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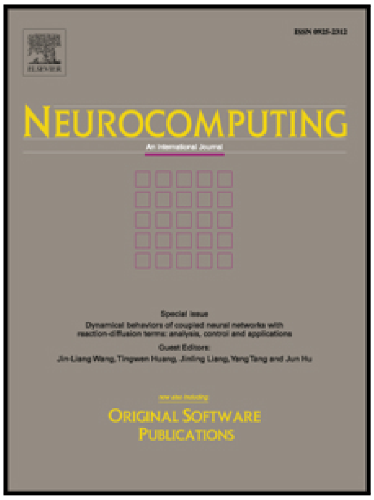

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Neural-Network-Based Synchronous Iteration Learning Method for Multi-Player Zero-Sum Games

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Abstract

In this paper, a synchronous solution method for multi-player zero-sum games without system dynamics is established based on neural network. The policy iteration (PI) algorithm is presented to solve the Hamilton-Jacobi-Bellman (HJB) equation. It is proven that the obtained iterative cost function is convergent to the optimal game value. For avoiding system dynamics, off-policy learning method is given to obtain the iterative cost function, controls and disturbances based on PI. Critic neural network (CNN), action neural networks (ANNs) and disturbance neural networks (DNNs) are used to approximate the cost function, controls and disturbances. The weights of neural networks compose the synchronous weight matrix, and the uniformly ultimately bounded (UUB) of the synchronous weight matrix is proven. Two examples are given to show that the effectiveness of the proposed synchronous solution method for multi-player ZS games.

Key words: Adaptive dynamic programming, Approximate dynamic programming, Adaptive critic designs, Multi-player, Iteration learning, Neural network
PACS:

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1 Introduction

The importance of strategic behavior in the human and social world is increasingly recognized in theory and practice. As a result, game theory has emerged as a fundamental instrument in pure and applied research [1]. Modern day society relies on the operation of complex systems, including aircraft, automobiles, electric power systems, economic entities, business organizations, banking and finance systems, computer networks, manufacturing systems, and industrial processes. Networked dynamical agents have cooperative team-based goals as well as individual selfish goals, and their interplay can be complex and yield unexpected results in terms of emergent teams. Cooperation and conflict of multiple decision-makers for such systems can be studied within the field of cooperative and noncooperative game theory [2]. It knows that many real-world systems are often controlled by more than one controller or decision maker with each using an individual strategy. These controllers often operate in a group with a general quadratic performance index function as a game. Therefore, some scholars research the multi-player games. In [3], off-policy integral reinforcement learning method was developed to solve nonlinear continuous-time multi-player non-zero-sum (NZS) games. In [4], a multi-player zero-sum (ZS) differential games for a class of continuous-time uncertain nonlinear systems were solved using upper and lower iterations. ZS game theory relies on solving the Hamilton-Jacobi-Isaacs (HJI) equations, a generalized version of the Hamilton-Jacobi-Bellman(HJB) equations appearing in optimal control problems. In the nonlinear case the HJI equations are difficult or impossible to solve, and may not have global analytic solutions even in simple cases. Therefore, many approximate methods are proposed to obtain the solution of HJI equations [5–8].

Adaptive dynamic programming (ADP) algorithm is an effective approximate method in optimal control field [9–13]. ADP algorithms include value iteration (VI) and policy iteration (PI) [14–17]. VI is a Lyapunov recursion, which is easy to implement and does not require Lyapunov equation solutions [18–21]. In [22], discrete-time VI was proposed to solve HJB equation approximately with convergence analysis. In [23], a novel non-model-based, data-driven adaptive optimal controller was presented by continuous-time VI. In [24], a class of continuous-time nonlinear two-player ZS differential games was considered, VI ADP method was proposed for the situations that the saddle point exists or does not exist. On the other hand, PI refers to a class of algorithms built as a two-step iteration: policy evaluation and policy improvement [25], starting from evaluating the performance index function of a given initial admissible (stabilizing) controller [26–28]. In [29], PI algorithm and convergence analysis were given for nonlinear systems with saturating actuators. In [30], optimal model-free output synchronization of heterogeneous systems using off-policy reinforcement learning was developed. In [31], a data-driven ADP method was proposed for a class of continuous-time unknown nonlinear systems ZS optimal control problems. In[32], an online solution method for two-player ZS games was presented by synchronous PI.

Although the progress on ADP algorithm is significant in the optimal control field, within the radius of our knowledge, it is still an open problem about how to solve multi-player ZS games for completely unknown continuous-time nonlinear systems. In this paper, this open problem will be explicitly figured out. The main contributions of this paper are summarized as follows.
1): A synchronous solution method based on PI algorithm and neural networks is established.
2): It is proven that the iterative cost function converges to the optimal game value with system dynamics for traditional PI algorithm.
3): Synchronous solution method is given to solve the off-policy HJB equation with convergence analysis, according to critic neural network (CNN), action neural networks (ANNs) and disturbance neural networks (DNNs).
4): The uniformly ultimately bounded (UUB) of the synchronous weight matrix is proven.

The rest of this paper is organized as follows. In Section 2, we present the motivations and preliminaries of the discussed problem. In Section 3, the synchronous solution of multi-player ZS games is developed and the convergence proof is given. In Section 4, two examples are given to demonstrate the effectiveness of the proposed scheme. In Section 5, the conclusion is drawn.

2 Motivations and Preliminaries

In this paper, we consider the continuous-time nonlinear system described by

$$\dot{x} = f(x) + g(x)\sum_{i=1}^{p} u_i + h(x)\sum_{j=1}^{q} d_j \tag{1}$$

where $x \in \Omega \in R^n$ is the system state, $u_i \in R^{m_1}$ and $d_j \in R^{m_2}$ are the control input and the disturbance input, respectively. $f(x) \in R^n$, $g(x)$ and $h(x)$ are unknown functions. $f(0) = 0$ and $x = 0$ is an equilibrium point of the system. Assume that $f(x)$, $g(x)$ and $h(x)$ are locally Lipschitz functions on the compact set Ω that contains the origin. The dynamical system is stabilizable on Ω. The performance index function is a generalized quadratic form given by

$$J(x(0), U_p, D_q) = \int_0^\infty \left\{x^T Q x + \sum_{i=1}^{p} u_i^T R_i u_i - \sum_{j=1}^{q} d_j^T S_j\, d_j\right\} dt \tag{2}$$

where Q, R_i and S_j are positive definite matrixes, $U_p = \{u_1, \cdots, u_p\}$ and $D_q = \{d_1, \cdots, d_q\}$. Then, we define the multi-player ZS differential game subject to (1) as

$$V^*(x(0)) = \inf_{u_1}\inf_{u_2}\cdots\inf_{u_p}\sup_{d_1}\sup_{d_2}\cdots\sup_{d_q} J(x(0), U_p, D_q) \tag{3}$$

The multi-player ZS differential game selects the minimizing player set U_p and the maximizing player set D_q such that the saddle point U_p^* and D_q^* satisfies the following inequalities:

$$J(x, U_p^*, D_q) \leq J(x, U_p^*, D_q^*) \leq J(x, U_p, D_q^*) \tag{4}$$

where $U_p^* = \{u_1^*, \cdots, u_p^*\}$ and $D_q^* = \{d_1^*, \cdots, d_q^*\}$.

In this paper, we assume that the multi-player optimal control problem has a unique solution if and only if the Nash condition holds [33]

$$V^*(x) = \inf_{U_p}\sup_{D_q} J(x, U_p, D_q) = \sup_{D_q}\inf_{U_p} J(x, U_p, D_q) \tag{5}$$

If we give the feedback policy $(U_p(x), D_q(x))$, then the value or cost of the policy is

$$V(x(t)) = \int_t^\infty \left\{x^T Q x + \sum\nolimits_{i=1}^p u_i^T R_i u_i - \sum\nolimits_{j=1}^q d_j^T S_j\, d_j\right\} dt \tag{6}$$

By using Leibniz's formula and differentiating, (6) has a differential equivalent. Then we can obtain the nonlinear ZS game Bellman equation, which is given in terms of the Hamiltonian function

$$\begin{aligned} &H(x, \nabla V, U_p, D_q) \\ &= x^T Q x + \sum\nolimits_{i=1}^p u_i^T R_i u_i - \sum\nolimits_{j=1}^q d_j^T S_j d_j \\ &\quad + \nabla V^T (f + g \sum\nolimits_{i=1}^p u_i + h \sum\nolimits_{j=1}^q d_j) \\ &= 0 \end{aligned} \tag{7}$$

where $\nabla V = \dfrac{\partial V}{\partial x}$. The stationary conditions are

$$\frac{\partial H}{\partial u_i} = 0, i = 1, 2, \cdots, p \tag{8}$$

and

$$\frac{\partial H}{\partial d_j} = 0, j = 1, 2, \cdots, q \tag{9}$$

According to (7), we have the optimal controls and the disturbances are

$$u_i^* = -\frac{1}{2} R_i^{-1} g^T \nabla V^*, i = 1, 2, \cdots, p \tag{10}$$

and

$$d_j^* = \frac{1}{2} S_j^{-1} h^T \nabla V^*, j = 1, 2, \cdots, q \tag{11}$$

From Bellman equation (7), we can derive V^* from the solution of the HJI equation

$$\begin{aligned} 0 = x^T Q x + \nabla V^T f - \frac{1}{4} \sum_{i=1}^{p} \nabla V^T g R_i^{-1} g^T \nabla V \\ + \frac{1}{4} \sum_{j=1}^{q} \nabla V^T h S_j^{-1} h^T \nabla V \end{aligned} \tag{12}$$

Note that if (12) is solved, then the optimal controls are obtained. In general case, the PI algorithm can be applied to get V^*. The algorithm implementation process is given in Algorithm 1.

Algorithm 1 PI for nonlinear multi-player ZS differential games

1: Start with stabilizing initial policies $u_1^{[0]}$, $u_2^{[0]}$, $\cdots$, $u_p^{[0]}$, and $d_1^{[0]}$, $d_2^{[0]}$, $\cdots$, $d_q^{[0]}$.

2: Let $k = 1, 2, 3, \cdots$, solve $V^{[k]}$ from

$$\begin{aligned} 0 = x^T Q x + \sum_{i=1}^{p} u_i^{[k]T} R_i u_i^{[k]} - \sum_{j=1}^{q} d_j^{[k]T} S_j d_j^{[k]} \\ + \nabla V^{[k]T} (f + g \sum_{i=1}^{p} u_i^{[k]} + h \sum_{j=1}^{q} d_j^{[k]}) \end{aligned} \tag{13}$$

3: Update control and disturbance using

$$u_i^{[k+1]} = -\frac{1}{2} R_i^{-1} g^T \nabla V^{[k]} \tag{14}$$

and

$$d_j^{[k+1]} = \frac{1}{2} S_j^{-1} h^T \nabla V^{[k]} \tag{15}$$

4: Let $k = k + 1$, return to Step 2 and continue.

The convergence of Algorithm 1 will be analyzed in the next theorem.

Theorem 1 *Define $V^{[k]}$ as in (13). Let control policy $u_i^{[k]}$ and disturbance policy $d_j^{[k]}$ be in (14) and (15), respectively. Then the iterative values $V^{[k]}$ converge to the optimal game values V^*, as $k \to \infty$.*

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Dùng Lyapunov

Proof: According to (13), we have

$$\dot{V}^{[k+1]} = -x^T Q x - \sum_{i=1}^{p} u_i^{[k+1]T} R_i u_i^{[k+1]} + \sum_{j=1}^{q} d_j^{[k+1]T} S_j d_j^{[k+1]} \tag{16}$$

Then

$$\begin{aligned}\dot{V}^{[k]} &= -x^T Q x - \sum_{i=1}^{p} u_i^{[k]T} R_i u_i^{[k]} + \sum_{j=1}^{q} d_j^{[k]T} S_j d_j^{[k]}\\ &\quad - \sum_{i=1}^{p} u_i^{[k+1]T} R_i u_i^{[k+1]} + \sum_{j=1}^{q} d_j^{[k+1]T} S_j d_j^{[k+1]}\\ &\quad + \sum_{i=1}^{p} u_i^{[k+1]T} R_i u_i^{[k+1]} - \sum_{j=1}^{q} d_j^{[k+1]T} S_j d_j^{[k+1]}\\ &= \dot{V}^{[k+1]} + \sum_{i=1}^{p} u_i^{[k+1]T} R_i u_i^{[k+1]} - \sum_{j=1}^{q} d_j^{[k+1]T} S_j d_j^{[k+1]}\\ &\quad - \sum_{i=1}^{p} u_i^{[k]T} R_i u_i^{[k]} + \sum_{j=1}^{q} d_j^{[k]T} S_j d_j^{[k]}\end{aligned} \tag{17}$$

By transformation, we have

$$\begin{aligned}\dot{V}^{[k]} &= \dot{V}^{[k+1]} - \sum_{i=1}^{p} (u_i^{[k+1]} - u_i^{[k]})^T R_i (u_i^{[k+1]} - u_i^{[k]})\\ &\quad +2\sum_{i=1}^{p} u_i^{[k+1]T} R_i (u_i^{[k+1]} - u_i^{[k]})\\ &\quad +\sum_{j=1}^{q} (d_j^{[k+1]} - d_j^{[k]})^T S_j (d_j^{[k+1]} - d_j^{[k]})\\ &\quad -2\sum_{i=1}^{p} d_j^{[k+1]T} S_j (d_j^{[k+1]} - d_j^{[k]})\end{aligned} \tag{18}$$

Let $\Delta u_i^{[k]} = u_i^{[k+1]} - u_i^{[k]}$ and $\Delta d_j^{[k]} = d_j^{[k+1]} - d_j^{[k]}$, then

$$\begin{aligned}\dot{V}^{[k]} &= \dot{V}^{[k+1]} - \sum_{i=1}^{p} \Delta u_i^{[k]T} R_i \Delta u_i^{[k]} + 2\sum_{i=1}^{p} u_i^{[k+1]T} R_i \Delta u_i^{[k]}\\ &\quad + \sum_{j=1}^{q} \Delta d_j^{[k]T} S_j \Delta d_j^{[k]} - 2\sum_{i=1}^{p} d_j^{[k+1]T} S_j \Delta d_j^{[k]}\end{aligned} \tag{19}$$

From (14) and (15), we have

$$\nabla V^{[k]T} g = -2u_i^{[k+1]T} R_i \tag{20}$$

and

$$\nabla V^{[k]T} h = 2d_j^{[k+1]T} S_j \tag{21}$$

Then (19) is expressed as

$$\begin{aligned}\dot{V}^{[k]} &= \dot{V}^{[k+1]} - \sum_{i=1}^{p} \Delta u_i^{[k]T} R_i \Delta u_i^{[k]} - \sum_{i=1}^{p} \Delta V^{[k]T} g \Delta u_i^{[k]}\\ &\quad + \sum_{j=1}^{q} \Delta d_j^{[k]T} S_j \Delta d_j^{[k]} - \sum_{i=1}^{p} \Delta V^{[k]T} h \Delta d_j^{[k]}\end{aligned} \tag{22}$$

Thus a sufficient conditions for $\dot{V}^{[k]} \leq \dot{V}^{[k+1]}$ are

$$\Delta u_i^{[k]T} R_i \Delta u_i^{[k]} - \Delta V^{[k]T} g \Delta u_i^{[k]} > 0 \tag{23}$$

and

$$\Delta d_j^{[k]T} S_j \Delta d_j^{[k]} - \Delta V^{[k]T} h \Delta d_j^{[k]} < 0 \tag{24}$$

Hence, if $\delta^H(S_j)||\Delta d_j^{[k]}|| \leq ||\Delta V^{[k]T} h||$ and $\Delta V^{[k]T} g \Delta u_i^{[k]} > 0$, or $\delta^H(S_j)||\Delta d_j^{[k]}|| \leq ||\Delta V^{[k]T} h||$ and $\delta_L(R_i)||\Delta u_i^{[k]}|| > ||\Delta V^{[k]T} g||$, where δ_L is the operator which takes the minimum singular value, and δ^H is the operator which takes the maximum singular value. Then $\dot{V}^{[k]} \leq \dot{V}^{[k+1]}$. The proof completes.

From Algorithm 1, we can see that the PI algorithm depends on system dynamics, which is unknown in this paper. Therefore, in the next section, off-policy PI algorithm will be presented which can solve the control and disturbance policies synchronously.

3 Synchronous Solution of Multi-player ZS Games

In this section, off-policy algorithm will be proposed based on Algorithm 1. The neural networks implementation process is also given. Based on that, the stability of the synchronous solution method is proven.

3.1 Derivation of off-policy algorithm

Let $u_i^{[k]}$ and $d_j^{[k]}$ be obtained by (14) and (15), then the original system (1) is rewritten as

$$\begin{aligned} \dot{x} = f + g \sum\nolimits_{i=1}^{p} u_i^{[k]} + h \sum\nolimits_{j=1}^{q} d_j^{[k]} \\ + g \sum\nolimits_{i=1}^{p} (u_i - u_i^{[k]}) + h \sum\nolimits_{j=1}^{q} (d_j - d_j^{[k]}) \end{aligned} \tag{25}$$

Substitute (25) into (6), we have

$$\begin{aligned} & V^{[k]}(x(t+T)) - V^{[k]}(x(t)) \\ & = \int_t^{t+T} \nabla V^{[k]T} \dot{x} d\tau \\ & = \int_t^{t+T} \nabla V^{[k]T} \left(f + g \sum\nolimits_{i=1}^{p} u_i^{[k]} + h \sum\nolimits_{j=1}^{q} d_j^{[k]} \right) d\tau \\ & \quad + \int_t^{t+T} \nabla V^{[k]T} \left(g \sum\nolimits_{i=1}^{p} (u_i - u_i^{[k]}) + h \sum\nolimits_{j=1}^{q} (d_j - d_j^{[k]}) \right) d\tau \end{aligned} \tag{26}$$

According to (13), (26) is

$$
\begin{aligned}
&V^{[k]}(x(t+T))-V^{[k]}(x(t))\\
&=-\int\limits_{t}^{t+T}\left(x^TQx+\sum\nolimits_{i=1}^{p}u_i^{[k]T}R_iu_i^{[k]}-\sum\nolimits_{j=1}^{q}d_j^{[k]T}\,S_jd_j^{[k]}\right)d\tau\\
&\quad+\int\limits_{t}^{t+T}\nabla V^{[k]T}\left(g\sum\nolimits_{i=1}^{p}(u_i-u_i^{[k]})+h\sum\nolimits_{j=1}^{q}(d_j-d_j^{[k]})\right)d\tau
\end{aligned}
\tag{27}
$$

Then (27) is the off-policy Bellman equation for multi-player ZS games, which is expressed as

$$
\begin{aligned}
&V^{[k]}(x(t+T))-V^{[k]}(x(t))\\
&=-\int\limits_{t}^{t+T}\left(x^TQx+\sum\nolimits_{i=1}^{p}u_i^{[k]T}R_iu_i^{[k]}-\sum\nolimits_{j=1}^{q}d_j^{[k]T}\,S_jd_j^{[k]}\right)d\tau\\
&\quad+\int\limits_{t}^{t+T}-2\Big(u_i^{[k+1]T}R_i\sum\nolimits_{i=1}^{p}(u_i-u_i^{[k]})\\
&\quad-d_j^{[k+1]T}S_j\sum\nolimits_{j=1}^{q}(d_j-d_j^{[k]})\Big)\,d\tau
\end{aligned}
\tag{28}
$$

It can be seen that (28) shows two points. First, the system dynamics is not necessary for obtaining $V^{[k]}$. Second, $u_i^{[k]}$, $d_j^{[k]}$ and $V^{[k]}$ can be obtained synchronously. In the next part, the implementation method for solving (28) will be presented.

3.2 Implementation method for off-policy algorithm

In this part, the method for solving off-policy Bellman equation (28) is given. Critic, action and disturbance networks are applied to approximate $V^{[k]}$, $u_i^{[k]}$ and $d_j^{[k]}$. The implementation block diagram is shown in Fig. 1. Here CNN, ANNs and DNNs are used to approximate the cost, control policies and disturbances.

In the neural network, if the number of hidden layer neurons is L, the weight matrix between the input layer and hidden layer is Y, the weight matrix between the hidden layer and output layer is W and the input vector of the neural network is X, then the output of three-layer neural network is represented by:

$$F_N(X,Y,W)=W^T\hat{\sigma}(YX), \tag{29}$$

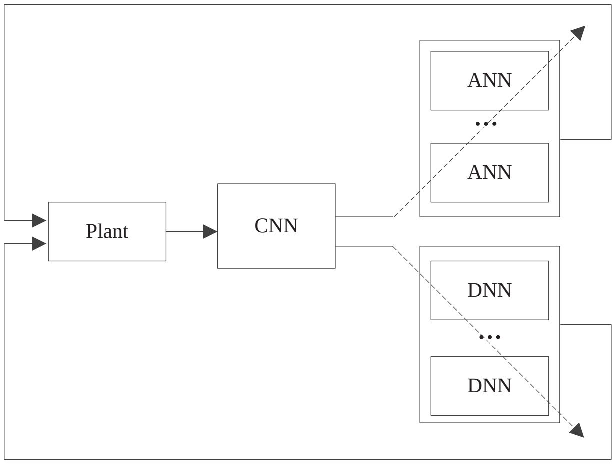

Fig. 1. Implementation block diagram

where $\hat{\sigma}(YX)$ is the activation function. For convenience of analysis, only the output weight W is updating during the training, while the hidden weight is kept unchanged. Hence, in the following part, the neural network function (29) can be simplified by the expression

$$F_N(X,W) = W^T\sigma(X). \tag{30}$$

The neural network expression of CNN is given as

$$V^{[k]}(x) = A^{[k]T}\phi_V(x) + \delta_V(x) \tag{31}$$

where $A^{[k]}$ is the ideal weight of critic network, $\phi_V(x)$ is the active function, and $\delta_V(x)$ is residual error. Let the estimation of $A^{[k]}$ is $\hat{A}^{[k]}$. Then the estimation of $V^{[k]}(x)$ is

$$\hat{V}^{[k]}(x) = \hat{A}^{[k]T}\phi_V(x) \tag{32}$$

and

$$\nabla\hat{V}^{[k]}(x) = \nabla\phi_V^T(x)\hat{A}^{[k]} \tag{33}$$

The neural network expression of ANN is

$$u_i^{[k]} = B_i^{[k]T}\phi_u(x) + \delta_u(x) \tag{34}$$

where $B_i^{[k]}$ is the ideal weight of action network, $\phi_u(x)$ is the active function, and $\delta_u(x)$ is residual error. Let $\hat{B}_i^{[k]}$ be the estimation of $B_i^{[k]}$, then the estimation of $u_i^{[k]}$ is

$$\hat{u}_i^{[k]} = \hat{B}_i^{[k]T}\phi_u(x) \tag{35}$$

The neural network expression of DNN is

$$d_j^{[k]} = C_j^{[k]T}\phi_d(x) + \delta_d(x) \tag{36}$$

where $C_j^{[k]}$ is the ideal weight of action network, $\phi_d(x)$ is the active function, and $\delta_d(x)$ is residual error. Let $\hat{C}_j^{[k]}$ be the estimation of $C_j^{[k]}$, then the estimation of $d_j^{[k]}$ is

$$\hat{d}_j^{[k]} = \hat{C}_j^{[k]T}\phi_d(x) \tag{37}$$

According to (28), we define the equation error as

$$\begin{aligned} e^{[k]} &= \hat{V}^{[k]}(x(t)) - \hat{V}^{[k]}(x(t+T)) \\ &\quad - \int_t^{t+T} \left(x^T Q x + \sum_{i=1}^{p} \hat{u}_i^{[k]T} R_i \hat{u}_i^{[k]} - \sum_{j=1}^{q} \hat{d}_j^{[k]T} S_j \hat{d}_j^{[k]}\right) d\tau \\ &\quad + \int_t^{t+T} -2\Big(\hat{u}_i^{[k+1]T} R_i \sum_{i=1}^{p} (u_i - \hat{u}_i^{[k]}) \\ &\quad -\hat{d}_j^{[k+1]T} S_j \sum_{j=1}^{q} (d_j - \hat{d}_j^{[k]})\Big)\, d\tau \end{aligned} \tag{38}$$

Therefore, substitute (32), (35) and (37) into (38), we have

$$\begin{aligned} e^{[k]} &= \hat{V}^{[k]}(x(t)) - \hat{V}^{[k]}(x(t+T)) \\ &\quad - \int_t^{t+T} \left(x^T Q x + \sum_{i=1}^{p} \hat{u}_i^{[k]T} R_i \hat{u}_i^{[k]} - \sum_{j=1}^{q} \hat{d}_j^{[k]T} S_j \hat{d}_j^{[k]}\right) d\tau \\ &\quad + \int_t^{t+T} -2\Big(\phi_u^T \hat{B}_i^{[k+1]} R_i \sum_{i=1}^{p} (u_i - \hat{u}_i^{[k]}) \\ &\quad - \phi_d^T \hat{C}_j^{[k+1]} S_j \sum_{j=1}^{q} (d_j - \hat{d}_j^{[k]})\Big)\, d\tau \end{aligned} \tag{39}$$

Since

$$\begin{aligned} &\phi_u^T \hat{B}_i^{[k+1]T} R_i \sum_{i=1}^{p} (u_i - \hat{u}_i^{[k]}) \\ &= \left(\left((\sum_{i=1}^{p} (u_i - \hat{u}_i^{[k]}))^T R_i\right) \otimes \phi_u^T\right) vec(\hat{B}_i^{[k+1]}) \end{aligned} \tag{40}$$

where $\otimes$ denotes kronecker product, and

$$\begin{aligned} &\phi_d^T \hat{C}_j^{[k+1]T} S_j \sum_{j=1}^{q} (d_j - \hat{d}_j^{[k]}) \\ &= \left(\left((\sum_{j=1}^{q} (d_j - \hat{d}_j^{[k]}))^T S_j\right) \otimes \phi_d^T\right) vec(\hat{C}_j^{[k+1]}) \end{aligned} \tag{41}$$

Substitute (40) and (41) into (39)

$$
\begin{aligned}
e^{[k]} = & \left((\phi_V(x(t)) - \phi_V(x(t+T)))^T \otimes I \right) \hat{A}^{[k]} \\
& - \int_t^{t+T} \left(x^T Q x + \sum\nolimits_{i=1}^{p} \hat{u}_i^{[k]T} R_i \hat{u}_i^{[k]} - \sum\nolimits_{j=1}^{q} \hat{d}_j^{[k]T} S_j \hat{d}_j^{[k]} \right) d\tau \\
& + \int_t^{t+T} -2 \left(\left(\left((\sum\nolimits_{i=1}^{p} (u_i - \hat{u}_i^{[k]}))^T R_i \right) \otimes \phi_u^T \right) vec(\hat{B}_i^{[k+1]}) \right. \\
& \left. - \left(\left((\sum\nolimits_{j=1}^{q} (d_j - \hat{d}_j^{[k]}))^T S_j \right) \otimes \phi_d^T \right) vec(\hat{C}_j^{[k+1]}) \right) d\tau
\end{aligned} \tag{42}
$$

Define

$$
\Pi_V = (\phi_V(x(t)) - \phi_V(x(t+T)))^T \otimes I \tag{43}
$$

$$
\Pi = \int_t^{t+T} \left(x^T Q x + \sum\nolimits_{i=1}^{p} \hat{u}_i^{[k]T} R_i \hat{u}_i^{[k]} - \sum\nolimits_{j=1}^{q} \hat{d}_j^{[k]T} S_j \hat{d}_j^{[k]} \right) d\tau \tag{44}
$$

$$
\Pi_u = \int_t^{t+T} -2 \left(\left(\left((\sum\nolimits_{i=1}^{p} (u_i - \hat{u}_i^{[k]}))^T R_i \right) \otimes \phi_u^T \right) d\tau \tag{45}
$$

$$
\Pi_d = -\int_t^{t+T} \left((\sum\nolimits_{j=1}^{q} (d_j - \hat{d}_j^{[k]}))^T S_j \right) \otimes \phi_d^T d\tau \tag{46}
$$

Then we have

$$
\begin{aligned}
e^{[k]} &= \Pi_V \hat{A}^{[k]} - \Pi + \Pi_u vec(\hat{B}_i^{[k+1]}) + \Pi_d vec(\hat{C}_j^{[k+1]}) \\
&= [\Pi_V \;\; \Pi_u \;\; \Pi_d] \begin{bmatrix} \hat{A}^{[k]} \\ vec(\hat{B}_i^{[k+1]}) \\ vec(\hat{C}_j^{[k+1]}) \end{bmatrix} - \Pi
\end{aligned} \tag{47}
$$

Define activation function matrix

$$
\Pi_\Pi = [\Pi_V \;\; \Pi_u \;\; \Pi_d] \tag{48}
$$

and the synchronous weight matrix

$$\hat{W}_{i,j}^{[k]} = \begin{bmatrix} \hat{A}^{[k]} \\ vec(\hat{B}_i^{[k+1]}) \\ vec(\hat{C}_j^{[k+1]}) \end{bmatrix} \tag{49}$$

Then (47) is

$$e^{[k]} = \Pi_\Pi \hat{W}_{i,j}^{[k]} - \Pi \tag{50}$$

Define $E^{[k]} = 1/2 e^{[k]T} e^{[k]}$, then according to gradient descent algorithm, the update method of the weight $\hat{W}_{i,j}^{[k]}$ is

$$\dot{\hat{W}}_{i,j}^{[k]} = -\eta_{i,j}^{[k]} \Pi_\Pi^T \left(\Pi_\Pi \hat{W}_{i,j}^{[k]} - \Pi \right) \tag{51}$$

where $\eta_{i,j}^{[k]}$ is a positive number.

According to gradient descent algorithm, the optimal weight $\hat{W}_{i,j}^{[k]}$ makes $E^{[k]}$ minimum, which can be obtained adaptively by (51). Therefore, the weights of critic, action and disturbance networks are solved simultaneously. In this proposed method, only one equation is necessary instead of (13)-(15) in Algorithm 1 to obtain the optimal solution for the multi-player ZS games.

3.3 Stability analysis

Theorem 2 *Let the update method for critic, action and disturbance networks be as in (51). Define the weight estimation error as $\tilde{W}_{i,j}^{[k]} = W_{i,j}^{[k]} - \hat{W}_{i,j}^{[k]}$, Then $\tilde{W}_{i,j}^{[k]}$ is UUB.*

Proof: Let Lyapunov function candidate be:

$$\Lambda_{i,j}^{[k]} = \frac{\alpha}{2\eta_{i,j}^{[k]}} \tilde{W}_{i,j}^{[k]T} \tilde{W}_{i,j}^{[k]}, \forall i, j, k \tag{52}$$

where $\alpha > 0$.

According to (51), we have

$$\begin{aligned} \dot{\tilde{W}}_{i,j}^{[k]} &= \eta_{i,j}^{[k]} \Pi_\Pi^T \left(\Pi_\Pi (W_{i,j}^{[k]} - \tilde{W}_{i,j}^{[k]}) - \Pi \right) \\ &= -\eta_{i,j}^{[k]} \Pi_\Pi^T \Pi_\Pi \tilde{W}_{i,j}^{[k]} + \eta_{i,j}^{[k]} \Pi_\Pi^T \Pi_\Pi W_{i,j}^{[k]} - \eta_{i,j}^{[k]} \Pi_\Pi^T \Pi \end{aligned} \tag{53}$$

Therefore, the gradient of (52) is

$$\begin{aligned}
\dot{\Lambda}_{i,j}^{[k]} &= \frac{\alpha}{\eta_{i,j}^{[k]}} \tilde{W}_{i,j}^{[k]T} \dot{\tilde{W}}_{i,j}^{[k]} \\
&= \alpha \tilde{W}_{i,j}^{[k]T} \left(-\Pi_\Pi^T \Pi_\Pi \tilde{W}_{i,j}^{[k]} + \Pi_\Pi^T \Pi_\Pi W_{i,j}^{[k]} - \Pi_\Pi^T \Pi \right) \\
&= -\alpha \tilde{W}_{i,j}^{[k]T} \Pi_\Pi^T \Pi_\Pi \tilde{W}_{i,j}^{[k]} + \alpha \tilde{W}_{i,j}^{[k]T} \Pi_\Pi^T \Pi_\Pi W_{i,j}^{[k]} - \alpha \tilde{W}_{i,j}^{[k]T} \Pi_\Pi^T \Pi \\
&\leq -\alpha ||\tilde{W}_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \alpha \tilde{W}_{i,j}^{[k]T} \Pi_\Pi^T \Pi_\Pi W_{i,j}^{[k]} - \alpha \tilde{W}_{i,j}^{[k]T} \Pi_\Pi^T \Pi \\
&\leq -\alpha ||\tilde{W}_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \frac{1}{2} ||\tilde{W}_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \frac{\alpha^2}{2} ||W_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 \\
&\quad + \frac{1}{2} ||\tilde{W}_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \frac{\alpha^2}{2} ||\Pi||^2
\end{aligned} \tag{54}$$

By transformation, (54) is

$$\dot{\Lambda}_{i,j}^{[k]} \leq (-\alpha + 1) ||\tilde{W}_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \frac{\alpha^2}{2} ||W_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \frac{\alpha^2}{2} ||\Pi||^2 \tag{55}$$

Define

$$\Sigma_{i,j}^{[k]} = \frac{\alpha^2}{2} ||W_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \frac{\alpha^2}{2} ||\Pi||^2 \tag{56}$$

Then (55) is

$$\dot{\Lambda}_{i,j}^{[k]} \leq (-\alpha + 1) ||\tilde{W}_{i,j}^{[k]}||^2 ||\Pi_\Pi||^2 + \Sigma_{i,j}^{[k]} \tag{57}$$

Thus, if

$$\alpha > 1 \tag{58}$$

and

$$||\tilde{W}_{i,j}^{[k]}||^2 > \frac{\Sigma_{i,j}^{[k]}}{(\alpha - 1) ||\Pi_\Pi||^2} \tag{59}$$

then $\tilde{W}_{i,j}^{[k]}$ is UUB. The proof completes.

According to Theorem 2, if the convergence condition is satisfied, then $\hat{V}^{[k]} \to V^{[k]}$, $\hat{u}_i^{[k]} \to u_i^{[k]}$ and $\hat{d}_j^{[k]} \to d_j^{[k]}$.

Remark 1 *This paper establishes a synchronous solution method based on PI algorithm to solve the multi-player zero-sum games. First, the method is different with other optimal control methods, such as the ones in [34], [35], [36] and [37]. In [34], an online reinforcement learning algorithm is proposed for a class of affine multiple input and multiple output (MIMO) nonlinear discrete-time systems with unknown functions and disturbances. In the paper, only two parameters are needed to be adjusted, and thus the number of the adaptation laws* *is smaller than the previous results. The updating parameters do not depend on the number of the subsystems for MIMO systems and the tuning rules are replaced by adjusting the norms on optimal weight vectors in both action and critic networks. In [35], an adaptive fuzzy optimal control design is addressed for a class of unknown nonlinear discrete-time systems. Fuzzy logic systems are employed to approximate the unknown functions in the systems. By applying the backsteppping design technique, a reinforcement learning algorithm is used to develop an optimal control signal. The adaptation auxiliary signal for unknown dead-zone parameters is established to compensate for the effect of nonsymmetric dead-zone on the control performance. In [36], an optimal control scheme-based adaptive neural network design for a class of unknown nonlinear discrete-time systems is proposed. The systems are transformed into an output predictor form. For the output predictor, the ideal control signal and the strategic utility function can be approximated by using an action network and a critic network, respectively. In [37], the optimal tracking control problem for the Henon Mapping chaotic system is solved using the direct heuristic dynamic programming setting with filtered tracking error. The fuzzy logic system is used to approximate the long-term utility function.*

Second, the method is different with the existing adaptive control method, such as the ones in [38], [39] and [40]. In [38], an approximation-based adaptive tracking control approach is proposed for a class of MIMO nonlinear systems. By introducing Nussbaum function, the issue of unknown control directions is handled. In the backstepping design process, the dynamic surface control technique is employed to avoid differentiating certain nonlinear functions repeatedly. Neural networks approximate the desired control signals directly. In [39], the adaptive neural network controller design is proposed for nonlinear MIMO discrete-time systems. In [40], an adaptive neural network tracking control method for uncertain nonlinear discrete-time systems with nonaffine dead-zone input is presented.

Therefore, this paper is the first time to discuss the solution method for the multi-player zero-sum games with unknown system dynamics.

4 Simulation Study

In this section, two examples will be provided to demonstrate the effectiveness of the optimal control scheme proposed in this paper.

4.1 Example 1

Consider the following linear system [41] with modifications

$$\dot{x} = x + u + d \tag{60}$$

In this paper, the initial state is $x(0) = 1$. We select hyperbolic tangent functions as the activation functions of critic, action and disturbance networks. The structures of critic, action and disturbance networks are $1-8-1$. The initial weight W is selected arbitrarily from $(-1, 1)$, the dimension of W is 24×1. For the cost function, Q, R and S in the utility function are identity matrices of appropriate dimensions. After 500 time steps, the simulation results are obtained. In Fig. 2, the cost function is shown, which converges to zero as time increasing. The control and disturbance trajectories are given in Figs. 3 and 4. Under the action of the obtained control and disturbance inputs, the state trajectory is displayed in Fig. 5. It is clear that the presented method in this paper is very effective and feasible.

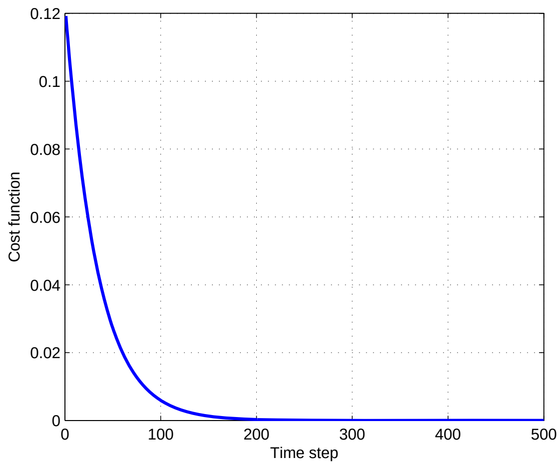

Fig. 2. Cost function

4.2 Example 2

Consider the following affine in control input nonlinear system [42]

$$\dot{x} = f(x) + g(x)\sum_{i=1}^{p} u_i + h(x)\sum_{j=1}^{q} d_j \tag{61}$$

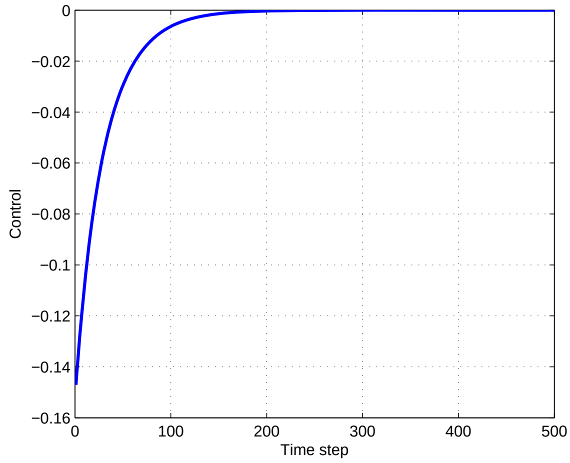

Fig. 3. Control

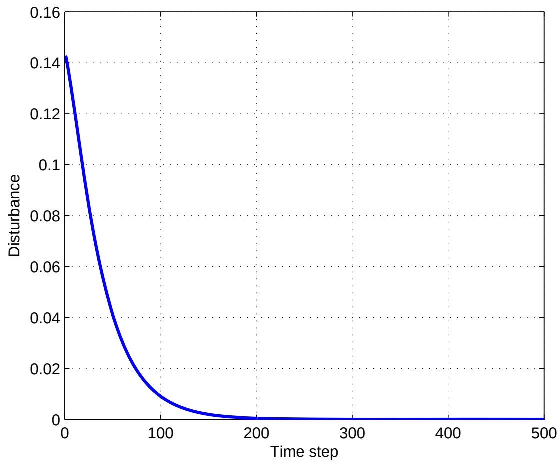

Fig. 4. Disturbance

where $f(x) = \begin{bmatrix} x_2 \\ -x_2 - \frac{1}{2}x_1 + \frac{1}{4}x_2(\cos(2x_1)+2)^2 + \frac{1}{4}x_2(\sin(4x_1^2)+2)^2 \end{bmatrix}$, $g(x) = \begin{bmatrix} 0 \\ \cos(2x_1)+2 \end{bmatrix}$, $h(x) = \begin{bmatrix} 0 \\ \sin(4x_1^2)+2 \end{bmatrix}$, $p = q = 1$.

In this simulation, the initial state is $x(0) = [1, -1]^T$. Hyperbolic tangent functions are used to be as the activation functions of critic, action and disturbance networks. The structures of the networks are $2-8-1$. The initial weight W is selected arbitrarily from $(-1, 1)$, the dimension of W is 24×1. For the cost function of (61), Q, R and S in the utility function are identity matrices of appropriate dimensions. The simulation results are obtained by 2500

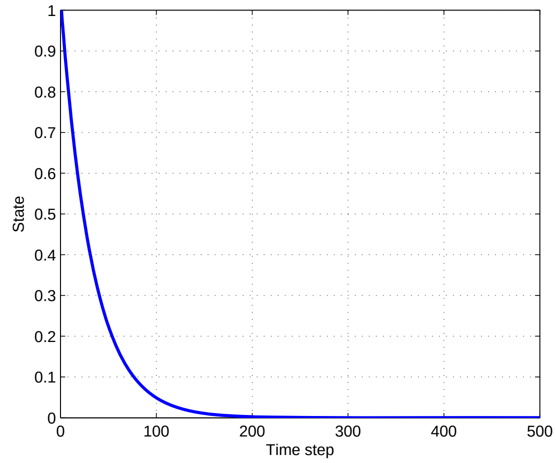

Fig. 5. State

time steps. The cost function is shown in Fig. 6, it is zero-sum. The control and disturbance trajectories are given in Figs. 7 and 8. The state trajectories are displayed in Fig. 9. We can see that the closed-loop system state, control and disturbance inputs converge to zero, as time step increasing. So the proposed synchronous method for multi-player zero-sum games in this paper is very effective.

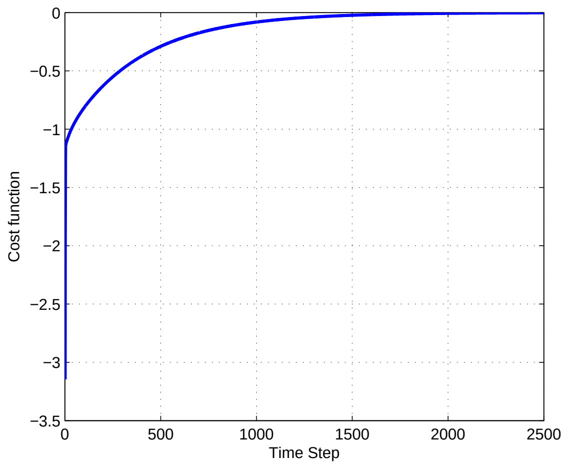

Fig. 6. Cost function

5 Conclusions

This paper proposed a synchronous solution method for multi-player zero-sum games without system dynamics based on neural network. PI algorithm is presented to solve the HJB equation with system dynamics. It is proven

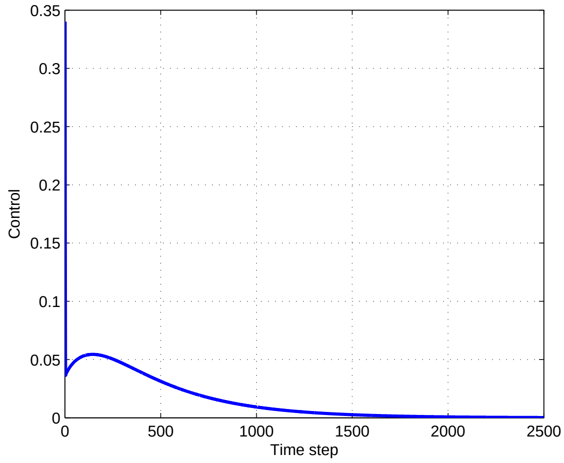

Fig. 7. Control

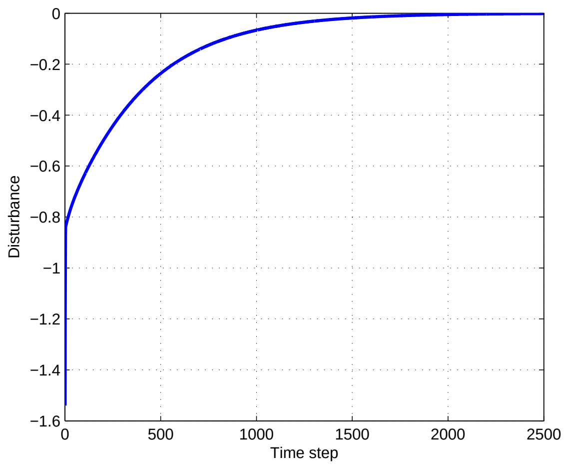

Fig. 8. Disturbance

that the obtained iterative cost function by PI is convergent to optimal game value. Based on PI, off-policy learning method is given to obtain the iterative cost function, controls and disturbances. The weights of CNN, ANNs and DNNs compose synchronous weight matrix, which is proven to be UUB by Lyapunov technique. Simulation study indicates the effectiveness of the proposed synchronous solution method for multi-player ZS games. A future research problem is to use the proposed approach to a class of systems with interconnection term.

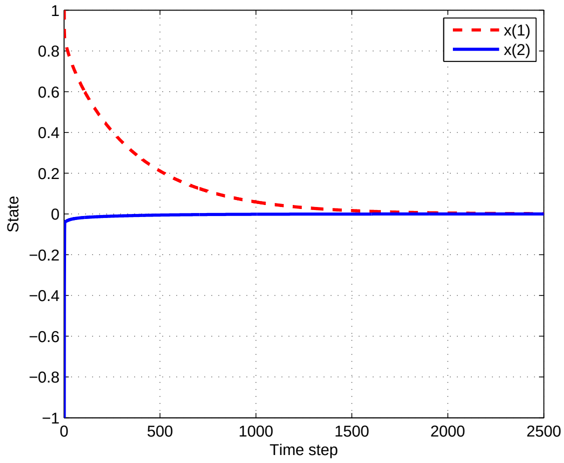

Fig. 9. State

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