1. The following table contains all treatments included in a certain 2^{5-2} fractional factorial experiment:

ad	ae	abcd	abce
b	c	bde	cde

(a) Determine the complete generating relation (i.e. include generalized interactions and signs with each "word") for this design.

$$I = +ABC = -ADE = -BCDE$$

(b) With what factorial effects is the main effect for factor B aliased? (If this effect is not aliased in this design, write "none".)

$$B = +AC = -ABDE = -CDE$$

2. A certain 2^3 experiment is to be performed in 6 blocks of size 4 as indicated below:

abc	ac	b	(1)

abc	ac	b	(1)

abc	ac	b	(1)

a	ab	bc	c

a	ab	bc	c

a	ab	bc	c

Write the "source" and degrees-of-freedom columns for an appropriate and complete ANOVA decomposition for this design, assuming random block effects. Represent each factorial effect with a single degree of freedom, and if multiple "error" lines are included, make clear which is appropriate for testing each factorial effect.

	source	df	
W.P.	AC	1	test with
	resid	4	
	c.t.	5	
S.P.	blocks	5	
	A	1	
	B	1	
	C	1	
	AB	1	test with
	BC	1	
	ABC	1	
	resid	12	
	c.t.	23	

3. A different 2^3 experiment in 6 blocks of size 4 is described below:

ab abc c (1)	bc abc a (1)	abc a b c
a b ac bc	b c ab ac	ac bc ab (1)

Assuming blocks have fixed effects,

(a) In terms of σ^2, the variance associated with units within a block, what is the variance of $\hat{\alpha}$.

$$\frac{1}{24}\sigma^2$$

(b) In terms of σ^2, what is the variance of $\widehat{(\alpha\beta)}$.

$$\frac{1}{16}\sigma^2$$

(c) Compute the number of degrees of freedom associated with the Mean Square Error.

$$\underset{(c.t.)}{23} - \underset{(blk)}{5} - \underset{(trt)}{7} = 11$$

(d) Given the 6 (physical) blocks, each containing 4 (physical) experimental units, how many equally probable ways can the treatment structure described above be applied for full randomization of this experiment? (Don't reduce your answer to a single number, but express it clearly and completely in terms of quantities like "6" and "4".)

$$6!\,(4!)^6$$

blocks

units within blocks

4. Yet another 2^3 factorial experiment is to be executed without blocks, with each treatment applied $r = 2$ times. Two different measures of the impact that factor A has on the response are:

$$\phi_1 = \tfrac{1}{4}(\mu_{211} + \mu_{212} + \mu_{221} + \mu_{222}) - \tfrac{1}{4}(\mu_{111} + \mu_{112} + \mu_{121} + \mu_{122})$$
$$\phi_2 = \mu_{211} - \mu_{111}$$

where μ_{ijk} is as we have defined it in class: the expectation of the response, in this experiment, at the indicated factor levels. For this experiment, in terms of σ^2:

(a) What is $Var(\hat{\phi}_1)$ if the full factorial model is used for inference?

$4 \times \frac{1}{16}\sigma^2 = \frac{1}{4}\sigma^2$

$\Phi_1 = 2\alpha$

$\Phi_2 = \alpha - \beta - \gamma - (\alpha\beta) - (\alpha\gamma) + (\beta\gamma) + (\alpha\beta\gamma)$
$\quad -(-\alpha - \beta - \gamma + (\alpha\beta) + (\alpha\gamma) + (\beta\gamma) - (\alpha\beta\gamma))$
$\quad 2(\alpha - (\alpha\beta) - (\alpha\gamma) + (\alpha\beta\gamma))$

(b) What is $Var(\hat{\phi}_1)$ if the 3-factor interaction is removed from the full model?

$4 \times \frac{1}{16}\sigma^2 = \frac{1}{4}\sigma^2$

(c) What is $Var(\hat{\phi}_2)$ if the full factorial model is used for inference?

$4 \times [\frac{1}{16}\sigma^2 \times 4] = \sigma^2$

(d) What is $Var(\hat{\phi}_2)$ if the 3-factor interaction is removed from the full model?

$4 \times [\frac{1}{16}\sigma^2 \times 3] = \frac{3}{4}\sigma^2$

5. Suppose you begin a study with an N-run OA(2) main-effects design; it could be a regular fraction of Resolution III or a "nongeometric" fraction like a Plackett-Burman design where N is not a multiple of 4. In any case, assume that it is an *orthogonal* design for the model containing only an intercept and main effects; i.e. $\mathbf{X}'\mathbf{X}$ for this model written in the full-rank parameterization we've been using is $N \times \mathbf{I}$. You decide to augment this initial design with its complete fold-over, e.g. N more runs selected by reversing the signs of all factors in all of the original runs. Prove that in the completed $2N$-run design, when the model containing the mean and main effects is fitted, estimates of main effects are not aliased by two-factor interactions. Assume that the two half-designs do not need to be treated as blocks in the overall experiment.

Let F be the $N \times f$ matrix corresponding to main effects in dsn #1
" S " " $N \times \binom{f}{2}$ " " " 2-factor int's " " "

For the 2N-run design:

model matrix for intercept + main effects is $X_1 = \begin{pmatrix} 1 & F \\ 1 & -F \end{pmatrix}$

model matrix for 2-factor interactions is $X_2 = \begin{pmatrix} S \\ S \end{pmatrix}$

So, $X_1'X_1 = 2N \times I \qquad (X_1'X_1)^{-1} = \frac{1}{2N} \times I$

$X_1'X_2 = \begin{pmatrix} 2 1'S \\ \hline 0 \end{pmatrix} \qquad A = \frac{1}{2N}\begin{pmatrix} 2 1'S \\ \hline 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{N} 1'S \\ \hline 0 \end{pmatrix}$ ← 1st row ← rows 2 thru f+1

So, while the intercept is biased by 2-factor interactions, the main effect estimates are not.

6. In a 2-level regular fractional factorial design of Resolution IV, with aberration index a (i.e. a "words" of length 4 in the generating relation):

(a) How many pairs of two-factor interactions are confounded?

$3a$

(b) How many (main effect, two-factor interaction) pairs are confounded?

0

(c) How many (main effect, three-factor interaction) pairs are confounded?

$4a$

7. A certain investigator wants to carry out a 2^2 experiment in blocks of size 4 (i.e. a CBD) for the *single purpose* of testing the hypothesis that the two factors do not interact, i.e. $H_0 : (\alpha\beta) = 0$ using the full-rank parameterization. He understands that, given the value of σ^2, the power of the test is determined by the number of blocks he can include and the size of the interaction if it is in fact non-zero. If he executes his design in b complete blocks:

(a) What is the noncentrality parameter associated with this test, in terms of σ^2 and $(\alpha\beta)$?

$$\frac{4b(\alpha\beta)^2}{\sigma^2}$$

(b) On how many denominator degrees of freedom will the test be based?

$$\underset{(c.t.)}{(4b-1)} - \underset{(b|c)}{(b-1)} - \underset{(trt)}{3} = 3b-3$$

8. An experimental design is needed for a 2^5 study to be carried out in 12 blocks of size 8, where blocks must be considered to have fixed effects. The design must satisfy the following requirements:

- All main effects and 2-factor interactions must be estimable with full efficiency, i.e. have variance $\frac{1}{96}\sigma^2$.
- All other factorial effects (3-, 4-, and 5-factor interactions) must be estimable with at least 2/3 efficiency, i.e. have variance $\frac{1}{64}\sigma^2$ or less.

(a) Use partial confounding to generate a design that meets these specifications. Specify your design by writing each identifying relation you use (in complete form).

e.g.

rep 1: $I = \pm ABC = \pm CDE = \pm ABDE$

rep 2: $I = \pm BCD = \pm ADE = \pm ABCE$

rep 3: $I = \pm ACE = \pm BDE = \pm ABCD$

(b) How many degrees of freedom are available for estimating σ^2 in your design (assuming no interaction between blocks and treatments)?

$$\underset{(c.t.)}{(96-1)} - \underset{(b|c)}{(12-1)} - \underset{(trt)}{31} = 53$$