52nd—24th INTERNATIONAL-RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS Problem 14

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Using the Ampere's law, we know that the magnetic field inside the soleloid (by the soleloid) is

$$B_0 = \frac{\mu_0 N I}{l}. \tag{1}$$

We can denote easily that in the homogeneous magnetic field, a tiny small iron sphere with relative permeability μ_r will have the magnetic dipole [1] :

$$m = V \frac{3\left(\mu_r - 1\right) B_0}{\mu_0 \left(\mu_r + 2\right)}. \tag{2}$$

The magnetic flux that the dipole effect on a circle wire distance z from the dipole is:

$$\frac{d\Phi(z)}{dN} = \int_0^r \frac{\mu_0}{4\pi} \frac{3z^2 - \left(z^2 + r^2\right)}{\left(z^2 + r^2\right)^{\frac{5}{2}}} \cdot 2\pi r dr = \frac{\mu_0 m r^2}{2\left(z^2 + r^2\right)^{\frac{3}{2}}}.$$

Where $dN = \frac{N}{l} dz$, so the total magnetic flux of the soleloid (caused by dipole) will be:

$$\Phi = \int_{-\infty}^{\infty} \frac{N}{l} \frac{\mu_0 m r^2}{2\left(z^2 + r^2\right)^{\frac{3}{2}}} dz = \frac{N \mu_0 m}{l}. \tag{3}$$

From (1), (2) and (3), we have:

$$\Phi = \frac{3\mu_0 \left(\mu_r - 1\right) V N^2}{\left(\mu_r + 2\right) l^2} I.$$

Therefore, the self-inductance change of the solenoid is;

$$\Delta L = \frac{3\mu_0 \left(\mu_r - 1\right) V N^2}{\left(\mu_r + 2\right) l^2}.$$

With $\mu_r \gg 1$, the result will be:

$$\Delta L = \frac{3\mu_0 V N^2}{l^2}.$$

References

[1] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (John Wiley & Sons, New York, NY, 1998).