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CS 225
4.4 Structural Induction
7th edition 5.3: {12, 26c, 43, 44}

12) Let P(n) be $f^2_1 + f^2_1 + ... + f^2_n = f_n f_{n+1}$
Basis step: $f^2_1 = f_1 * f_2$ is true. $f_1 = 1, f_2 = 1$ $1 * 1 = 1 = f_1 * f_2$

Recursive step: Assume P(n) is true. P(n+1) has to be true to complete this step.

$$\begin{aligned} f^2_1 + f^2_1 + ... + f^2_n + f_{n+1} &= (f^2_1 + f^2_2 + ... + f^2_n) + f_{n+1} \\ &= f_n f_{n+1} + f^2_{n+1} \\ &= f_{n+1}(f_n + f_{n+1}) \\ &= f_{n+1} f_{n+2} \end{aligned}$$

26 c) basis step: 5 | (0 + 0) = 0
Recursive step: Let a+ b = 5k, k being an integer.
case 1: 5|(a+2) + (b+3) = a + b + 5 = 5k + 5 = 5(k+1), k+1 is also an integer
case 2: 5|(a+2) + (b + 2) = a + b + 4 = 5k + 5 = 5(k+1). This is the same as case 1

43) basis step: For a tree with only the root, it is true: n(T) = 1, h(T) = 0, 1 ≥ 2*0 + 1

Recursive step: Need to show that n(T) ≥ 2h(T) + 1 for the full tree T. T is formed by T_1 and T_2 with addition, where T_1 and T_2 are smaller than T. The induction hypothesis shows that $n(T_1) \geq 2h(T_1) + 1$ and $n(T) = 1+n(T_1) + n(T_2)$ and $h(T) = 1 + \max(h(T_1, h(T_2))$

n(T) = 1 + n(T1) + n(T2)
≥ 1 + 2h(T1) + 1 + 2h(T2) + 1
≥ 1 + 2max(h(T1), h(T2)) + 2
= 1 +2 (max(h(T1) , h(T2)) + 1)
= 1 + 2h(T)

44) Basis step: The single root r is the smallest tree. Since it has no internal vertices, it is a leaf. l(T) = 1 = 1 + i(T).
Recursive step: T is formed by T1 and T2.
l9T) = l(T1) + l(T2)
= i(T1) + 1 + i (T2) + 1
= i(T) + 1