1 Probability distributions

1.1 Beta distribution

1.2 Binomial distribution

2 Bayesian parameter estimation

2.1 Beta-Bernoulli model

2.1.1 Summary

The model

$$X_i \sim \text{Ber}(\theta), \text{for } i \in \{1, \ldots, N\} \tag{2.1}$$

$$\mathcal{D} = \{x_1, \ldots, x_N\} \tag{2.2}$$

$$N_1 = \sum_{i=1}^{N} \mathbb{I}(x_i = 1) \tag{2.3}$$

$$N_0 = \sum_{i=1}^{N} \mathbb{I}(x_i = 0) \tag{2.4}$$

Likelihood

$$p(\mathcal{D}|\theta) = \theta^{N_1}(1 - \theta)^{N_0} \tag{2.5}$$

Prior

$$p(\theta) = \text{Beta}(\theta|a, b) \tag{2.6}$$

Posterior

$$p(\theta|\mathcal{D}) = \text{Beta}(\theta|N_1 + a, N_0 + b) \tag{2.7}$$

Posterior predictive

$$p(x|\mathcal{D}) = \frac{a}{a + b} \tag{2.8}$$

2.1.2 Derivations

2.2 Beta-binomial model

2.2.1 Summary

The model

$$N_1 \sim \text{Bin}(N, \theta) \tag{2.9}$$
$$\mathcal{D} = \{N_1, N\} \tag{2.10}$$
$$N_1 = \text{number of successes} \tag{2.11}$$
$$N = \text{total number of trials} \tag{2.12}$$
$$\mathcal{D}' = \{N_1', N'\} \tag{2.13}$$
$$N_1' = \text{number of successes in a new batch of data} \tag{2.14}$$
$$N' = \text{total number of trials in a new batch of data} \tag{2.15}$$

Likelihood

$$p(\mathcal{D}|\theta) = \text{Bin}(N_1|N, \theta) \tag{2.16}$$

Prior

$$p(\theta) = \text{Beta}(\theta|a, b) \tag{2.17}$$

Posterior

$$p(\theta|\mathcal{D}) = \text{Beta}(\theta|N_1 + a, N_0 + b) \tag{2.18}$$

Posterior predictive

$$p(\mathcal{D}'|\mathcal{D}) = \text{Bb}(N_1'; a, b, N') \tag{2.19}$$

2.2.2 Derivations