4.7 Exercises

Conceptual

2. In LDA method, ~~with~~ with $p=1$, the Bayesian posterior is given by (4.12):

$$P_K(x) = \frac{\pi_K \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(x-\mu_K)^2\right]}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(x-\mu_l)^2\right]} \qquad (4.12)$$

Bayes' classifier assign x to K^{Bayes} such that $P_{K^{Bayes}}(x)$ is maximum:

$$K^{Bayes} = \arg\max_K P_K(x) = \arg\max_K \ln P_K(x)$$

since ln is monotonically increasing.

~~$\ln P_K(x) \Rightarrow$~~ because the denominator of (4.12) is constant w.r.t. K. So

$$\arg\max_K \ln P_K(x) = \arg\max_K \ln\left\{\pi_K \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(x-\mu_K)^2\right]\right\}$$

$$= \arg\max_K \left\{\ln \pi_K - \underbrace{\ln \sqrt{2\pi}\sigma}_{const} - \frac{1}{2\sigma^2}(x-\mu_K)^2\right\}$$

$$= \arg\max_K \left\{\ln \pi_K - \frac{1}{2\sigma^2}(\underbrace{x^2}_{const} - 2x\mu_K + \mu_K^2)\right\}$$

$$= \arg\max_K \underbrace{\left\{\frac{x\mu_K}{\sigma^2} - \frac{\mu_K^2}{2\sigma^2} + \ln \pi_K\right\}}_{\text{linear discriminant}}$$