Discrete-Time Signals in the Time Domain

1

1.1 Introduction

Digital signal processing is concerned with the processing of a discrete-time signal, called the *input signal*, to develop another discrete-time signal, called the *output signal*, with more desirable properties. In certain applications, it may be necessary to extract some key properties of the original signal using specific digital signal processing algorithms. It is also possible to investigate the properties of a discrete-time system by observing the output signals for specific input signals. It is thus important to learn first how to generate in the time domain some basic discrete-time signals in MATLAB and perform elementary operations on them, which are the main objectives of this first exercise. A secondary objective is to learn the application of some basic MATLAB commands and how to apply them in simple digital signal processing problems.

1.2 Getting Started

The CD provided with this book contains all of the MATLAB programs and the partially written reports for both the PC and the Macintosh computers. In particular, it includes both PC and Macintosh versions of the MATLAB M-files of the first 10 exercises in folders grouped by chapters and report documents written in Microsoft Word in folders also grouped by chapters. After the completion of a project of a laboratory exercise, you record in the report of that exercise the answers to questions referring to this project at their designated locations.

Installation Instructions for a PC

To copy the program and the report folders onto the hard disk of a PC running Windows XP follow the steps given below:

1. Insert the CD.
2. Open the **My Computer** window by double-clicking on its icon displayed on the Desktop.
3. Open the window of the CD by double-clicking on its icon.
4. Open the window of the desired hard drive by double-clicking on its icon. Depending on your setup, it may be necessary to open another window by double-clicking

on **My Computer** icon before you can select the destination hard drive.

5. In the CD drive window, select the folder marked **PC** and drag it to the directory displayed in the hard drive window where you would like to copy the files.

Installation Instructions for a Macintosh computer

To copy the program and the report folders on the hard disk of a Macintosh computer running Mac OS X follow the steps given below:

1. Insert the CD.
2. Open the hard drive window by double-clicking on its icon displayed on the Desktop.
3. Open the window of the CD by double-clicking on its icon.
4. In the CD window, select the folder marked **MAC** and drag it to the directory displayed in the hard drive window where you would like to copy the files.

Downloading from the World Wide Web

The web site for downloading the files to a computer is **http://iplserv.ece.ucsb.edu**. The directories containing the files for the PC, Macintosh computer, and UNIX workstation are as follows:

pub/mitra/Labs/pc
pub/mitra/Labs/mac
pub/mitra/Labs/unix (M-files only)

To download the files from this site to your computer, follow the steps given below:

1. Open the available Internet web browser.
2. Type **http://iplserv.ece.ucsb.edu** in the URL window.
3. Double-click on the desired directory (the directory for the PC and Macintosh versions are shown above).
4. Double-click on the desired file for downloading. You will get a dialog box asking where you would like to save the file.

1.3 Background Review

R1.1 A discrete-time signal is represented as a sequence of numbers, called *samples* . A sample value of a typical discrete-time signal or sequence $\{x[n]\}$ is denoted as $x[n]$ with the argument n being an integer in the range $-\infty$ and ∞. For convenience, the sequence $\{x[n]\}$ is often denoted without the curly brackets.

R1.2 The discrete-time signal may be a finite length or an infinite length sequence. A finite length (also called *finite duration* or *finite extent*) sequence is defined only for a finite time interval:

$$N_1 \le n \le N_2, \tag{1.1}$$

where $-\infty < N_1$ and $N_2 < \infty$ with $N_2 \ge N_1$. The length or duration N of the finite length sequence is

$$N = N_2 - N_1 + 1. \tag{1.2}$$

R1.3 A sequence $\tilde{x}[n]$ satisfying

$$\tilde{x}[n] = \tilde{x}[n + kN] \qquad \text{for all} \quad n, \tag{1.3}$$

is called a *periodic sequence* with a period N where N is a positive integer and k is any integer.

R1.4 The *energy* of a sequence $x[n]$ is defined by

$$\mathcal{E} = \sum_{n=-\infty}^{\infty} |x[n]|^2. \tag{1.4}$$

The energy of a sequence over a finite interval $-K \le n \le K$ is defined by

$$\mathcal{E}_K = \sum_{n=-K}^{K} |x[n]|^2. \tag{1.5}$$

R1.5 The *average power* of an aperiodic sequence $x[n]$ is defined by

$$\mathcal{P}_{av} = \lim_{K\to\infty} \frac{1}{2K+1}\mathcal{E}_K = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |x[n]|^2. \tag{1.6}$$

The average power of a periodic sequence $\tilde{x}[n]$ with a period N is given by

$$\mathcal{P}_{av} = \frac{1}{N} \sum_{n=0}^{N-1} |\tilde{x}[n]|^2. \tag{1.7}$$

R1.6 The *unit sample sequence*, often called the *discrete-time impulse* or the *unit impulse*, denoted by $\delta[n]$, is defined by

$$\delta[n] = \begin{cases} 1, & \text{for } n = 0, \\ 0, & \text{for } n \neq 0. \end{cases} \tag{1.8}$$

The *unit step sequence* , denoted by $\mu[n]$, is defined by

$$\mu[n] = \begin{cases} 1, & \text{for } n \ge 0, \\ 0, & \text{for } n < 0. \end{cases} \tag{1.9}$$

R1.7 The *exponential sequence* is given by

$$x[n] = A\alpha^n, \tag{1.10}$$

where A and α are real or complex numbers. By expressing

$$\alpha = e^{(\sigma_o + j\omega_o)}, \quad \text{and} \quad A = |A|e^{j\phi},$$

we can rewrite Eq. (1.10) as

$$x[n] = |A|e^{\sigma_o n + j(\omega_o n + \phi)} = |A|e^{\sigma_o n}\cos(\omega_o n + \phi) + j|A|e^{\sigma_o n}\sin(\omega_o n + \phi). \tag{1.11}$$

R1.8 The *real sinusoidal sequence* with a constant amplitude is of the form

$$x[n] = A\cos(\omega_o n + \phi), \tag{1.12}$$

where A, ω_o, and ϕ are real numbers. The parameters A, ω_o, and ϕ in Eqs. (1.11) and (1.12) are called, respectively, the *amplitude* , the *angular frequency* , and the *initial phase* of the sinusoidal sequence $x[n]$. $f_o = \omega_o/2\pi$ is the *frequency*.

R1.9 The complex exponential sequence of Eq. (1.11) with $\sigma_o = 0$ and the sinusoidal sequence of Eq. (1.12) are periodic sequences if $\omega_o N$ is an integer multiple of 2π, that is,

$$\omega_o N = 2\pi r, \tag{1.13}$$

where N is a positive integer and r is any integer. The smallest possible N satisfying this condition is the *period* of the sequence.

R1.10 The *product* of two sequences $x[n]$ and $h[n]$ of length N yields a sequence $y[n]$, also of length N, as given by

$$y[n] = x[n] \cdot h[n]. \tag{1.14}$$

The *addition* of two sequences $x[n]$ and $h[n]$ of length N yields a sequence $y[n]$, also of length N, as given by

$$y[n] = x[n] + h[n]. \tag{1.15}$$

The *multiplication* of a sequence $x[n]$ of length N by a scalar A results in a sequence $y[n]$ of length N as given by

$$y[n] = A \cdot x[n]. \tag{1.16}$$

The *time-reversal* of a sequence $x[n]$ of infinite length results in a sequence $y[n]$ of infinite length as defined by

$$y[n] = x[-n]. \tag{1.17}$$

The *delay* of a sequence $x[n]$ of infinite length by a positive integer M results in a sequence $y[n]$ of infinite length given by

$$y[n] = x[n - M]. \tag{1.18}$$

If M is a negative integer, the operation indicated in Eq. (1.18) results in an *advance* of the sequence $x[n]$.

A sequence $x[n]$ of length N can be *appended* by another sequence $g[n]$ of length M resulting in a longer sequence $y[n]$ of length $N + M$ given by

$$\{y[n]\} = \{\{x[n]\}, \{g[n]\}\}. \tag{1.19}$$

1.4 MATLAB Commands Used

The MATLAB commands you will encounter in this exercise are as follows:

Operators and Special Characters

`:` `.` `+` `-` `*` `/` `;` `%`

Elementary Matrices and Matrix Manipulation

`i` `ones` `pi` `rand` `randn` `zeros`

Elementary Functions

`cos` `exp` `imag` `real`

Data Analysis

`sum`

Two-Dimensional Graphics

`axis` `grid` `legend` `plot` `stairs`
`stem` `title` `xlabel` `ylabel`

General Purpose Graphics Functions

`clf` `subplot`

Signal Processing Toolbox

`sawtooth` `square`

For additional information on these commands, see the *MATLAB Reference Guide* [Mat05] or type `help commandname` in the Command window. A brief explanation of the MATLAB functions used here can be found in Appendix B.

1.5 Generation of Sequences

The purpose of this section is to familiarize you with the basic commands in MATLAB for signal generation and for plotting the generated signal. MATLAB has been designed to operate on data stored as vectors or matrices. For our purposes, sequences will be stored as vectors. Therefore, all signals are limited to being causal and of finite length . The steps to follow to execute the programs listed in this book depend on the platform being used to run the MATLAB.