Minh Nguyen
CS 225
4.3 Recursive Definitions
7th edition 5.3: {2(a,b), 8, 24, 26a, 28a, 32a}

2) a) f(n+1) = -2f(n)

f(1) = -2f(0) = -2*3 = -6
f(2) = -2f(1) = -2*-6 = 12
f(3) = -2f(2) = -2*12 = -24
f(4) = -2f(3) = -2*-24 = 48
f(5) = -2f(4) = -2*48 = --96

b) f(n+1) = 3f(n) + 7

f(1) = 3f(0) + 7 = 3*3 + 7 = 16
f(2) = 3f(1) + 7 = 3*16 + 7 = 55
f(3) = 3f(2) + 7 = 3*55 + 7 = 172
f(4) = 3f(3) + 7 = 3*172 + 7 = 523
f(5) = 3f(4) + 7 = 3*523 + 7 = 1576

8) a) a_n = 4n - 2

base case: a(1) = (4(1)-2 = 2
Recursive step: a_n = 4(n+1) -2
= 4n +4 - 2
= 4n - 2 + 4
= a_n + 4

b) a_n= $1+(-1)^n$

base case: $a_1 = 1+(-1)^1 = 2$
a(n+1) = a(n) + ?
? = $a_{n+1} - a_n$
= $1+(-1)^{n+1} - 1+(-1)^n$
= $-(1)^{n+1} -1^n$
= -2
a(n+1)_ = a^n - 2

c) a_n = n(n+1)

base case: a_1 = 1(1+1) = 2
$a_{n+1} = a_n$ + ?
? = $a_{n+1} - a_n$
= (n+1)(n+2) - n(n+1)
= $n^2 + 3n + 2 - n^2 - n$
? = 2n + 2
= a_n + 2n + 2

d) $a_n = n^2$

base case: $a_1 = 1^2 = 1$

$a_{n+1} = a_n + ?$

$? = (n+1)^2 - n^2$

$= n^2 + 2n + 1 - n^2$

$?= 2n + 1$

$= a_n + 2n + 1$

24) a) Set of odd integers

Basis step: $1 \in S$

Recursive step: if $x \in S$, then $x+2 \in S$

b) Set of positive integer power of 3

basis step: $3 \in S$

Recursive step: if $x \in S$, then $3x \in S$

c) Set of polynomials with integer coefficients

basis step: $0 \in S$

Recursive step: $A(n) \in S$, then $A(n) + B(n)^k \in S$, $B \in Z$, $k \in Z$

Z is the set of integers.

26) a)

1: (2,3), (3,2)

2: (4,6), (5,5), (6,4)

3: (6,9), (7,8), (8, 7), (9, 6)

4: (8, 12), (9,11), (10,10) (11,9), (12,8)

5: (10, 15), (11, 14),(12,13),(13,12),(14,11),(15,10)

28 a)

basis step: $(0,0) \in S$

Recursive step: if $(a,b) \in S$, then $(a, b+2) \in S$, $(a+2, b) \in S$, $(a+1, b+1) \in S$

32a) Basis step: ones(λ) = 0, λ is the empty string

Recursive step = if $x \in S$, and $w \in \Sigma^*$, then ones(wx) = ones(w) + x