Lecture 6

Plane Waves

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ΔE_x
ΔE_y
ΔE_z

$\Delta E_x \rightarrow \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_z}{\partial z^2}$

$x^2 + 4x + 3$

$\frac{\partial x}{\partial t} = \frac{\partial x}{\partial t} + C \rightarrow x = 2, 3, \ldots$

Review of the Previous Lecture

$E_x = E(x, y, z, t)$

$$\Delta \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\Delta \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$\vec{D} = ---$

Wave Equation

Skin Depth Equation

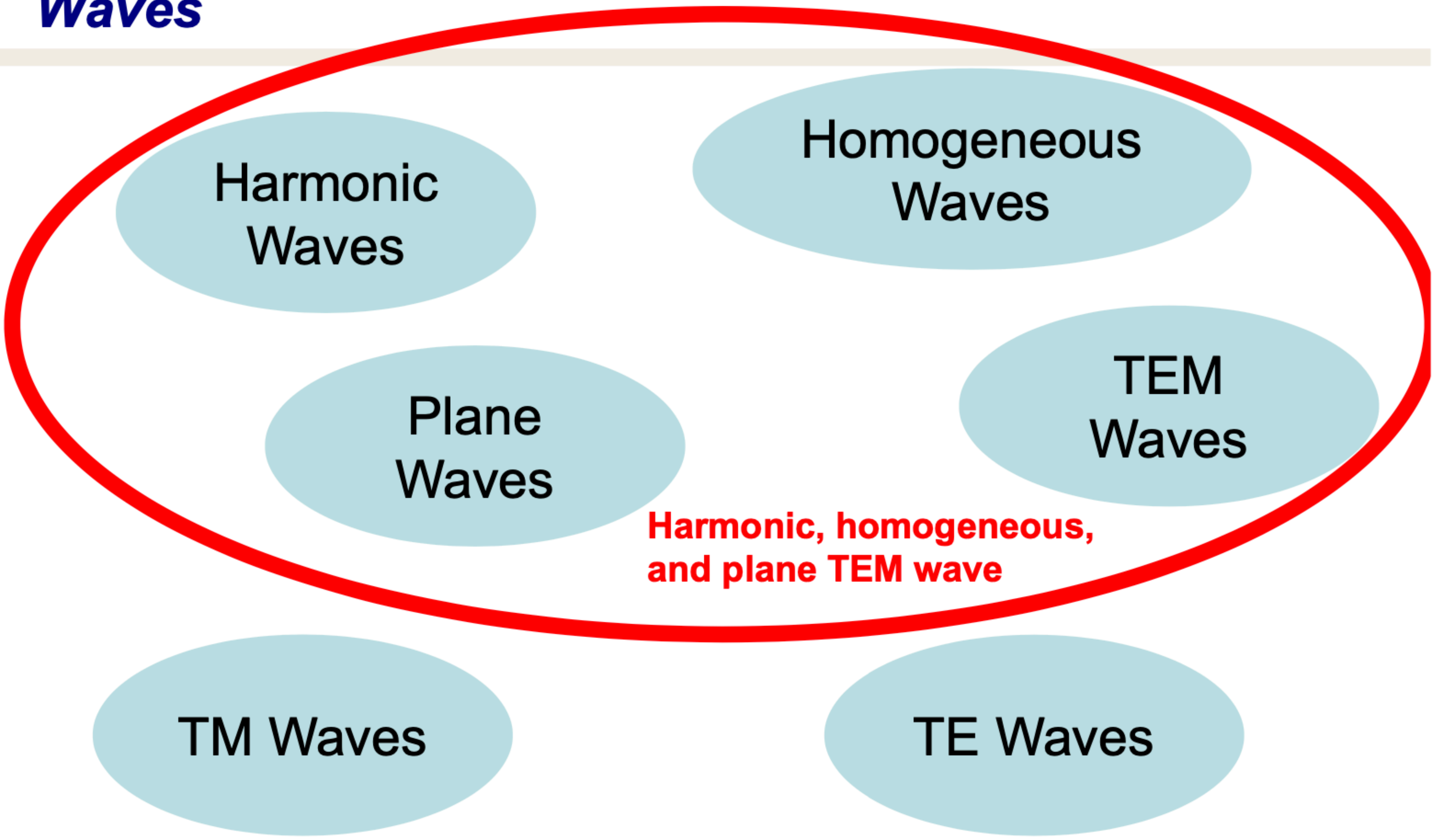
Good δ

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}}$$

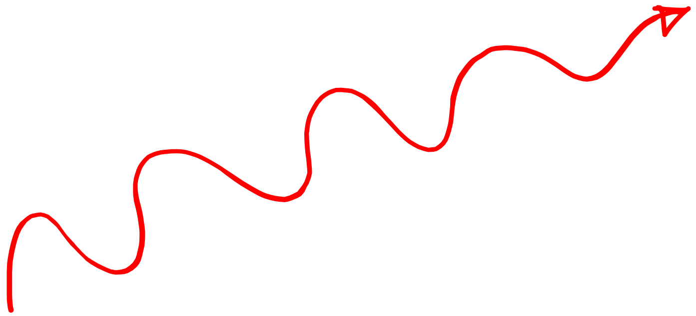
Bad δ

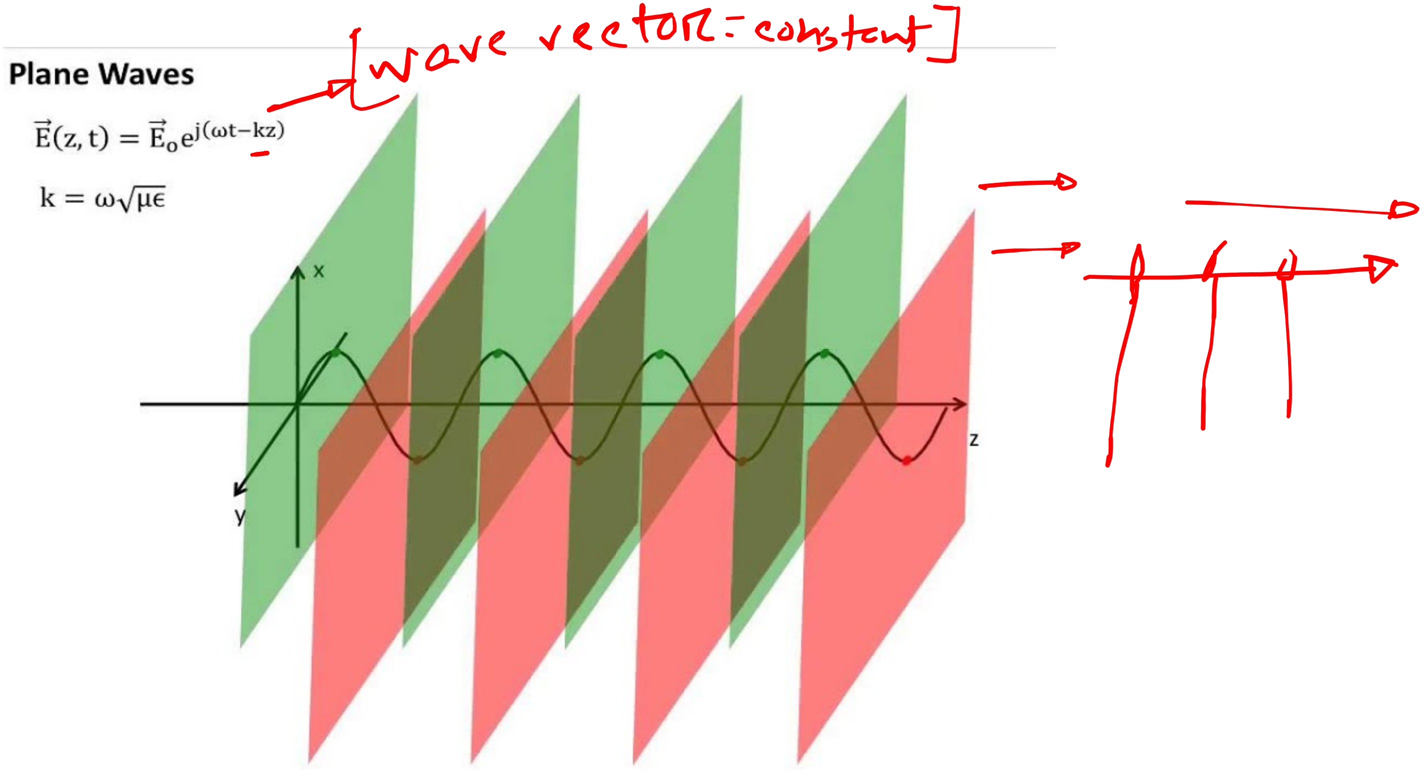
$$\delta_s = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Waves


Harmonic Waves
Homogeneous Waves
Plane Waves
TEM Waves
Harmonic, homogeneous, and plane TEM wave
TM Waves
TE Waves

Plane Waves

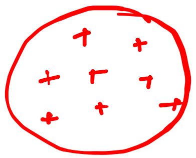




- A plane wave is a special case of wave or field: a physical quantity whose value, at any moment, is constant over any plane that is perpendicular to a fixed direction in space.
- The phase surfaces (i.e., surfaces on which the phase of the wave is constant in any point of such a surface) are planes if the direction of wave propagation is unchanged.
- The wave vector **k** is the same at any position and at any time.

[Review>> Wave Vector: A wave vector is a vector indicating the direction of wave propagation and the phase delay per unit length.]

Homogeneous Waves



- The vector E0(x,y,z) is unchanged on a phase surface; this means that the wave’s intensity is the same in any point of such a surface.

constant

$$\underline{\vec{E}}(x,y,z) = \boxed{\underline{\vec{E}}_0(x,y,z)}\, \mathrm{e}^{-j\underline{\vec{k}}\cdot\vec{r}}$$

Harmonic Waves

Wave Eq^n Sol^n

$\partial/\partial z$

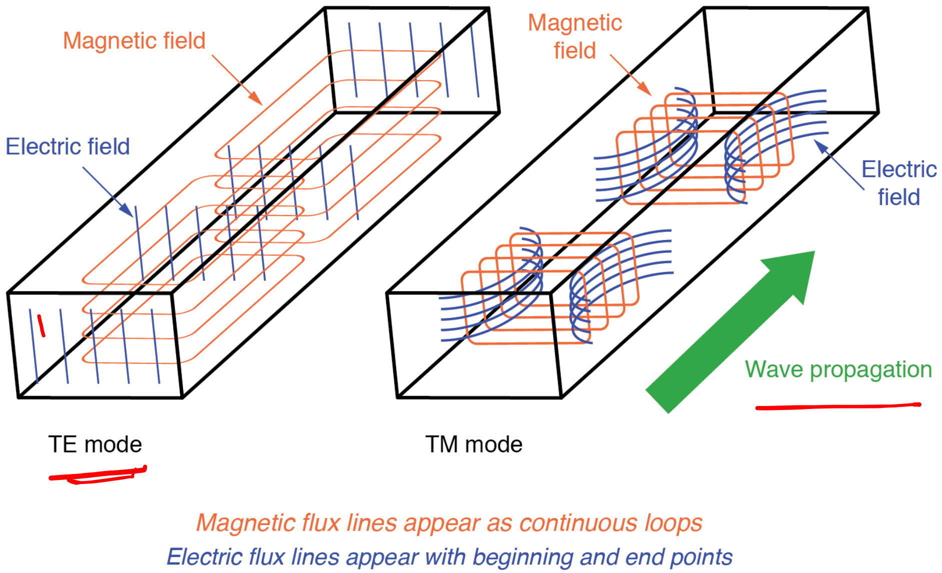
$\partial^2 E_x/\partial z^2 = 0$

Not harmonic

$$\boxed{E_x = f(\omega t - \vec{k} \cdot \vec{r}) + g(\omega t + \vec{k} \cdot \vec{r})}$$

- A wave is called a harmonic wave if the general d'Alembert solutions f and g are harmonic functions (varying with angular frequency ω).
- [Review: **A harmonic function** is a twice continuously differentiable function]

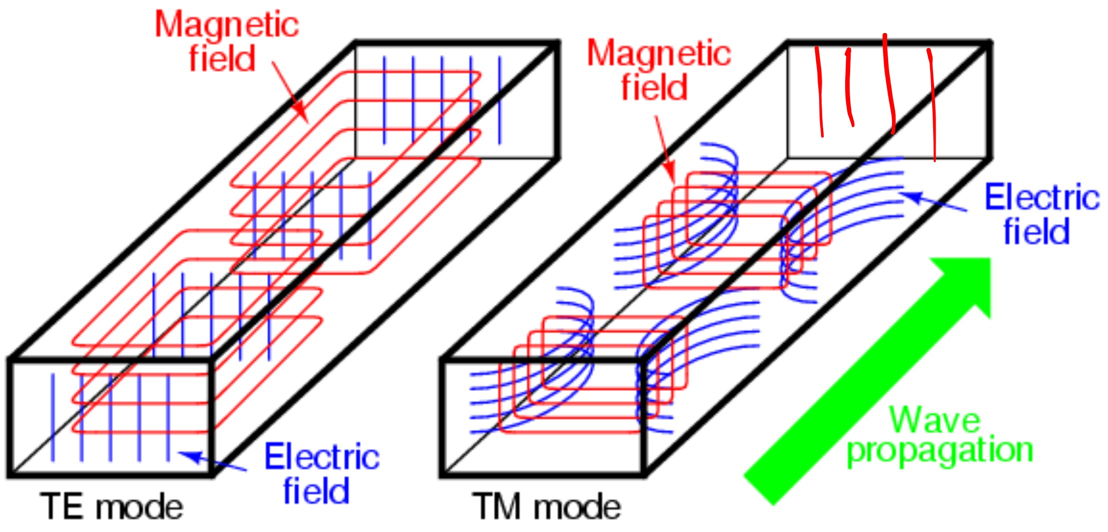
Transverse Electric (TE) Waves



Electric

- The electric field vector does not have a field component directed in the direction of propagation, **E0**(x,y,z)·**k** = 0.
- The magnetic field vector can have a field component directed in the direction of propagation.
- The electric field is parallel to the phase planes and perpendicular to the direction of propagation, **E0**(x,y,z) ⊥ **k**.

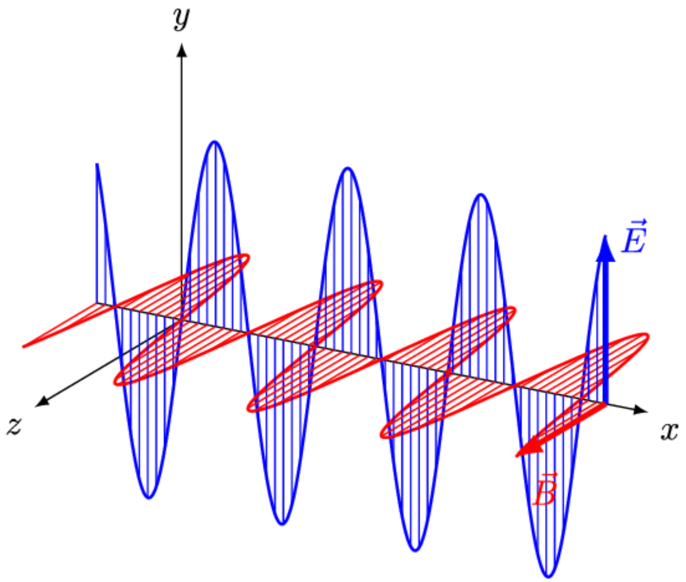
Transverse Magnetic (TM) Waves



Magnetic flux lines appear as continuous loops
Electric flux lines appear with beginning and end points

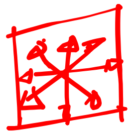
- The magnetic field vector does not have a field component directed in the direction of propagation, **H0**(x,y,z)·**k** = 0
- The electric field vector can have a field component directed in the direction of propagation.
- The magnetic field is parallel to the phase planes and perpendicular to the direction of propagation, **H0**(x,y,z) ⊥ k.

Transverse Electro-Magnetic (TEM) Waves

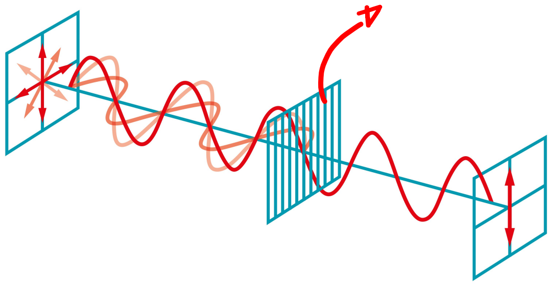


- A TEM wave has neither an electric nor a magnetic field component directed in the direction of propagation.
- Both the electric and the magnetic field vectors are parallel to the phase surface (i.e., perpendicular to the direction of propagation, **E0**(x, y, z) ⊥ **k**, **H0**(x, y, z) ⊥ **k**).

Linearly Polarized Waves



360°



- There exists a constant vector **e** such that $|\mathbf{E0}(x,y,z)\cdot\ \mathbf{e}| = |\mathbf{E0}(x,y,z)|$ holds for all times and all positions.
- The electric field vector does not change its orientation while propagating.
- For the indication of a linearly polarized wave, the direction of the electric field vector **E0** is used as the reference.
- **Example:** The waves of satellite TV are typically vertically or horizontally polarized TEM waves.

Plane Waves in Free Space

Assume a homogeneous and plane TE wave propagating harmonically in the z- direction. The wave should be linearly polarized in the y-direction. The medium is an insulator and free of charges.

$\vec{P} = \vec{E} \times \vec{H}$

$$\vec{E}(z,t) = \Re\left\{\underline{\vec{E}}_0\, \mathrm{e}^{\,j(\omega t - k_z z)}\right\}$$

$$\underline{E}_x = 0$$

$$\underline{E}_y = E_0\, \mathrm{e}^{\,j(\omega t - k_z z)}$$ $\boxed{2}$ [because linearly polarized]

$$\underline{E}_z = 0$$

where E_0 is the magnitude of the electric vector field, $E_0 = |\underline{\vec{E}}_0|$. Due to the homogeneity of the wave the partial derivatives in x and y vanish:

Intensity constant

$$\frac{\partial \underline{E}_y}{\partial x} = 0, \qquad \frac{\partial \underline{E}_y}{\partial y} = 0$$

Plane Waves in Free Space

$\vec{\nabla} \times \vec{E}$

$$\text{curl}\,\underline{\vec{E}} = -j\omega\mu\underline{\vec{H}}$$ [Maxwell's Second Equation]

$$\text{curl}\underline{\vec{E}} = \begin{pmatrix} \frac{\partial \underline{E}_z}{\partial y} - \frac{\partial \underline{E}_y}{\partial z} \\ \frac{\partial \underline{E}_x}{\partial z} - \frac{\partial \underline{E}_z}{\partial x} \\ \frac{\partial \underline{E}_y}{\partial x} - \frac{\partial \underline{E}_x}{\partial y} \end{pmatrix}$$ ✓

$$\underline{\vec{E}} = \begin{pmatrix} 0 \\ \underline{E}_y(z) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial \cancel{\underline{E}_z}}{\partial y} - \frac{\partial \underline{E}_y}{\partial z} \\ \frac{\partial \cancel{\underline{E}_x}}{\partial z} - \frac{\partial \cancel{\underline{E}_z}}{\partial x} \\ \cancel{\frac{\partial \underline{E}_y}{\partial x}} - \frac{\partial \cancel{\underline{E}_x}}{\partial y} \end{pmatrix} = -j\omega\mu \begin{pmatrix} \underline{H}_x \\ \cancel{\underline{H}_y} \\ \cancel{\underline{H}_z} \end{pmatrix}$$ ✓

$$\frac{\partial E_y}{\partial x} = 0, \quad \frac{\partial E_y}{\partial y} = 0$$

It must be a TEM Wave!

Plane Waves in Free Space

$$\frac{\partial \underline{E}_y}{\partial z} = -j\,k_z\,\underline{E}_y$$

$$j\,k_z\,\underline{E}_y = -j\omega\mu\underline{H}_x$$

$$\underline{H}_x = -\frac{k_z}{\omega\mu}\underline{E}_y = -\frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu}\,\underline{E}_y$$

$$= -\sqrt{\frac{\varepsilon}{\mu}}\,\underline{E}_y$$

$$= -\frac{1}{Z_F}\underline{E}_y$$

$Z_f = \sqrt{\mu/\varepsilon}$

characteristic wave impedance

$$\begin{pmatrix} \frac{\partial \cancel{\underline{E}_z}}{\partial y} - \frac{\partial \underline{E}_y}{\partial z} \\ \cancel{\frac{\partial \underline{E}_x}{\partial z}} - \cancel{\frac{\partial \underline{E}_z}{\partial x}} \\ \cancel{\frac{\partial \underline{E}_y}{\partial x}} - \cancel{\frac{\partial \underline{E}_x}{\partial y}} \end{pmatrix} = -j\omega\mu \begin{pmatrix} \underline{H}_x \\ \cancel{\underline{H}_y} \\ \cancel{\underline{H}_z} \end{pmatrix}$$

$$\underline{E}_y = E_0\,\mathrm{e}^{j(\omega t - k_z z)}$$

- The ratio of the *Electric (E)* and Magnetic (H) field components is seen to have units of impedance, known as the *characteristic wave impedance.*
- For planes waves the wave impedance is equal to the intrinsic impedance of the medium. In free-space the intrinsic impedance 377 Ohm.
- Note that the *Electric (E)* and *H* vectors are orthogonal to each other and orthogonal to the direction of propagation.

Plane Waves in Free Space – TEM Waves

$\vec{E} \perp \vec{H}$ **and** $\vec{E}$ **and** $\vec{H}$ **are in phase!** $\left|\vec{E}\right| = Z_F \left|\vec{H}\right|$

$$Z_F = \sqrt{\frac{\mu}{\varepsilon}} = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\boxed{Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega}$$

characteristic wave impedance in vaccum

$$\vec{H} = \frac{\vec{e}_z \times \vec{E}}{Z_F}$$

Orthogonal System

Plane Waves in Free Space – TEM Waves

The Poynting vector describes the flow of electromagnetic power and directed in the direction of propagation.

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & E_y & 0 \\ H_x & 0 & 0 \end{vmatrix}$$

$$H_x = -\frac{1}{Z_F} E_y$$

From previous equation

$$\begin{aligned}
\vec{S} &= \vec{E} \times \vec{H} \\
&= \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} \times \begin{pmatrix} H_x \\ 0 \\ 0 \end{pmatrix} \\
&= -E_y H_x \vec{e}_z \\
&= \frac{1}{Z_F} E_y^2 \; \vec{e}_z
\end{aligned}$$

$$\vec{S} = \vec{E} \times \vec{H} \qquad = \frac{1}{Z_F} E_y^2 \; \vec{e}_z$$

4.1 Problem 1

A plane wave propagates in a lossless medium. The electric vector field is given as:

$$\underline{\vec{E}} = \underline{E}_0 \mathrm{e}^{-jkz}\, \vec{e}_x$$

a) Relate k, ω, μ, and ε.

b) Give an expression for the wave impedance and the magnetic vector field $\underline{\vec{H}}$.

c) Using eq. (4.1.1), give an expression for the wavelength λ.

d) Express and discuss the phase velocity v_{ph} of the wave.

e) Determine the phase velocity v_{ph} of an electromagnetic wave in free space.

a) wave number $k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$

ε_0 : permittivity of free space

$= 8.8542 \cdot 10^{-12} \frac{As}{Vm}$

μ_0: permeability of free space

$= 4\pi\, 10^{-7} \frac{Vs}{Am}$

4.1 Problem 1

$e^{j\theta}$

A plane wave propagates in a lossless medium. The electric vector field is given as:

$\underline{\vec{E}}$ $\vec{E}$ $\qquad \underline{\vec{E}} = E_0 e^{-jkz}\, \vec{e}_x \qquad e^{j\omega t}$

a) Relate k, ω, μ, and ε.

b) Give an expression for the wave impedance and the magnetic vector field $\underline{\vec{H}}$.

c) Using eq. (4.1.1), give an expression for the wavelength λ.

d) Express and discuss the phase velocity v_{ph} of the wave. ① $\frac{E_0}{Z_F}$ ⑪ $\vec{e}_y$

e) Determine the phase velocity v_{ph} of an electromagnetic wave in free space.

$\vec{E}(x,t) = \underline{\vec{E}}\, e^{j\omega t} \qquad \vec{E} = E_0 \cos(\omega t - kz)\, \hat{e}_x$

- The ratio of the *Electric (E)* and Magnetic (H) field components is seen to have units of impedance, known as the *characteristic wave impedance.*

$\frac{2\pi}{T}$

k = wave number

($\vec{k}$ = wave vector) $\rightarrow$ wave propagation

b) $Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$

magnetic vector field $\vec{H}$: $\operatorname{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's Law)

$= -j\omega\mu \vec{H}$

$\Rightarrow \vec{H} = -\frac{1}{j\omega\mu} \operatorname{curl} \vec{E}$

here: $\vec{E} = E_x \vec{e}_x \qquad \operatorname{curl} \vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ \frac{\partial E_x}{\partial z} \\ -\frac{\partial E_x}{\partial y} \end{pmatrix}$

(x,t)

here: $\vec{E} = E_0 e^{j(\omega t - kz)} \vec{e}_x$

$\frac{\partial E_x}{\partial y} = 0 \; ; \; \frac{\partial E_x}{\partial z} = -jk E_0 e^{j(\omega t - kz)}$

$\Rightarrow \vec{H}(x,t) = \begin{pmatrix} 0 \\ -jkE_0 e^{j(\omega t - kz)} \\ 0 \end{pmatrix} \cdot \left(-\frac{1}{j\omega\mu}\right) = \begin{pmatrix} 0 \\ \frac{k}{\omega\mu} E_0 e^{j(\omega t - kz)} \\ 0 \end{pmatrix}$

$\frac{|\vec{E}|}{|\vec{H}|} = \frac{E_0}{\frac{k}{\omega\mu} E_0} = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = Z$

$\underline{\vec{H}} = \begin{pmatrix} 0 \\ \frac{k}{\omega\mu} E_0 e^{-jkz} \\ 0 \end{pmatrix}$

4.1 Problem 1

A plane wave propagates in a lossless medium. The electric vector field is given as:

$$\underline{\vec{E}} = E_0 e^{-jkz}\,\vec{e}_x$$

$\cos(\omega t - kz)$

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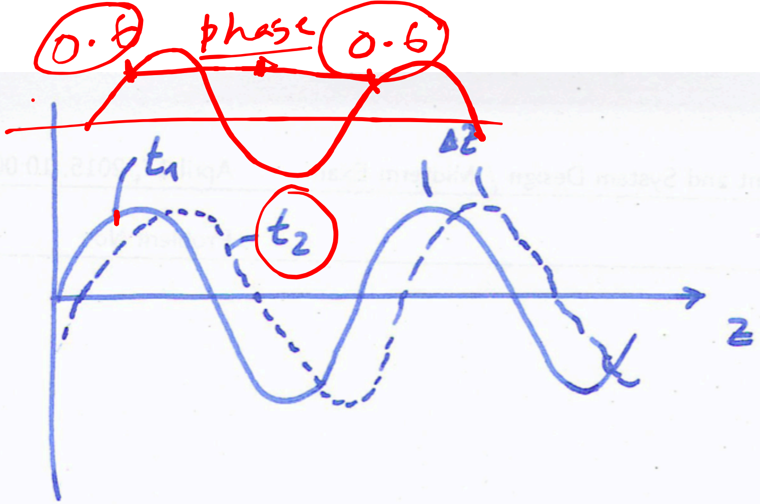
d)



t_1, t_2 same phase

$t_2 = t_1 + \Delta t$

Phase at t_1 : $\omega t_1 - k z_0$

Phase at t_2 : $\omega t_2 - k(z_0 + \Delta z) = \omega(t_1 + \Delta t) - k(z_0 + \Delta z)$

$\omega(\cancel{t_1} + \Delta t) - k(\cancel{z_0} + \Delta z) = \cancel{\omega t_1} - \cancel{k z_0}$

$\omega \Delta t - k \Delta z = 0$

$$\frac{\Delta z}{\Delta t} = v_{ph} = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

c) $\cancel{\omega t} - k\cancel{z} = \cancel{\omega t} - k(\cancel{z} + \lambda) + 2\pi$

$0 = -k\lambda + 2\pi$

$k = \frac{2\pi}{\lambda}$; $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{f\sqrt{\mu\varepsilon}}$

wave vector

Phase সমান

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e) Determine the phase velocity v_{ph} of an electromagnetic wave in free space.

$$v_{ph} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}}$$

e) Free space : $\varepsilon = \varepsilon_0 = 8.8542 \cdot 10^{-12} \frac{As}{Vm}$

$\mu = \mu_0 = 4\pi 10^{-7} \frac{Vs}{Am}$

$v_{ph} = c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \frac{m}{s}$

$\approx 300\,000 \frac{km}{s}$

$\approx 300 \frac{km}{ms}$

$\approx 300 \frac{m}{\mu s}$

$\approx 30\, cm/ns$

$3 \times 10^8\ ms^{-1}$