1. Consider the following BIBD for 3 treatments (indicated in the following table) with blocks of size 2:

	Block					
	1	2	3	4	5	6
	1 3	2 3	1 2	1 3	2 3	1 2

5 (a) What are the values of r and λ for this design?

$r = 4$ $\quad \lambda = 2$ ✓ ✓

5 (b) Under the assumptions that $\tau_1 = \tau_2 + 1 = \tau_3 + 2$ and $\sigma = 2$, completely characterize the distribution of the F statistic used to test for equality of the three treatments.

blocks 5 $\quad \sum(\bar{Y} - \bar{\bar{Y}})^2 = 2$, times $r = 8$, times $\frac{t(k-1)}{k(t-1)} = 6$

trt (2) $\quad$ divided by $\sigma^2 = (1\frac{1}{2})$

resid (4)

tot 11

$F'(2, 4, 1\frac{1}{2})$ ✓

5 (c) Suppose the sum-of-squares for "blocks ignoring treatments" and "treatments after correction for blocks" are each equal to (100), and the corrected total sum-of-squares is (240). Compute the F-statistic that would be used to test $Hyp_0 : \tau_1 = \tau_2 = \tau_3$.

blocks	5	100	
trt/b	2	100	← $\frac{100/2}{40/4} = 5$ ✓
resid	4	40	←
tot	11	240	

5 (d) Suppose we had fitted the model "backward", finding sums-of-squares for "treatments ignoring blocks" and "blocks after correction for treatments" (e.g. acting as if the τ's were nuisance parameters and the β's were the parameters of interest to the experimenter). Given the information in the previous problem part, what can you say about the values of these two sums-of-squares?

$SS(trt) \geq 100$

$SS(blocks \mid t) \leq 100$ } sum = 200

5 (e) Suppose now that an inter-block analysis had been performed for this experiment.

i. What is the fundamental assumption that is necessary for this analysis, that is not required for the intra-block analysis?

random block effects

ii. How many degrees of freedom would be used for a t-quantile needed for a confidence interval for $\tau_1 - \tau_2$ based only on the inter-block analysis?

$U = \begin{pmatrix} 1 & 1 & & 1 \\ 1 & & 1 & 1 \\ 1 & 1 & 1 & \\ 1 & 1 & & 1 \\ 1 & & 1 & 1 \\ 1 & 1 & 1 & \end{pmatrix}$, rank = 3, df = "N" − 3 = 3 ✓

5 (f) Suppose that only blocks 1, 3, 4, and 6 were actually executed (or that the data from blocks 2 and 5 were lost). What treatment contrasts (i.e. contrasts of τ_1, τ_2 and τ_3) are estimable in an inter-block analysis? Write the *full* model matrix that would be used for this analysis and defend your answer using this.

$U = \begin{pmatrix} 1 & 1 & & 1 \\ 1 & & 1 & 1 \\ 1 & 1 & & 1 \\ 1 & & 1 & 1 \end{pmatrix}$ ← only $\tau_2 - \tau_3$ (or equivalently, $\tau_3 - \tau_2$) ✓

2. Recall that in Homework 6, we discussed a split-plot experiment involving steel bars. The final part of that problem involved expanding the experiment to include three different levels or *strata* of experimental units. Here is a short description of that setting, with some of the details changed:

   - Experimental factors of interest are the type of steel used in the bar (4 levels), the temperature at which the steel is treated in the furnace (3 levels), and the type of coating applied to each bar (2 levels).
   - For each type of steel, two batches of uncoated bars are received. (For clarity, each "batch" contains 12 bars.) ↳ 8 batches → 48 groups → 96 bars
   - Each batch of bars is divided into 6 groups of bars (2 bars each). A group of bars is processed one run of the furnace, which has been randomly set to one of the 3 temperatures, under the constraint that each temperature is used for 2 of the 6 groups of bars in a batch.
   - For the pair of bars from the same batch and treated with the same run of the furnace, a coin-flip is used to determine which is coated with coating 1 and which with coating 2.
   - Hint: Note that *coating* is applied to an *individual bar*, *temperature* is applied to a *batch of bars*, and that *steel type* is a property of a *batch of bars*. *Steel type* wasn't really "applied to" the batch as required in a true experiment, but answer the questions here as if this were the case.

20 (a) Construct the "source" and "degrees of freedom" columns for the appropriate split-split-plot ANOVA table. Indicate which residual mean square would be used in the denominator for testing wach of the 7 treatment main effects and interactions in the $4 \times 3 \times 2$ factorial treatment structure.

Stratum	Source	df	
1	steel (S)	3	
	resid	4	
	ct. (batch)	7	
2	temp (T)	2	(15)
	S×T	6	
	resid	32	
	ct. (group)	47	
3	coat (C)	1	
	S×C	3	(59)
	T×C	2	
	S×T×C	6	
	resid	36	
	ct. (bar)	95	

5 (b) Suppose it were determined that *batch of bars* could not be regarded as random, but could still be assumed not to interact with treatments. What factorial effects (e.g. groups of main effects and interactions) could still be tested?

T S×T C S×C T×C S×T×C

5 (c) Suppose it were determined that neither *batch of bars* nor *furnace run* could be regarded as random, but could still be assumed not to interact with treatments. What factorial effects (e.g. groups of main effects and interactions) could still be tested?

C S×C T×C S×T×C

N=64

3. Consider a 2^5 factorial experiment in which each treatment is applied to $r = 2$ experimental units. The investigators select a reduced model for their analysis; the estimates of the factorial effects they will include in their model are:

$$\hat{\alpha} = 2 \quad \hat{\beta} = 2 \quad \widehat{(\alpha\beta)} = 1 \quad \widehat{(\alpha\gamma)} = 1 \quad \widehat{(\alpha\delta)} = 1 \quad \widehat{(\alpha\beta\gamma)} = -1$$

and the corrected total sum-of-squares from their analysis is 968.

10 (a) Complete the following partial ANOVA decomposition as indicated:

Source	degrees of freedom	sum of squares
Treatments	6	768
Residual	57	200
Corrected Total	63	968

$64 \cdot [2^2 + 2^2 + 1^2 + 1^2 + 1^2 + (-1)^2]$

10 (b) Using the proposed model, what is the variance of the least-squares estimate of $\mu_{22222} - \mu_{11122}$? (Write this as a number times σ^2.)

$\frac{1}{64}\sigma^2 \underset{\sim}{c}'M, M_1'\underset{\sim}{c}$

$\mu \quad \alpha \quad \beta \quad \alpha\beta \quad \alpha\gamma \quad \alpha\delta \quad \alpha\beta\gamma$

$+ \quad (+ \quad + \quad + \quad + \quad + \quad + \quad +)$

$- \quad (+ \quad - \quad - \quad + \quad + \quad - \quad -)$

$(0 \quad 2 \quad 2 \quad 0 \quad 0 \quad 2 \quad 2) \rightarrow$ sum of squares $= 16$

$\Rightarrow 16\frac{\sigma^2}{64} = \frac{\sigma^2}{4}$

10 (c) What effects would have to be added to the proposed model in order to satisfy:

i. the hierarchy principle?

$\gamma \quad \delta \quad (\beta\gamma)$

ii. the heredity principle?

none