Discrete-Time Systems in the Time Domain 2

2.1 Introduction

A discrete-time system processes an input signal in the time-domain to generate an output signal with more desirable properties by applying an algorithm composed of simple operations on the input signal and its delayed versions. The aim of this second exercise is to illustrate the simulation of some simple discrete-time systems on the computer using MATLAB and investigate their time domain properties.

2.2 Background Review

R2.1 For a *linear* discrete-time system , if $y_1[n]$ and $y_2[n]$ are the responses to the input sequences $x_1[n]$ and $x_2[n]$, respectively, then for an input

$$x[n] = \alpha\, x_1[n] + \beta\, x_2[n], \tag{2.1}$$

the response is given by

$$y[n] = \alpha\, y_1[n] + \beta\, y_2[n]. \tag{2.2}$$

The superposition property of Eq. (2.2) must hold for any arbitrary constants α and β and for all possible inputs $x_1[n]$ and $x_2[n]$. If Eq. (2.2) does not hold for at least one set of nonzero values of α and β, or one set of nonzero input sequences $x_1[n]$ and $x_2[n]$, then the system is *nonlinear* .

R2.2 For a *time-invariant* discrete-time system , if $y_1[n]$ is the response to an input $x_1[n]$, then the response to an input

$$x[n] = x_1[n - n_o]$$

is simply

$$y[n] = y_1[n - n_o].$$

where n_o is any positive or negative integer. The above relation between the input and output must hold for any arbitrary input sequence and its corresponding output. If it does not hold for at least one input sequence and its corresponding output sequence, the system is *time-varying* .

R2.3 A *linear time-invariant* (LTI) discrete-time system satisfies both the linearity and the time-invariance properties.

R2.4 If $y_1[n]$ and $y_2[n]$ are the responses of a *causal* discrete-time system to the inputs $u_1[n]$ and $u_2[n]$, respectively, then

$$u_1[n] = u_2[n] \qquad \text{for } n < N$$

implies also that

$$y_1[n] = y_2[n] \qquad \text{for } n < N.$$

R2.5 A discrete-time system is said to be *bounded-input, bounded-output* (BIBO) *stable* if, for any bounded input sequence $x[n]$, the corresponding output $y[n]$ is also a bounded sequence , that is, if

$$|x[n]| < B_x \qquad \text{for all values of } n,$$

then the corresponding output $y[n]$ is also bounded, that is,

$$|y[n]| < B_y \qquad \text{for all values of } n,$$

where B_x and B_y are finite constants.

R2.6 The response of a discrete-time system to a unit sample sequence $\{\delta[n]\}$ is called the *unit sample response* or, simply, the *impulse response* , and denoted as $\{h[n]\}$. Correspondingly, the response of a discrete-time system to a unit step sequence $\{\mu[n]\}$, denoted as $\{s[n]\}$, is its *unit step response* or, simply the *step response*.

R2.7 The response $y[n]$ of a linear, time-invariant discrete-time system characterized by an impulse response $h[n]$ to an input signal $x[n]$ is given by

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k], \tag{2.3}$$

which can be alternately written as

$$y[n] = \sum_{k=-\infty}^{\infty} h[n-k]\, x[k], \tag{2.4}$$

by a simple change of variables. The sum in Eqs. (2.3) and (2.4) is called the *convolution sum* of the sequences $x[n]$ and $h[n]$, and is represented compactly as:

$$y[n] = h[n] \circledast x[n], \tag{2.5}$$

where the notation $\circledast$ denotes the *convolution sum*.

R2.8 The overall impulse response $h[n]$ of the LTI discrete-time system obtained by a cascade connection of two LTI discrete-time systems with impulse responses $h_1[n]$ and $h_2[n]$, respectively, and as shown in Figure 2.1, is given by

$$h[n] = h_1[n] \circledast h_2[n]. \tag{2.6}$$

If the two LTI systems in the cascade connection of Figure 2.1 are such that

$$h_1[n] \circledast h_2[n] = \delta[n], \tag{2.7}$$

then the LTI system $h_2[n]$ is said to be the *inverse* of the LTI system $h_1[n]$ and vice-versa .

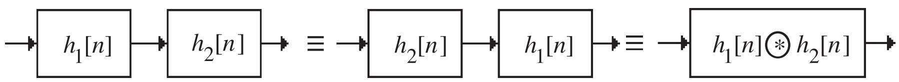

Figure 2.1 The cascade connection.

R2.9 An LTI discrete-time system is BIBO *stable* if and only if its impulse response sequence $\{h[n]\}$ is absolutely summable , that is,

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty. \tag{2.8}$$

R2.10 An LTI discrete-time system is *causal* if and only if its impulse response sequence $\{h[n]\}$ satisfies the condition

$$h[k] = 0 \quad \text{for} \quad k < 0. \tag{2.9}$$

R2.11 The class of LTI discrete-time systems with which we shall be mostly concerned in this book is characterized by a linear constant-coefficient difference equation of the form

$$\sum_{k=0}^{N} d_k\, y[n-k] \sum_{k=0}^{M} p_k\, x[n-k], \tag{2.10}$$

where $x[n]$ and $y[n]$ are, respectively, the input and the output of the system, and $\{d_k\}$ and $\{p_k\}$ are constants. The *order* of the discrete-time system is max(N, M), which is the order of the difference equation characterizing the system. If we assume the system to be causal, then we can rewrite Eq. (2.10) to express $y[n]$ explicitly as a function of $x[n]$:

$$y[n] = -\sum_{k=1}^{N} \frac{d_k}{d_0} y[n-k] + \sum_{k=0}^{M} \frac{p_k}{d_0} x[n-k], \tag{2.11}$$

provided $d_0 \neq 0$. The output $y[n]$ can be computed using Eq. (2.11) for all $n \geq n_o$ knowing $x[n]$ and the *initial conditions* $y[n_o - 1], y[n_o - 2], \ldots, y[n_o - N]$.

R2.12 A discrete-time system is called a *finite impulse response* (FIR) system if its impulse response $h[n]$ is of finite length. Otherwise, it is an *infinite impulse response* (IIR) system. The causal system of Eq. (2.11) represents an FIR system if $d_k = 0$ for $k > 0$. Otherwise, it is an IIR system.

2.3 MATLAB Commands Used

The MATLAB commands you will encounter in this exercise are as follows:

General Purpose Commands

`disp`

Operators and Special Characters

`:` `.` `+` `-` `*` `/` `;` `%` `<`

Language Constructs and Debugging

`break` `end` `for` `if` `input`

Elementary Matrices and Matrix Manipulation

`ones` `pi` `zeros`

Elementary Functions

`abs` `cos`

Polynomial and Interpolation Functions

`conv`

Two-Dimensional Graphics

`axis` `plot` `stem` `title` `xlabel` `ylabel`

General Purpose Graphics Functions

`clf` `subplot`

Character String Functions

`num2str`