52nd—24th INTERNATIONAL-RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
Problem 4

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a) The time that a body need to soar and turn back to the horizontal plane is $T = \frac{2v}{g}$ where v is the initial vertical velocity of that body.

In this problem, we call v_x and v_y are the initial horizontal velocity and the initial vertical velocity of the ball. After hiting the wall, the ball will have the horizontal velocity is kv_x. So time for the ball hit the wall will be k times to bounce back A. So that, we have:

$$\frac{v_y}{g} = k\left(\frac{v_y}{g} + 2k\frac{v_y}{g} + 2k^2\frac{v_y}{g} + 2k^3\frac{v_y}{g} + ... + 2k^N\frac{v_y}{g}\right)$$

$$\Leftrightarrow 1 = k\left(2\frac{1-k^{N+1}}{1-k} - 1\right).$$

$$\Leftrightarrow N = \log_k\left(\frac{k^2+2k-1}{2}\right) - 2.$$

Choose $f(x) = \log_x\left(\frac{x^2+2x-1}{2}\right) - 2$ with $x < 1$ and $f^{-1}(x)$ is the inverse function of $f(x)$.

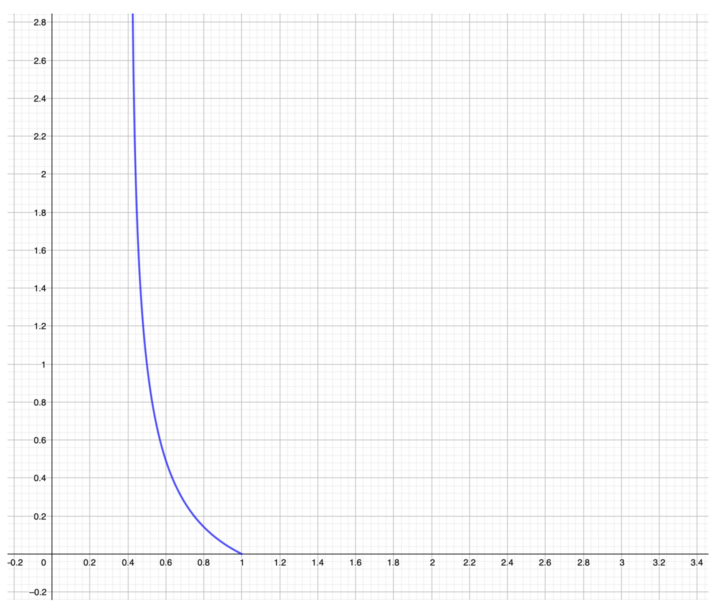

Figure 1: Function $y = f(x)$

So we can write:

$$k = f^{-1}(N).$$

When $N \to \infty$, $k \to -1 + \sqrt{2} \approx 0.414$ and when $N = 1$, $k = \frac{1}{2}$.

b) Cause the time the ball in the air only depend on v_y and k, so we let $v_y = v$. This time is:

$$t = 2\frac{v}{g} + 2k\frac{v}{g} + 2k^2\frac{v}{g} + 2k^3\frac{v}{g} + ... = \frac{2v}{g(1-k)}.$$

When $N = \infty$, time the ball in the air is $t = \left(2 + \sqrt{2}\right)\frac{v}{g} \approx 3.414\frac{v}{g}$.