1/ Chứng minh

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

1/ Khẳng định

$p \to q$

p

$\therefore q$

$\to \boxed{(p \to q) \wedge p \to q}$

2/ Phủ định

$p \to q$

$\neg q$

$\therefore \neg p$

$[(p \to q) \wedge \neg q] \to \neg p$.

3/ Tam đoạn luận

3/ Tam đoạn luận

$p \to q$

$q \to r$

$\therefore p \to r$

$$[(p \to q) \land (q \to r)] \to p \to r$$

4/ Tam đoạn luận rời

$p \lor q$

$\neg q$

$\therefore p$

$$[(p \lor q) \land \neg q] \to p.$$

$p \to q$

$\neg q$

$\therefore \neg p.$

5/ C/m theo trường hợp

$p \to r$

$q \to r$

$$[(p \to r) \land (q \to r)] \to [(p \lor q) \to r].$$

$\therefore (p \vee q) \to r$

6/ Mâu thuẫn

$$\frac{\begin{array}{l} p_1 \\ p_2 \\ \dots \\ p_n \end{array}}{\therefore q} \to \frac{\begin{array}{l} p_1 \\ p_2 \\ \dots \\ p_n \\ \neg q \end{array}}{\therefore 0}$$

7/ Rút gọn

$$\frac{\begin{array}{l} p \wedge q \\ \therefore q \end{array}}{\therefore p}$$

8/ Nối liền

$$\frac{\begin{array}{l} p \\ q \end{array}}{\therefore p \wedge q}$$

$$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$$

$p \lor q$

$\cancel{p \to r}$

$q \to r$

$\therefore r$

5/ $p \to r$

$q \to r$

$\therefore (p \lor q) \to r$

$p \to r$ (Tiền đề)

$q \to r$

$\therefore (p \lor q) \to r$ (C/m theo trường hợp)

$(p \lor q) \to r$ (Tiền đề)

$p \lor q$

$\therefore r$ (Khẳng định)

$p \to q$

p

$\therefore q$

$\therefore\ r$ (Khẳng định) | $\therefore\ q$

$p \to q$
$r \to \neg q$
r
$\therefore\ \neg p$

$r \to \neg q$, r $\to$ $\neg q$

$p \to q$
p
$\therefore\ q$

$p \to q$
$\neg q$
$\therefore\ \neg p$

Phủ định

$p \to (q \to r)$
$\neg q \to \neg p$
p
$\therefore\ r$

$p \to (q \to r)$
p
$\therefore\ q \to r$
$\neg q \to \neg p \to p \to q$
r · $\neg p \vee q$

r. $\qquad \underline{\neg p \vee q}$.

$\underline{\neg q \to \neg p}$ $\quad \neg(\neg q) \vee \neg p \Leftrightarrow q \vee \neg p$.

$p \to q = \underline{\neg p \vee q}$. $\quad \neg q \to \neg p \equiv \underline{p \to q}$.

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array} \qquad \begin{array}{l} p \to r \\ \underline{p} \\ \therefore r \end{array}$$

b/ $P(n)$ $\dfrac{1}{2!} + \dfrac{2}{3!} + \dots + \dfrac{n}{(n+1)!} = 1 - \dfrac{1}{(n+1)!}$

Bước cơ sở $P(1)$: $\dfrac{1}{(1+1)!} = 1 - \dfrac{1}{(1+1)!}$

$$\Leftrightarrow \frac{1}{2} = 1 - \frac{1}{2}$$

$$\Rightarrow VT = VP.$$

Giả sử $P(K)$ đúng:

$$P(K): \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{K}{(K+1)!} = 1 - \frac{1}{(K+1)!}$$

Cần chứng minh $P(K+1)$ đúng:

$$P(K+1): \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{K}{(K+1)!} + \frac{K+1}{(K+2)!} = 1 - \frac{1}{(K+2)!}$$

$$\Leftrightarrow \cancel{1} - \frac{1}{(K+1)!} + \frac{K+1}{(K+2)!} = \cancel{1} - \frac{1}{(K+2)!}$$

$$n! = n\,(n-1)!$$

$$(n+2)! = (n+2)(n+1)!$$

$n! = n.(n-1)!$

$(n+1)! = (n+1).n!$

$(n+2)! = (n+2)(n+1)!$

$$\Leftrightarrow -\frac{1.(k+2)}{\cancel{(k+1)!(k+2)}} + \frac{k+1}{\cancel{(k+2)(k+1)!}} = -\frac{1}{\cancel{(k+2)(k+1)!}}$$

$$\Leftrightarrow -(k+2) + k+1 = -1$$

$$\Leftrightarrow \cancel{-k} - 2 + \cancel{k} + 1 = -1$$

$$\Leftrightarrow -1 = -1 \Rightarrow VT = VP$$

$\Rightarrow P(k+1)$ đúng $\Rightarrow P(n)$ đúng $\forall n$.

$$1.1! + 2.2! + \ldots + n.n! = (n+1)! - 1$$

Bước cơ sở: $P(1)$: $1.1! = (1+1)! - 1$

$$\Leftrightarrow 1 = 1$$

$\Rightarrow VT = VP \Rightarrow P(1)$ đ

$\Rightarrow VT = VP \Rightarrow P(1)$ đúng

Giả sử $P(K)$ đúng:

$P(K): 1.1! + 2.2! + \ldots + K.K! = (K+1)! - 1$

Cần chứng minh $P(K+1)$ đúng:

$P(K+1): \boxed{1.1! + 2.2! + \ldots + K.K!} + (K+1)(K+1)! = (K+2)! - 1$

$\Leftrightarrow (K+1)! - \cancel{1} + (K+1)(K+1)! = (K+2)! - \cancel{1}$

$\Leftrightarrow (K+1)! + (K+1)(K+1)! = (K+2)(K+1)!$

$\Leftrightarrow 1 + K + 1 = K + 2$

$\Leftrightarrow K + 2 = K + 2$

$\Rightarrow VT = VP$

$\Rightarrow P(k+1)$ đúng $\Rightarrow P(n)$ đúng tn.

2/ $x+y+z+t=20$ (1)

$x, y, z, t \geqslant 0$ $\qquad K_a^b = C_{b+a-1}^b$

Đặt $a=x$, $b=y$, $c=z$, $d=t$

$\Rightarrow (1): \underline{a+b+c+d} = \underline{20}$

Số nghiệm nguyên dương $K_4^{20} = C_{23}^{20}$

b/ $\underline{x \geqslant 3}$, $\underline{y \geqslant 4}$, $\underline{z > 2}$, $\underline{t \geqslant 1}$.

Viết lại điều kiện:

$\Rightarrow x \geqslant 3$, $y \geqslant 4$, $z \geqslant 3$, $\cancel{t \geqslant 1}$

Đặt $a = x-3$ $\qquad \begin{cases} x = a+3 \end{cases}$

Đặt $a = x - 3$
$$\begin{cases} b = y - 4 \\ c = z - 3 \\ d = t - 1 \end{cases} \Rightarrow \begin{cases} x = a + 3 \\ y = b + 4 \\ z = c + 3 \\ d = t + 1 \end{cases}$$

$\Rightarrow (1): \underline{a + b + c + d} = 20 - 11 = \underline{9}.$

$\Rightarrow$ Nghiệm: $K_4^9 = C_{12}^9$

$\underline{III}: x \geq 3,\ y \geq 4,\ z \geq 2,\ \cancel{t < 1}$

$\Rightarrow \underline{b - c = a}$

TH1: $\geq$ (Giải như bt)

TH_2: $>$ (Đổi $> \Rightarrow \geq +1$

Ví dụ: $x > 2 \Rightarrow x \geq 3$)

TH_3: $<$

B_1: Đổi $< \rightarrow \geq 0$: Tìm b.

B_2: Đổi $< \rightarrow \geq$: Tìm c

B_3: $b - c \rightarrow$ đáp án

TH_4: $\leq$

B_1: Đổi $\geq 0 \rightarrow$ Tìm b

B_2: Đổi $\geq +1$ VD: $x \leq 1 \Rightarrow x \geq 2$: Tìm c

B_3: $b - c$ $\boxed{0 < x < 3}$ $a \leq y \leq b$

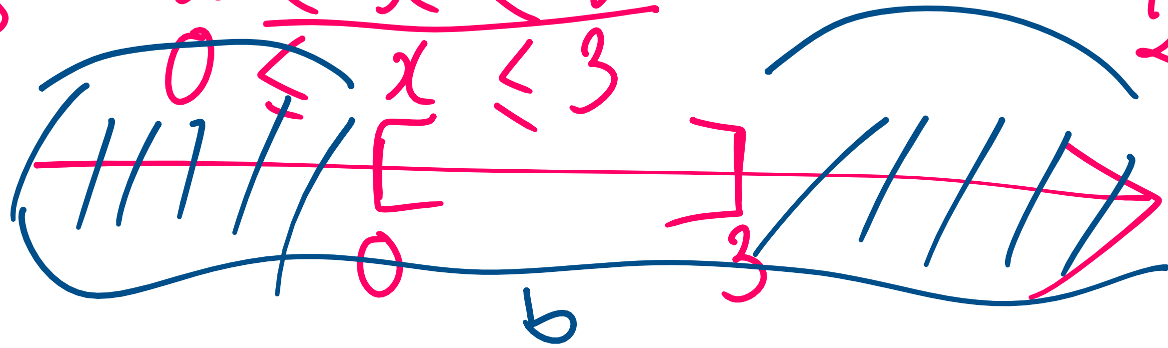
$\leq \quad \geq \quad \geq d$

TH_5: $c\ a \leq x \leq b$: $\quad$ " $\quad$ l/

Ht_5: $a \le x \le b$

$0 \le x \le 3$

2^n 16



c/ $\begin{cases} x_n - 5x_{n-1} + 6x_{n-2} = 2n+1 \quad (n \geq 2) \quad (2) \\ x_0 = 2, \; x_1 = 3 \end{cases}$

Giải hệ thức đệ quy tuyến tính ko thuần nhất (2) bằng cách giải phương trình đặc trưng (3): $\lambda^2 - 5\lambda + 6 = 0$

$\Leftrightarrow \left[\begin{array}{l} \lambda_1 = 3 \\ \lambda_2 = 2 \end{array}\right.$

$\Rightarrow x_n = C_1\lambda_1^n + C_2\lambda_2^n = \underline{C_1 . 3^n + C_2 . 2^n}$

Ta có: $f = 2n + 1$

Ta có: $f_n = 2n+1$

$x_n = \beta^n . Q_r(n) = 1^n (2n+1) = 2n+1$

Vì $\beta = 1$ không là nghiệm của ptđt

$\Rightarrow x_{f_n} = \beta^n . (an+b) = 1^n . (an+b) = an+b.$

Thế $x_{f_n} = an+b$ vào pt (2)

$\Rightarrow x_n - 5x_{n-1} + 6x_{n-2} = 2n+1$

$\Leftrightarrow (an+b) - 5[a(n-1)+b] + 6[a(n-2)+b] = 2n+1$

Cho $n=1$, $n=2$

$$\Rightarrow \begin{cases} a+b-5b+6b-6a = 3. \\ 2a+b-5a-5b+6b = 5. \end{cases}$$

$$\Leftrightarrow \begin{cases} -5a + 2b = 3 \\ -3a + 2b = 5 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 4 \end{cases}$$

$$\Rightarrow x_{fn} = n + 4$$

$$\Rightarrow x_n = x_n + x_{fn}$$

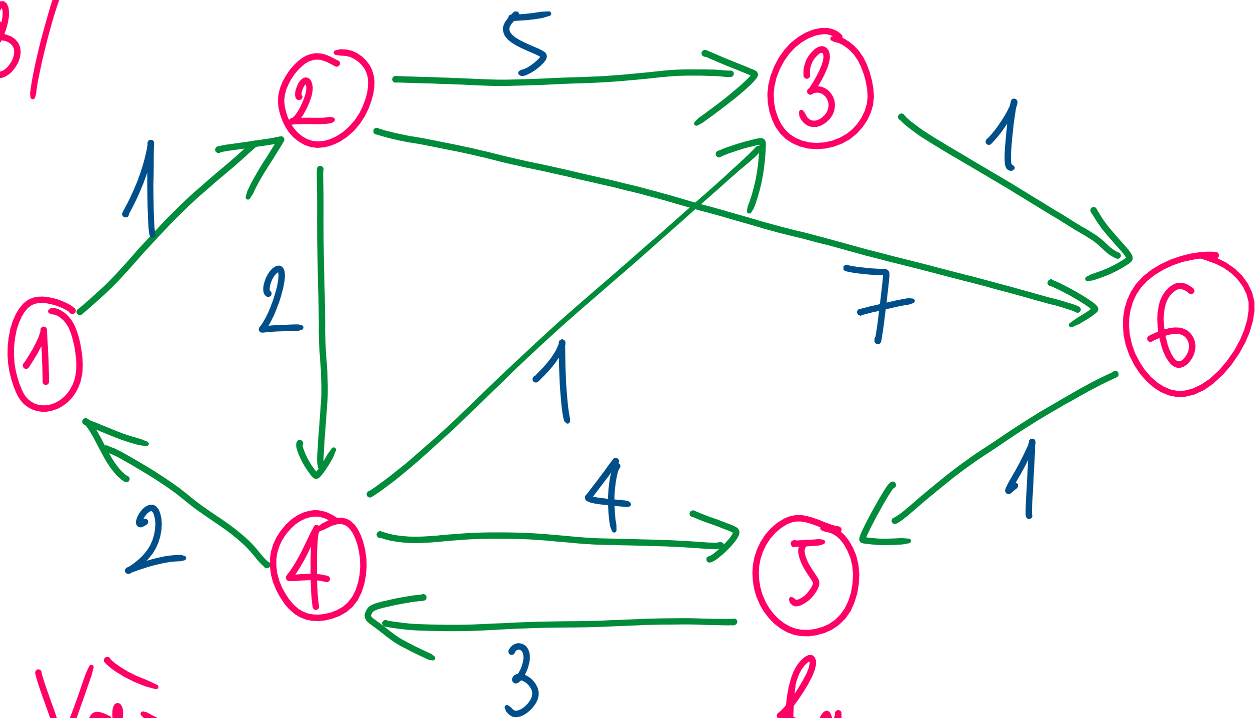
$$\Rightarrow \begin{cases} x_n = C_1 . 3^n + C_2 . 2^n + n + 4 \\ x_0 = 2 \\ x_1 = 3 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2 = C_1 + C_2 + 4 \\ 3 = 3C_1 + 2C_2 + 5 \end{cases} \Leftrightarrow \begin{cases} C_1 + C_2 = -2 \\ 3C_1 + 2C_2 = -2 \end{cases}$$

$$\Leftrightarrow \begin{cases} C_1 = 2 \\ C_2 = -4 \end{cases}$$

$$\Rightarrow x_n = 2 . 3^n - 4 . 2^n + n + 4.$$

3/



Vào	Ra
$deg_-(1) = 1$	$deg_+(1) = 1$
$deg_-(2) = 1$	$deg_+(2) = 3$
$deg_-(3) = 2$	$deg_+(3) = 1$
$deg_-(4) = 2$	$deg_+(4) = 2$
$deg_-(5) = 2$	$deg_+(5) = 1$
$deg_-(6) = 2$	$deg_+(6) = 1$

1	2	3	4	5	6

0	$(\infty;-)$	$(\infty;-)$	$(\infty;-)$	$(\infty;-)$	$(\infty;-)$
—	$(1;1)$	$(\infty;-)$	$(\infty;-)$	$(\infty;-)$	$(\infty;-)$
— $1 \to 2$	—	$(6;2)$	$(3;2)$	$(\infty;-)$	$(8;2)$
—	—	$(4;4)$	—	$(7;4)$	$(8;2)$
—	—	—	—	$(7;4)$	$(5;3)$
—	—	—	—	$(6;6)$	—

Xuất đường đi:

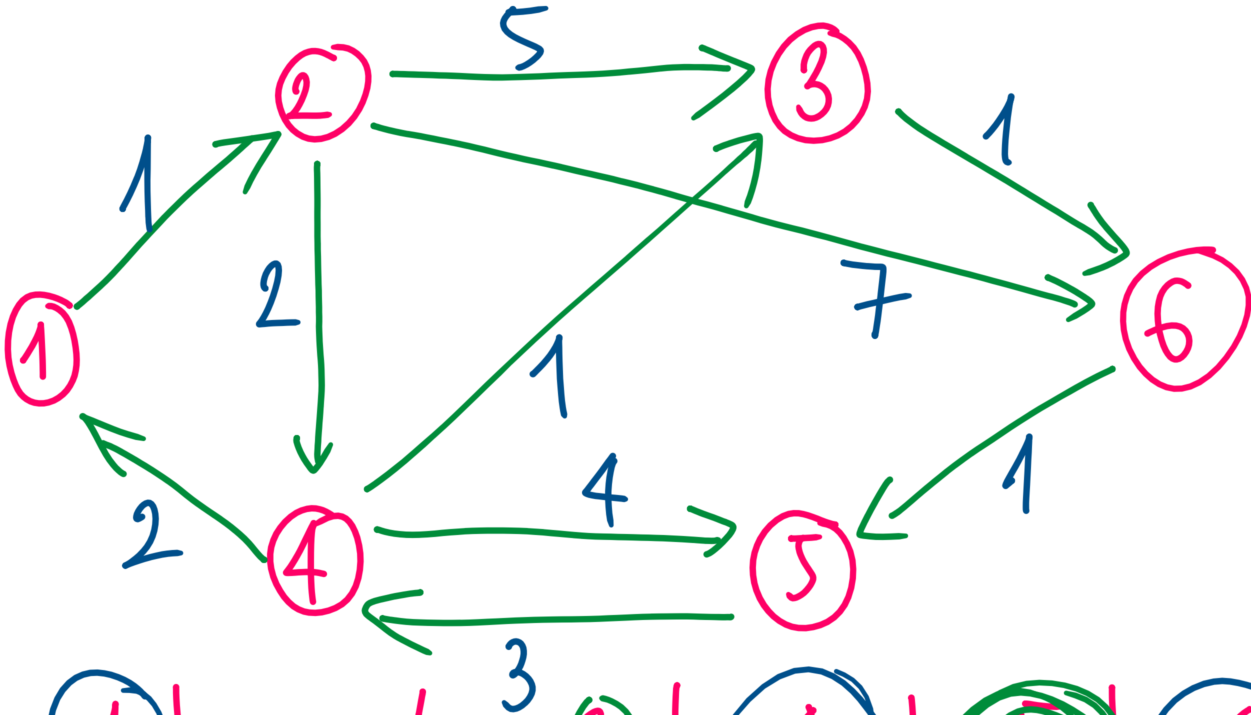
$1 \to 2 : 1$

$1 \to 2 \to 4 \to 3 : 4$

$1 \to 2 \to 4 : 3$

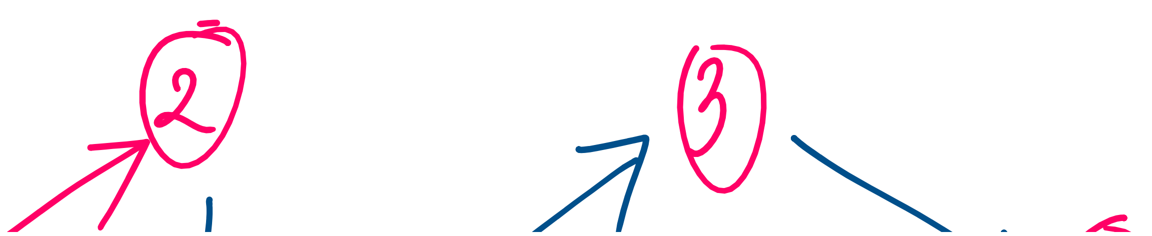
$1 \to 2 \to 4 \to 3 \to 6 \to 5 : 6$

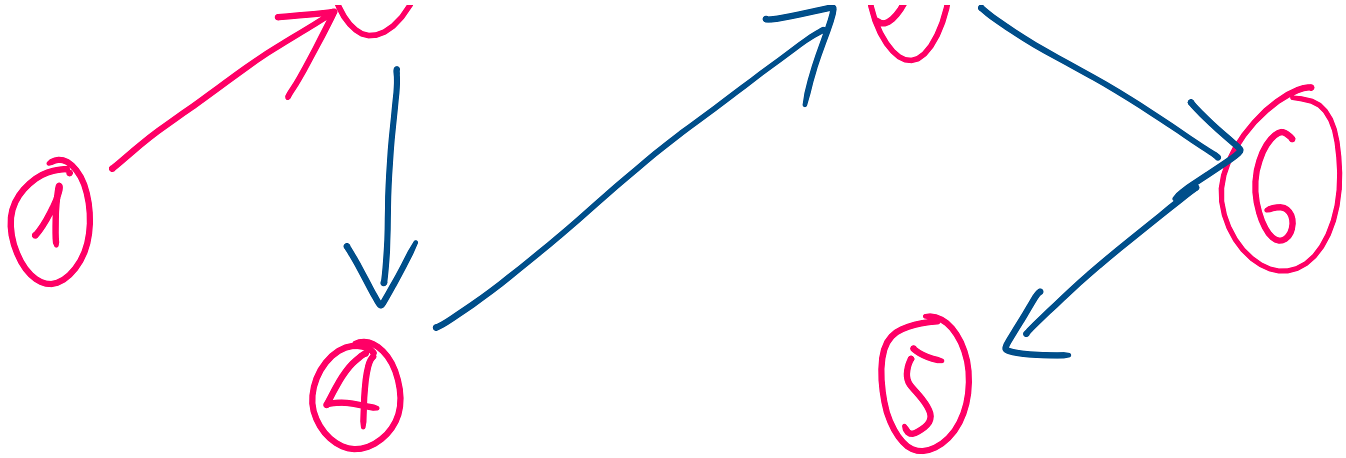
$1 \to 2 \to 4 \to 3 \to 6 : 5$



1	2	3	4	5	6
0	(∞;-)	(∞;-)	(∞;-)	(∞;-)	(∞;-)
—	(1;1)	(∞;-)	(∞;-)	(∞;-)	(∞;-)
—	—	(6;2)	(3;2)	(∞;-)	(8;2)
—	—	(4;4)	—	(7;4)	(8;2)
—	—	—	—	(7;4)	(5;3)
—	—	—	—	(6;6)	—

1 → 2 → 4 → 3





$1 \to 2 : 1$

$1 \to 2 \to 4 \to 3 : 4$

$1 \to 2 \to 4 : 3$

$1 \to 2 \to 4 \to 3 \to 6 \to 5 : 6$

$1 \to 2 \to 4 \to 3 \to 6 : 5$

$7{,}5 \cdot 30\% + 8.$ $\quad$ $8{,}4 \to B.$

$10 \cdot 30\% + \underline{7 \cdot 70\%} =$

$3 + \overline{4,9} = 7,9.$

$\underline{8,6} = A.$

$2,4 + 5,6$

$= 8,8$