$$\lim_{x\to 0} \frac{\sqrt{\cos x} - \sqrt[3]{\cos x}}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{\sqrt{\cos x} - 1 + 1 - \sqrt[3]{\cos x}}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{\sqrt{\cos x} - 1}{\sin^2 x} + \lim_{x\to 0} \frac{1 - \sqrt[3]{\cos x}}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{(\sqrt{\cos x} - 1)(\sqrt{\cos x} + 1)}{\sin^2 x(\sqrt{\cos x} + 1)} \qquad (A^2 - B^2)$$

$$= \lim_{x\to 0} \frac{\cos x - 1}{\sin^2 x(\sqrt{\cos x} + 1)}$$

$$= \lim_{x\to 0} \frac{-\frac{x^2}{2}}{x^2 \cdot (\sqrt{\cos x} + 1)} = \lim_{x\to 0} \frac{-1}{2(\sqrt{\cos x} + 1)} = -\frac{1}{4}$$

$f(x)$ liên tục $[a, b]$, kv (a, b)

1/ 0 $c \in (a, b)$

2/ $\sqrt{\ } \to$ L'H $f'(c) = \frac{f(b) - f(a)}{b - a}$

3/ $x \to 0$? VCBTĐ

4/ $\frac{f(x)}{g(x)}$; L'H $= \frac{f(a) - f(b)}{a - b}$

5/ Taylor 6/ ĐLGHK

$$\lim_{x\to 0} \frac{1 - \sqrt[3]{\cos x}}{\sin^2 x} \qquad A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$= \lim_{x\to 0} \frac{(1 - \sqrt[3]{\cos x})\left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2\right]}{\sin^2 x\left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2\right]}$$

$$= \lim_{x\to 0} \frac{1 - \cos x}{\sin^2 x\left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2\right]}$$

$$= \lim_{x\to 0} \frac{\frac{x^2}{2}}{x^2\left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2\right]}$$

$$= \lim_{x\to 0} \frac{1}{2\left[1 + \sqrt[3]{\cos x} + (\sqrt[3]{\cos x})^2\right]} = \frac{1}{6}$$

$1 + 1 + 1$

Vậy $L = -\frac{1}{\ } + \frac{1}{\ } = -\frac{1}{\ }$, Vậy $L = -\frac{1}{4} + \frac{1}{6} = -\frac{1}{12}$

$$\lim_{x\to 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3} \quad \left(\sqrt{1+\tan x} - \sqrt{1+\sin x}\right)$$

$$\lim_{x\to 0} \frac{1+\tan x - 1 - \sin x)}{\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right) x^3}$$

$$\lim_{x\to 0} \frac{\sin x\left(\frac{1}{\cos x} - 1\right)}{\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right) x^3}$$

$$\lim_{x\to 0} \frac{\sin x\left(\frac{1}{\cos x} - 1\right) \quad \frac{1-\cos x}{\cos x}}{\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right) x^3}$$

$$= \lim_{x\to 0} \frac{\sin x\,(1-\cos x)}{\cos x \,.\, x^3\left(1+\sqrt{\tan x} + \sqrt{1+\sin x}\right)}$$

$$= \lim_{x\to 0} \frac{x \,.\, \frac{x^2}{2}}{\cos x \,.\, x^3\left(1+\sqrt{\tan x} + \sqrt{1+\sin x}\right)}$$

$$= \lim_{x\to 0} \frac{1}{2 \,.\, \cos x \,.\left(1+\sqrt{\tan x} + \sqrt{1+\sin x}\right)}$$

$$= \frac{1}{4}$$

$$\lim_{x\to 0} \frac{\sin x - \tan x}{x^3} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\sin x - \frac{\sin x}{\cos x}}{x^3}$$

$$\lim_{x \to 0} \frac{\quad}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin x \left(1 - \frac{1}{\cos x}\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{-\sin x (1 - \cos x)}{\cos x \cdot x^3}$$

$$= \lim_{x \to 0} \frac{-x \cdot \frac{x^2}{2}}{\cos x \cdot x^3} = \lim_{x \to 0} \frac{-1}{2\cos x} = -\frac{1}{2}$$

7 dạng vô định

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0.\infty, 1^{\infty}, \infty^{0}, 0^{\infty}$$

Quy tắc L'Hospital

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}$$

a/ $\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{x} = 1$

b/ $\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$

c/ $\lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{x \to 0} \frac{x}{x} = 1$

d/ $\lim_{x \to 0} \frac{a^x - 1}{x} \underset{=}{\overset{\text{L'Hospital}}{}} \lim_{x \to 0} \frac{a^x \cdot \ln a}{1}$

$(a^x - 1)' = a^x \cdot \ln a$ $\quad = \lim_{x \to 0} (a^x \cdot \ln a)$

$(x^a)' = a \cdot x^{a-1}$

$(a^x)' = a^x \cdot \ln a$ $\quad = \ln a$

e/ $\lim_{x \to 0} \frac{(1+x)^p - 1}{x}$ $\quad \frac{f(x)}{g(x)}$ $\quad$ P e/ $\lim_{x\to 0} \frac{(1+x)^p - 1}{x}$ $\frac{f(x)}{g(x)}$ p

$(x^a)' = a.x^{a-1}$

$(u^a)' = u'.a.u^{a-1}$

$= \lim_{x\to 0} \frac{1.p.(1+x)^{p-1}}{1}$

$= \lim_{x\to 0} p.(1+x)^{p-1} = p$ $1^a = 1$

$\lim_{x\to x_0} \frac{f(x)}{g(x)}$ $\frac{\infty}{\infty}$ $\frac{0}{0}$

Khai triển Taylor của hàm $f(x)$ tại x_0.

$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2$

$x_0 = 0$. Maclaurin

1/ 0
2/ VCB
3/ L'Hospital

$(\tan x)' = \frac{1}{\cos^2 x}$

1/ $C' = 0$

2/ $Cx' = C$

3/ $(x^\alpha)' = \alpha . x^{\alpha-1}$

4/ $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

5/ $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

6/ $(u.v)' = u'v + v'u$.

7/ $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

8/ $(Cv)' = Cv'$

9/ $\left(\frac{C}{v}\right)' = \frac{-Cv'}{v^2}$

10/ $(\sin x)' = \cos x$

11/ $(\cos x)' = -\sin x$

12/ $(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

13/ $(\cot x)' = -\frac{1}{\sin^2 x} = -(1+\cot^2 x)$

14/ $(a^x)' = a^x \ln a$ / $(u^x)' = u'.u^x.\ln u$

16/ $(e^x)' = e^x$

17/ $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

18/ $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

14/ $(a^x)' = a^x . \ln a$ | $(u^x)' = u' . u^x . \ln u$

15/ $(\ln x)' = \frac{1}{x}$ | $(\ln u)' = \frac{u'}{u}$

$(u^\alpha)' = u' . u^{\alpha - 1} . \alpha$

18/ $(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$

19/ $(\arctan x)' = \frac{1}{x^2+1}$

20/ $(\text{arccot}\, x)' = -\frac{1}{x^2+1}$

34/a/ $\lim_{x \to 0^+} \frac{\tan x - x}{x - \sin x}$

$= \lim_{x \to 0^+} \frac{\frac{1}{\cos^2 x} - 1}{1 - \cos x}$

$= \lim_{x \to 0^+} \frac{1 - \cos^2 x}{\cos^2 x . (1 - \cos x)}$

$= \lim_{x \to 0^+} \frac{\sin^2 x}{\cos^2 x . (1 - \cos x)}$

$= \lim_{x \to 0^+} \frac{x^2}{\cos^2 x . \frac{x^2}{2}}$

$= \lim_{x \to 0^+} \frac{1 . 2}{\cos^2 x} = 2$

b/ $\lim_{x \to +\infty} \frac{\ln x}{x^{0,0001}}$

$a^{m-n} = \frac{a^m}{a^n}$

$= \lim_{x \to +\infty} \frac{1}{x . 0,0001 . x^{0,0001 - 1}}$

$= \lim_{x \to +\infty} \frac{1}{\cancel{x} . 0,0001 . \frac{x^{0,0001}}{\cancel{x}}}$

$= \lim_{x \to +\infty} \frac{1}{0,0001 . x^{0,0001}} = 0$

c/ $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin x - x}{x\sin x}\right) = \lim_{x\to 0}\left(\frac{\sin x - x}{x^2}\right)$

Dùng khai

$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2$

$x_0 = 0$

$\sin x = \sin 0 + \frac{\cos 0}{1!}\cdot(x-0)^1 + \frac{-\sin 0}{2!}(x-0)^2 + \dots$

$= 0 + x - 0 + \frac{1.x^3}{3!} \quad \frac{-\cos}{3!}(x-0)^3 \quad \frac{x^5}{3!}$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = (\cos x)' = -\sin x$

$f'''(x) = (-\sin x)' = -\cos x$

$f^{(4)}(x) = (-\cos x)' = \sin x$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$

Dùng khai triển Maclaurin với sinx tới bậc 3

$\sin x = x - \frac{x^3}{3!} + o(x^3)$

$\lim_{x\to 0}\left(\frac{\sin x - x}{x^2}\right) = \lim_{x\to 0}\left(\frac{x - \frac{x^3}{3!} + o(x^3) - x}{x^2}\right)$

$= \lim_{x\to 0}\left(\frac{x^3 + o(x^3)}{3!\,.\,x^2}\right) = \lim_{x\to 0}\frac{x}{3!} = 0$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^k\cdot\frac{x^{2k-1}}{(2k-1)!}$

$= x - x^3 \quad x^5 \quad x^7 \quad x^9 \quad (-1)$

$(2k-1)!$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + o(x^9)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + o(x^8)$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

f/ $\lim\limits_{x\to 0} \dfrac{\sin(x-\sin x)}{\sqrt{1+x^3}-1}$

1/ $\frac{0}{0}$

2/ VCB

3/ L'H

4/

$$= \lim_{x\to 0} \frac{2\sin(x-\sin x)}{x^3}.$$

Dùng Khai triển Maclaurin với sinx đến bậc 3

$$\sin x = x - \frac{x^3}{3!} + o(x^3)$$

$$x - \sin x = x - \left[x - \frac{x^3}{3!} + o(x^3)\right]$$

$$= \frac{x^3}{3!} + o(x^3)$$

$(3!)^3 . 3!$

$$\sin(x-\sin x) = \frac{x^3}{3!} + o(x^3) - \frac{\left(\frac{x^3}{3!}\right)^3}{3!} + o(x^3) + o(x^3)$$

$$= \frac{x^3}{3!} + o(x^3) - \frac{x^9}{(3!)^4} + o(x^3) + o(x^3)$$

$$\lim_{x\to 0} \frac{2\sin(x-\sin x)}{x^3} = \lim_{x\to 0} 2 \cdot \frac{\left(\frac{x^3}{3!} + o(x^3) - \frac{x^9}{(3!)^4} + o(x^3) + o(x^3)\right)}{x^3}$$

$0, 3\ 3\ 3\ 3$

$$= \lim_{x\to 0} 2 \cdot \frac{1}{3!} = \frac{2}{3!} = \frac{1}{3}.$$

d/ $\lim \quad x^{\frac{1}{x}}$

d/ $\lim_{x \to +\infty} x^{\frac{1}{x}}$

$= \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}}$

$= \lim_{x \to +\infty} e^{\frac{1}{x}\ln x} = e^0 = 1$

$= \lim_{x \to +\infty} e^{\frac{\ln x}{x}} = \lim_{x \to +\infty} e^{\frac{1}{x \cdot 1}}$

g/ $\lim_{x \to 1} \dfrac{1 - x + \ln x}{1 - \sqrt{2x - x^2}}$

$\left(\sqrt{u}\right)' = \dfrac{u'}{2\sqrt{u}}$

$\sqrt{2x - x^2} = \dfrac{\cancel{2} - \cancel{2}x}{\cancel{2}\sqrt{2x - x^2}}$

$= \lim_{x \to 1} \dfrac{-x + 1}{\dfrac{x - 1}{\sqrt{2x - x^2}} \cdot x}$

$\dfrac{1 - x}{\sqrt{2x - x^2}}$

$= \lim_{x \to 1} \dfrac{-x + 1}{\dfrac{x^2 - x}{\sqrt{2x - x^2}}}$

$= \lim_{x \to 1} \dfrac{\left(\sqrt{2x - x^2}\right) \cdot (1 - x)}{-x(1 - x)}$

$= \lim_{x \to 1} \dfrac{\sqrt{2x - x^2}}{-x}$

$= -1.$

35/ $\lim_{x \to 0} \dfrac{x^2 \sin \frac{1}{x}}{\sin x}$

13/e/ $\lim\limits_{n\to+\infty} \frac{1}{n}\cos\frac{n\pi}{2}$

3 dãy số $(u_n), (v_n), (w_n)$ thỏa mãn

1/ $u_n < v_n < w_n \quad \forall n \in \mathbb{N}^*$

2/ $\lim\limits_{n\to\infty}(u_n) = \lim\limits_{n\to\infty}(w_n) = L$

$\Rightarrow \lim\limits_{n\to\infty}(v_n) = L \qquad -1 \le \cos x \le 1$

13/e/ $\lim\limits_{n\to+\infty} \frac{1}{n}\cos\frac{n\pi}{2} \quad -1 \le \sin x \le 1$

Ta có: $-1 \le \cos\frac{n\pi}{2} \le 1$

$\Leftrightarrow -\frac{1}{n} \le \frac{1}{n}\cos\frac{n\pi}{2} \le \frac{1}{n}$

Vì $\lim\limits_{n\to+\infty}\left(-\frac{1}{n}\right) = \lim\limits_{n\to+\infty}\frac{1}{n} = 0 \Rightarrow \lim\limits_{n\to+\infty}\frac{1}{n}\cos\frac{n\pi}{2} = 0$

$\lim\limits_{n\to+\infty}\sqrt[n]{3+\sin n} = 1$

Ta có: $-1 \le \sin n \le 1$

$$\Leftrightarrow 2 \leq 3+\sin \leq 4$$

$$\Leftrightarrow \sqrt[n]{2} \leq \sqrt[n]{3+\sin x} \leq \sqrt[n]{4}$$

Vì $\lim\limits_{n\to\infty} \sqrt[n]{2} = \lim\limits_{n\to+\infty} \sqrt[n]{4} = 1 \Rightarrow$

$$\frac{(a^x)', (x^a)'}{(x^x)'}$$

3/ $(x^x)'$

$(\ln x)' = \frac{1}{x}$

$(\ln u)' = \frac{u'}{u}$

$f(x) = x^x \Leftrightarrow \ln f(x) = \ln x^x$

$\Leftrightarrow \boxed{\ln f(x)} = \boxed{x \ln x}$

$$\boxed{\frac{f'(x)}{f(x)}} = (x \ln x)' = \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} = \ln x + 1$$

$$f'(x) = (\ln x + 1) \cdot f(x) = x^x \cdot (\ln x + 1)$$

$$f(x) = \sqrt[x]{x} = x^{\frac{1}{x}}$$

$$\Rightarrow \ln f(x) = \ln x^{\frac{1}{x}} = \frac{1}{x} \ln x$$

$\Rightarrow \ln f(x) = \ln x^{\frac{1}{x}} = \frac{1}{x} \ln x$

$$\Rightarrow \frac{f'(x)}{f(x)} = -\frac{1}{x^2} \cdot \ln x + \frac{1}{x} \cdot \frac{1}{x} = \frac{1-\ln x}{x^2}$$

$$\Rightarrow f'(x) = f(x) \cdot \frac{1-\ln x}{x^2} = \sqrt[x]{x} \cdot \frac{1-\ln x}{x^2}$$

11/a/ $f(x) = \begin{cases} \dfrac{x+x^2}{x^2-1}, & x \neq \pm 1 \\ m, & x = \pm 1 \end{cases}$

$x - 1$

$1 + x$

$-1(1+x)$

$-1-x$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x+x^2}{x^2-1} = \lim_{x\to 1^+} \frac{x(\cancel{1+x})}{(x-1)(\cancel{x+1})}$$

1,000001

$$= \lim_{x\to 1^+} \frac{x}{x-1} \quad \frac{1}{0,000001} = +\infty$$

$\neq$

$$\lim_{x\to 1^-} \frac{x}{x-1} \quad \frac{1}{-0,9999} = -\infty$$

1 2

$\Rightarrow f(x)$ ko lt tại $x = 1$

$$\lim_{x\to -1^+} \frac{x}{x-1} = \frac{1}{2} = \frac{-1}{-1-1} = \frac{1}{2}$$

$$\lim_{x\to -1^-} \frac{x}{x-1} = \frac{1}{2} \cdot \frac{-1}{-1-1} = \frac{1}{2}$$

$\Rightarrow f(x)$ liên tục tại $x = -1$

$\Rightarrow f(x)$ liên tục tại $x = -1$.

Vậy với $x = -1$ — $m = \frac{1}{2}$ thì hs liên tục

$m \neq \frac{1}{2}$ thì hs ko liên tục

b/ $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

$\lim\limits_{x \to 0^+} \frac{\sin x}{x} = \lim\limits_{x \to 0^+} \frac{x}{x} = 1$. $\alpha = 1$

$x \neq 1$

c/ $f(x) = x^{\frac{1}{x-1}}$ / 1^+ 1^-

$\lim\limits_{x \to 1^+} x^{\frac{1}{x-1}}$ /

$\lim\limits_{x \to 0} (1+x)^{\frac{1}{x}}$

$\lim\limits_{u \to 0} (1+u)^{\frac{1}{u}}$

$u = x - 1$

$= \lim\limits_{x \to 1^+} (1 + \underline{x-1})^{\frac{1}{x-1}}$

$= \lim\limits_{x \to 1^+} [1 + (x-1)]^{\frac{1}{x-1}} = \underline{e}$

2b/ $f(x) = \frac{1}{\ldots}$ $\frac{1}{\ldots}$ A B

26/ $f(x) = \frac{1}{x^2-3x+2} = \frac{1}{(x-2)(x-1)} = \frac{A}{(x-2)} + \frac{B}{(x-1)}$

$f^{(n)}(x) = \left(\frac{1}{x-2} - \frac{1}{x-1}\right) \qquad = \frac{1}{x-2} - \frac{1}{x-1}$

$\left(\frac{1}{x-2}\right)' = \frac{-1}{(x-2)^2}, \qquad ((x-2)^{-1})' \qquad (u^{\alpha})' = u'\alpha.u^{\alpha-1}$

$\left(\frac{1}{x-2}\right)'' = -1\left[\frac{1}{(x-2)^2}\right]' = 1.2\,\frac{1}{(x-2)^3}$

$1.2\left[\frac{1}{(x-2)^3}\right]' = -1.2.3.\frac{1}{(x-2)^4}$

$(-1)^n.n!.\frac{1}{(x-2)^{n+1}}$

1/ $\left(\frac{1}{x+a}\right)^{(n)} = \frac{(-1)^n.n!}{(x+a)^{n+1}}\quad \forall n\in N, a\in R$

2/ $(e^x)^{(n)} = e^x, \ \forall n\in N$

3/ $(e^{ax})^{(n)} = a^n.e^{ax}, \forall \in N, a\neq 0$

4/ $(a^x)^{(n)} = a^x(\ln a)^n, \forall \in N, \ 0<a\neq 1$

5/ $(a^{\alpha x})^{(n)} = a^{\alpha x}.(\alpha.\ln a)^n \ \forall n\in N \quad 0<a\neq 1, \ \alpha\neq 0$

6/ $\left(\sqrt[k]{x+a}\right)^{(n)} = \frac{1}{k}\left(\frac{1}{k}-1\right)\left(\frac{1}{k}-2\right)\ldots\left(\frac{1}{k}-n+1\right)(x+a)^{\frac{1}{k}-n} \ \forall n\in N, a\in R$

7/ $[\sin(ax+b)]^{(n)} = a^n\sin\left(ax+b+n.\frac{\pi}{2}\right)$

8/ $[\cos(ax+b)]^{(n)} = a^n\cos(ax+b+n.\frac{\pi}{2})$

$7/\ [\sin(ax+b)]^{(n)} = a^n \sin(ax+b+n\cdot\frac{\pi}{2})$

$8/\ [\cos(ax+b)]^{(n)} = a^n \cos(ax+b+n\frac{\pi}{2})$

Số phức: $e^{ix} = \cos x + i\sin x$

$\Rightarrow \cos x = \frac{1}{2}(e^{ix} + e^{-ix})$

$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$

$(x^2)^{(3)} = 0$

Công thức Leibnitz

$$(u.v)^{(n)} = \sum_{k=0}^{n} C_n^k . u^{(k)} . v^{(n-k)}$$

$$(x^2.e^x)^{(100)} = C_{100}^0 . x^2 . (e^x)^{(100)}$$
$$+ C_{100}^1 . (x^2)^{(1)} . (e^x)^{(99)}$$
$$+ C_{100}^2 . (x^2)^{(2)} . (e^x)^{(98)}$$
$$= 1.x^2.e^x + 100.2x.e^x + C_{100}^2 . 2.e^x$$

TÍCH PHÂN SUY RỘNG

$$\int_a^{+\infty} f(x)dx = \lim_{A\to+\infty} \int_a^A f(x)dx \begin{cases} b: \text{Hội tụ} \\ \nexists, \infty: \text{Phân kỳ} \end{cases}$$

$$\int_{-\infty}^{a} f(x)dx = \lim_{A\to-\infty} \int_A^a f(x)dx$$

$$\int_{-\infty}^{+\infty} f(x)dx = \lim_{\substack{A\to+\infty, \\ A'\to-\infty}} \int_{A'}^{A} f(x)dx$$

$+\infty$ a

$$= \int_a f(x)dx + \int_{-\infty} f(x)dx$$

10 CT mở rộng

1/ $\int \frac{dx}{x} = \ln|x| + C$ — $\int \frac{du}{u} = \ln|u| + C$

2/ $\int \frac{dx}{x^2} = -\frac{1}{x} + C$ — $\int \frac{du}{u^2} = -\frac{1}{u} + C$

3/ $\int \frac{dx}{x^2+1} = \arctan x + C$ — $\int \frac{du}{u^2+1} = \arctan u + C$

4/ $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$ $\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

5/ $\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$ $\int \frac{du}{u^2-1} = \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + C$

6/ $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$ $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

7/ $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$ $\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$

8/ $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$ $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

9/ $\int \frac{dx}{\sqrt{x^2+a}} = \ln\left|x + \sqrt{x^2+a}\right| + C$

10/ $\int \sqrt{x^2+a}\,dx = \frac{x}{2}\sqrt{x^2+a} + \frac{a}{2}\cdot\ln\left|x+\sqrt{x^2+a}\right| + C$

1/ Tính tích phân:

$$\int_{e^2}^{+\infty} \frac{dx}{x.\ln x\left[\ln(\ln x)\right]^2} = \lim_{A\to+\infty} \int_{e^2}^{A} \frac{dx}{x.\ln x\left[\ln(\ln x)\right]^2}$$

Xét tích phân $J = \int_{e^2}^{A} \frac{dx}{x\ln x\left[\ln(\ln x)\right]^2}$

Đặt $t = \ln(\ln x)$

$\Rightarrow dt = \frac{dx}{x.\ln x}$

Đổi cận

$x = e^2 \Rightarrow t = \ln 2$

$x = A \Rightarrow t = \ln(\ln A)$

$J = \int^{\ln(\ln A)} dt \quad = -\frac{1}{\ } \Big|^{\ln(\ln A)}$

$$J = \int_{\ln 2}^{\ln(\ln A)} \frac{dt}{t^2} = -\frac{1}{t}\Big|_{\ln 2}^{\ln(\ln A)}$$

$$= -\frac{1}{\ln(\ln A)} + \frac{1}{\ln 2}$$

$$I = \lim_{A\to+\infty} J = \lim_{A\to+\infty}\left(\frac{-1}{\ln(\ln A)} + \frac{1}{\ln 2}\right)$$

$$= 0 + \frac{1}{\ln 2} = \frac{1}{\ln 2}$$

2/ $$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^2} = \lim_{\substack{A\to+\infty,\\ A'\to-\infty}} \int_{A'}^{A} \frac{dx}{(x^2+1)^2}$$

Đặt $x = \tan t$

$\Rightarrow dx = \frac{dt}{\cos^2 t} = (1+\tan^2 t)dt$

Đổi cận $x = A \Rightarrow t = \arctan A$

$x = A' \Rightarrow t = \arctan A'$

$$J = \int_{\arctan A'}^{\arctan A} \frac{1+\tan^2 t}{(\tan^2 t+1)^2} dt = \int_{\arctan A'}^{\arctan A} \frac{dt}{1+\tan^2 t}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$= \int_{\arctan A'}^{\arctan A} \cos^2 t\, dt = \int_{\arctan A'}^{\arctan A} \frac{1+\cos 2t}{2} dt$$

$$= \left(\frac{t}{2} + \frac{1}{4}\sin 2t\right)\Big|_{\arctan A'}^{\arctan A}$$

$$= \left(\frac{\arctan A}{2} + \frac{1}{4}\cdot\sin 2(\arctan A) - \left(\frac{\arctan A'}{2} + \frac{1}{4}\sin 2(\arctan A')\right)\right)$$

$$= \frac{\pi}{4} + \frac{1}{4}\sin\pi - \left(-\frac{\pi}{4} - \frac{1}{4}\sin\pi\right)$$

$$= \frac{\pi}{4} + \frac{\pi}{4} + \quad = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$= \frac{\pi}{4} + \frac{\pi}{4} + \; = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\int \frac{dx}{\tan^3 x} = \int \frac{\cos^3 x}{\sin^3 x} dx$$

Đặt $t = \sin x$

$dt = \cos x \, dx$

$$= \int \frac{\cos^2 x . \cos x \, dx}{\sin^3 x}$$

$$= \int \frac{(1-\sin^2 x) . \cos x \, dx}{\sin^3 x}$$

$$= \int \frac{(1-t^2) . dt}{t^3} = \int \frac{dt}{t^3} - \int \frac{dt}{t}$$

$$= -\frac{1}{2t^2} - \ln|t| = -\frac{1}{2\sin^2 x} - \ln|\sin x| + C$$

a/ $\displaystyle\int \frac{dx}{x^4 - x^2 - 2}$

$$= \int \frac{dx}{(x^2-2)(x^2+1)}$$

$$= \frac{1}{3}\int \left(\frac{1}{x^2-2} - \frac{1}{x^2+1} \right) dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2-2} - \frac{1}{3}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{3} . \frac{1}{2\sqrt{2}} . \ln\left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{1}{3}\arctan x + C .$$

$$\int x^2 \ln x\,dx$$

Đặt $\begin{cases} u = \ln x \\ dv = x^2 dx \end{cases} \Rightarrow \begin{cases} du = \dfrac{dx}{x} \\ v = \dfrac{x^3}{3} \end{cases}$

$$\Rightarrow \underline{I} = \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3}\frac{dx}{x}$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3}$$

$$= \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$$

$$\int \frac{\arcsin x}{x^2}dx$$

Đặt $\begin{cases} u = \arcsin x \\ dv = \dfrac{dx}{x^2} \end{cases} \Rightarrow \begin{cases} du = \dfrac{1}{\sqrt{1-x^2}}dx \\ v = -\dfrac{1}{x} \end{cases}$

$$\Rightarrow \underline{I} = \arcsin x \cdot \frac{1}{x} + \int \frac{dx}{x\sqrt{1-x^2}} = \int \frac{x\,dx}{x^2\sqrt{1-x^2}}$$

Đặt $t = \sqrt{1-x^2}$

$\Rightarrow t^2 = 1 - x^2$

$\Rightarrow x^2 = 1 - t^2$

$\Rightarrow 2x\,dx = -2t\,dt$

$\Rightarrow x\,dx = -t\,dt$

$$\arcsin\frac{1}{x} + \int \frac{-t\,dt}{(1-t^2).t}$$

$$= \arcsin\frac{1}{x} - \int \frac{dt}{1-t^2}$$

$$= \arcsin\frac{1}{x} + \int \frac{dt}{t^2-1}$$

$$1 - t^2 = -(t^2 - 1) = \arcsin\frac{1}{x} + \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|$$

$$= \arcsin\frac{1}{x} + \frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right|$$

4/a/ $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \underbrace{\frac{1}{K.(K+1)}}$

$$\frac{1}{K(K+1)} = \frac{A}{K} + \frac{B}{K+1} = \frac{1}{K} - \frac{1}{K+1}$$

$$\frac{1}{1.2} = \frac{1}{1} - \frac{1}{1+1}$$

$$\frac{1}{2.3} = \frac{1}{2} - \frac{1}{2+1}$$

$$\frac{1}{3.4} = \frac{1}{3} - \frac{1}{3+1}$$

....

$$\frac{1}{(n-1)n} = \frac{1}{n-1} - \frac{1}{n}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \underbrace{\frac{1}{n+1}}$$

$$\sum = \underbrace{1} - \frac{1}{n+1}$$

$$\lim_{n\to+\infty}\left(1 - \frac{1}{n+1}\right) = 1$$

Vậy chuỗi ht

b/ $\sum^{\infty}$ 1 A B C

$$h/\ \sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)} \quad \frac{A}{k}+\frac{B}{k+1}+\frac{C}{k+2}$$

$$A(k+1)(k+2)+Bk(k+2)+C(k+1)k$$

$$=A(k^2+2k+k+2)+B(k^2+2k)+C(k^2+k)$$

$$=Ak^2+2Ak+Ak+2A+Bk^2+2Bk+Ck^2+Ck=1$$

$$\begin{cases} A+B+C=0 \\ 3A+2B+C=0 \\ 2A=1 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{2}+B+C=0 \\ 3\frac{1}{2}+2B+C=0 \\ A=\frac{1}{2} \end{cases}$$

$$B=-1 \quad C=\frac{1}{2}$$

$$\frac{1}{k(k+1)(k+2)}=\frac{1}{2}\left(\frac{1}{k}-\frac{2}{k+1}+\frac{1}{k+2}\right)$$

$$\frac{1}{1.2.3}=\frac{1}{2}\left(\frac{1}{1}-\frac{2}{1+1}+\frac{1}{1+2}\right) \qquad \frac{1}{3}+\frac{1}{3}-\frac{2}{3}=0$$

$$\frac{1}{2.3.4}=\frac{1}{2}\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)$$

$$\frac{1}{3.4.5}=\frac{1}{2}\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)$$

$$\cdots$$

$$\frac{1}{(n-1)n.(n+1)}=\frac{1}{2}\left(\frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}\right)$$

$$\frac{1}{n(n+1)(n+2)}=\frac{1}{2}\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)$$

$$\sum = \frac{1}{\ }\left(\frac{1}{\ }-\frac{1}{\ }+\frac{1}{\ }\right)$$

$$\sum = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}\right)$$

$$\lim_{n\to+\infty} = \frac{1}{4}$$

$$t = 1 - x$$

$$\int \frac{x^2\,dx}{(1-x)^{100}} \Rightarrow x = 1 - t$$

$$\Rightarrow dx = -dt$$

$$\int \frac{(1-t)^2 . (-dt)}{t^{100}} = -\int \frac{1 - 2t + t^2}{t^{100}}\,dt$$

$$= -\left(\int \frac{dt}{t^{100}} - \int \frac{2dt}{t^{99}} + \int \frac{dt}{t^{98}}\right)$$

$$= -\left(\frac{t^{-99}}{-99} - 2 . \frac{t^{-98}}{-98} + \frac{t^{-97}}{-97}\right)$$

$$= \frac{1}{99t^{99}} - \frac{1}{49 . t^{98}} + \frac{1}{97t^{97}}$$

$$I = \int \frac{(x+2)\,dx}{\sqrt{x^2 - 5x + 6}}$$

$$= \int \frac{x\,dx}{\sqrt{x^2-5x+6}} + \int \frac{2\,dx}{\sqrt{x^2-5x+6}}$$

$$= \quad K \quad + \quad (A+B)$$

$+\,\_^{2}$

$$A = 2\int \frac{dx}{\sqrt{x^2-5x+6}} \qquad x^2-5x+6$$

$$= 2\int \frac{dx}{\sqrt{\left(x-\frac{5}{2}\right)^2-\frac{1}{4}}} = 2\int \frac{\left(x-\frac{5}{2}\right)'dx}{\sqrt{\left(x-\frac{5}{2}\right)^2-\frac{1}{4}}}$$

$$= 2\int \frac{d\left(x-\frac{5}{2}\right)}{\sqrt{\left(x-\frac{5}{2}\right)^2-\frac{1}{4}}} = 2\ln\left|x-\frac{5}{2}+\sqrt{x^2-5x+6}\right| + C$$

$$K = \int \frac{x\,dx}{\sqrt{x^2-5x+6}} = \frac{1}{2}\int \frac{2x-5+5}{\sqrt{x^2-5x+6}}\,dx$$

$$= \frac{1}{2}\int \frac{2x-5}{\sqrt{x^2-5x+6}} + \frac{1}{2}\int \frac{5\,dx}{\sqrt{x^2-5x+6}}$$

$$= \quad J \quad + \frac{1}{2}\cdot 5\cdot \ln\left|x-\frac{5}{2}+\sqrt{x^2-5x+6}\right| + C$$

$$\frac{1}{}\int \frac{\left(x^2-5x+6\right)'}{\ldots}$$

$$= \frac{1}{2}\int \frac{(x^2-5x+6)'}{\sqrt{x^2-5x+6}}\,dx$$

$$\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C$$

$$= \frac{1}{2}\int \frac{d(x^2-5x+6)}{\sqrt{x^2-5x+6}}$$

$$\frac{du}{\sqrt{u}} = \frac{du}{u^{\frac{1}{2}}} = \int u^{-\frac{1}{2}} du$$

$$= \int u^{-\frac{1}{2}} du$$

$$= \frac{1}{2}\, 2\,(x^2-5x+6)^{-\frac{1}{2}} + C$$

$$= \boxed{(x^2-5x+6)^{-\frac{1}{2}} + C}$$

ĐÁNH GIÁ SỰ HỘI TỤ CỦA CHUỖI SỐ

1/ Đối với chuỗi tuỳ ý

Nếu chuỗi $\sum_{n=1}^{\infty} u_n$ hội tụ thì $\lim_{n\to\infty} u_n = 0$

$\sum_{n=1}^{\infty} u_n$ và $\sum_{n=1}^{\infty} v_n$ hội tụ thì $\sum_{n=1}^{\infty} u_n \cdot \alpha + \sum_{n=1}^{\infty} v_n \cdot \beta$ hội tụ

$\sum_{n=1}^{\infty}$ hội tụ khi và chỉ khi $\sum_{n=n_0}^{\infty}$ cũng hội tụ $n_0 \in \mathbb{N}^*$

$\sum^{\infty} |u_n|$ hội tụ thì chuỗi $\sum^{\infty} u_n$ cũng ht

$\sum_{n=1}^{\infty} |u_n|$ hội tụ thì chuỗi $\sum_{n=1}^{\infty} u_n$ cũng ht

2/ Đối với chuỗi dương

Chuỗi số dương $\sum_{n=1}^{\infty} u_n$ hội tụ khi và chỉ khi dãy tổng riêng $S_k = u_1 + u_2 + u_3 + \dots + u_k$ bị chặn trên

4 tiêu chuẩn đánh giá hội tụ:

1/ Tiêu chuẩn so sánh

Nếu 2 chuỗi số dương $\sum_{n=1}^{\infty} u_n$ và $\sum_{n=1}^{\infty} v_n$ mà thoả mãn điều kiện $u_n \leq v_n$, $\forall n \geq n_0 \in \mathbb{Z}^+$

Nếu $\sum_{n=1}^{\infty} v_n$ hội tụ $\sum_{n=1}^{\infty} u_n$ cũng hội tụ

$\sum_{n=1}^{\infty} v_n$ phân kỳ $\sum_{n=1}^{\infty} u_n$ cũng phân kỳ

2/ Tiêu chuẩn Riemann để so sánh $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$

Nếu $\alpha > 1$, thì hội tụ

$\alpha \leq 1$, thì phân kỳ

Và dựa vào kết quả của giới hạn $\lim\limits_{n\to\infty} \frac{u(n)}{v(n)} = K \geq 0$

Nếu $0 < K < +\infty$ thì sự hội tụ của 2 chuỗi $u(n)$ và $v(n)$ là như nhau

$K = 0$ thì $\sum\limits_{n=1}^{\infty} v(n)$ hội tụ $\Rightarrow \sum\limits_{n=1}^{\infty} u(n)$ hội tụ

$K = +\infty$ thì $\sum\limits_{n=1}^{\infty} (v)$ phân kỳ $\Rightarrow \sum\limits_{n=1}^{\infty} u(n)$ pkỳ.

3/ Tiêu chuẩn D'alembert

Cho chuỗi số dương $\sum\limits_{n=1}^{\infty} u(n)$ và $\lim\limits_{n\to\infty} \frac{u(n+1)}{u(n)} = K \geq 0$

Nếu $K < 1$ thì chuỗi $\sum\limits_{n=1}^{\infty} u(n)$ hội tụ

$K > 1$ thì chuỗi $\sum\limits_{n=1}^{\infty} u(n)$ phân kỳ.

4/ Tiêu chuẩn Cauchy

Cho chuỗi số dương $\sum\limits_{n=1}^{\infty} u(n)$ có $\lim\limits_{n\to\infty} \sqrt[n]{u(n)} = K \geq 0$

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$\sum_{n=1}^{\infty} u(n)$ có $\lim_{n\to\infty} \sqrt[n]{u(n)} = K > 0$

Nếu $K < 1$ thì chuỗi $\sum_{n=1}^{\infty} u(n)$ hội tụ

$K > 1$ thì chuỗi $\sum_{n=1}^{\infty} u(n)$ phân kỳ.

1 số giới hạn dãy

1/ $\lim_{n\to\infty} C = 0$

2/ $\lim_{n\to\infty} n^{\alpha} = +\infty \quad (\alpha > 0)$

3/ $\lim_{n\to\infty} \frac{a}{n^{\alpha}} = 0 \quad (\alpha > 0)$

4/ $\lim_{n\to\infty} q^n = \begin{cases} 0 & |q| < 1 \\ 1 & |q| = 1 \\ +\infty & |q| > 1 \end{cases}$

5/ $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$

6/ $\lim_{n\to+\infty} \sqrt[n]{a} = 1 \ (a > 0)$

7/ $\lim_{n\to+\infty} \sqrt[n]{n} = 1$

8/ $\lim_{n\to+\infty} \sqrt[n]{n!} = +\infty$

9/ $\lim_{n\to+\infty} \frac{n^p}{a^n} = 0 \ (a > 1)$

1/ Xét sự hội tụ của chuỗi

$\sum_{n=1}^{\infty} \frac{n+1}{2n} \quad \left(\lim_{n\to\infty} u(n) = 0 \ ???\right)$

$$\sum_{n=1} \frac{n+1}{2n} \quad (\lim_{n\to\infty} u(n) = 0 \; ???)$$

Xét $\lim\limits_{n\to\infty} \frac{n+1}{2n} = \lim\limits_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{2n} = \frac{1}{2} \neq 0$

2/ Xét sự hội tụ của chuỗi số $\sum\limits_{n=1}^{\infty} \left|\frac{(-1)^n . n}{2^n}\right|$

Xét giới hạn $\lim\limits_{n\to\infty} \sqrt[n]{\frac{(-1)^n . n}{2^n}}$

$$= \lim_{n\to\infty} \frac{1 . \sqrt[n]{n}}{2} = \frac{1}{2} < 1$$

Vậy chuỗi đã cho hội tụ

3/ ĐGSHT $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[3]{n} . 3^n}$

Xét $\lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{\sqrt[3]{n} . 3^n}} = \lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{\sqrt[n]{n}} . 3} = \frac{1}{3}$

Vậy chuỗi hội tụ

Vậy chuỗi hội tụ

$$\lim_{n\to\infty}\frac{u(n+1)}{u(n)} = \lim_{n\to\infty}\frac{\dfrac{1}{\sqrt[3]{n+1}\cdot 3^{n+1}}}{\dfrac{1}{\sqrt[3]{n}\cdot 3^{n}}} = \frac{1}{\sqrt[3]{n+1}\cdot 3^{n+1}}\cdot\frac{\sqrt[3]{n}\cdot 3^{n}}{1}$$

$$= \frac{1}{3}\sqrt[3]{\frac{n}{n+1}} = \frac{1}{3}\sqrt[3]{\frac{\cancel{n}}{\cancel{n}\left(1+\frac{1}{n}\right)}} = \frac{1}{3}\sqrt{\frac{1}{1}} < 1$$

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$$\sum_{n=1}^{\infty}\left(\frac{2n+1}{n-3}\right)^n$$

Xét chuỗi số $\sum_{n=4}^{\infty}\left(\frac{2n+1}{n-3}\right)^n$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{2n+1}{n-3}\right)^n} = \lim_{n\to\infty}\frac{n\left(2+\frac{1}{n}\right)}{n\left(1-\frac{3}{n}\right)} = 2 > 1$$

Chuỗi phân kỳ.

6/ $\sum_{n=1}^{\infty} \sqrt{\tan\frac{1}{n} - \sin\frac{1}{n}}$ $\quad 0 - 0$

$\tan\frac{1}{n} - \sin\frac{1}{n}$ $\qquad \tan = \frac{s}{c}$

$\to s = c \oplus$

$= \tan\frac{1}{n}\left(1 - \cos\frac{1}{n}\right)$

$= \frac{1}{n} \cdot \frac{1}{2n^2} = \frac{1}{2n^3}$ $\qquad \frac{\left(\frac{1}{n}\right)^2}{2} = \frac{1}{2n^2}$

$\sum \sqrt{\frac{1}{2n^3}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{2}\cdot n^{3/2}} \to$ Hội tụ

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\qquad \sqrt{n^3} =$

$\frac{1}{1 \to \infty} = 0$ $\qquad n!$

$\frac{1}{n^\alpha} = 0$ $\qquad ! \; n^n$

$n^\alpha \to \infty$ $\quad \alpha > 1 \to HT$

$\frac{1}{n^\alpha} \to \infty$ $\quad \alpha < 0$ $\quad \frac{1}{\to 0} = \infty \to PK.$

$\sum^{\infty} \frac{(n!)^\alpha}{\dots}$

$$\sum_{n=1}^{\infty} \frac{(n!)^{\alpha}}{n^n}$$

$$\lim_{n\to\infty} \frac{u(n+1)}{u(n)} = \frac{[(n+1)!]^{\alpha}}{(n+1)^{n+1}} \cdot \frac{n^n}{(n!)^{\alpha}}$$

$$= \left[\frac{(n+1)!}{n!}\right]^{\alpha} \cdot \frac{n^n}{(n+1)^{n+1}}$$

$$= \left[\frac{(n+1)n!}{n!}\right]^{\alpha} \cdot \frac{n^n}{(n+1)^n \cdot (n+1)}$$

$$= \frac{(n+1)^{\alpha}}{(n+1)^1} \cdot \frac{n^n}{(n+1)^n} \qquad \lim_{n\to\infty} u(n) = \underline{k > 1} \;\; \overline{PK}$$

$\alpha = 1$

$\alpha < 1$ HT

$\alpha > 1$ PK

$$= \frac{(n+1)^{\alpha - 1}}{(n+1)} \cdot \boxed{\left(\frac{n}{n+1}\right)^n} \qquad \frac{1}{(n+1)} \qquad \frac{<1}{HT}$$

$$e \qquad \frac{1}{\infty} = 0$$

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \qquad \left(\frac{n+1}{n}\right)^{-n}$$

$$= \left(1 + \frac{1}{n}\right)^{-n} = e^{-1}$$

$$1 \cdot \frac{1}{e} = K \quad HT \qquad \underline{\frac{1}{e} < 1}$$

$$\alpha \leq 1 \Rightarrow HT$$