CS 331 **Introduction to Artificial Intelligence** Fall 2023

OREGON STATE UNIVERSITY

School of Electrical Engineering and Computer Science

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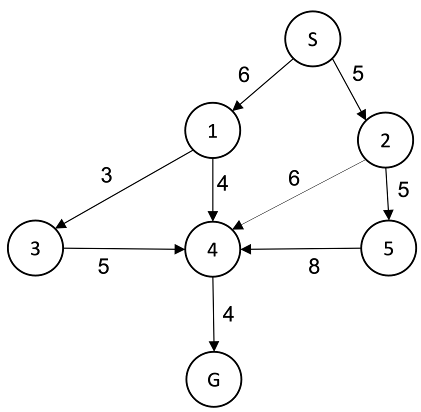
Total points: 100 HW 2: **Search and Logic** Due date: Oct 30 2023

**Instructions**: This homework assignment consists of a written portion and a programming portion. Collaboration is not allowed on any part of this assignment. Solutions must be typed (hand written and scanned submissions will not be accepted) and saved as a .pdf file. You will submit a single .zip file that contains the the code base and solutions as a .pdf file.

1. (**25 points**) For the graph in Figure 1, we want to find the shortest path between nodes $S$ and $G$. Implement A* for the graph in Figure 1, using the heuristic values provided. Write the following: (i) the shortest path; (ii) solution cost of the shortest path; (iii) the number of nodes expanded; and (iv) time taken to solve. Include a screenshot of your code displaying the answers to (i) - (iv). You are provided with a skeleton code in Python. You will need to fill in the missing lines in the code. You can also choose to implement A* in a different programming language of your choice.




| Node | h value |
|---|---|
| S | 7 |
| 1 | 6 |
| 2 | 5 |
| 3 | 4 |
| 4 | 2 |
| 5 | 3 |

Figure 1: Graph for informed search

2. (**15 points**) For the graph in Figure 1, (i) provide an admissible heuristic that is NOT consistent. (ii) Explain or prove how it satisfies admissibility and how it violates consistency. (iii) Solve the graph (you can solve it manually; no need to implement) using your heuristic and provide the solution path and cost. Does it find an optimal solution?

3. (**15 points**) Consider a graph of a country with each node denoting a city and the edge weights between the nodes denoting the distance between them. Let two friends $A$ and $B$ live in different cities in the map. On every turn, we can simultaneously move each friend to a neighboring city on the map. The amount of time needed to move from city $i$ to city $j$ is equal to the road distance $d(i, j)$ between the cities, but on each turn the

friend that arrives first must wait until the other one arrives (and calls the friend on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible.

(i) Let $D(i, j)$ denote the straight line distance between cities $i$ and $j$. Which of the following heuristic function is admissible? (a) $D(i, j)$ (b)$2 \cdot D(i, j)$ (c) $D(i, j)/2$. Briefly explain your answer.

(ii) Does the solution cost vary depending on the heuristic for this problem? Briefly explain your answer.

(iii) Are there completely connected maps for which no solution exists? Briefly explain your answer.

4. (**25 points**) Consider the graph in Figure 2 with blue nodes denoting MAX nodes and red nodes denoting MIN nodes.

   (i) Solve the graph using minimax algorithm. Clearly show your calculations at each step. Identify what strategy A must choose and the corresponding payoff it will receive (value).

   (ii) Solve the graph using alpha-beta pruning. Clearly show your calculations at each step. What strategy should A choose and what is the corresponding payoff?

   (iii) Which nodes are pruned using alpha-beta pruning? Did it change A's strategy? Why or why not?

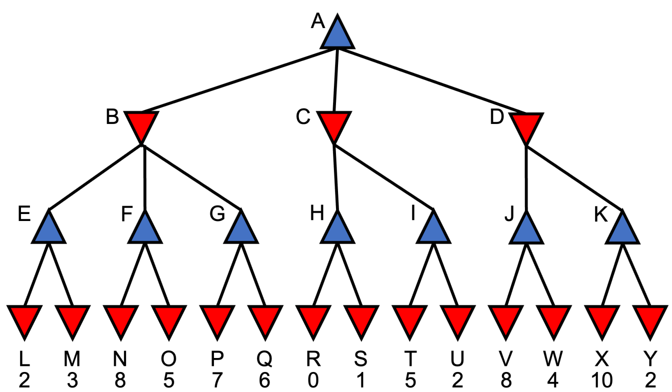



Figure 2:

5. (**15 points**) For each of the following statements, prove if it is true or false.

   (i) $(A \wedge B) \models (A \Leftrightarrow B)$ [3.5 points]

   (ii) $A \Leftrightarrow B \models A \vee B$ [3.5 points]

   (iii) $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$ [4points]

   (iv) $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable [4 points]

6. (**5 points**) Decide if the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules in Figure 11 (3rd edition) or Figure 7.11 (4th edition) in the textbook. For those of you who don't have access to the book, I am including a snapshot of the figure below.

   $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg \alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \beta \Rightarrow \neg \alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg \alpha \vee \neg \beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg \alpha \wedge \neg \beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

**Figure 7.11** Standard logical equivalences. The symbols $\alpha$, $\beta$, and $\gamma$ stand for arbitrary sentences of propositional logic.