Minh Nguyen

CS 225

Asn 3.2: Set Operations

Section 2.2 (7th edition) = {2,4,12,16,18,20}

2) a) A ∩ B

b) A - B

c) A U B

d) $\overline{A}$ U $\overline{B}$

4) Let A = {a,b,c,d,e} and B = {a,b,c,d,e,f,g,h}

a) A U B = {a,b,c,d,e,f,g,h} b) A ∩ B = {a,b,c,d,e} c) A - B = Ø d) B - A = {f,g,h}

12) Prove A U (A ∩ B) = A

This identity can be proven by being able to show that each side of the solution is the subset of the other.

We start off by trying to show that A U (A ∩ B) is a subset of A. If x € A U (A ∩ B), then (x € A) or (x € A ∩ B) by the law of unions. With the definition of intersections, we get (x € A) OR (x € A and x € B). Since the element x is in both cases of A no matter what, then we know that A U (A ∩ B) is a subset of A.

Now to show that A is a subset of A U (A ∩ B), let x € A. Then, (x € A) or (x € A and x € B). This means that x € A U (A ∩ B), by the definition of unions. This shows that subset A has an element x, that is also in A U (A ∩ B), which means A is a subset of A U (A ∩ B).

16) a) Show (A ∩ B) ⊆ A

let x € (A ∩ B). this also means x € A and x € B using the definition of interections. If x € A, that means (A ∩ B) is a subset of A.

b) A ⊆ (A U b)

let x € A. By the definition of unions, x € A or x € B. so x € A U B, which shows that A is a subset of A U B

c) A - B ⊆ A

let x € A - B. This is a difference of A and B, which can be written as x € A and x ~~€~~ B. That mean x € A , so A - B must be a subset of A.

d) A ∩ (B - A) = Ø

The approach here is to use contradiction. We can assume that A ∩ (B - A) is not Ø, and that there is an element x that such that x € A, x € B and x ~~€ A~~, By the definition of intersection and differences. But that is a contradiction since cannot be x € A and x ~~€ A~~ . So our proof for contradiction was false, which means A ∩ (B - A) = Ø is true.

e) A U (B - A) = A U B

A	B	B - A	A U (B - A)	A U B
1	1	0	1	1
1	0	0	1	1
0	1	1	1	1
0	0	0	0	0

The proof is complete, The two collumns are the same.

18) a) show (A U B) ⊆ (A U B U C)

let x € (A U B). Then x € A or x € B, by definition of union. Then that means for

(A U B U C), x € A, x € B, or x € C. We can show that x € (A U B U C) by using union definition, which means that (A U B) ⊆ (A U B U C).

b) show (A ∩ B ∩ C) ⊆ (A ∩ B)

let x € (A ∩ B ∩ C). We can distribute this out by the law of intersection, x € A, x € B, and

x € C. Using this same method for (A ∩ B), we get x € A, and x € B. It doesn't matter that x € C, because as long as x € A, x € B is true in (A ∩ B ∩ C), then x € (A ∩ B) as well. Which concludes our proof that (A ∩ B ∩ C) ⊆ (A ∩ B)

c) show (A - B) - C ⊆ (A - C)

let x € (A-B) - C). The definition of difference allows us to write € A, x ~~€ B,~~ and x ~~€ C, .~~ For the (A ∩ B), we use the same rules to show that x € A and x ~~€ C,~~ This completes our proof since we have shown that x is an element in A, but not C on both accounts (A - B) - C ⊆ (A - C).

d) (A - C) ∩ (C - B) = Ø

We prove this by contradiction. Let's assume that (A - C) ∩ (C - B) contains an x that the case is x € (A - C) ∩ (C 0 B). This means x € (A - C) and x € (C-B) by definition of intersection. Then by the definition of differences, x € A, and x is not € C, and x € C, and x is not € B. x cannot both be in C and NOT be in C at the same time, which is a contradiction. Which proves our original hypothesis (A - C) ∩ (C - B) = Ø is true.

e. (B - A) U (C - A) = (B U C) - A

A	B	C	C - A	B - A	B U C	(B - A) U (C- A)	(B U C) - A
1	1	1	0	0	1	0	0
1	1	0	0	0	1	0	0
1	0	1	0	0	1	0	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	0	0	1	1	1
0	0	1	1	1	1	1	1
0	0	0	0	0	0	0	0

20) Show (A ∩ B) U (A ∩ $\overline{B}$) = A

A	B	(A ∩ B)	$\overline{B}$	(A ∩ $\overline{B}$)	(A ∩ B) U (A ∩ $\overline{B}$)
1	1	1	0	0	1
1	0	0	1	1	1
0	1	0	0	0	0
0	0	0	1	0	0