STAT 512 Homework Assignment 3: Due in class, Friday September 26, 2014

Bauchart et al. (1989) conducted two experiments to compare populations of bacteria that form in the rumen of Holstein × Friesian dairy cattle when fed different diets. In one experiment, a Latin Square design was used to compare the 4 diets (the control diet C1, and experimental diets R5, R10, and T). Four animals were used in the study, which was carried out of 4 time periods (over which time, each animal received a unique rotation of the 4 diets). Hence the design was intended to support inferences about the effects of diet, after correcting for nuisance effects that could be attributed to animals or time periods. The diet-specific averages reported for one response, total lipid (g/kg dry matter) of solid-adherent bacteria, were reported as follows:

diet	C1	R5	R10	T
ave. response	158	240	282	279

Suppose that the MSE for the analysis of this response is 400.

1. Which pairs of diets are significantly different? Establish this by performing 6 pair-wise t-tests (although, as we know, this doesn't control overall error rate), reporting a p-value for each.

2. Suppose the experiment could actually have been carried out over 8 weeks (rather than 4), but that only 4 animals were available. The experiment could then be planned according to a more general row-column design, testing 4 treatments within a block structure of 4 rows (animals) and 8 columns (weeks). ONE way to do this is by direct application of one of the replicated Latin Square strategies we talked about in class. Suppose this had, in fact, been done, and the 5 statistics reported to you above had been calculated for the data collected from this larger experiment. Repeat the part (1); does this change the answers?

3. Return now to the original Latin Square design. Suppose that after the conclusion of the experiment, it was discovered that diet T was actually contaminated, and that none of the resulting data could be used (not even as reflecting an "altered" and uninteresting diet). That is, the remaining treatments might have been assigned to units as follows:

	time 1	time 2	time 3	time 4
animal 1	R5	-	C1	R10
animal 2	-	R10	R5	C1
animal 3	C1	R5	R10	-
animal 4	R10	C1	-	R5

   Use R, or another computer package that supports numerical linear algebra, to determine whether this design is "Condition E" equivalent to a completely randomized design for $t = 3$ and each $n_i = 4$.

4. Here's another potential experimental nightmare. Suppose the container that was to have contained the T diet *actually* contained the C1 diet, so that the actual assignment of diets to experimental units was:

	time 1	time 2	time 3	time 4
animal 1	R5	C1	C1	R10
animal 2	C1	R10	R5	C1
animal 3	C1	R5	R10	C1
animal 4	R10	C1	C1	R5

As a result, the experiment involved only 3 treatments, rather than 4, but the blocking format remained that of an order-4 Latin Square. Using pencil-and-paper algebra (not computer numerics this time), determine whether this design is "Condition E" equivalent to a completely randomized design for $t = 3$, with $n_1 = 8$ and $n_2 = n_3 = 4$. Note that the matrix $\mathbf{H}_1$ here is exactly that of an order-4 Latin Square. Use the same approach we used in class to show that a regular Latin Square is "Condition E" equivalent to its corresponding CRD (i.e., avoid explicit construction of $\mathbf{H}_1$).

Reference: Bauchart, D., F. Legay-Carmier, M. Doreau, B. Gaillard (1990). "Lipid Metabolism of Liquid-Associated and Solid-Adherent Bacteria in Rumen Contents of Daiy Cows Offered Lipid-Duppliemented Diets," *British Journal of Nutrition* **63**, 563-578.