All of C

$t=12$

$a=4 \quad b=3$

STATISTICS 512, PRACTICE QUIZ #2

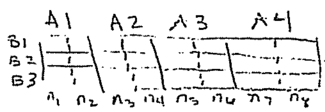
A. Consider an experiment executed to compare a collection of 12 treatments, where each treatment is defined by selecting one level of a 4-level factor A, and one level of a 3-level factor B. The experimental "material" in this experiment is a group of 8 people, or "subjects". The study is carried out as follows:

- The 8 subjects are randomly divided into 4 groups, each of size 2.
- Both subjects in group i are treated with the ith level of factor A ($i = 1, 2, 3, 4$)
- Each subject is subsequently treated with all 3 levels of factor B, and one value of the response is measured to represent each of these treatments. (So, each of 8 subjects yields 3 data values, for a total of 24 data values.) Denote the data collected by y_{ilj} where $i = 1, 2, 3, 4$ (level of factor A), $\boxed{l = 1...8}$ (subject), and $j = 1, 2, 3$ (level of factor B).

$r=2 \qquad l=1,2$

The investigator paid no attention to the possible effects associated with subjects, and analyzed the data in a way that would be appropriate for a CRD design for a 4×3 factorial experiment with 2 observations in each cell. Part of his ANOVA table looks like:

$r=2$



$N=24$

source	sum-of-squares
A	20
B	30
A×B	30
residual	48
	128

source	df	SS
A	3	
ind	$a(r-1)=4$	
B	2	
AB	6	
resid	8	
c. total	23	

He further calculated the quantity $\sum_l (\bar{y}_{.l.} - \bar{y}_{...})^2$ and found it to be 10.

The investigator's analysis is incorrect, but you have enough information to determine some of what he should have done. Given what you've been told, what should be:

whole plot part of exp.

1. the denominator sum-of-squares for testing the main effect of factor A

$\sum_i \sum_l b(\bar{y}_{.l.} - \bar{y}_{...})^2$

$= 3(10) = 30 - 20 = \boxed{10}$

WP (CRD) c. total

A 20

resid diff.

c.t. $= b\sum(\bar{y}_{.l.}-\bar{y}_{...})^2$

$= 3(10) = 30$

so $30 - 20 = 10$

2. the denominator degrees-of-freedom for testing the main effect of factor A

4

3. the denominator sum-of-squares for testing the AB interaction

$48 - 10 = 38$

$128 - [20 + 10 + 30 + 30] = 38$

4. the denominator degrees-of-freedom for testing AB interaction

8

$BT = B_2 - 2$
$B_2 = B_3 - 2$
$B_3 =$

$Q = \sum n_i (\tau_i - \bar{\tau})^2$

$$\boxed{E(\beta_1) = E(\beta_2) - 2 = E(\beta_3) - 4}$$

B. Suppose now that the 4 × 3 factorial experiment described in A had instead been carried out using only two subjects, with each subject being exposed to all 12 treatments. We know that subjects can have an effect on the response, but we are at least initially willing to assume that this effect is additive, i.e. that subjects and treatments don't interact. The investigator strongly suspects that factor A does not have any effect on the response, i.e. that the main effect for A and the AB interaction are absent. His best guess is that the expected responses at level 1 of factor B are 2 response measurement units less than those at level 2 of factor B, which are in turn 2 response measurement units less than those at level 3 of factor B. He also believes that for this experiment, the random noise associated with each measurement will be i.i.d. with variance of 4. $\sigma^2 = 4$

1. What will be the value of the noncentrality parameter governing the power of the F test of:

   CBD which is Conf. E equiv.

   - the test for no main effect for factor A

     0

     $E(\bar{Y}) = N$

   - the test for no main effect for factor B

     $n_i = \boxed{8}$

     $8[(-2)^2 + 0 + 2^2] = \frac{64}{4} = 16$

   - the test for no difference between any two treatments

     $4 \left[2 \left[(-2)^2 + 0 + (2^2) \right] \right] = \frac{64}{4} = 16$

     # tmt combos $\quad n_i$

     $ncp = \frac{r}{\sigma^2} \left[4 \left[(-2)^2 + 0^2 + 2^2 \right] \right]$, where $r = 2$, $\sigma^2 = 4$

2. How many denominator degrees of freedom will be involved in the test for no main effect for factor B?

	block	1
tmt = 12-1	~~A~~	3
	B	2
	~~AB~~	6
	resid	11
		23

$\boxed{11}$ ✓

3. Suppose you learn that one (but not both) of the two subjets has an allergy that produces an unusual reaction related to the response variable *only* when the subject is treated with the second level of factor A. Does this have possible implications for the validity of the "standard" data analysis for this experiment? Why? (Be specific.)

   A2 v/c 2

   Yes b/c no result w/ person 1 A2? + occurred over all levels of B?

   assumes no tmt block interaction

2

Q b

C. Consider an unreplicated, unblocked 2^6 complete factorial experiment. The "corrected total sum of squares",

$$\sum_{ijklmn}(y_{ijklmn} - \bar{y}_{......})^2$$

has the value of 2856. Using Lenth's method, an informal analysis of the data suggests that there are only three "active" factorial effects, with least-squares estimates: $\hat{\alpha} = 3$, $\widehat{(\alpha\beta)} = 4$, $\widehat{(\alpha\beta\gamma)} = 2$.

1. One "validation" procedure (that is of very questionable value) following Lenth's procedure is to calculate an F statistic for testing the selected effects, given the reduced model – in this case, for:

$$Hyp_0 : E(y) = \mu \Leftrightarrow \alpha = \alpha\beta = \alpha\beta\gamma = 0 \quad \text{r.d.}$$
$$Hyp_A : E(y) = \mu \pm \alpha \pm (\alpha\beta) \pm (\alpha\beta\gamma) \quad \text{c.u.}$$

where the appropriate selection of $\pm$ is for each response and term depends on the factor levels. For this statistic, what is:

- the numerator sum of squares

  $N(\hat{\alpha}^2 + \widehat{\alpha\beta}^2 + \widehat{\alpha\beta\gamma}^2) = 2^6(9+16+4) = 2^6(29)$
  $= 64(29)$
  $= 1856$

- the numerator degrees of freedom

  3

- the denominator sum of squares

  $SST - SS_{num} = 2856 - 1856 = 1000$

- the denominator degrees of freedom

  $2^6 - 1 = \underbrace{63}_{N-1} - \underbrace{3}_{trt} = \boxed{60}$

  trt 3
  error $\boxed{60}$
  total 63

2. What minimal collection of additional effects (besides the three listed above) would have to be re-introduced into the model in order to satisfy:

   $\alpha\beta\gamma$
   $\alpha\beta \quad \alpha\gamma \quad \beta\gamma$

- effect heredity? none

- effect hierarchy? $\hat{\beta}, \hat{\gamma}, \widehat{\alpha\gamma}, \widehat{\beta\gamma}$

  12 12.5 13
  13.5 13.5
  15 14.5 14

  β: $-$ $+$
  γ: $+$ $+$
  $\beta\gamma$: 12 13 15 14

3. Suppose you are given that

   $\hat{\gamma}, \hat{\delta} + \widehat{\gamma\delta}$

   $\bar{y}_{..21..} = 14$, $\bar{y}_{..11..} = 15$, $\bar{y}_{..22..} = 13$, $\bar{y}_{..12..} = 12$

Identify and give the value of any additional factorial effects that can be computed based on this information.

$\bar{y}_{..2} = \frac{14+13}{2} = 13.5$ $\quad \bar{y}_{...1} = \frac{14+15}{2} = 14.5$
$\bar{y}_{..1} = \frac{15+12}{2} = 13.5$ $\quad \bar{y}_{...2} = \frac{13+12}{2} = 12.5$

$\hat{\gamma} = \frac{1}{2}(13.5 - 13.5) = 0$, $\hat{\delta} = \frac{1}{2}(-2) = \frac{-1}{32}$, $\widehat{\gamma\delta} = \frac{1}{64}[\ ]$ $\boxed{0}$ $\boxed{\frac{2}{64}}$

$(r-1)$ — $r = \frac{bk}{t}$ $\lambda = \frac{r(k-1)}{t-1}$

$bk = N$

D. An investigator would like to compare six treatments using a experiment arranged as a BIBD.

1. She would like to execute her experiment in six blocks, each of size four. Can this be done for a BIBD? State clearly why it can or cannot.

$t = 6$ wms $r = \frac{bk}{t}$ and $\lambda = \frac{r(k-1)}{t-1}$ are integers

$b = 6$ $r = \frac{6(4)}{6} = 4$ ✓

$k = 4$

$\lambda = \frac{4(3)}{5} = \frac{12}{5} = 2\frac{3}{5}$

2. She decides instead to use six blocks, each of size five. Complete the figure below by filling in each blank with a treatment number (1-6), so that the six blocks form a BIBD.

$r = \frac{6(5)}{6} = 5$

$\lambda = \frac{5(4)}{5} = 4$

1	2	3	4	5
1	2	3	4	6
1	2	3	5	6
1	2	4	5	6
1	3	4	5	6
2	3	4	5	6

3. Apart from a factor of σ^2, what is the value of

$$Var(2\tau_1 - \widehat{\tau_2} - \tau_3)$$

for this experiment, assuming that blocks represent fixed effects?

$\underline{c} = (2, -1, -1, 0, 0, 0)$

$Var[c'\tau] = \frac{k\sigma^2}{\lambda t} c'c$

$= \sigma^2 \left(\frac{5}{4(6)}\right) [4+1+1]$

$= \boxed{\frac{5}{4}\sigma^2}$

Solution notes:

Want $\Sigma(\bar{y}_{il.} - \bar{y}_{i..})^2$

A

$\Sigma(\bar{y}_{il.} - \bar{y}_{...})^2 - \Sigma(\bar{y}_{.l.} - \bar{y}_{...})^2$

=

1. 3×10 (total between subjects) - 20 (factor A main effect) = 10
2. $4(2-1) = 4$
3. $48 - 10 = 38$
4. 23 - 7(subjects) - 2(B) - 6(AB) = 8

B

- -
  - 0
  - $8(2^2 + 0 + 2^2)/4 = 16$
  - $2(4 \times 2^2 + 4 \times 0 + 4 \times 2^2)/4 = 16$
- 23 - 1 - 11 = 11
- block-by-treatment interaction

C

- -
  - $64(3^2 + 4^2 + 2^2) = 1856$
  - 3
  - 2856 - 1856 = 1000
  - 60
- -
  - none
  - β, γ, $(\alpha\gamma)$, $(\beta\gamma)$
- $\hat{\beta} = (13.5 - 13.5)/2 = 0$, $\hat{\gamma} = (12.5 - 14.5)/2 = -1$, $\widehat{(\beta\gamma)} = (14 - 13)/2 = 0.5$

D

- $r = 4$, $\lambda = 4 * (4-1)/(6-1)$, no
- (12345),(12346),(12356),(12456),(13456),(23456)
- apart from σ^2, $\frac{k}{\lambda t}\mathbf{c}'\mathbf{c} = \frac{5}{4\times 6}6 = 5/4$