52nd—24th INTERNATIONAL-RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
Problem 1

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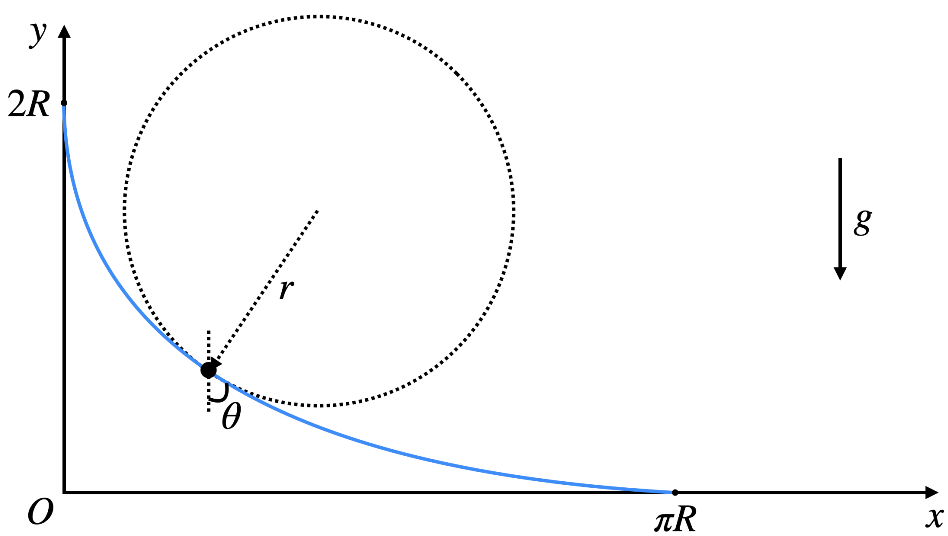
We note the graph of a semi-cycloid-shaped can be performed by only one degree of freedom θ:

$$x = R\left(2\theta - \sin 2\theta\right)$$
$$y = R\left(1 + \cos 2\theta\right).$$



We take differential of these equation and receive some result about this graph:

- Angle formed by the tangent of the graph to the vertical is θ.
- Radius of curvature:

$$\begin{aligned} r(\theta) &= \left|\frac{\left[x'(\theta)^2 + y'(\theta)^2\right]^{\frac{3}{2}}}{x'(\theta)y"(\theta) - y'(\theta)x"(\theta)}\right| \\ &= 4R\sin\left(\theta\right). \end{aligned}$$

- The speed of the body:

$$\begin{aligned} v &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \\ &= 4R\sin(\theta)\,\dot{\theta} \\ \Rightarrow \dot{v} &= 4R\cos(\theta)\,\dot{\theta}^2 + 4R\sin(\theta)\,\ddot{\theta}. \end{aligned}$$

When the body slides down the slope, θ run from 0 (i.e $x = 0$, $y = 2R$) to $\frac{\pi}{2}$ (i.e $x = \pi R$, $y = 0$). The differential equation governing the motion of the body is:

- Perpendicular to the orbital tangent: $\frac{mv^2}{r} = N - mg\sin\theta$.
- Orbital tangen: $m\dot{v} = mg\cos\theta - \mu N$.

$$\Rightarrow \dot{v} = g\left(\cos\theta - \mu\sin\theta\right) - \mu\frac{v^2}{r}.$$

Put $\dot{v}$, v, α and r in this equation, we have:

$$\ddot{\theta} + \left(\cot\theta - \mu\right)\dot{\theta}^2 = \frac{g}{4R}\left(\cot\theta - \mu\right).$$

Multiply this equation with $\sin^2(\theta)e^{-2\mu\theta}$:

$$\begin{aligned} \frac{d}{d\theta}\left[\dot{\theta}^2\sin^2(\theta)e^{-2\mu\theta}\right] &= \frac{g}{2R}\left(\cot\theta - \mu\right)\sin^2(\theta)e^{-2\mu\theta}. \\ \Rightarrow \dot{\theta}^2\sin^2(\theta)e^{-2\mu\theta} &= \frac{g}{4R}\sin^2(x)e^{-2\mu\theta} + C \\ \Rightarrow \dot{\theta} &= \sqrt{\frac{g}{4R} + \frac{Ce^{2\mu\theta}}{\sin^2(\theta)}} \end{aligned}$$

At the bottom of slope ($\theta = \frac{\pi}{2}$), the body is stop (i.e $\dot{\theta} = 0$), so we can find the constant $C = -\frac{ge^{-\mu\pi}}{4R}$.

Because when we choose the top of slope is the place to slide the body down, we can't receive a meaningful result, so we will choose a very close place with $\theta = \theta_0$ (with $\theta_0 \ll 1$) to solve this problem. Thus, we find μ to the body stop at the bottom of slope is:

$$\mu = -\frac{2\ln\left(\theta_0\right)}{\pi}.$$