

# A note on Mathematical Methods

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These notes are a review of the mathematical methods course, focusing on the content most relevant for physics. The primary sources were mostly come from David Skinner's [lecture notes on Methods](#).

## 1 Fourier series

We begin by reviewing Fourier series. Fourier series are defined for functions  $f : S^1 \rightarrow \mathbb{C}$ , parametrized by  $\theta \in [-\pi, \pi]$ . We defined the Fourier coefficients by an inner product

$$\hat{f}_n = \frac{1}{2\pi} \left( e^{in\theta}, f \right) \equiv \frac{1}{2\pi} \int_0^{2\pi} e^{-in\theta} f(\theta) d\theta.$$

We then claim that

$$f(\theta) = \sum_{n \in \mathbb{Z}} \hat{f}_n e^{in\theta}.$$

In particular, via reality condition, the Fourier series can be obtained by

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\theta + b_n \sin n\theta,$$

where

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(\theta) d\theta, \\ a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} \cos n\theta f(\theta) d\theta, \\ b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} \sin n\theta f(\theta) d\theta. \end{aligned}$$

We then investigate whether this sum converges to  $f$ , if it converges at all. One can

show that the Fourier series converges to  $f$  for continuos functions with bounded continuos derivaties. When  $f$  has a discontinuity, the Fourier series converges to the average of the left and right limits.

### Fejer's theorem

Fejer's theorem state that one can always recover  $f$  from the  $\hat{f}_n$  as long as  $f$  is continuos except at finitely many points, though it makes no statement about the convergence of the Fourier series. Also, the Fourier series converges to  $f$  as long as  $\sum_n |\hat{f}_n|$  converges. In other words,

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= 0, \\ \lim_{n \rightarrow \infty} b_n &= 0. \end{aligned}$$

### Sawtooth function

The sawtooth function defined by

$$f(\theta) = \theta \quad \text{for } \theta \in [-\pi, \pi),$$

and the Fourier coefficients for it are

$$\begin{aligned} a_0 &= 0, \quad n = 0, \\ a_n &= \frac{1}{in} (-1)^{n+1}, \quad n \neq 0. \end{aligned}$$

## Parseval's identity

$$\int_{-\pi}^{\pi} |f(\theta)|^2 d\theta = 2\pi \sum_{k \in \mathbb{Z}} |\hat{f}_k|^2.$$

Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Nunc ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam. The Fourier transform is linear, and obeys

$$\mathcal{F}[f(x - a)] = e^{-ika} \tilde{f}(k),$$

$$\mathcal{F}[e^{ilx} f(x)] = \tilde{f}(x),$$

$$\mathcal{F}[f(cx)] = \frac{\tilde{f}(k/c)}{|c|}$$