1- $G_c(s) = \frac{K(s+z)}{s+p}$

$$G_{lead}(s) = \frac{K(1+\alpha\tau s)}{\alpha(1+\tau s)} = \frac{K(1+\alpha\tau wj)}{\alpha(1+\tau wj)} \longrightarrow \phi(w) = \tan^{-1}(\alpha\tau w) - \tan^{-1}(\tau w) = \tan^{-1}\left(\frac{\alpha w\tau - w\tau}{1+(w\tau)^2\alpha}\right)$$

$$\frac{d}{dw}\phi(w) = 0 \Rightarrow \frac{d}{dw}\left(\frac{\alpha-1}{1+w^2\tau^2\alpha} w\tau\right) = 0 \Rightarrow (\alpha-1)\tau(1+w^2\tau^2\alpha) - (2\tau^2\alpha w)(\alpha-1)\tau w = 0 \Rightarrow 1+w^2\tau^2\alpha - 2\tau^2\alpha w^2 = 0$$

$$\Rightarrow w = \sqrt{\frac{1}{\tau^2\alpha}} = \frac{1}{\tau\sqrt{\alpha}}$$

$$\phi_m(w) = \tan^{-1}\left(\frac{\sqrt{\alpha} - \frac{1}{\sqrt{\alpha}}}{2}\right) = \tan^{-1}\left(\frac{\alpha-1}{2\sqrt{\alpha}}\right) \Rightarrow \tan(\phi_m) = \frac{\alpha-1}{2\sqrt{\alpha}} \Rightarrow \sin(\phi_m) = \frac{\alpha-1}{\sqrt{(\alpha-1)^2+4\alpha}} = \frac{\alpha-1}{\alpha+1} \Rightarrow \alpha = \frac{1+\sin(\phi_m)}{1-\sin(\phi_m)}$$

$$G_{lag}(s) = \frac{K\alpha(1+\tau s)}{1+\alpha\tau s} = \frac{K\alpha(1+\tau wj)}{(1+\alpha\tau wj)} \longrightarrow \phi(w) = \tan^{-1}(\tau w) - \tan^{-1}(\alpha\tau w) = \tan^{-1}\left(\frac{w\tau - \alpha\tau w}{1+(w\tau)^2\alpha}\right)$$

$$\frac{d}{dw}\phi(w) = 0 \Rightarrow \frac{d}{dw}\left(\frac{1-\alpha}{1+w^2\tau^2\alpha} w\tau\right) = 0 \Rightarrow (1-\alpha)\tau(1+w^2\tau^2\alpha) - (2\tau^2\alpha w)(1-\alpha)\tau w = 0 \Rightarrow 1+w^2\tau^2\alpha - 2\tau^2\alpha w^2 = 0$$

$$\Rightarrow w = \sqrt{\frac{1}{\tau^2\alpha}} = \frac{1}{\tau\sqrt{\alpha}}$$

$$\phi_m(w) = \tan^{-1}\left(\frac{\frac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{2}\right) = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right) \Rightarrow \tan(\phi_m) = \frac{1-\alpha}{2\sqrt{\alpha}} \Rightarrow \sin(\phi_m) = \frac{1-\alpha}{\sqrt{(1-\alpha)^2+4\alpha}} = \frac{1-\alpha}{\alpha+1} \Rightarrow \alpha = \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)}$$

2- $G(s) = \frac{K}{s(s+10)(s+14)}$ $\qquad G_c(s) = \underbrace{K_1\alpha}_{K_1'} \frac{1+\tau s}{1+\alpha\tau s}$

$$e_{ss} = 10\% A = \frac{A}{K_V} \longrightarrow K_V = 10 \Rightarrow K_V = \lim_{s\to 0} sL(s) = \frac{K}{140}K_1' = 10 \Rightarrow K_1' = \frac{1400}{K}$$

$$L(s) = \frac{1400}{s(s+10)(s+14)} = \frac{10}{s(\frac{1}{10}s+1)(\frac{1}{14}s+1)}$$

$$\left.\begin{array}{l} |L(wj)| = 1400\left((140-w^2)^2w^2 + (24w^2)^2\right)^{-\frac{1}{2}} = 1 \longrightarrow w = 7.21 \\ \phi(w) = -90^\circ - \tan^{-1}(\frac{1}{10}w) - \tan^{-1}(\frac{1}{14}w) \end{array}\right\} \phi(w) = -153^\circ \Rightarrow PM = 27^\circ$$

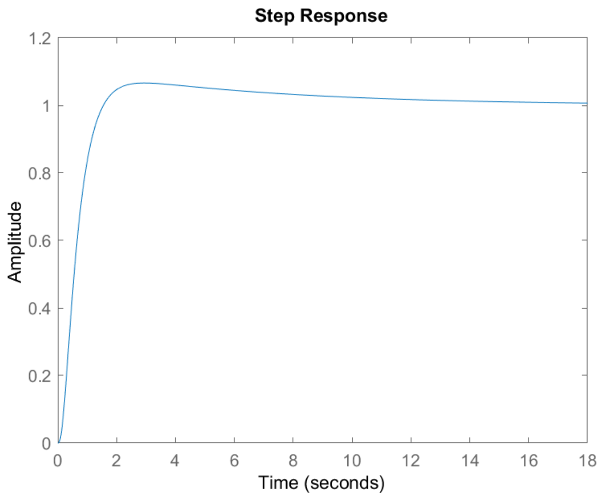
$$\zeta = 0.707 \longrightarrow PM = 70.7^\circ \rightsquigarrow 76^\circ \Rightarrow \phi(w) = -104^\circ \longrightarrow w = 1.43 \Rightarrow |L(wj)| = 6.88, \quad \alpha = 6.88$$

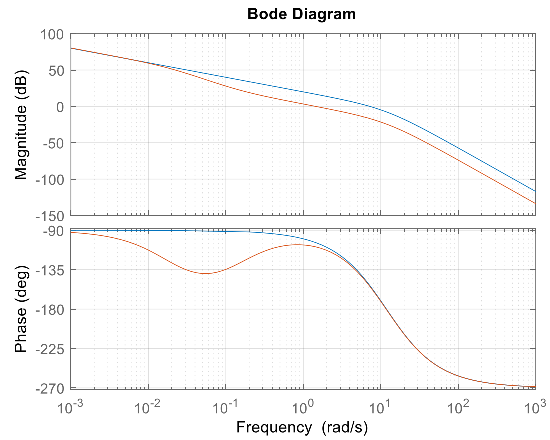
$$z = \frac{w_c'}{10} = 0.143, \quad p = \frac{z}{\alpha} = 0.02, \quad K_1 = \frac{K_1'}{\alpha} = \frac{203.5}{K}$$

$$\Rightarrow G_c = \frac{203.5}{K}\frac{s+0.143}{s+0.02}, \quad L(s) = \frac{203.5(s+0.143)}{s(s+10)(s+14)(s+0.02)}$$

2-

Plots for our system with controller :

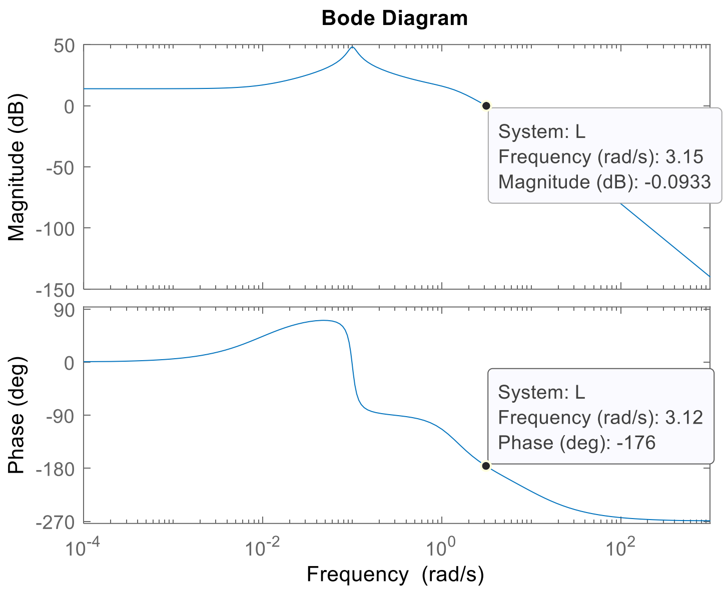




Actual properties of our system with controller:

RiseTime	0.9467
TransientTime	11.0703
SettlingTime	11.0703
SettlingMin	0.9014
SettlingMax	1.0662
Overshoot	6.6236
Undershoot	0
Peak	1.0662
PeakTime	2.9459

3-



3_ a) $P.O. < 10\% \longrightarrow \zeta > 0.59$

$T_s < 2 \longrightarrow \frac{4}{\zeta\omega_n} < 2 \Rightarrow \zeta\omega_n > 2$

$\Rightarrow \zeta = 0.6, \quad \omega_n = 3.5 \longrightarrow \quad 2.1 \pm 2.8j \Rightarrow z = -2.1$

$\cos^{-1}\zeta = \cos^{-1} 0.6 = 53^\circ$

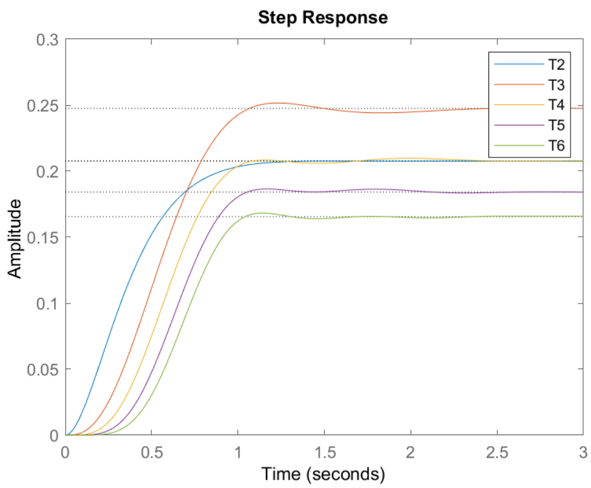
$$53^\circ + \underset{21.5^\circ}{\underbrace{\tan^{-1}\left(\frac{2.8}{7.1}\right)}} - \underset{30^\circ}{\underbrace{\tan^{-1}\left(\frac{1.8}{3.1}\right)}} - \underset{50.8^\circ}{\underbrace{\tan^{-1}\left(\frac{3.8}{3.1}\right)}} - \underset{13^\circ}{\underbrace{\tan^{-1}\left(\frac{2.8}{12.1}\right)}} - \theta_p = -180 \Rightarrow \theta_p = 160.7^\circ \Rightarrow p = -8 - 2.1 = -10.1$$

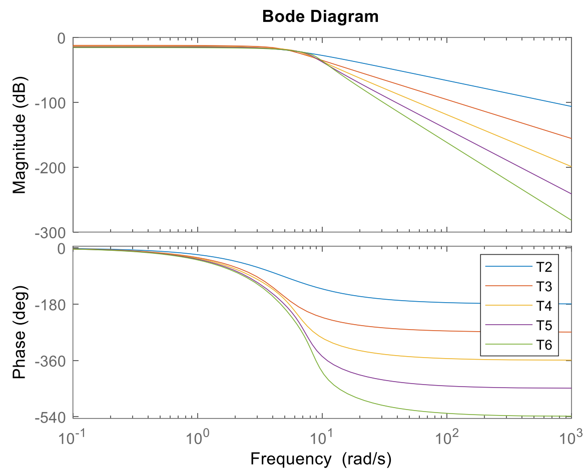
4- $T_s = \frac{\beta}{W_0} = 1 \Rightarrow W_0 = \beta$

$s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \qquad (a_0 = W_0^n)$

$\bar{s} = \frac{s}{W_0}$

بی‌بعدسازی : $\frac{s^n}{W_0^n} + \bar{a}_{n-1} \frac{s^{n-1}}{W_0^{n-1}} + \dots + \bar{a}_1 \frac{s}{W_0} + 1 = \bar{s}^n + \bar{a}_{n-1} \bar{s}^{n-1} + \dots + a_1 \bar{s} + 1$

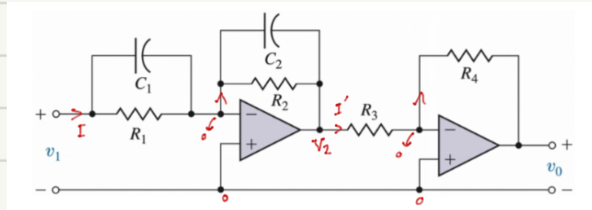




As we see rise time and settling time of different optimized transfer functions are different. Except n = 2 in general settling time is getting greater and rise time getting smaller.

Also in bode plots we could see that magnitude is decreasing faster by increasing n and phase become more negative. (90 deg every step)

5-



$$I = \frac{V_1}{R_1} + sC_1V_1$$

$$-V_2 = \left(\frac{1}{R_2} + sC_2\right)^{-1} I$$

$$I' = \frac{V_2}{R_3}$$

$$-V_0 = I'R_4$$

$$\longrightarrow V_0 = -R_4\left(\frac{V_2}{R_3}\right) = \frac{R_4}{R_3}\left(\frac{1}{R_2} + sC_2\right)^{-1} I = \frac{R_4}{R_3}\,\frac{\frac{1}{R_1}+sC_1}{\frac{1}{R_2}+sC_2} V_1 \longrightarrow G(s) = \frac{R_4C_1}{R_3C_2}\,\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}} \qquad \left(\frac{1}{R_1C_1} < \frac{1}{R_2C_2} \Rightarrow R_2C_2 < R_1C_1\right)$$

6-

a) $L(s) = \dfrac{4000\,K}{s(s+20)(s+10)}$ $\qquad E(s) = \dfrac{1}{1+L(s)}R(s) - \dfrac{G(s)}{1+L(s)}T_d(s)$

$$\frac{\frac{40}{s(s+20)}}{1+\frac{4000\,K}{s(s+20)(s+10)}}\,T_d(s) = \frac{40(s+10)}{s(s+20)(s+10)+4000\,K}\cdot\frac{1}{s} = H(s)$$

$$\lim_{s\to 0} sH(s) = \frac{1}{10k} \leq 5\% \Rightarrow 2 < K \longrightarrow K = 2$$

b) $\Delta(s) = s(s+20)(s+10) + 4000\,K = s^3 + 30s^2 + 200s + 8000$

3	1	200
2	30	8000
1	$\frac{-200}{3}$	0
0	8000	

$\longrightarrow$ not stable

c) $G_{lead}(s) = \dfrac{K(1+\alpha\tau s)}{\alpha(1+\tau s)}$

$$|L(wj)| = 4000\left(w^2(900w^2 + (200-w^2)^2)\right)^{-\frac{1}{2}} = 1 \longrightarrow w = 11.43$$

$$\Phi(w) = -90 - \tan^{-1}\left(\tfrac{1}{10}w\right) - \tan^{-1}\left(\tfrac{1}{20}w\right) = -168° \Rightarrow P.M. = 12°$$

$$PM = 30° - 12° = 18° \longrightarrow 30°$$

$$\alpha = \frac{1+\sin(\Phi_m)}{1-\sin(\Phi_m)} = 3 \longrightarrow 20\log|L(wj)| = -10\log(\alpha) \Rightarrow |L(wj)| = \frac{1}{\sqrt{3}} \Rightarrow W_m = 15.2$$

$$p = W_m\sqrt{\alpha} = 26.3 \qquad z = \frac{p}{\alpha} = 8.7 \qquad \Rightarrow G_c(s) = 2\,\frac{s+8.7}{s+26.3}$$

$$\frac{\frac{40}{s(s+20)}}{1+\frac{8000\,(s+8.7)\,K'}{s(s+20)(s+10)(s+26.3)}}\,T_d(s) = \frac{40(s+10)(s+26.3)}{s(s+20)(s+10)(s+26.3)+8000(s+8.7)K'}\cdot\frac{1}{s} = H'(s)$$

$$\lim_{s\to 0} sH(s) = \frac{26.3}{20\times 8.7\,K'} \leq 5\% \Rightarrow 3 < K' \longrightarrow K' = 3$$

$$PM = 50° - 40° = 10° \longrightarrow 20°$$

$$\alpha = \frac{1+\sin(\Phi_m)}{1-\sin(\Phi_m)} = 2 \longrightarrow 20\log|L'(wj)| = -10\log(\alpha) \Rightarrow |L'(wj)| = \frac{1}{\sqrt{2}} \Rightarrow W'_m = 14.6$$

$$p = W_m\sqrt{\alpha} = 20.65 \qquad z = \frac{p}{\alpha} = 10.32 \qquad \Rightarrow G_c(s) = 3\,\frac{s+20.65}{s+10.32}$$

MATLAB Assignments

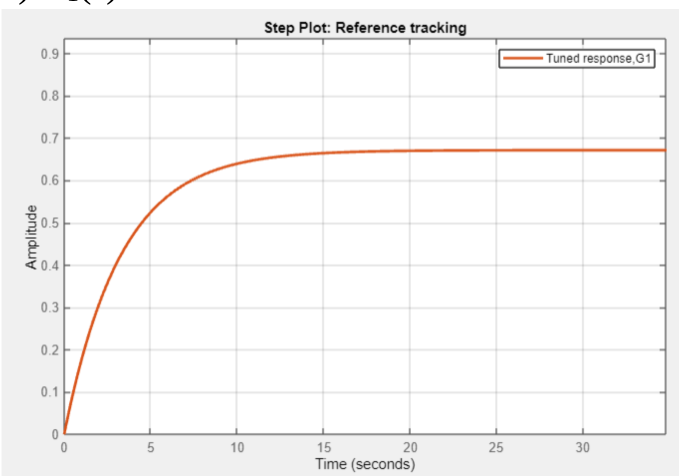
7 PID Tuner Toolbox

First we define transfer functions as below:

```
%% 7

G1 = tf([1],[10,1]);
G2 = tf([1],[1,0.1,1]);
G3 = tf([1],[1,0.1,1,0]);
G4 = tf([1],[1,0,0,0,-1,0]);
```

a) $G_1(s)$



Controller Parameters

	Tuned
Kp	2.0426
Ki	n/a
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	7.22 seconds
Settling time	12.9 seconds
Overshoot	0 %
Peak	0.671
Gain margin	Inf dB @ NaN rad/s
Phase margin	119 deg @ 0.178 rad/s
Closed-loop stability	Stable

(P):
By decreasing response time we will have less settling time and greater value for Kp but overshoot will be 0% all the time so it doesn't effect of which value we choose.

Controller Parameters

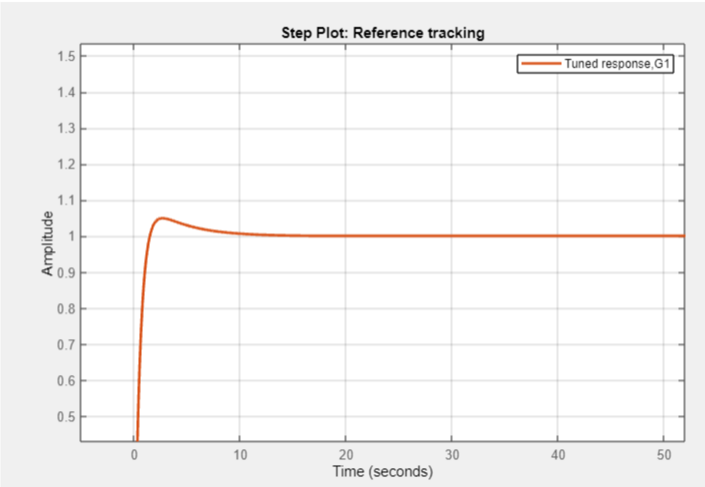
	Tuned
Kp	37.8562
Ki	n/a
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.565 seconds
Settling time	1.01 seconds
Overshoot	0 %
Peak	0.974
Gain margin	Inf dB @ NaN rad/s
Phase margin	91.5 deg @ 3.78 rad/s
Closed-loop stability	Stable

We choose value of Kp in the way that settling time will be 1 sec. (Kp = 37.8562)

(PI):



Controller Parameters

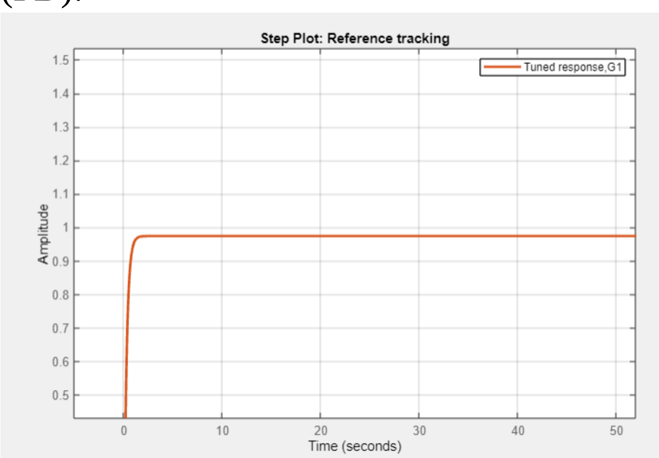
	Tuned
Kp	19.8508
Ki	5.2708
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.921 seconds
Settling time	6.35 seconds
Overshoot	4.97 %
Peak	1.05
Gain margin	Inf dB @ NaN rad/s
Phase margin	85.3 deg @ 2 rad/s
Closed-loop stability	Stable

We set response time to 1 sec and then change other parameters for reaching our needed values for settling time an overshoot. (Kp = 19.8508 , Ki = 5.2708)

(PD):



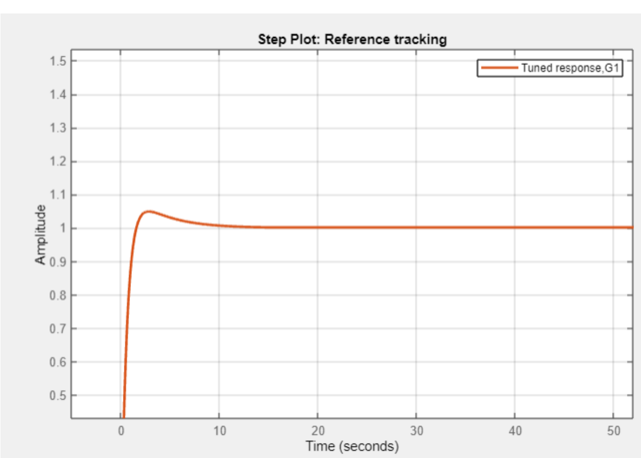
Controller Parameters

	Tuned
Kp	37.7206
Ki	n/a
Kd	0
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.567 seconds
Settling time	1.01 seconds
Overshoot	0 %
Peak	0.974
Gain margin	Inf dB @ NaN rad/s
Phase margin	91.5 deg @ 3.77 rad/s
Closed-loop stability	Stable

(PID):



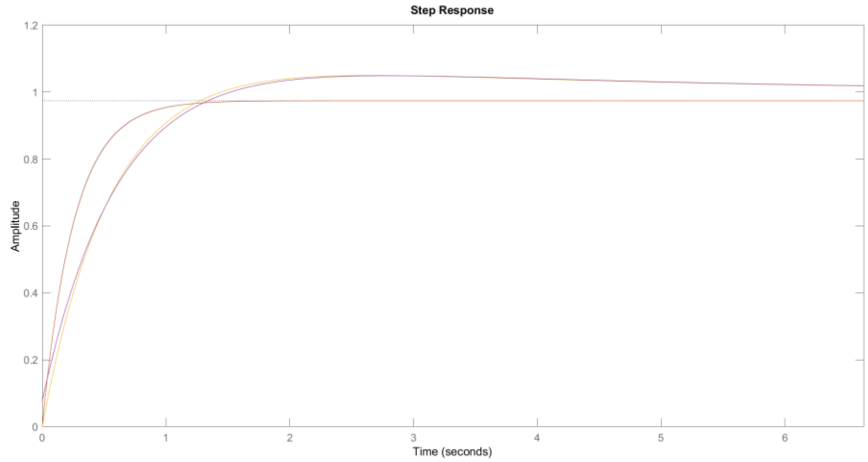
Controller Parameters

	Tuned
Kp	20
Ki	5.3493
Kd	0.83733
Tf	n/a

Performance and Robustness

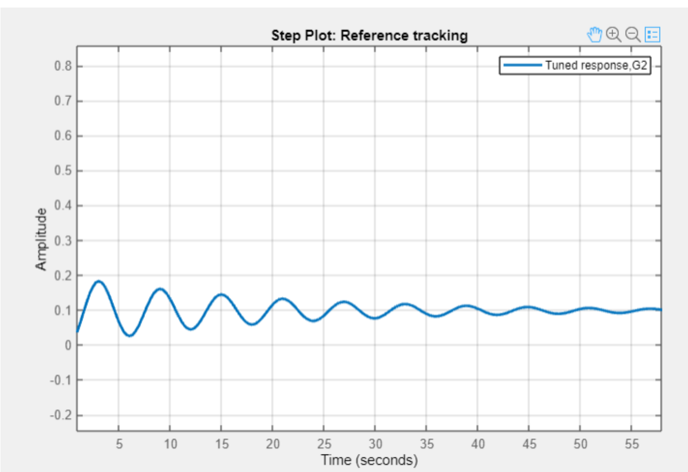
	Tuned
Rise time	0.998 seconds
Settling time	6.43 seconds
Overshoot	4.9 %
Peak	1.05
Gain margin	Inf dB @ NaN rad/s
Phase margin	90 deg @ 2 rad/s
Closed-loop stability	Stable

By increasing values of Kp, KI and KD in different situations we could see that settling time will decrease and overshoot will increase. Step response of systems are like below:



b) $G_2(s)$

(P):



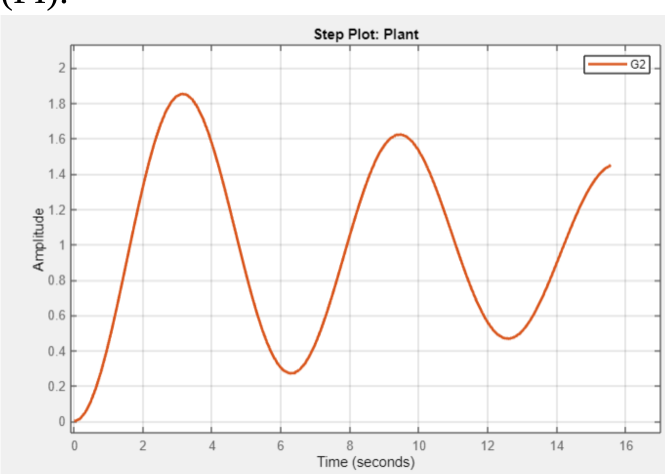
Controller Parameters

	Tuned
Kp	0.11006
Ki	n/a
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	1.03 seconds
Settling time	77.8 seconds
Overshoot	86.1 %
Peak	0.185
Gain margin	Inf dB @ Inf rad/s
Phase margin	68 deg @ 1.02 rad/s
Closed-loop stability	Stable

(PI):



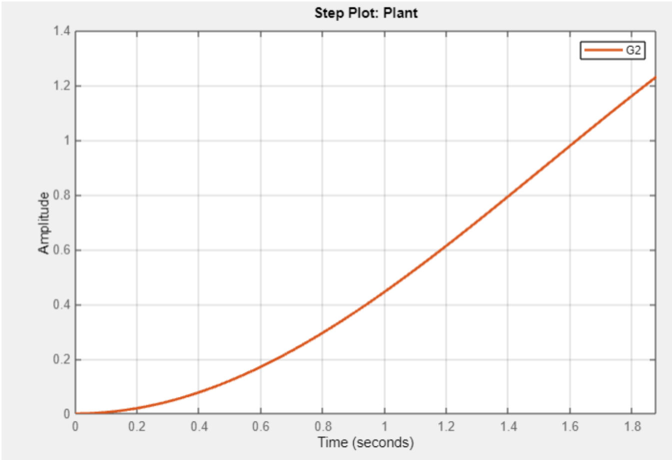
Controller Parameters

	Tuned
Kp	1.1813
Ki	0.03047
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	1.19 seconds
Settling time	224 seconds
Overshoot	4.93 %
Peak	1.05
Gain margin	Inf dB @ Inf rad/s
Phase margin	6.16 deg @ 1.47 rad/s
Closed-loop stability	Stable

(PD):



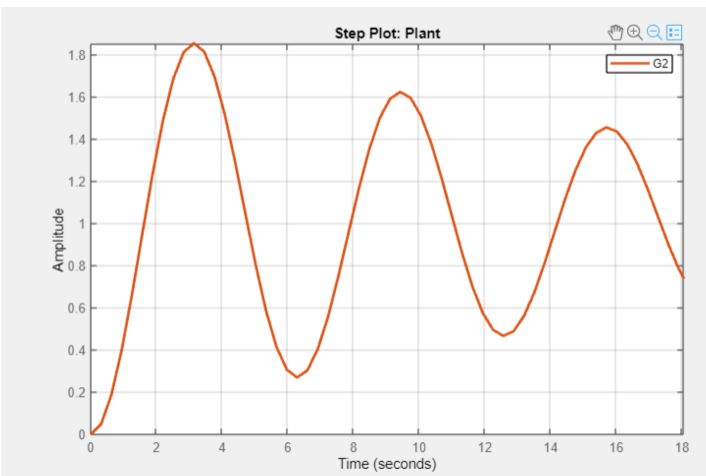
Controller Parameters

	Tuned
Kp	24.8074
Ki	n/a
Kd	37.6326
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.049 seconds
Settling time	1.53 seconds
Overshoot	5 %
Peak	1.01
Gain margin	Inf dB @ NaN rad/s
Phase margin	89.1 deg @ 37.7 rad/s
Closed-loop stability	Stable

(PID):



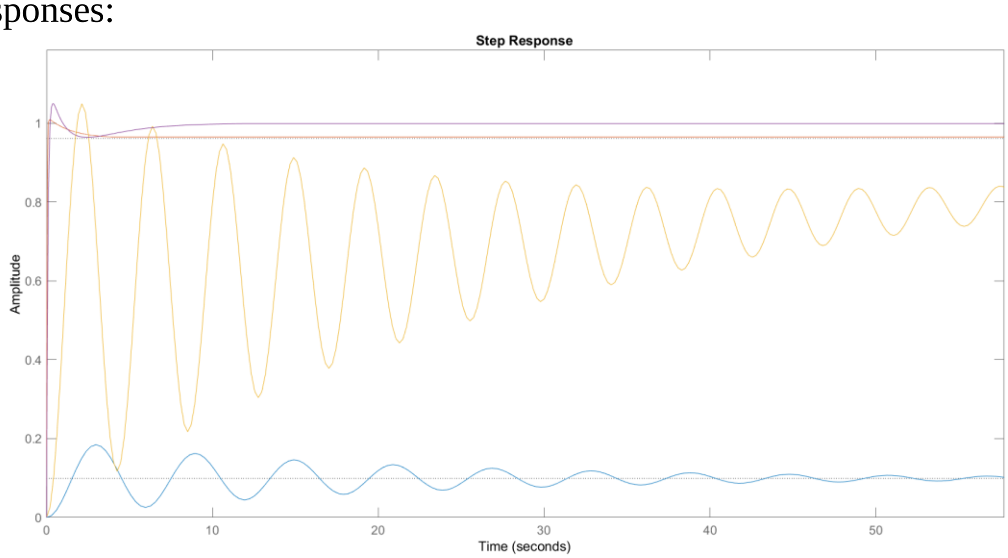
Controller Parameters

	Tuned
Kp	15.8581
Ki	4.9356
Kd	12.7378
Tf	n/a

Performance and Robustness

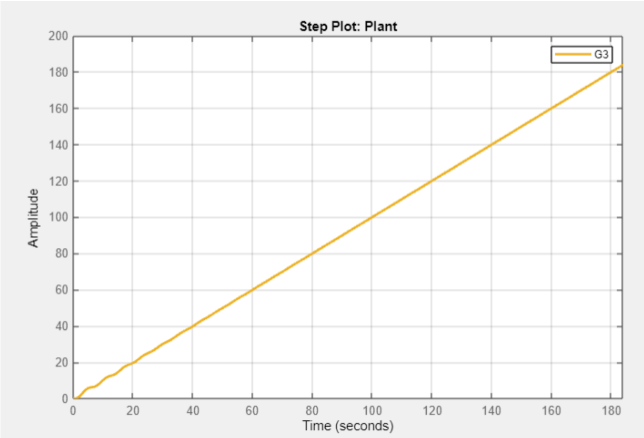
	Tuned
Rise time	0.142 seconds
Settling time	4.85 seconds
Overshoot	4.97 %
Peak	1.05
Gain margin	Inf dB @ NaN rad/s
Phase margin	84.9 deg @ 12.8 rad/s
Closed-loop stability	Stable

Step responses:



c) $G_3(s)$

(P):



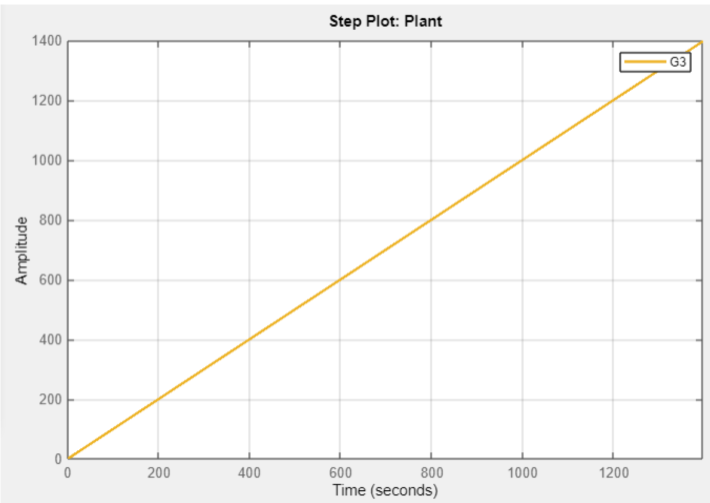
Controller Parameters

	Tuned
Kp	0.084992
Ki	n/a
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	20.5 seconds
Settling time	193 seconds
Overshoot	4.82 %
Peak	1.05
Gain margin	1.41 dB @ 1 rad/s
Phase margin	89.5 deg @ 0.0856 rad/s
Closed-loop stability	Stable

(PI):



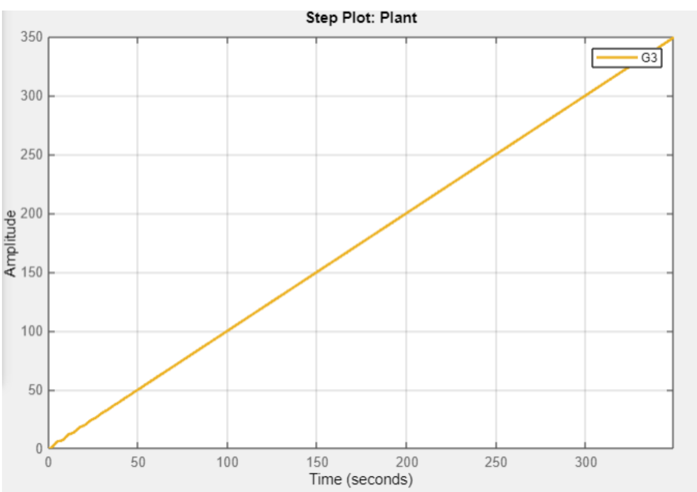
Controller Parameters

	Tuned
Kp	0.057078
Ki	5.7215e-05
Kd	n/a
Tf	n/a

Performance and Robustness

	Tuned
Rise time	33.1 seconds
Settling time	65.1 seconds
Overshoot	1.97 %
Peak	1.02
Gain margin	4.87 dB @ 1 rad/s
Phase margin	88.7 deg @ 0.0573 rad/s
Closed-loop stability	Stable

(PD):



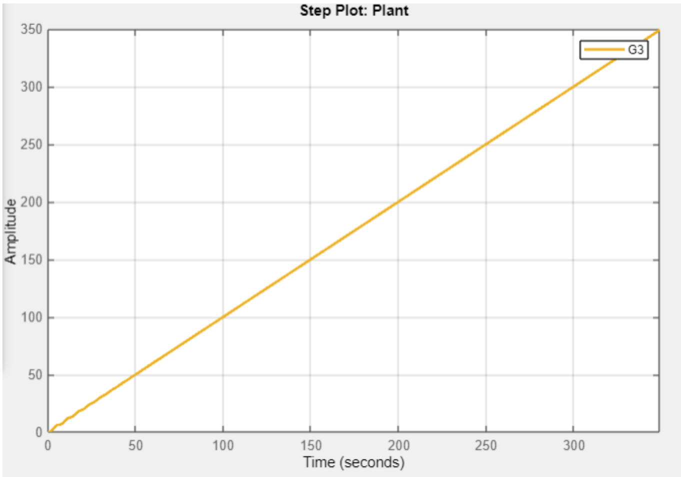
Controller Parameters

	Tuned
Kp	0.062469
Ki	n/a
Kd	0.0062716
Tf	n/a

Performance and Robustness

	Tuned
Rise time	32.5 seconds
Settling time	77.2 seconds
Overshoot	0.763 %
Peak	1.01
Gain margin	4.17 dB @ 1.01 rad/s
Phase margin	90 deg @ 0.0627 rad/s
Closed-loop stability	Stable

(PID):



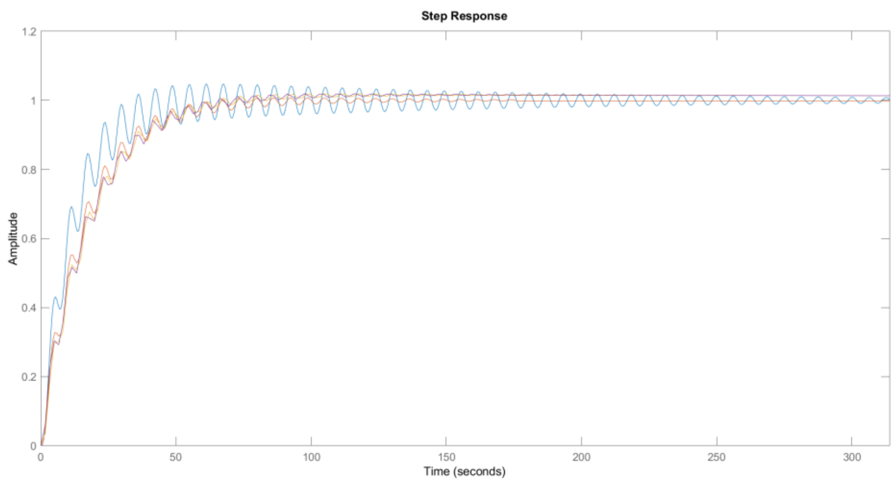
Controller Parameters

	Tuned
Kp	0.057776
Ki	5.8596e-05
Kd	0.023233
Tf	n/a

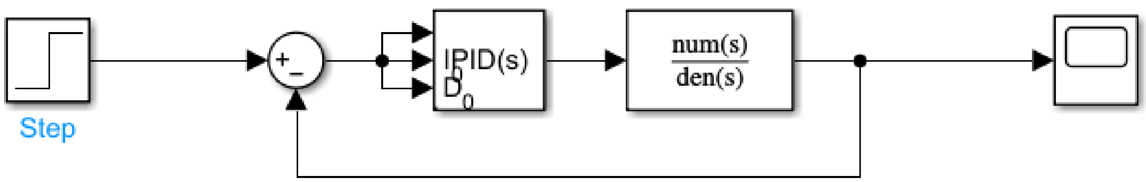
Performance and Robustness

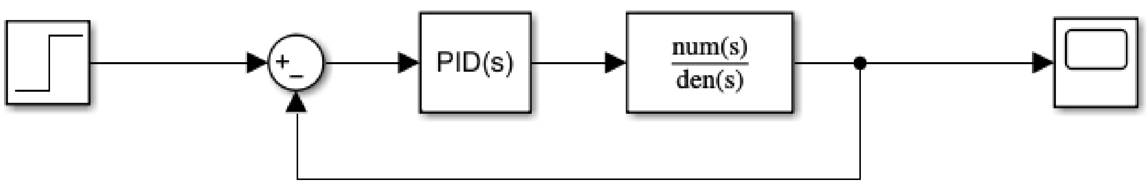
	Tuned
Rise time	32.9 seconds
Settling time	64.2 seconds
Overshoot	1.98 %
Peak	1.02
Gain margin	5.12 dB @ 1.02 rad/s
Phase margin	90 deg @ 0.058 rad/s
Closed-loop stability	Stable

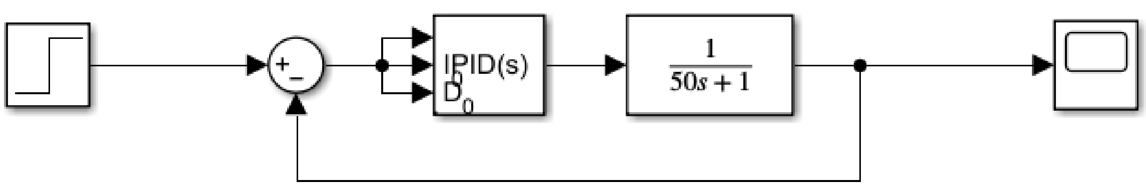
Step responces:



8 Cascade Control System







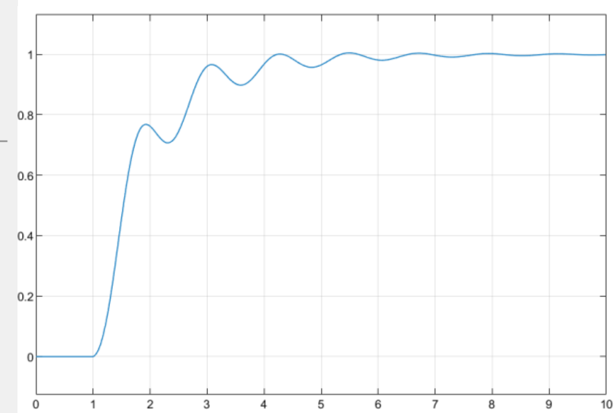
First we determine PID controller for loop1. Following table shows Coefficents and properties of the controller. Output of system is like below:

Controller Parameters

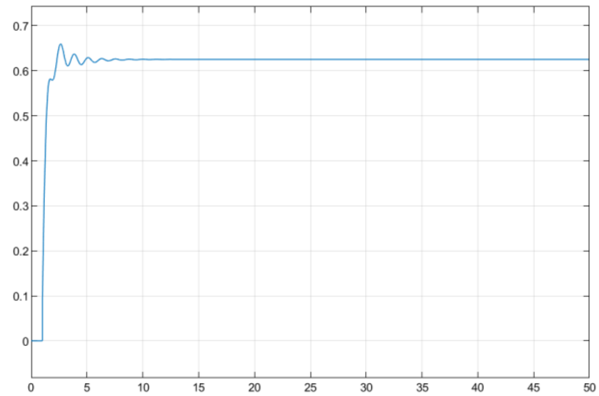
	Tuned	Block
P	0.51401	0.90838
I	0.7008	0.74679
D	-0.20028	-0.0035839
N	2.1227	253.4595

Performance and Robustness

	Tuned	Block
Rise time	1.6 seconds	2.25 seconds
Settling time	4.14 seconds	5.05 seconds
Overshoot	0.451 %	0 %
Peak	1	0.999
Gain margin	8.83 dB @ 5.72 rad/s	Inf dB @ Inf rad/s
Phase margin	90 deg @ 1.23 rad/s	47.5 deg @ 5.91 rad/s
Closed-loop stability	Stable	Stable



Also we should check u1:



It is suitable for our demand.

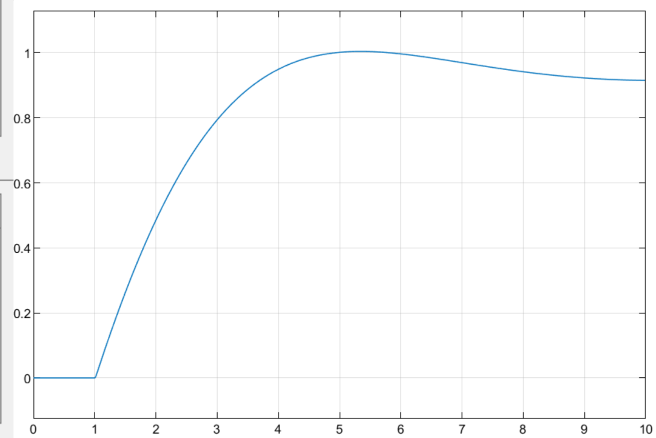
For loop2 we have:

Controller Parameters

	Tuned	Block
P	3.3652	1.8194
I	0.48861	0.27283
D	5.7477	3.0111
N	84.0006	56.3047

Performance and Robustness

	Tuned	Block
Rise time	2.4 seconds	4.32 seconds
Settling time	18.3 seconds	25 seconds
Overshoot	0.423 %	0 %
Peak	1	1
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	72.9 deg @ 0.735 rad/s	72.9 deg @ 0.493 rad/s
Closed-loop stability	Stable	Stable



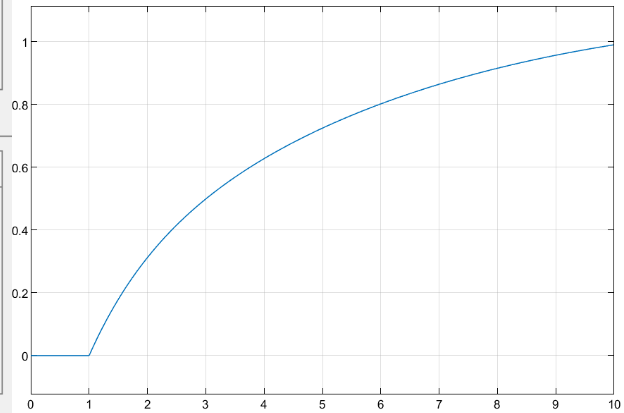
And loop3:

Controller Parameters

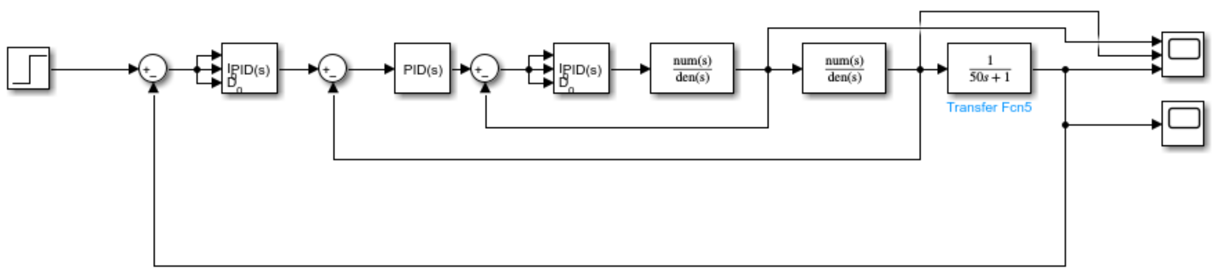
	Tuned	Block
P	13.4241	0.90838
I	0.98817	0.74679
D	9.7094	-0.0035839
N	0.7756	253.4595

Performance and Robustness

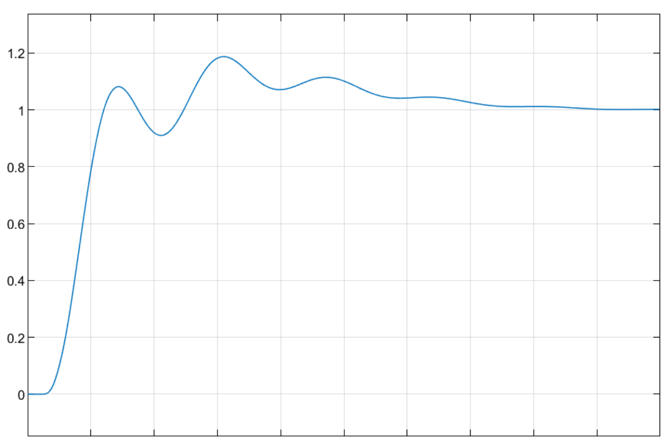
	Tuned	Block
Rise time	6.42 seconds	9.35 seconds
Settling time	38.3 seconds	188 seconds
Overshoot	8.5 %	61.3 %
Peak	1.08	1.61
Gain margin	Inf dB @ NaN rad/s	Inf dB @ Inf rad/s
Phase margin	90 deg @ 0.287 rad/s	17.7 deg @ 0.122 rad/s
Closed-loop stability	Stable	Stable

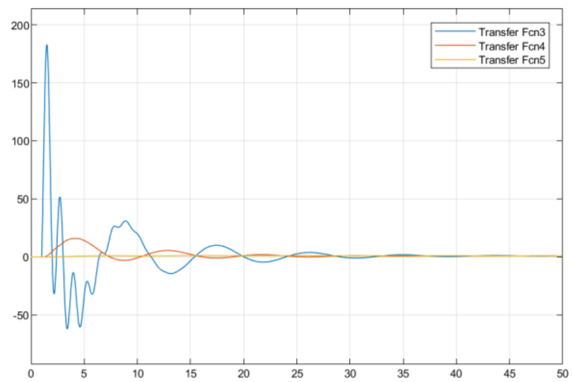


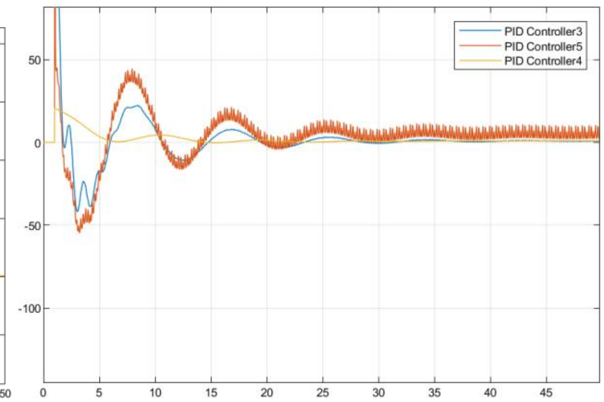
Everything seems to be appropriate for our system. So now we connect them together:



Following plot is the output of the system.







Output of every loop   output of controllers

We should consider that settling time of each loop should be in a way that it gets stable before the outer loop gets stable.

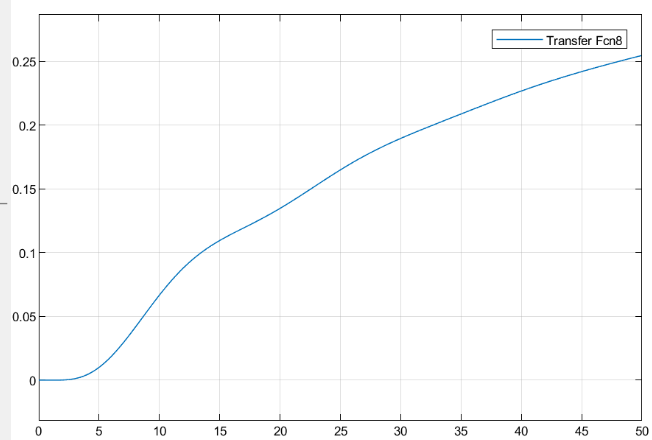
e) Now we just want to use one controller instead of cascade one:

Controller Parameters

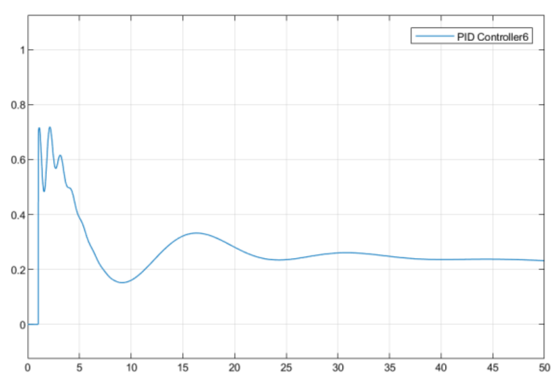
	Tuned	Block
P	0.67654	0.67654
I	1.1134	1.1134
D	-0.0036289	-0.0036289
N	186.432	186.432

Performance and Robustness

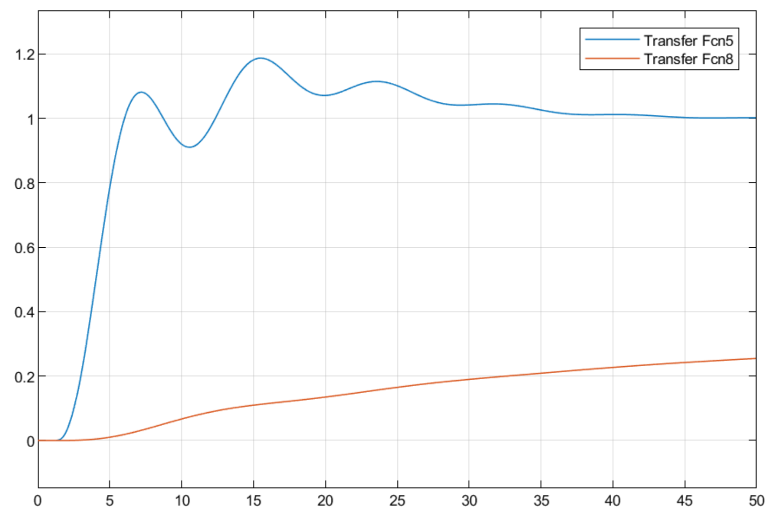
	Tuned	Block
Rise time	1.29 seconds	1.29 seconds
Settling time	14.1 seconds	14.1 seconds
Overshoot	10.4 %	10.4 %
Peak	1.1	1.1
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	52 deg @ 5.46 rad/s	52 deg @ 5.46 rad/s
Closed-loop stability	Stable	Stable



Checking u1:

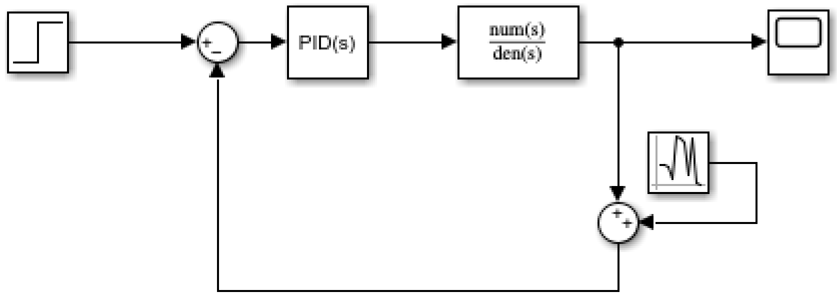


f) Output of two types of controllers:



We see that the system with cascade controller will be stable faster (settling time is smaller). So we reach our appropriate response faster but as the question mentioned we can’t do some measurements in cascade controller.

9 Reducing The Effect of Measurement Noise



First we find PID controllers for aggressive setting by changing noise variance:

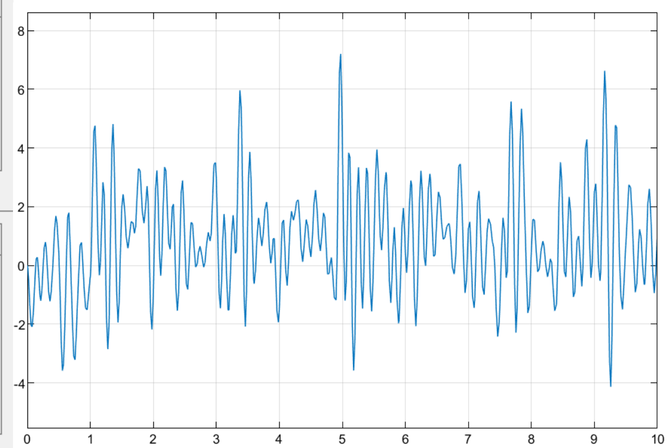
variance = 1

Controller Parameters

	Tuned	Block
P	1657.3831	1
I	642.5164	1
D	13.3566	0
N	44.4444	100

Performance and Robustness

	Tuned	Block
Rise time	0.0237 seconds	1.05 seconds
Settling time	1.01 seconds	74.7 seconds
Overshoot	83.8 %	41.8 %
Peak	1.84	1.42
Gain margin	Inf dB @ Inf rad/s	3.52 dB @ 1.58 rad/s
Phase margin	9 deg @ 44.4 rad/s	6.4 deg @ 1.38 rad/s
Closed-loop stability	Stable	Stable



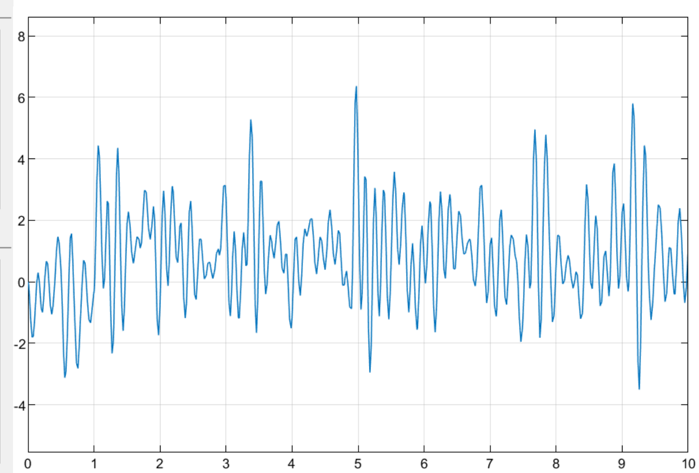
variance = 0.75

Controller Parameters

	Tuned	Block
P	1649.3795	1657.3831
I	637.855	642.5164
D	13.3211	13.3566
N	44.3361	44.4444

Performance and Robustness

	Tuned	Block
Rise time	0.0237 seconds	0.0237 seconds
Settling time	1.01 seconds	1.01 seconds
Overshoot	83.8 %	83.8 %
Peak	1.84	1.84
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	9 deg @ 44.3 rad/s	9 deg @ 44.4 rad/s
Closed-loop stability	Stable	Stable



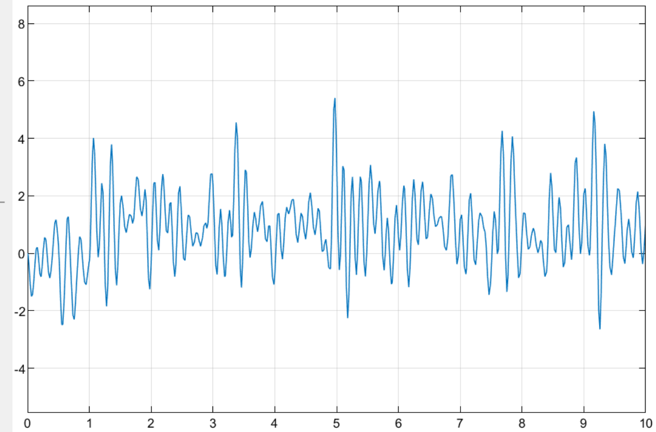
variance = 0.5

Controller Parameters

	Tuned	Block
P	1632.3494	1649.3795
I	2088.2493	637.855
D	14.842	13.3211
N	44.5434	44.3361

Performance and Robustness

	Tuned	Block
Rise time	0.0235 seconds	0.0237 seconds
Settling time	1 seconds	1.01 seconds
Overshoot	84.7 %	83.8 %
Peak	1.85	1.84
Gain margin	-26.9 dB @ 8.7 rad/s	Inf dB @ Inf rad/s
Phase margin	9 deg @ 44.5 rad/s	9 deg @ 44.3 rad/s
Closed-loop stability	Stable	Stable



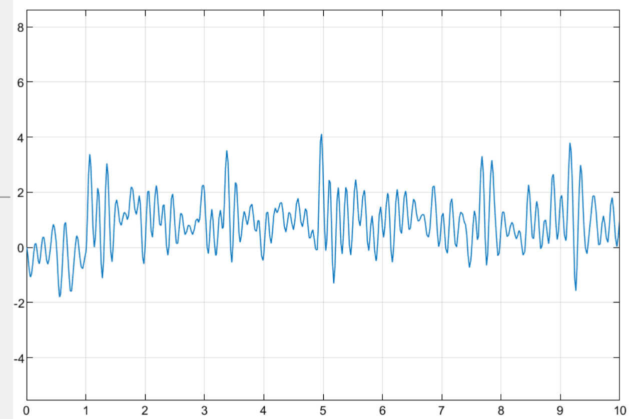
variance = 0.25

Controller Parameters

	Tuned	Block
P	1634.5281	1632.3494
I	2091.7933	2088.2493
D	14.852	14.842
N	44.5732	44.5434

Performance and Robustness

	Tuned	Block
Rise time	0.0234 seconds	0.0235 seconds
Settling time	1 seconds	1 seconds
Overshoot	84.7 %	84.7 %
Peak	1.85	1.85
Gain margin	-26.9 dB @ 8.7 rad/s	-26.9 dB @ 8.7 rad/s
Phase margin	9 deg @ 44.6 rad/s	9 deg @ 44.5 rad/s
Closed-loop stability	Stable	Stable



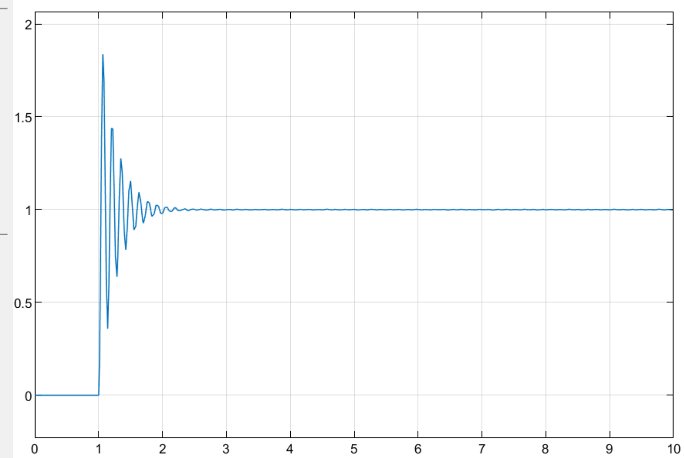
variance = 0

Controller Parameters

	Tuned	Block
P	1639.6287	1634.5281
I	2100.0961	2091.7933
D	14.8756	14.852
N	44.6429	44.5732

Performance and Robustness

	Tuned	Block
Rise time	0.0234 seconds	0.0234 seconds
Settling time	1 seconds	1 seconds
Overshoot	84.7 %	84.7 %
Peak	1.85	1.85
Gain margin	-26.9 dB @ 8.72 rad/s	-26.9 dB @ 8.7 rad/s
Phase margin	9 deg @ 44.6 rad/s	9 deg @ 44.6 rad/s
Closed-loop stability	Stable	Stable



We should mention that we considered settling time is equal to 1.

Now we find PID controllers for robust setting by changing noise variance:

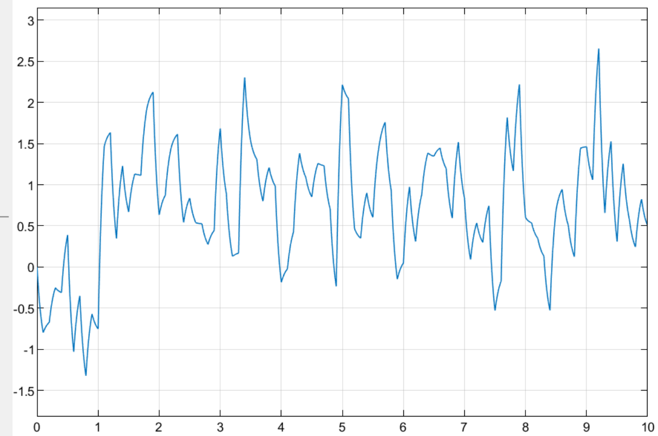
variance = 1

Controller Parameters

	Tuned	Block
P	5.8047	1
I	0.4712	1
D	11.5531	0
N	1329.6428	100

Performance and Robustness

	Tuned	Block
Rise time	0.135 seconds	1.05 seconds
Settling time	1.8 seconds	74.7 seconds
Overshoot	8.17 %	41.8 %
Peak	0.967	1.42
Gain margin	Inf dB @ Inf rad/s	3.52 dB @ 1.58 rad/s
Phase margin	90 deg @ 11.6 rad/s	6.4 deg @ 1.38 rad/s
Closed-loop stability	Stable	Stable



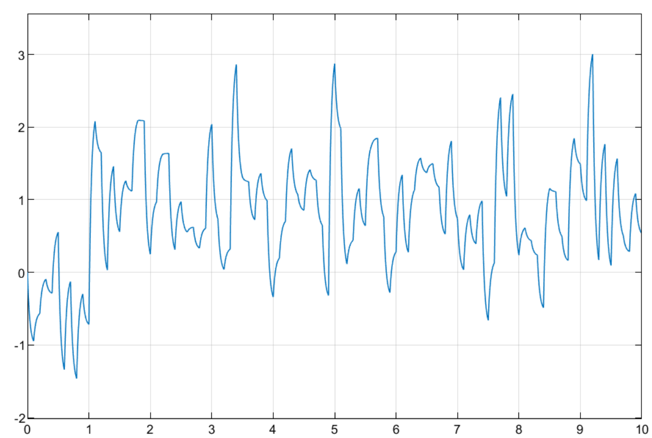
variance = 0**.75**

Controller Parameters

	Tuned	Block
P	19.7173	5.8047
I	3.1545	0.4712
D	27.3832	11.5531
N	3134.0408	1329.6428

Performance and Robustness

	Tuned	Block
Rise time	0.0788 seconds	0.135 seconds
Settling time	0.139 seconds	1.8 seconds
Overshoot	0 %	8.17 %
Peak	0.997	0.967
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	89.2 deg @ 27.4 rad/s	90 deg @ 11.6 rad/s
Closed-loop stability	Stable	Stable



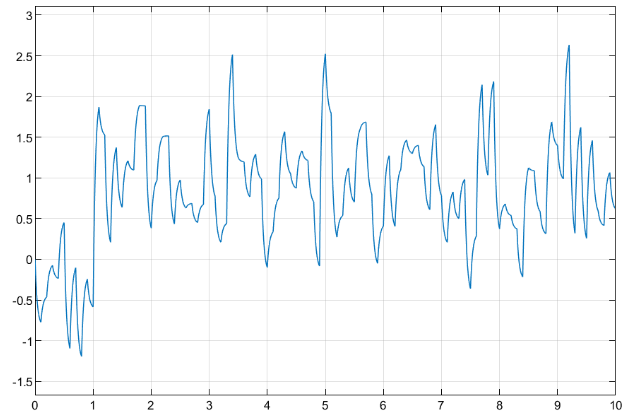
variance = 0.5

Controller Parameters

	Tuned	Block
P	19.7173	5.8047
I	3.1545	0.4712
D	27.3832	11.5531
N	3134.0408	1329.6428

Performance and Robustness

	Tuned	Block
Rise time	0.0788 seconds	0.135 seconds
Settling time	0.139 seconds	1.8 seconds
Overshoot	0 %	8.17 %
Peak	0.997	0.967
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	89.2 deg @ 27.4 rad/s	90 deg @ 11.6 rad/s
Closed-loop stability	Stable	Stable



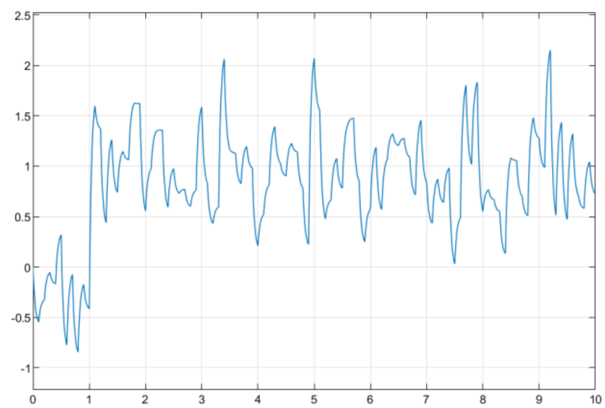
variance = 0.25

Controller Parameters

	Tuned	Block
P	19.7173	5.8047
I	3.1545	0.4712
D	27.3832	11.5531
N	3134.0408	1329.6428

Performance and Robustness

	Tuned	Block
Rise time	0.0788 seconds	0.135 seconds
Settling time	0.139 seconds	1.8 seconds
Overshoot	0 %	8.17 %
Peak	0.997	0.967
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	89.2 deg @ 27.4 rad/s	90 deg @ 11.6 rad/s
Closed-loop stability	Stable	Stable



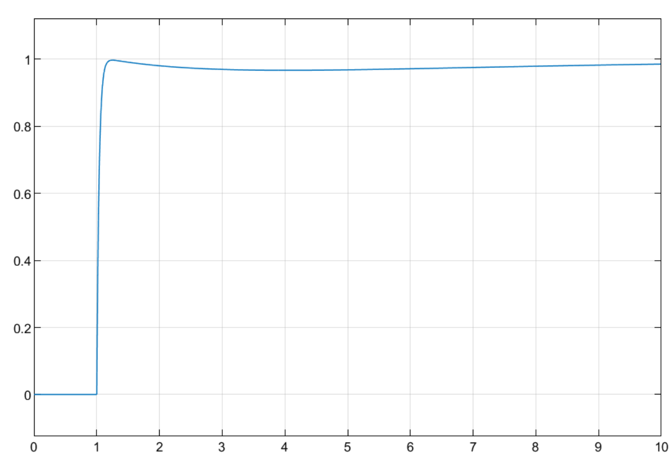
variance = 0

Controller Parameters

	Tuned	Block
P	19.7173	5.8047
I	3.1545	0.4712
D	27.3832	11.5531
N	3134.0408	1329.6428

Performance and Robustness

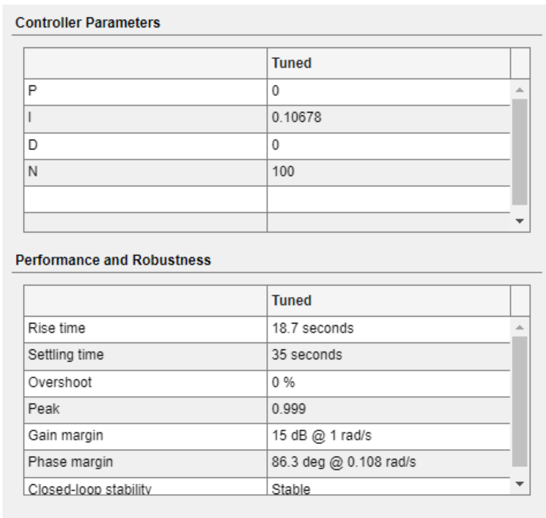
	Tuned	Block
Rise time	0.0788 seconds	0.135 seconds
Settling time	0.139 seconds	1.8 seconds
Overshoot	0 %	8.17 %
Peak	0.997	0.967
Gain margin	Inf dB @ Inf rad/s	Inf dB @ Inf rad/s
Phase margin	89.2 deg @ 27.4 rad/s	90 deg @ 11.6 rad/s
Closed-loop stability	Stable	Stable



Now we add a low-pass filter. Here we have a problem, our close loop system after applying filter will be unstable. For solving this problem we ignore the amount of settling time to be 1 second and we focus on stability of system.

Our new PID controller and the output for aggressive setting will be like below:

(noise variance =1)

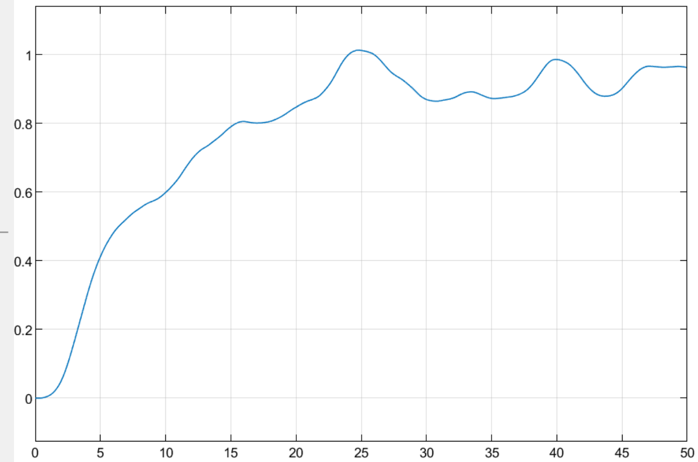


Controller Parameters

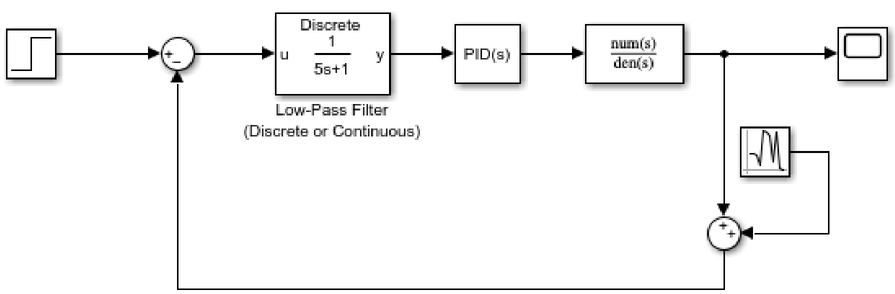
	Tuned
P	0
I	0.10678
D	0
N	100

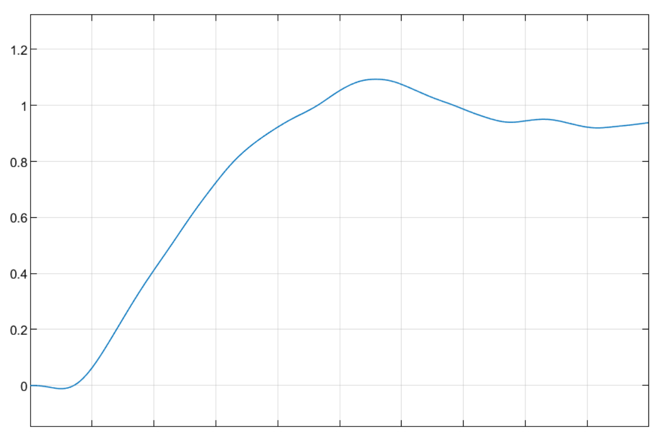
Performance and Robustness

	Tuned
Rise time	18.7 seconds
Settling time	35 seconds
Overshoot	0 %
Peak	0.999
Gain margin	15 dB @ 1 rad/s
Phase margin	86.3 deg @ 0.108 rad/s
Closed-loop stability	Stable



Now we add filter:





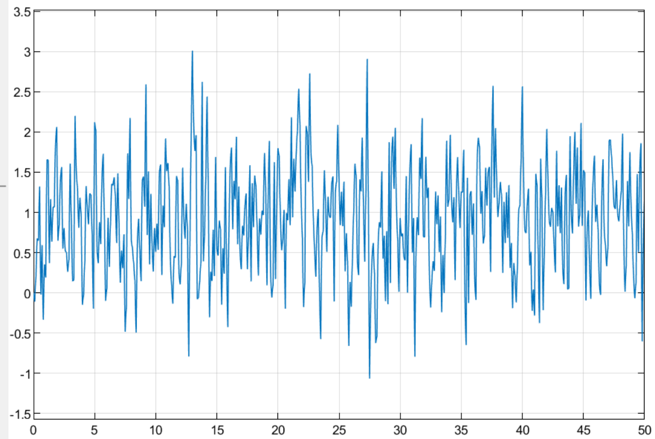
For robust setting we have:

Controller Parameters

	Tuned	Block
P	5.4598	0
I	0.41988	0.10678
D	10.6922	0
N	1232.0254	100

Performance and Robustness

	Tuned	Block
Rise time	0.216 seconds	18.7 seconds
Settling time	30.9 seconds	35 seconds
Overshoot	0 %	0 %
Peak	0.998	0.999
Gain margin	Inf dB @ Inf rad/s	15 dB @ 1 rad/s
Phase margin	90 deg @ 10.8 rad/s	86.3 deg @ 0.108 rad/s
Closed-loop stability	Stable	Stable



After adding filter: