Particle filter

Particle filter

We want to target the family of distributions $p(\mathbf{x}_{1:t} \mid \mathbf{y}_{1:t}, \theta), t = 1, \ldots, T$.

Assume we can sample from a proposal $q(\cdot \mid \mathbf{x}_{t-1}, \mathbf{y}_t \theta), t = 1, \ldots, T, q(\cdot \mid \mathbf{x}_0, \mathbf{y}_1, \theta) = q(\cdot \mid \mathbf{y}_1, \theta$.

Assume we can only evaluate p and q up to some multiplicative factors.

Particle filter

$$t = 1$$

Sample from proposal

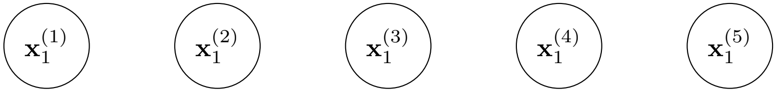
$$\mathbf{x}_1^{(k)} \sim q(\cdot \mid \mathbf{y}_1^{(k)}, \theta)$$

Particle filter

$t = 1$

Sample from proposal

$$\mathbf{x}_1^{(k)} \sim q(\cdot \mid \mathbf{y}_1^{(k)}, \theta)$$



Particle filter

$t = 1$

Compute weights

$$w_1^{(k)} \propto \frac{p\left(\mathbf{x}_1^{(k)} | \mathbf{y}_1, \theta\right)}{q\left(\mathbf{x}_1^{(k)} | \mathbf{y}_1, \theta\right)}$$

$\mathbf{x}_1^{(1)}$ $\mathbf{x}_1^{(2)}$ $\mathbf{x}_1^{(3)}$ $\mathbf{x}_1^{(4)}$ $\mathbf{x}_1^{(5)}$

Particle filter

$t = 1$

Compute weights

$$w_1^{(k)} \propto \frac{p\left(\mathbf{x}_1^{(k)}|\mathbf{y}_1,\theta\right)}{q\left(\mathbf{x}_1^{(k)}|\mathbf{y}_1,\theta\right)}$$

$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
0	2	2	0	1

Particle filter

$t = 1$

Normalise weights

$$W_1^{(k)} = \frac{w_1^{(k)}}{\sum_{k'} w_1^{(k')}}$$

$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
0	2	2	0	1

Particle filter

$t = 1$

Normalise weights

$$W_1^{(k)} = \frac{w_1^{(k)}}{\sum_{k'} w_1^{(k')}}$$

$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
$0/5$	$2/5$	$2/5$	$0/5$	$1/5$

Particle filter

$t = 1$

Normalise weights

$$W_1^{(k)} = \frac{w_1^{(k)}}{\sum_{k'} w_1^{(k')}}$$

$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
$0/5$	$2/5$	$2/5$	$0/5$	$1/5$

Can resample from

$$\hat{p}(\mathrm{d}\mathbf{x}_1 \mid \mathbf{y}_1, \theta) = \sum_k W_1^{(k)} \delta_{\mathbf{x}_1^{(k)}}(\mathbf{x}_1)$$

to estimate

$$p(\mathbf{x}_1 \mid \mathbf{y}_1, \theta)$$

Particle filter

$t = 2$

Sample parents' indices of 2nd generation

$$A_1^{(k)} \sim \mathrm{Cat}(W_1^{(1)}, \ldots, W_1^{(5)})$$

$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
$0/5$	$2/5$	$2/5$	$0/5$	$1/5$

Particle filter

$t = 2$

Sample parents' indices of 2nd generation

$$A_1^{(k)} \sim \mathrm{Cat}(W_1^{(1)}, \ldots, W_1^{(5)})$$

$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
$0/5$	$2/5$	$2/5$	$0/5$	$1/5$

$A_1^{(1)} = 2$ $\quad A_1^{(2)} = 3$ $\quad A_1^{(3)} = 5$ $\quad A_1^{(4)} = 3$ $\quad A_1^{(5)} = 2$

Particle filter

$t = 2$

Sample 2nd generation using corresponding parents

$$\mathbf{x}_2^{(k)} \sim q(\cdot \mid \mathbf{y}_2, \mathbf{x}_1^{(A_1^{(k)})}, \theta)$$

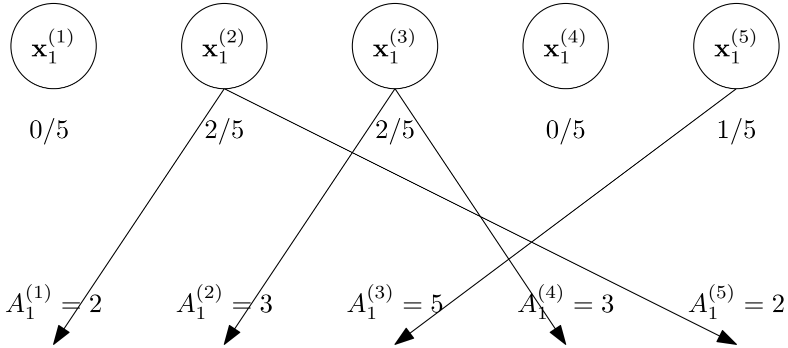
$\mathbf{x}_1^{(1)}$	$\mathbf{x}_1^{(2)}$	$\mathbf{x}_1^{(3)}$	$\mathbf{x}_1^{(4)}$	$\mathbf{x}_1^{(5)}$
$0/5$	$2/5$	$2/5$	$0/5$	$1/5$
$A_1^{(1)} = 2$	$A_1^{(2)} = 3$	$A_1^{(3)} = 5$	$A_1^{(4)} = 3$	$A_1^{(5)} = 2$

Particle filter

$t = 2$

Sample 2nd generation using corresponding parents

$$\mathbf{x}_2^{(k)} \sim q(\cdot \mid \mathbf{y}_2, \mathbf{x}_1^{(A_1^{(k)})}, \theta)$$

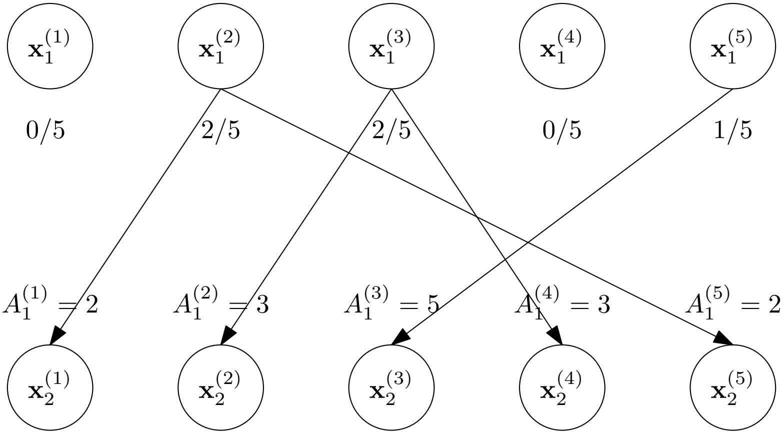


Particle filter

$t = 2$

Sample 2nd generation using corresponding parents

$$\mathbf{x}_2^{(k)} \sim q(\cdot \mid \mathbf{y}_2, \mathbf{x}_1^{(A_1^{(k)})}, \theta)$$

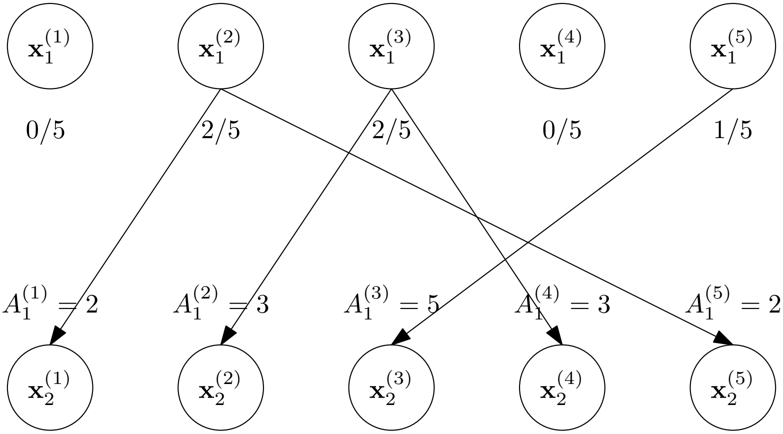


Particle filter

$t = 2$

Compute weights

$$w_2^{(k)} \propto \frac{p\left(\mathbf{x}_{1:2}^{(k)} | \mathbf{y}_{1:2}, \theta\right)}{q\left(\mathbf{x}_{1:2}^{(k)} | \mathbf{y}_{1:2}, \theta\right)} = w_1^{(k)} \frac{p\left(\mathbf{y}_2 | \mathbf{x}_2^{(k)}\right) p\left(\mathbf{x}_2^{(k)} | \mathbf{x}_1^{(k)}\right)}{q\left(\mathbf{x}_2^{(k)} | \mathbf{x}_1^{(k)}, \mathbf{y}_2\right)}$$

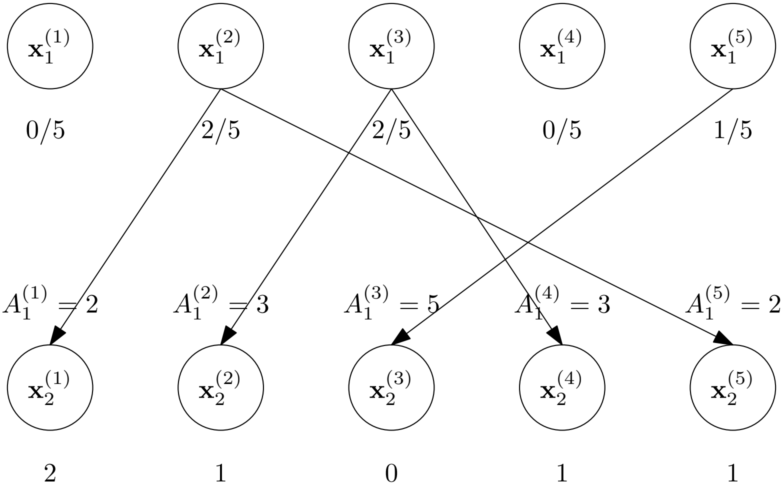


Particle filter

$t = 2$

Compute weights

$$w_2^{(k)} \propto \frac{p\left(\mathbf{x}_{1:2}^{(k)}|\mathbf{y}_{1:2},\theta\right)}{q\left(\mathbf{x}_{1:2}^{(k)}|\mathbf{y}_{1:2},\theta\right)} = w_1^{(k)} \frac{p\left(\mathbf{y}_2|\mathbf{x}_2^{(k)}\right)p\left(\mathbf{x}_2^{(k)}|\mathbf{x}_1^{(k)}\right)}{q\left(\mathbf{x}_2^{(k)}|\mathbf{x}_1^{(k)},\mathbf{y}_2\right)}$$

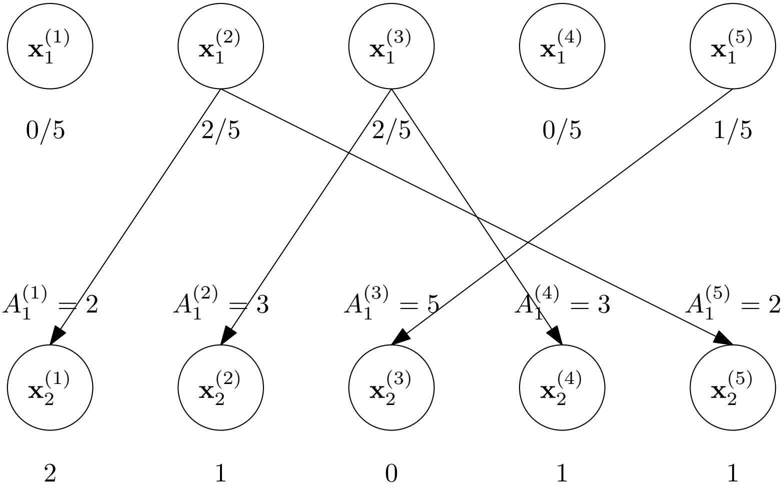


Particle filter

$t = 2$

Normalise weights

$$W_2^{(k)} = \frac{w_2^{(k)}}{\sum_{k'} w_2^{(k')}}$$



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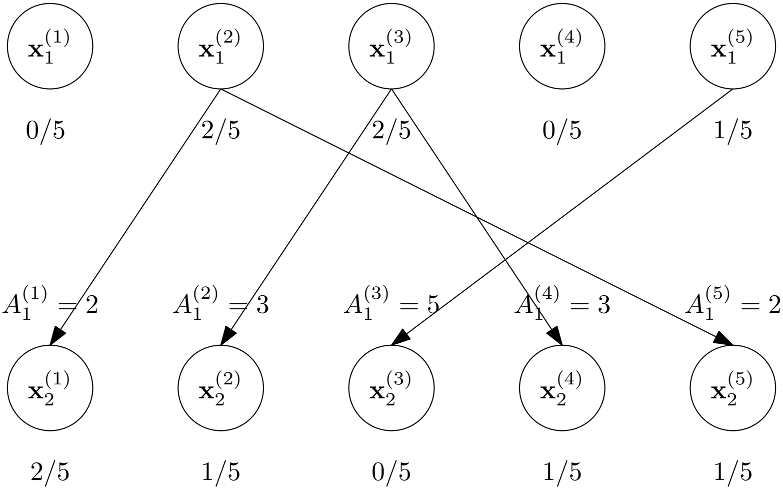

Particle filter

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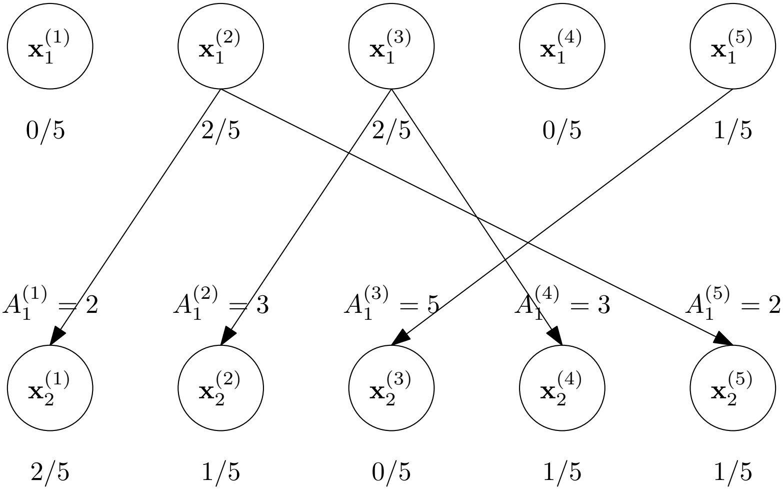
$$W_2^{(k)} = \frac{w_2^{(k)}}{\sum_{k'} w_2^{(k')}}$$



Can resample from

$$\hat{p}(\mathrm{d}\mathbf{x}_{1:2} \mid \mathbf{y}_{1:2}, \theta) = \sum_k W_2^{(k)} \delta_{\mathbf{x}_{1:2}^{(k)}}(\mathbf{x}_{1:2})$$

to estimate

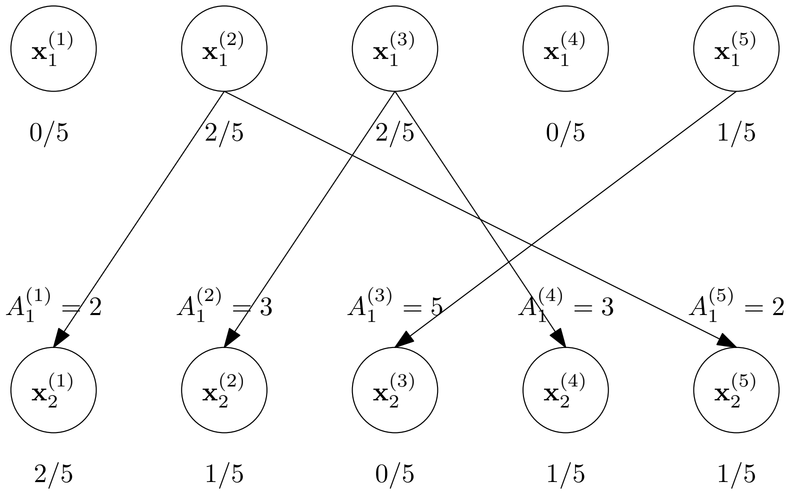
$$p(\mathbf{x}_{1:2} \mid \mathbf{y}_{1:2}, \theta)$$

Particle filter

$t = 3$

Sample parents' indices of 3rd generation

$$A_2^{(k)} \sim \mathrm{Cat}(W_2^{(1)}, \ldots, W_2^{(5)})$$

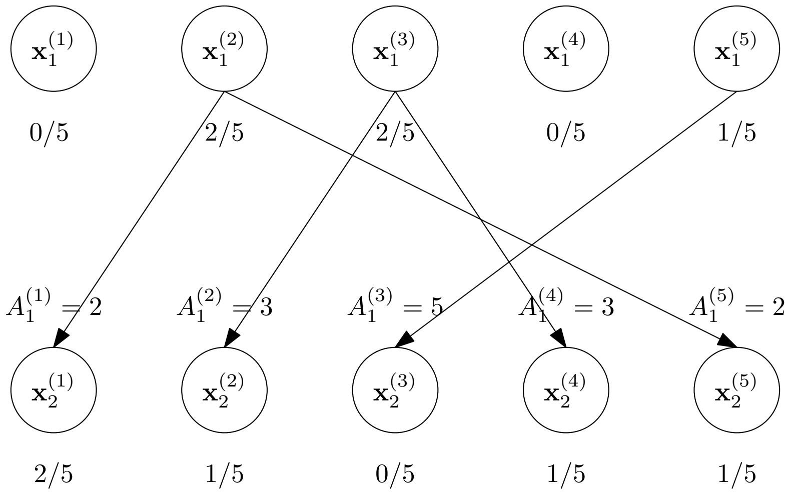


Particle filter

$t = 3$

Sample parents' indices of 3rd generation

$$A_2^{(k)} \sim \mathrm{Cat}(W_2^{(1)}, \ldots, W_2^{(5)})$$



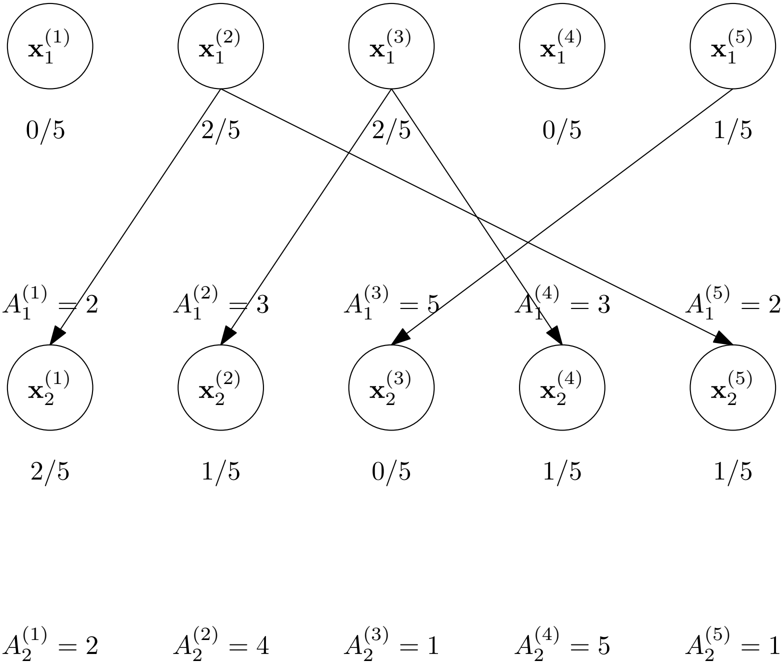
$A_2^{(1)} = 2$ $\quad A_2^{(2)} = 4$ $\quad A_2^{(3)} = 1$ $\quad A_2^{(4)} = 5$ $\quad A_2^{(5)} = 1$

Particle filter

$t = 3$

Sample 3rd generation using corresponding parents

$$\mathbf{x}_3^{(k)} \sim q(\cdot \mid \mathbf{y}_2, \mathbf{x}_2^{(A_2^{(k)})}, \theta)$$



Particle filter

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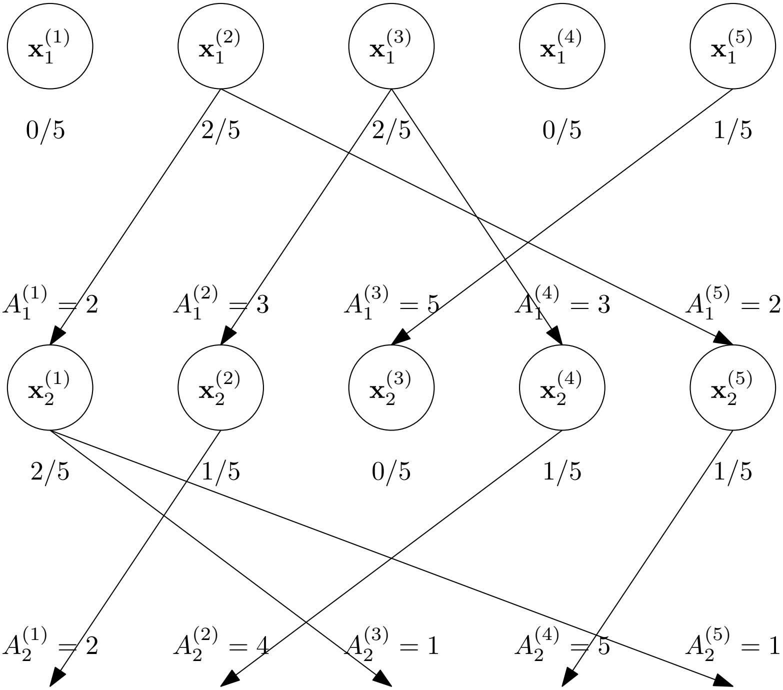

Particle filter

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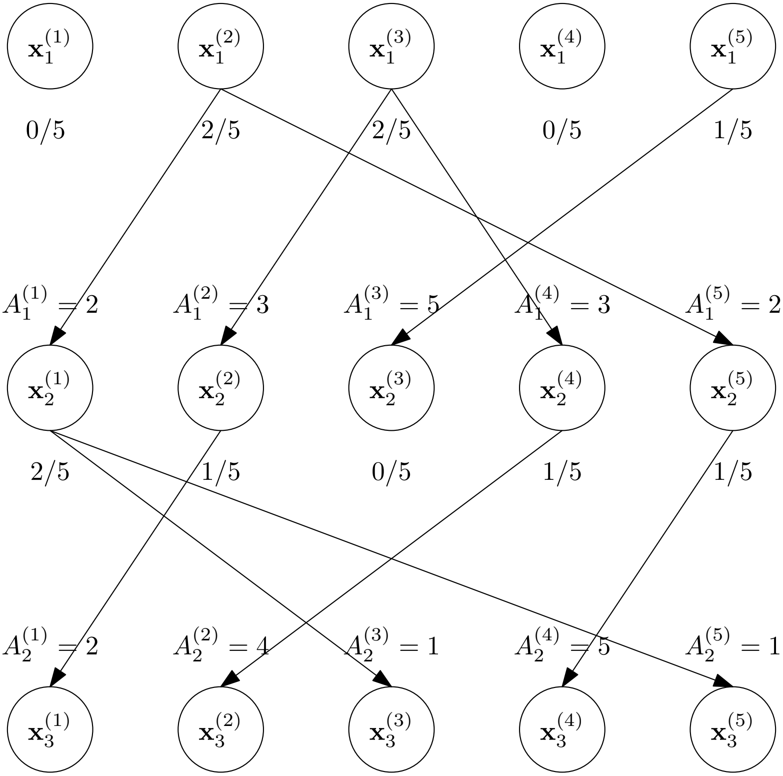
Sample 3rd generation using corresponding parents

$$\mathbf{x}_3^{(k)} \sim q(\cdot \mid \mathbf{y}_2, \mathbf{x}_2^{(A_2^{(k)})}, \theta)$$

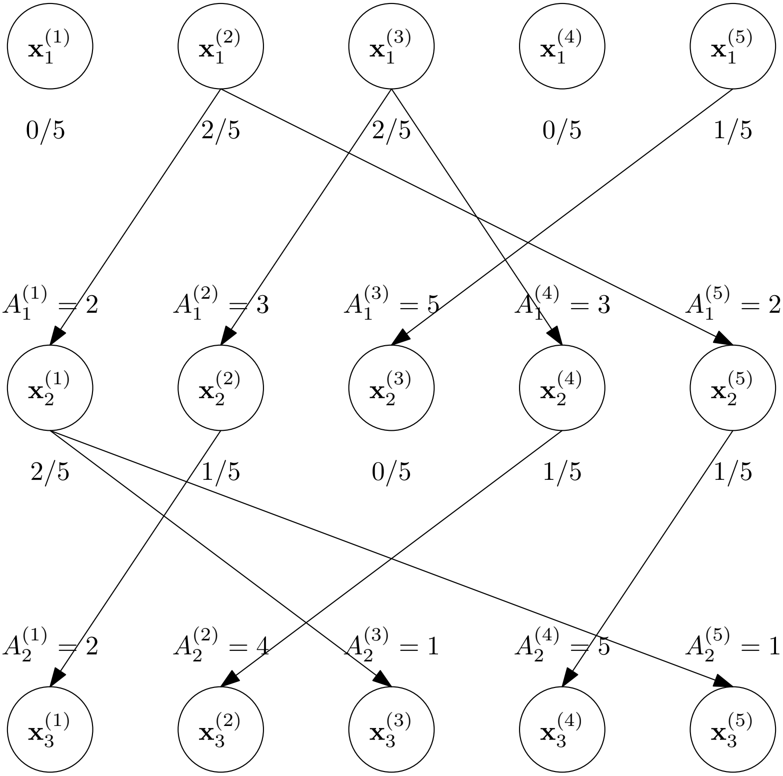


Particle filter

$t = 3$

Compute weights

$$w_3^{(k)} \propto \frac{p\left(\mathbf{x}_{1:3}^{(k)} | \mathbf{y}_{1:3}, \theta\right)}{q\left(\mathbf{x}_{1:3}^{(k)} | \mathbf{y}_{1:3}, \theta\right)} = w_2^{(k)} \frac{p\left(\mathbf{y}_3 | \mathbf{x}_3^{(k)}\right) p\left(\mathbf{x}_3^{(k)} | \mathbf{x}_2^{(k)}\right)}{q\left(\mathbf{x}_3^{(k)} | \mathbf{x}_2^{(k)}, \mathbf{y}_3\right)}$$

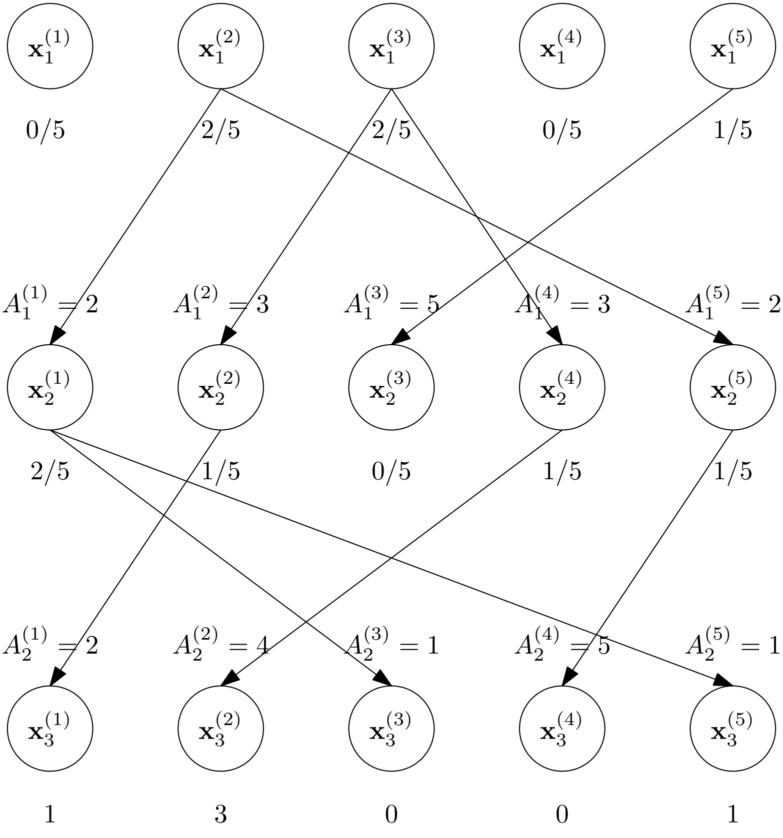


Particle filter

$t = 3$

Compute weights

$$w_3^{(k)} \propto \frac{p\left(\mathbf{x}_{1:3}^{(k)}|\mathbf{y}_{1:3},\theta\right)}{q\left(\mathbf{x}_{1:3}^{(k)}|\mathbf{y}_{1:3},\theta\right)} = w_2^{(k)} \frac{p\left(\mathbf{y}_3|\mathbf{x}_3^{(k)}\right) p\left(\mathbf{x}_3^{(k)}|\mathbf{x}_2^{(k)}\right)}{q\left(\mathbf{x}_3^{(k)}|\mathbf{x}_2^{(k)},\mathbf{y}_3\right)}$$

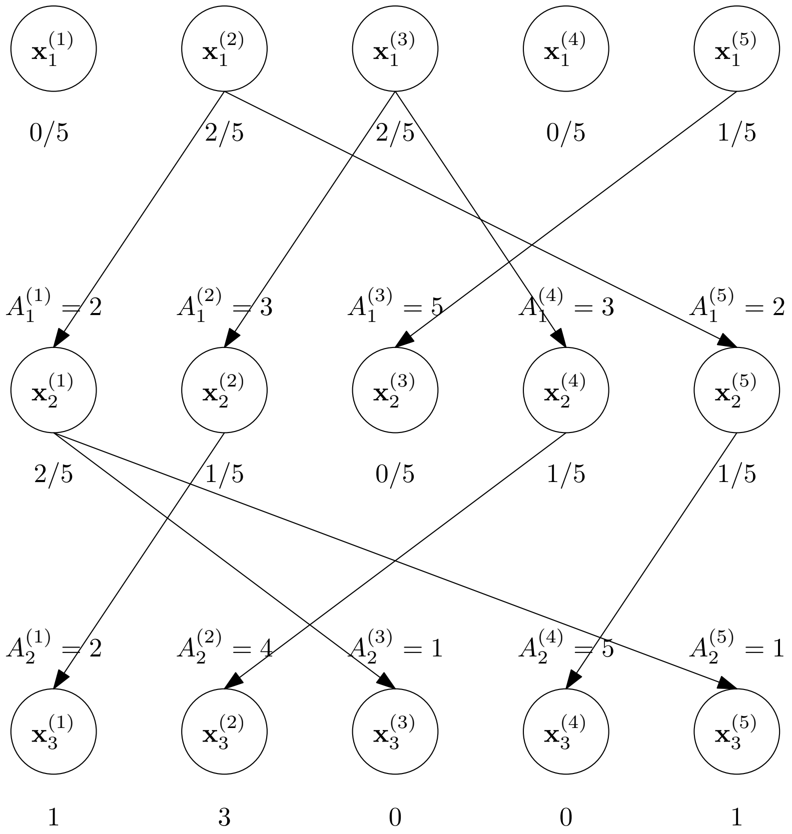


Particle filter

$t = 3$

Normalise weights

$$W_3^{(k)} = \frac{w_3^{(k)}}{\sum_{k'} w_3^{(k')}}$$



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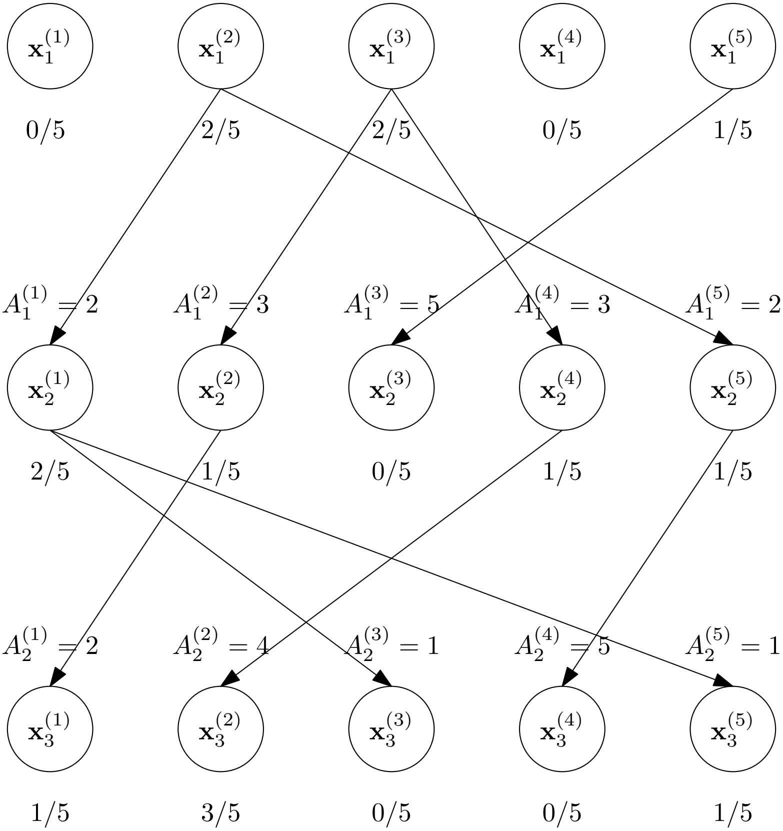
$$W_3^{(k)} = \frac{w_3^{(k)}}{\sum_{k'} w_3^{(k')}}$$

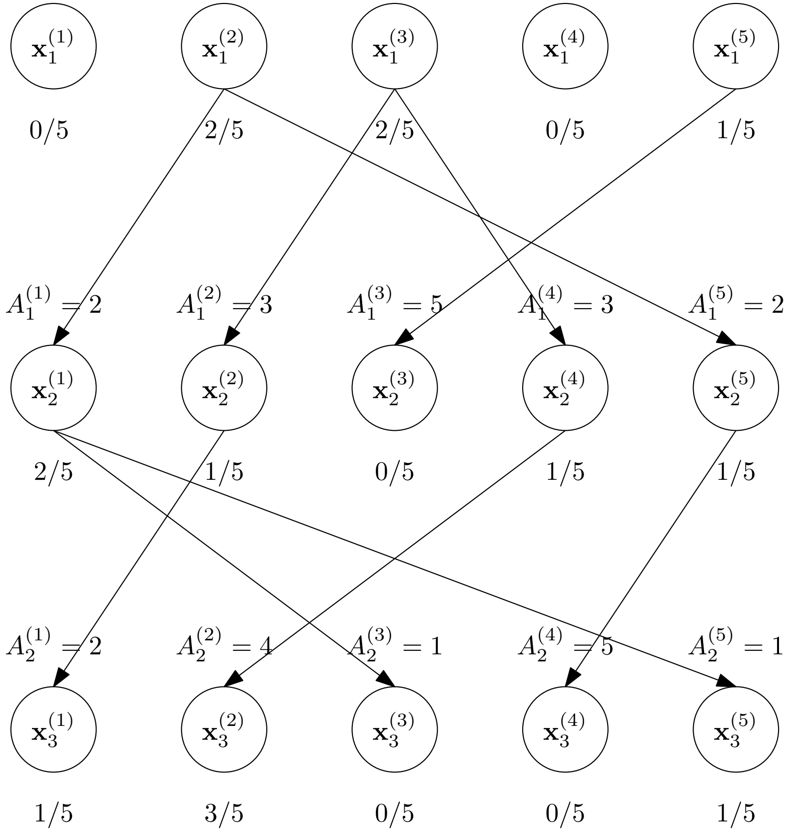


Particle filter

$t = 3$

Normalise weights

$$W_3^{(k)} = \frac{w_3^{(k)}}{\sum_{k'} w_3^{(k')}}$$



Can resample from

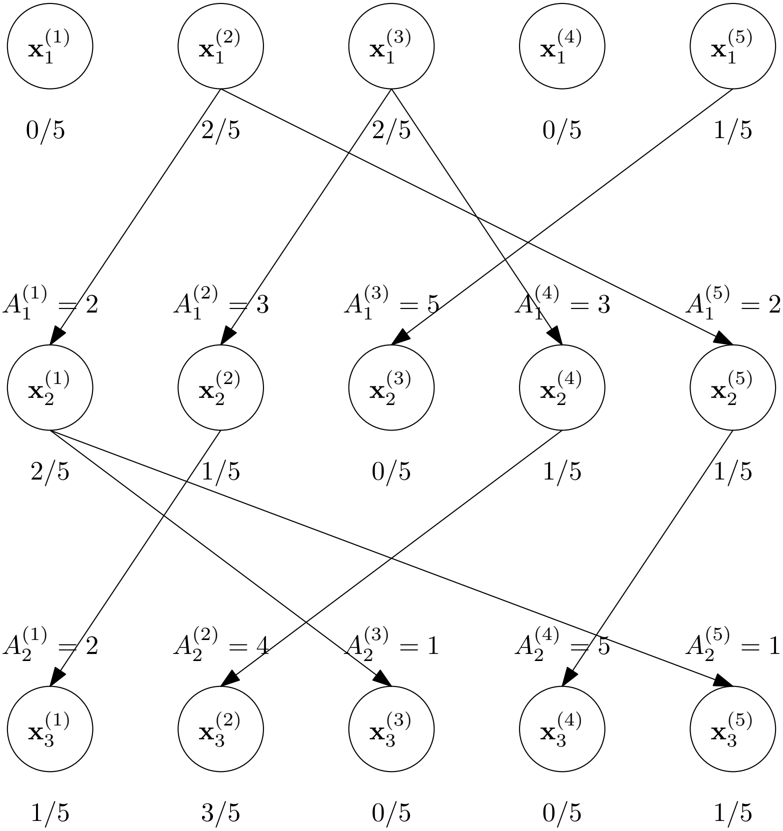
$$\hat{p}(\mathrm{d}\mathbf{x}_{1:3} \mid \mathbf{y}_{1:3}, \theta) = \textstyle\sum_k W_3^{(k)} \delta_{\mathbf{x}_{1:3}^{(k)}}(\mathbf{x}_{1:3})$$

to estimate

$$p(\mathbf{x}_{1:3} \mid \mathbf{y}_{1:3}, \theta)$$

Particle filter

Calculate lineage of $\mathbf{x}_3^{(1)}$.



Particle filter

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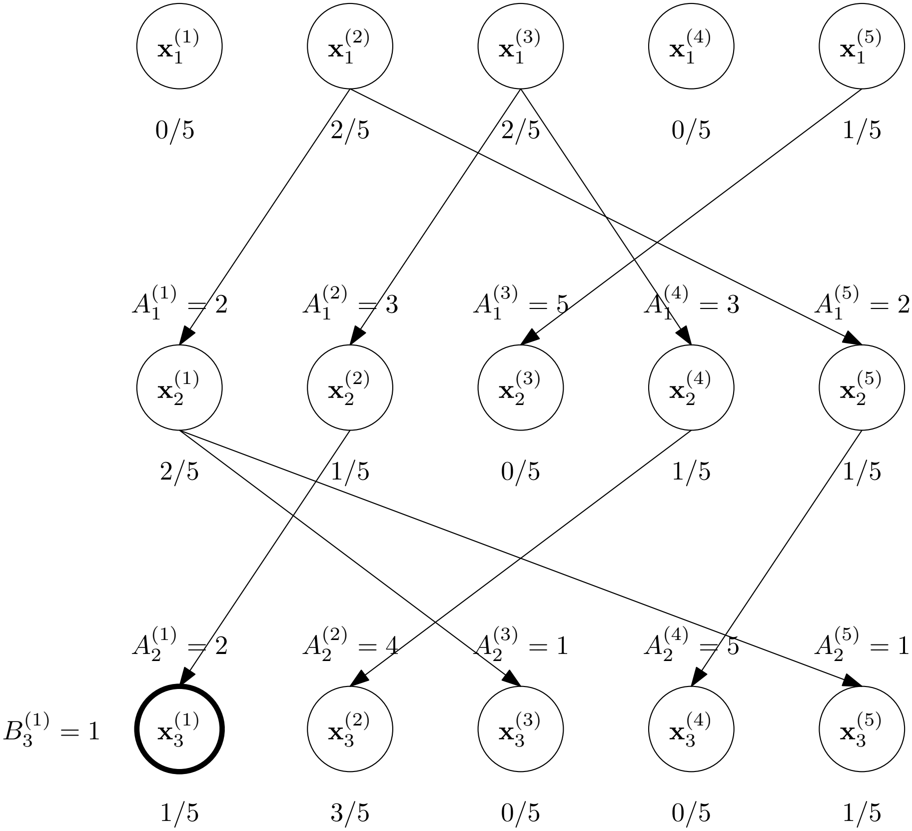
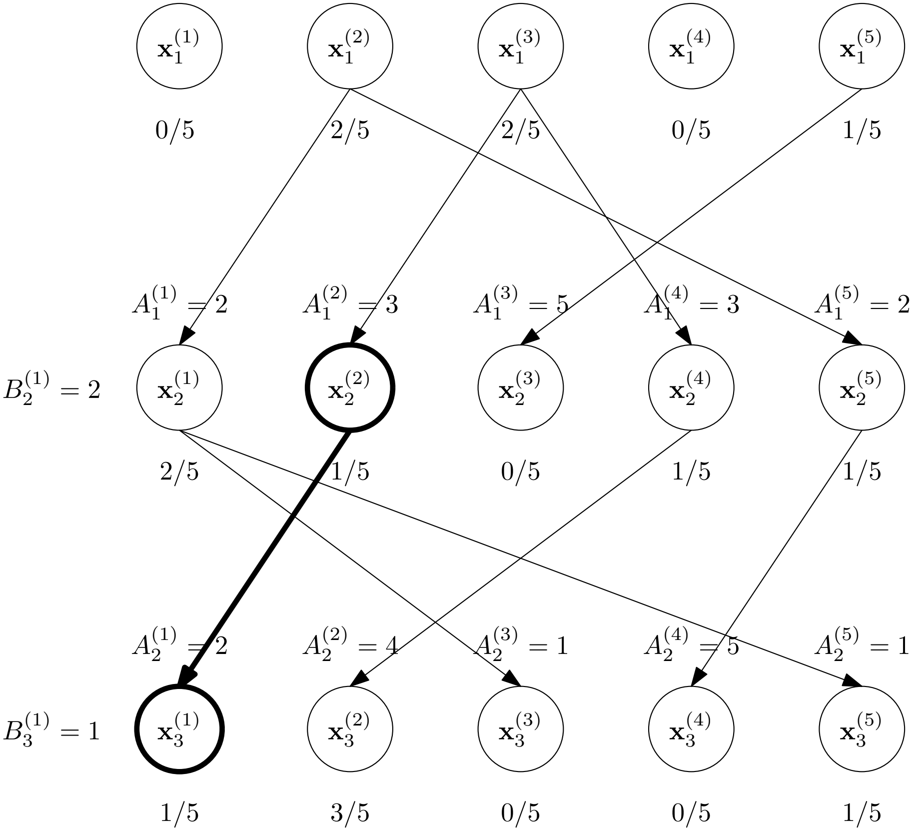


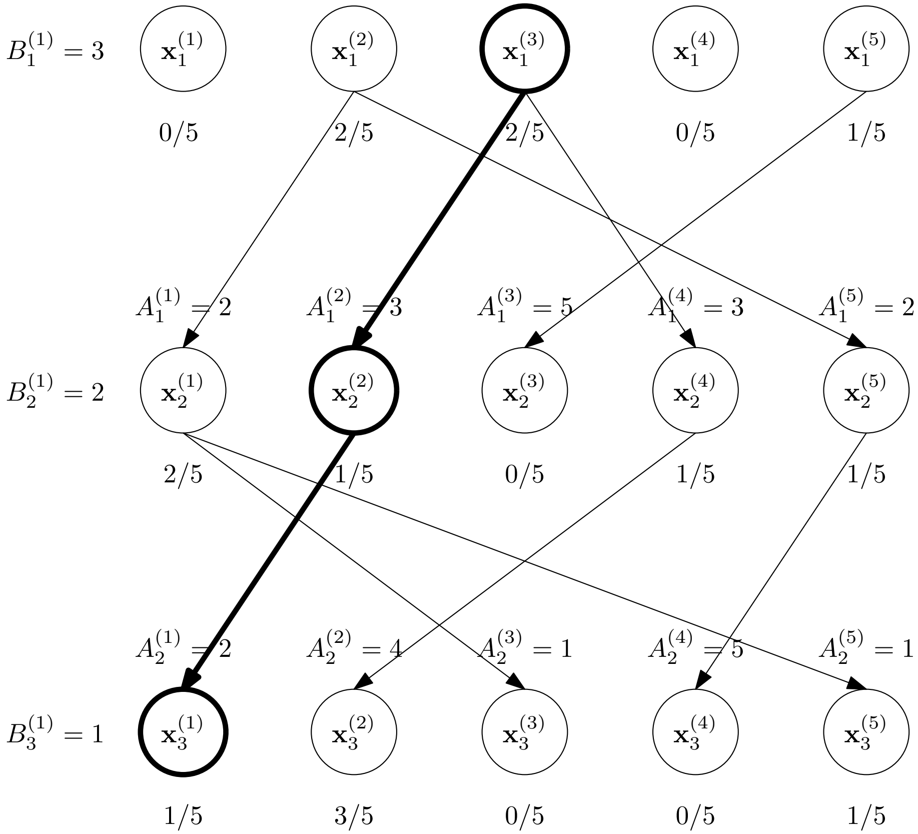
Particle filter

Calculate lineage of $\mathbf{x}_3^{(1)}$.



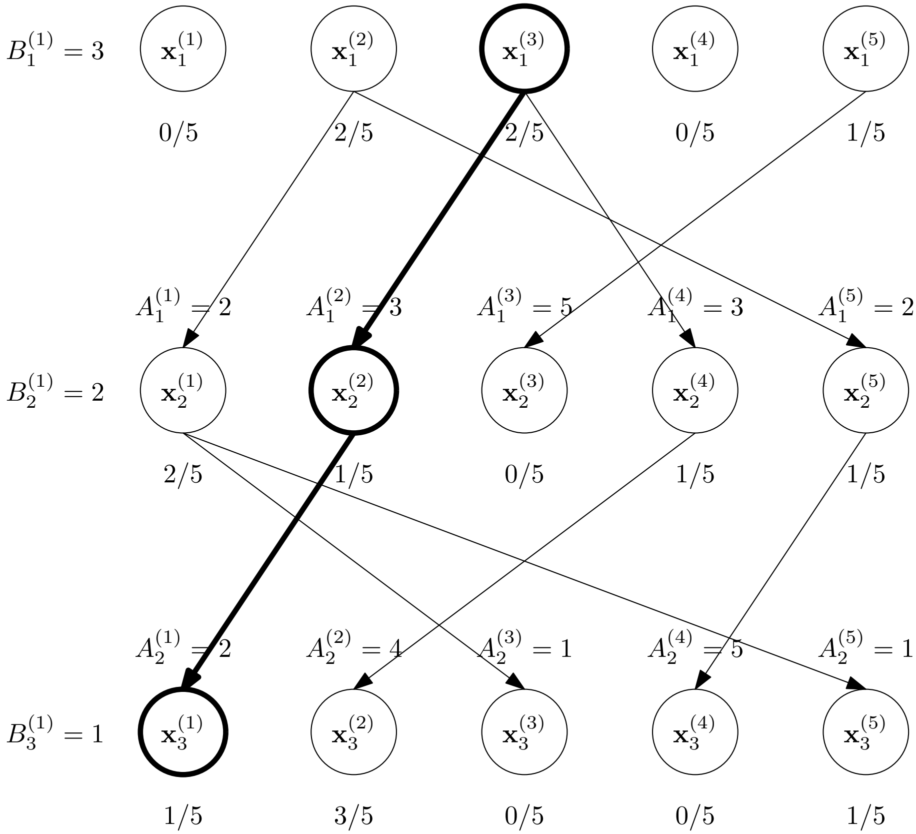
Particle filter

Calculate lineage of $\mathbf{x}_3^{(1)}$.



Particle filter

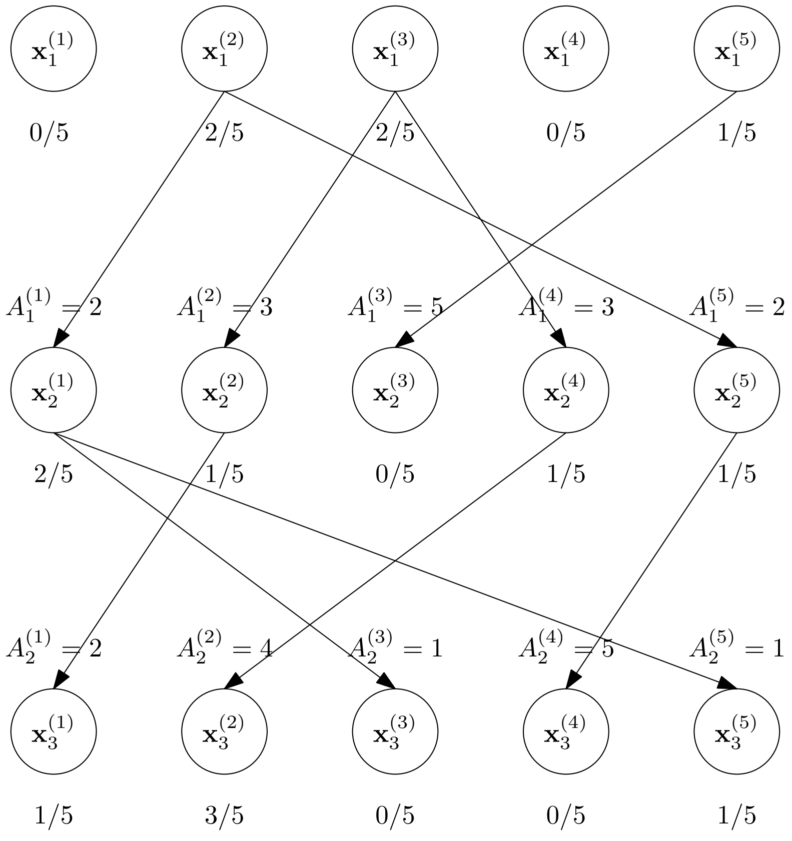
Calculate lineage of $\mathbf{x}_3^{(1)}$.



$\mathbf{B}^{(1)} = (3, 2, 1)$

Particle filter

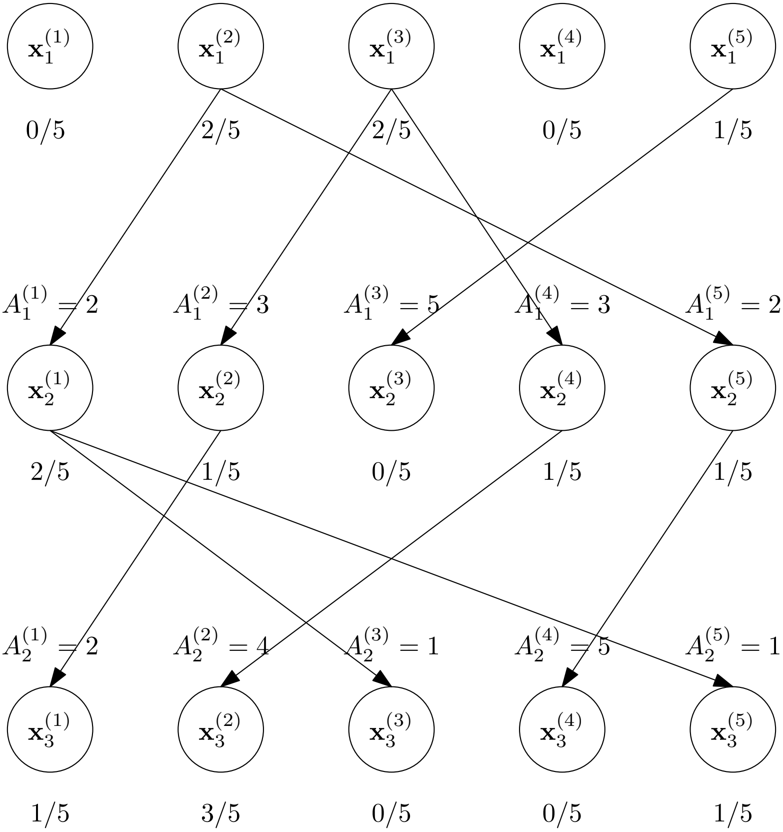
Calculate linage of the rest in similar way.



$$\mathbf{B}^{(1)} = (3, 2, 1)$$

Particle filter

Calculate linage of the rest in similar way.



$$\mathbf{B}^{(1)} = (3, 2, 1)$$
$$\mathbf{B}^{(2)} = (3, 4, 2)$$
$$\mathbf{B}^{(3)} = (2, 1, 3)$$
$$\mathbf{B}^{(4)} = (2, 5, 4)$$
$$\mathbf{B}^{(5)} = (2, 1, 5)$$