4.7 Exercises
conceptual

3. Same as exercise 2, but the variance is class-specific σ_k^2.

$$p_k(x) = \frac{\pi_k (2\pi\sigma_k)^{-1/2} \exp\left[-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right]}{\sum_{l=1}^{K} \pi_k (2\pi\sigma_k)^{-1/2} \exp\left[\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right]}$$

$$k^{Bayes} = \arg\max_k p_k(x) = \arg\max_k \ln p_k(x)$$

$$= \arg\max_k \ln\left\{\pi_k (2\pi\sigma_k)^{-1/2} \exp\left[-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right]\right\}$$

$$= \arg\max_k \left\{\ln \pi_k - \frac{1}{2}\ln 2\pi\sigma_k - \frac{1}{2\sigma_k^2}(x-\mu_k)^2\right\}$$

$$= \arg\max_k \left\{\ln \pi_k - \frac{1}{2}\ln 2\pi\sigma_k - \frac{1}{2\sigma_k^2}(x^2 - 2\mu_k x + \mu_k^2)\right\}$$

$$= \arg\max_k \underbrace{\left\{-\frac{x^2}{2\sigma_k^2} + \frac{\mu_k}{\sigma_k^2}x + \ln \pi_k - \frac{1}{2}\ln 2\pi\sigma_k - \frac{\mu_k^2}{2\sigma_k^2}\right\}}$$

discriminant is a quadratic function of x.