Quiz #2 Sol'n Notes

1. Consider the following balanced incomplete block design for 5 treatments (represented by numbers) set in 10 blocks (represented by rectangles) of 3 units each:

1	1	1	1	1	1	2	2	2	3
2	2	2	3	3	4	3	3	4	4
3	4	5	4	5	5	4	5	5	5

5 (a) What are the values of r and λ for this design?

$r = 6$ $\lambda = 3$

5 (b) Given 10 physical blocks of 3 units each, how many actual assignments of units to treatments are possible for this design? (In application, each of these possible arrangements should receive equal probability in the randomization process. You do not need to reduce this answer to a single number, but clearly express the answer as a fully numeric expression.)

$10! \times (3!)^{10}$ ... all blocks

$(3!)$ ways to assign units within one block

$10!$ ways to assign physical blocks to design blocks

5 (c) For an intra-block (i.e. fixed block) analysis, how many degrees of freedom are available for estimating σ^2?

$$\text{c.t. } (N-1) - \text{blocks } (b-1) - \text{trt's } (t-1)$$

$$29 - 9 - 4 = 16$$

5 (d) Again, for the intra-block analysis, what is the variance of $\widehat{\tau_1 - \tau_2}$? (The factor of σ^2 should be the only non-numeric factor in your answer.)

$$\sigma^2 \frac{k}{\lambda t} c'c = \sigma^2 \frac{3}{3 \cdot 5} 2 = \sigma^2 \frac{2}{5}$$

5 (e) For the inter-block (additional random block) analysis, how many degrees of freedom are available in the "residual" line?

$$\text{total } (N) - \text{model} = $$

6 param's, but one linear dep'cy

$$10 - 5 = 5$$

5 (f) Does a smaller BIBD (i.e. fewer blocks) for 5 treatments in blocks of size 3 exist? If yes, construct one. If no, prove that no smaller BIBD exists.

$$r = b\frac{k}{t} = b\frac{3}{5}$$ ← integer for mult's of 5

$$\lambda = b\frac{k(k-1)}{t(t-1)} = b\frac{6}{20} = b\frac{3}{10}$$ ← integer for mult's of 10

$\Rightarrow$ $b = 10$ is the smallest possible # of blocks

2. Consider a 2-factor experiment in which factor A has 3 levels, and factor B has 4 levels, with each treatment applied to 2 experimental units. Throughout this problem, the factorial notation used assumes the "overparameterized" notation (with over-dots in the book).

5 (a) First, suppose that the experiment is executed as a completely randomized design. Completely specify the statistical distribution (by its name and any parameter values) that would be needed to determine the critical value for a test of:

$$H_0 : \alpha_1 = \alpha_2 = \alpha_3$$

2 df.

c.t. - trt

resid df = 23 - 11 = 12

F(2,12)

5 (b) Next (and for all remaining sections of this problem), suppose the experiment is executed as a block design as follows, with (i, j) representing the treatment at level i of factor A and level j of factor B, and with blocks represented by rectangles:

(1,1)	(1,2)	(1,3)	(1,4)
(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)
(3,1)	(3,2)	(3,3)	(3,4)

For an analysis that assumes fixed block effects, what degrees of freedom are associated with 'block', 'treatment', and 'residual'?

c.t. - block - trt = resid

23 - 5 - 9 = 9

"B" = 3

"AB" = 6

"A" is confounded w/ blocks!

5 (c) Again assuming fixed block effects, suppose $\beta_1 = \beta_2 = -1$, $\beta_3 = \beta_4 = +1$ and $\sigma^2 = 2$. Compute the (numerical) value of the noncentrality parameter associate with the test of:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4$$

# of obs at each level of "B"

$$\frac{6 \sum_{i}^{4} (\beta_i - \bar{\beta})^2}{\sigma^2} = \frac{6 \cdot 4}{2} = 12$$

5 (d) Suppose blocks are regarded as random effects. Completely specify the statistical distribution (by its name and any parameter values) that would be needed to determine a critical value for a test of:

$$H_0 : \alpha_1 = \alpha_2 = \alpha_3$$

2 df

For whole plot analysis:

resid df = 5 (c.t.) - 2 (trt) = 3

F(2,3)

5 (e) Again assuming random block effects, completely specify the statistical distribution (by its name and any parameter values) that would be needed to determine a critical value for a test of $H_0 : \sigma_\delta^2 = 0$ (i.e. that there really are no block effects).

$$\frac{\text{WP resid. MS} \quad (d)}{\text{SP resid. MS} \quad (b)} \Rightarrow F(3,9)$$

3. Consider an unblocked 2^4 factorial experiment, executed as a complete block design with 2 complete replicates (i.e. two blocks, each consisting of 16 units, with one unit assigned to each treatment in each block). Throughout this problem, the factorial notation used assumes the "full rank" notation (with no over-dots in the book).

10 (a) Suppose that $\hat{\alpha} = 2$, $\widehat{(\alpha\beta)} = 2$, $\widehat{(\alpha\beta\gamma)} = -2$, and the residual sum of squares $= 120$. Compute the F statistic for testing:

$$H_0 : \alpha = (\alpha\beta) = (\alpha\beta\gamma) = 0 \qquad df = 3$$

Num. SS $= 32 \times (2^2 + 2^2 + (-2)^2) = 384$

resid df $= \underset{(ct)}{31} - \underset{(blk)}{1} - \underset{(trt)}{15} = 15$

$$F = \frac{384/3}{120/15} = 16$$

10 (b) Suppose now that a model containing an intercept, all 4 main effects, and all 6 two-factor interactions, but no higher-order terms, is used to analyzed the data. Under this model, compute the variance (with the factor of σ^2 the only non-numeric quantity) of the least-squares estimator of:

$$2\mu_{1111} - \mu_{1112} - \mu_{1121}$$

$\frac{\sigma^2}{N} c' M_1 M_1' c$

	μ	α	β	γ	δ	$(\alpha\beta)$	$(\alpha\gamma)$	$(\alpha\delta)$	$(\beta\gamma)$	$(\beta\delta)$	$(\gamma\delta)$
2	+	−	−	−	−	+	+	+	+	+	+
−1	+	−	−	−	+	+	+	−	+	−	−
−1	+	−	−	+	−	+	−	+	−	+	−
	0	0	0	−2	−2	0	2	2	2	2	4

sum of squares $= 40$

$\frac{40}{32}\sigma^2 = \frac{5}{4}\sigma^2$

10 (c) Continue to use the model specified in part (b). Suppose also that the difference between the two block averages is 2, and that *each* least-squares estimate of a factorial effect has absolute value of 2. Complete the following partial ANOVA decomposition for the data.

Source	degrees of freedom	sum of squares	
blocks	$b-1 = 1$	$16 \times \sum_1^2 (\bar{y}_{block} - \bar{y})^2$ ±1	$= 32$
treatments	10 fact. effects	$32 \times \sum_1^{10} (\cdot)^2$ → 40	$= 1280$
residual	20		
corr. total	$N-1 = 31$		