Problem 1

HW3 Sol^n Notes

part (a)

```
b <- 8
df1 <- 4-1
df2 <- (8-1)*(4-1)
crit <- qf(.90,df1,df2)
answers <- matrix(0,ncol=2,nrow=10)
for(i in 1:10) {
  ncp <- 4+(i-1)/10
  answers[i,1] <- ncp
  pII <- pf(crit,df1,df2,ncp)
  answers[i,2] <- pII
}
answers

      [,1]      [,2]
 [1,]  4.0 0.5645101
 [2,]  4.1 0.5565864
 [3,]  4.2 0.5487181
 [4,]  4.3 0.5409064
 [5,]  4.4 0.5331529
 [6,]  4.5 0.5254586
 [7,]  4.6 0.5178249
 [8,]  4.7 0.5102528
 [9,]  4.8 0.5027434 <- NCP = 4.8
[10,]  4.9 0.4952977
```

part (b)

```
oldncp <- 4.9
answers <- matrix(0,ncol=2,nrow=10)
for (i in 1:10) {
  b <- 15+i
  answers[i,1] <- b
  ncp <- b*(oldncp/8)
  df1 <- 4-1
  df2 <- (b-1)*(4-1)
  crit <- qf(.90,df1,df2)
  power <- 1-pf(crit,df1,df2,ncp)
  answers[i,2] <- power
}
answers

      [,1]      [,2]
 [1,]   16 0.8146817
 [2,]   17 0.8385738
 [3,]   18 0.8597764
 [4,]   19 0.8785164
 [5,]   20 0.8950171
 [6,]   21 0.9094947 <- 21 blocks
 [7,]   22 0.9221551
 [8,]   23 0.9331917
 [9,]   24 0.9427843
[10,]   25 0.9510986
```

Problem 2

a) $H_1 = \operatorname{diag}\left(\frac{1}{4}J_{4\times4} \;\; \frac{1}{4}J_{4\times4} \;\; \frac{1}{4}J_{4\times4}\right)$

b.) $X_{2|1} = X_2 - H_1X_2 = \begin{pmatrix} I \\ I \\ I \end{pmatrix} - \begin{pmatrix} \frac{1}{4}J \\ \frac{1}{4}J \\ \frac{1}{4}J \end{pmatrix}$

For CRD, $H_1 = \frac{1}{12}J$, so $H_1X_2 = \frac{1}{12}\begin{pmatrix} 3J \\ 3J \\ 3J \end{pmatrix} = \begin{pmatrix} \frac{1}{4}J \\ \frac{1}{4}J \\ \frac{1}{2}J \end{pmatrix} \Rightarrow$ yes

c.) $X_2 = \left(\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 1 \\ & & 1 \\ \hline 1 & & \\ & 1 & \\ & & 1 \\ & & 1 \\ \hline 1 & & \\ & 1 & \\ & & 1 \\ & & 1 \end{array}\right) \quad X_{2|1} = X_2 - H_1X_2 = \; - \left(\frac{1}{4}\underset{\sim}{1} \;\; \frac{1}{4}\underset{\sim}{1} \;\; \frac{1}{2}\underset{\sim}{1}\right)$

For CRD, $H_1 = \frac{1}{12}J$, so $H_1X_2 = \frac{1}{12}\left(3\underset{\sim}{1} \;\; 3\underset{\sim}{1} \;\; 6\underset{\sim}{1}\right) = \left(\frac{1}{4}\underset{\sim}{1} \;\; \frac{1}{4}\underset{\sim}{1} \;\; \frac{1}{2}\underset{\sim}{1}\right)$

$\Rightarrow$ yes

d.) $X_2 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \\ \hline 1 & & \\ & 1 & \\ & & 1 \\ \hline 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$ $\quad X_{2|1} = X_2 - H_1 X_2 = \quad - \frac{1}{4} \begin{pmatrix} 2\underset{\sim}{1} & \underset{\sim}{1} & \underset{\sim}{1} \\ \hline \underset{\sim}{1} & 2\underset{\sim}{1} & \underset{\sim}{1} \\ \hline \underset{\sim}{1} & \underset{\sim}{1} & 2\underset{\sim}{1} \end{pmatrix}$

For CRD, $H_1 = \frac{1}{12} J$, so $H_1 X_2 = \frac{1}{3} J \Rightarrow \underset{\sim}{\text{no}}$

e.) $X_{2|1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ \hline \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ \hline \frac{3}{4} & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{pmatrix}$

so $X_{2|1}' X_{2|1} \hat{\underset{\sim}{\tau}} = X_{2|1}' \underset{\sim}{y}$

$\Downarrow$

$$\begin{pmatrix} 2\frac{1}{2} & -1\frac{1}{4} & -1\frac{1}{4} \\ -1\frac{1}{4} & 2\frac{1}{2} & -1\frac{1}{4} \\ -1\frac{1}{4} & -1\frac{1}{4} & 2\frac{1}{2} \end{pmatrix} \begin{pmatrix} \hat{\tau}_1 \\ \hat{\tau}_2 \\ \hat{\tau}_3 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{2}(2\bar{y}_{11} - y_{12} - y_{13}) + \frac{1}{4}(3y_{21} - 2\bar{y}_{22} - y_{23}) + \frac{1}{4}(3y_{31} - y_{32} - 2\bar{y}_{33}) \\ \frac{1}{4}(3y_{12} - 2\bar{y}_{11} - y_{13}) + \frac{1}{2}(2\bar{y}_{22} - y_{21} - y_{23}) + \frac{1}{4}(3y_{32} - y_{31} - 2\bar{y}_{33}) \\ \frac{1}{4}(3y_{13} - 2\bar{y}_{11} - y_{12}) + \frac{1}{4}(3y_{23} - y_{21} - 2\bar{y}_{22}) + \frac{1}{2}(2\bar{y}_{33} - y_{31} - y_{32}) \end{pmatrix}$$

cont $\Rightarrow$

So, for example, the 1st eqn minus the 2nd is:

$$\frac{15}{4}(\hat{\tau}_1 - \hat{\tau}_2) = \frac{3}{2}\bar{y}_{11} - \frac{5}{4}y_{12} - \frac{1}{4}y_{13} + \frac{5}{4}y_{21} - \frac{3}{2}\bar{y}_{22} + \frac{1}{4}y_{23} + y_{31} - y_{32}$$

Multiply both sides by $\frac{4}{15}$ & group right-side terms by block:

$$\begin{aligned}\widehat{\tau_1 - \tau_2} &= \left(\tfrac{6}{15}\bar{y}_{11} - \tfrac{5}{15}y_{12} - \tfrac{1}{15}y_{13}\right) && \leftarrow \text{linear combinations} \\ &\quad + \left(\tfrac{5}{15}y_{21} - \tfrac{6}{15}\bar{y}_{22} + \tfrac{1}{15}y_{23}\right) && \leftarrow \text{within each block are} \\ &\quad + \tfrac{4}{15}\left(y_{31} - y_{32}\right) && \leftarrow \underline{\text{contrasts}}\text{, so } \beta\text{'s} \\ & && \quad\ \text{cancel out of expectations}\end{aligned}$$

$$\Rightarrow \quad \left.\begin{aligned} E(\widehat{\tau_1 - \tau_2}) &= \left(\tfrac{6}{15}\tau_1 - \tfrac{5}{15}\tau_2 - \tfrac{1}{15}\tau_3\right) \\ &\quad + \left(\tfrac{5}{15}\tau_1 - \tfrac{6}{15}\tau_2 + \tfrac{1}{15}\tau_3\right) \\ &\quad + \left(\tfrac{4}{15}\tau_1 - \tfrac{4}{15}\tau_2\right) \end{aligned}\right\} \quad \tau_1 - \tau_2$$