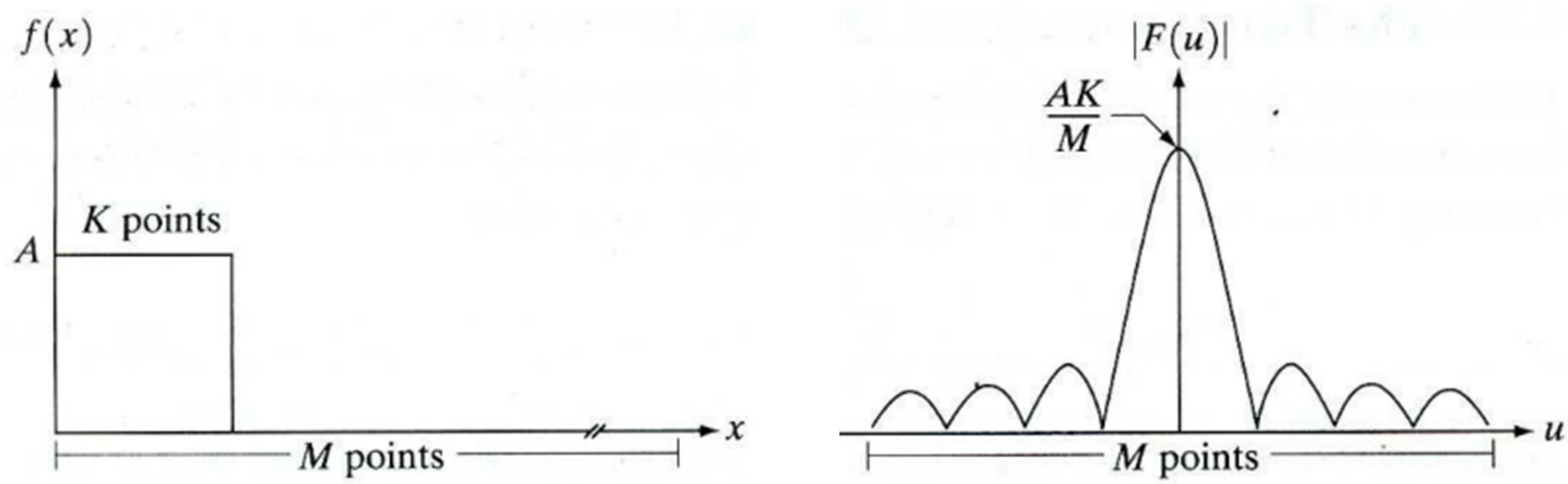

FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum.

Image Transforms

One-Dimensional Discrete Fourier Transform

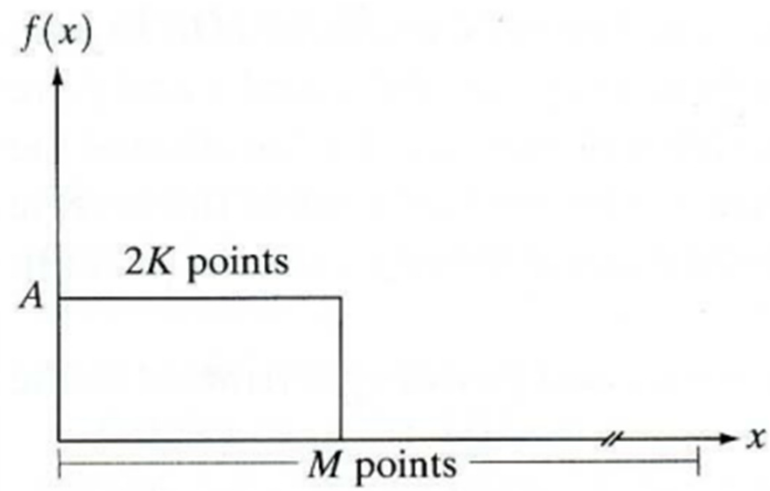

FIGURE 4.2 (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

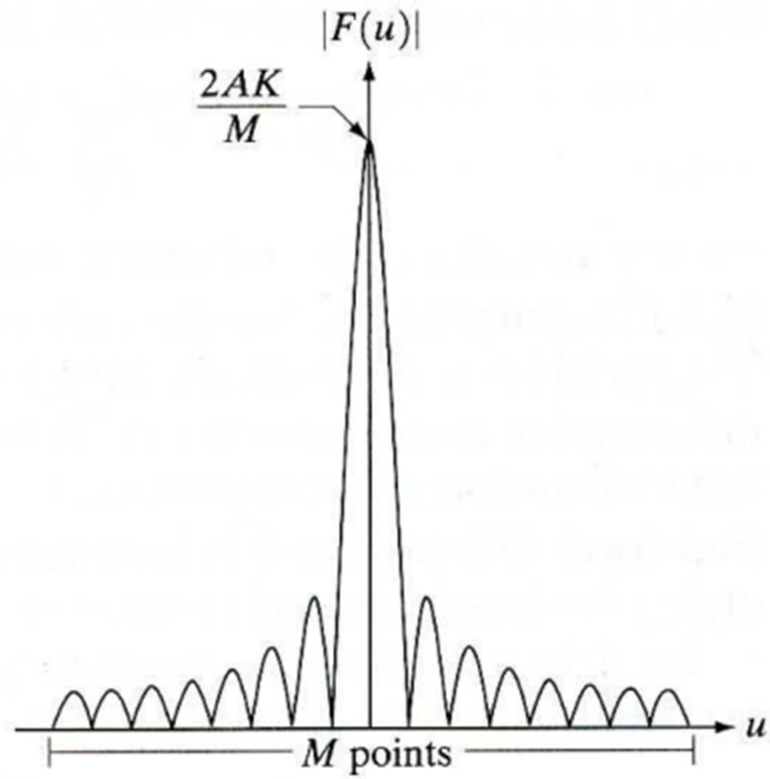

Image Transforms

when dealing with discrete variables. The variable u has a similar interpretation, but the sequence always starts at true zero frequency. Thus, the sequence for the values of u is $0, \Delta u, 2\Delta u, \ldots, [M-1]\Delta u$. Then, $F(u)$ is understood to mean

$$F(u) \triangleq F(u\Delta u)$$

for $u = 0, 1, 2, \ldots, M-1$. This type of shorthand notation simplifies equations considerably and is much easier to follow.

Given the inverse relationship between a function and its transform illustrated in Fig. 4.2, it is not surprising that Δx and Δu are inversely related by the expression

$$\Delta u = \frac{1}{M\Delta x}.$$

This relationship is useful when measurements are an issue in the images being processed. For instance, in an application of electron microscopy the image samples may be spaced 1 micron apart, and certain characteristics in the frequency domain (like periodicity components) may have implications regarding the structure of the physical sample. For the most part in subsequent discussions in this book we use the variables x and u without making reference to specific sampling or other measurement considerations.

Image Transforms

Example : Consider the time signal shown in Fig. 2.1. The DFT of this signal is shown in Fig. 2.2.

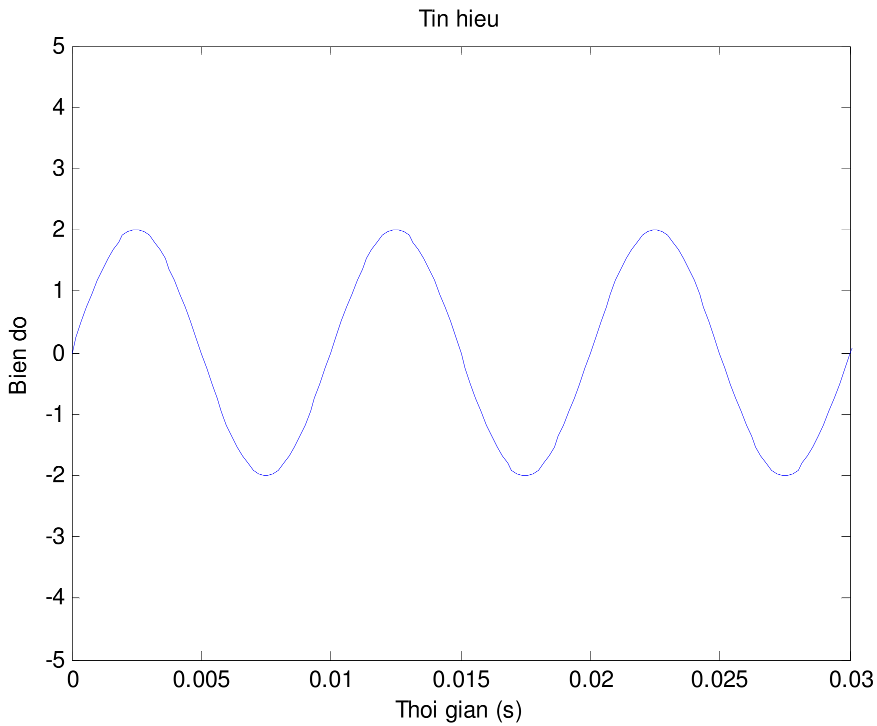

Fig. 2.1: Signal x, defined in the time domain

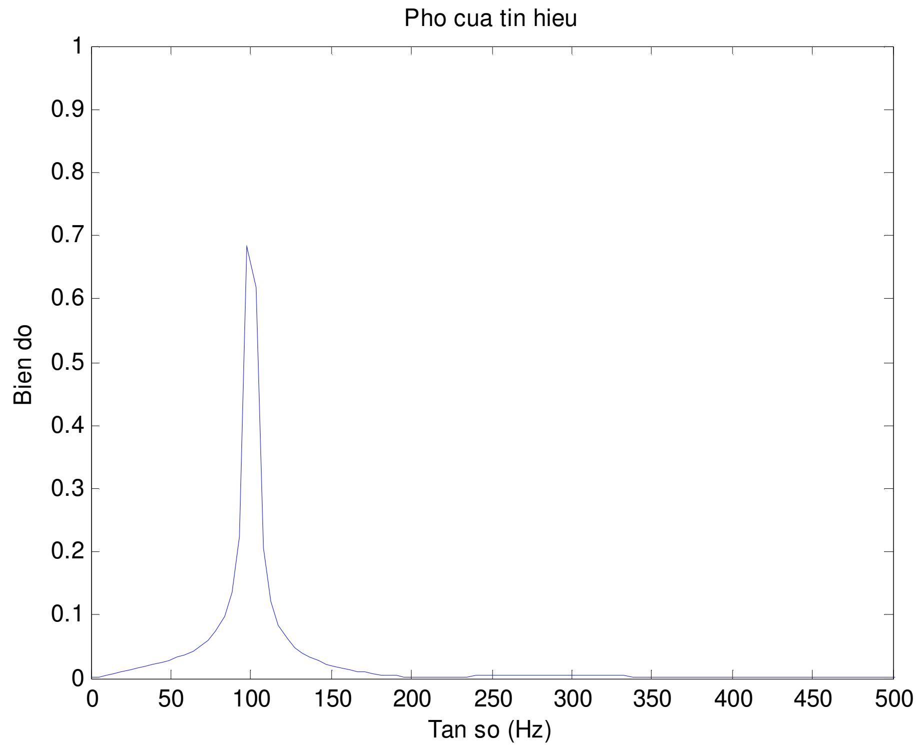

Fig. 2.2: Magnitude of the DFT of signal x

Image Transforms

Two-Dimensional Discrete Fourier Transform

The 2D FT is a rather straightforward extension of the 1D transform. Mathematically, the 2D DFT is defined as:

$$G(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} g(x,y)e^{-j\frac{2\pi(ux)}{M}}e^{-j\frac{2\pi(vy)}{N}} \qquad (3.1)$$

where $u=0,1,\ldots,M-1$ and $v=0,1,\ldots,N-1$ are the frequency axes, in which $G(u,v)$ is described as a Fourier image.

Two-Dimensional Discrete Fourier Transform

The inverse transformation, the 2D IDFT is denoted as:

$$g(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} G(u,v) e^{j\frac{2\pi(ux)}{M}} e^{j\frac{2\pi(vy)}{N}} \qquad (3.2)$$

where $x=0,1,\ldots,M\text{-}1$ and $y=0,1,\ldots,N\text{-}1$ and $g(x,y)$ is an image after inverse transform.

Image Transforms

The variables u and v are the *transform* or *frequency variables*, and x and y are the *spatial* or *image variables*. As in the one-dimensional case, the location of the $1/MN$ constant is not important. Sometimes it is located in front of the inverse transform. Other times it is found split into two equal terms of $1/\sqrt{MN}$ multiplying the transform and its inverse.

Image Transforms

Example of Fourier transform

$G(u,v)$ is the Fourier transformation of an image $g(x,y)$

$$G = U^T * g * V \tag{3.3}$$

Where U, V are the matrices:

$$U(x,u) = e^{-\frac{j2\pi xu}{M}}; x,u = 0 : M-1 \tag{3.4}$$

$$V(y,v) = e^{-\frac{j2\pi yv}{N}}; y,v = 0 : N-1 \tag{3.5}$$

Image Transforms

Example of Fourier transform

If $U(x,u)=U(u,x)$ ==> $U^T=U$, *one can re-write as follows:*

$$G = U * g * V \qquad (3.6)$$

$g(x,y)$ is the inverse Fourrier transform of $G(u,v)$

$$g = U^{-1} * G * V^{-1} \qquad (3.7)$$

where

$$G = U^T * g * V \Leftrightarrow \left(U^T\right)^{-1} * G * V^{-1} = \left(U^T\right)^{-1} U^T * g * V * V^{-1}$$

$$\Leftrightarrow g = \left(U^T\right)^{-1} * G * V^{-1}$$

Image Transforms

Example: Find the Fourier transform of the following 4x4 image

$$g(x,y)=\begin{bmatrix}0&0&0&0\\0&1&1&0\\0&1&1&0\\0&0&0&0\end{bmatrix}$$

Solution:

- One has the row, M=4 ; the column, N=4
- Transform matrix $U(x,u)$ using the formula:

$$U(x,u)=e^{-\frac{j2\pi xu}{M}};x,u=0:M-1$$

$$U=\begin{bmatrix}e^{\frac{-j\times2\times\pi\times0\times0}{4}}&e^{\frac{-j\times2\times\pi\times0\times1}{4}}&e^{\frac{-j\times2\times\pi\times0\times2}{4}}&e^{\frac{-j\times2\times\pi\times0\times3}{4}}\\e^{\frac{-j\times2\times\pi\times1\times0}{4}}&e^{\frac{-j\times2\times\pi\times1\times1}{4}}&e^{\frac{-j\times2\times\pi\times1\times2}{4}}&e^{\frac{-j\times2\times\pi\times1\times3}{4}}\\e^{\frac{-j\times2\times\pi\times2\times0}{4}}&e^{\frac{-j\times2\times\pi\times2\times1}{4}}&e^{\frac{-j\times2\times\pi\times2\times2}{4}}&e^{\frac{-j\times2\times\pi\times2\times3}{4}}\\e^{\frac{-j\times2\times\pi\times3\times0}{4}}&e^{\frac{-j\times2\times\pi\times3\times1}{4}}&e^{\frac{-j\times2\times\pi\times3\times2}{4}}&e^{\frac{-j\times2\times\pi\times3\times3}{4}}\end{bmatrix}\Rightarrow U=\begin{bmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{bmatrix}$$

Using the formula of Euler, we have $e^{-j\varpi}=\cos\varpi-j\sin\varpi$

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Image Transforms

$$U=\begin{bmatrix} e^{\frac{-j2\pi 0x0}{4}} & e^{\frac{-j2\pi 0x1}{4}} & e^{\frac{-j2\pi 0x2}{4}} & e^{\frac{-j2\pi 0x3}{4}} \\ e^{\frac{-j2\pi 1x0}{4}} & e^{\frac{-j2\pi 1x1}{4}} & e^{\frac{-j2\pi 1x2}{4}} & e^{\frac{-j2\pi 1x3}{4}} \\ e^{\frac{-j2\pi 2x0}{4}} & e^{\frac{-j2\pi 2x1}{4}} & e^{\frac{-j2\pi 2x2}{4}} & e^{\frac{-j2\pi 2x3}{4}} \\ e^{\frac{-j2\pi 3x0}{4}} & e^{\frac{-j2\pi 3x1}{4}} & e^{\frac{-j2\pi 3x2}{4}} & e^{\frac{-j2\pi 3x3}{4}} \end{bmatrix}$$

Image Transforms

Similarly, one has the matrix, V

$$V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$G = U * g * V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$G = \begin{bmatrix} 4 & -2-j2 & 0 & -2+j2 \\ -2-j2 & j2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ -2+2j & 2 & 0 & -j2 \end{bmatrix}$$

DFT image

The kernel of DFT is a complex function → these images are complex:

- Real parts
- Imaginary parts

Display the DFT of an image

- Coefficients of the high frequency are small.
- Logarithmic function

$$d\ (u, v) = \log(1 + |G(u, v)|)$$

Image Transforms

We define the Fourier spectrum, phase angle, and power spectrum as in the previous section:

$$|F(u, v)| = \left[R^2(u, v) + I^2(u, v)\right]^{1/2}$$

$$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)}\right]$$

$$\begin{aligned} P(u, v) &= |F(u, v)|^2 \\ &= R^2(u, v) + I^2(u, v) \end{aligned}$$

where $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively.

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Image Transforms

It is common practice to multiply the input image function by $(-1)^{x+y}$ prior to computing the Fourier transform. Due to the properties of exponentials, it is not difficult to show (see Section 4.6) that

$$\Im\left[f(x, y)(-1)^{x+y}\right] = F(u - M/2, v - N/2)$$

where $\Im[\cdot]$ denotes the Fourier transform of the argument. This equation states that the origin of the Fourier transform of $f(x, y)(-1)^{x+y}$ [that is, $F(0, 0)$] is located at $u = M/2$ and $v = N/2$. In other words, multiplying $f(x, y)$ by $(-1)^{x+y}$ shifts the origin of $F(u, v)$ to frequency coordinates $(M/2, N/2)$, which is the center of the $M \times N$ area occupied by the 2-D DFT. We refer to this area of the frequency domain as the *frequency rectangle*. It extends from $u = 0$ to $u = M - 1$, and from $v = 0$ to $v = N - 1$ (keep in mind that u and v are integers). In order to guarantee that these shifted coordinates are integers, we require that M and N be even numbers. When implementing the Fourier transform in a computer, the limits of summations are from $u = 1$ to M and $v = 1$ to N. The actual center of the transform will then be at $u = (M/2) + 1$ and $v = (N/2) + 1$.

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Image Transforms

The value of the transform at $(u, v) = (0, 0)$ is, from Eq. (4.2-16),

$$F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y),$$

which we see is the average of $f(x, y)$. In other words, if $f(x, y)$ is an image, the value of the Fourier transform at the origin is equal to the average gray level of the image. Because both frequencies are zero at the origin, $F(0, 0)$ sometimes is called the *dc* component of the spectrum. This terminology is from electrical engineering, where "dc" signifies direct current (i.e., current of zero frequency).

If $f(x, y)$ is real, its Fourier transform is conjugate symmetric; that is,

$$F(u, v) = F^*(-u, -v)$$

where "*" indicates the standard conjugate operation on a complex number. From this, it follows that

$$|F(u, v)| = |F(-u, -v)|,$$

which says that the spectrum of the Fourier transform is symmetric. Conjugate symmetry and the centering property discussed previously truly simplify the specification of circularly symmetric filters in the frequency domain, as shown in the following section.

Image Transforms

Finally, as in the 1-D case, we have the following relationships between samples in the spatial and frequency domains:

$$\Delta u = \frac{1}{M\Delta x}$$

$$\Delta v = \frac{1}{N\Delta y}.$$

Image Transforms

■ Figure 4.3(a) shows a white rectangle of size 20×40 pixels superimposed on a black background of size 512×512 pixels. This image was multiplied by $(-1)^{x+y}$ prior to computing the Fourier transform in order to center the spectrum, which is shown in Fig. 4.3(b). (Note the location, labels, and origin of the axes in both figures. We follow this convention throughout all discussions of images and their corresponding Fourier spectra.) In Fig. 4.3(b), the separation of spectrum zeros in the u-direction is exactly twice the separation of zeros in the v direction. This corresponds inversely to the 1-to-2 size ratio of the rectangle in the image. The spectrum was processed prior to displaying by using the log transformation in Eq. (3.2-2) to enhance gray-level detail. A value of $c = 0.5$ was used in the transformation in order to decrease overall intensity. Most Fourier spectra shown in this chapter are similarly processed by a log transformation. ■

Image Transforms

FIGURE 4.3
(a) Image of a 20×40 white rectangle on a black background of size 512×512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.

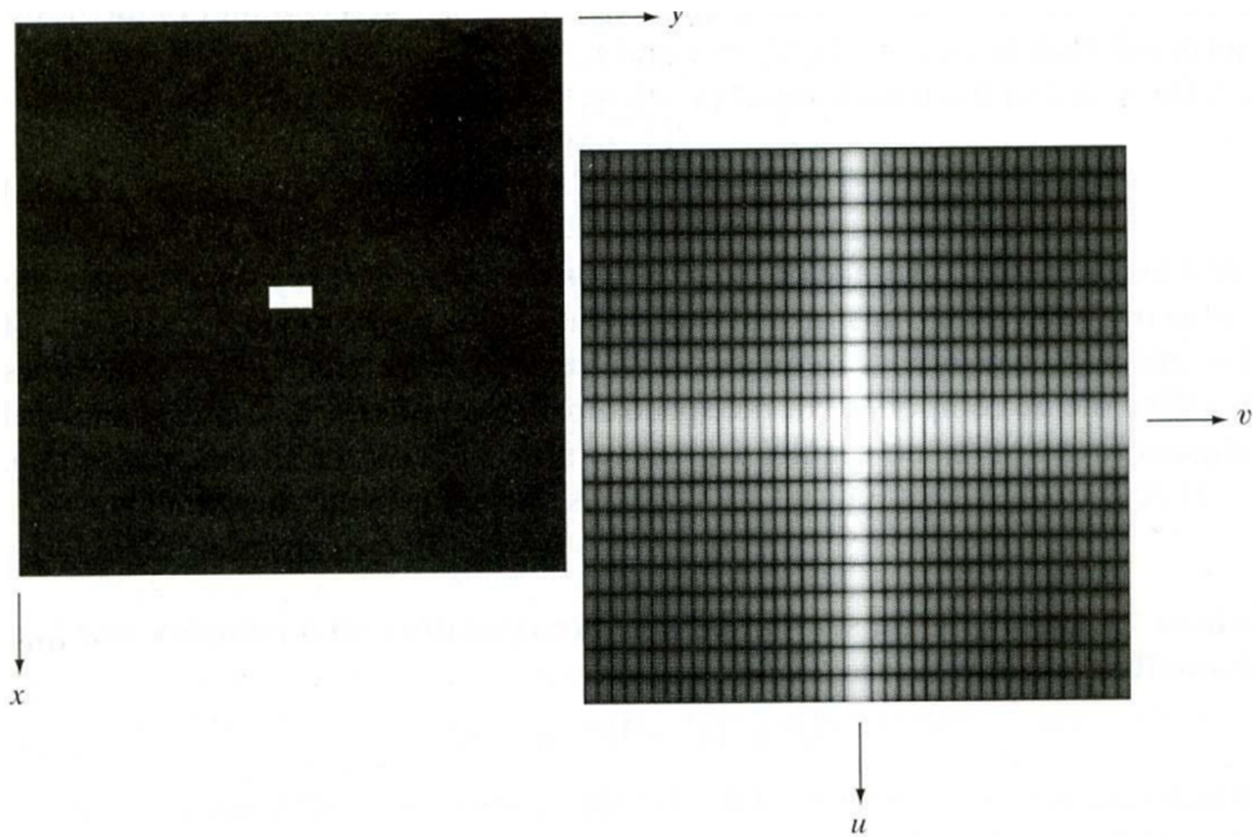
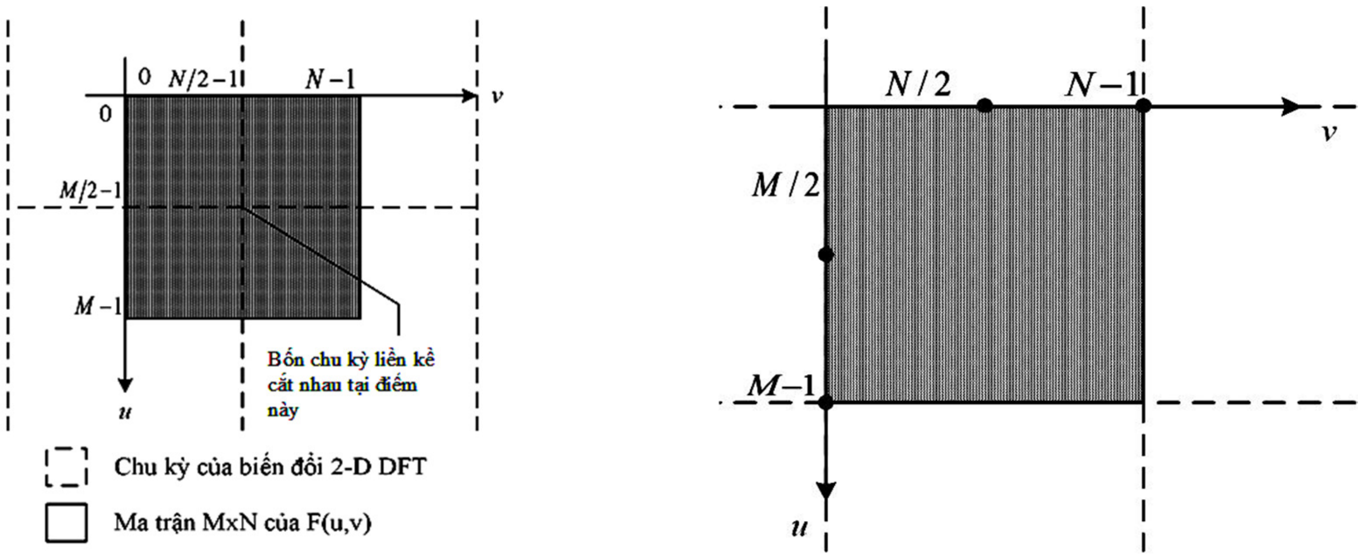


Image Transforms

F=fft2(f)
Hàm này sẽ trả về biến đổi Fourier cũng với kích thước có dữ liệu được sắp xếp như trong hình 3.2(a), khi đó, gốc tọa độ nằm ở góc trên bên trái của ma trận và chu kỳ kết thúc tại trung tâm của hình chữ nhật tần số.



Hình 3.2. *(a) Phổ Fourier có kích thước và chỉ ra bốn góc tư của các chu kỳ tiếp giáp nhau trong phổ. (b) Phổ có được bằng cách nhân hàm với ( ) trước khi tính biến đổi Fourier. Khi đó trọn vẹn 1 chu kỳ được xem xét bằng cách thực hiện biến đổi Fourier thông thường*

Image Transforms

với các giá trị bằng không khi sử dụng biến đổi Fourier để lọc ảnh. Trong trường hợp này, cú pháp được cho như sau:

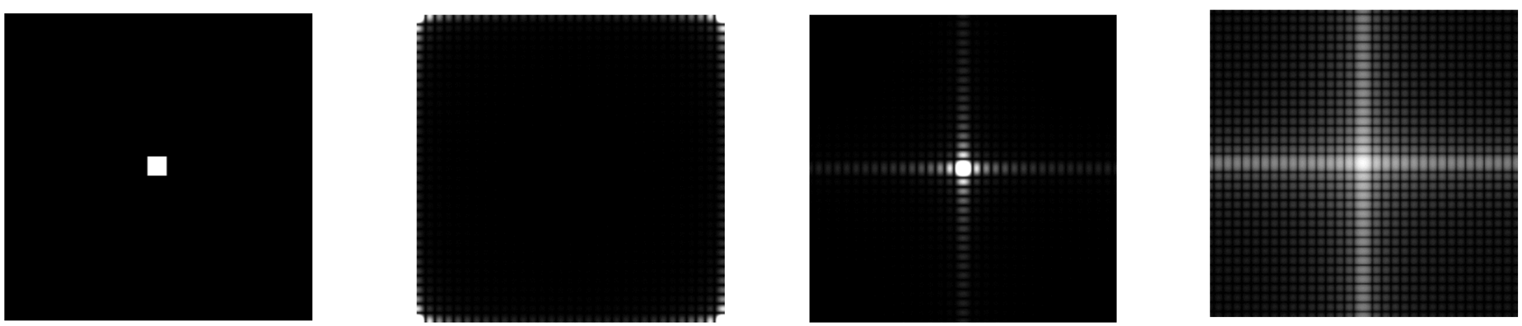
F=fft2(f,P,Q)

Với cú pháp này, fft2 mở rộng ngõ vào với số lượng giá trị không cho phép sao cho hàm kết quả có kích thước .

Phổ biên độ Fourier đạt được bằng cách sử dụng hàm abs:

S=abs(F)

Hàm abs tính biên độ (căn bậc hai của tổng bình phương phần thực và ảo) của từng phần tử trong mảng. Hình 3.3(b) trình bày phổ Fourier, trong đó, các điểm sáng tại các góc có biên độ lớn nhất tương ứng với 4 góc phần tư của 4 chu kỳ tiếp giáp nhau.



Hình 3.3. *Biến đổi Fourier cho ảnh và phổ của nó:*
(a). Ảnh gốc; (b) Phổ Fourier; (c) Phổ được định vị trí trung tâm
(d) Phổ được tăng cường để hiển thị bằng biến đổi log

Image Transforms

Phân tích phổ bằng cách biểu diễn nó như một ảnh độc lập được xem là một khía cạnh quan trọng trong miền tần số. Hàm fftshift trong Toolbox được sử dụng để chuyển gốc tọa độ của biến đổi đến trung tâm của hình chữ nhật hay nói đúng hơn thì nó thực hiện chức năng tương tự việc nhân $(-1)^{x+y}$. Cú pháp của hàm fftshift được đưa ra như sau:

Fc=fftshift(F)

với F là biến đổi Fourier đã được tính toán bằng hàm fft2 và Fc là biến đổi trung tâm. Hàm fftshift thực hiện hoán đổi các gốc phần tư của F như trong hình 3.3(c), khi đó, việc tái sắp xếp các góc phần tư này làm cho các điểm sáng đều được đặt tại vị trí trung tâm và cho phép hiển thị trọn vẹn một chu kỳ hoàn chỉnh. Hình 3.3(d) biểu diễn phổ như hình 3.3(c) nhưng được tăng cường bằng biến đổi log.

CMR: $(-1)^{x+y} = e^{j\pi(x+y)}$

$$e^{-j\varpi} = \cos\varpi - j\sin\varpi$$

Image Transforms

Ví dụ 3.2: Thực hiện biến đổi Fourier cho ảnh và tính phổ của nó

```
clear all;
close all;
f=imread('TestDFT.tif');
F=fft2(f);
S=abs(F);
Fc=fftshift(F);
Sc=abs(Fc);
S2=log(1+abs(Fc));
figure;
subplot(2,2,1);imshow(f);
subplot(2,2,2);imshow(S);
subplot(2,2,3);imshow(Sc);
subplot(2,2,4);imshow(S2);
```

Ngược với hàm fftshift là hàm ifftshift sẽ chuyển các góc phần tư về vị trí ban đầu, hàm này có cú pháp như sau:

```
F=ifftshift(Fc)
```

biến đổi Fourier ngược, hàm ifft được sử dụng với cú pháp như sau:

```
f=ifft2(F)
```

Với F là biến đổi Fourier sau khi đã chuyển các góc phần tư về vị trí ban đầu, f là ảnh trên miền không gian. Nếu ngõ vào tính toán F là số thực thì tương ứng ngõ ra cũng là dạng số thực. Tuy nhiên, trong thực tế, hàm ifft2 thường có một vài thành phần ảo rất nhỏ là kết quả từ việc làm tròn vốn là đặc trưng trong tính toán dấu chấm động. Do đó, tốt nhất là ta sẽ lấy phần thực và dùng hàm như sau:

```
f=real(ifft2(F))
```

Exercise

Compute the DFT of the following image

$$g = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Solution

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Image Transforms

Exercise

Verify the relationship of the DFTs of the following images

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

A B

Image Transforms

Exercise

Find the DFT of the following image

$$\begin{matrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{matrix}$$

Image Transforms

Exercise

Solution

8.0000 + 0.0000i	-4.0000 - 4.0000i	0.0000 + 0.0000i	-4.0000 + 4.0000i
0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i

Image Transforms

Creat the function *fft2_new.m* to calculate Fourier transform of an image

function G=fft2_new(g)
% G is fourier tranform of g
%=====================
% Get size of matrix f
 	
M=size(g,1); % get number of rows
N=size(g,2) ; % get number of columns
%=====================
	%create matrix U
	%U(*x*,*u*)=exp(-j*2*pi**x***u*/M)
	% *x*, *u* run from 0 to M-1
	U=[ ];
	for *x*=0:M-1
	  for *u*=0:M-1
	    U(*x*+1,*u*+1)=exp(-j*2*pi*m*p/M);
	  end
	end

Image Transforms

Creat the function *fft2_new.m* to calculate Fourier transform of an image

```
%=====================
% create matrix V
%V(y,v)=exp(-j*2*pi*y*v/N)
% y, v run from 0 to N-1
V=[ ];
for y=0:N-1
   for v=0:N-1
       V(y+1,v+1)=exp(-
j*2*pi*y*v/N);
   end
end
```

```
%=====================
%Fourier transform
G=U*g*V;
```

Image Transforms

Creat the inverse function *ifft2_new.m* to calculate inverse Fourier transform of an image

```
function g=ifft2_new(G)
% g is invert fourier tranform of G
%=====================
%Get size of matrix G
M=size(G,1);  % get number of rows
N=size(G,2) ; % get number of columns
%=====================
```

```
%=====================
%create matrix U
%U(m,p)=exp(-j*2*pi*x*u/M)
% x, u run from 0 to M-1
U=[ ];
for x=0:M-1
   for u=0:M-1
       U(x+1,u+1)=exp(-j*2*pi*x*u/M);
   end
end
```

Image Transforms

Test and compare two results on Matlab *fft2* and *ifft2*

```
% Test and compare together G1 and G2, g1 và g2
g=[0 0 0 0; 0 1 1 0; 0 1 1 0; 0 0 0 0]
G1=fft2(g)
G2=fft2_new(g)
g1=ifft2(G1)
g2=ifft2_new(G1)
```

- MAGNITUDE(G) = SQRT(REAL(G)^2+IMAGINARY(G)^2)
- PHASE(F) = ATAN(IMAGINARY(G)/REAL(G))

Image Transforms

Example 3.6: Consider the image g(x,y) shown in Fig. 3.3. Calculate the 2D DFT of this image.

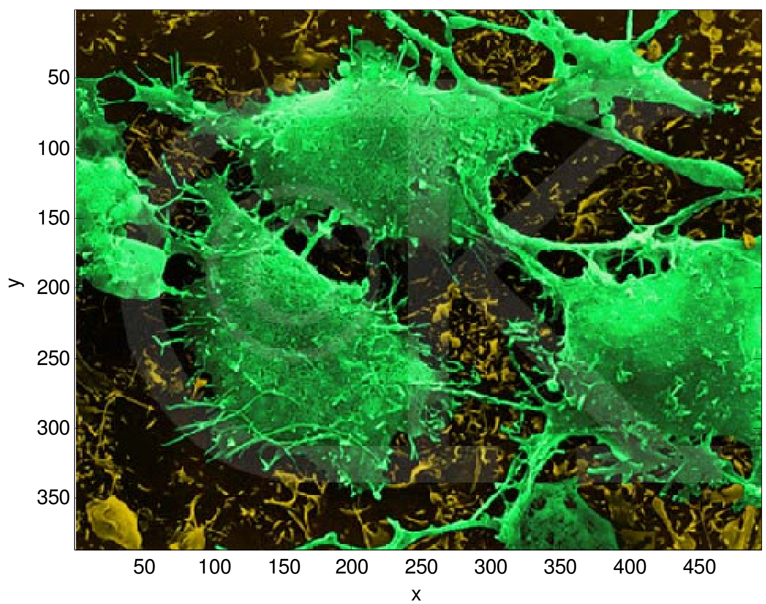


Fig. 3.3: Image in the time domain

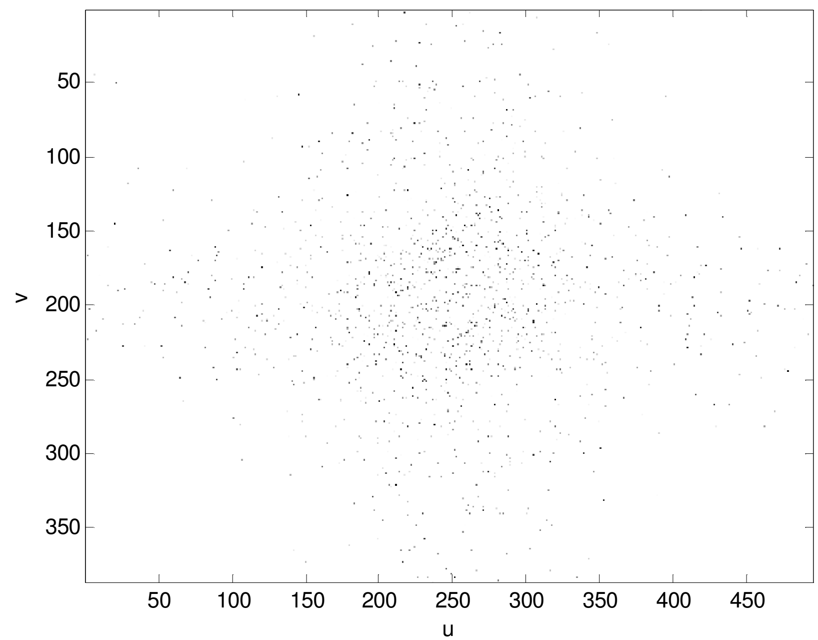


Fig. 3.4: Magnitude of the DFT of image

Image Transforms

Example:

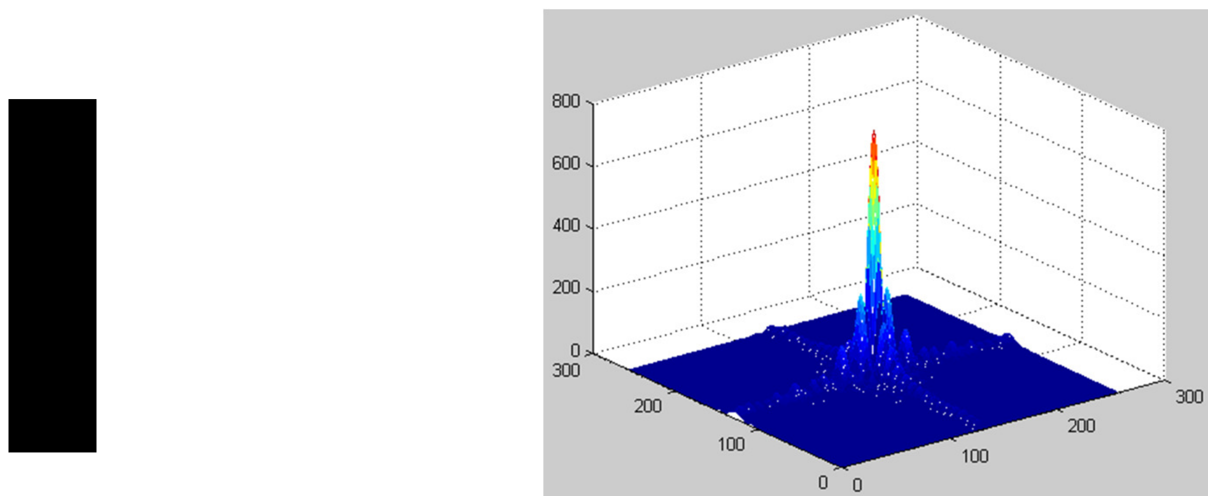


Fig. 3.5: Image and Magnitude of the DFT of image in 3D display

Image Transforms

The End