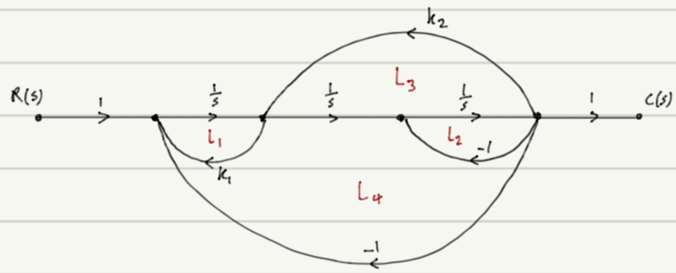
1_



$P_1(s) = \frac{1}{s^3}$

$L_1(s) = \frac{K_1}{s} \quad L_2(s) = \frac{-1}{s} \quad L_3(s) = \frac{K_2}{s^2} \quad L_4(s) = \frac{-1}{s^3}$

$\Delta(s) = 1 - \left(L_1(s) + L_2(s) + L_3(s) + L_4(s)\right) + \left(L_1(s)\,L_2(s)\right) = 1 - \left(\frac{K_1}{s} + \frac{-1}{s} + \frac{K_2}{s^2} + \frac{-1}{s^3}\right) + \left(\frac{K_1}{s}\,\frac{-1}{s}\right) = 1 - \frac{(K_1-1)}{s} - \frac{(K_2+K_1)}{s^2} + \frac{1}{s^3}$

$\Delta_1(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{P_1(s)\,\Delta_1(s)}{\Delta(s)} = \frac{\frac{1}{s^3}}{1 - \frac{(K_1-1)}{s} - \frac{(K_2+K_1)}{s^2} + \frac{1}{s^3}} = \frac{1}{s^3 - (K_1-1)s^2 - (K_2+K_1)s + 1}$$

s^3	1	$-K_1-K_2$
s^2	$1-K_1$	1
s^1	$\frac{-1}{1-K_1} - (K_1+K_2)$	0
s^0	1	0

$\longrightarrow$ سطر تمام صفر برای قطب موهومی خالص $\Rightarrow \frac{-1}{1-K_1} - K_1 - K_2 = 0 \Rightarrow K_2 = \frac{K_1^2 - K_1 + 1}{1-K_1}$

$K_1 = \frac{1}{2}\left(\pm\sqrt{(K_2+1)^2+4} - K_2 + 1\right)$

$1 - K_1 > 0 \Rightarrow K_1 < 1$

چند جمله‌ای کمکی : $a(s) = (1-K_1)s^2 + 1$

$\frac{d\,a(s)}{ds} = 2(1-K_1)\,s \Rightarrow 2(1-K_1) > 0 \Rightarrow K_1 < 1$

ریشه‌های $a(s)$: $(1-K_1)s^2 + 1 = 0 \Rightarrow s = \pm j\sqrt{\frac{1}{1-K_1}} \Rightarrow W_n = \sqrt{\frac{1}{1-K_1}}$

2 - 1) $G_H(s) = \frac{20}{(s+3)(s+30)} = \frac{1}{\frac{1}{20}s^2+\frac{33}{20}s+\frac{9}{2}}$

$G_L(s) = \frac{k}{s+p_1} = \frac{1}{\frac{1}{k}s+\frac{p_1}{k}}$

$\frac{G_H(s)}{G_L(s)} = \frac{\frac{1}{k}s+\frac{p_1}{k}}{\frac{1}{20}s^2+\frac{33}{20}s+\frac{9}{2}} = \frac{M(s)}{\Delta(s)} \Rightarrow \begin{matrix} M(s) = \frac{1}{k}s+\frac{p_1}{k} \\ \Delta(s) = \frac{1}{20}s^2+\frac{33}{20}s+\frac{9}{2} \end{matrix} \longrightarrow \left[\begin{matrix} M^{(0)}(0) = \frac{p_1}{k} & \Delta^{(0)}(0) = \frac{9}{2} \\ M^{(1)}(0) = \frac{1}{k} & \Delta^{(1)}(0) = \frac{33}{20} \\ M^{(2)}(0) = 0 & \Delta^{(2)}(0) = \frac{1}{10} \end{matrix} \right.$

$q=0$: $M^{(0)}(0)\,M^{(0)}(0) = \left(\frac{p_1}{k}\right)^2$

$\Delta^{(0)}(0)\,\Delta^{(0)}(0) = \left(\frac{9}{2}\right)^2 \Rightarrow \frac{p_1}{k} = \frac{9}{2} \Rightarrow p_1 = \frac{9}{2}k$

$q=1$: $(-1)\frac{M^{(0)}(0)M^{(2)}(0)}{2} + \frac{M^{(1)}(0)M^{(1)}(0)}{1} + (-1)\frac{M^{(2)}(0)M^{(0)}(0)}{2} = 0 + \left(\frac{1}{k}\right)^2 + 0 = \frac{1}{k^2}$

$\Rightarrow \frac{1}{k} = \frac{3\sqrt{101}}{20} \Rightarrow k = 0.663$

$(-1)\frac{\Delta^{(0)}(0)\Delta^{(2)}(0)}{2} + \frac{\Delta^{(1)}(0)\Delta^{(1)}(0)}{1} + (-1)\frac{\Delta^{(2)}(0)\Delta^{(0)}(0)}{2} = -\frac{1}{2}\left(\frac{9}{2}\times\frac{1}{10}\right) + \left(\frac{33}{20}\right)^2 - \frac{1}{2}\left(\frac{1}{10}\times\frac{9}{2}\right) = \frac{-9}{20} + \frac{1089}{400} = \frac{909}{400}$

$\Rightarrow p_1 = 2.983$

$\downarrow$

$\frac{0.663}{s+2.983}$

2) $G_H(s) = \frac{-0.2s+1}{(0.5s+1)(0.25s+1)(0.2s+1)} = \frac{-0.2s+1}{0.025s^3+0.275s^2+0.95s+1}$

$G_L(s) = \frac{k}{(s+p_1)(s+p_2)} = \frac{k}{s^2+(p_1+p_2)s+p_1p_2} = \frac{1}{\frac{1}{k}s^2+\frac{(p_1+p_2)}{k}s+\frac{p_1p_2}{k}}$

$\frac{G_H(s)}{G_L(s)} = \frac{(-0.2s+1)\left(\frac{1}{k}s^2+\frac{(p_1+p_2)}{k}s+\frac{p_1p_2}{k}\right)}{0.025s^3+0.275s^2+0.95s+1} = \frac{M(s)}{\Delta(s)} \Rightarrow \begin{matrix} M(s) = (-0.2s+1)\left(\frac{1}{k}s^2+\frac{(p_1+p_2)}{k}s+\frac{p_1p_2}{k}\right) \\ \Delta(s) = 0.025s^3+0.275s^2+0.95s+1 \end{matrix} \longrightarrow \left[\begin{matrix} M^{(0)}(0) = \frac{p_1p_2}{k} & \Delta^{(0)}(0) = 1 \\ M^{(1)}(0) = \frac{-0.2p_1p_2+p_1+p_2}{k} & \Delta^{(1)}(0) = 0.95 \\ M^{(2)}(0) = \frac{-0.4(p_1+p_2)+2}{k} & \Delta^{(2)}(0) = 0.55 \\ M^{(3)}(0) = \frac{-1.2}{k} & \Delta^{(3)}(0) = 0.15 \end{matrix} \right.$

$q=0$: $M^{(0)}(0)\,M^{(0)}(0) = \left(\frac{p_1p_2}{k}\right)^2$

$\Delta^{(0)}(0)\,\Delta^{(0)}(0) = 1 \Rightarrow \frac{p_1p_2}{k} = 1 \Rightarrow p_1p_2 = k$

$q=1$: $(-1)\frac{M^{(0)}(0)M^{(2)}(0)}{2} + \frac{M^{(1)}(0)M^{(1)}(0)}{1} + (-1)\frac{M^{(2)}(0)M^{(0)}(0)}{2} = -\frac{1}{2}\left(\frac{p_1p_2}{k}\,\frac{-0.4(p_1+p_2)+2}{k}\right) + \left(\frac{-0.2p_1p_2+p_1+p_2}{k}\right)^2 - \frac{1}{2}\left(\frac{p_1p_2}{k}\,\frac{-0.4(p_1+p_2)+2}{k}\right)$

$= \frac{0.4(p_1+p_2)-2}{k} + \left(\frac{p_1+p_2}{k} - 0.2\right)^2$

$(-1)\frac{\Delta^{(0)}(0)\Delta^{(2)}(0)}{2} + \frac{\Delta^{(1)}(0)\Delta^{(1)}(0)}{1} + (-1)\frac{\Delta^{(2)}(0)\Delta^{(0)}(0)}{2} = -\frac{1}{2}(1\times 0.55) + (0.95)^2 - \frac{1}{2}(1\times 0.55) = 0.3525$

$\Rightarrow \frac{(p_1+p_2)^2}{k^2} - \frac{2}{k} = 0.3125 \Rightarrow \frac{p_1^2+p_2^2}{k^2} = 0.3125$

$q=2$: $\frac{M^{(0)}(0)M^{(4)}(0)}{4!} + (-1)\frac{M^{(1)}(0)M^{(3)}(0)}{3!} + \frac{M^{(2)}(0)M^{(2)}(0)}{2!\,2!} + (-1)\frac{M^{(3)}(0)M^{(1)}(0)}{3!} + \frac{M^{(4)}(0)M^{(0)}(0)}{4!} =$

$0 - \frac{1}{6}\,\frac{-0.2p_1p_2+p_1+p_2}{k}\,\frac{-1.2}{k} + \frac{1}{4}\left(\frac{-0.4(p_1+p_2)+2}{k}\right)^2 - \frac{1}{6}\,\frac{-0.2p_1p_2+p_1+p_2}{k}\,\frac{-1.2}{k} + 0 = \frac{0.04(p^2+q^2)+1}{k^2}$

$\frac{\Delta^{(0)}(0)\Delta^{(4)}(0)}{4!} + (-1)\frac{\Delta^{(1)}(0)\Delta^{(3)}(0)}{3!} + \frac{\Delta^{(2)}(0)\Delta^{(2)}(0)}{2!\,2!} + (-1)\frac{\Delta^{(3)}(0)\Delta^{(1)}(0)}{3!} + \frac{\Delta^{(4)}(0)\Delta^{(0)}(0)}{4!} = 0 - \frac{1}{6}(0.95\times 0.15) + \frac{1}{4}(0.55)^2 - \frac{1}{6}(0.95\times 0.15) + 0 = 0.028125$

$\Rightarrow 0.04(0.3125) + \frac{1}{k^2} = 0.028125 \Rightarrow k^2 = 64 \Rightarrow k = 8$

$\begin{matrix} p_1p_2 = 8 \\ p_1^2+p_2^2 = 20 \end{matrix} \Rightarrow \begin{matrix} p_1 = 2\ ,\ p_2 = 4 \\ p_1 = 4\ ,\ p_2 = 2 \end{matrix} \longrightarrow \frac{8}{(s+4)(s+2)}$

3 –

$$T(s) = \frac{\frac{K}{s+a}\,\frac{1}{s+2}}{1+\frac{K}{s+a}\,\frac{1}{s+2}} = \frac{K}{s^2+(2+a)s+2a+k}$$

$$\left\{\frac{w_n^2}{s^2+2\zeta w_n s+w_n^2}\right\}$$

$$P.O. = 100\, e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} < 5 \Rightarrow \zeta > 0.69$$

2% [illegible] : $T_s = \frac{4}{\zeta w_n} < 2 \Rightarrow \zeta w_n > 2$

$$2+a = 2\zeta w_n \Rightarrow \frac{2+a}{2} = \zeta w_n > 2 \Rightarrow \boxed{a>2}$$

under damped system : $0<\zeta<1 \Rightarrow 0.69<\zeta<1$

$w_n < \frac{2}{\zeta} \Rightarrow w_n < 2 \quad (w_n^2 < 4) \Rightarrow 2a+k<4 \longrightarrow k<4-2a$

$a>2 \rightarrow -2a<-4 \Rightarrow \boxed{k<0}$

4 — $E(s) = \frac{1}{1+L(s)} R(s)$, $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

1) $L_1(s) = \frac{20}{s(s+1)(s+3)}$ $\Rightarrow$ $E(s) = \frac{1}{1+\frac{20}{s(s+1)(s+3)}}\frac{1}{s}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{1+\frac{20}{s(s+1)(s+3)}} = 0$

$E(s) = \frac{1}{1+\frac{20}{s(s+1)(s+3)}}\frac{10}{s^2}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{10}{s+\frac{20}{(s+1)(s+3)}} = \frac{3\times 10}{20} = 1.5$

$E(s) = \frac{1}{1+\frac{20}{s(s+1)(s+3)}}\frac{1}{s^3}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{s^2+\frac{20s}{(s+1)(s+3)}} = \infty$

2) $L_2(s) = \frac{100}{s(s+2)(s+4)(s+6)}$ $\Rightarrow$ $E(s) = \frac{1}{1+\frac{100}{s(s+2)(s+4)(s+6)}}\frac{1}{s}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{1+\frac{100}{s(s+2)(s+4)(s+6)}} = 0$

$E(s) = \frac{1}{1+\frac{100}{s(s+2)(s+4)(s+6)}}\frac{10}{s^2}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{10}{s+\frac{100}{(s+2)(s+4)(s+6)}} = \frac{48\times 10}{100} = 4.8$

$E(s) = \frac{1}{1+\frac{100}{s(s+2)(s+4)(s+6)}}\frac{1}{s^3}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{s^2+\frac{100s}{(s+2)(s+4)(s+6)}} = \infty$

3) $L_3(s) = \frac{10(s+7)}{(s+1)(s+2)}$ $\Rightarrow$ $E(s) = \frac{1}{1+\frac{10(s+7)}{(s+1)(s+2)}}\frac{1}{s}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{1+\frac{10(s+7)}{(s+1)(s+2)}} = \frac{1}{1+\frac{70}{2}} = \frac{1}{36}$

$E(s) = \frac{1}{1+\frac{10(s+7)}{(s+1)(s+2)}}\frac{10}{s^2}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{10}{s+\frac{10(s+7)s}{(s+1)(s+2)}} = \infty$

$E(s) = \frac{1}{1+\frac{10(s+7)}{(s+1)(s+2)}}\frac{1}{s^3}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{10}{s^2+\frac{10(s+7)s^2}{(s+1)(s+2)}} = \infty$

4) $L_4(s) = \frac{5(s+1)}{s^2(s+3)}$ $\Rightarrow$ $E(s) = \frac{1}{1+\frac{5(s+1)}{s^2(s+3)}}\frac{1}{s}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{1+\frac{5(s+1)}{s^2(s+3)}} = 0$

$E(s) = \frac{1}{1+\frac{5(s+1)}{s^2(s+3)}}\frac{10}{s^2}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{10}{s+\frac{5(s+1)}{s(s+3)}} = \lim_{s\to 0} \frac{10s(s+3)}{s^2(s+3)+5(s+1)} = 0$

$E(s) = \frac{1}{1+\frac{5(s+1)}{s^2(s+3)}}\frac{1}{s^3}$ $\Rightarrow$ $e_{ss} = \lim_{s\to 0} \frac{1}{s^2+\frac{5(s+1)}{(s+3)}} = \frac{1}{\frac{5}{3}} = 0.6$

5 - $G(s) = \dfrac{\frac{k}{s^2+6s}}{1+\frac{k}{s^2+6s}} = \dfrac{k}{s^2+6s+k}$

1) $w_n^2 = k$

$2\zeta w_n = 6 \rightarrow \zeta = \frac{3}{\sqrt{k}}$

$\frac{1}{\beta} e^{-\zeta w_n t} = \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta w_n t} \Rightarrow \text{minimize}\left(\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta w_n T_s} = a\right) \longrightarrow T_s = \frac{1}{-\zeta w_n} \ln a\sqrt{1-\zeta^2} = \frac{-1}{3} \ln a\sqrt{1-\frac{9}{k}} \longrightarrow T_s = \frac{-1}{3} \ln a \quad (a = 2\%)$

$\Rightarrow T_s = \frac{-1}{3} \ln \frac{2}{100} = 1.3$

2) $w_n T_s \simeq 6 \Rightarrow T_s \simeq \frac{6}{3} = 2$

$\zeta = 1 \Rightarrow w_n = 3$

3) minimize $\left(\int_0^\infty e^2(t)\, dt\right)$

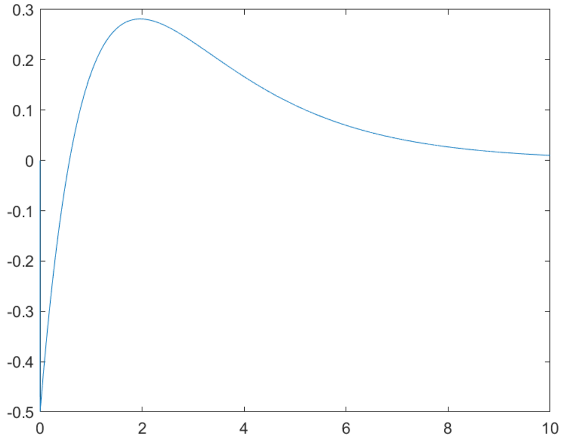
$e(t) = \underbrace{\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta w_n t}}_{\text{توان دو}} \sin(\beta w_n t + \cos^{-1}\zeta)$

$\frac{1}{(1-\zeta^2)} e^{-2\zeta w_n t} = \frac{1}{(1-\zeta^2)} e^{-6t} = \frac{k}{k-9} e^{-6t} \Rightarrow \int_0^\infty e^2(t)\, dt = \frac{k}{6(k-9)} \xrightarrow{k\to\infty} = \frac{1}{6}$

MATLAB Assignments

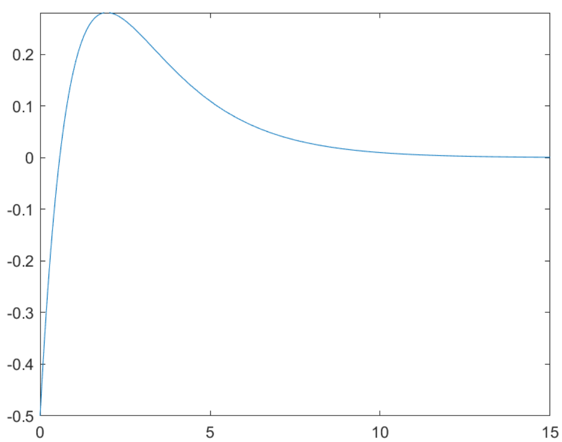
6 Symbolic e^{At}

1)



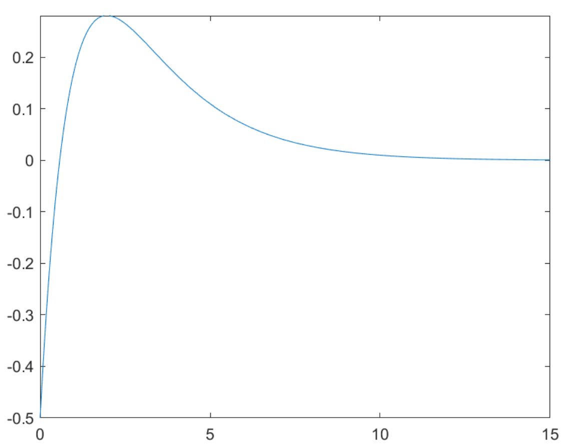

```
%% 6_1

clc; clear;

dt = 0.001;

t = 0:dt:10;

a = [-1 1];

b = [2 3 1];


impls = (1/dt)*(t==0);

sys = tf(a,b);

y1 = lsim(sys, impls, t, 0);

plot(t,y1)
```

2)



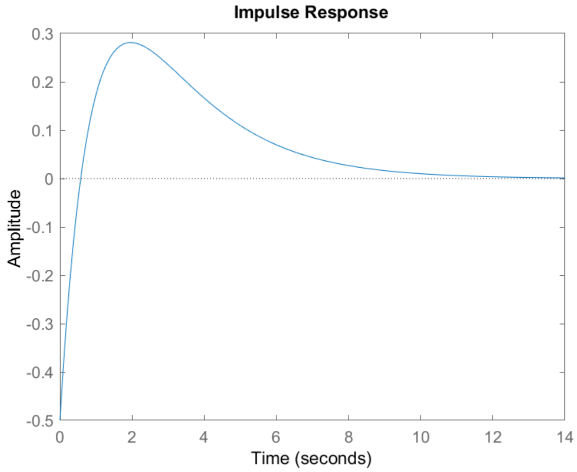

```
%% 6_2

syms s
num = poly2sym(a, s);
den = poly2sym(b, s);

isys = ilaplace(num/den);

fplot(isys, [0 15])
```

3)




```
%% 6_3

syms t
[A,B,C,D] = tf2ss(a,b);
e = expm(A.*t);
y2 = C*e*B;

fplot(y2, [0 15])
```

4)

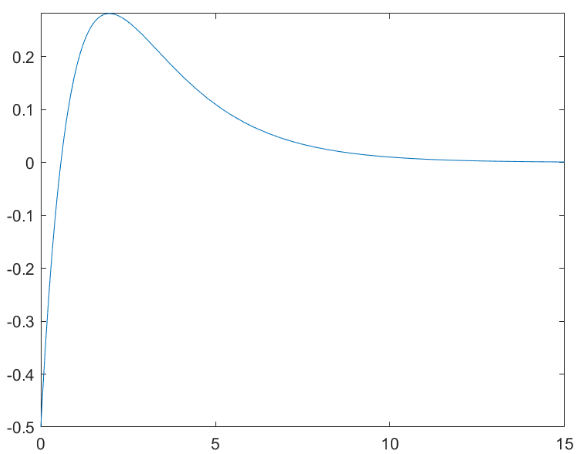



```
%% 6_4

impulse(sys)
```

5)

As we see, all of responses from different methods are the same.

6)




```
%% 6_6

eA = ilaplace( inv(s*eye(size(A))-A) );
y3 = C*eA*B;
fplot(y3 , [0 15])
```

This method also has the same response compare to part 3.

```
e =

[  2*exp(-t) - exp(-t/2),    exp(-t) - exp(-t/2)]
[2*exp(-t/2) - 2*exp(-t), 2*exp(-t/2) - exp(-t)]
```

(part 3)

```
eA =

[  2*exp(-t) - exp(-t/2),    exp(-t) - exp(-t/2)]
[2*exp(-t/2) - 2*exp(-t), 2*exp(-t/2) - exp(-t)]
```

(part 6)

Simulation Assignments

7 System 1

7- 1) چون ضرایب $x(t)$ و $u(t)$ در معادلات حالت وابستگی به زمان ندارند و همچنین معادلات خطی هستند، پس سیستم LTI است.

2)
$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \frac{-k}{m} & \frac{-1}{m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

3)
$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ \frac{k}{m} & s+\frac{1}{m} \end{bmatrix}^{-1} = \frac{1}{s^2+\frac{1}{m}s+\frac{k}{m}} \begin{bmatrix} s+\frac{1}{m} & 1 \\ \frac{-k}{m} & s \end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B + D = \frac{1}{s^2+\frac{1}{m}s+\frac{k}{m}} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+\frac{1}{m} & 1 \\ \frac{-k}{m} & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} = \frac{\frac{1}{m}}{s^2+\frac{1}{m}s+\frac{k}{m}}$$

4) $u(t) = K_p\big(r(t) - y(t)\big)$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{-k}{m}x_1(t) + \frac{-1}{m}x_2(t) + \frac{1}{m}u(t) \\ y(t) = x_1(t) \end{cases} \rightarrow \begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{-k}{m}x_1(t) + \frac{-1}{m}x_2(t) + \frac{K_p}{m}\big(r(t) - \underbrace{y(t)}_{x_1(t)}\big) \\ y(t) = x_1(t) \end{cases} \rightarrow \begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{-(k+K_p)}{m}x_1(t) + \frac{-1}{m}x_2(t) + \frac{K_p}{m}r(t) \\ y(t) = x_1(t) \end{cases}$$

5)
$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \frac{-(k+K_p)}{m} & \frac{-1}{m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{K_p}{m} \end{bmatrix} r(t)$$

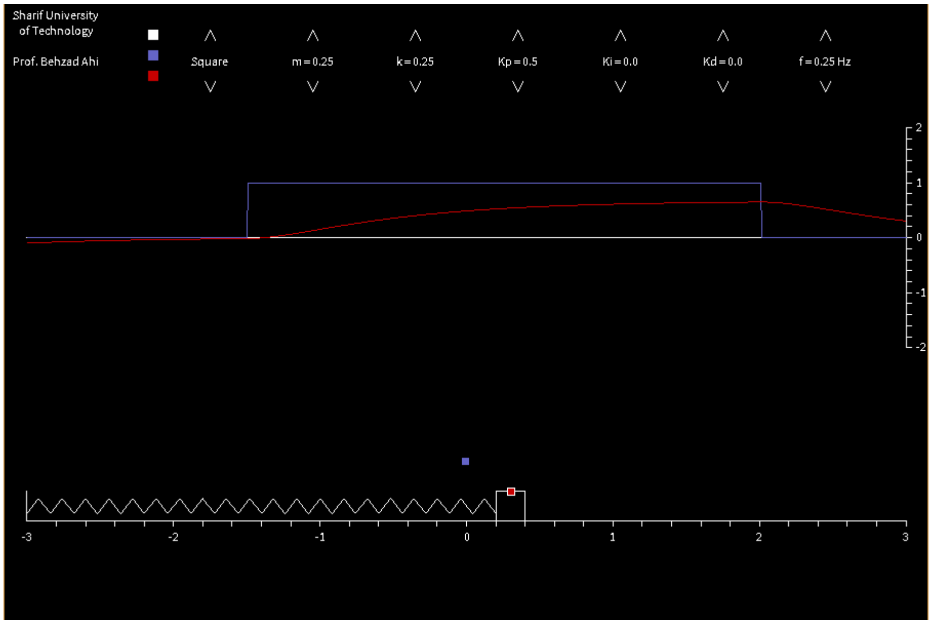
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

6)
$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ \frac{k+K_p}{m} & s+\frac{1}{m} \end{bmatrix}^{-1} = \frac{1}{s^2+\frac{1}{m}s+\frac{k+K_p}{m}} \begin{bmatrix} s+\frac{1}{m} & 1 \\ \frac{-(k+K_p)}{m} & s \end{bmatrix}$$

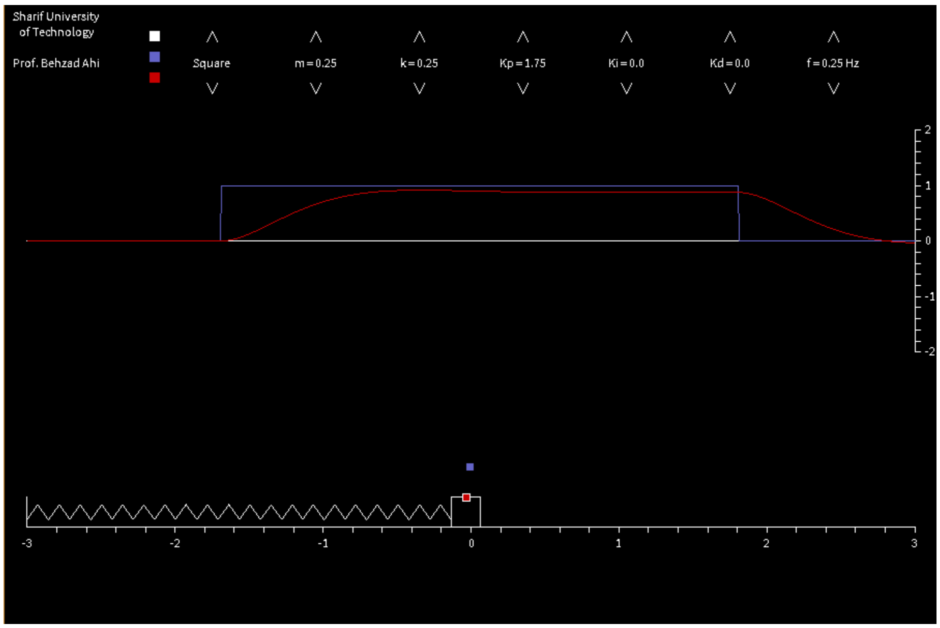
$$T(s) = C(sI-A)^{-1}B + D = \frac{1}{s^2+\frac{1}{m}s+\frac{k+K_p}{m}} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+\frac{1}{m} & 1 \\ \frac{-(k+K_p)}{m} & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{K_p}{m} \end{bmatrix} = \frac{\frac{K_p}{m}}{s^2+\frac{1}{m}s+\frac{k+K_p}{m}}$$

7) $m = 0.25$, $k = 0.25$ $\Rightarrow T(s) = \dfrac{4K_p}{s^2+4s+1+4K_p}$

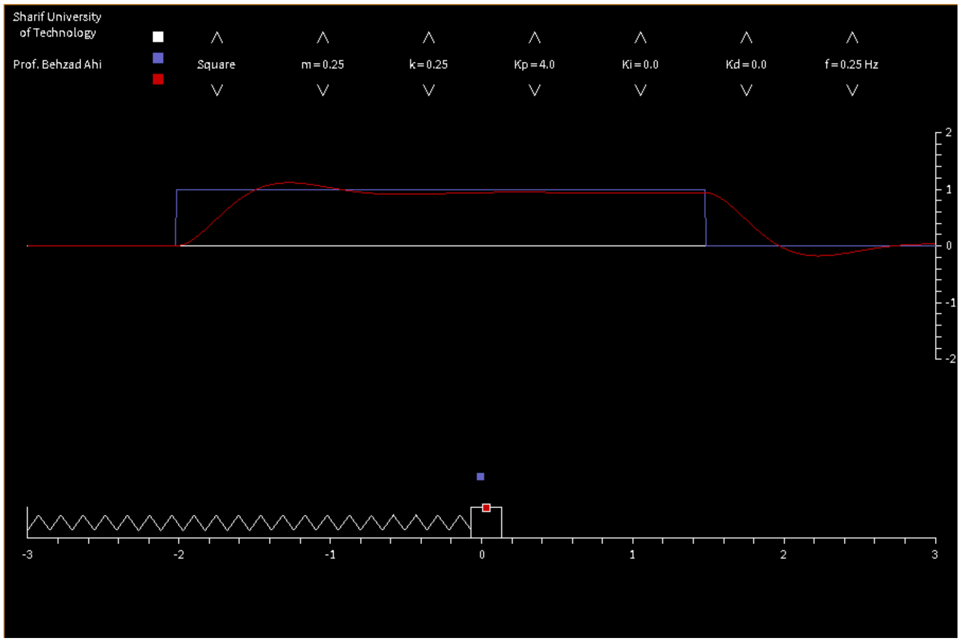
8. The image below shows the response of system when K_p=0.5

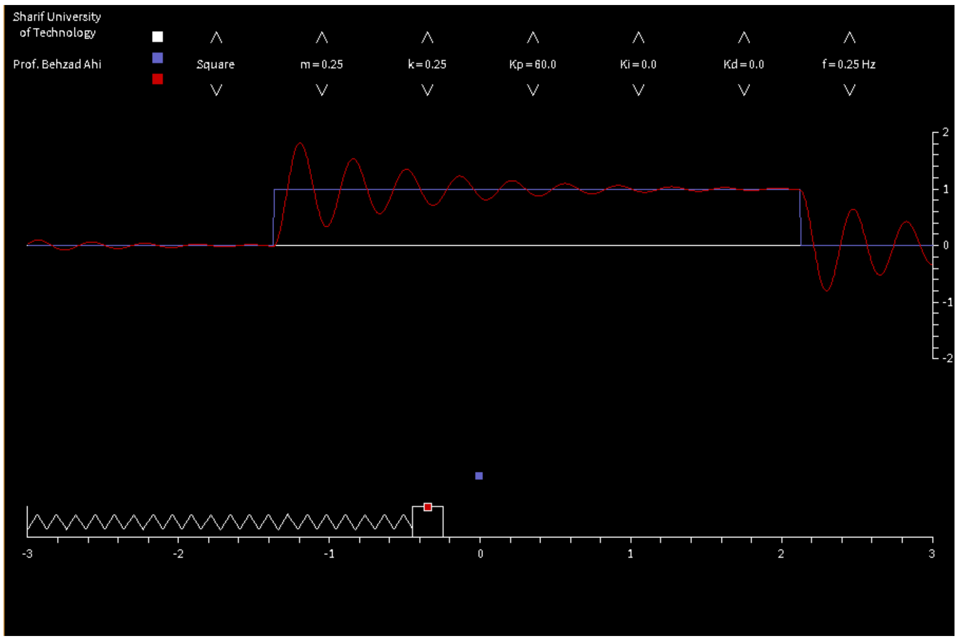


As we increasing K_p, system is changing from overdamped to critically damped. (now K_p is 1.75)



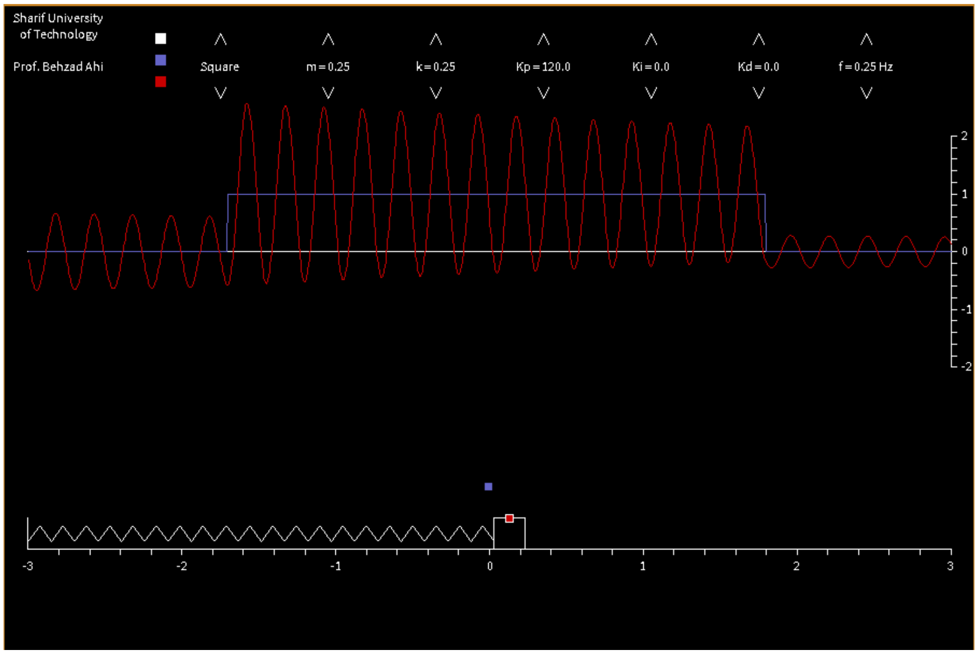
Around K_p = 4 we can clearly see that the system is underdamped and the response has changed.



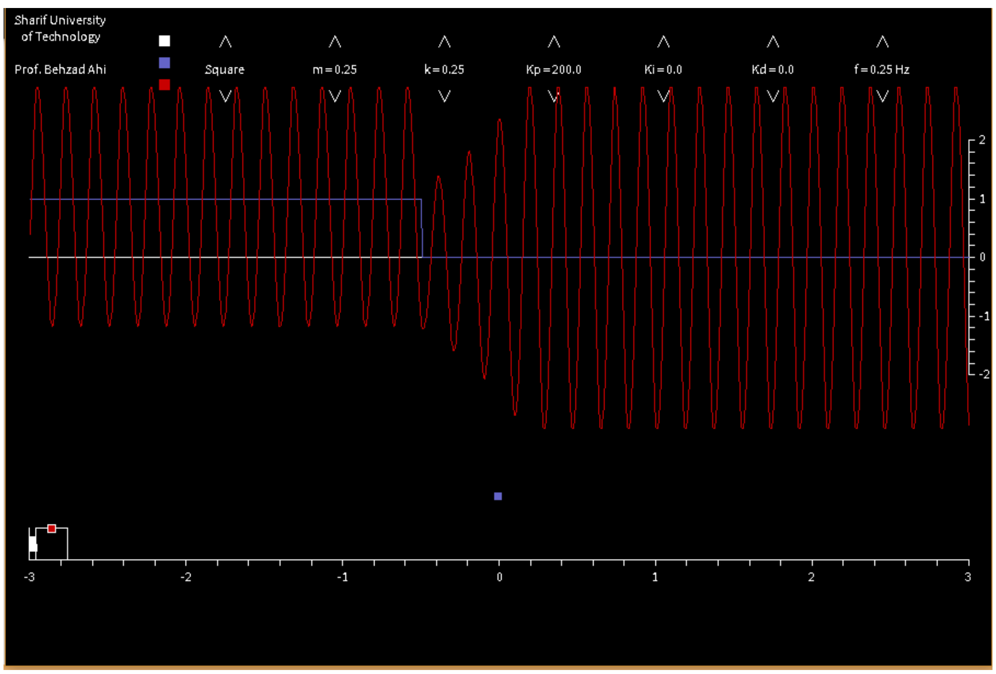


(K_p = 60)

When we reach to K_p = 100, we can realize that the system is undamped.

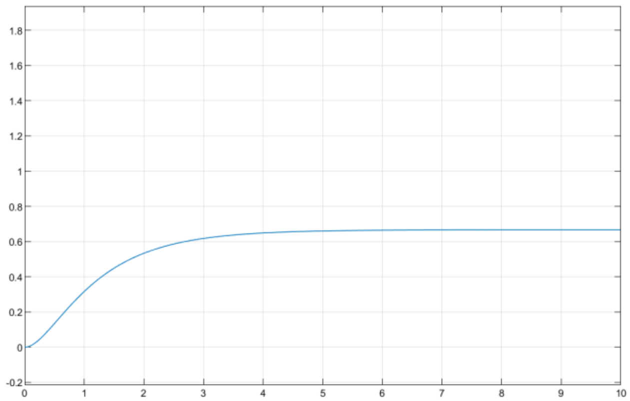


Further than that the system is getting unstable.

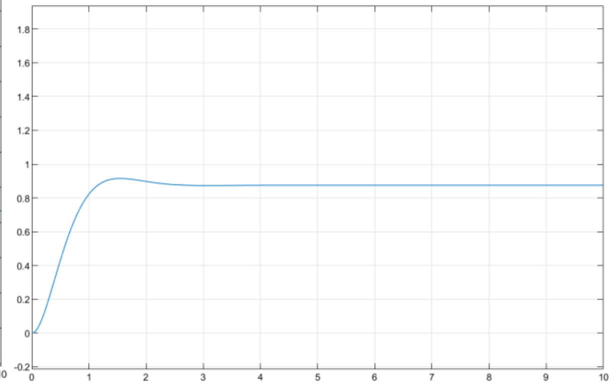


Simulation has limited horizontal axis of [-3 , 3], so the response is between these numbers but in fact the system id unstable.

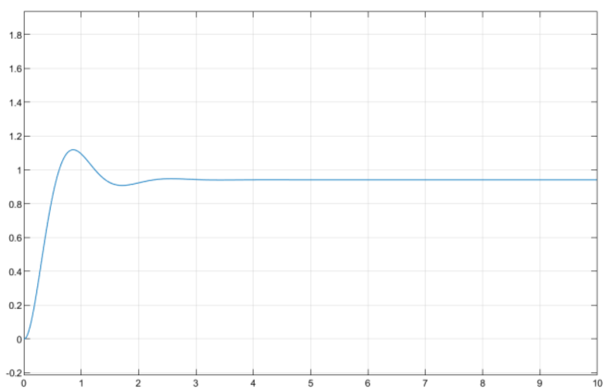
9.



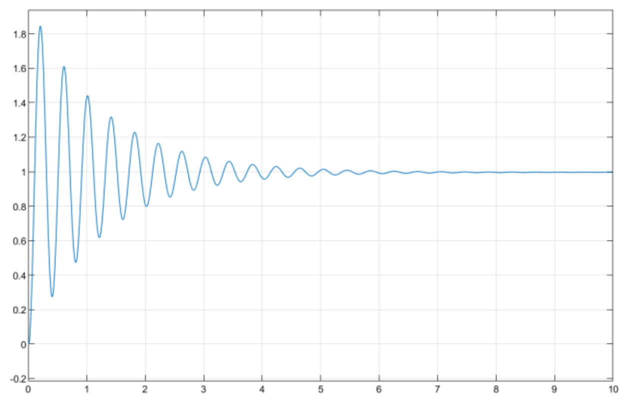
$K_p = 0.5$



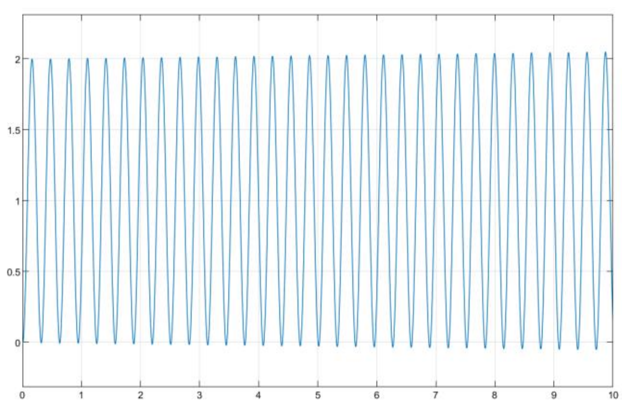
$K_p = 1.75$



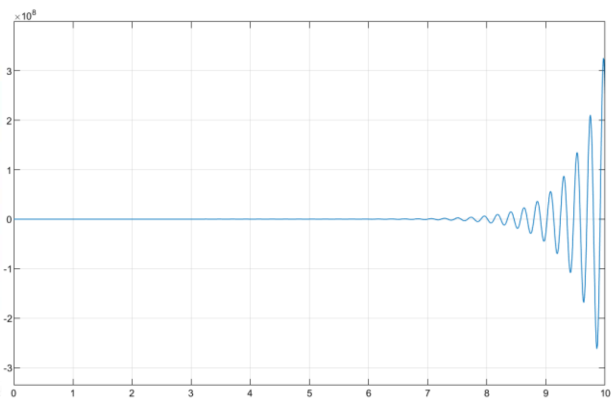
$K_p = 4$



$K_p = 60$

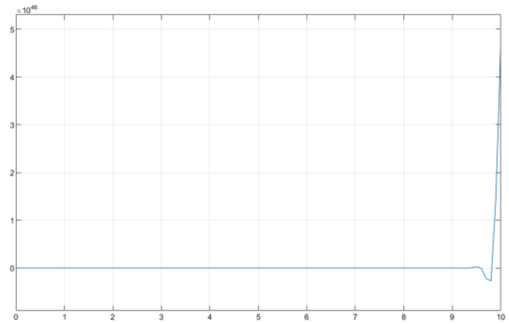


$K_p = 100$

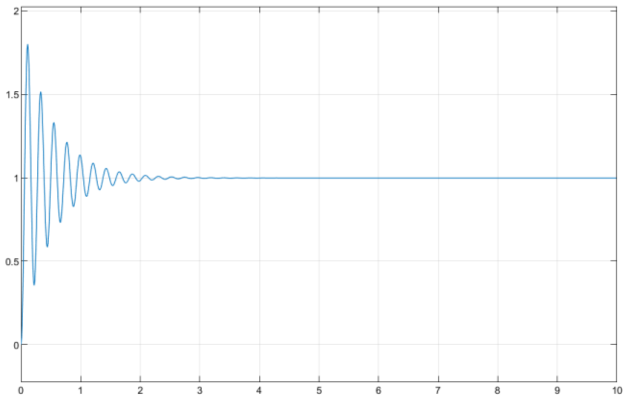


$K_p = 200$

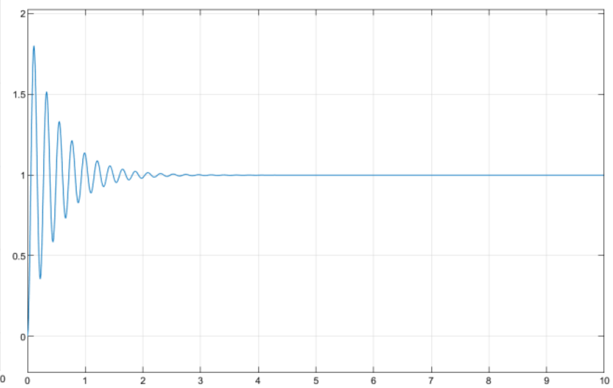
10.



Ts = 0.01



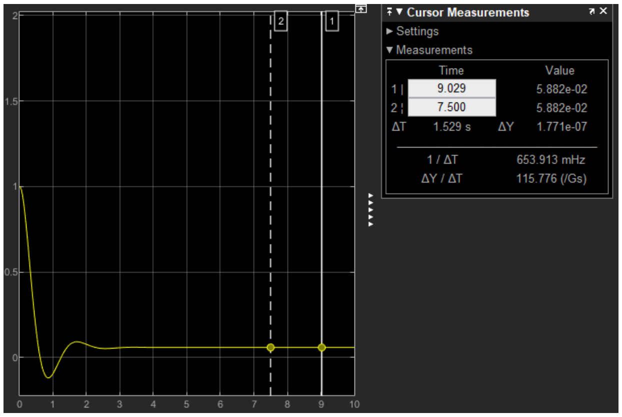
Ode2



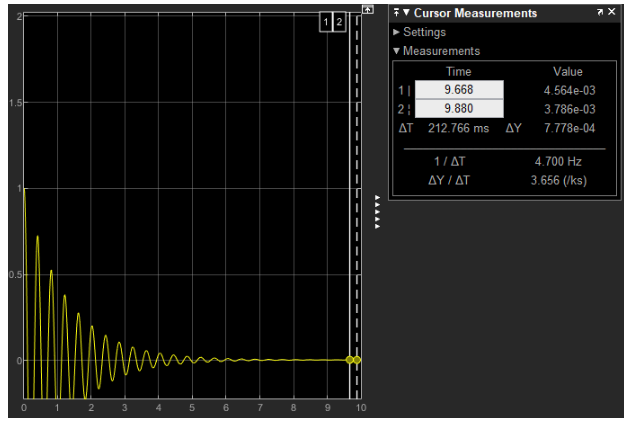
Ode3

As we see by changing the solver the system is stabilized. This may have happened due to the method of other solvers.

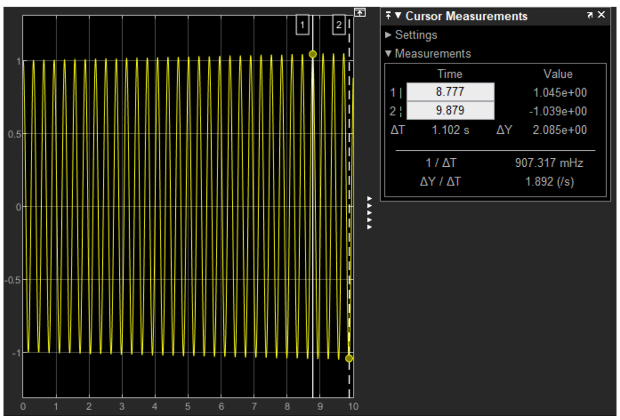
11.



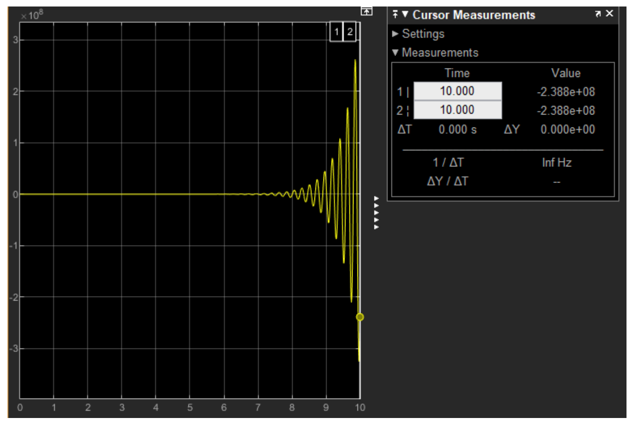
$K_p = 4$



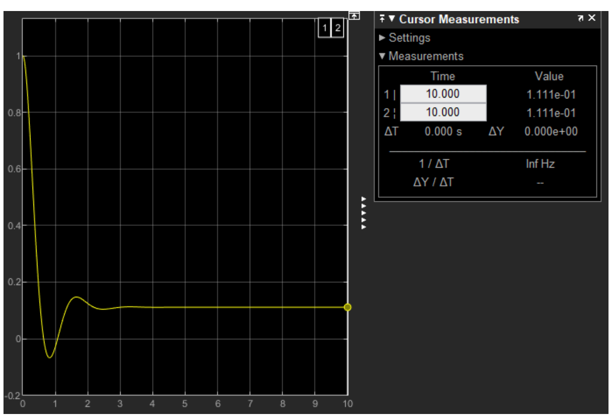
$K_p = 60$



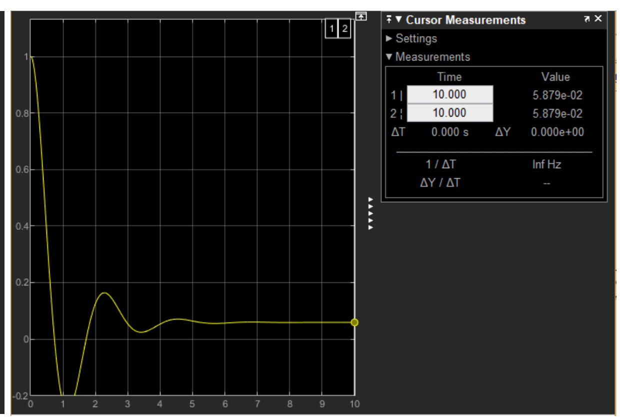
$K_p = 100$



$K_p = 200$



$K_p = 4$, $k = 0.5$, $m = 0.25$



$K_p = 4$, $k = 0.25$, $m=0.5$

12) critically damped $\rightarrow$ $\zeta = 1$ $\Rightarrow 2\zeta\omega_n = 4 \Rightarrow \omega_n = 2$

$$\Rightarrow \omega_n^2 = 1 + 4K_p \Rightarrow K_p = \frac{3}{4}$$

13) $T_s = \frac{1}{-\zeta\omega_n} \ln a\sqrt{1-\zeta^2} = \frac{-1}{2} \ln a\sqrt{1-\zeta^2} = \frac{-1}{2} \ln \frac{2}{100} = 1.95$

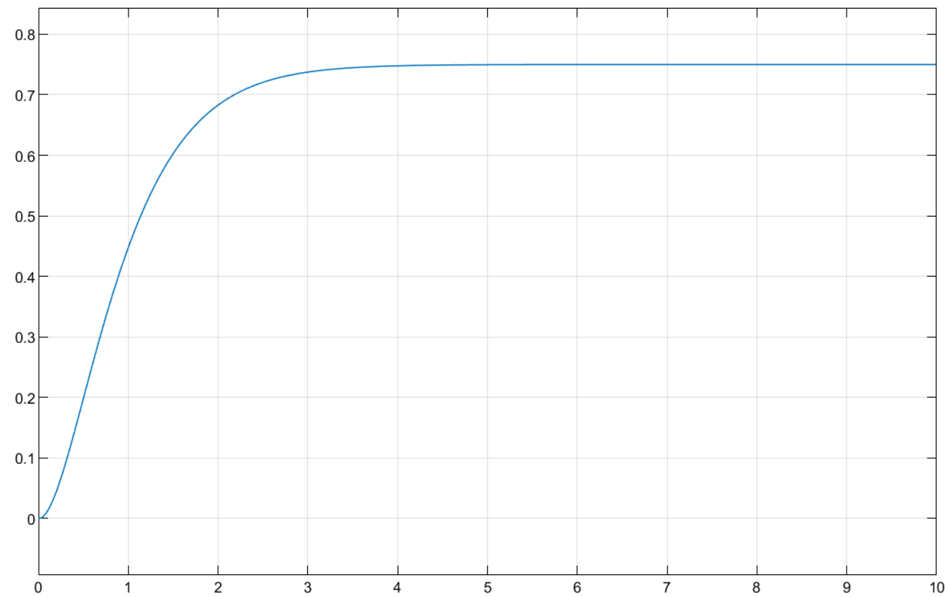
$$\zeta = \frac{2}{\omega_n} = 2/(1+4K_p)^{\frac{1}{2}} = 0 \rightarrow K_p = \infty$$

14) $T(s) = \dfrac{\frac{K_p}{m}}{s^2 + \frac{1}{m}s + \frac{K+K_p}{m}} = \dfrac{1}{\frac{m}{K_p}s^2 + \frac{1}{K_p}s + \frac{K}{K_p} + 1}$

$$\begin{array}{c|cc} s^2 & \frac{m}{K_p} & \frac{K}{K_p}+1 \\ s^1 & \frac{1}{K_p} & 0 \\ s^0 & \frac{K}{K_p}+1 & 0 \end{array} \Rightarrow \begin{array}{l} \frac{1}{K_p} > 0 \rightarrow K_p > 0 \\ \frac{m}{K_p} > 0 \rightarrow m > 0 \\ \frac{K}{K_p} + 1 > 0 \Rightarrow K > -K_p \end{array}$$

سطر تمام صفر برای
قطب موهومی خالص
: $\begin{cases} \frac{1}{K_p} = 0 \Rightarrow K_p \rightarrow \infty \\ \text{یا} \\ \frac{K}{K_p} + 1 = 0 \Rightarrow \frac{K}{K_p} = -1 \Rightarrow K = -K_p \end{cases}$

12.



$K_p = 0.75$

13.

```
K = 2.0200e-07
settlingTime = 14.8789
K = 4.3064e-05
settlingTime = 14.8762
K = 0.0121
settlingTime = 14.1549
K = 4.5286
settlingTime = 1.9036
K = 2.2994e+03
settlingTime = 1.9361
K = 1.6068e+06
settlingTime = 1.9554
K = 1.5710e+09
settlingTime = 1.9559
K = 2.1861e+12
settlingTime = 1.9549
```

8 system2

8_ 1)

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2k}{m} & \frac{-1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & 0 & \frac{-2k}{M} & \frac{-1}{M} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M} \end{bmatrix} u(t)$$

$$y_1(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(t)$$

$$y_2(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x(t)$$

2)

$$(sI - A)^{-1} = \begin{bmatrix} s & 1 & 0 & 0 \\ \frac{2k}{m} & s+\frac{1}{m} & -\frac{2k}{m} & 0 \\ 0 & 0 & s & -1 \\ -\frac{2k}{M} & 0 & \frac{2k}{M} & s+\frac{1}{M} \end{bmatrix}^{-1} = \frac{1}{det} \begin{bmatrix} s^3+(\frac{1}{m}+\frac{1}{M})s^2+(\frac{2k}{M}+\frac{1}{mM})s+\frac{2k}{mM} & s^2+\frac{1}{M}s+\frac{2k}{M} & \frac{2k}{m}s+\frac{2k}{mM} & \frac{2k}{m} \\ \frac{-2k}{m}s^2+\frac{-2k}{mM}s & s^3+\frac{1}{M}s^2+\frac{2k}{M}s & \frac{2k}{m}s^2+\frac{k}{mM}s & \frac{2k}{m}s \\ \frac{2k}{m}s+\frac{2k}{mM} & \frac{2k}{m} & s^3+(\frac{1}{m}+\frac{1}{M})s^2+(\frac{2k}{m}+\frac{1}{mM})s+\frac{2k}{mM} & s^2+\frac{1}{M}s+\frac{2k}{M} \\ \frac{2k}{m}s^2+\frac{k}{mM}s & \frac{2k}{m}s & \frac{-2k}{m}s^2+\frac{-2k}{mM}s & s^3+\frac{1}{M}s^2+\frac{2k}{M}s \end{bmatrix}$$

$$det = \frac{s\left(2k(ms+Ms+2)+s(ms+1)(Ms+1)\right)}{mM}$$

$$\Rightarrow G_1(s) = C_1(sI-A)^{-1}B + D_1 = \frac{1}{det}\left(\frac{1}{M} \times \frac{2k}{m}\right) = \frac{2k}{s\left(2k(ms+Ms+2)+s(ms+1)(Ms+1)\right)}$$

$$G_2(s) = C_2(sI-A)^{-1}B + D_2 = \frac{1}{det}\left(\frac{1}{M} \times \frac{2k}{m}s\right) = \frac{2k}{2k(ms+Ms+2)+s(ms+1)(Ms+1)}$$

3) $u(t) = K_p\left(r(t) - y_1(t)\right)$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2k}{m} & \frac{-1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & 0 & \frac{-2k}{M} & \frac{-1}{M} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_p}{M} \end{bmatrix} r(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-K_p}{M} \end{bmatrix} y_1(t) \quad \Rightarrow \quad \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2k}{m} & \frac{-1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k-K_p}{M} & 0 & \frac{-2k}{M} & \frac{-1}{M} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_p}{M} \end{bmatrix} r(t)$$

$$y_1(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(t)$$

$$(sI - A')^{-1} = \frac{1}{det} \begin{bmatrix} \cdots & \frac{2k}{m}s \\ \vdots & \frac{2k}{m}s^2 \\ \vdots & \vdots \end{bmatrix} \Rightarrow T_1(s) = C_1(sI-A')^{-1}B' + D_1 = \frac{1}{det}\left(\frac{K_p}{M} \times \frac{2k}{m}s\right) = \frac{2kK_p}{2k(ms+Ms+2+\frac{K_p}{s}) + s(ms+1)(Ms+1)}$$

$$det_1 = \frac{s\left(2k(ms+Ms+2+\frac{K_p}{s}) + s(ms+1)(Ms+1)\right)}{mM}$$

$$= \frac{2kK_p s}{2k(ms^2+Ms^2+2s+K_p)+s^2(ms+1)(Ms+1)}$$

4) $u(t) = K_p\left(r(t) - y_1(t)\right)$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2k}{m} & \frac{-1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & 0 & \frac{-2k}{M} & \frac{-1}{M} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_p}{M} \end{bmatrix} r(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-K_p}{M} \end{bmatrix} y_2(t) \quad \Rightarrow \quad \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2k}{m} & \frac{-1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & \frac{-K_p}{M} & \frac{-2k}{M} & \frac{-1}{M} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_p}{M} \end{bmatrix} r(t)$$

$$y_2(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x(t)$$

$$(sI - A')^{-1} = \frac{1}{det} \begin{bmatrix} \cdots & \frac{2k}{m} \\ \vdots & \frac{2k}{m}s \\ \vdots & \vdots \end{bmatrix} \Rightarrow T_1(s) = C_1(sI-A')^{-1}B' + D_1 = \frac{1}{det}\left(\frac{K_p}{M} \times \frac{2k}{m}s\right) = \frac{2kK_p}{2k(ms+Ms+K_p+2) + s(ms+1)(Ms+1)}$$

$$det_1 = \frac{s\left(2k(ms+Ms+K_p+2) + s(ms+1)(Ms+1)\right)}{mM}$$

5. for different k , m and Kp: