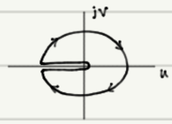
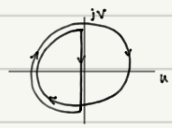
1 – a) $G_1(s) = \frac{1}{s^2} \Rightarrow G_1(wj) = \frac{1}{-w^2}$



	Rel	Im
$W=0$	$\to \infty$	0
$W=\infty$	$\to 0$	0

ریشه در صفر : $s = re^{i\theta} \to \underset{(r\to 0)}{r^{-2}e^{-2i\theta}} \rightsquigarrow \theta: \frac{-\pi}{2} \to \frac{\pi}{2} \Rightarrow \pi \to -\pi$

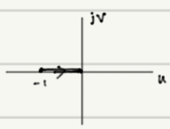
b) $G_2(s) = \frac{1}{s^3} \Rightarrow G_2(wj) = \frac{1}{-jw^3} = \frac{j}{w^3}$



	Rel	Im
$W=0$	$\to 0$	∞
$W=\infty$	$\to 0$	0

ریشه در صفر : $\underset{(r\to 0)}{r^{-3}e^{-3i\theta}} \rightsquigarrow \theta: \frac{-\pi}{2} \to \frac{\pi}{2} \Rightarrow \frac{3\pi}{2} \to \frac{-3\pi}{2}$

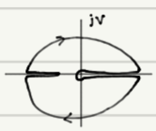
c) $G_3(s) = \frac{1}{s^2-1} \Rightarrow G_3(wj) = \frac{1}{-w^2-1} = \frac{-1}{w^2+1}$



	Rel	Im
$W=0$	$\to -1$	0
$W=\infty$	$\to 0$	0

d) $G_4(s) = \frac{-1}{s^2+1} = \frac{-1}{(s+j)(s-j)} \Rightarrow G_4(wj) = \frac{1}{w^2-1}$

	Rel	Im
$W=0$	$\to -1$	0
$W=\infty$	$\to 0$	0



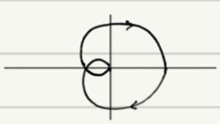
ریشه در j : $s = re^{j\theta} + j \to \frac{-1}{re^{i\theta}(re^{i\theta}+2j)} = \frac{-1}{2j} r^{-1}e^{-j\theta} \Rightarrow \pi \to 0$

$\rightsquigarrow \theta: \frac{-\pi}{2} \to \frac{\pi}{2}$

ریشه در -j : $s = re^{j\theta} - j \to \frac{-1}{re^{i\theta}(re^{i\theta}-2j)} = \frac{1}{2j} r^{-1}e^{-j\theta} \Rightarrow 0 \to -\pi$

$(r \to 0)$

e) $G_5(s) = \frac{2}{(s+1)(s+2)(s+3)} = \frac{\frac{1}{3}}{(s+1)(\frac{1}{2}s+1)(\frac{1}{3}s+1)} \Rightarrow G_5(wj) = \frac{2}{6(1-w^2)j + w(11-w^2)}$

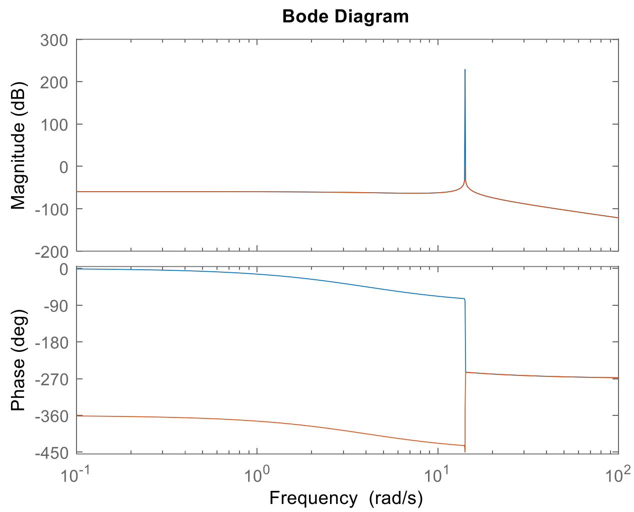


	θ	Rel	Im
$W \to 0$	: 0	0	$\frac{-1}{3}$
$W \to \infty$	: $-\frac{3\pi}{2}$	0	0

$\phi(w) = -\tan^{-1}(w) - \tan^{-1}(\frac{w}{2}) - \tan^{-1}(\frac{w}{3})$

2-

a) $G(s) = \dfrac{1}{(0.1s^2+20)(12s+48)}$



c) designing a controller with positive phase margin will provide a safety margin against uncertainties and variations in the system. It will ensure that the system remains stable even in presence of disturbances, parameter variations or modeling inaccuracies. But nyquist stability does not guarantee stability in these situations. So the first method is closer to real-world situations.

3_

b)

1) $G_1(s) = k$ $\qquad$ $Re\{G_1(jw)\} = k \geqslant \epsilon$ $\qquad$ (ISP)

2) $G_2(s) = \frac{1}{s} \longrightarrow G_2(jw) = \frac{-j}{w}$ $\qquad$ $Re\{G_2(jw)\} = 0 \geqslant 0$ $\qquad$ (Passive)

3) $G_3(s) = s \longrightarrow G_3(jw) = jw$ $\qquad$ $Re\{G_3(jw)\} = 0 \geqslant 0$ $\qquad$ (Passive)

4) $G_4(s) = \frac{k}{\tau s+1} \longrightarrow G_4(jw) = \frac{k}{\tau wj+1} = \frac{k}{1+(\tau w)^2}(1-\tau wj)$

$$\left.\begin{array}{l} Re\{G_4(jw)\} = \frac{k}{1+\tau^2 w^2} \geqslant \epsilon > 0 \\ |G(wj)| = k\,\frac{\sqrt{1+\tau^2 w^2}}{1+\tau^2 w^2} \Rightarrow |G(wj)|^2 = \frac{k^2}{1+\tau^2 w^2} \end{array}\right\} \rightarrow Re\{G_4(jw)\} \geqslant \epsilon\,|G(jw)|^2$$

$$\downarrow$$

$$\frac{1}{k} \geqslant \epsilon > 0 \qquad (OSP)$$

5) $G_5(s) = \frac{s^2+1}{(s+1)^2} \longrightarrow G_5(jw) = \frac{1-w^2}{1-w^2+2jw} = \frac{1-w^2}{(1-w^2)^2+4w^2}(1-w^2-2jw)$

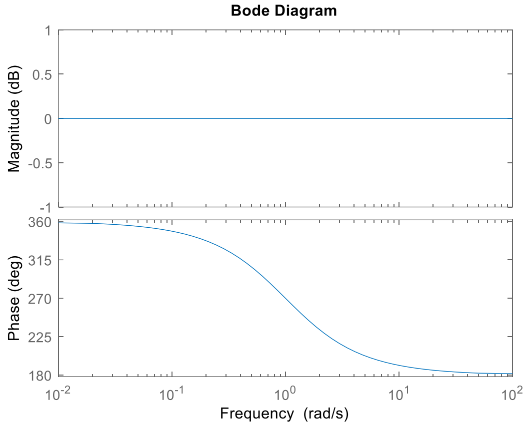
$$\left.\begin{array}{l} Re\{G_5(jw)\} = \frac{(1-w^2)^2}{(1-w^2)^2+4w^2} \geqslant \epsilon > 0 \\ |G(wj)| = \frac{1-w^2}{(1-w^2)^2+4w^2}\sqrt{(1-w^2)^2+(2w)^2} \Rightarrow |G(wj)|^2 = \frac{(1-w^2)^2}{(1-w^2)^2+4w^2} \end{array}\right\} \rightarrow (ISP)$$

C) series: $\frac{1}{s+1}\,\frac{1}{s+1} = \frac{1}{(s+1)^2} \longrightarrow G(jw) = \frac{1}{1-w^2+2jw} = \frac{(1-w^2-2jw)}{(1-w^2)^2+4w^2}$ $\qquad$ $Re\{G(jw)\} = \frac{1-w^2}{(1-w^2)^2+4w^2} \not\geqslant 0$ $\qquad$ ممکن است بزرگتر از صفر نباشد

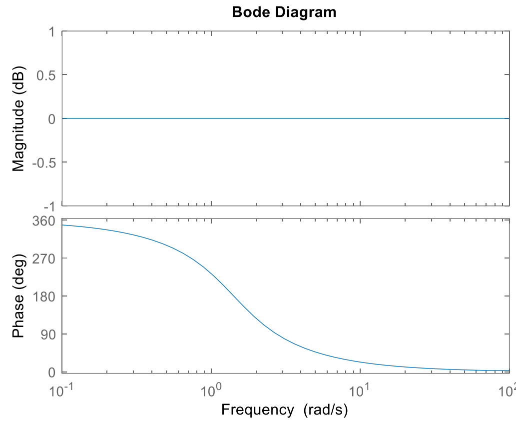
parallel: $\frac{2}{s+1} \longrightarrow G(jw) = \frac{2}{wj+1} = \frac{2}{w^2+1}(1-wj)$ $\qquad$ $Re\{G(jw)\} = \frac{2}{1+w^2} \geqslant 0$

چون در حالت موازی دو سیستم با یکدیگر جمع می شوند پس سیستم در ترکیب موازی همواره پسیو خواهد ماند.

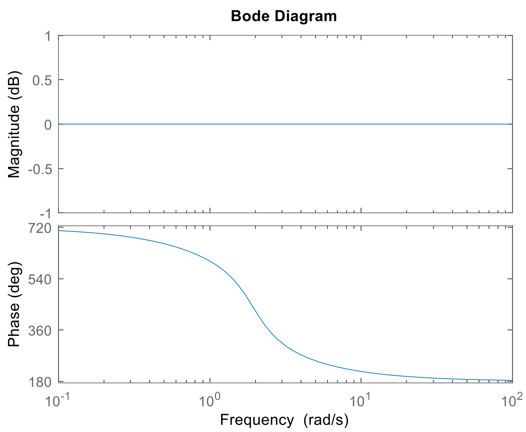
4 Time Delay Approximations



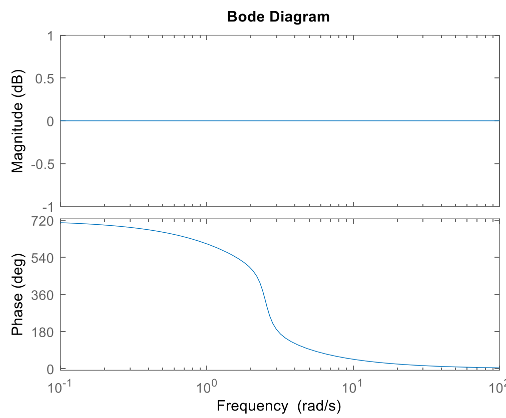
n=1



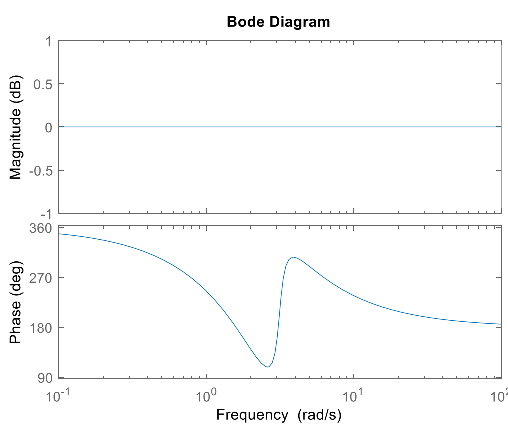
n=2



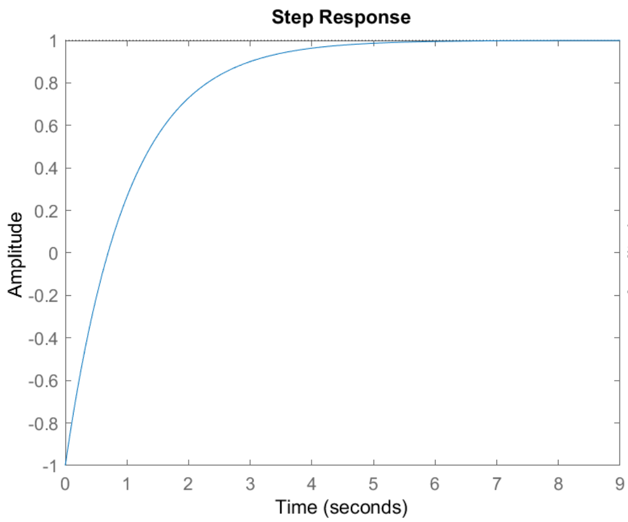
n=3



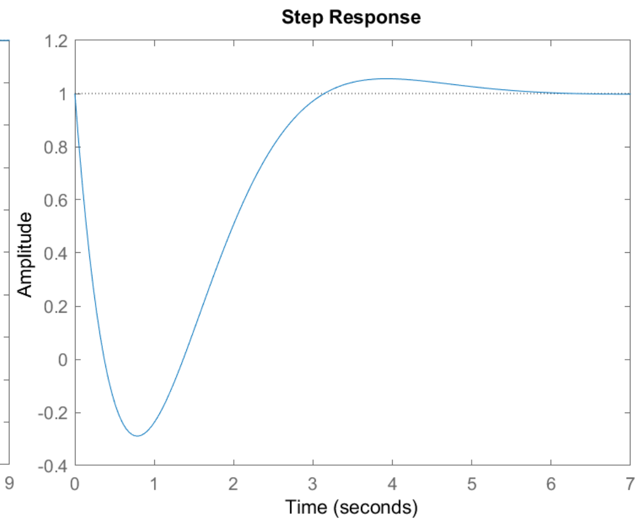
n=4



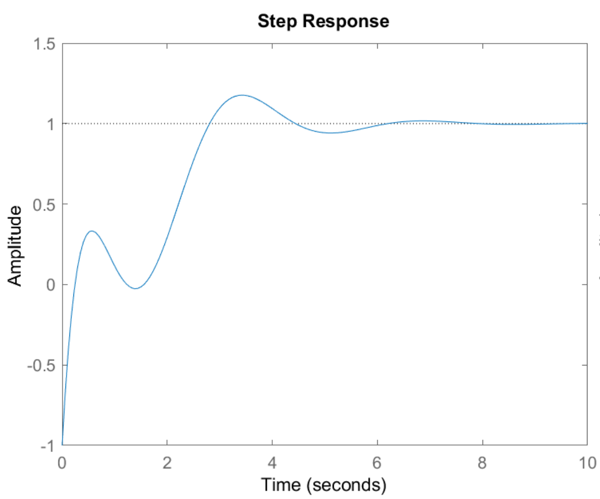
n=5


Step Response
Amplitude
Time (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 1 2 3 4 5 6 7 8 9

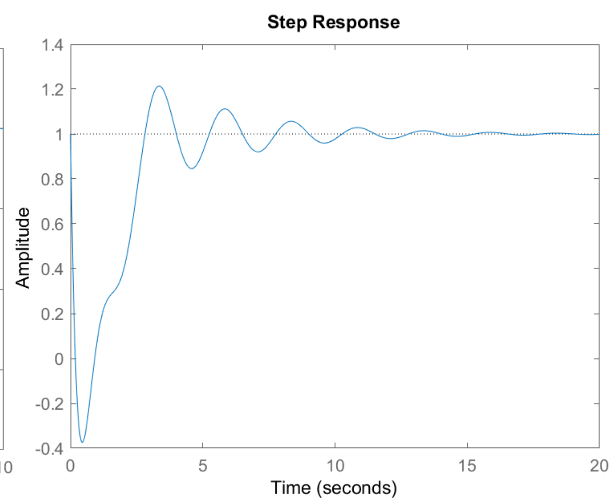
n=1


Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0 1 2 3 4 5 6 7

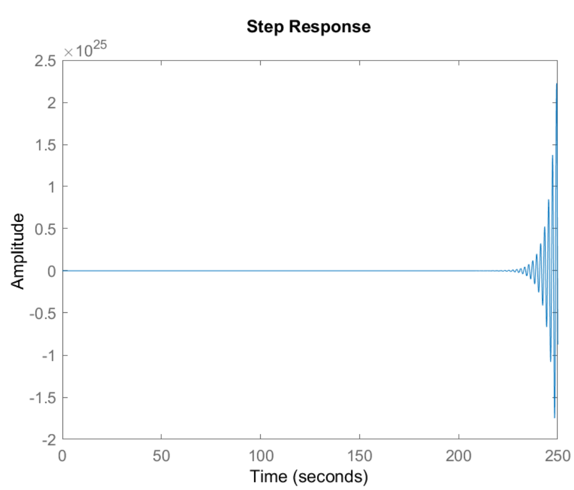
n=2


Step Response
Amplitude
Time (seconds)
1.5
1
0.5
0
-0.5
-1
0 2 4 6 8 10

n=3


Step Response
Amplitude
Time (seconds)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0 5 10 15 20

n=4


Step Response
×10^25
Amplitude
Time (seconds)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 50 100 150 200 250

n=5

c) as we see the equation with degree n=5 is unstable. That would because of the phase margin of system as we saw in bode diagram plots.

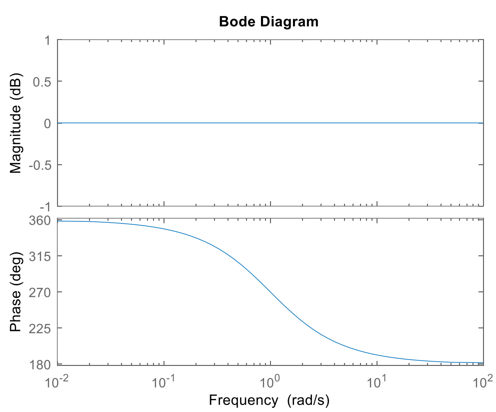
d) $$R_{1,1}(Ts) = \frac{\sum_{i=0}^{1} p_i (Ts)^i}{\sum_{i=0}^{1} q_i (Ts)^i} = \frac{1 + p_1 Ts}{1 + q_1 Ts} \quad , \quad \text{Pade} : \frac{-\frac{T}{2}s + 1}{\frac{T}{2}s + 1} \quad \Rightarrow p_1 = -\frac{1}{2} \; , \; q_1 = \frac{1}{2}$$

e) $$R_{2,2}(Ts) = \frac{\sum_{i=0}^{2} p_i (Ts)^i}{\sum_{i=0}^{2} q_i (Ts)^i} = \frac{1 + p_1 Ts + p_2 T^2 s^2}{1 + q_1 Ts + q_2 T^2 s^2} \quad , \quad \text{Pade} : \frac{\frac{T^2}{12}s^2 - \frac{T}{2}s + 1}{\frac{T^2}{12}s^2 + \frac{T}{2}s + 1} \quad \Rightarrow p_1 = -\frac{1}{2} \; , \; q_1 = \frac{1}{2} \; , \; p_2 = \frac{1}{12} \; , \; q_2 = \frac{1}{12}$$

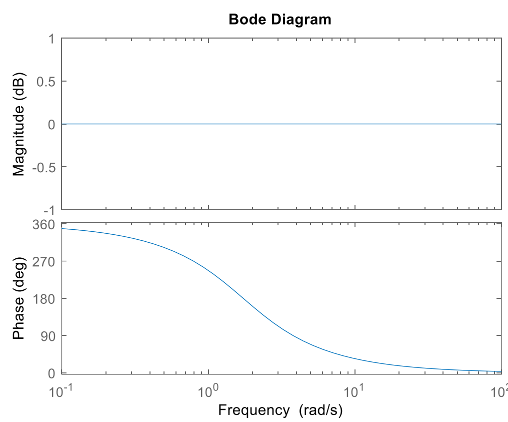
ضرایب بدست آمده در حالت کلی مشابه بسط تیلور نیستند. برای حالت $T=2$ و $n=m=1$ هر دو تقریب یکسان هستند.

$$R_{m,n}(Ts) = \frac{\sum_{i=0}^{m} p_i (Ts)^i}{\sum_{i=0}^{n} q_i (Ts)^i} \xrightarrow{m=n} \frac{1 + p_1 Ts + \dots + p_n T^n s^n}{1 + q_1 Ts + \dots + q_n T^n s^n} = \underset{\substack{\downarrow \\ t=0 \text{ ضربه در }}}{1} + \frac{(p_1 - q_1)Ts + \dots + (p_n - q_n)T^n s^n}{1 + q_1 Ts + \dots + q_n T^n s^n}$$

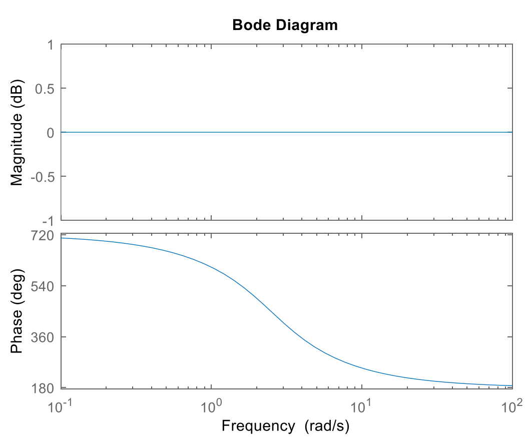
f)



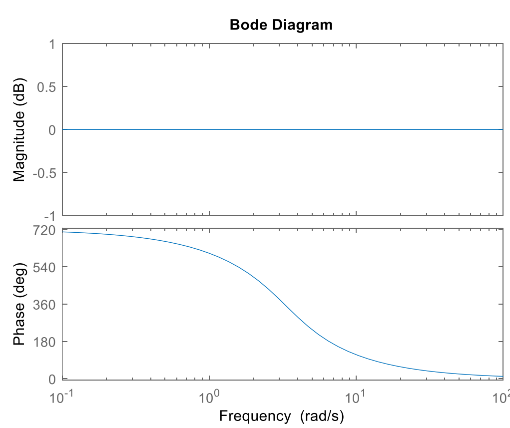
n=m=1



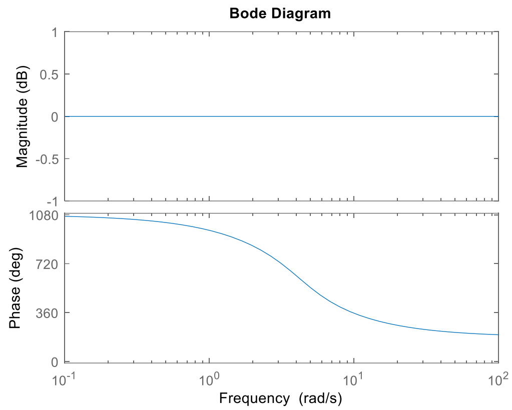
n=m=2



n=m=3

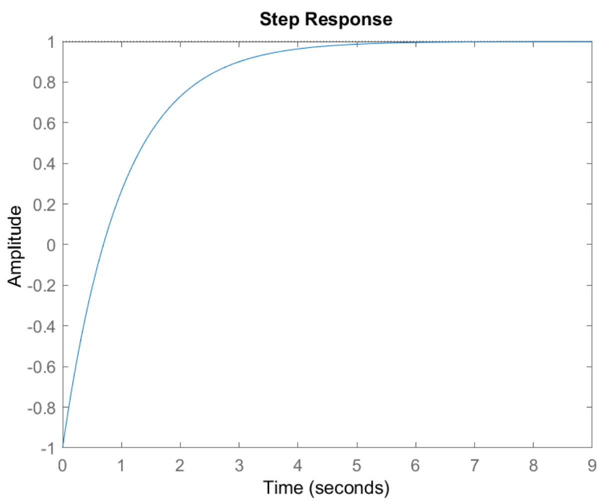


n=m=4

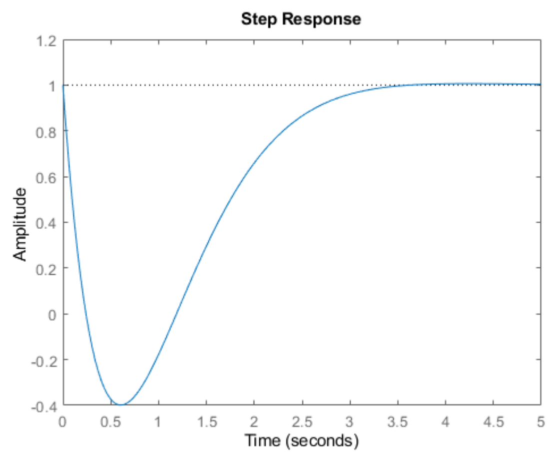


n=m=5

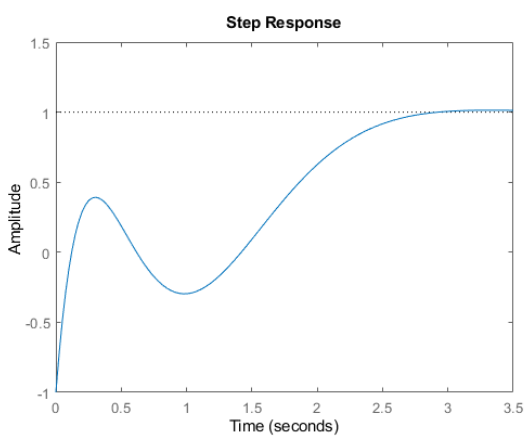
g)



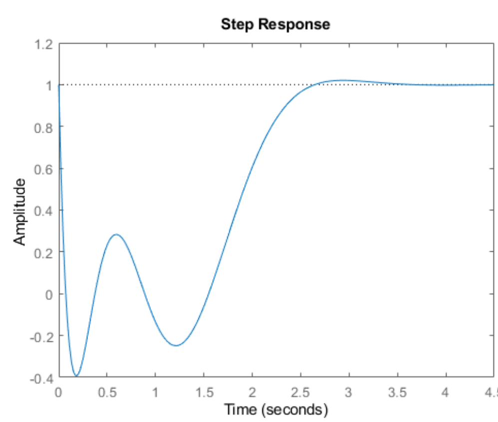
n=m=1



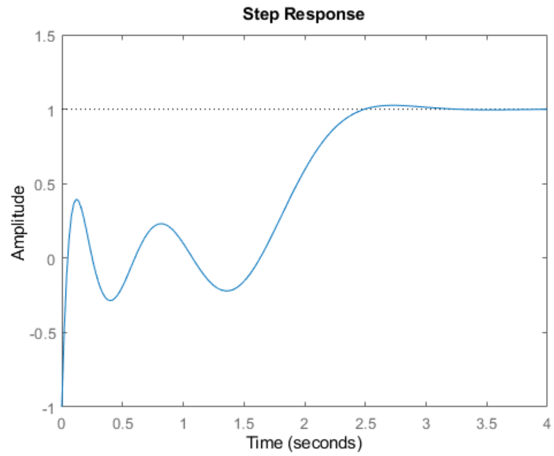
n=m=2



n=m=3

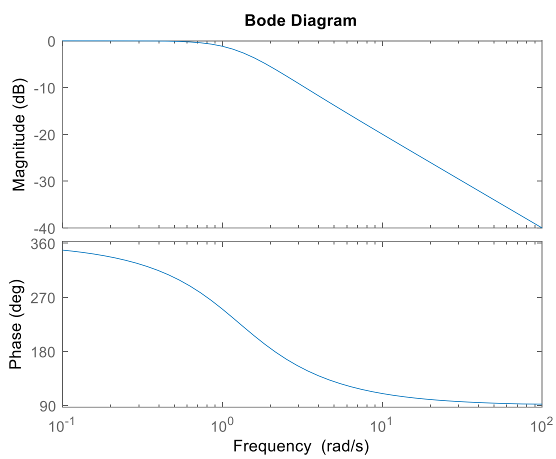


n=m=4

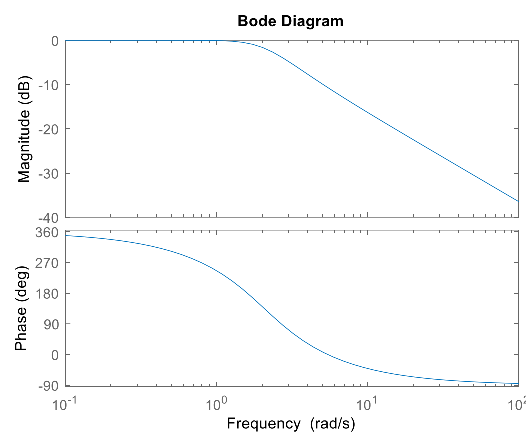


n=m=5

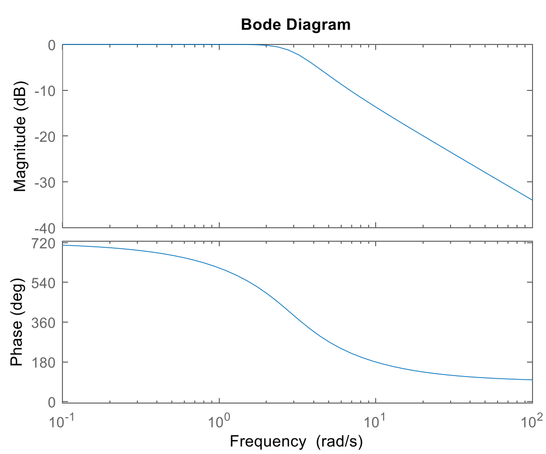
h)



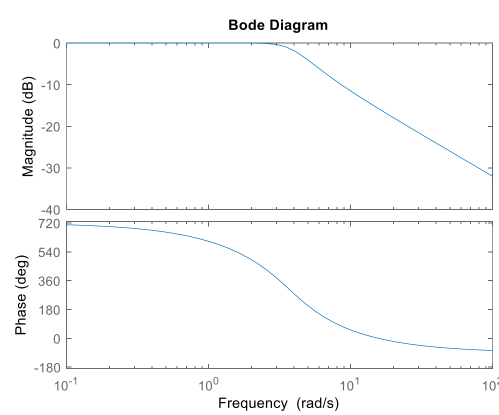
n=2 m=1



n=3 m=2

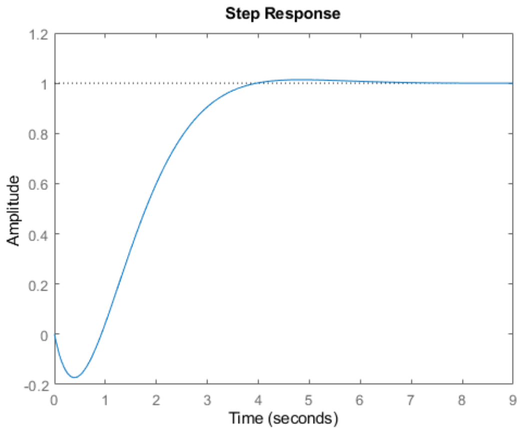


n=4 m=3

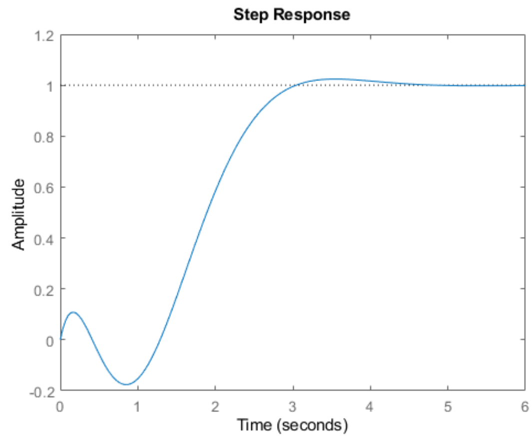


n=5 m=4

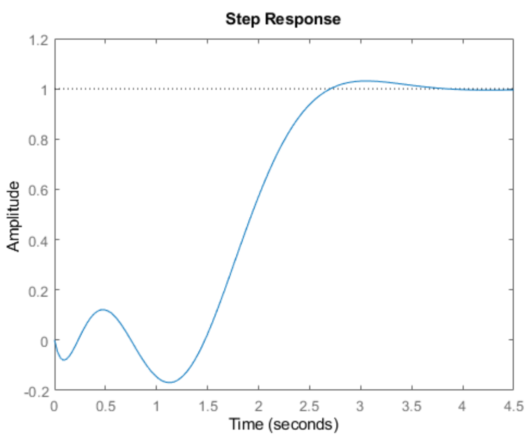
i)



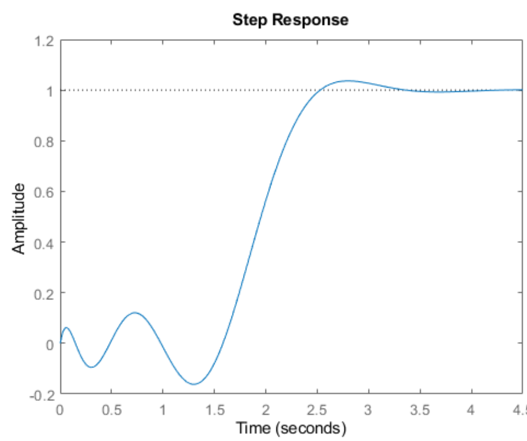
n=2 m=1



n=3 m=2



n=4 m=3



n=5 m=4

```
%% 4
clear; clc;

syms s
T = 2;

for n=1:4
    f1 = exp(-s*T);
    T1 = pade(f1,'Order',[n n]); % pade(f1,'Order',[n n+1]);
    [NT, DT] = numden(sym(T1));
    transferF = tf(sym2poly(NT),sym2poly(DT));
    figure
    %bode(transferF)
    %step(transferF)
end
```

j) We use Padé approximations with an equal degree of numerator and denominator because it helps to accurately capture the behavior of the system over a wide range of frequencies as we see in bode plots.

When the degrees of the numerator and denominator are equal, the Padé approximation can better represent the high-frequency and low-frequency behavior of the system, resulting in a more accurate frequency response.

Padé approximations with a lower degree of numerator than denominator are used because they provide a better representation of the transient response of the system.

MATLAB Assignments

5 Root-Locus of Time Delayed Systems

a) Magnitude of L(s) should remain constant for a point belong to the root-locus.

$$|L(s)| = |\frac{e^{-Ts}}{s+P}|$$

Phase angle of L(s) should satisfy the angle condition.

$$\theta = \arg(e^{-Ts}) - \arg(\, s + P)$$

b)

```
%% 5
clear; clc;

s = tf('s');
T = 0.1;
P = 2;
k = 1;
L = exp(-T*s)/(s+P);
T = feedback(k*L,1);
poles = pole(T);
```

pole = -3

```
%% 5
clear; clc;

syms s k
T = 0.1;
P = 2;
L = exp(-T*s)/(s+P);


H = 1+k*L;

poles = solve(H,s);
```

Another way to find poles is with this code, and then with subs function we determine the range of k to find poles. Also by using rlocus and give k as second input we can see root locus and poles of the system but in first part it errors that can't find solution.

c)

```
%% 5
clear; clc;

s = tf('s');
T = 0.1;
P = 2;
poles=[];
L = exp(-T*s)/(s+P);

for k = 0:0.1:20
    T = feedback(k*L,1);
    poles = [poles;pole(T)];
end
```

poles = from -2.1 to -22

d) …

```
%% 5_d
r= 1./(s*ones(200,1)+poles);
% rlocus(r)
...
```