**STAT 512 Homework Assignment 1: Due in class, Friday September 12, 2014**

**I.** Butler et al. (2009) conducted an study to investigate the strength and toughness of textile-reinforced concrete. Read the article (not for all the details about concrete, but for information about how the experiment was conducted).

1. What were the units used in this study?
2. What were the treatments investigated in this study?
3. Briefly discuss what you can tell from the article about how each of the following design principles were used/addressed:
   - experimental control (not in the sense of "control group", but elimination of variation)
   - randomization
   - replication

**II.** Paiva et al. (2010) conducted an experiment to investigate the effectiveness of a commercially available sealant in preventing the growth or survival of the pathogen *Listeria monocytogenes* within concrete. Specifically, they made a number of "ice-cube sized" bricks from a standard concrete mix and divided these randomly into 4 groups of equal size for treatment as follows:

- Treatment 1: no sealant applied
- Treatment 2: sealant applied before inoculation
- Treatment 3: sealant applied after inoculation
- Treatment 4: sealant applied before and after inoculation

Here, "inoculation" refers to submerging the bricks in a bath containing a fixed concentration of *L. monocytogenes 3C*. Where sealant was applied (either before or after), it was done in a standardized way to make the sealant coatings as uniform as possible.

After 30 minutes, each brick was broken in half, and the exposed internal surface was swabbed, after which the swabs were processed in a standard way, leading eventually to cell cultures grown on plates. The response variable analyzed was the surviving *L. monocytogenes 3C* cell concentration ($\log_{10}$ cfu/cm$^2$) determined from each plate. Means recorded for each group were:

Trt 1: 2.64 Trt 2: 2.28 Trt 3: 1.82 Trt 4: 1.40

Suppose that a total of 12 bricks were made, that 3 were randomly allocated to each of the 4 treatments in the experiment, and that the residual mean square for a one-way analysis of variance for these data was 0.3675.

1. Test the null hypothesis of no difference among the 4 treatments using the standard ANOVA-based F-test. Include the value of the test statistic and and achieved significance level ("p-value").

2. Consider an "effects" linear model for this setting, including one parameter, $\alpha$, to represent the effects common to all units in the experiment (e.g. the particular batch of concrete used, the lab conditions on the day of the experiment, et cetera), and 4 additional parameters, $\tau_1$ through $\tau_4$, representing the treatments. Prove or disprove (don't just say what the basis of that proof would be) that each of the following is estimable:

   (a) $\alpha$

   (b) $\alpha + \tau_1$

   (c) $\alpha - \tau_1$

   (d) $\tau_1 + \tau_2$

   (e) $\tau_1 - \tau_2$

3. Suppose that in executing this experiment, a non-trivial amount of the expense is actually the application of the sealant. Let's say that production of each brick and submersion in the inoculum costs \$10, and that coating a brick costs \$10. So each unit to which treatment 1 is applied costs \$10, each unit to which either treatment 2 or treatment 3 is applied costs \$20, and each unit to which treatment 4 is applied costs \$30. The investigator wants to be able to estimate 3 treatment contrasts well:

$$\begin{gathered}\tau_1 - \tfrac{1}{3}(\tau_2 + \tau_3 + \tau_4)\\ \tfrac{1}{2}(-\tau_1 + \tau_2 - \tau_3 + \tau_4)\\ \tfrac{1}{2}(-\tau_1 - \tau_2 + \tau_3 + \tau_4)\end{gathered}$$

   and has a budget of \$600 with which to carry out this experiment. Use the Method of Lagrangian Multipliers to find the design that minimizes the average variance for the 3 contrasts of interest, under the cost constraints of the experiment. First derive the optimal design as if the $n_i$ are continuous variables, and then round your result to integer-valued sample sizes that satisfy the const constraint. (Note that the contrasts described above are equivalent to their counterparts with $\mu$'s substituted for $\tau$'s, so you can use the cell means model to work this problem if you prefer.)

4. Suppose that the investigator had considered 3 different unit allocation patterns for the Completely Randomized Design for this study:

| | design | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $n_1$ | 3 | 2 | 4 |
| $n_2$ | 3 | 4 | 2 |
| $n_3$ | 3 | 4 | 4 |
| $n_4$ | 3 | 2 | 2 |
| $N$ | 12 | 12 | 12 |

   Suppose further that before the experiment, the investigator actually *thought* that

$$\tau_1 - 0.5 = \tau_2 = \tau_3 = \tau_4 + 0.5$$

and that $\sigma = 0.5$. Under these conditions, what is the power of the F-test (with type I error probability of 0.10) of no treatment difference for each of the 3 experimental designs? (Note that because the equation above is equivalent to the same expression with $\mu$'s substituted for their respective $\tau$'s, you can work this problem using the cell means model if you prefer.)

**References:**


- Butler, M., V. Mechtcherine, and S. Hempel (2009). "Experimental Investigations on the Durability of Fibre-Matrix Interfaces in Textile-Reinforced Concrete," *Cement & Concrete Composites* **31**, 221-231.

- Paiva, D.M., K.S. Macklin, S.B. Price, J.B. Hess, D.E. Conner, and M. Singh (2010). "Efficacy of a Commercial Concrete Sealant against Listeria spp.: A Model for Poultry Processing Facilities," *Journal of Applied Poultry Research* **19**, 146-151.