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# Tilted Magnetic Field Studies of Spin-Splitting of the Landau-Levels in Modulation-Doped n-Channel $Si/Si_{1-x}Ge_x$ Quantum Well Structures

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Studies of Shubnikov-de Haas (SdH) oscillations corresponding to the spin-Landau levels of a two-dimensional electron gas at 100 mK have been performed on modulation-doped n-type $Si/Si_{1-x}Ge_x$ heterostructures. The method used to obtain the effective Landé $g$-factor can be described as the method of coincidences, where the tilt angle that causes adjacent Shubnikov-de Haas (SdH) minima to be equal is used to determine the effective $g$-factor. The effective $g$-factor for 6⇔8 was 3.34±0.05. The results demonstrated that the effective $g$-factor oscillated as a function of the filling factor.

## I. INTRODUCTION

In the past three decades, there has been considerable interest in the magnetic-field dependences of the spin and the valley degeneracies. Various groups [1–6] have extensively studied the magnetic-field-induced lifting of the spin and the valley degeneracies in metal-oxide-silicon (MOS) systems. Such systems exhibit properties that are very different from the bulk material. One such property is that the Landé $g$-factor is considerably larger than the bulk value of 2 [7]. This fact was discovered using the tilted magnetic-field method first developed by Fang and Stiles [1]. Thus, the tilted magnetic-field increases the Landé $g$-factor with decreasing electron concentration ($n_s$). Its enhancement and inverse dependence on $n_s$ have been attributed to the exchange interaction, as proposed by Janak [8]. Since then, a number of groups [9–12] have studied the interesting consequences of $g$-factor enhancement, $g^*$, in a two-dimensional system, in particular Ando and Uemura [10], who first pointed out that $g$-factor enhancement should be an oscillatory function of the magnetic-field. The physical idea behind this periodic $g$-factor enhancement is the following: Zeeman spin-splitting has been shown [8] to be increased by the exchange interaction, which is greatest where the difference between the occupancies of the spin-up and spin-down Landau levels is large. This population difference depends on the position of the Fermi level, which changes relative to the Landau levels as the field is changed. Resistivity or conductivity minima, which occur when the Fermi level lies between two spin-split states of the same Landau-level, will correspond to a maximum enhancement, leading to a larger $g^*$-factor, while for a Fermi level between the spin states originating from different levels, the population difference will be much smaller, leading to a smaller $g^*$-factor. This gives rise to the observed oscillatory enhancement of the effective $g$-factor.

The most commonly employed method to obtain the magnitude of the $g$-factor involves tilting of the sample relative to the magnetic-field direction. The Landau-level splitting is only determined by the perpendicular component of the magnetic-field, $B_\perp$, whereas the spin-splitting is proportional to the total magnetic-field, $B$. Therefore, in tilted magnetic-fields, the spin-splitting can be enlarged in comparison with the Landau- level splitting and can be determined with respect to the Landau-level splitting, by using, for example, a measurement of the corresponding change in the Shubnikov-de Haas (SdH) oscillations.

Another unsolved property of two-dimensional electron gases (2DEGs) formed on (100) Si is the nature of the two-fold valley degeneracy. Several models [13,14] have been proposed to describe the occurrence of valley-splitting, but agreement between theory and experiment remains unsatisfactory. The magnitude of this splitting is expected to increase with the electric field, and thus with the carrier density, within the quantum well [15] although the opposite has been observed experimentally in Si MOSFET structures. Köhler *et al.* [16,17] tried to determine the magnitude of the valley- splitting, $\triangle E_V$, by using a slightly different argument for the phase change of the SdH oscillations. They tilted the magnetic field and realized a situation $\eta\omega_c \sim \triangle E_v$. From this method, they obtained $\triangle E_v$∼0.69 meV at $B_\perp$=15.2 T for an electron concentration of $n_s$=2.4×$10^{12}$ $cm^{-2}$. Most theoretical calculations in the literature have only considered bare ($B$=0) valley-splitting [13,14,18] and predict values

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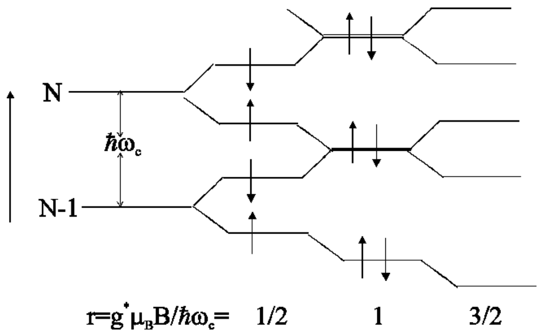


Fig. 1. A schematic view of the energy levels for Landau quantum numbers $(N-1)$ and $N$.

for the magnitude of $E_v$ that are much smaller than those that are observed experimentally [16,17]. This situation strongly suggests that valley-splitting is also enhanced by exchange interactions in high magnetic-fields, as proposed by Ando and Uemura [10].

In this paper, a study of the $g^*$-factor by using the tilted magnetic-field method is presented for $Si/Si_{0.7}Ge_{0.3}$ heterostructures and shows clear evidence for both enhancement and oscillatory behavior. However, the value of the valley-splitting cannot be determined due to the limited mobility in our samples.

## II. THEORETICAL BACKGROUND

As discussed in the previous section, the tilted magnetic-field method allows separate control of the spin-level and Landau-level splittings, and this can be applied to measure the $g$-factor enhancement ($g^*$). It is also worth discussing in some detail the measurement conditions used since these can be quite complex for a system with an oscillating $g^*$-factor. In a tilted field, the energy levels are given by

$$\begin{aligned} E &= \left(n+\frac{1}{2}\right)\hbar\omega_c \pm g^*\mu_B B \\ &= \left(n+\frac{1}{2}\right)\frac{\hbar eB}{m^*}\cos\theta \pm \frac{1}{2}g^*\mu_B B \end{aligned} \tag{1}$$

where $\theta$ is the tilt angle. Making the simplifying assumption of a constant $g^*$-factor will lead to an evenly spaced ladder of levels whenever the condition

$$|g^*\mu_B B| = r\hbar\omega_c = r\frac{\hbar eB}{m^*}\cos\theta \tag{2}$$

is satisfied, where $r$=1/2, 3/2, and 5/2 correspond to a ladder of alternating spin levels, and $r$=1, 2, 3, $\cdots$ correspond to a ladder with coincident spin-up and spin-down levels from different Landau-levels. These conditions are shown schematically in Fig. 1, where the energy levels are shown for a constant perpendicular field component and increasing total field, hence for spin-splitting. Since the resistivity values of the SdH oscillation minima depend on the energy spacing of the broadened Landau-levels, the level spacings are approximately equal when the resistivity of adjacent minima are equal, assuming that the change in Landau-level half-width is small over field range involved. At this point, Eq. (2) is described by the following formula [19]:

$$\frac{g^*\mu_B B_{\perp(n=\nu)}}{\cos\theta_{n\Leftrightarrow n\pm 2}} = \frac{\hbar eB_\perp}{m^*} - \frac{g^*\mu_B B_\perp}{\cos\theta_{n\Leftrightarrow n\pm 2}} \tag{3}$$

where $B_{(\nu=n)}$ is the perpendicular component of the magnetic field at the $\nu=n$ SdH minima (Landau-level occupancy, *i.e.*, $\nu$=4, 8, *etc.*) and $\theta_{n\Leftrightarrow n\pm 2}$ is the tilt angle at which the resistivities of the $\nu=n$ and $\nu=n\pm 2$ minima are equal. Since, for a constant carrier density,

$$nB_{n=\nu} = (n\pm 2)B_{\nu=n\pm 2}\ , \tag{4}$$

Eq. (3) can be solved for $g^*$ and simplified to

$$g^* = \left(\frac{n\pm 2}{n\pm 1}\right)\left(\frac{m_0}{m^*}\right)\cos\theta_{n\Leftrightarrow n\pm 2} \tag{5}$$

where the definition $\mu_B$ is $e\hbar/2m_0$. Therefore, from Eq. (3), one can obtained the value of $g^*$ by determining the tilt angle at which the resistivities of adjacent minima are equal [19].

## III. EXPERIMENTAL METHOD

The n-type modulation-doped $Si/Si_{1-x}Ge_x$ samples used in this study were grown at the Interdisciplinary Research Center for Semiconductor Materials (IRC) by using gas source molecular beam epitaxy (MBE) as described in Ref. 20. The gas sources were disilane ($Si_2H_6$) and germane ($GeH_4$) for the semiconductor matrix and arsine ($AsH_3$) for the arsenic-doped n-channel supply layers. Growth rates varied between 0.1 and 1 Å/s, depending on the alloy composition. After depositing a 0.3-$\mu$m-thick Si buffer layer onto a high-resistivity p-type Si substrate, a 1.5-$\mu$m-graded $Si_{1-x}Ge_x$ layer, in which the Ge content increased linearly from 5 to 35 %, was grown; next, a 0.5−1 $\mu$m layer of constant composition $Si_{0.7}Ge_{0.3}$ alloy was grown. These layers produced a strain-relaxed, but low dislocation density, surface onto which the tensilely-strained Si channel, 110 Å thick, was deposited, followed by an undoped $Si_{0.7}Ge_{0.3}$ spacer layer, a doped $Si_{0.7}Ge_{0.3}$ supply layer, and finally undoped $Si_{0.7}Ge_{0.3}$ and Si cap layers. For the supply layer, the As dopant was supplied as a delta- ($\delta$-) doped layer at the start of the layer, although surface segregation was thought to be likely to produce a redistribution of the As through the subsequently deposited material [21]. To measure the magnetoresistivity of the sample, we photo-lithographically processed the layers into 400×80 $\mu m^2$ Hall bar samples. The mesa pattern for the Hall bars was etched by using a reactive ion etching to define the channel geometry. The alloyed AuSb ohmic

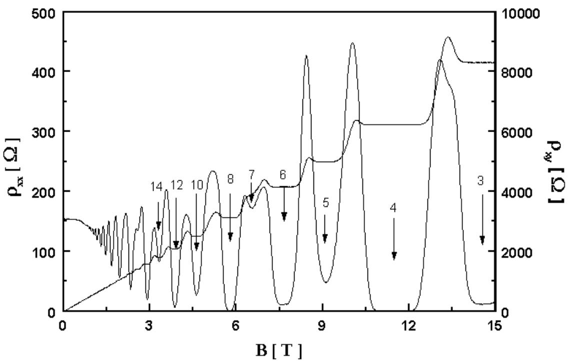


Fig. 2. Longitudinal resistivity and Hall resistivity versus magnetic field $B$ ($T$=0.1 K). Filling factors are indicated with arrows and numbers.

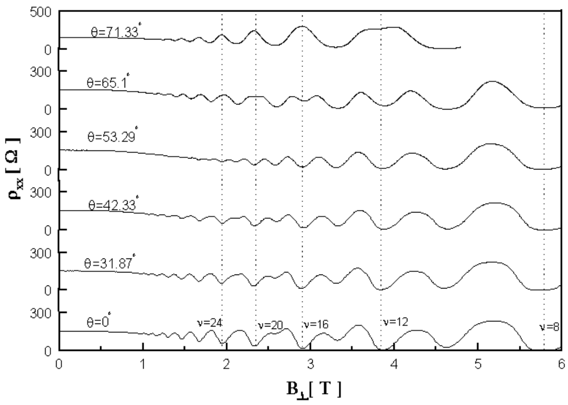


Fig. 3. Longitudinal resistivity $xx$ for different angles (71.33, 65.1, 53.29, 42.33, 31.87°, and 0°) versus the normal component of the magnetic field $B$. The positions of some integer-filling factors are indicated by the dotted lines.

contacts to the Hall bars were prepared by using thermal evaporation. New experiment in the tilted magnetic-field for our sample were performed at low temperatures down to 100 mK in magnetic fields up to 15 T by using a helium dilution refrigerator. At these temperatures, the samples showed an electron mobility as high as $\mu$=35810 cm$^2$/Vs with a carrier density $n_s$=1.13×10$^{12}$ cm$^{-2}$, as discussed [22]. The sample holder allowed the tilt angle between the magnetic field and the sample normal to be changed in the range 0−90°. The magnetic field dependencies of the longitudinal resistivity $\rho_{xx}$ and the Hall resistivity $\rho_{xy}$ were measured for different tilt angles at 0.1 K in magnetic-fields up to 15 T. All measurements were performed using phase-locked measurement techniques with a driving current of 50 nA.

## IV. RESULTS AND DISCUSSION

In Fig. 2, data from the SdH and the Hall measurements at $\theta$=0° are displayed. The large plateau, visible around $B$=15 T, corresponds to a filling factor $\nu$=3, indicating valley-splitting. The plateau around $B$=11.5 T represents the first filled Landau-level ($\nu$=4). Between 12 T and 6 T, $\rho_{xx}$ exhibits three minima. At 9.1 T and 6.54 T valley-splitting can be observed at $\nu$=5 and at $\nu$=7, respectively. Spin splitting is resolved at around 7.53 T ($\nu$=6). At lower fields, SdH oscillations can be observed as the spin degenerate Landau-levels are depopulated, but higher-index valley- degenerate Landau-levels can be not resolved due to the limited mobility in this sample. Spin-splitting is observable down to $B$=2.6 T, with the SdH oscillations themselves beginning at $B$=0.8 T.

Typical $B_\perp$ dependencies of the longitudinal resistivity at different values of are presented in Fig. 3. For the traces at $\theta$=65.1°, the minima corresponding to the Landau-level splitting become shallower, while the minima of spin splitting get deeper. For the traces at $\theta$=71.33°, the minima corresponding to the Landau-level splitting disappear. Instead, maxima are observable at filling factors $\nu$=16, 20, 24 $\cdots$, due to the coincidence of the spin levels of adjacent Landau levels, as shown Fig. 1 (condition $r$=1). As mentioned earlier, since the spin-splitting is an isotropic effect, it is determined by the total magnetic-field, $B$, while the Landau-level splitting depends only upon the perpendicular component of the magnetic-field. The spin-splitting, thus, becomes relatively more pronounced as the sample is rotated toward higher magnetic field, as can be seen in Fig. 3. This fact may be used to measure $g^*$, which can be analyzed from the relative amplitudes of the oscillations and their angular dependence by using Eq. (2) with $r$=1, as first reported by Fang and Stiles [1]. As shown in Fig. 3, the $\rho_{xx}$ at integer filing factors rises until the coincidence is reached at a certain angle $\theta$. Since Landau-level and spin-splitting have the same size at this angle [from Eq. (2)], it is possible to extract the effective $g$-factor. For this condition, Weitz *et al.* [23] obtained the results that $g^*$ stayed almost constant for $28 \geq \nu \geq 16$ in high mobility Si/$Si_{0.75}Ge_{0.25}$ heterostructures. One can see that the resistivities of the adjacent minima $\nu$=8⇔10 and 10⇔12 are equally resolved. On the other hand, the relation in Eq. (5) can be used to determine the value of $g^*$ when the resistivities of the minima are equal, as discussed in the previous section. We observed equal minima of adjacent resistivities for the following situations: 42.33, 60.72, 50.1, 58, 50.7°, and 57.1° for $\nu$=6⇔8, 8⇔10, 10⇔12, 12⇔14, 14⇔16, and 16⇔18, respectively. Typical traces for two of these are shown in Fig. 4.

The effective $g$-factor has been measured as a function of the Landau-level index for a fixed electron concentration at $T$=0.1 K. As shown in Fig. 4, since $\rho_{xx}$ at $\nu$=10⇔12 are equal at a tilt angle of 50.1°, it is possible to extract the effective $g$-factor from Eq. (5). The effective $g$-factor obtained from such method was 3.07±0.01. The resulting values of $g^*$ determined from Eq. (5), at which equal resistivity minima for $\nu$=6⇔8, 8⇔10, 10⇔12, 12⇔14, 14⇔16, and 16⇔18 occur, are

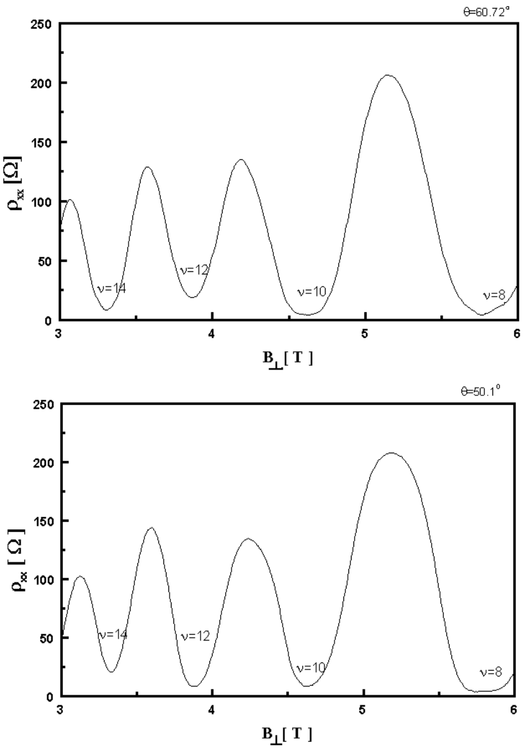


Fig. 4. Typical results for $\rho_{xx}$ at $\theta$=60.72° and 50.1° versus the normal component of magnetic field $B$ between 3 and 6 T. One can see that the resistivities of the adjacent minma $\nu$=8⇔10 and 10⇔12 are equally resolved.

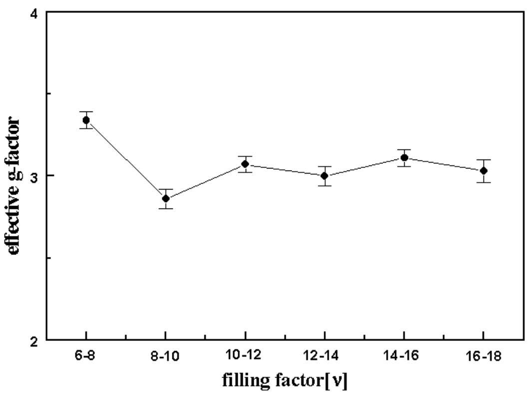


Fig. 5. Plot of the effective $g$-factor for various Landau coincidences.

plotted in Fig. 5. The $g^*$ from the data is seen to oscillate with greater variation as $\nu$ is reduced. The value of $g^*$=3.34±0.05 for $\nu$=6⇔8 decreases to 2.86±0.06 for $\nu$=8⇔10 and then increases again to 3.07±0.01 for $\nu$=10⇔12. The value of $g^*$ below $\nu$=6⇔8 cannot be observed because of the limited field range of the magnetic-field or because of lower mobility.

The effective $g^*$-factors obtained by using our method should be average values of the $g^*$-factor for the two Landau-levels. However, that does not contradict the

Table 1. Calculated values of the maximum $g^*$-factor under the assumption of a constant minimum $g^*$-factor.

| | $g^*_{min}$=2 | $g^*_{min}$=2.62 | $g^*_{min}$=2.7 | $g^*_{min}$=2.8 |
|---|---|---|---|---|
| $g^*_8$(6⇔8) | 4.35 | 3.88 | 3.82 | 3.75 |
| $g^*_{10}$(6⇔10) | 3.90 | 3.15 | 3.1 | 3.95 |
| $g^*_{10}$(10⇔12) | 3.96 | 3.45 | 3.37 | 3.3 |
| $g^*_{14}$(12⇔14) | 4.2 | 3.47 | 3.35 | 3.23 |
| $g^*_{14}$(14⇔16) | 4.08 | 3.54 | 3.47 | 3.38 |

behavior of an oscillatory $g$-factor because the value of $g^*_{n\Leftrightarrow n\pm2}$ determined by this method is a weighted average of $g^*$-factors for $\nu = n$ and $\nu=\nu \pm 2$. For instance, if we assume that each minimum $\nu = n$ in the SdH characteristic has a certain value $g^*$ associated with it, for example, then the $g$-factor determined from Eq. (5) is equal to [19]

$$g_{n\Leftrightarrow n\pm2} = \frac{1}{2}\Big[\frac{n\pm2}{n\pm1}g^*_n + \Big(\frac{n}{n\pm1}\Big)g^*_{n\Leftrightarrow2}\Big]. \quad (6)$$

Now, for example, considering the cases $\nu$=10⇔12 and $\nu$=12⇔14, the following result can be obtained:

$$g^*_{8\Leftrightarrow10} = \frac{1}{9}[5g^*_8+4g^*_{10}], \quad g^*_{10\Leftrightarrow12} = \frac{1}{10}[6g^*_{10}+5g^*_{12}]. \quad (7)$$

From Eq. (7) it is clear that $g^*_{8\Leftrightarrow10}$ is weighted toward $g^*_8$ while $g^*_{10\Leftrightarrow12}$ is weighted toward $g^*_{10}$. Since the Fermi level lies between two spin-split states of the same Landau level at $\nu$=10, $g^*_{10}$ is expected to be larger than it would be due to the exchange interaction, as discussed previous chapter. From the experimental results, the above relations are satisfied since $g^*_{10\Leftrightarrow12}$=3.07 and $g^*_{8\Leftrightarrow10}$=2.86. From Eq. (6), assuming a value for the minimum $g^*$ at $\nu$=4, 8, 12, $\cdots$, it is possible to evaluate the maximum values for $\nu$=6, 10, 14, $\cdots$. The results for the bare $g^*$-value of 2 and for the values estimated from the activation analysis are given in Table 1 [24].

Table 1 shows the maximum $g^*$ calculated under the assumption of a constant minimum $g^*$-factor. Hence, the oscillatory features observed in this sample agree with the theoretical prediction [10]. If valley splitting is significant in this field range, one cannot obtain an effective $g$-factor from a study of the angular dependence of $\rho_{xx}$ near coincidences in this magnetic-field range. However, level widths of spin-splitting levels estimated from Eq. (2) are typically ∼1.75 meV at $B$=7.6 T, which is much larger than the values calculated theoretically for valley-splitting; *i.e.*, Ohkawa and Uemura [25] determined values of $\triangle E_v$=0.2×$(n_s/10^{12})$ meV for a Si inversion layer giving, 0.2 meV for our sample. Thus, it can be expected that the influence of the valley splitting can be neglected.

## V. CONCLUSIONS

In summary, SdH oscillations corresponding to the spin-split Landau-levels of a 2DEG in a $Si/Si_{0.7}Ge_{0.3}$

heterostructure have been studied with magnetic-fields of up to 15 T. Unfortunately, the information could not be obtained about valley-splitting in our experiment because of limits on the mobility and the magnetic-field. The experiment used to obtain the effective $g$-factor employed the method of the tilted magnetic-field, where the tilt angle that causes adjacent SdH minima to be equal is used to determine $g^*$. The effective $g$-factor for $\nu$=10⇔12 was found to be 3.07±0.01. The results indicated that the effective $g$-factor oscillated as a function of the filling factor, with greater variations as $\nu$ is reduced. The values of $g^*$ obtained from this method are weighted average values; thus the actual individual oscillatory behavior is likely to be much stronger. From the data, the oscillatory behavior of $g^*$ exists throughout the whole range of the filling factor, with greater variations as is reduced.


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