HW5 - STAT 580 - Sp 2015

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1.

(a)
```
#1.
#a)
set.seed(1)
p <- .3
lambda <- 2
n <- 100
r <- rbinom(n, 1, p)
y <- rpois(n, lambda)
x <- r*y
x

##   [1] 0 0 0 6 0 1 0 0 0 0 0 0 0 0 1 0 3 0 0 2 6 0 0 0 0 0 0 0 1 0 0 0 0 0 4
##  [36] 0 2 0 6 0 3 0 1 0 0 2 0 0 0 0 0 2 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 1
##  [71] 0 3 0 0 0 4 2 0 2 4 0 1 0 0 4 0 3 0 0 0 0 0 0 4 1 2 0 0 1 0
```

(c)
```
#b)
a<- 1;b <- 1
gibbs <- function(a,b){
  r <- rep(1, n)
  cnt <- 0
  burn.in <- 10000
  keep <- 0
  ns <- 10000
  pkeep <- lambdakeep <- NULL
  repeat{
    lambda <- rgamma(1,a +sum(x), b +sum(r))
    p <- rbeta(1,1+sum(r), n+1-sum(r))
    pnew <- p*exp(-lambda)/(p*exp(-lambda) + (1-p)*(x==0))
    r <- rbinom(n, 1, pnew)
    cnt <- cnt +1
    if (cnt >burn.in){
      keep <- keep +1
      pkeep[keep] <- p
      lambdakeep[keep] <- lambda
    }
    if (cnt >burn.in + ns) break
  }

  cip <- quantile(pkeep, prob = c(.025, .975))
  cilambda <- quantile(lambdakeep, prob = c(.025, .975))
  res <- rbind(p=cip, lambda=cilambda)
  res
}

gibbs(1,1) # the confidence intervals do contain the true values
```

(1.b) $p(X_i, R_i \mid \lambda, p) = p(X_i \mid R_i, \lambda, p) \cdot p(R_i \mid p)$

$$= e^{-\lambda r_i} \cdot \frac{(\lambda r_i)^{x_i}}{x_i!} \cdot p^{r_i} (1-p)^{1-r_i}$$

$$\rightarrow \prod_{i=1}^{n} p(X_i, R_i \mid \lambda, p) = e^{-\lambda \sum r_i} \prod \frac{(\lambda r_i)^{x_i}}{x_i!} \cdot p^{\sum r_i} (1-p)^{n-\sum r_i}$$

$$\rightarrow p(\underset{\sim}{X}, \underset{\sim}{R}, \lambda, p) = \left[\prod p(X_i, R_i \mid \lambda, p)\right] p(\lambda)\, p(p)$$

$$= e^{-\lambda \sum r_i} \prod \frac{(\lambda r_i)^{x_i}}{x_i!}\, p^{\sum r_i} (1-p)^{n-\sum r_i} \frac{b^a \lambda^{a-1} e^{-b\lambda}}{\Gamma(a)}$$

i) $p(\lambda \mid \cdot) \propto \lambda^{(\sum x_i)+a-1} e^{-\lambda(b+\sum r_i)} \rightarrow \lambda \mid p, \underset{\sim}{R}, \underset{\sim}{X} \sim \text{Gamma}(a+\sum x_i,\ b+\sum r_i)$

ii) $p(p \mid \cdot) \propto p^{\sum r_i} (1-p)^{n-\sum r_i} \Rightarrow p \mid \lambda, \underset{\sim}{R}, \underset{\sim}{X} \sim \text{Beta}(1+\sum r_i,\ n+1-\sum r_i)$

iii) $p(r_i \mid \cdot) \propto r_i^{x_i}\, p^{r_i} (1-p)^{1-r_i} e^{-\lambda r_i}$

$$\propto r_i^{x_i} (p e^{-\lambda})^{r_i} (1-p)^{1-r_i} \propto \left(\frac{p e^{-\lambda}}{1-p}\right)^{r_i} r_i^{x_i}$$

$$\rightarrow r_i \sim \text{Bernoulli}\left(\frac{p e^{-\lambda}}{p e^{-\lambda} + (1-p)\, I\{x_i = 0\}}\right)$$

```
##             2.5%     97.5%
## p      0.2355416 0.4386853
## lambda 1.6742864 2.8436237

gibbs(1,2) # the confidence intervals do contain the true values

##             2.5%     97.5%
## p      0.2387191 0.4471769
## lambda 1.6022768 2.7320347

gibbs(1000,1000) # the confidence intervals do not contain the true values

##             2.5%     97.5%
## p      0.3290509 0.6016109
## lambda 0.9682580 1.0897239

gibbs(2, 1000)  # the confidence intervals do not contain the true values

##              2.5%      97.5%
## p      0.86273583 0.99907441
## lambda 0.05486017 0.08580084
```

2.

```
#2.
theta1 <- 1.5
theta2 <- 2

f <- function(z, theta1 = 1.5, theta2 = 2){
  z^(-3/2)*exp(-theta1*z-theta2/z +2*sqrt(theta1*theta2) + log(sqrt(2*theta2)))
}

mh <- function(a, b){
    i <- 1
    x <- NULL
    x[1] <- 1
    repeat{
      y <- rgamma(1, a, b)
      gy <-dgamma(y, a, b)
      r <- f(y)*dgamma(x[i], a, b)/(f(x[i])*dgamma(y, a, b))
      u <- runif(1,0,1)
      x[i+1] <- ifelse(u <r, y, x[i])
      i <- i+1
      if (i == 10000) break
    }
#     hist( x, nclass = 100, prob = T)
    abs(c(mx.diff = mean(x), m1x.diff = mean(1/x))-c(mz = sqrt(theta2/theta1),
                                              m1z=sqrt(theta1/theta2) + 1/(2*theta2)))
  }

mh(1,.1)

##     mx.diff    m1x.diff
## 0.008498937 0.015303931

mh(1,1) # better
```

```
##      mx.diff     m1x.diff
## 0.0008119234 0.0078215705
```

```r
mh(2,2)
```

```
##     mx.diff    m1x.diff
## 0.006390013 0.013114352
```