Minh Nguyen

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4.2: Strong Induction

7th edition 4.2: { 2, 4, 12, 30}

2) Solution: Let P(n) be the statement that dominoes n, n+1, and n+2 will fall.

Basis step: P(1) is true, as stated in the original problem

Inductive Step: Assume the inductive hypothesis P(j) is true for integers $j < k$, show that P(k+1) is true. if the (k-2) domino falls, then the domino in the 4th position will fall, since k-2 will cause the 3rd one down the lane to fall. We know that P(2) is true since we are told the first 3 dominoe is pushed down. So since the $k>2$ at this point, P(k-2) is true, because stated earlier that k-2 will cause the 4th one in the arrangement to fall. Then we know that dominoes at k-2, k-1, and k will will. This shows that P(n) is true for all positive integers. We've shown that all dominoes in the infinite arrangement will fall.

4) a) P(18) is true = one 4-cent stamp, and two 7-cent stamps

P(19) is true = one 7-cent stamp, and three 4-cent stamps

P(20) is true = five 4-cent stamps

P(21) is true = three 7-cent stamps

b) The inductive hypothesis P(k) is using just 4-cent and 7-cent stamps, we can form j cent stamps for all j with $18 \le j \le k$, assuming $k \ge 18$.

c) in the inductive step, we need to prove that we can form the k+1 stamp using only 4 and 7 - cent stamps

d) Since we know that all the k's that are $k \le 21$ are true, then we know that P(k-3) will also be true, meaning we can form the (k-3)th -cent postage. And to form the next one from this, we simply add one more 4-cent stamp to form the (k+1) cent postage.

e) We have completed both the basis and inductive step, so by the definition of strong induction, the original statement is true as long as it's for postages that are greater than or equal to 18 postages.

12) Let P(n) be that "a positive integer n can be written as a sum of distinct powers of two."

Basis step: P(1) is $1 = 2^0$, P(2) is $2 = 2^1$

Inductive step: k+1 can be either even or odd. We can assume that P(j) is true for all j≤k.

If k+1 is even, (k+1)/2 is an integer. This means (k+1)/2 ≤ k. This shows that (k+1)/2 can be shown as the sum of distinct powers of 2. if k+1 is even, then P(k+1) is true.

If k+1 is odd, that means the previous number k is even. the only odd number of the all the dinstinct powers of 2 is $2^0 = 1$. So $k+1 = k + 2^0$ which is still the sum of distinct powers of 2.

This completes the inductive steps, showing that if the inductive hypothesis P(j) (j<k) is true, then P(k+1) is true.

30) Basis step: $a^0 = 1$ is true

inductive step: we assume that $a^j = 1$ for all non negative integers j with j ≤ k.

$a^{k+1} = \frac{a^k * a^k}{a^{k-1}} = \frac{1*1}{1} = 1$

The basis step is wrong since we a^1 does not always = 1.