5.4 Exercises

1. Derive (5.6) $\alpha = \dfrac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$

where $\sigma_X^2 = Var(X)$, $\sigma_Y^2 = Var(Y)$, $\sigma_{XY} = Cov(X,Y)$

α is a solution to the equation:

$$\frac{d}{d\alpha} Var[\alpha X + (1-\alpha)Y] = 0$$

$$Var[\alpha X + (1-\alpha)Y] = \alpha^2\sigma_X^2 + (1-\alpha)^2\sigma_Y^2 + 2\alpha(1-\alpha)\sigma_{XY}$$

$$\frac{d}{d\alpha} Var[\alpha X + (1-\alpha)Y] = 2\alpha\sigma_X^2 - 2(1-\alpha)\sigma_Y^2 + 2(1-2\alpha)\sigma_{XY}$$

$$= 2\alpha\sigma_X^2 - 2\sigma_Y^2 + 2\alpha\sigma_Y^2 + 2\sigma_{XY} - 4\alpha\sigma_{XY}$$

$$= 2\alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}) - 2\sigma_Y^2 + 2\sigma_{XY}$$

$$\frac{d}{d\alpha} Var[\alpha X + (1-\alpha)Y] = 0$$

$$\Rightarrow 2\alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}) - 2\sigma_Y^2 + 2\sigma_{XY} = 0$$

$$\Rightarrow \alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$$