52nd—24th INTERNATIONAL-RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
Problem 18

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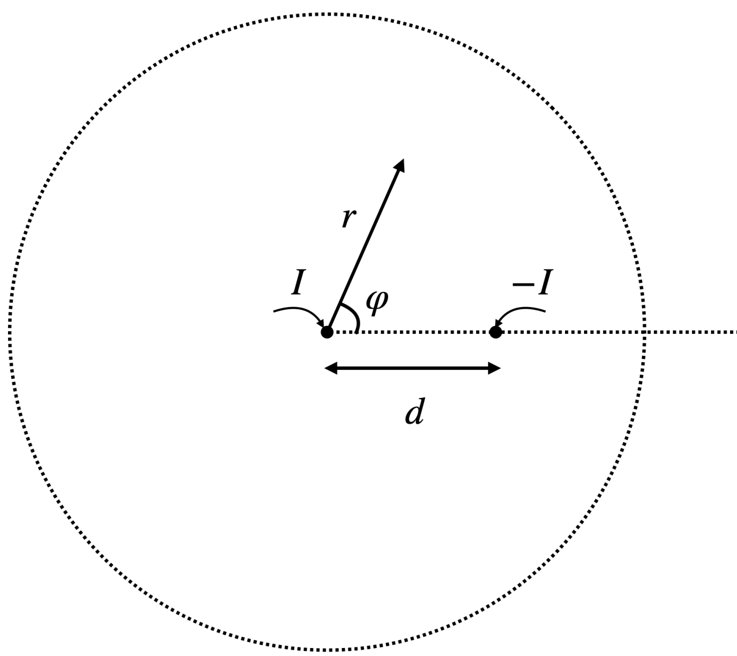
We determine the coordinate of a point on conical shell with the spherical coordinate system (r, θ, φ) where $\theta = \arcsin\left(\frac{1}{\pi}\right) = const$, so we actually only need (r, φ). Choose this coordinate system so that $\varphi = 0$ at B and $\varphi = \pi$ at C.
With our coordinate system, if we let $j'_r = \frac{j_r}{\sin\theta} = \pi j_r$, the solution of this problem will be the same as when we consider a material of a large, thin, homogeneous metal plate.



In that case, we have:

$$j'_r = \frac{I}{2\pi\delta}\left(\frac{1}{r} - \frac{r - d\cos\varphi}{d^2 + r^2 - 2dr\cos\varphi}\right).$$
$$j_\varphi = \frac{I}{2\pi\delta}\frac{d\sin\varphi}{d^2 + r^2 - 2dr\cos\varphi}.$$

Return to our problem where $d = \pi R$, at C, $r = \pi R$ and $\varphi = \pi$ so the density current is:

$$j_C = \pi j'_C = \frac{I}{4\pi\delta R}.$$