1) $G_1(s) = \dfrac{s^2 - 2s + 4}{(s+1)(s+10)(s+30)}$

a) $N - M = 3 - 2 = 1$

poles : $-1, -10, -30$

zeros : $1 \pm j\sqrt{3}$

$\sigma = \dfrac{((-1)+(-10)+(-30)) - (1+1)}{1} = -43$

$\phi = \dfrac{2k+1}{1}\, 180^\circ \longrightarrow \{180\}$

b) $a(s) = s^2 - 2s + 4$

$b(s) = (s+1)(s+10)(s+30) = s^3 + 41s^2 + 340s + 300$

$a\dfrac{db}{ds} = b\dfrac{da}{ds}$

$\longrightarrow (s^2 - 2s + 4)(3s^2 + 82s + 340) = (s^3 + 41s^2 + 340s + 300)(2s - 2)$

$\longrightarrow s^4 - 4s^3 - 410s^2 - 272s + 1960 = 0$

$\longrightarrow \{-17.81, -2.6, 1.87, 22.54\}$
$\quad\;\; \times \quad \checkmark \quad \times \quad \times$

c) $1 + KG_1(s) = 0$

[illegible] : $s^3 + (41+k)s^2 + (340-2k)s + 300 + 4k$

s^3	1	$340-2k$
s^2	$41+k$	$300+4k$
s^1	c	0
s^0	$300+4k$	0

$41 + k > 0 \longrightarrow k > -41$

$c = 340 - 2k - \dfrac{300+4k}{41+k} > 0 \longrightarrow \begin{cases} k < -41 \quad \times \\ -40.67 < k < 167.67 \end{cases}$

$300 + 4k > 0 \longrightarrow k > -75$

$\Longrightarrow \underset{\text{(stable)}}{0 < k < 167.67} \longrightarrow \underset{\text{for being unstable}}{(k<0) \cup (167.67 < k)}$

d) $90 + \theta - \left(\tan^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1}\left(\frac{\sqrt{3}}{11}\right) + \tan^{-1}\left(\frac{\sqrt{3}}{31}\right)\right) = \pm 180(2k+1)$

$90 + \theta - (40.89 + 8.94 + 3.19) = 180 \Longrightarrow \theta = 143.02^\circ$

e) $K_0 = \dfrac{-b(s_0)}{a(s_0)} = \dfrac{324.416}{15.96} = 20.32$

$G_2(s) = \dfrac{s+4}{s(s+6)(s+8)(s^2+3s+4)}$

a) $N - M = 5 - 1 = 4$

poles : $0, -6, -8, -1.5 \pm \frac{\sqrt{7}}{2}j$

zeros : -4

$\sigma = \dfrac{((-6)+(-8)+(0)+(-1.5)+(-1.5)) - (-4)}{4} = -3.25$

$\phi = \dfrac{2k+1}{4}\, 180^\circ \longrightarrow \{45, 135, 225, 315\}$

b) $a(s) = s + 4$

$b(s) = s(s+6)(s+8)(s^2+3s+4) = s^5 + 17s^4 + 94s^3 + 200s^2 + 192s$

$a\dfrac{db}{ds} = b\dfrac{da}{ds}$

$\longrightarrow (s+4)(5s^4 + 68s^3 + 282s^2 + 400s + 192) = (s^5 + 17s^4 + 94s^3 + 200s^2 + 192s)$

$\longrightarrow 4s^5 + 71s^4 + 460s^3 + 1328s^2 + 1600s + 768 = 0$

$\longrightarrow \{-7.07, -4.35 \pm 1.44j, -0.98 \pm 0.56j\}$
$\qquad\qquad\quad \times \qquad\qquad \times$

c) $1 + KG_1(s) = 0$

[illegible] : $s^5 + 17s^4 + 94s^3 + 200s^2 + (192+k)s + 4k$

s^5	1	94	$192+K$
s^4	17	200	$4K$
s^3	82.23	$192+0.76k$	0
s^2	$160.3-0.15k$	$4K$	0
s^1	c	0	0
s^0	$4K$	0	0

$160.3 - 0.15k > 0 \Rightarrow K < 1068.66$

$C = 192 + 0.76k - \dfrac{82.23(4k)}{160.3 - 0.15k} > 0 \Rightarrow \begin{cases} -2192.37 < k < 123.14 \\ K > 1068.67 \quad \times \end{cases}$

$4k > 0 \Rightarrow k > 0$

$\Longrightarrow \underset{\text{(stable)}}{0 < k < 123.14} \longrightarrow \underset{\text{for being unstable}}{(k<0) \cup (123.14 < k)}$

d) $\tan^{-1}\left(\frac{\frac{\sqrt{7}}{2}}{3.5}\right) - \left(\tan^{-1}\left(\frac{\frac{\sqrt{7}}{2}}{-1.5}\right) + 90 + \tan^{-1}\left(\frac{\frac{\sqrt{7}}{2}}{4.5}\right) + \tan^{-1}\left(\frac{\frac{\sqrt{7}}{2}}{6.5}\right) + \theta\right) = \pm 180(2k+1)$

$20.7 - (138.6 + 90 + 16.38 + 11.5 + \theta) = -180 \Longrightarrow \theta = -55.78^\circ$

e) $K_0 = \dfrac{-b(s_0)}{a(s_0)} = \dfrac{-230.58}{-3.07} = 75.1$

2) $(s-z_1)(s-z_2)\dots(s-z_n) = s^n - \left(\sum_{i=1}^{n} z_i\right) s^{n-1} + \dots$

$$\lim_{s\to\infty}\left(1+KG(s)H(s)\right) = \lim_{s\to\infty} 1 + K\frac{b_0\left(s^m - \left(\sum_{i=1}^{m} z_i\right)s^{m-1} + \dots\right)}{a_0\left(s^n - \left(\sum_{i=1}^{n} p_i\right)s^{n-1} + \dots\right)} \simeq \lim_{s\to\infty} K \frac{b_0\left(s^m - \left(\sum_{i=1}^{m} z_i\right)s^{m-1}\right)}{a_0\left(s^n - \left(\sum_{i=1}^{n} p_i\right)s^{n-1}\right)} \simeq \lim_{s\to\infty} K\frac{b_0\left(1 - \left(\sum_{i=1}^{m} z_i\right)s^{-1}\right)}{a_0\left(s^{n-m} - \left(\sum_{i=1}^{n} p_i\right)s^{n-m-1}\right)}$$

$$\left(\lim_{x\to 0}(1-x) \simeq \frac{1}{1+x}\right)$$

$$\Rightarrow \lim_{s\to\infty}\left(1+KG(s)H(s)\right) = K\frac{b_0}{a_0}\frac{1}{\left(s^{n-m} - \left(\sum_{i=1}^{n} p_i\right)s^{n-m-1}\right)\left(1 + \left(\sum_{i=1}^{m} z_i\right)s^{-1}\right)} = K\frac{b_0}{a_0}\frac{1}{\left(s^{n-m} + \left(\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i\right)s^{n-m-1} - \left(\sum_{i=1}^{m} z_i\right)\left(\sum_{i=1}^{n} p_i\right)s^{n-m-2}\right)}$$

$$\lim_{s\to\infty}\left(1+KG(s)H(s)\right) \simeq K\frac{b_0}{a_0}\frac{1}{\left(\underbrace{s^{n-m}}_{s^q} + \underbrace{\left(\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i\right)}_{\alpha} s^{n-m-1}\right)} = K\frac{b_0}{a_0}\frac{1}{s^q - \alpha s^{q-1}}$$

$$\left(\lim_{s\to\infty}(s-\sigma)^q = s^q - q\sigma s^{q-1}\right)$$

$$\Rightarrow -q\sigma = \alpha \quad \Rightarrow \quad \sigma = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m}$$

3) $G(s) = \dfrac{s+2}{(s+1)^2}$

$N-M = 2-1 = 1$

zeros : -2

poles : $-1, -1$

$\sigma = \dfrac{((-1)+(-1))-(-2)}{1} = 0$

$\phi = \dfrac{2k+1}{1}180^\circ \longrightarrow \{180\}$

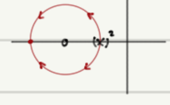
$a(s) = s+2 \qquad b(s) = s^2+2s+1$

$a\dfrac{db}{ds} = b\dfrac{da}{ds}$

$(s+2)(2s+2) = (s^2+2s+1) \longrightarrow s^2+4s+3=0 \longrightarrow s : \{-1, -3\}$

$0 - 2\theta = \pm(2k+1)180 \longrightarrow \theta = \pm 90$

$$K_0 = \left.\frac{\prod_{j=1}^{N}|s+p_j|}{\prod_{i=1}^{M}|s+z_i|}\right|_{s=s_0} \longrightarrow 4 = \frac{|s_0-1|^2}{|s_0-2|} \Rightarrow s_0 = 3 \qquad \Rightarrow$$



4) $G(s) = \dfrac{s+0.1}{s(s-0.2)(s^2+s+0.6)}$

a) $N - M = 4 - 1 = 3$

poles : $0,\ 0.2,\ -0.5 \pm 0.59\,j$

zeros : -0.1

$$\sigma = \frac{((0.2)+(-0.5)+(-0.5)+(0)) - (-0.1)}{3} = \frac{-0.7}{3} \simeq -0.23$$

$$\phi = \frac{2k+1}{3}\,180^\circ \longrightarrow \{60,\ 180,\ 300\} \qquad \{0,\ 120,\ 240\}$$

b) $a(s) = s + 0.1$

$b(s) = s(s-0.2)(s^2+s+0.6) = s^4 + 0.8s^3 + 0.4s^2 - 0.12s$

$$a\frac{db}{ds} = b\frac{da}{ds}$$

$$(s+0.1)(4s^3 + 2.4s^2 + 0.8s - 0.12) = (s^4 + 0.8s^3 + 0.4s^2 - 0.12s)$$

$$3s^4 + 2s^3 + 0.64s^2 + 0.08s - 0.012 = 0$$

$$s : \{\underset{\substack{\checkmark \\ (k>0)}}{-0.36},\ \underset{\substack{\checkmark \\ (k>0)}}{0.08},\ \underset{\times}{-0.19 \pm 0.31j}\}$$

c) معادله مشخصه : $s^4 + 0.8s^3 + 0.4s^2 + (K - 0.12)s + 0.1K$

s^4	1	0.4	$0.1K$
s^3	0.8	$K - 0.12$	0
s^2	$0.55 - 1.25k$	$0.1k$	0
s^1	c	0	0
s^0	$0.1k$	0	0

s^2 row $\longrightarrow$ $0.55 - 1.25k > 0 \Rightarrow k < 0.44$

s^1 row $\longrightarrow$ $c > 0 \begin{cases} k > 0.44 & \times \\ 0.15 < k < 0.34 \end{cases} \Longrightarrow$ Stable : $0.15 < k < 0.34$

$\Downarrow$

for being unstable : $(k < 0.15) \vee (0.34 < k)$

$$\left(c = k - 0.12 - \frac{0.08k}{0.55 - 1.25k}\right)$$

d) $\tan^{-1}\left(\frac{0.59}{-0.4}\right) - \left(\tan^{-1}\left(\frac{0.59}{-0.5}\right) + \tan^{-1}\left(\frac{0.59}{-0.7}\right) + 90 + \theta\right) = \pm(2k+1)\,180$

$124.13 - (130.27 + 139.87 + 90 + \theta) = -180$

$\theta = -56.01$

$\theta = 123.99$

e) $K_0 = \dfrac{-b(s_0)}{a(s_0)} \begin{cases} s_0 = -0.36 & : \ \dfrac{-0.0745}{-0.26} = 0.28 \quad \text{(stable)} \\ s_0 = 0.08 & : \ \dfrac{0.0065}{0.18} = 0.036 \quad \text{(unstable)} \end{cases}$

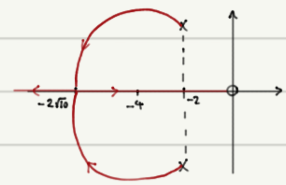
5) $G(s) = \frac{4s+40}{s}$, $H(s) = \frac{1}{s+p}$

$\frac{G(s)}{1+G(s)H(s)} \longrightarrow$ [illegible] : $s(s+p) + 4s + 40 = s^2 + 4s + 40 + ps = 0 \implies 1 + p\frac{s}{s^2+4s+40} = 0 \longrightarrow$ zeros : 0
poles : $-2 \pm 6j$

$N - M = 2 - 1 = 1$

$\sigma = \frac{((-2)+(-2)) - (0)}{1} = -4$

$\phi = \frac{2k+1}{1} 180^\circ \longrightarrow \{180\}$



$a(s) = s$

$b(s) = s^2 + 4s + 40$

$a\frac{db}{ds} = b\frac{da}{ds}$

$s(2s+4) = (s^2+4s+40) \longrightarrow s^2 = 40 \rightarrow s = \pm 2\sqrt{10}$

s^2	1	40
s^1	$4+p$	0
s^0	40	0

$4+p > 0 \implies p > -4 \implies$ always stable when $0 < p < \infty$

$\tan^{-1}\left(\frac{6}{2}\right) - (\theta + 90) = \pm(2k+1)\,180$

$71.56 - \theta - 90 = -180$

$\theta = 161.56$

6) $G(s) = \dfrac{k(b_1 s+1)(1-b_2 s)\,w_n^2}{(\tau_1 s+1)(\tau_2 s+1)(s^2+2\zeta w_n s+w_n^2)}$

$$\begin{bmatrix} \hat{k} = 1 \\ \hat{\tau}_1 = 0.5 \\ \hat{\tau}_2 = 5 \\ \hat{\zeta} = 0.2 \\ \hat{w}_n = 2 \\ \hat{b}_1 = 3 \\ \hat{b}_2 = 1 \end{bmatrix}$$

1) when we have input noise we will see our nominal output with the noise which is effected by transfer function. so plot D is appropriate for this.

2) when we have output noise we have our nominal output effected by noised so plot H is appropriate for this.

3) K acts like gain value for system. so plot C is appropriate for this.

5,4) changing the amplitude of poles effects on slower or faster response. higher values will effect less on response by changing them. so plot A and B are appropriate for τ_2 and τ_1.

6) less value for ζ system has higher overshoot and lower rise time. so plot E is appropriate for this.

7) w_n will effect of output frequeney. so plot I is appropriate for this.

8) if left zero is near to polars it will effect the response of system. so plot F is appropriate for this.

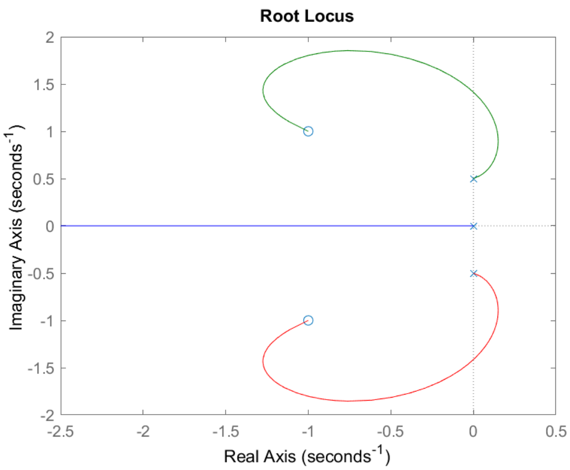
9) right zero will effect undershoot. so plot G is appropriate for this.

MATLAB Assignments

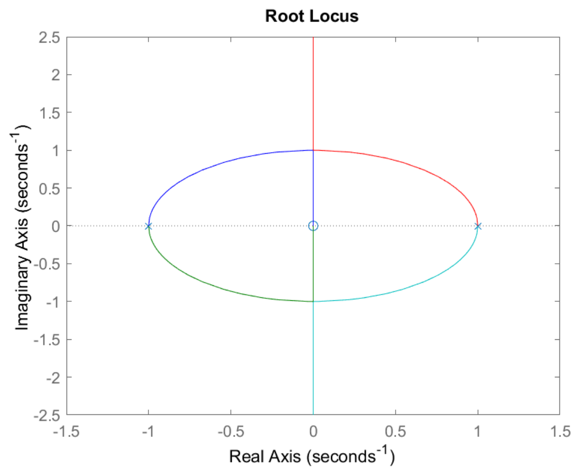
7 Breakaway Points

a)

$G_1(s)$



$G_2(s)$




```
baway =

 - 2.0291289456597803634180201034811 - 1.3357918919869738626969712269717i
 - 2.0291289456597803634180201034811 + 1.3357918919869738626969712269717i
0.029128945659780363418020103481059 - 0.28960800211921472630369875126637i
0.029128945659780363418020103481059 + 0.28960800211921472630369875126637i
```

```
baway =

 -1.0
    0
  1.0
-1.0i
 1.0i
```

```
function Baway= breakaway(Gs)

   TF = Gs;

   num = TF.Numerator;
   den = TF.Denominator;

   syms s
   aS = poly2sym(num{1,1},s);
   bS = poly2sym(den{1,1},s);

   Baway = vpasolve(aS*diff(bS)-bS*diff(aS));

end
```

b)

closed loop transfer function is like below:

$$1 + K\,G(s)\,H(s) = 0$$

Where K varies from zero to infinity. So if we consider $K' = -K$, it varies from zero to minus infinity.

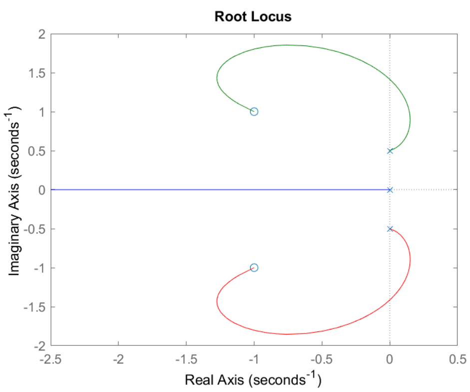
$$1 + K'\,G(s)\,H(s) = 0 \;\Rightarrow\; 1 + (-K)\,G(s)\,H(s) = 0 \;\Rightarrow\; 1 + K\,(-G(s))\,H(s) = 0$$

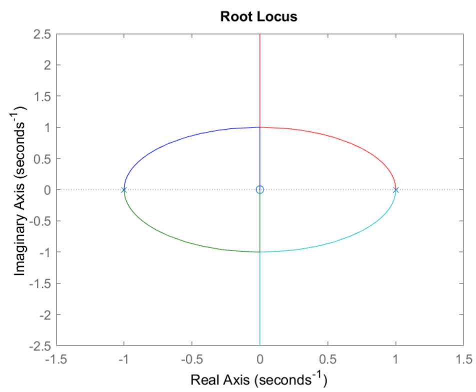
So for plotting complementary root-locus we can use rlocus function but instead of using G(s) as input we give –G(s) to this function so the output plot will be complementary root-locus.

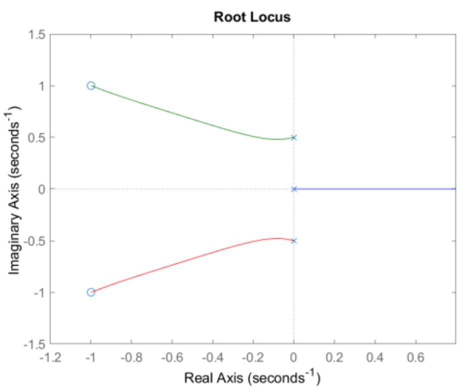
```
function [] = complementRlocus(Gs)
    rlocus(-Gs);
end
```

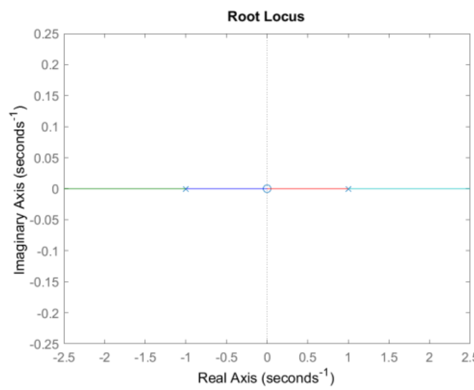
c)

G1(s)　　　　　　　　　　　　G2(s)










```
rootl =

[-1.0, 0, 1.0]

crootl =

     []

notbelong =

[-1.0i, 1.0i]
```

```
rootl =

     []

crootl =

     []

notbelong =

[- 2.0291289456597803634180201034811 - 1.3357918919869738626969712269717i, - 2.0291289456597803634]
```

We used this fact that in root locus each part of the real axis which has odd numbers of zeros and poles right side of it is part of our root locus. For complementary root locus there should be even numbers of zeros and poles right side of it.

```
function [Baway, rootl, crootl ,notbelong] =
breakaway(Gs)

    TF = Gs;

    num = TF.Numerator;
    den = TF.Denominator;

    syms s
    aS = poly2sym(num{1,1},s);
    bS = poly2sym(den{1,1},s);

    Baway = vpasolve(aS*diff(bS)-bS*diff(aS));


    realZP = [real( zero(Gs) ) ; real( pole(Gs) )];
    notbelong = [];
    rootl = [];
    crootl = [];
    for i = 1:size(Baway)
        k = 0;
        if imag(Baway(i)) ~= 0
            notbelong = [notbelong Baway(i)];
        else
            for j = 1:size(realZP)
                k = k+(Baway(i) <= realZP(j));
            end
        if mod(k,2)==0
            crootl = [crootl Baway(i)];
        else
            rootl = [rootl Baway(i)];
        end
        end
    end
end
```