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CS 225

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asn: 2.2 Contradiction & other techniques. (7th edition) section 1.7: 8,22,24,26, 30; section 1.8: 8,30,36

Section 1.7

8) Prove that if n is a perfect square, then n + 2 is not a perfect square.

let p be "if n is a perfect square", and q be "n + 2 is not a perfect square". By contradiction, both p and ¬q is true. ¬q would be "n+2 is a perfect square". If n+2 is a perfect square, then an integer k exists that is $n+2=k^2(k \geq 2)$. When $k \geq 2$, we can write $n < (k+1)^2 = k^2 + 2k + 1$. This shows that n is not a perfect square, which makes the statement ¬p not true. So having p and ¬q true, is a contradiction, completing the proof.

22) Show that if you pick three socks from a drawer containing just b lue socks and black socks, you must get either a pair of block socks or a pair of block socks.

let p be "if you pick three socks from a drawer containing just blue and black socks", and q be "you will get either a pair of blue or black socks." Assuming that both p and ¬q are true, you will not be getting a pair of either blue or black socks if you picked from a drawer that only has blue and black socks. Since picking out a pair means you will get at least 2 socks of the same color, you will indeed get a black or blue pair, since the hypothesis had said if you picked three socks from a drawer. Since there is no sizes involved, this assumes that all the socks are the same size, but just merely different colors. So indeed, you will get either a blue or black pair of socks, and one extra one that doesn't matter.

24) Show that at least 3 of any 25 days chosen must fall in the same month of the year.

Let p be "at least 3 of any 25 days chosen must fall in the same month of the year". Assume that ¬p is true, then at most 2 of 25 days fall on the same month. Since there are 12 months in a year, that means at most 24 days could be chosen due to the fact that at most two of the chosen days could fall on the same month. The original statement says 25 days, which contradicts the 24 days that we have if ¬p is true. if r is the statement that 25 days are chosen, and ¬r means 25 days aren't chosen (which in our case is 24 days), then ¬p -> (r ^ ¬r). We know that ¬cannot be true, so p is true. This ends the case using contradiction.

26) prove that if n is a positive integer, then n is even if and only if 7n + 4 is even.

let p be "n is even" and q is "7n + 4 is even". The statement contains an "if and only if" statement, meaning show that p - > q and q -> p are both true to prove this statement. An even integer n by definition is n = 2k, k being a positive integer. so 7n = 4 = 7(2k) + 4, factoring it out we get 2(7k+2). Thus, we can see that 7n+4 is an even integer. Our statement p -> q has been proven. The other part now is to show that q->p is true. Since we are saying that 7n+4 is an even integer, then by definition of an even integer, $7n+4 = 2k$, if k is a positive integer. Doing some algebra, we can get $7n = 2k-4 = 2(k-2)$ which is also a definition of an even integer. n is an even integer, which shows that q -> p is true.

30) Show that these three statements are equivalent, where a and be are real numbers:

1) a is less than b

2) the average of a and b is greater than a

3) the average of a and b is less than b

To prove all of these statements are equivalent, we must show that conditions 1 -> 2, 2 -> 1, 1 -> 3, and 3 -> 1 are true in their contraposition. By showing all of these to be true, we will have 1 <-> 2, and 1 <-> 3, which will show that all these statements are equivalent.

for 1->2, the contraposition of this is that assume that the average of a and b is smaller than a, which is: $a+b / 2 \leq a$; then, obviously $b \leq a$. Both contrapositions are true, which means 1 -> 2 is true.

for 2 -> 1, The contraposition of this is that a is greater than b, $a \geq b$, so $a+b \leq 2a$, which we can conclude $a+b/2 \leq a$. Both contrapositions are true, which means 2 -> 1 is true.

For 1-> 3, the contraposition of this is that suppose the average of a and b is GREATER than b, $a+b/2 \geq b$, so we can also say $a \geq b$. Both contrapositions are true, which means 1 -> 3 is true.

for 3 -> 1, the contraposition of this is that suppose $a \geq b$, so $a + b \geq 2b$. Then, $a+b/2 \geq 2$. Both contrapositions are true, which means 3->1 is also true.

Section 1.8

8) Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

1 is the positive integer that equals the sum of the positive integers not exceeding it. This is constructive proof.

30) Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

Using proof by case, we check the possible cases that this statement can be true first. $2x^2 > 14$ when

$|x|$ is ≥ 3, and $5y^2 > 14$ when $|y| \geq 2$. This means the values that x can be is in the range of -2,-1,0, 1 and 2, and for y it is -1,0, and 1. The possible values that $2x^2$ can produce is 0,2, and 8, and for $5y^2$ it is only 0 and 5. The closest we get to 14 is 8+5 which is 13, so it is not possible for $2x^2 + 5y^2$ to ever b e 14 if x and y are integers.

36. Prove that between every rational number and every irrational number there is an irrational number.

let r be a rational number, and let i be an irrational number. We can prove by contradition that $x+i/2$ is an irrational number.

if $x+i/2$ is a rational number, then $x+i/2 = a/b$, and $i = p/q$. a, b, p, and q are all integers and t and q are not 0.

$x+i/2 = x+i/2$

$x = 2(x+i / 2) - i$

$= 2a/b - p/q$

$= (2aq - pb) / qb$

(2aq - pb)is an integer, as well as qt. qt is not 0. So then x here is shown as a rational number, contradicting the original statement that x is irrational.