

52nd—24th INTERNATIONAL-RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS

Problem 1

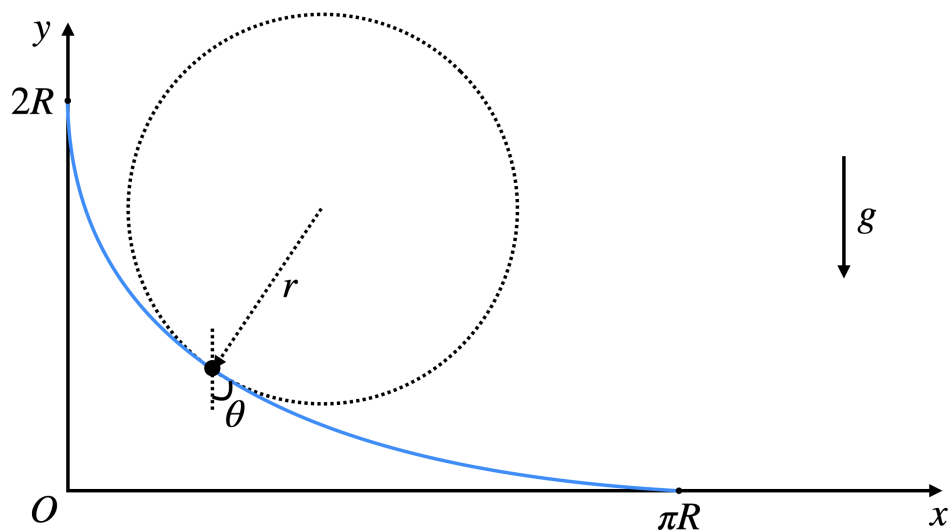
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We note the graph of a semi-cycloid-shaped can be performed by only one degree of freedom θ :

$$x = R(2\theta - \sin 2\theta)$$

$$y = R(1 + \cos 2\theta).$$



We take differential of these equation and receive some result about this graph:

- Angle formed by the tangent of the graph to the vertical is θ .
- Radius of curvature:

$$\begin{aligned} r(\theta) &= \left| \frac{[x'(\theta)^2 + y'(\theta)^2]^{\frac{3}{2}}}{x'(\theta)y''(\theta) - y'(\theta)x''(\theta)} \right| \\ &= 4R \sin(\theta). \end{aligned}$$

- The speed of the body:

$$\begin{aligned}
v &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \\
&= 4R \sin(\theta) \dot{\theta} \\
\Rightarrow \dot{v} &= 4R \cos(\theta) \ddot{\theta} + 4R \sin(\theta) \dot{\theta}.
\end{aligned}$$

When the body slides down the slope, θ run from 0 (i.e $x = 0$, $y = 2R$) to $\frac{\pi}{2}$ (i.e $x = \pi R$, $y = 0$). The differential equation governing the motion of the body is:

- Perpendicular to the orbital tangent: $\frac{mv^2}{r} = N - mg \sin \theta$.
- Orbital tangen: $m\dot{v} = mg \cos \theta - \mu N$.

$$\Rightarrow \dot{v} = g(\cos \theta - \mu \sin \theta) - \mu \frac{v^2}{r}.$$

Put \dot{v} , v , α and r in this equation, we have:

$$\ddot{\theta} + (\cot \theta - \mu) \dot{\theta}^2 = \frac{g}{4R} (\cot \theta - \mu).$$

Multiply this equation with $\sin^2(\theta)e^{-2\mu\theta}$:

$$\begin{aligned}
\frac{d}{d\theta} \left[\dot{\theta}^2 \sin^2(\theta) e^{-2\mu\theta} \right] &= \frac{g}{2R} (\cot \theta - \mu) \sin^2(\theta) e^{-2\mu\theta}. \\
\Rightarrow \dot{\theta}^2 \sin^2(\theta) e^{-2\mu\theta} &= \frac{g}{4R} \sin^2(x) e^{-2\mu\theta} + C \\
\Rightarrow \dot{\theta} &= \sqrt{\frac{g}{4R} + \frac{C e^{2\mu\theta}}{\sin^2(\theta)}}
\end{aligned}$$

At the bottom of slope ($\theta = \frac{\pi}{2}$), the body is stop (i.e $\dot{\theta} = 0$), so we can find the constant $C = -\frac{ge^{-\mu\pi}}{4R}$.

Because when we choose the top of slope is the place to slide the body down, we can't receive a meaningful result, so we will choose a very close place with $\theta = \theta_0$ (with $\theta_0 \ll 1$) to solve this problem. Thus, we find μ to the body stop at the bottom of slope is:

$$\mu = -\frac{2 \ln(\theta_0)}{\pi}.$$