**STAT 512 Homework Assignment 6: Due in class, Wednesday October 17, 2012**

Box, Hunter, and Hunter (2005) described an experiment conducted to assess the influence of two experimental factors on the corrosion resistance of steel bars. The experimental factors of interest were:

- the temperature of the furnace in which the bars are cured (3 levels), and
- the type of coating applied to the bars (4 levels).

Hence the experiment was carried out to evaluate a total of 12 treatments. The study was (apparently) conducted using a single furnace, and re-setting the temperature of the furnace could not be done quickly. Therefore, once a partiular temperature was set, four steel bars, one with each of the coatings, were simultaneously cured. One measurement, reflecting the degree of corrosion resistance, was made on each bar at the conclusion of the experiment; these data are reported in the following table:

| Furnace Run | Temperature (°C) | Coating 1 | Coating 2 | Coating 3 | Coating 4 |
|---|---|---|---|---|---|
| 1 | 360 | 67 | 73 | 83 | 89 |
| 2 | 370 | 65 | 91 | 87 | 86 |
| 3 | 380 | 155 | 127 | 147 | 212 |
| 4 | 380 | 108 | 100 | 90 | 153 |
| 5 | 370 | 140 | 142 | 121 | 150 |
| 6 | 360 | 33 | 8 | 46 | 54 |

1. Construct an ANOVA table and test the null hypotheses of no main effect for temperature, no main effect for coating, and no temperature-by-coating interaction, *ignoring* the split-plot nature of this study, i.e. as if the data came from a simple CRD. For each of the three tests, give the $F$ statistic, degrees of freedom, and $p$-value.

2. Construct a correct (i.e. split-plot) ANOVA table and test the null hypotheses of no main effect for temperature, no main effect for coating, and no temperature-by-coating interaction. For each of the three tests, give the $F$ statistic, degrees of freedom, and $p$-value.

3. Suppose that the experiment was actually larger than described here, and that in addition to the two factors described above, a third experimental factor was included:

   - the type of steel from which the bars are made (2 levels).

   Suppose that for each type of steel, two "batches" of bars are made available (and that there may be some random difference associated with batch). The experiment as described above (i.e. 6 furnace runs) was actually executed 4 times; the temporal order of these 4 "sub-experiment" was randomized, resulting in an overall study schedule as follows:

   sub-experiment 1: 1st batch of bars made with the first type of steel
   sub-experiment 2: 1st batch of bars made with the second type of steel
   sub-experiment 3: 2nd batch of bars made with the second type of steel
   sub-experiment 4: 2nd batch of bars made with the first type of steel

Construct the "source" and "degrees of freedom" columns for the appropriate split-split-plot ANOVA table. Indicate which residual mean square would be used in the denominator for testing each of the 7 treatment mean effects and interactions in the $2 \times 3 \times 4$ factorial treatment structure.

**Reference**: Box, G., W. Hunter, and S. Hunter (2005). *Statistics for Experimenters: Design, Innovation, and Discovery*, 2nd edition. New York, NY: Wiley-Interscience.