STAT 512 Homework Assignment 3: Due in class, Friday September 14, 2011

1. Cline et al. (1997) investigated the effect of the level of vitamin A on bone density in dogs using a complete block design. From their abstract:

   > "*Thirty-two random-source dogs were assigned to four treatments in a randomized complete block design. The diets contained 15,000, 50,000, 116,000, or 225,000 IU vitamin A/1,000 kcal ME. Diets were fed up to 1 yr. Computed tomography was used to determine bone density of the right tibia at ... 12 mo. ... There was no difference ($p > .10$) in tibia bone or marrow density in any of the dogs fed the various concentrations of vitamin A.*"

   Assume the investigators used blocks of 4 similar dogs, and assigned one dog from each block to each treatment.

   (a) Suppose that the diets actually *do* influence bone density, i.e. that the investigators made a type II error in this study, testing at the 0.10 level. What would the value of the non-centrality parameter for this test have to be in order for the probability of this error to be 0.5? (Determine this to 2 significant places numerically by trial and error.)

   (b) Now suppose a larger follow-up experiment of the same kind (CBD in blocks of size 4) might be performed with the hope of establishing differences among the 4 diets. If the non-centrality parameter for this first study actually was the value you calculated above, how many blocks would be needed in the follow-up study to result in a power of 0.9?

2. Consider a block experiment with 3 blocks of 4 units each ($N = 12$). Using the notation developed in class:

   (a) What is $\mathbf{H}_1$?

   (b) Suppose $t = 4$, and one unit is assigned to each treatment in each block (i.e. our definition of a standard CBD). What is $\mathbf{X}_{2|1}$? Show why this design is or is not "Condition E-equivalent" to a CRD with $n_1 = n_2 = n_3 = n_4 = 3$.

   (c) Suppose $t = 3$, and in each block one unit is assigned to each of treatments 1 and 2 and two units are assigned to treatment 3. What is $\mathbf{X}_{2|1}$? Show why this design is or is not "Condition E-equivalent" to a CRD with $n_1 = n_2 = 3$ and $n_3 = 6$.

   (d) Suppose $t = 3$, and in block i ($i = 1, 2, 3$), two units are assigned to treatment i and one unit is assigned to each of the other two treatments. What is $\mathbf{X}_{2|1}$? Show why this design is or is not "Condition E-equivalent" to a CRD with $n_1 = n_2 = n_3 = n_4 = 3$.

   (e) Continuing with the design specified in part (d) of this problem, derive the reduced normal equations for $\hat{\boldsymbol{\tau}}$. (Note that you will have to be careful about the way you define notation for data here. I suggest using $y_{i,j}$ for the datum from block i and treatment j when $i \neq j$, and $\bar{y}_{i,i}$ for the average of the two responses with common treatment in each block.)

Reference: Cline, J.L., G.L.Czarnecki-Maulden, J.M. Losonsky, C.R. Sipe, and R.A. Easter (1997). "Effect of increasing dietary vitamin A on bone density in adult dogs," *J. Anim. Sci.* **75**, 2980-5.