

STAT 512 Homework Assignment 6, Due in class Friday, Oct. 31, 2014.

Sharma et al. (1996) conducted an experiment to determine the effects of 5 different controllable manufacturing process factors on the optical properties of paper. Five factors were varied systematically in an unreplicated, full 2^f experiment, and a number of response measurements were recorded for each treatment, including indices of brightness (%), opacity (%), light-scattering coefficient (m^2/kg), and light-absorption coefficient (m^2/kg). The two levels (values) for each factor are given in the table below, and coded factors (in the order given, with 1 = “low” and 2 = “high”) along with the recorded brightness response are tabulated in the attached data file.

Factor	Factors levels in Paper Experiment		
	Low Level	High Level	Physical Units
Pulping chemical NaOH	0	2	% of dry paper by weight
Pulping chemical Na_2SO_3	0	2	% of dry paper by weight
Pulping pressure	200	400	psig
Pulping time	2	4	min
Dispersant	0	0.25	% of dry paper by weight

For parts 1-5, assume that the experiment was executed as a CRD.

1. Since the experiment was unreplicated, formal inference cannot be undertaken based on a fit of the full model. Instead, use R (or another computer package if you prefer) to produce a half-normal plot of the effect estimates, and indicate which factorial effects you think are non-zero based on your plot. Include your code script, along with the plot you produce.
2. Use Lenth’s method to continue your analysis of the effect estimates from part 1. Show how you calculate the pseudo standard error, and indicate which factorial effects should be considered significant based on a t value of 1.8.
3. If each of the “significant” effects from part 2 are included in the model, what other effects (if any) should also be included to satisfy the hierarchy principle? the heredity principle?
4. Suppose now, and in parts 5 and 6, that each data value listed is actually the average of 2 independent measurements (i.e. that the experimental design was actually a *replicated* full factorial with a total of 64 observations), and that the residual sum-of-squares for the full data set is 1000 (on a $\%^2$ scale, whatever that is!). What is the value of the actual standard error against which each factorial effect estimate should be compared?
5. Test the hypothesis that all interactions involving 3 or more factors are zero. Give values for the degrees of freedom, the F -statistic, and the p -value.
6. Finally, suppose this experiment had actually been performed in the following way:
 - Four batches of pulp were produced, 2 were treated with high NaOH and two with low NaOH.
 - Each batch was divided in half, and the two halves were treated with high or low Na_2SO_3 , respectively.

- Each half-batch was divided in half, and the two quarters were treated with high or low dispersant, respectively.
- Each quarter-batch was divided into 4 parts, which were treated with the 4 combinations of pressure and time levels.

How many degrees of freedom would have been available to construct a standard error for each of the 5 main effects in this study?

Reference: Sharma, A.K., W.K. Forester, and E.H. Shriver (1996). "Physical and Optical Properties of Steam-Exploded Laser-Printed Paper," *TAPPI Journal* **79**, 211-221.