(8)

$$C_q = -\frac{1}{\frac{k^2}{4m}+\frac{q^2}{2m}-E}\sum_{q_1} V_{qq_1} C_{q_1}$$

Let us consider $V_{qq_1} = \text{const} = V = -|V| < 0$

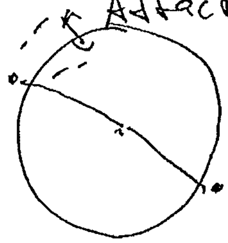
$$\cancel{\sum_q C_q} = |V| \sum_q \frac{1}{\frac{k^2}{4m}+\frac{q^2}{2m}-E} \cancel{\sum_{q_1} C_{q_1}}$$

$$\boxed{1 = |V| \sum_q \frac{1}{\frac{k^2}{4m}+\frac{q^2}{2m}-E}}$$

$k_1^2 = q^2 + qkt + \frac{k^2}{4}$ $\quad t = \cos \vec{q}\,\vec{k}$

$k_2^2 = q^2 - qkt + k^2/4$ $\quad k_1 \simeq -k_2$ $\quad q = k_F + \Delta q$

Attractive



$q^2 = k_F^2 + 2k_F \Delta q$

Both electrons are close to k_F

$k_1^2 > k_F^2 \qquad k_2^2 > k_F^2$

$k_F^2 + 2k_F\Delta q + qkt \geq k_F^2$ $\quad k^2/4$

$k_F^2 + 2k_F\Delta q - qkt \geq k_F^2$ $\quad k^2/4$

$q \equiv k_F$

$\cos \hat{kq} = t > 0$

$$\boxed{\Delta q_{min} = \frac{qk}{2k_F}|t|} = \frac{k|t|}{2}$$

$\frac{k^2}{8k_F}$

$|\epsilon_k - \epsilon_{k-q}| < \omega_0$

$\frac{k_1^2}{2m} < \epsilon_F + \omega_0 \quad \rightarrow \quad 2k_F\Delta q + qkt < 2m\omega_0$

$\frac{k_2^2}{2m} < \epsilon_F + \omega_0 \qquad\qquad 2k_F\Delta q - qkt < 2m\omega_0$

$$\Delta q_{max} = \frac{m\omega_0}{k_F} - \frac{qk|t|}{2k_F}$$

3

Equation

$$1 = |V| \int \frac{d^3q}{(2\pi)^3} \frac{1}{\frac{k_F^2}{m} + \frac{2k_F\Delta q}{m} - \mathcal{E} + \frac{K^2}{4M}}$$

$$2\epsilon_F - \mathcal{E} = \mathcal{E}$$

$$1 = \frac{|V|}{(2\pi)^3} \cdot \frac{4\pi}{2 \cdot 2\pi} \int_0^1 d\cos\theta \int_{\Delta q_{min}}^{\Delta q_{max}} \frac{k_F^2 \, d\Delta q}{\frac{2k_F\Delta q}{m} + \mathcal{E}}$$

$t>0$

$$1 = \frac{|V|}{2\pi^2} k_F^2 \frac{m}{2k_F} \int_0^1 \ln \left| \left| \frac{2k_F\Delta q}{m} + \mathcal{E} \right| \right|_{\frac{qKx}{2k_F}}^{\frac{m\omega_0}{k_F} - \frac{qKx}{2k_F}} dx$$

$$\leftarrow -\frac{K^2}{8K_F}$$

$$+ K^2/4M$$

$$1 = \frac{|V|}{2\pi^2} \frac{k_F^2 m}{2k_F} \int_0^1 \ln \left| \frac{2\omega_0 - \frac{k_F K t}{m} + \mathcal{E} - \frac{K^2}{4M}}{\frac{k_F K t}{m} + \mathcal{E} - \frac{K^2}{4M}} \right| dt \quad + K^2/4M$$

Assumptions: 1) $\mathcal{E} \ll \omega_0$

2) $\frac{K}{2} \ll \frac{m}{k_F}\omega$

$$1 = \frac{|V|}{2\pi^2} k_F^2 \cdot \frac{m}{2k_F} \int_0^1 dt \ln \left| \frac{2\omega_0}{\frac{k_F K}{m} t + \mathcal{E}} \right|$$

$$1 = \boxed{\underbrace{\frac{1}{2} |V| \frac{m k_F}{2\pi^2}}_{\nu(\epsilon_F)}} \int_0^1 dt \ln \left| \frac{2\omega_0}{\frac{k_F K}{m} t + \mathcal{E}} \right|$$

Another factor 2 due to hole's contribut.

$$I = \int_0^1 dt \ln \frac{2\omega_0}{\frac{k_F k t}{m} + \mathcal{E}} = t \ln\left(\frac{2\omega_0}{\frac{k_F k}{m} t + \mathcal{E}}\right)\Big|_0^1$$

$$+ \int_0^1 dt \frac{t \cdot \frac{k_F k}{m} \pm \mathcal{E}}{\frac{k_F k t}{m} + \mathcal{E}} =$$

$$= \ln\left|\frac{2\omega_0}{\frac{k_F k}{m} + \mathcal{E}}\right| + 1 - \frac{\mathcal{E}}{\frac{k_F k}{m}} \ln\left|\frac{\frac{k_F k}{m} + \mathcal{E}}{\mathcal{E}}\right|$$

$$= \cancel{1+} \ln\frac{2\omega_0}{\mathcal{E}} \cancel{- \frac{\mathcal{E}}{\frac{k_F k}{m}} \ln\left|1 + \frac{k_F k}{m\mathcal{E}}\right|} \approx \ln\left(\frac{2\omega_0}{\mathcal{E}}\right)$$

$$\boxed{\mathcal{E} = 2\omega_0 \, e^{-\frac{1}{|V| D(\epsilon_F)}}}$$

$\mathcal{E} > 0$ $\qquad$ $2\epsilon_F - E > 0$!!!

$E < 2\epsilon_F$ $\qquad$ Singlet pair $\uparrow\downarrow$

$k = 0$ $\qquad$ Problems $\qquad$ 2D well

$k \neq 0$!!!

Let us consider critical velocity:

$$\mathcal{E} = 0 \qquad k = k_{cr} = 2m v_{cr}$$

$$I = \ln\left|\frac{2\omega_0}{\frac{k_F k_{cr}}{m}}\right| + 1$$

$$= \ln\left|\frac{2\omega_0 e}{\frac{k_F k_{cr}}{m}}\right|$$

$$g = \frac{1}{2}|U|\,\nu(\epsilon_F)$$

$$\mathcal{E}_0 = 2\omega_0 \exp\left(-\frac{2}{|U|\,\nu(\epsilon_F)}\right)$$

$$= 2\omega_0 e^{-\frac{1}{g}}$$

$$I = \frac{1}{g}$$

$$\ln\left|\frac{\cancel{2}\omega_0 \cdot e}{\cancel{2}k_F v_{cr}}\right| = \frac{1}{g}$$

$$v_{cr} = \omega_0 e^{-1/g} \cdot \frac{e}{k_F}$$

$$\sim \frac{\mathcal{E}_0}{k_F}$$