Brief introduction to Probability

Def. (deterministic event)

- an event whose outcome/value is completely known

Ex: Tomorrow is Wed

Def: (random event)

- an event whose outcome/value is not completely known.

Ex: - rolling a (fair) dice: the outcome could be $\{1, 2, .., 6\}$

- flipping a (fair) coin: out_come could be either head or tail.

Def (probability space) $\Omega, \mathcal{F}, \mathbb{P}$

a) Ω : sample space : contain all the outcomes or values of a (elementary) random events

Ex: 1) flipping a coin: element event is "flipping". $\Omega = \{ \{H\}, \{T\} \}$

2) Rolling dice: (elementary event)

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

non-elementary event: "event of getting face less than 3"

2) $\mathcal{F}$ = σ-algebra = the set of all "measurable" events $A \subseteq \Omega$. Here measurable means that we can assign A with a probability. with additional conditions

i) Complementation: $A \in \mathcal{F} \Rightarrow A' = \Omega \setminus A \in \mathcal{F}$

ii) Countable union:

$A_i \in \mathcal{F}$, then $\left(\bigcup_{i=1}^{\infty} A_i \right) \in \mathcal{F}$

3) <u>Probabity measure $\mathbb{P}$</u>: is a function

$\mathbb{P}: \Omega \rightarrow [0,1]$:

with the conditions

1) $\mathbb{P}[\emptyset] = 0$, $\mathbb{P}[\Omega] = 1$

2) $\forall A \in \mathcal{F}$: $0 \leq \mathbb{P}[A] \leq 1$

3) $A_i \in \mathcal{F}$, $A_i \cap A_j = \emptyset$ (mutually disjoint, mutually exclusive events), then

$$\mathbb{P}\left[\bigcup_{i=1}^{n} A_i \right] = \sum_{i=1}^{n} \mathbb{P}[A_i]$$

Ex: flipping a coin: $\mathbb{P}[\{H\}] = \mathbb{P}[\{T\}] = 1/2$

i) $\Omega = \{ \{H\}, \{T\} \}$

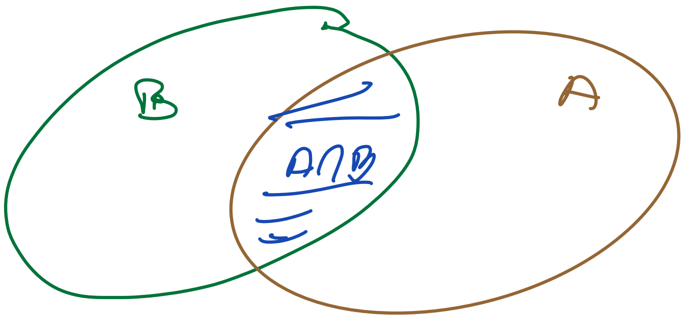
ii) $\mathcal{F} = \{ \phi, \Omega, \{H\}, \{T\} \}$

Proposition: i) $\mathbb{P}[A'] = 1 - \mathbb{P}[A]$

ii) $A \subseteq B$: then $\mathbb{P}[A] \leq \mathbb{P}[B]$

iii) **Sum rule:**

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B]$$



iv). **Union bound:** $A, B \in \mathcal{F}$

$$\mathbb{P}[A \cup B] \leq \mathbb{P}[A] + \mathbb{P}[B]$$

Ex: A: conducting the class virtually

B: attending " virtually

suppose $\mathbb{P}[A] \leq \varepsilon_1$

$\mathbb{P}[B] \leq \varepsilon_2$

Consider the event of attending class in person.

$C = A' \cap B'$

$$\mathbb{P}[C] = 1 - \mathbb{P}[C']$$
$$= 1 - \mathbb{P}[(A' \cap B')']$$
$$= 1 - \mathbb{P}[[(A')'] \cup [(B')']]$$
$$= 1 - \mathbb{P}[A \cup B]$$

$$\geq 1 - \mathbb{P}[A] - \mathbb{P}[B]$$

$$= 1 - \varepsilon_1 - \varepsilon_2$$