

BTVN 2

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Chúng minh $\sum_{s=1}^2 v_{\vec{k}}^{(s)} \bar{v}_k^{(s)} = -\not{k} + m$

$$\begin{aligned} RHS &= \sqrt{E_{\vec{k}} + m} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(s)} \\ \chi^{(s)} \end{pmatrix} \sqrt{E_{\vec{k}} + m} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(r)\dagger} & \chi^{(r)\dagger} \end{pmatrix} \gamma^0 \\ &= (E_{\vec{k}} + m) \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(s)} \\ \chi^{(s)} \end{pmatrix} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(r)\dagger} & -\chi^{(r)\dagger} \end{pmatrix} \\ &= (E_{\vec{k}} + m) \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(s)} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(r)\dagger} & -\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(s)} \chi^{(r)\dagger} \\ \chi^{(s)} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(r)\dagger} & -\chi^{(s)} \chi^{(r)\dagger} \end{pmatrix} \\ &= (E_{\vec{k}} + m) \begin{pmatrix} \frac{k^2}{(E_{\vec{k}}+m)^2} \mathbb{1}_{2 \times 2} \delta_{s,r} & -\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \mathbb{1}_{2 \times 2} \delta_{s,r} \\ \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \mathbb{1}_{2 \times 2} \delta_{s,r} & -\mathbb{1}_{2 \times 2} \delta_{s,r} \end{pmatrix} \\ &= \begin{pmatrix} \frac{k^2}{E_{\vec{k}}+m} & -\vec{k}\vec{\sigma} \\ \vec{k}\vec{\sigma} & -E_{\vec{k}} - m \end{pmatrix} \otimes \mathbb{1}_{2 \times 2} \\ &= E_{\vec{k}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \mathbb{1}_{2 \times 2} - m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \mathbb{1}_{2 \times 2} + \vec{k} \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \otimes \mathbb{1}_{2 \times 2} \\ &= E_{\vec{k}} \gamma^0 - \vec{k} \vec{\gamma} - m \\ &= \not{k} - m \end{aligned}$$

Chứng minh $\bar{v}_k^{(s)} v_k^{(r)} = -2m\delta_{s,r}$

$$\begin{aligned}
RHS &= \sqrt{E_{\vec{k}} + m} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(s)\dagger} & \chi^{(s)\dagger} \end{pmatrix} \gamma^0 \sqrt{E_{\vec{k}} + m} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(r)} \\ \chi^{(r)} \end{pmatrix} \\
&= (E_{\vec{k}} + m) \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(s)\dagger} & -\chi^{(s)\dagger} \end{pmatrix} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m} \chi^{(r)} \\ \chi^{(r)} \end{pmatrix} \\
&= (E_{\vec{k}} + m) \left[\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}} + m} \chi^{(s)\dagger} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}} + m} \chi^{(r)} - \chi^{(s)\dagger} \chi^{(r)} \right] \\
&= \left[\frac{k^2}{E_{\vec{k}} + m} \delta_{s,r} - (E_{\vec{k}} + m) \delta_{s,r} \right] \\
&= [(E_{\vec{k}} - m) \delta_{s,r} - (E_{\vec{k}} + m) \delta_{s,r}] \\
&= -2m\delta_{s,r} (\text{DPCM})
\end{aligned}$$

$$\frac{d^n f}{dx^n} \frac{df}{dx}$$