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4.1 Weak Induction

5.1 (7th edition): { 6,8,14,18,28,38,40}

6) let P(n) be the sum of the the n terms is correct.

Basis Step: P(1) is true, 1 * 1! - (1 + 1)! - 1 -1

Inductive Step: The statement that P(k) is true, where k is a positive integer is the inductive hypothesis.

So P(k) is 1 * 1! + 2 * 2! + ... + k * k! - (k + 1)!

To complete this, we must show that P(k+1) is also true if P(k) is true. Add (k +1) * (k+1)! to both sides.

(k+1)! 1+ (k+1) (k+1)! = (k + 1)! * 1 + (k + 1)! *(k + 1) - 1

= (k+1)![k+2] - 1

=(k + 2)! - 1

This completes the inductive argument showing P(k+1) is true.

8)Solution: Let P(n) be the sum of the first n terms

Basis Step: P(0) is true because 2 = $\frac{1-(-7)^{0+1}}{4}$ = 2

Inductive Step: To prove that the inductive step is true, if P(k) is true, then P(k+1) is also true.

P(k) is in this case: $2 - 2*7 + 2* 7^2 - .... + 2(-7)^k = (1 - (-7)^{k+1}) / 4$

Adding $2(-7)^{k+1}$ to both sides of the equality of P(k):

$2 - 2*7 + 2* 7^2 - .... + 2(-7)^k + 2(-7)^{k+1} = (1 - (-7)^{k+1}) / 4 + 2(-7)^{k+1}$

$$= \frac{(1-(-7)^{k+1}}{4} + \frac{2(-7)^{k+1}}{4}$$

$$= \frac{(1-(-7)^{k+1}}{4} + \frac{8(-7)^{k+1}}{4}$$

$$= \frac{1-(-7)^{K+1} + 8(-7)^{k+1}}{4}$$

$= \frac{1+7\ (-7)^{k+1}}{4}$

$= \frac{1+(-1)(-7)\ (-7)^{k+1}}{4}$

$= \frac{1+(-1)(-7)^{k+2}}{4}$

$= \frac{1-(-7)^{k+2}}{4}$

This shows P(k+1) is also true when P(k) is true, thus completing the inductive step

14) Solution: Let P(n) be the sum of the n terms

Basis step: P(1) is true, because $1(2)^1 = (1\text{-}1)2^{1+1} + 2$

$2 = 2$

Inductive step: To prove the inductive argument, the inductive hypothesis is that P(k) is true, where k is a postive integer. P(k) is the statement $\sum_{i-1}^{k} i2^i = (k-1)^{k+1} + 2$

If P(k) is true, then we must show that P(k+1) is also true. So we add $(k+1)*\ 2^{k+1}$ to both sides of the equation of P(k):

$$\sum_{i-1}^{k} i2^i + (k+1)2^{k+1} = (k-1)^{k+1} + 2 + (k+1)2^{k+1}$$

$= 2^{k+1} * (k - 1) + 2^{k+1} * (k+1) + 2$

$= 2^{k+1} [k -1 + k + 1] + 2$

$= 2^{k+1} * 2k + 2$

$= k * 2^{k+2} + 2$

$= \sum_{i-1}^{k+1} k2^k - k * 2^{k+2} + 2$

Here we finish the inductive argument, and it is true for P(k+1) so our argument is complete.

18) a) $P(2) = (2)! < 2^2$ ::::: $2 < 4$

b) 2 is less than 4

c) Inductive hypothesis $P(k)$ is $k! < k^k$

d) For all $k > 2$ that $P(k)$ means $P(k+1)$ is true; so show that $(k+1) < (k+1)^{k+1}$

e) $k! * (k+1) < k^k\ (k+1) < (k+1)^k(k+1) - (k+1)^{k+1}$

f) We've completed the basis step and the inductive step, proving that P(k) and P(k+1) is true, meaning that we've completed the proof through mathematical induction.

28) Prove $n^2 - 7n + 12$ is nonnegative whenever $n >= 3$

Solution: Let P(n) be the statement that $n^2 - 7n + 12$ is nonnegative whenever $n >= 3$

Basis Step: $P(3) = 3^2 - 7(3) + 12 = 9 - 21 + 12 = 0$. 0 is a nonnegative number. Basis step complete

Inductive Step: The inductive hypothesis P(k) is true, where k is a positive integer greater than 3. P(k) is

$k^2 - 7k +12 >= 0$ ($k >= 3$)

If P(k) is true, then we must show P(k+1) is also true to complete the inductive step.

$(k+1)^2 - 7(k+1) + 12 = k^2 + 2k + 2 - 7k - 7 + 12$

$= (k^2 - 7k + 12) + 2(k-3)$

Since whenever k>=3, we have a nonnegative number in 2(k-3), and the other portion of the equation is the same as the original equation, then we know that adding to it will always lead to a nonnegative number, thus proving that P(k)is true, and makes P(k+1) also true.

Completing the basis step and the induction step, we have prove that P(n) is true for all n>= 3.

38) Solution: Let P(n) be $\bigcup_{j-1}^{n} A_j \subseteq \bigcup_{j-1}^{n} B_j$ if $A_i \subseteq B_i$ for i -1,2,...,n

Basis step: P(1) is proof that $A_1 \subseteq B_1$, so it implies that $\bigcup_{j-1}^{n} A_j \subseteq \bigcup_{j-1}^{n} B_j$

Inductive Step: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer. P(k) is then the statement if $A_j \subseteq B_j$ for j -1,2,...,k ,then $\bigcup_{j-1}^{k+1} A_j \subseteq \bigcup_{j-1}^{k+1} B_j$

if x is an element in the above subsets, then $x \in \bigcup_{j-1}^{k+1} A_j$ or $x \in A_{k+1}$. if $x \in \bigcup_{j-1}^{k+1}$, so is $x \in \bigcup_{j-1}^{k+1} B_j$

But if $x \in A_{k+1}$, then we also know that $A_{k+1} \subseteq B+_{k+1}$, which also means $x \in B_{k+1}$. So with both possibility leading to the fact that x is an element of both cases for B, it shows that P(k+1) is true, which completes our inductive case.

40) Solution: Let P(n) be $(A_1 \cap A_2 \cap \ldots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cap B) \cap \ldots \cap (A_n \cup B)$

Basis step: P(1) is true because $A_1 \cup B = A_1 \cup B$

Inductive step: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer. The statement P(k) is $(A_1 \cap A_2 \cap \ldots \cap A_k) \cup B = (A_1 \cup B) \cap (A_2 \cap B) \cap \ldots \cap (A_k \cup B)$

If P(k) is true, we must show that P(k+1) is also true.

$(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) \cup B = [(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1})] \cup B$ - by associative law

$= [(A_1 \cap A_2 \cap \ldots \cap A_k) \cup B] \cap (A_{k+1} \cup B)$ - distributive law

$= (A_1 \cup B) \cap (A_2 \cap B) \cap \ldots \cap (A_k \cup B) \cap (A_{k+1} \cup B)$ by the IH

So, P(k+1) is true. Completing our inductive argument.