

Total points: 100

HW 3: **Logic, CSP, Probability**

Due date: Nov 22 2023

Instructions: This homework assignment consists of a written portion only. Collaboration is not allowed on any part of this assignment. Solutions must be typed (hand written and scanned submissions will not be accepted) and saved as a .pdf file.

1. **(15 points)** Use the full joint probability distribution in the table below to calculate the following.

Toothache	Cavity	Catch	P(Toothache, Cavity, Catch)
false	false	false	0.576
false	false	true	0.144
false	true	false	0.008
false	true	true	0.072
true	false	false	0.064
true	false	true	0.016
true	true	false	0.012
true	true	true	0.108

- (i) $P(\text{Toothache} = \text{true})$
 - (ii) $P(\text{Cavity} = \text{true})$
 - (iii) $P(\text{Cavity} = \text{false})$
 - (iv) $P(\text{Toothache} = \text{false} | \text{Cavity} = \text{false})$
 - (v) $P(\text{Toothache} = \text{true} | \text{Cavity} = \text{false})$
 - (vi) $P(\text{Toothache} = \text{false})$
 - (vii) $P(\text{Toothache} = \text{true} | \text{Cavity} = \text{true})$
2. **(10 points)** After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (ie. the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?
3. **(15 points)** For each of the following statements, prove if it is true or false using truth tables or equivalence rules.
- (i) $(A \wedge B) \models (A \Leftrightarrow B)$ [3.5 points]

- (ii) $A \Leftrightarrow B \models A \vee B$ [3.5 points]
- (iii) $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$ [4points]
- (iv) $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable [4 points]
4. **(5 points)** Decide if the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules on Slide 31 in Lecture 9.
- $$((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$$
5. **(20 points)** (a) Convert the following to CNF:
- (i) S1: $A \Leftrightarrow (B \vee E)$
- (ii) S2: $E \Rightarrow D$
- (iii) S3: $(C \wedge F) \Rightarrow \neg B$
- (iv) S4: $E \Rightarrow B$
- (v) S5: $B \Rightarrow F$
- (vi) S6: $B \Rightarrow C$
- (b) Using the sentences above as the knowledge base KB, prove $\neg A \wedge \neg B$ using resolution.
6. **(15 points)** Convert the following sentences to first order logic sentences. We use the following predicates:
- $E(x)$ means “ x is an easy course”
- $H(y)$ means “ y is a happy student”
- $T(y, x)$ means “student y takes the course x ”
- $F(x)$ means “ x is a course with a final”
- (i) “If a course is easy, some students are happy”
- (ii) “If a course has a final, no students are happy”
- (iii) “if a course has a final, the course is not easy”
7. **(20 points)** An incoming freshman starting in the Fall is trying to plan the classes she will take in order to graduate after 4 years (8 semesters). There is a subset R of required courses out of the complete set of courses C that must all be taken to graduate with a degree in her desired major. Additionally, for each course $c \in C$, there is a set of prerequisites $Prereq(c) \subset C$ and a set of semesters $Semesters(c) \subseteq S$ that it will be offered, where $S = \{1, \dots, 8\}$ is the complete set of 8 semesters. A maximum load of 4 courses can be taken each semester.
- (a) Specify the set of variables, the domain of each variable, and the set of constraints. Your constraints need not be limited to unary and binary constraints. You may use any precise and unambiguous mathematical notation.
- (b) The student managed to find a schedule of classes that will allow her to graduate in 8 semesters using the CSP formulation, but now she wants to find a schedule that will allow her to graduate in as few semesters as possible. With this additional objective, formulate this problem as an uninformed search problem. Specify the start state, goal test, successor function and cost function.