Ôn tập cơ lượng tử 3

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1 Bài tập chương 6

6.28

Khai triển Taylor cho $\Psi(x,t)$ ta có

$$\begin{aligned}\Psi(x,t_0+\delta) &= \Psi(x,t_0) + \delta\frac{\partial\Psi}{\partial t}\bigg|_{t_0} + ... \\ &= \Psi(x,t_0) + \delta\frac{1}{i\hbar}\hat{H}(t_0)\Psi(x,t_0) \\ &= \left[1 - \frac{i\delta}{\hbar}\hat{H}(t_0) + ...\right]\Psi(x,t_0)\end{aligned}$$

2 Bài tập chương 11

11.18

Problem 11.18 A particle of mass m is in the ground state of the infinite square well (Equation 2.22). Suddenly the well expands to twice its original size—the right wall moving from a to $2a$—leaving the wave function (momentarily) undisturbed. The energy of the particle is now measured.

(a) What is the most probable result? What is the probability of getting that result?

(b) What is the *next* most probable result, and what is its probability? Suppose your measurement returned this value; what would you conclude about conservation of energy?

(c) What is the *expectation value* of the energy? *Hint:* If you find yourself confronted with an infinite series, try another method.

(a) Năng lượng

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}; \quad \psi(x,0) = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$$

Khi tăng bề rộng giếng lên gấp đôi ngay lập tức

$$E_n = \frac{n^2\pi^2\hbar^2}{2m(2a)^2}; \quad \psi_n(x,0) = \sqrt{\frac{2}{2a}}\sin\frac{n\pi x}{2a}$$

Ta khai triển hàm sóng ban đầu theo các ψ_n

$$\psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)$$

Ta đi tính các hệ số c_n cho $[0,a]$

$$\begin{aligned} c_n &= \frac{\sqrt{2}}{a}\int_0^a \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{n\pi x}{2a}\right)dx \\ &= \frac{\sqrt{2}}{2a}\int_0^a \cos\left[\left(\frac{n}{2}-1\right)\frac{\pi x}{a}\right] + \cos\left[\left(\frac{n}{2}+1\right)\frac{\pi x}{a}\right] \\ &= \frac{4\sqrt{2}}{\pi}\frac{\sin\left[(\frac{n}{2}+1)\pi\right]}{n^{2-4}} = \begin{cases} 0 \quad if\ n\ even \\ \pm\dfrac{4\sqrt{2}}{\pi(n^2-4)} \quad if\ n\ odd \end{cases} \end{aligned}$$

Ta có được xác suất

$$P_n = |c_n|^2 \begin{cases} \dfrac{1}{2}\ n=2 \\ \dfrac{32}{\pi^2(n^2-4)}\ n\ odd \end{cases}$$

11.20

Problem 11.19 A particle is in the ground state of the harmonic oscillator with classical frequency ω, when suddenly the spring constant quadruples, so $\omega' = 2\omega$, without initially changing the wave function (of course, Ψ will now *evolve* differently, because the Hamiltonian has changed). What is the probability that a measurement of the energy would still return the value $\hbar\omega/2$? What is the probability of getting $\hbar\omega$? *Answer:* 0.943.

Năng lượng ban đầu

$$E_n = (n+1/2)\hbar\omega$$

Năng lượng khi tăng tần số $\omega' = 2\omega$

$$E_n^{'} = 2(n + 1/2)\hbar\omega$$

Vậy xác suất để đo được năng lượng $E_n = \frac{1}{2}\hbar\omega$ là bằng 0.