$\boxed{\bullet \text{ In summation notation}}$

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2} \quad , \quad SE(\hat{\beta}) = \sqrt{\frac{\sum (y_i - x_i \hat{\beta})^2}{(n-1)\sum x_i^2}}$$

$$\text{t-statistic} = \frac{\hat{\beta}}{SE(\hat{\beta})} = \frac{\hat{\beta}\sqrt{n-1}\sqrt{\sum x_i^2}}{\sqrt{\sum (y_i - x_i\hat{\beta})^2}}$$

$$= \frac{\sqrt{n-1}\sqrt{\sum x_i^2}}{\sqrt{\sum \left(\frac{y_i - \hat{\beta} x_i}{\hat{\beta}}\right)^2}} = \frac{\sqrt{n-1}\sqrt{\sum x_i^2}}{\sqrt{\sum y_i^2 \frac{1}{\hat{\beta}^2} \underbrace{- 2\frac{1}{\hat{\beta}}\sum x_i y_i}_{-2\sum x_i^2} + \sum x_i^2}}$$

$$= \frac{\sqrt{n-1}\sqrt{\sum x_i^2}}{\sqrt{\frac{\sum y_i^2 (\sum x_i^2)^2}{(\sum x_i y_i)^2} - \sum x_i^2}}$$

$$= \frac{\sqrt{n-1}\sqrt{\sum x_i^2}}{\sqrt{\frac{\sum x_i^2}{(\sum x_i y_i)^2}\left[\sum y_i^2 \sum x_i^2 - (\sum x_i y_i)^2\right]}} = \frac{\sqrt{n-1}\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2 - (\sum x_i y_i)^2}} \quad (*)$$

$(*)$

$\boxed{\bullet \text{ In vector notation}}$

$$\hat{\beta} = \frac{x^T y}{x^T x} \quad , \quad SE(\hat{\beta}) = \frac{\sqrt{(y - \hat{\beta}x)^T (y - \hat{\beta}x)}}{\sqrt{n-1}\sqrt{x^T x}}$$

$$\text{t-statistic} = \frac{\hat{\beta}}{SE(\hat{\beta})} = \frac{\hat{\beta}\sqrt{n-1}\sqrt{x^T x}}{\sqrt{(y - \hat{\beta}x)^T (y - \hat{\beta}x)}} = \frac{\sqrt{n-1}\sqrt{x^T x}}{\sqrt{\left(\frac{y}{\hat{\beta}} - x\right)^T \left(\frac{y}{\hat{\beta}} - x\right)}}$$

$$= \frac{\sqrt{n-1}\sqrt{x^T x}}{\sqrt{\frac{y^T y}{\hat{\beta}^2} - \frac{y^T x}{\hat{\beta}} - \frac{x^T y}{\hat{\beta}} + x^T x}} = \frac{\sqrt{n-1}\sqrt{x^T x}}{\sqrt{\frac{y^T y}{\hat{\beta}^2} - 2\frac{x^T y}{\hat{\beta}} + x^T x}}$$

$$= \frac{\sqrt{n-1}\sqrt{x^T x}}{\sqrt{\frac{y^T y}{\hat{\beta}^2} - x^T x}} = \frac{\sqrt{n-1}\sqrt{x^T x}}{\sqrt{\frac{y^T y (x^T x)^2}{(x^T y)^2} - x^T x}} = \frac{\sqrt{n-1}\sqrt{x^T x}}{\sqrt{\frac{x^T x}{(x^T y)^2}\left[(y^T y)(x^T x) - (x^T y)^2\right]}}$$

$$= (*)$$