

3.

- (a) corrected total sum of squares  $= N(\hat{\alpha}^2 + (\hat{\alpha\beta})^2 + (\hat{\alpha\beta\gamma})^2) = 2856 - 64(3^2 + 4^2 + 2^2) = 1000$ .
- (b) No additional effects are needed to satisfy the heredity principle. The hierarchy principle requires the addition of  $\beta$ ,  $\gamma$ ,  $(\alpha\gamma)$ , and  $(\beta\gamma)$ .
- (c)  $\hat{\gamma} = \frac{1}{2}(\bar{y}_{..2..} - \bar{y}_{..1..}) = \frac{1}{2}(13.5 - 13.5) = 0$ .  $\hat{\delta} = \frac{1}{2}(\bar{y}_{...2..} - \bar{y}_{...1..}) = \frac{1}{2}(12.5 - 14.5) = -1$ .  
 $(\hat{\gamma\delta}) = \frac{1}{4}(\bar{y}_{..22..} - \bar{y}_{..21..} - \bar{y}_{..12..} - \bar{y}_{..11..}) = \frac{1}{4}(13 - 14 - 12 + 15) = 0.5$ .

5.

- (a)  $\mu_{111} - \mu_{122} = -2\beta - 2\gamma + 2(\alpha\beta) + 2(\alpha\gamma)$ .  $Var(\mu_{111} - \mu_{122}) = \frac{4}{16}4 \times \sigma^2 = \sigma^2$ .
- (b)  $\mu_{111} - \mu_{122} = -2\beta - 2\gamma$ .  $Var(\mu_{111} - \mu_{122}) = \frac{2}{16}4 \times \sigma^2 = 0.50\sigma^2$ .
- (c) Numerator degrees of freedom = 2, numerator mean square =  $N((\hat{\alpha\beta})^2 + (\hat{\alpha\beta\gamma})^2)/2 = 104$ , denominator degrees of freedom = 8, denominator mean square =  $\frac{16}{8} = 2$ .

6.

- (a) and (b)

stratum	source	d.f. (a)	d.f. (b)
whole-plots	blocks (days)	0	$r - 1$
	temperature	1	1
	residual	0	$r - 1$
	corrected total (oven runs, conf.'d w/ temp)	1	$2r - 1$
split-plots	blocks (oven runs, conf.'d w/ temp)	1	$2r - 1$
	oil concentration	1	1
	temp $\times$ oil conc.	1	1
	residual	0	$2r - 2$
	corrected total	3	$4r - 1$

- (c) The oil concentration main effect will be tested against "noise" associated with batch-to-batch variability (less than run-to-run variability), which will be estimated with  $2r - 2$  split-plot degrees of freedom (more than  $r - 1$  whole-plot degrees of freedom).