BTVN 2

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Chứng minh $\displaystyle\sum_{s=1}^{2} v_{\vec{k}}^{(s)} \bar{v}_{k}^{(s)} = -\not{k} + m$

$$
\begin{aligned}
RHS &= \sqrt{E_{\vec{k}} + m} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)} \\ \chi^{(s)} \end{pmatrix} \sqrt{E_{\vec{k}} + m} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)^\dagger} & \chi^{(r)^\dagger} \end{pmatrix} \gamma^0 \\
&= (E_{\vec{k}} + m) \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)} \\ \chi^{(s)} \end{pmatrix} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)^\dagger} & -\chi^{(r)^\dagger} \end{pmatrix} \\
&= (E_{\vec{k}} + m) \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)}\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)^\dagger} & -\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)}\chi^{(r)^\dagger} \\ \chi^{(s)}\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)^\dagger} & -\chi^{(s)}\chi^{(r)^\dagger} \end{pmatrix} \\
&= (E_{\vec{k}} + m) \begin{pmatrix} \frac{k^2}{(E_{\vec{k}}+m)^2}\mathbb{1}_{2\times 2}\delta_{s,r} & -\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\mathbb{1}_{2\times 2}\delta_{s,r} \\ \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\mathbb{1}_{2\times 2}\delta_{s,r} & -\mathbb{1}_{2\times 2}\delta_{s,r} \end{pmatrix} \\
&= \begin{pmatrix} \frac{k^2}{E_{\vec{k}}+m} & -\vec{k}\vec{\sigma} \\ \vec{k}\vec{\sigma} & -E_{\vec{k}} - m \end{pmatrix} \otimes \mathbb{1}_{2\times 2} \\
&= E_{\vec{k}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \mathbb{1}_{2\times 2} - m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \mathbb{1}_{2\times 2} + \vec{k} \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \otimes \mathbb{1}_{2\times 2} \\
&= E_{\vec{k}}\gamma^0 - \vec{k}\vec{\gamma} - m \\
&= \not{k} - m
\end{aligned}
$$

Chứng minh $\bar{v}_k^{(s)} v_k^{(r)} = -2m\delta_{s,r}$

$$\begin{aligned}
RHS &= \sqrt{E_{\vec{k}}+m}\begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)^\dagger} & \chi^{(s)^\dagger} \end{pmatrix} \gamma^0 \sqrt{E_{\vec{k}}+m}\begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)} \\ \chi^{(r)} \end{pmatrix} \\
&= (E_{\vec{k}}+m)\begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)^\dagger} & -\chi^{(s)^\dagger} \end{pmatrix} \begin{pmatrix} \frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)} \\ \chi^{(r)} \end{pmatrix} \\
&= (E_{\vec{k}}+m)\left[\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(s)^\dagger}\frac{\vec{k}\vec{\sigma}}{E_{\vec{k}}+m}\chi^{(r)} - \chi^{(s)^\dagger}\chi^{(r)}\right] \\
&= \left[\frac{k^2}{E_{\vec{k}}+m}\delta_{s,r} - (E_{\vec{k}}+m)\delta_{s,r}\right] \\
&= \left[(E_{\vec{k}}-m)\delta_{s,r} - (E_{\vec{k}}+m)\delta_{s,r}\right] \\
&= -2m\delta_{s,r} (\text{ĐPCM})
\end{aligned}$$