

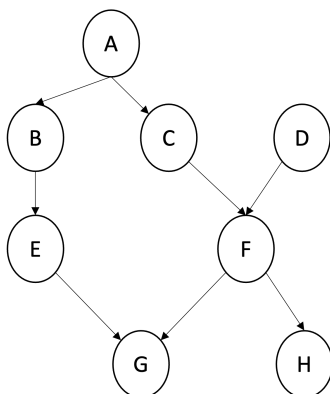
Total points: 100

HW 4: **Probability, Bayes Nets, Game Theory**

Due date: Dec 6 2023

**Instructions:** This homework assignment consists of a written portion only. Collaboration is not allowed on any part of this assignment. Solutions must be typed (hand written and scanned submissions will not be accepted) and saved as a .pdf file.

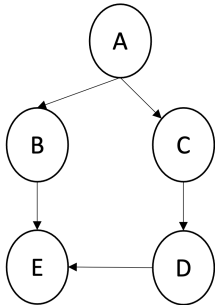
1. **(20 points)** Consider the Bayesian network below. Answer true or false for the following questions on d-separation. Show the blocked paths and explain your answer.



- $I(B, C | \{A, G\})$
- $I(C, D | H)$
- $I(G, H | F)$
- $I(A, H | F)$
- $I(E, D | C)$

2. (25 points) Calculate the following probabilities using the Bayesian network below. You may use various probability formulas such as marginalization, the chain rule, conditional independence, Bayes rule, etc.

- $P(A=\text{true}, B=\text{true}, C=\text{false}, D=\text{true}, E=\text{true})$
- $P(A=\text{true}, B=\text{true}, D=\text{false})$
- $P(D=\text{true} | A=\text{false})$
- Write  $P(D=\text{true})$  using variable elimination. Define factors as needed. Write your answers using probabilities and factors (no need to substitute numerical values).
- Write  $P(E=\text{true})$  using variable elimination. Define factors as needed. Write your answers using probabilities and factors (no need to substitute numerical values).



A	P(A)
false	0.4
true	0.6

A	B	P(B A)
false	false	0.9
false	true	0.1
true	false	0.25
true	true	0.75

C	D	P(D C)
false	false	0.75
false	true	0.25
true	false	0.9
true	true	0.1

A	C	P(C A)
false	false	0.25
false	true	0.75
true	false	0.8
true	true	0.2

B	D	E	P(E B,D)
false	false	false	0.1
false	false	true	0.9
false	true	false	0.2
false	true	true	0.8
true	false	false	0.3
true	false	true	0.7
true	true	false	0.4
true	true	true	0.6

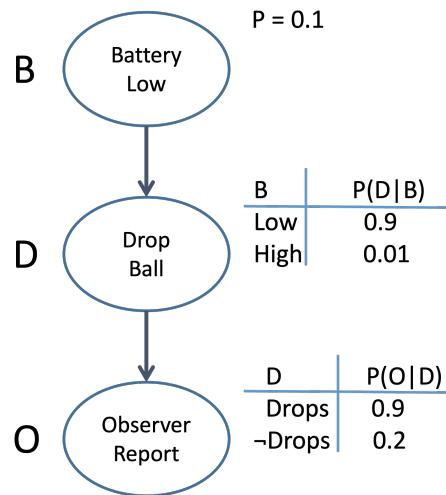
3. (25 points) Consider the game with two players A and B, with three strategies each, represented as a normal form game with the payoffs in the table.

	B:S1	B:S2	B:S3
A:S1	A = 1, B = -4	A = -3, B = 1	A = 7, B = 4
A:S2	A = 9, B = 10	A = 3, B = 8	A = 1, B = 5
A:S3	A = 0, B = 2	A = 5, B = 3	A = 2, B = 0

(a) List all pure strategy Nash equilibrium

(b) Does Player A have a strictly dominant strategy? If yes, state what it is and explain why. If no, explain why.

- (c) What is the Pareto optimal outcome in this game?
- (d) Let us assume that B plays a mixed strategy with  $p_1, p_2, p_3$  denoting the probabilities of selecting strategies  $S_1, S_2, S_3$  respectively. Player A always responds with strategy  $S_2$ . What is A's expected payoff? Write your answer in terms of  $p_1, p_2, p_3$ .
- (e) Let  $p$  denote the probability that player B plays  $S_1$  and  $q$  denote the probability that player A plays  $S_2$ . What is the expected payoff for player A in this case ( $S_2, S_1$ )?
4. **(20 points)** Orville, the robot juggler, drops balls quite often when its battery is low. A robot observer, with a somewhat unreliable vision system, reports that Orville dropped a ball. For the graph below, calculate the probability that the battery is low, given the observer's report.



5. **(10 points)** Prove the conditionalized version of general product rule:  $P(A, B|E) = P(A|B, E)P(B|E)$ .  
 Hint: The first step of this derivation is  $P(A, B|E) = \frac{P(A, B, E)}{P(E)}$ .