I provided the heuristic method of AB_custom AB_Custom2, AB_Custom_3,
The

    own = len(game.get_legal_moves(player))
    opp = len(game.get_legal_moves(game.get_opponent(player)))
The AB_Improved heuristic is in fact custom_2

The test result as below
Round 1

    custom_score: own - 1.5*opp
    custom_2:   own - opp
    custom-3:   1.5*own - opp

```
          ************************
                 Playing Matches
          ************************

Match #    Opponent    AB_Improved    AB_Custom    AB_Custom_2    AB_Custom_3
                       Won | Lost    Won | Lost    Won | Lost    Won | Lost
   1       Random      10  |  0      9  |  1       10  |  0      10  |  0
   2       MM_Open      7  |  3      6  |  4        4  |  6       6  |  4
   3      MM_Center     7  |  3      5  |  5        6  |  4       6  |  4
   4     MM_Improved    8  |  2      7  |  3        6  |  4       5  |  5
   5       AB_Open      5  |  5      4  |  6        6  |  4       5  |  5
   6      AB_Center     2  |  8      6  |  4        6  |  4       6  |  4
   7     AB_Improved    7  |  3      8  |  2        5  |  5       6  |  4
-------------------------------------------------------------------------
          Win Rate:     65.7%        64.3%         61.4%         62.9%
```

Round 2

    custom_score: own - 2*opp
    custom_2:   own - opp
    custom-3:   2*own - opp

```
          ************************
                 Playing Matches
          ************************

Match #    Opponent    AB_Improved    AB_Custom    AB_Custom_2    AB_Custom_3
                       Won | Lost    Won | Lost    Won | Lost    Won | Lost
   1       Random       8  |  2      10 |  0        9  |  1       9  |  1
   2       MM_Open      6  |  4      5  |  5        7  |  3       7  |  3
   3      MM_Center     5  |  5      7  |  3        9  |  1       9  |  1
   4     MM_Improved    7  |  3      6  |  4        5  |  5       5  |  5
   5       AB_Open      5  |  5      5  |  5        4  |  6       5  |  5
   6      AB_Center     6  |  4      6  |  4        7  |  3       6  |  4
   7     AB_Improved    5  |  5      5  |  5        4  |  6       5  |  5
-------------------------------------------------------------------------
          Win Rate:     60.0%        62.9%         64.3%         65.7%
```

Round 3
  custom_score: own - 2.5*opp
    custom_2:   own - opp
    custom-3:   2.5*own – opp

```
************************
       Playing Matches
************************
```

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	10	0	10	0	10	0	10	0
2	MM_Open	7	3	5	5	6	4	5	5
3	MM_Center	8	2	7	3	6	4	8	2
4	MM_Improved	5	5	7	3	7	3	7	3
5	AB_Open	5	5	6	4	3	7	6	4
6	AB_Center	5	5	6	4	5	5	6	4
7	AB_Improved	3	7	6	4	5	5	3	7
Win Rate:		61.4%		67.1%		60.0%		64.3%	

Round 4
    custom_score: own - 3*opp
    custom_2:   own - opp
    custom-3:   3*own – opp

```
************************
       Playing Matches
************************
```

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	9	1	10	0	10	0	8	2
2	MM_Open	5	5	7	3	7	3	8	2
3	MM_Center	8	2	7	3	9	1	8	2
4	MM_Improved	6	4	7	3	5	5	7	3
5	AB_Open	7	3	2	8	6	4	6	4
6	AB_Center	6	4	5	5	6	4	5	5
7	AB_Improved	5	5	4	6	6	4	5	5
Win Rate:		65.7%		60.0%		70.0%		67.1%	

Round 5
    custom_score: own**2 - opp**2

custom_2:   own - opp
custom-3:   -opp/own

```
************************
       Playing Matches
************************
```

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	9	1	9	1	9	1	8	2
2	MM_Open	7	3	6	4	6	4	8	2
3	MM_Center	7	3	8	2	7	3	7	3
4	MM_Improved	6	4	5	5	5	5	6	4
5	AB_Open	6	4	6	4	5	5	5	5
6	AB_Center	6	4	7	3	5	5	6	4
7	AB_Improved	5	5	7	3	5	5	3	7
	Win Rate:	65.7%		68.6%		60.0%		61.4%	

Round	Heuristic	AB_improved Win Rate	Win Rate	Gap
	own - 1.5*opp	65.7	64.3	-1.4
	own - opp	65.7	61.4	-4.3
1	1.5*opp-own	65.7	62.9	-2.8
	own - 2*opp	60	62.9	2.9
	own - opp	60	64.3	4.3
2	2*opp-own	60	65.7	5.7
	own - 2.5*opp	61.4	67.1	5.7
	own - opp	61.4	60	-1.4
3	2.5*own – opp	61.4	64.3	2.9
	own – 3*opp	65.7	60	-5.7
	own - opp	65.7	70	4.3
4	3*own – opp	65.7	67.1	1.4
	own**2 – opp**2	65.7	68.7	3
	own – opp	65.7	60	-5.7
5	-opp/own	65.7	61.4	-4.3
	own**2 – opp**2	58.6	64.3	5.7
	own – opp	58.6	64.3	5.7
6	2*own – opp	58.6	64.3	5.7

From round 1~5, I got the win rate gap against the AB_improved, the green shows the top 5
heuristics, Among them, I choose 3 to conduct test round #6.
You can see the data of Round # 6 and # Round 2 are stably outperform the AB_improved.

However, you should discuss a little *why* the results are the way they are. Why do you think your final combination of
heuristics performs best of all?
From Round 1 to 5, I was testing the value of t, which varies in (1.5, 2, 2,5, 3), and test it against the
AB_Improved. And the final results shows 2 are optimal to my machine and my code.

The reasoning is based on the test data. As for the performance and computational cost, they have
been covered by the lecture.

Round 6

```
        *************************
                Playing Matches
        *************************
```

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	9	1	9	1	10	0	9	1
2	MM_Open	6	4	7	3	6	4	7	3
3	MM_Center	7	3	7	3	8	2	7	3
4	MM_Improved	6	4	6	4	7	3	6	4
5	AB_Open	5	5	6	4	5	5	5	5
6	AB_Center	3	7	6	4	5	5	5	5
7	AB_Improved	5	5	4	6	4	6	6	4
	Win Rate:	58.6%		64.3%		64.3%		64.3%	